

Tunneling magnetoresistance in a junction made of X -wave magnets with $X = p, d, f, g, i$

Motohiko Ezawa 

Department of Applied Physics, *The University of Tokyo*, 7-3-1 Hongo, Tokyo 113-8656, Japan



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We investigate the tunneling magnetoresistance (TMR) of a bilayer system made of X -wave magnets with $X = p, d, f, g, i$, where $X = d, g, i$ corresponds to altermagnets. A universal analytic formula is derived for the TMR ratio. It is proportional to $|J|/(N_X \Gamma)$ for small Γ , where N_X is the number of the nodes of the X -wave magnet, J is the strength of the X -wave magnet, and Γ is the self-energy. It is contrasted with the TMR ratio made of ferromagnets, where it is proportional to J^2/Γ^2 for small Γ . Therefore, the TMR ratio is larger in ferromagnets for $|J| > \Gamma$. However, the X -wave magnets are expected to achieve high-speed and ultradense memory owing to the zero net magnetization.

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I. INTRODUCTION

Magnetic memory is a basis of a computer, where the spin direction acts as a memory. It is necessary to observe the spin direction to read out the memory. A magnetic tunneling junction is a most successful way to do this [1,2]. It consists of the bilayer ferromagnets spaced by an insulator. The resistance is low (large) if the directions of spins are identical (opposite). This phenomenon is called the tunneling magnetoresistance (TMR). The spin direction of the memory can be read out by using the TMR. There are limitations of magnetic devices based on ferromagnets. One limitation is that it is hard to integrate ferromagnetic memories in an ultradense way due to the stray field induced by ferromagnets. The other limitation is that it is hard to flip a spin direction quickly. Antiferromagnets are expected to solve these limitations and achieve high-speed and ultradense memories because antiferromagnets have no net magnetization and are free from a stray field [3–9]. The Néel vector is used as a memory for antiferromagnets. The problem of antiferromagnets is that it is hard to read out the direction of the Néel vector.

Recently discovered altermagnets overcome this problem and form a new class of antiferromagnets. They are the d -wave, g -wave, and i -wave altermagnets [10,11], which have the d -wave, g -wave, and i -wave spin-split band structures in the electronic band structure. In altermagnets, the direction of the Néel vector may also be used as a memory. The Néel vector can be readout by measuring the anomalous Hall conductivity [12–20], which may occur because their band structures break time-reversal symmetry. There are similar magnets called the p -wave magnet [21] and the f -wave magnet [10] possessing the corresponding spin-split band structures, although they are not altermagnets because their band structures preserve time-reversal symmetry. In these magnets the spin-split direction can be used as a memory.

It is important to investigate the d -wave, g -wave, and i -wave altermagnets as well as the p -wave and f -wave magnets in the universal notion of the higher symmetric X -wave magnets [22] with $X = p, d, f, g, i$. The characteristic feature is that the band structure has N_X nodes with $N_X = 1, 2, 3, 4, 6$, respectively. There is a report [22] that they share a universal physics in the planar Hall effect characterized by the node number N_X irrespective of the absence or the presence of time-reversal symmetry. Analytic results on the TMR for these X -wave magnets have yet to be presented, although magnetic tunneling junctions have been discussed in the d -wave altermagnet [11,23–28] and the p -wave magnet [29].

In this paper, we study the TMR for the bilayer junction made of X -wave magnets and derive analytic formulas. We consider two cases, where the spin-splitting vector of the X -wave magnet is parallel and antiparallel as in Fig. 1. The main result is that the differential conductivity for the parallel configuration is given by

$$\lim_{\Gamma \rightarrow 0} \frac{G_P}{4e\pi^3} = \frac{2m\pi^2}{\hbar^2 \Gamma}, \quad (1)$$

while that for the antiparallel configuration is given by

$$\lim_{\Gamma \rightarrow 0} \frac{G_{AP}}{4e\pi^3} = \sqrt{\frac{m}{2\mu}} \frac{N_X \pi^2}{\hbar a |J|}, \quad (2)$$

where J is the strength of the X -wave magnet, Γ is the self-energy, μ is the chemical potential, a is the lattice constant, and m is the mass of electrons. Hence, the TMR ratio is given by

$$\lim_{\Gamma \rightarrow 0} \text{TMR}_{\text{ratio}} \equiv \lim_{\Gamma \rightarrow 0} \frac{G_P - G_{AP}}{G_{AP}} = \frac{2a|J|\sqrt{2m\mu}}{N_X \hbar \Gamma}. \quad (3)$$

We compare these analytic formulas to numerical results based on the tight-binding model.

It is to be contrasted with the TMR ratio based on ferromagnets, where it is given by

$$\lim_{\Gamma \rightarrow 0} \text{TMR}_{\text{ratio}} = \frac{2J^2}{\Gamma^2}. \quad (4)$$

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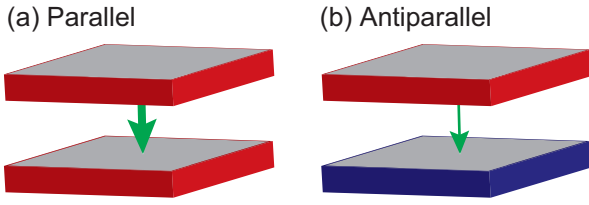


FIG. 1. Illustration of a bilayer magnetic tunneling junction made of magnets. (a) Parallel configuration, where the spin directions are identical at each lattice site between the two layers. (b) Antiparallel configuration, where the spin directions are opposite at each lattice site between the two layers. The green arrow indicates the tunneling current, which is larger in the parallel configuration.

Therefore, the TMR ratio is larger in ferromagnets for $|J| > \Gamma$. Nevertheless, the X -wave magnets are expected to achieve high-speed and ultradense memory owing to the zero net magnetization.

This paper is composed as follows. In Sec. II, the continuum model and the tight-binding model describing a metal coupled with the X -wave magnet are introduced. We derive analytic formulas based on the continuum model, while we use the tight-binding model for numerical calculations. In Sec. III, we investigate the current through a junction made of two layers. Simplified formulas for small bias voltage and zero temperature are derived. Then, we derive a universal formula of the TMR ratio applicable to all the X -wave magnets. In Sec. IV, materials realizing X -wave magnets are presented. Section V is devoted to discussion.

II. MODEL

A. Continuum model

Electrons coupled with the X -wave magnet are well described by the two-band Hamiltonian given by [22,30]

$$H = \frac{\hbar^2 \mathbf{k}^2}{2m} - \mu + J f_X(\mathbf{k}) \mathbf{n} \cdot \boldsymbol{\sigma}, \quad (5)$$

where the last term represents the momentum-dependent spin-split effect induced by the X -wave magnet, J is its strength, and $\mathbf{n}$ is the spin-splitting vector describing the direction of the spin split of the band structure. We assume the direction is along the z axis by taking $\mathbf{n} = (0, 0, \pm 1)$. The spin-split structure function $f_X(\mathbf{k})$ is determined by the symmetry analysis of the crystal structure and the spin group, which is given by [10,11,22,30]

$$f_s = 1, \quad (6)$$

$$f_p = k_x = ak \cos \phi, \quad (7)$$

$$f_d = 2a^2 k_x k_y = a^2 k^2 \sin 2\phi, \quad (8)$$

$$f_f = a^3 k_x (k_x^2 - 3k_y^2) = a^3 k^3 \cos 3\phi, \quad (9)$$

$$f_g = 4a^4 k_x k_y (k_x^2 - k_y^2) = a^4 k^4 \sin 4\phi, \quad (10)$$

$$f_i = 2a^6 k_x k_y (3k_x^2 - k_y^2)(k_x^2 - 3k_y^2) = a^6 k^6 \sin 6\phi, \quad (11)$$

where $k_x = k \cos \phi$ and $k_y = k \sin \phi$. Especially, f_s describes a ferromagnet, which is a constant; f_d , f_g , and f_i describe the d -wave, g -wave, and i -wave altermagnets, respectively; f_p and

f_f describe the p -wave and f -wave magnets, respectively. The X -wave magnet has N_X nodes in the band structure, where $N_X = 1, 2, 3, 4, 6$ for $X = p, d, f, g, i$, respectively. The effective Hamiltonian describes the lowest two bands in the vicinity of the Γ point [31].

The eigenenergy is given by

$$\varepsilon_s(\mathbf{k}) = \frac{\hbar^2 \mathbf{k}^2}{2m} - \mu + sJ f_X(\mathbf{k}), \quad (12)$$

with the spin index $s = \pm 1$.

B. Tight-binding model

The tight-binding model [22,30] is constructed from the Hamiltonian (5) by replacing $k_j \mapsto \sin ak_j$ and $k_j^2 \mapsto 2(1 - \cos ak_j)$, where $j = x, y$. The Hamiltonian is defined on the square lattice for the p -wave magnet, the d -wave magnet, and the g -wave magnet, while it is defined on the triangular lattice for the f -wave magnet and the i -wave magnet.

The tight-binding Hamiltonian is given by

$$H = H_{\text{Kine}} + H_X. \quad (13)$$

The kinetic term reads

$$H_{\text{Kine,Sq}} = \frac{\hbar^2}{ma^2} (2 - \cos ak_x - \cos ak_y) \quad (14)$$

on the square lattice, while it reads

$$H_{\text{Kine,Tri}} = \frac{\hbar^2}{ma^2} \left(3 - \cos ak_x - \sum_{\pm} \cos a \frac{k_x \pm \sqrt{3}k_y}{2} \right) \quad (15)$$

on the triangular lattice. We use Eq. (14) for the p -wave magnet, the d -wave altermagnet, and the g -wave altermagnet, while we use Eq. (15) for the f -wave magnet and the i -wave altermagnet.

The X -wave term H_X is given by

$$H_p = J\sigma_z \sin ak_x \quad (16)$$

for the p -wave magnet [21,31–36],

$$H_d = J\sigma_z \sin ak_x \sin ak \quad (17)$$

for the d -wave magnet [10,11,37–43],

$$H_f = 4J\sigma_z \sin ak_x \sin \frac{ak_x + \sqrt{3}ak_y}{2} \sin \frac{-ak_x + \sqrt{3}ak_y}{2} \quad (18)$$

for the f -wave magnet [22,30],

$$H_g = 2J\sigma_z \sin ak_x \sin ak_y (\cos ak_y - \cos ak_z) \quad (19)$$

for the g -wave magnet [22,30], and

$$H_i = -\frac{16}{3\sqrt{3}} J\sigma_z \sin ak_x \sin \frac{ak_x + \sqrt{3}ak_y}{2} \sin \frac{-ak_x + \sqrt{3}ak_y}{2} \\ \times \sin \sqrt{3}ak_y \sin \frac{3ak_x + \sqrt{3}ak_y}{2} \sin \frac{-3ak_x + \sqrt{3}ak_y}{2} \quad (20)$$

for the i -wave magnet [30].

III. TUNNELING MAGNETORESISTANCE

We consider a bilayer system made of X -wave magnets spaced by an insulator, as illustrated in Fig. 1. Electrons coupled with the X -wave magnet in each layer are described by the Hamiltonian (5) but may have a different parameter J . We denote it as J^T in the top layer and J^B in the bottom layer. There are two configurations. One is the parallel configuration, where the spin directions are identical at each lattice site between the two layers as in Fig. 1(a). Namely, both X -wave magnets are described by the Hamiltonian (5) together with $J^T = J^B = J$ as in Fig. 1(a). The other is the antiparallel configuration, where the spin directions are opposite at each lattice site between the two layers as in Fig. 1(b). Namely, the top X -wave magnet is described by the Hamiltonian (5) while the bottom X -wave magnet is described by the Hamiltonian (5) with the replacement of J by $-J$, or $J^T = -J^B = J$. Large (small) current flows perpendicularly if the spin directions are identical (opposite) between the two layers. We calculate the TMR analytically based on the continuum model and numerically based on the tight-binding model.

The current I under the bias voltage V is calculated based on the Green's function method [29,44],

$$\begin{aligned} \frac{I}{4e\pi^3} = & \int_{-\infty}^{\infty} d\omega \sum_{s=\pm 1} \sum_{\mathbf{k}_1, \mathbf{k}_2} |T_{\mathbf{k}_1, \mathbf{k}_2}|^2 \\ & \times \text{Tr}[\text{Im}\mathcal{G}_s^T(\hbar\omega; \mathbf{k}_1) \text{Im}\mathcal{G}_s^B(\hbar\omega - eV; \mathbf{k}_2)] \\ & \times [f(\hbar\omega - eV) - f(\hbar\omega)], \end{aligned} \quad (21)$$

where

$$f(\hbar\omega) = 1/\{\exp[\hbar\omega/(k_B T)] + 1\} \quad (22)$$

is the Fermi distribution function. While $\mathcal{G}_s^T$ ($\mathcal{G}_s^B$) is the retarded Green's function of the top (bottom) layer defined by

$$\mathcal{G}_s^T(\omega; \mathbf{k}) \equiv \mathcal{G}_s^B(\omega; \mathbf{k}) \equiv \mathcal{G}_s(\omega; \mathbf{k}) \quad (23)$$

for the parallel configuration and

$$\mathcal{G}_s^T(\omega; \mathbf{k}) \equiv \mathcal{G}_s(\omega; \mathbf{k}), \quad \mathcal{G}_s^B(\omega; \mathbf{k}) \equiv \mathcal{G}_{-s}(\omega; \mathbf{k}) \quad (24)$$

for the antiparallel configuration, where we have defined

$$\mathcal{G}_s(\omega; \mathbf{k}) \equiv \frac{1}{\hbar\omega - \varepsilon_s + i\Gamma}, \quad (25)$$

with the self-energy Γ and Eq. (12) for ε_s . The self-energy is induced, for instance, by the Coulomb interactions and impurities. The imaginary part of the Green's function is explicitly given by

$$\text{Im}\mathcal{G}_s(\omega; \mathbf{k}) = \frac{\Gamma}{(\hbar\omega - \varepsilon_s)^2 + \Gamma^2}. \quad (26)$$

It is reduced to the δ function,

$$\lim_{\Gamma \rightarrow 0} \text{Im}\mathcal{G}_s(\omega; \mathbf{k}) = -\pi\delta(\omega - \varepsilon_s), \quad (27)$$

in the small Γ limit.

For small bias V , Eq. (21) is expanded as

$$\begin{aligned} \frac{I}{4e\pi^3} = & - \int_{-\infty}^{\infty} d\omega \sum_{s=\pm 1} \sum_{\mathbf{k}_1, \mathbf{k}_2} |T_{\mathbf{k}_1, \mathbf{k}_2}|^2 \text{Tr}[\text{Im}\mathcal{G}_s^T(\hbar\omega; \mathbf{k}_1) \\ & \times \text{Im}\mathcal{G}_s^B(\hbar\omega; \mathbf{k}_2)] eV \frac{\partial f(\hbar\omega)}{\partial \hbar\omega}. \end{aligned} \quad (28)$$

It is linear as a function of V for small bias V .

At the zero temperature ($T = 0$) and the small bias voltage limit ($V = 0$), the differential conductance dI/dV is simplified as

$$\frac{G}{4e\pi^3} = \sum_{s=\pm 1} \sum_{\mathbf{k}_1, \mathbf{k}_2} |T_{\mathbf{k}_1, \mathbf{k}_2}|^2 \text{Tr}[\text{Im}\mathcal{G}_s^T(0; \mathbf{k}_1) \text{Im}\mathcal{G}_s^B(0; \mathbf{k}_2)]. \quad (29)$$

A comment is in order with respect to the retarded Green's functions (24) in the antiparallel configuration. The eigenenergy (12) appears in the Green's function, where J appears only in the combination J_s . Hence, the effect of $J^T = -J^B = J$ is equivalent to taking $-s$ instead of s in the bottom layer, leading to Eq. (24).

The differential conductance G_P for the parallel configuration is given by

$$\frac{G_P}{4e\pi^3} = \sum_{s=\pm 1} \sum_{\mathbf{k}_1, \mathbf{k}_2} |T_{\mathbf{k}_1, \mathbf{k}_2}|^2 \text{Tr}[\text{Im}\mathcal{G}_s(0; \mathbf{k}_1) \text{Im}\mathcal{G}_s(0; \mathbf{k}_2)], \quad (30)$$

while the differential conductance G_{AP} for the antiparallel configuration is given by

$$\frac{G_{AP}}{4e\pi^3} = \sum_{s=\pm 1} \sum_{\mathbf{k}_1, \mathbf{k}_2} |T_{\mathbf{k}_1, \mathbf{k}_2}|^2 \text{Tr}[\text{Im}\mathcal{G}_s(0; \mathbf{k}_1) \text{Im}\mathcal{G}_{-s}(0; \mathbf{k}_2)]. \quad (31)$$

In Eq. (29), $|T_{\mathbf{k}_1, \mathbf{k}_2}|^2$ is the tunneling coefficient given by

$$|T_{\mathbf{k}_1, \mathbf{k}_2}|^2 = T_0^2 \delta(\mathbf{k}_1 - \mathbf{k}_2) \quad (32)$$

for a bilayer system because the momentum is conserved. By using the WKB approximation, T_0^2 is given by

$$T_0^2 \propto \exp\left[-\frac{4}{\hbar} \int_{x_1}^{x_2} \sqrt{2m[V(x) - E]} dx\right]. \quad (33)$$

For a rectangular potential barrier $V(x) = V$, it is given by

$$T_0^2 \propto \exp\left[-\frac{4}{\hbar} \sqrt{2m(V - E)} L\right], \quad (34)$$

where $L \equiv x_2 - x_1$ is the width of the potential barrier, and V is the energy of the potential barrier.

The TMR ratio is defined by using the differential conductances G_P (G_{AP}) for the parallel (antiparallel) configuration as [45]

$$\text{TMR}_{\text{ratio}} \equiv \frac{G_P - G_{AP}}{G_{AP}}. \quad (35)$$

In the absence of the magnetic order $J = 0$, the differential conductance (29) is exactly calculated as

$$\frac{G}{4e\pi^3} = \frac{m\pi}{\hbar^2 \Gamma} \left(\frac{2\Gamma\mu}{\mu^2 + \Gamma^2} + 2 \arctan \frac{\mu}{\Gamma} + \pi \right). \quad (36)$$

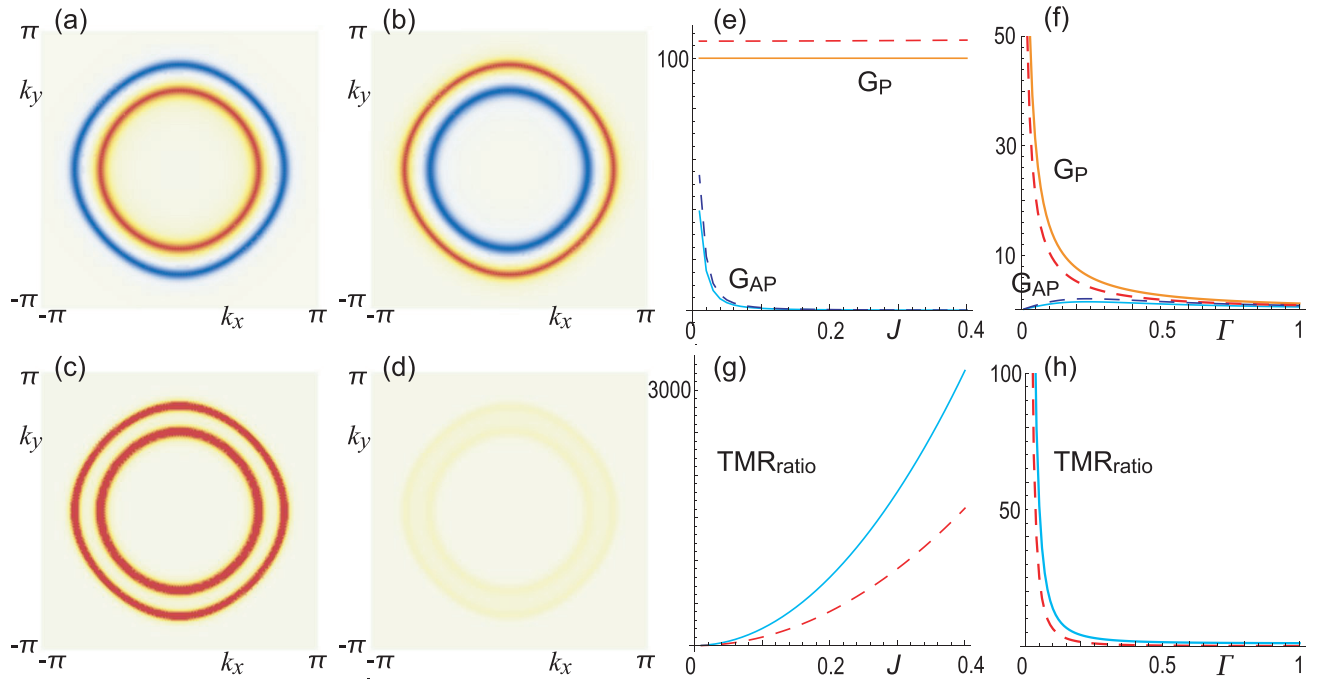


FIG. 2. Ferromagnet. (a) Spin density (S_z) when $J = 0.25\varepsilon_0$, and (b) that when $J = -0.25\varepsilon_0$ in one layer. Red (blue) color indicates up (down) spin. (c) Overlap $\mathcal{O}_P$ for the parallel configuration, and (d) overlap $\mathcal{O}_{AP}$ for the antiparallel configuration. Red color indicates a large overlap. (e) Differential conductance $G/4e\pi^5$ as a function of J , and (f) as a function of Γ . Dashed red (blue) curves indicate numerical results based on the tight-binding model for the parallel (antiparallel) configuration. Solid orange (cyan) curves indicate analytic results based on the continuum model for the parallel (antiparallel) configuration. (g) TMR ratio as a function of J , and (h) as a function of Γ . Dashed red (solid cyan) curves indicate numerical results based on the tight-binding (continuum) model. We have set $\hbar^2 k_0^2/2m = \varepsilon_0$, $J = 0.25\varepsilon_0$, and $k = 1/a$. We have set $\mu = \varepsilon_0$ and $\Gamma = 0.1\varepsilon_0$ in panels (a), (b), (c), and (d), while we have set $\mu = \varepsilon_0/2$ and $\Gamma = 0.01\varepsilon_0$ in panels (e), (f), (g), and (h).

For $\Gamma/\mu \ll 1$, it is expanded as

$$\frac{G}{4e\pi^3} = \frac{2m\pi^2}{\hbar^2\Gamma} - \frac{4\pi m\Gamma^2}{3\mu^3} + \frac{8m\pi\Gamma^4}{5\mu^5} - \dots, \quad (37)$$

and it is simplified as

$$\lim_{\Gamma \rightarrow 0} \frac{G}{4e\pi^3} = \frac{2m\pi^2}{\hbar^2\Gamma} \quad (38)$$

in the small- Γ limit.

We define the overlap between the Fermi surfaces in the top layer and the bottom layer by

$$\mathcal{O}_P(\mathbf{k}) \equiv \sum_{s=\pm 1} \text{Tr} [\text{Im}\mathcal{G}_s(0; \mathbf{k}) \text{Im}\mathcal{G}_s(0; \mathbf{k})], \quad (39)$$

$$\mathcal{O}_{AP}(\mathbf{k}) \equiv \sum_{s=\pm 1} \text{Tr} [\text{Im}\mathcal{G}_s(0; \mathbf{k}) \text{Im}\mathcal{G}_{-s}(0; \mathbf{k})]. \quad (40)$$

With the use of them, the differential conductance (29) is simply given by

$$\frac{G_P}{4e\pi^3} = \sum_{\mathbf{k}} T_0^2 \mathcal{O}_P(\mathbf{k}), \quad \frac{G_{AP}}{4e\pi^3} = \sum_{\mathbf{k}} T_0^2 \mathcal{O}_{AP}(\mathbf{k}). \quad (41)$$

We also define the spin density by

$$\langle S_z(\mathbf{k}) \rangle \equiv \text{Im}\mathcal{G}_+(0; \mathbf{k}) - \text{Im}\mathcal{G}_-(0; \mathbf{k}) \quad (42)$$

in each layer.

We calculate these for the ferromagnet and each X -wave magnet in the following sections.

A. Ferromagnet

For the sake of comparison, we first revisit the ordinary ferromagnetic magnetic tunneling junction illustrated in Fig. 1 and obtain exact formulas for the TMR. The Hamiltonian is given by setting $f_X = 1$ in Eq. (5). The spin density (42) is shown in the momentum space in Fig. 2(a) when $J > 0$ and in Fig. 2(b) when $J < 0$. The inner Fermi surface is up-spin polarized, while the outer Fermi surface is down-spin polarized in Fig. 2(a). On the other hand, the inner Fermi surface is down-spin polarized, while the outer Fermi surface is up-spin polarized in Fig. 2(b).

1. Parallel configuration

The overlap $\mathcal{O}_P$ for the parallel configuration in Eq. (39) is shown in Fig. 2(c). The effect of the ferromagnet is only to shift the chemical potential from μ to $\mu - sJ$. By replacing $\mu \mapsto \mu - sJ$ in Eq. (36), the differential conductance with the parallel configuration is exactly calculated as

$$\frac{G_P}{4e\pi^3} = \frac{m\pi}{\hbar^2\Gamma} \left(\frac{2\Gamma(\mu + J)}{(\mu + J)^2 + \Gamma^2} + 2 \arctan \frac{\mu + J}{\Gamma} + \frac{2\Gamma(\mu - J)}{(\mu - J)^2 + \Gamma^2} + 2 \arctan \frac{\mu - J}{\Gamma} + 2\pi \right). \quad (43)$$

For small Γ , it is simplified as

$$\lim_{\Gamma \rightarrow 0} \frac{G_P}{4e\pi^3} = \frac{m\pi^2}{2\hbar^2\Gamma} (2 + \text{sgn}[\mu - J] + \text{sgn}[\mu + J]). \quad (44)$$

The differential conductance G_P is shown as a function of J in Fig. 2(e). It is almost flat for small J , but it is slightly different between the tight-binding model and the continuum model even at $J = 0$. This is because the Fermi surface of the tight-binding model is deformed from the circle.

The differential conductance G_P is shown as a function of J in Fig. 2(e) and as a function of Γ in Fig. 2(f).

2. Antiparallel configuration

The overlap $\mathcal{O}_{AP}$ for the antiparallel configuration in Eq. (40) is shown in Fig. 2(d). There is almost no overlap for small Γ .

The differential conductance with the antiparallel configuration is exactly calculated as

$$\begin{aligned} \frac{G_{AP}}{4e\pi^3} &= \frac{\Gamma m}{8\hbar^2 J(J^2 + \Gamma^2)} \\ &\times \left[2J \left(\arctan \frac{J + \mu}{\Gamma} - \arctan \frac{J - \mu}{\Gamma} + \pi \right) \right. \\ &\left. + \Gamma \log \frac{\Gamma^2 + (J + \mu)^2}{\Gamma^2 + (J - \mu)^2} \right]. \end{aligned} \quad (45)$$

It is simplified as

$$\lim_{\Gamma \rightarrow 0} \frac{G_{AP}}{4e\pi^3} = \frac{\Gamma m}{4\hbar^2 J^2} (2 + \text{sgn}[\mu - J] + \text{sgn}[\mu + J]) \quad (46)$$

for small Γ . The differential conductance G_{AP} is shown as a function of J in Fig. 2(e) and as a function of Γ in Fig. 2(f).

3. Tunneling magnetoresistance ratio

The TMR ratio is obtained as

$$\text{TMR}_{\text{ratio}} = \frac{2J^2}{\Gamma^2} - 1 \simeq \frac{2J^2}{\Gamma^2}. \quad (47)$$

It is antiproportional to Γ^2 .

The tunneling magnetoresistance ratio is shown as a function of J in Fig. 2(g) and as a function of Γ in Fig. 2(h). It rapidly decreases as the increase of Γ . It is necessary to make Γ smaller to achieve larger TMR.

The current $I/4e\pi^3$ is shown as a function of the bias voltage V in Fig. 3(a). It is linear as a function of V for small V as explained in Eq. (28). The differential conductance $G/4e\pi^3$ is shown as a function of temperature T in Fig. 3(b). It decreases with the increase of temperature.

B. p -wave magnet

The spin density (42) is shown in the momentum space in Fig. 4(a) when $J > 0$ and (b) when $J < 0$. The Fermi surfaces shift along the k_x axis without changing their shapes, where the shifts are opposite depending on the spins. The overlap $\mathcal{O}_P$ for the parallel configuration is shown in Fig. 4(c).

The Hamiltonian is rewritten as

$$\frac{\hbar^2 k^2}{2m} - \mu \pm Jk_x = \frac{\hbar^2 k_x'^2}{2m} + \frac{\hbar^2 k_y^2}{2m} - \mu', \quad (48)$$

with

$$k_x' \equiv k_x \pm mJ/\hbar^2, \quad \mu' \equiv \mu + \frac{mJ^2}{2\hbar^2}. \quad (49)$$

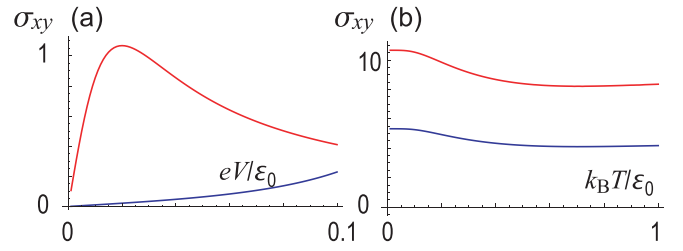


FIG. 3. Ferromagnet. (a) Conductance $G/4e\pi^3$ as a function of V . (b) Differential conductance $G/4e\pi^3$ as a function of temperature T . Red (blue) curves indicate numerical results based on the tight-binding model for the parallel (antiparallel) configuration. We have set $\hbar^2 k_0^2/2m = \varepsilon_0$, $J = 0.1\varepsilon_0$, $k = 1/a$, and $\mu = \varepsilon_0/2$. We have set $\Gamma = 0.01\varepsilon_0$ in panel (a), while we have set $\mu = \varepsilon_0/2$ and $\Gamma = 0.1\varepsilon_0$ in panel (b).

The Fermi surface is given by [30]

$$\hbar\sqrt{k_x'^2 + k_y^2} = \sqrt{2m\mu'}. \quad (50)$$

It is shifted by $-smJ/\hbar^2$ for $s = \pm 1$ in the p -wave magnet as in Fig. 4(a).

1. Parallel configuration

By inserting Eq. (49) into Eq. (36), the differential conductance is exactly calculated as

$$\frac{G_P}{4e\pi^3} = \frac{m\pi}{2\hbar^2\Gamma} \left(\frac{2\Gamma\mu'}{\mu'^2 + \Gamma^2} + 2 \arctan \frac{\mu'}{\Gamma} + \pi \right). \quad (51)$$

The differential conductance G_P is shown as a function of J in Fig. 4(e). It depends scarcely on J because the shapes of the Fermi surfaces do not change as a function of J . The differential conductance G_P is shown as a function of Γ in Fig. 4(f).

2. Antiparallel configuration

The overlaps $\mathcal{O}_{AP}$ are shown in Fig. 4(d). There are two peaks where the two Fermi surfaces with up and down spins overlap. By solving equations $\varepsilon_+ = \varepsilon_- = 0$ with Eq. (12), or

$$\varepsilon_{\pm}(k_x, k_y) = \frac{\hbar^2 k^2}{2m} - \mu \pm Jk_x = 0, \quad (52)$$

the momenta of the peaks are obtained as

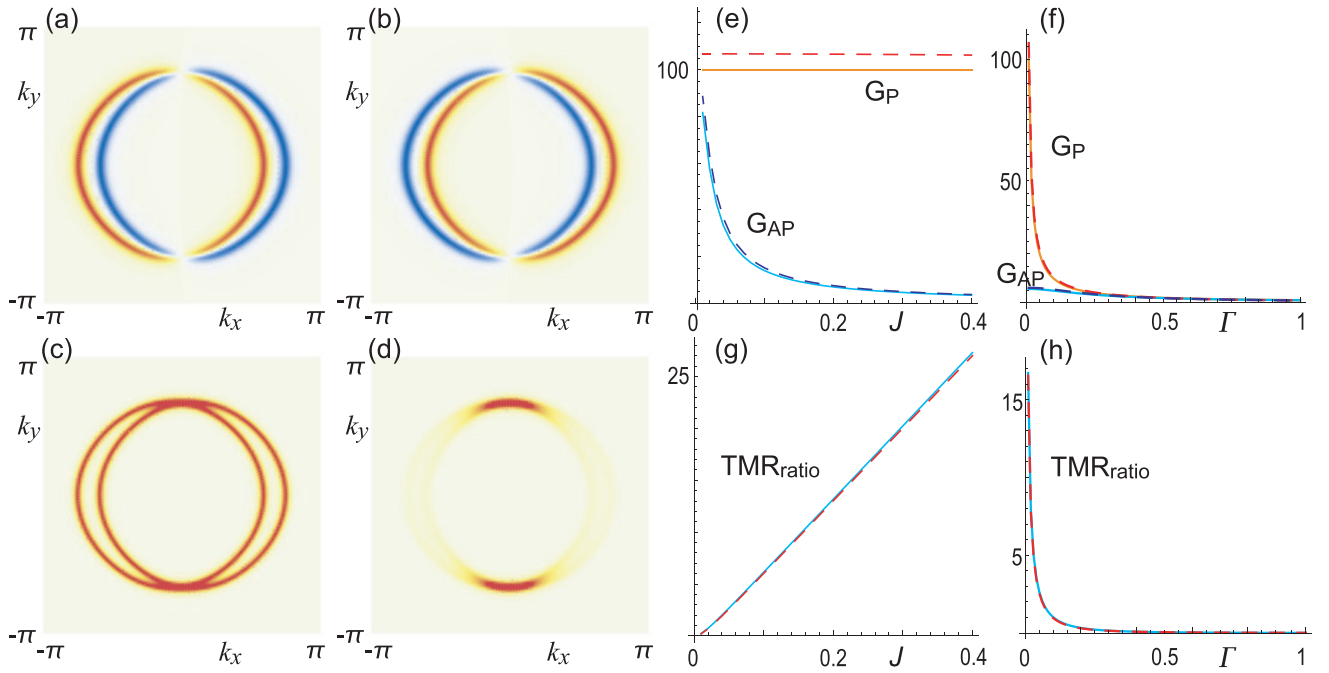
$$(k_x, k_y) = (0, \pm\sqrt{2m\mu}). \quad (53)$$

We first take the limit $\Gamma \rightarrow 0$ in the differential conductance (29). It is given by

$$\frac{G_{AP}}{4e\pi^3} = \pi^2 \sum_{\mathbf{k}} \delta(\varepsilon_+) \delta(\varepsilon_-) = \pi^2 \sum_{\mathbf{k}} \frac{\delta(k_x - k_+)}{\left| \frac{\partial \varepsilon_+}{\partial k_x} \right|} \delta(\varepsilon_-), \quad (54)$$

with

$$\frac{\partial \varepsilon_+}{\partial k_x} = \frac{\hbar^2 k_x}{m} + Ja, \quad (55)$$

FIG. 4. *p*-wave magnet. There are two peaks in panel (d). See the caption of Fig. 2.

and

$$k_{\pm} = -aJ \frac{m}{\hbar^2} \pm \sqrt{2m\mu + a^2 J^2 \frac{m^2}{\hbar^2} - k_y^2}. \quad (56)$$

It is further calculated as

$$\frac{G_{AP}}{4e\pi^3} = \sum_{k_y, \pm} \frac{\pi^2 \delta(k_y \pm \sqrt{2m\mu})}{\left| \frac{\partial \varepsilon_{\pm}}{\partial k_x} \right|_{k_y=k_{\pm}} \left| \frac{\partial \varepsilon_{\pm}}{\partial k_y} \right|_{k_y=k_{\pm}}} = \sqrt{\frac{2m}{\mu}} \frac{\pi^2}{\hbar a |J|}.$$

By requiring the Lorentzian form, we recover Γ in this equation as

$$\frac{G_{AP}}{4e\pi^3} = \sqrt{\frac{2m}{\mu}} \frac{\pi^2}{\hbar \sqrt{a^2 J^2 + \frac{\hbar^2}{2m\mu} \Gamma^2}}, \quad (57)$$

where we have determined the coefficient of Γ to recover the result with $J = 0$ as

$$\lim_{J \rightarrow 0} \frac{G_{AP}}{4e\pi^3} = \frac{2m\pi^2}{\hbar^2 \Gamma}. \quad (58)$$

The differential conductance G_{AP} is shown as a function of J in Fig. 4(e). It monotonically decreases as the increase of J because the overlap between the Fermi surfaces with up and down spins decreases with the increase of J . The differential conductance G_{AP} is shown as a function of Γ in Fig. 4(f). The numerical result and the analytic result well agree.

3. Tunneling magnetoresistance ratio

The TMR ratio is obtained as

$$\text{TMR}_{\text{ratio}} = \frac{\sqrt{2m\mu a^2 J^2 + \hbar^2 \Gamma^2}}{\hbar \Gamma} - 1. \quad (59)$$

It is simplified as

$$\lim_{\Gamma \rightarrow 0} \text{TMR}_{\text{ratio}} = \frac{\sqrt{2m\mu a} |J|}{\hbar \Gamma} \quad (60)$$

for small Γ . The TMR ratio is shown as a function of J in Fig. 4(g). It monotonically increases as a function of J . They rapidly decrease with the increase of Γ . The TMR ratio is shown as a function of Γ in Fig. 4(h). It also rapidly decreases with the increase of Γ .

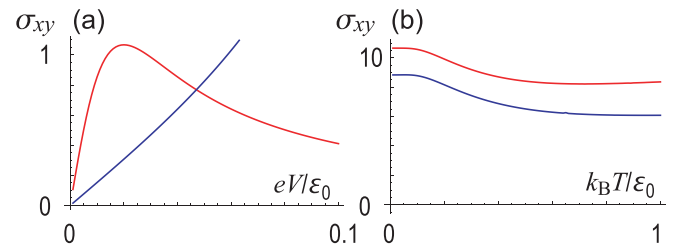
The current $I/4e\pi^3$ is shown as a function of the bias voltage V in Fig. 5(a). The differential conductance $G/4e\pi^3$ is shown as a function of temperature T in Fig. 5(b).

C. *d*-wave magnet

By solving $\varepsilon_s = 0$ in Eq. (12), the Fermi surface is analytically obtained [30] as a function of ϕ ,

$$\hbar k_s^F(\phi) = \sqrt{\frac{2\mu}{\frac{1}{m} + sa^2 J \sin 2\phi}}. \quad (61)$$

The spin density (42) is shown in the momentum space in Fig. 6(a) when $J > 0$ and Fig. 6(b) when $J < 0$. The Fermi

FIG. 5. *p*-wave magnet. See the caption of Fig. 3.

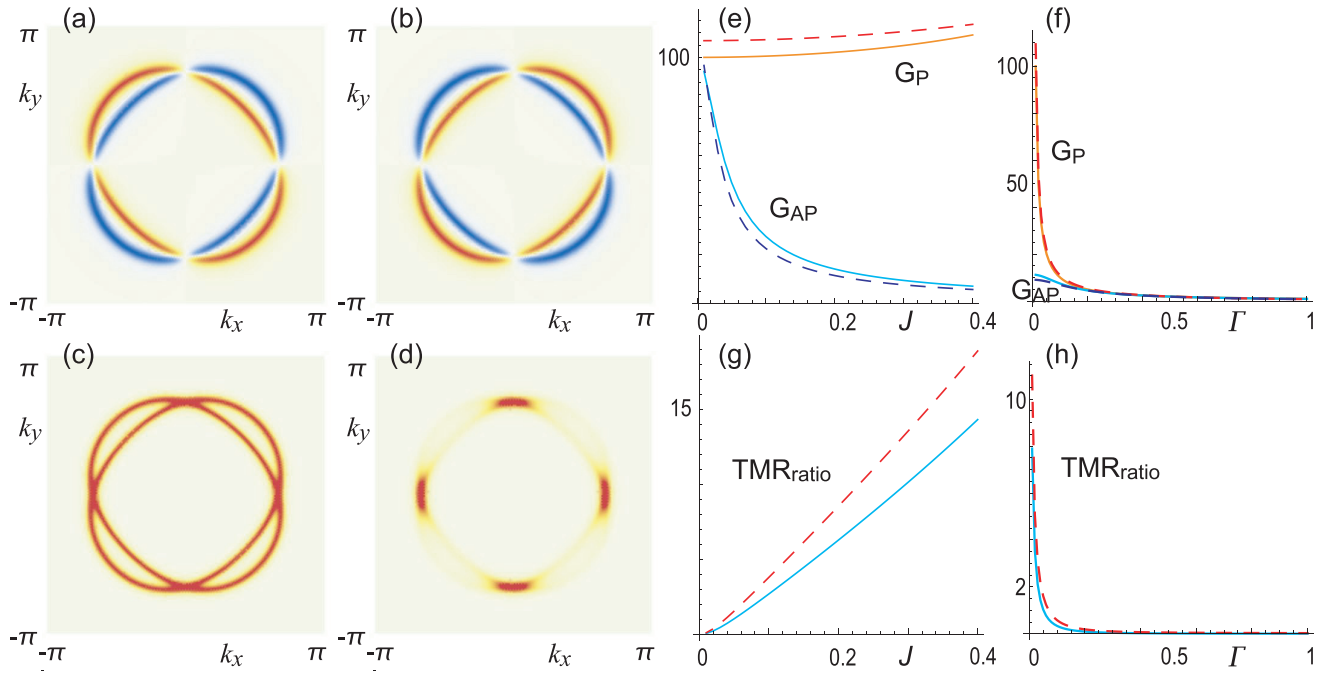


FIG. 6. *d*-wave magnet. There are four peaks in panel (d). See the caption of Fig. 2.

surface is deformed to an elliptic shape for each spin $s = \pm 1$ in the *d*-wave magnet.

1. Parallel configuration

The overlaps $\mathcal{O}_p$ for the parallel configuration are shown in Fig. 6(c). The differential conductance is exactly calculated as

$$\frac{G_p}{4e\pi^3} = \frac{m\pi}{2\hbar^2\Gamma} \frac{1}{\sqrt{1 - 4\frac{m^2a^2J^2}{\hbar^4}}} \left(\frac{2\Gamma\mu}{\mu^2 + \Gamma^2} + 2\arctan \frac{\mu}{\Gamma} + \pi \right). \quad (62)$$

It is valid only for $4m^2J^2/\hbar^4 < 1$, which is identical to the condition that the parabola is convex upward, or

$$\frac{\hbar^2\mathbf{k}^2}{2m} > |J|\mathbf{k}^2. \quad (63)$$

It is simplified as

$$\lim_{\Gamma \rightarrow 0} \frac{G_p}{4e\pi^3} = \frac{2m\pi^2}{\hbar^2\Gamma} \frac{1}{\sqrt{1 - 4\frac{m^2a^2J^2}{\hbar^4}}} \quad (64)$$

for small Γ . The differential conductances G_p are shown as a function of J in Fig. 6(e) and as a function of Γ in Fig. 6(f).

2. Antiparallel configuration

The overlaps $\mathcal{O}_{AP}$ for the antiparallel configuration are shown in Fig. 6(d), respectively. There are four peaks at $(k_x, k_y) = (0, \pm\sqrt{2m\mu}), (\pm\sqrt{2m\mu}, 0)$, where the two Fermi surfaces with up and down spins are overlapped. Then, the differential conductance is twice of that of the *p*-wave magnet (57) and is given by

$$\frac{G_{AP}}{4e\pi^3} = \sqrt{\frac{2m}{\mu}} \frac{2\pi^2}{\hbar a|J|}. \quad (65)$$

By requiring the Lorentzian form, we recover Γ in this equation as

$$\frac{G_{AP}}{4e\pi^3} = \sqrt{\frac{2m}{\mu}} \frac{\pi^2}{\hbar \sqrt{\frac{a^2J^2}{4} + \frac{\hbar^2}{2m\mu} \Gamma^2}}. \quad (66)$$

The differential conductances G_{AP} are shown as a function of J in Fig. 6(e) and as a function of Γ in Fig. 6(f).

3. Tunneling magnetoresistance ratio

The TMR ratio is shown as a function of J in Fig. 6(g). The tunneling magnetoresistance ratio is shown as a function of Γ in Fig. 6(h). The current $I/4e\pi^3$ is shown as a function of the bias voltage V in Fig. 7(a). The differential conductance $G/4e\pi^3$ is shown as a function of temperature T in Fig. 7(b).

D. *f*-wave magnet

By solving $\varepsilon_s = 0$ in Eq. (12), the Fermi surface is described by [30]

$$\hbar k_s^F(\phi) = \sqrt{2\mu m} - 2m^2\mu s a^3 J \cos 3\phi \quad (67)$$

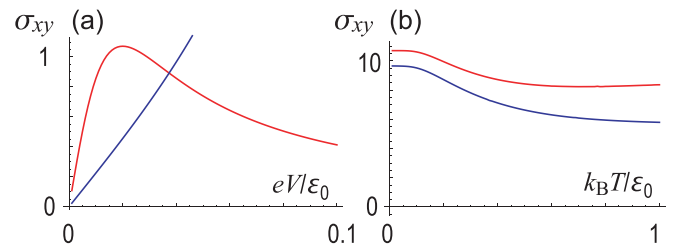
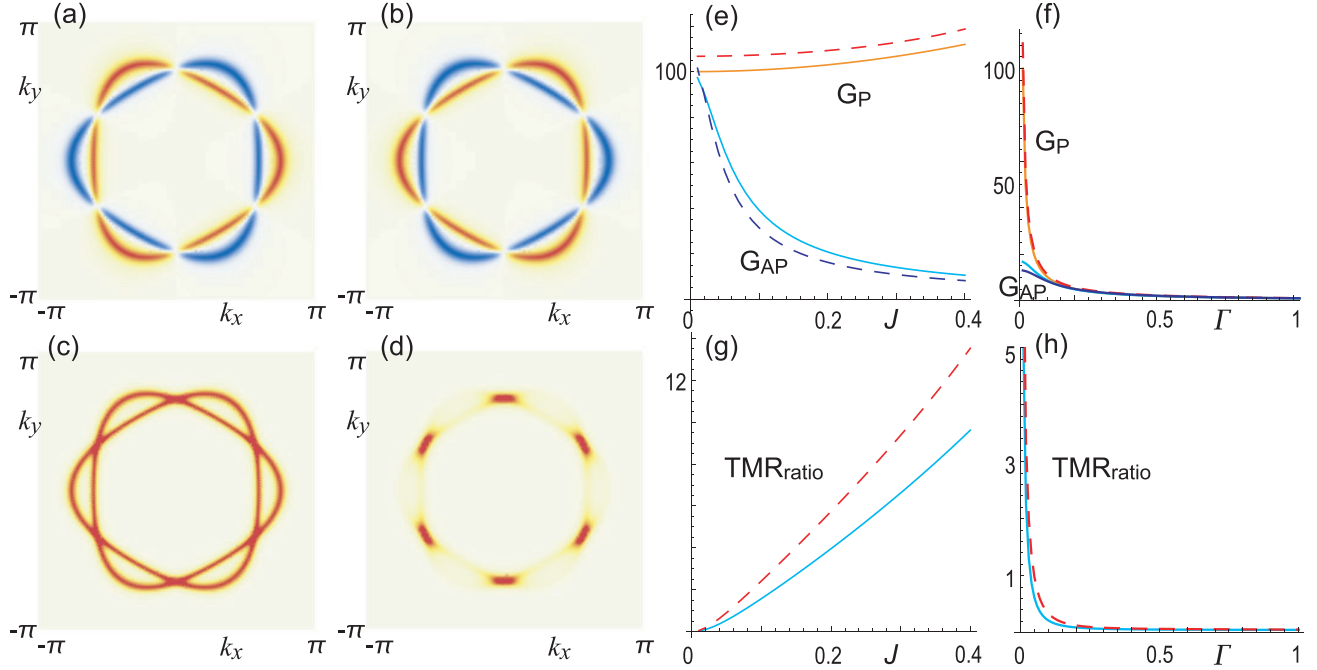


FIG. 7. *d*-wave altermagnet. See the caption of Fig. 3.

FIG. 8. *f*-wave magnet. There are six peaks in panel (d). See the caption of Fig. 2.

up to the first order in J . The spin density (42) is shown in the momentum space in Fig. 8(a) when $J > 0$ and (b) when $J < 0$. The Fermi surface is deformed to a triangular shape for each spin $s = \pm 1$ in the *f*-wave magnet.

1. Parallel configuration

The overlaps $\mathcal{O}_P$ for the parallel configuration are shown in Fig. 8(c). The differential conductance is given by

$$\frac{G_P}{4e\pi^3} = \frac{2m\pi^2}{\hbar^2\Gamma} \left[1 + 12 \left(\frac{m}{\hbar^2} \right)^3 \mu a^2 J^2 \right] \quad (68)$$

up to the second order in J for small Γ . The differential conductances G_P are shown as a function of J in Fig. 8(e) and as a function of Γ in Fig. 8(f).

2. Antiparallel configuration

The overlaps $\mathcal{O}_{AP}$ for the antiparallel configuration are shown in Fig. 8(d). There are six peaks at $(k_x, k_y) = \sqrt{2m\mu}(\cos \frac{n\pi}{3}, \sin \frac{n\pi}{3})$, with $n = 0, 1, 2, 3, 4, 5$, where the two Fermi surfaces with up and down spins are overlapped. Then, the differential conductance is three times larger than that of the *p*-wave magnet (57) and is given by

$$\frac{G_{AP}}{4e\pi^3} = \sqrt{\frac{2m}{\mu}} \frac{3\pi^2}{\hbar a |J|}. \quad (69)$$

By assuming the Lorentzian form, we recover Γ as

$$\frac{G_{AP}}{4e\pi^3} = \sqrt{\frac{2m}{\mu}} \frac{\pi^2}{\hbar \sqrt{\frac{a^2 J^2}{9} + \frac{\hbar^2}{2m\mu} \Gamma^2}}. \quad (70)$$

The differential conductances G_{AP} are shown as a function of J in Fig. 8(e) and as a function of Γ in Fig. 8(f).

3. Tunneling magnetoresistance ratio

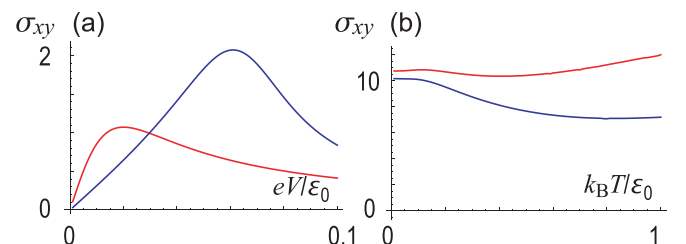
The tunneling magnetoresistance ratio is shown as a function of J in Fig. 8(g). The tunneling magnetoresistance ratio is shown as a function of Γ in Fig. 8(h). The current $I/4e\pi^3$ is shown as a function of the bias voltage V in Fig. 9(a). The differential conductance $G/4e\pi^3$ is shown as a function of temperature T in Fig. 9(b).

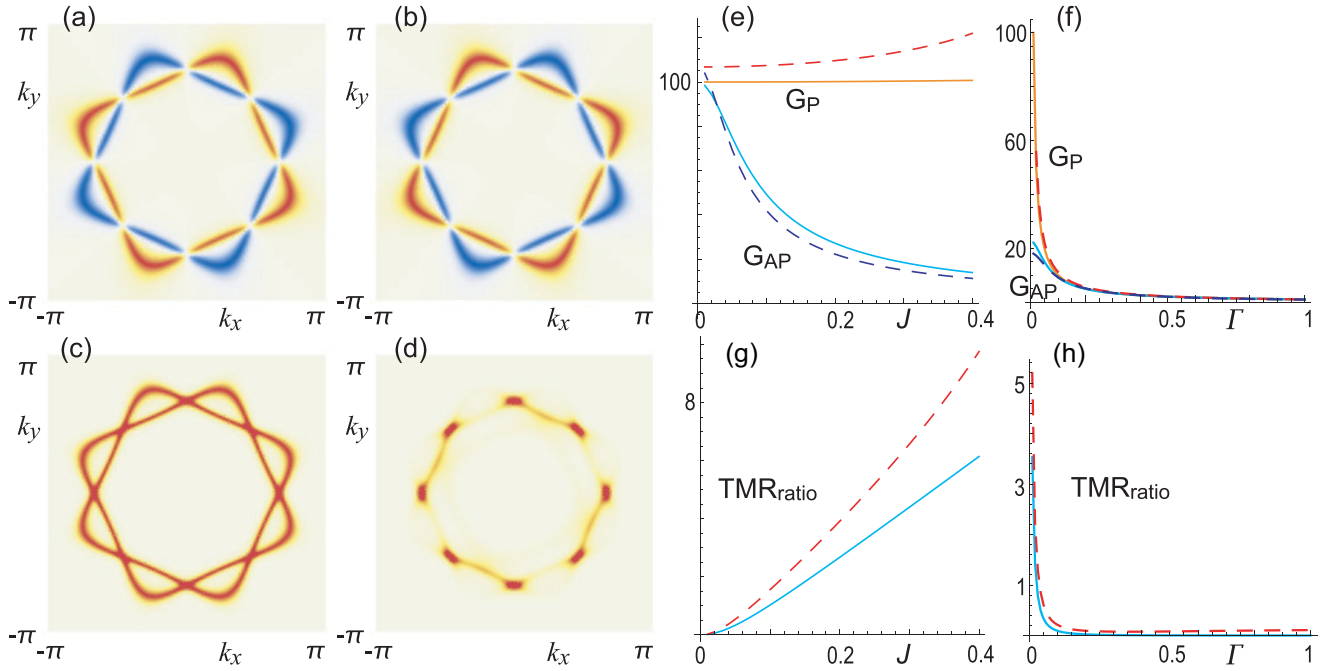
E. g-wave magnet

By solving $\varepsilon_s = 0$ in Eq. (12), the Fermi surface is described by [30]

$$\hbar k_s^F(\phi) = \sqrt{\frac{-1 + \sqrt{1 + 4\mu s a^4 J m^2 \sin 4\phi}}{s J m \sin 4\phi}}. \quad (71)$$

The spin density (42) is shown in the momentum space in Fig. 10(a) when $J > 0$ and (b) when $J < 0$. The Fermi surface is deformed to a square shape for each spin $s = \pm 1$ in the *g*-wave magnet.

FIG. 9. *f*-wave magnet. See the caption of Fig. 3.

FIG. 10. *g*-wave magnet. There are eight peaks in panel (d). See the caption of Fig. 2.

1. Parallel configuration

The overlaps $\mathcal{O}_P$ for the parallel configuration are shown in Fig. 10(c). The differential conductance is given by

$$\lim_{\Gamma \rightarrow 0} \frac{G_P}{4e\pi^3} = \frac{2m\pi^2}{\hbar^2\Gamma} \left[1 + 3 \left(\frac{m}{\hbar^2} \right)^4 \mu^2 J^2 \right] \quad (72)$$

up to the second order in J for small Γ . The differential conductances G_P are shown as a function of J in Fig. 10(e) and as a function of Γ in Fig. 10(f).

2. Antiparallel configuration

The overlaps $\mathcal{O}_{AP}$ for the antiparallel configuration are shown in Fig. 10(d). There are eight peaks at $(k_x, k_y) = \sqrt{2m\mu}(\cos \frac{n\pi}{4}, \sin \frac{n\pi}{4})$, with $n = 0, 1, \dots, 7$. Then, the differential conductance is four times larger than that of the *p*-wave magnet (57) and is given by

$$\frac{G_{AP}}{4e\pi^3} = \sqrt{\frac{2m}{\mu}} \frac{4\pi^2}{\hbar a |J|}. \quad (73)$$

By requiring the Lorentzian form, we recover Γ in this equation as

$$\frac{G_{AP}}{4e\pi^3} = \sqrt{\frac{2m}{\mu}} \frac{\pi^2}{\hbar \sqrt{\frac{a^2 J^2}{16} + \frac{\hbar^2}{2m\mu} \Gamma^2}}. \quad (74)$$

The differential conductances G_{AP} are shown as a function of J in Fig. 10(e) and as a function of Γ in Fig. 10(f).

3. Tunneling magnetoresistance ratio

The tunneling magnetoresistance ratio is shown as a function of J in Fig. 10(g). The tunneling magnetoresistance ratio is shown as a function of Γ in Fig. 10(h). The current $I/4e\pi^3$ is shown as a function of the bias voltage V in Fig. 11(a).

The differential conductance $G/4e\pi^3$ is shown as a function of temperature T in Fig. 11(b).

F. *i*-wave magnet

By solving $\varepsilon_s = 0$ in Eq. (12), the Fermi surface is described by [30]

$$\hbar k_s^F(\phi) = \sqrt{2m\mu}(1 - 2\mu^2 m^3 s a^6 J \sin 6\phi) \quad (75)$$

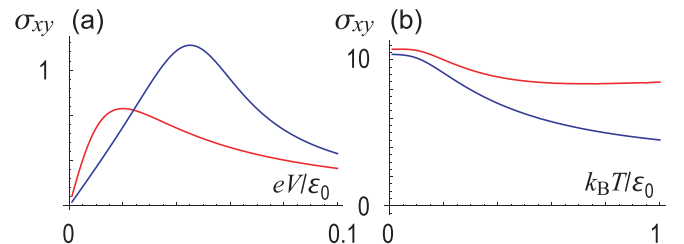
up to the first order in J . The spin density (42) is shown in the momentum space in Fig. 12(a) when $J > 0$ and in Fig. 12(b) when $J < 0$. The Fermi surface is deformed to a hexagonal shape for each spin $s = \pm 1$ in the *i*-wave magnet.

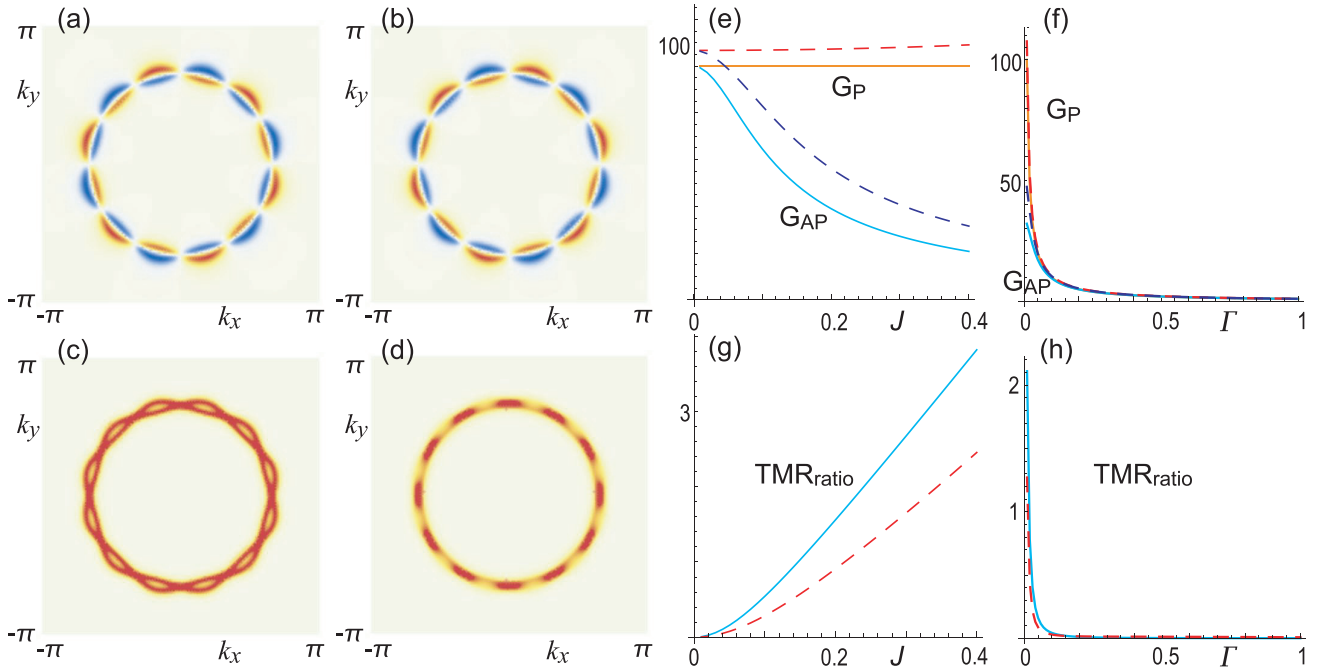
1. Parallel configuration

The overlaps $\mathcal{O}_P$ for the parallel configuration are shown in Fig. 12(c). The differential conductance is given by

$$\lim_{\Gamma \rightarrow 0} \frac{G_P}{4e\pi^3} = \frac{2m\pi^2}{\hbar^2\Gamma} \quad (76)$$

up to the first order in J for small Γ . The differential conductances G_P are shown as a function of J in Fig. 12(e) and as a function of Γ in Fig. 12(f).

FIG. 11. *g*-wave alternomagnet. See the caption of Fig. 3.

FIG. 12. *i*-wave magnet. There are 12 peaks in panel (d). See the caption of Fig. 2.

2. Antiparallel configuration

The overlaps $\mathcal{O}_{AP}$ for the antiparallel configuration are shown in Fig. 12(d). There are 12 peaks at $(k_x, k_y) = \sqrt{2m\mu}(\cos \frac{n\pi}{6}, \sin \frac{n\pi}{6})$, with $n = 0, 1, \dots, 11$. Then, the differential conductance is six times larger than that of the *p*-wave magnet (57) and is given by

$$\frac{G_{AP}}{4e\pi^3} = \sqrt{\frac{2m}{\mu}} \frac{6\pi^2}{\hbar a |J|}. \quad (77)$$

By requiring a Lorentzian form, we recover Γ in this equation as

$$\frac{G_{AP}}{4e\pi^3} = \sqrt{\frac{2m}{\mu}} \frac{\pi^2}{\hbar \sqrt{\frac{a^2 J^2}{36} + \frac{\hbar^2}{2m\mu} \Gamma^2}}. \quad (78)$$

The differential conductances G_{AP} are shown as a function of J in Fig. 12(e) and as a function of Γ in Fig. 12(f).

3. Tunneling magnetoresistance ratio

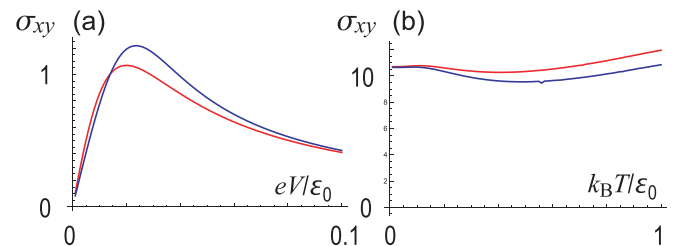
The tunneling magnetoresistance ratio is shown as a function of J in Fig. 12(g). The tunneling magnetoresistance ratio is shown as a function of Γ in Fig. 12(h). The current $I/4e\pi^3$ is shown as a function of the bias voltage V in Fig. 13(a). The differential conductance $G/4e\pi^3$ is shown as a function of temperature T in Fig. 13(b).

IV. MATERIALS

We have so far studied electrons coupled with the *X*-wave magnet because they are relevant in the tunneling phenomenon. Actually, the last term in Eq. (5) is induced

by the *X*-wave magnet, which has local spins. We now discuss the relation between the spin-splitting vector $\mathbf{n}$ and the local spin $\mathbf{S}$ of the *X*-wave magnet. Concerning the spin texture, altermagnets are collinear antiferromagnets [10,11] while the *p*-wave magnet [21,46] and the *f*-wave magnet are noncollinear antiferromagnets [32,47]. *p*-wave magnets are realized by the helix structure of spins. They are characterized by the spin-splitting vector $\mathbf{n}$, which is perpendicular to the spin-rotation direction. In Eq. (5), the spin-splitting vector $\mathbf{n}$ is identical to the Néel vector for altermagnets, where $\mathbf{n}$ is parallel (antiparallel) to the spin directions of altermagnets at the *A* (*B*) site, while $\mathbf{n}$ is perpendicular to the direction of the spin rotation of the *p*-wave and *f*-wave magnets.

It has been theoretically proposed [21] that CeNiAsO is a *p*-wave magnet and experimentally observed [48]. It has also been theoretically proposed [49] that a *p*-wave magnet is realized in graphene by introducing the spin nematic order. Experiments on *p*-wave magnets have been reported [46] in $\text{Gd}_3\text{Ru}_4\text{Al}_{12}$ and reported [50] in NiI_2 . A *d*-wave magnet has been realized in RuO_2 [14,51–54], Mn_5Si_3 [16,55], FeSb_2 [56], organic materials [57], perovskite materials [58,59], and

FIG. 13. *i*-wave altermagnet. See the caption of Fig. 3.

twisted magnetic Van der Waals bilayers [60]. It has been theoretically proposed [49] that an f -wave magnet can be realized in the rhombohedral trilayer graphene by introducing the spin nematic order. A g -wave altermagnet and an i -wave altermagnet have been realized in twisted magnetic Van der Waals bilayers [60].

V. DISCUSSION

We have analytically derived a formula of the TMR ratio based on the X -wave magnet, which is proportional to $|J|/(N_X\Gamma)$. On the other hand, the TMR ratio based on ferromagnets is proportional to J^2/Γ^2 . Hence, the TMR ratio is larger in ferromagnets for $|J| > \Gamma$. Nevertheless, the X -wave magnets are expected to achieve high-speed and ultradense memory owing to the zero net magnetization.

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DATA AVAILABILITY

The data that support the findings of this article are not publicly available upon publication because it is not technically feasible and/or the cost of preparing, depositing, and hosting the data would be prohibitive within the terms of this research project. The data are available from the authors upon reasonable request.

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- [1] M. Julliere, Tunneling between ferromagnetic films, *Phys. Lett. A* **54**, 225 (1975).
 - [2] J. S. Moodera, L. R. Kinder, T. M. Wong, and R. Meservey, Large magnetoresistance at room temperature in ferromagnetic thin film tunnel junctions, *Phys. Rev. Lett.* **74**, 3273 (1995).
 - [3] T. Jungwirth, X. Marti, P. Wadley, and J. Wunderlich, Antiferromagnetic spintronics, *Nat. Nanotechnol.* **11**, 231 (2016).
 - [4] V. Baltz, A. Manchon, M. Tsoi, T. Moriyama, T. Ono, and Y. Tserkovnyak, Antiferromagnetic spintronics, *Rev. Mod. Phys.* **90**, 015005 (2018).
 - [5] J. Han, R. Cheng, L. Liu, H. Ohno, and S. Fukami, Coherent antiferromagnetic spintronics, *Nat. Mater.* **22**, 684 (2023).
 - [6] Z. Ni, A. V. Haglund, H. Wang, B. Xu, C. Bernhard, D. G. Mandrus, X. Qian, E. J. Mele, C. L. Kane, and L. Wu, Imaging the Neel vector switching in the monolayer antiferromagnet MnPSe₃ with strain-controlled Ising order, *Nat. Nanotechnol.* **16**, 782 (2021).
 - [7] J. Godinho, H. Reichlov, D. Kriegner, V. Novak, K. Olejnik, Z. Kašpar, Z. Šoban, P. Wadley, R. P. Campion, R. M. Otxoa, P. E. Roy, J. Železný, T. Jungwirth, and J. Wunderlich, Electrically induced and detected Neel vector reversal in a collinear antiferromagnet, *Nat. Commun.* **9**, 4686 (2018).
 - [8] K. Kimura, Y. Otake, and T. Kimura, Visualizing rotation and reversal of the Neel vector through antiferromagnetic trichroism, *Nat. Commun.* **13**, 697 (2022).
 - [9] Y.-H. Zhang, T.-C. Chuang, D. Qu, and S.-Y. Huang, Detection and manipulation of the antiferromagnetic Neel vector in Cr₂O₃, *Phys. Rev. B* **105**, 094442 (2022).
 - [10] L. Šmejkal, J. Sinova, and T. Jungwirth, Beyond conventional ferromagnetism and antiferromagnetism: A phase with nonrelativistic spin and crystal rotation symmetry, *Phys. Rev. X* **12**, 031042 (2022).
 - [11] L. Šmejkal, J. Sinova, and T. Jungwirth, Emerging research landscape of altermagnetism, *Phys. Rev. X* **12**, 040501 (2022).
 - [12] Z. Feng, X. Zhou, L. Šmejkal, L. Wu, Z. Zhu, H. Guo, R. Gonzalez-Hernandez, X. Wang, H. Yan, P. Qin, X. Zhang, H. Wu, H. Chen, Z. Meng, L. Liu, Z. Xia, J. Sinova, T. Jungwirth, and Z. Liu, An anomalous Hall effect in altermagnetic ruthenium dioxide, *Nat. Electron.* **5**, 735 (2022).
 - [13] A. Fakhredine, R. M. Sattigeri, G. Cuono, and C. Autieri, Interplay between altermagnetism and nonsymmorphic symmetries generating large anomalous Hall conductivity by semi-Dirac points induced anticrossings, *Phys. Rev. B* **108**, 115138 (2023).
 - [14] T. Tschirner, P. Keler, R. D. G. Betancourt, T. Kotte, D. Kriegner, B. Buechner, J. Dufouleur, M. Kamp, V. Jovic, L. Šmejkal, J. Sinova, R. Claessen, T. Jungwirth, S. Moser, H. Reichlova, and L. Veyrat, Saturation of the anomalous Hall effect at high magnetic fields in altermagnetic RuO₂, *APL Mater.* **11**, 101103 (2023).
 - [15] T. Sato, S. Haddad, I. C. Fulga, F. F. Assaad, and J. van den Brink, Altermagnetic anomalous Hall effect emerging from electronic correlations, *Phys. Rev. Lett.* **133**, 086503 (2024).
 - [16] M. Leiviskä, J. Rial, A. Bad'ura, R. L. Seeger, I. Kounta, S. Beckert, D. Kriegner, I. Joumard, E. Schmoranzarová, J. Sinova, O. Gomonay, A. Thomas, S. T. B. Goennenwein, H. Reichlová, L. Šmejkal, L. Michez, T. Jungwirth, and V. Baltz, Anisotropy of the anomalous Hall effect in the altermagnet candidate Mn₅Si₃ films, *Phys. Rev. B* **109**, 224430 (2024).
 - [17] M. Wang, K. Tanaka, S. Sakai, Z. Wang, K. Deng, Y. Lyu, C. Li, D. Tian, S. Shen, N. Ogawa, N. Kanazawa, P. Yu, R. Arita, and F. Kagawa, Emergent zero-field anomalous Hall effect in a reconstructed rutile antiferromagnetic metal, *Nat. Commun.* **14**, 8240 (2023).
 - [18] L. Attias, A. Levchenko, and M. Khodas, Intrinsic anomalous Hall effect in altermagnets, *Phys. Rev. B* **110**, 094425 (2024).
 - [19] R. Takagi, R. Hirakida, Y. Settai, R. Oiwa, H. Takagi, A. Kitaori, K. Yamauchi, H. Inoue, J.-I. Yamaura, D. Nishio-Hamane, S. Itoh, S. Aji, H. Saito, T. Nakajima, T. Nomoto, R. Arita, and S. Seki, Spontaneous Hall effect induced by collinear antiferromagnetic order at room temperature, *Nat. Mater.* **24**, 63 (2025).
 - [20] S. Sheoran and P. Dev, Spontaneous anomalous Hall effect in two-dimensional altermagnets, *Phys. Rev. B* **111**, 184407 (2025).
 - [21] A. B. Hellenes, T. Jungwirth, and J. Sinova, and Libor Šmejkal, Unconventional p-wave magnets, *arXiv:2309.01607*.
 - [22] M. Ezawa, Almost half-quantized planar Hall effects in X -wave magnets with $X = p, d, f, g, i$, *Phys. Rev. B* **112**, 235307 (2025).
 - [23] F. Liu, Z. Zhang, X. Yuan, Y. Liu, S. Zhu, Z. Lu, and R. Xiong, Giant tunneling magnetoresistance in insulated

- altermagnet/ferromagnet junctions induced by spin-dependent tunneling effect, *Phys. Rev. B* **110**, 134437 (2024).
- [24] B. Chi, L. Jiang, Y. Zhu, G. Yu, C. Wan, J. Zhang, and X. Han, Crystal-facet-oriented altermagnets for detecting ferromagnetic and antiferromagnetic states by giant tunneling magnetoresistance, *Phys. Rev. Appl.* **21**, 034038 (2024).
- [25] R.-W. Zhang, C. Cui, R. Li, J. Duan, L. Li, Z.-M. Yu, and Y. Yao, Predictable gate-field control of spin in altermagnets with spin-layer coupling, *Phys. Rev. Lett.* **133**, 056401 (2024).
- [26] Z. Yan, R. Yang, C. Fang, W. Lu, and X. Xu, Giant electrode effect on tunneling magnetoresistance and electroresistance in van der Waals intrinsic multiferroic tunnel junctions using VS₂, *Phys. Rev. B* **109**, 205409 (2024).
- [27] Z. Yang, X. Yang, J. Wang, R. Peng, C. H. Lee, L. K. Ang, J. Lu, Y. S. Ang, and S. Fang, Unconventional tunnel magnetoresistance scaling with altermagnets, [arXiv:2505.17192](https://arxiv.org/abs/2505.17192).
- [28] Y.-F. Sun, Y. Mao, Y.-C. Zhuang, and Q.-F. Sun, Tunneling magnetoresistance effect in altermagnets, *Phys. Rev. B* **112**, 094411 (2025).
- [29] B. Brekke, P. Sukhachov, H. G. Giil, A. Brataas, and J. Linder, Minimal models and transport properties of unconventional *p*-wave magnets, *Phys. Rev. Lett.* **133**, 236703 (2024).
- [30] M. Ezawa, Third-order and fifth-order nonlinear spin-current generation in *g*-wave and *i*-wave altermagnets and perfectly nonreciprocal spin current in *f*-wave magnets, *Phys. Rev. B* **111**, 125420 (2025).
- [31] M. Ezawa, Purely electrical detection of the spin-splitting vector in *p*-wave magnets based on linear and nonlinear conductivities, *Phys. Rev. B* **112**, 125412 (2025).
- [32] S. Okumura, T. Morimoto, Y. Kato, and Y. Motome, Quadratic optical responses in a chiral magnet, *Phys. Rev. B* **104**, L180407 (2021).
- [33] M. Ezawa, Topological insulators based on *p*-wave altermagnets: Electrical control and detection of the altermagnetic domain wall, *Phys. Rev. B* **110**, 165429 (2024).
- [34] T. Jungwirth, R. M. Fernandes, E. Fradkin, A. H. MacDonald, J. Sinova, and L. Šmejkal, Altermagnetism: An unconventional spin-ordered phase of matter, [arXiv:2411.00717](https://arxiv.org/abs/2411.00717).
- [35] M. Ezawa, Out-of-plane Edelstein effects: Electric-field induced magnetization in *p*-wave magnets, *Phys. Rev. B* **111**, L161301 (2025).
- [36] M. Ezawa, Quantum geometry and elliptical optical dichroism in *p*-wave magnets, *Phys. Rev. B* **112**, 045302 (2025).
- [37] L. Šmejkal, A. H. MacDonald, J. Sinova, S. Nakatsuji, and T. Jungwirth, Anomalous Hall antiferromagnets, *Nat. Rev. Mater.* **7**, 482 (2022).
- [38] D. Zhu, Z.-Y. Zhuang, Z. Wu, and Z. Yan, Topological superconductivity in two-dimensional altermagnetic metals, *Phys. Rev. B* **108**, 184505 (2023).
- [39] S. Ali, A. Ghorashi, T. L. Hughes, and J. Cano, Altermagnetic routes to Majorana modes in zero net magnetization, *Phys. Rev. Lett.* **133**, 106601 (2024).
- [40] Y.-X. Li and C.-C. Liu, Majorana corner modes and tunable patterns in an altermagnet heterostructure, *Phys. Rev. B* **108**, 205410 (2023).
- [41] M. Ezawa, Detecting the Neel vector of altermagnets in heterostructures with a topological insulator and a crystalline valley-edge insulator, *Phys. Rev. B* **109**, 245306 (2024).
- [42] M. Ezawa, Intrinsic nonlinear conductivity induced by quantum geometry in altermagnets and measurement of the in-plane Neel vector, *Phys. Rev. B* **110**, L241405 (2024).
- [43] M. Ezawa, Bulk photovoltaic effects in altermagnets, *Phys. Rev. B* **111**, L201405 (2025).
- [44] J. R. Schrieffer, D. J. Scalapino, and J. W. Wilkins, Effective tunneling density of states in superconductors, *Phys. Rev. Lett.* **10**, 336 (1963).
- [45] E. Y. Tsymbal, O. N. Mryasov, and P. R. LeClair, Spin dependent tunneling in magnetic tunnel junctions, *J. Phys.: Condens. Matter* **15**, R109 (2003).
- [46] R. Yamada, M. T. Birch, P. R. Baral, S. Okumura, R. Nakano, S. Gao, M. Ezawa, T. Nomoto, J. Masell, Y. Ishihara, K. K. Kolincio, I. Belopolski, H. Sagayama, H. Nakao, K. Ohishi, T. Ohhara, R. Kiyonagi, T. Nakajima, Y. Tokura, T.-h. Arima, *et al.*, Metallic *p*-wave magnet with commensurate spin helix, *Nature (London)* **646**, 837 (2025).
- [47] S. Hayami, Y. Yanagi, and H. Kusunose, Bottom-up design of spin-split and reshaped electronic band structures in antiferromagnets without spin-orbit coupling: Procedure on the basis of augmented multipoles, *Phys. Rev. B* **102**, 144441 (2020).
- [48] H. Zhou, M. Wang, X. Ma, G. Li, D.-F. Shao, B. Liu, and S. Li, Anisotropic resistivity of a *p*-wave magnet candidate CeNiAsO, [arXiv:2509.07351](https://arxiv.org/abs/2509.07351).
- [49] S. K. Das and B. Roy, From local spin nematicity to altermagnets: Footprints of band topology, *Phys. Rev. B* **111**, L201102 (2025).
- [50] Q. Song, S. Stavríć, P. Barone, A. Droghetti, D. S. Antonenko, J. W. F. Venderbos, C. A. Occhialini, B. Ilyas, E. Ergecen, N. Gedik, S.-W. Cheong, R. M. Fernandes, S. Picozzi, and R. Comin, Electrical switching of a *p*-wave magnet, *Nature (London)* **642**, 64 (2025).
- [51] K.-H. Ahn, A. Hariki, K.-W. Lee, and J. Kunes, Antiferromagnetism in RuO₂ as *d*-wave Pomeranchuk instability, *Phys. Rev. B* **99**, 184432 (2019).
- [52] L. Šmejkal, R. Gonzalez-Hernandez, T. Jungwirth, and J. Sinova, Crystal time-reversal symmetry breaking and spontaneous Hall effect in collinear antiferromagnets, *Sci. Adv.* **6**, eaaz8809 (2020).
- [53] O. Fedchenko, J. Minar, A. Akashdeep, S. W. D' Souza, D. Vasilyev, O. Tkach, L. Odenbreit, Q. L. Nguyen, D. Kutnyakhov, N. Wind, L. Wenthous, M. Scholz, K. Rosnagel, M. Hoesch, M. Aeschlimann, B. Stadtmueller, M. Klauui, G. Schoenhense, G. Jakob, T. Jungwirth, *et al.*, Observation of time-reversal symmetry breaking in the band structure of altermagnetic RuO₂, *Sci. Adv.* **10**, eadj4883 (2024).
- [54] Z. Lin, D. Chen, W. Lu, X. Liang, S. Feng, K. Yamagami, J. Osiecki, M. Leandersson, B. Thiagarajan, J. Liu, C. Felser, and J. Ma, Observation of giant spin splitting and *d*-wave spin texture in room temperature altermagnet RuO₂, [arXiv:2402.04995](https://arxiv.org/abs/2402.04995).
- [55] H. Reichlova, R. L. Seeger, R. Gonzalez-Hernandez, I. Kounta, R. Schlitz, D. Kriegner, P. Ritzinger, M. Lammel, M. Leiviskä, A. B. Hellenes, K. Olejnik, V. Petříček, P. Doležal, L. Horak, E. Schmoranzero, A. Badura, S. Bertaina, A. Thomas, V. Baltz, L. Michez, J. Sinova, *et al.*, Observation of a spontaneous anomalous Hall response in the Mn₅Si₃ *d*-wave altermagnet candidate, *Nat. Commun.* **15**, 4961 (2024).

- [56] I. I. Mazin, K. Koepernik, M. D. Johannes, R. Gonzalez-Hernandez, and L. Šmejkal, Prediction of unconventional magnetism in doped FeSb₂, [Proc. Natl. Acad. Sci. USA](#) **118**, e2108924118 (2021).
- [57] M. Naka, S. Hayami, H. Kusunose, Y. Yanagi, Y. Motome, and H. Seo, Spin current generation in organic antiferromagnets, [Nat. Commun.](#) **10**, 4305 (2019).
- [58] M. Naka, Y. Motome, and H. Seo, Perovskite as a spin current generator, [Phys. Rev. B](#) **103**, 125114 (2021).
- [59] M. Naka, Y. Motome, and H. Seo, Altermagnetic perovskites, [npj Spintron.](#) **3**, 1 (2025).
- [60] Y. Liu, J. Yu, and C.-C. Liu, Twisted magnetic Van der Waals bilayers: An ideal platform for altermagnetism, [Phys. Rev. Lett.](#) **133**, 206702 (2024).