

Spin-selective elliptic optical dichroism and perfectly spin-polarized third-order nonlinear photocurrent in altermagnets

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(Received 8 March 2026; revised 15 June 2026; accepted 21 July 2026; published 13 August 2026)

We show that the low-energy theory of a d -wave altermagnet is characterized by anisotropic Dirac cones with spin-split band structures based on a recently proposed tight-binding model. In this system, spin-selective perfect elliptic dichroism emerges, enabling exclusive excitation of either up-spin or down-spin electrons by tuning the ellipticity of the incident light. We further derive a formula for the third-order photocurrent generated under simultaneous application of elliptically polarized light and a static electric field, expressed in terms of the quantum metric and Berry curvature. Using this formula, we demonstrate that only an up-spin polarized current is induced. This third-order response constitutes the leading photocurrent, as second-order processes such as injection and shift currents are forbidden by the inversion symmetry inherent to altermagnets. They lead to the prediction that, in insulating d -wave altermagnets, a spin-polarized third-order photocurrent arises as the leading electric response when elliptically polarized light and a static electric field are applied simultaneously. These findings provide a foundation for future developments in photoinduced spintronics based on altermagnets.

DOI: [10.1103/5rrq-5g1k](https://doi.org/10.1103/5rrq-5g1k)

Introduction. Altermagnets [1,2] are one of the hottest topics in condensed matter physics because they will be useful for future spintronics applications. They are collinear antiferromagnets but break time reversal symmetry, which allows nonzero anomalous Hall effect. Especially, d -wave altermagnets are prominent, where spin currents are induced by electric field [3–8] and temperature gradient [3,8]. d -wave altermagnets have a combinatorial symmetry $C_{4z}\mathcal{T}$ made of time-reversal symmetry and the fourfold rotational symmetry [1,2]. This symmetry enforces a spin-split band structure, where spins are opposite by the 90-degree momentum rotation. In addition, altermagnets have inversion symmetry. There is a simple four-band tight-binding model possessing all these symmetry properties of d -wave altermagnets and confirmed by the density-functional theory [9].

Circular dichroism is an intriguing phenomenon, which was originally proposed in honeycomb systems [10–13]. The tight-binding model based on the honeycomb lattice possesses two Dirac cones assigned by different valley degrees of freedom. By irradiating circularly polarized light to the system, only electrons at one valley are excited. Furthermore, circular dichroism is generalized to elliptic dichroism for anisotropic Dirac cones [8,14,15], where only one valley is excited by irradiating elliptically polarized light. Elliptic dichroism is formulated in terms of quantum metric and the Berry curvature [8,15].

The leading photoinduced current is the second order in electric field, i.e., $j \propto E^2$, where there are injection current

[16–25] and the shift current [16–20,23–33]. However, they are prohibited if the system has inversion symmetry because current and electric field are odd under inversion symmetry. Hence, there is no second-order photocurrent in altermagnets. On the other hand, the third-order photocurrent $j \propto E^3$ is allowed even in the presence of inversion symmetry. A typical example is the jerk current [34–38], which has been discussed so far only for linearly polarized light.

In this paper, we derive a low-energy theory of altermagnets characterized by anisotropic Dirac cones. Based on this theory, we demonstrate that elliptic dichroism arises in altermagnets, whereby electrons with either up or down spin can be selectively excited by appropriately tuning the ellipticity of the incident light. Furthermore, we derive a formula for higher-order photocurrents under elliptically polarized light, expressed in terms of the quantum metric and Berry curvature, analogous to the formula for elliptic optical dichroism. We then show that a perfectly spin-polarized third-order photocurrent is generated when elliptically polarized light and a static electric field are applied simultaneously. If the system is an insulator, the third-order photocurrent is the leading-order current. Our findings provide a foundation for future developments in photoexcited spintronics based on altermagnets.

Model. We start with the four-band tight-binding model for d -wave altermagnets defined on the square lattice [9,39,40],

$$\begin{aligned}
 H(k_x, k_y) = & A(\cos ak_x + \cos ak_y)\tau_0\sigma_0 \\
 & + B(\cos ak_x - \cos ak_y)\tau_z\sigma_0 \\
 & + t \cos \frac{ak_x}{2} \cos \frac{ak_y}{2} \tau_x\sigma_0 + \lambda \sin \frac{ak_x}{2} \sin \frac{ak_y}{2} \tau_y\sigma_z \\
 & + C(\cos ak_x - \cos ak_y)\tau_0\sigma_z \\
 & + (u + D(\cos ak_x + \cos ak_y))\tau_z\sigma_z,
 \end{aligned} \tag{1}$$

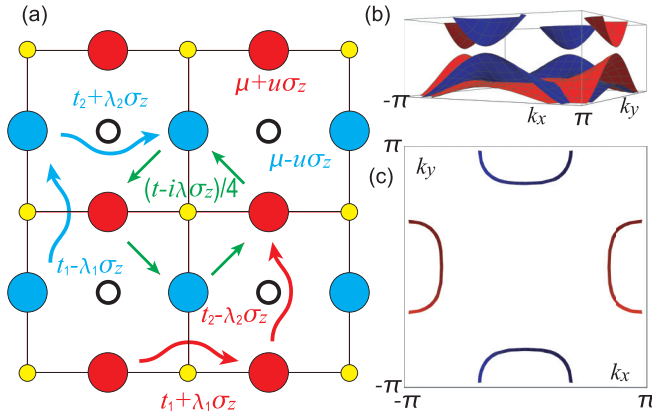


FIG. 1. (a) Illustration of the Lieb lattice model. Red (cyan) disks represent sites with up (down) spin. Small yellow disks represent nonmagnetic sites. The tight-binding model is constructed only with the use of red and blue disks, where the hopping terms are modified in the presence of yellow disks. Red (cyan) arrows represent hoppings between the next-nearest neighbor up(down)-spin sites. Green arrows represent hoppings between the nearest-neighbor sites between opposite spins. We have set $A = t_1 + t_2$, $B = t_1 - t_2$, $C = \lambda_1 + \lambda_2$, and $D = \lambda_1 - \lambda_2$ in the Hamiltonian, Eq. (1). (b) Bird's eye's view of the band structure containing two anisotropic Dirac cones. (c) Top view of the band structure. Red (blue) color represents the up-spin (down-spin) band structure.

where σ_j is the Pauli matrix for the spin, and τ_j is that for the sublattice degrees of freedom with $j = x, y, z$. a is the lattice constant, and u is a real number proportional to the spin order. A and B are the isotropic and anisotropic hoppings of the next-nearest neighbor sites, while C and D are the isotropic and anisotropic spin-dependent hoppings of the next-nearest neighbor sites. t is the hopping and λ is the spin-dependent hopping of nearest-neighbor sites. We assume $t > 0$ and $\lambda > 0$. The model was derived from the Lieb lattice as shown in Fig. 1(a). The Hamiltonian has such a symmetry [9] that $C_{4z}\mathcal{T} = \frac{i}{\sqrt{2}}(\sigma_y - \sigma_x)\tau_x\mathcal{K}$ with

$$C_{4z}\mathcal{T}H(k_x, k_y)C_{4z}\mathcal{T} = H(-k_y, k_x), \quad (2)$$

which guarantees the existence of opposite spin-split band structure by 90° rotation of the momentum, $C_{4z}\mathcal{T}(k_x, k_y)C_{4z}\mathcal{T} = (-k_y, k_x)$.

Because σ_z is a good quantum number, we may set $\sigma_z = s$ with $s = \pm 1$ in the Hamiltonian (1). Then, the Hamiltonian is reduced to a direct sum of 2 two-band models, $H = H_\uparrow \oplus H_\downarrow$. The band structure is shown in Figs. 1(b) and 1(c). With respect to the band with up spin $s = 1$, there is a Dirac cone at the X point, where the Hamiltonian is expanded as

$$H_\uparrow^X = -4C + tak'_x\tau_x + \lambda ak_y\tau_y - (4B - 2u)\tau_z \quad (3)$$

with $k'_x \equiv k_x - \pi/a$. It represents an anisotropic Dirac cone. Similarly, with respect to the band with down spin $s = -1$, there is a Dirac cone at the Y point, where the Hamiltonian is expanded as

$$H_\downarrow^Y = -4C + tak'_y\tau_x - \lambda ak_x\tau_y + (4B - 2u)\tau_z \quad (4)$$

with $k'_y \equiv k_y - \pi/a$. The two valleys are connected by the $C_{4z}\mathcal{T}$ symmetry, where spins are opposite between two valleys.

The Hamiltonians $H_\uparrow^X$ and $H_\downarrow^Y$ are rewritten in the form of $H(\mathbf{k}) = h_0(\mathbf{k}) + \mathbf{h}(\mathbf{k}) \cdot \boldsymbol{\tau}$. The energy spectrum is obtained as

$$\varepsilon_\pm(\mathbf{k}) = h_0(\mathbf{k}) \pm \sqrt{|\mathbf{h}(\mathbf{k})|}. \quad (5)$$

We define the normalized vector $\mathbf{n}(\mathbf{k})$ as $\mathbf{n}(\mathbf{k}) \equiv \mathbf{h}(\mathbf{k})/|\mathbf{h}(\mathbf{k})| = (\sin\theta\cos\phi, \sin\theta\sin\phi, \cos\theta)$. The quantum metric for the two-band system is explicitly given [41–48] by $g_{\pm}^{\mu\nu}(\mathbf{k}) = \pm \frac{1}{4}(\partial_{k_\mu}\mathbf{n}) \cdot (\partial_{k_\nu}\mathbf{n})$ with the normalized vector $\mathbf{n}(\mathbf{k})$.

The Berry curvature for the two-band system is explicitly given [41,44,49–51] by $\Omega_{\pm}^{xy}(\mathbf{k}) = \mp \frac{1}{2}\mathbf{n} \cdot (\partial_{k_x}\mathbf{n} \times \partial_{k_y}\mathbf{n})$. It is a solid angle spanned by the vector $\mathbf{n}$. The quantum metrics $g_{xx}^X(\mathbf{k})$, $g_{yy}^X(\mathbf{k})$ and the Berry curvature $\Omega_{xy}^X(\mathbf{k})$ at the X point are given by

$$g_{xx}^X(k'_x, k_y) = \frac{a^2 t^2 ((\lambda a k_y)^2 + \Delta^2)}{4((tak'_x)^2 + (\lambda a k_y)^2 + \Delta^2)^2}, \quad (6)$$

$$g_{yy}^X(k'_x, k_y) = \frac{a^2 \lambda^2 ((tak'_x)^2 + \Delta^2)}{4((tak'_x)^2 + (\lambda a k_y)^2 + \Delta^2)^2}, \quad (7)$$

$$\Omega_{xy}^X(k'_x, k_y) = \frac{a^2 \Delta t \lambda}{2((tak'_x)^2 + (\lambda a k_y)^2 + \Delta^2)^{3/2}} \quad (8)$$

with the Dirac mass $\Delta = 4B - 2u$. Similarly, we have

$$g_{xx}^Y(k_x, k'_y) = g_{xx}^X(k'_x, k_y), \quad (9)$$

$$g_{yy}^Y(k_x, k'_y) = g_{yy}^X(k'_x, k_y), \quad (10)$$

$$\Omega_{xy}^Y(k_x, k'_y) = \Omega_{xy}^X(k'_x, k_y) \quad (11)$$

at the Y point. The integration of the Berry curvature gives the half-quantized Chern number for a single Dirac cone,

$$\frac{1}{2\pi} \int \Omega_{xy}^X(\mathbf{k}) d\mathbf{k} = \frac{1}{2\pi} \int \Omega_{xy}^Y(\mathbf{k}) d\mathbf{k} = \frac{\Delta}{2|\Delta|}. \quad (12)$$

By summing up two Chern numbers from the two Dirac cones, we obtain a quantized Chern number,

$$\frac{1}{2\pi} \int (\Omega_{xy}^X(\mathbf{k}) + \Omega_{xy}^Y(\mathbf{k})) d\mathbf{k} = \frac{\Delta}{|\Delta|}. \quad (13)$$

It shows that the system is a Chern insulator, where quantum anomalous Hall effects occur.

Elliptic optical dichroism. We next analyze optical absorption. We apply elliptically polarized light with the electric field

$$\mathbf{E}(\omega) = E(\omega)(\mathbf{e}_x \cos \vartheta + i\mathbf{e}_y \sin \vartheta), \quad (14)$$

where ϑ is the ellipticity of injected light $0 < \vartheta < \pi$ for the right polarization, and $-\pi < \vartheta < 0$ for the left polarization. $\vartheta = \pi/4$ corresponds to the right circularly polarized right, while $\vartheta = -\pi/4$ corresponds to the left circularly polarized right. The optical conductivity is calculated as [8,15]

$$\sigma(\omega; \vartheta) = \hbar \sigma_H \int d\mathbf{k} f(\mathbf{k}) g(\mathbf{k}; \vartheta) \delta[\varepsilon_+(\mathbf{k}) - \varepsilon_-(\mathbf{k}) - \hbar\omega] \quad (15)$$

with the Hall conductivity unit $\sigma_H \equiv e^2/\hbar$, and

$$g(\mathbf{k}; \vartheta) \equiv g_{xx}(\mathbf{k}) \cos^2 \vartheta + g_{yy}(\mathbf{k}) \sin^2 \vartheta + \Omega_{xy}(\mathbf{k}) \sin \vartheta \cos \vartheta, \quad (16)$$

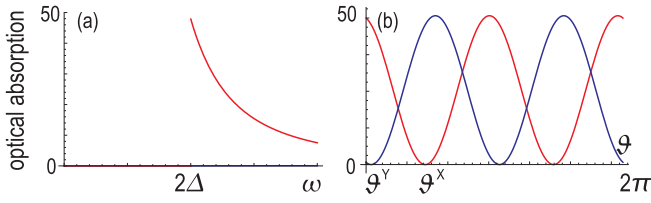


FIG. 2. (a) Optical absorption as a function of ω in units of $\hbar\sigma_0|\Delta|/(a^2t^2)$. Red (blue) curve corresponds to the spin up (down) polarization. We have set $\vartheta = \vartheta^Y$, where only electrons with up spin is excited. (b) Optical absorption as a function of ϑ at the optical band edge $\hbar\omega = 2|\Delta|$. We have set $t = 4$; $\lambda = 0.5$; $B = -1$ and $u = -2.2$.

where $g_{xx}(\mathbf{k})$ and $g_{yy}(\mathbf{k})$ are quantum metrics, $\Omega_{xy}(\mathbf{k})$ is the Berry curvature, while $f(\mathbf{r}, \mathbf{k}) = 1/(\exp((\varepsilon(\mathbf{k}) - \mu)/(k_B T)) + 1)$ is the Fermi distribution function. The detailed derivation is shown in the Supplemental Material [52]. We denote Eq. (16) by $g^X(\mathbf{k}; \vartheta)$ at the X point and $g^Y(\mathbf{k}; \vartheta)$ at the Y point. It is observed from Eq. (15) that the minimum of the excitation frequency ω is given by the optical band edge $\hbar\omega = 2|\Delta|$, where excitation occurs at the X or Y point: See Fig. 2(a). There is no optical absorption below the optical band edge. In the following, we consider the zero-temperature limit and we take the chemical potential within the Dirac gap, $\mu = -4C$, where the system is an insulator.

We calculate Eq. (15) based on the Dirac model around the X and Y points. Namely, we set $\mathbf{k} = (k'_x, k_y)$ at the X point and $\mathbf{k} = (k_x, k'_y)$ at the Y point. After integrating the momentum integration, we obtain

$$\sigma(\omega; \vartheta) = \hbar\sigma_H \int_0^{2\pi} \frac{k_F g(k_F)}{|\partial_k \varepsilon(k)|_{k_F}} d\phi = \frac{\hbar^2 \omega \sigma_H}{2a^2 t^2} \int_0^{2\pi} g(k_F) d\phi, \quad (17)$$

where $ak_F = \sqrt{(\hbar\omega/2)^2 - \Delta^2}/t$ is the solution of $\hbar\omega = \varepsilon_+(k_F) - \varepsilon_-(k_F)$. We have set $k_x = k \cos \phi$, $k_y = \frac{t}{\lambda} k \sin \phi$ at the X point and $k_x = \frac{t}{\lambda} k \cos \phi$, $k_y = k \sin \phi$ at the Y point. We have also used $\partial_k \varepsilon(k) = t^2 a^2 k / \sqrt{t^2 a^2 k^2 + \Delta^2}$. The optical conductivity at the optical band edge $\hbar\omega = 2|\Delta|$ is given by

$$\sigma\left(\omega = \frac{2|\Delta|}{\hbar}; \vartheta\right) = \frac{\hbar\sigma_H |\Delta|}{a^2 t^2} g(\mathbf{0}; \vartheta), \quad (18)$$

where we have used $k_F = 0$ at the optical band edge.

The condition that no electrons are excited at the Y point is given by

$$g^Y(\mathbf{0}; \vartheta^Y) = 0. \quad (19)$$

By solving this equation, the ellipticity of the light ϑ^Y is explicitly determined as

$$\vartheta^Y = \eta \arccos \frac{t}{\sqrt{\lambda^2 + t^2}}, \quad (20)$$

with $\eta = \pm 1$, where $\eta = 1$ for $\lambda\Delta > 0$ and $\eta = -1$ for $\lambda\Delta < 0$. Then, it follows from Eqs. (16) and (18) that the optical conductivity at the optical band edge $\hbar\omega = 2|\Delta|$ is given by

$$\frac{\sigma^X(\omega = \frac{2|\Delta|}{\hbar}; \vartheta^Y)}{\frac{\hbar\sigma_H |\Delta|}{a^2 t^2}} = \pi \frac{(\lambda^2 + \eta \frac{\lambda\Delta}{|\Delta|} t^2)^2}{2t^2 |\Delta| (\lambda^2 + t^2)}, \quad (21)$$

implying that electrons are excited at the X point. Hence, elliptic optical dichroism occurs.

In general, the ω dependence of the optical conductivity is obtained as

$$\frac{\sigma^Y(\omega; \vartheta^Y)}{\frac{\hbar\sigma_H |\Delta|}{a^2 t^2}} = \frac{\pi \lambda^2 (\hbar\omega - \eta \frac{\lambda}{|\Delta|} 2\Delta)^2}{(\hbar\omega)^3 (\lambda^2 + t^2)} \quad (22)$$

around the Y point, and

$$\frac{\sigma^X(\omega; \vartheta^Y)}{\frac{\hbar\sigma_H |\Delta|}{a^2 t^2}} = \frac{\pi}{2\lambda t (\hbar\omega)^3 (\lambda^2 + t^2)} (8\eta \Delta \hbar\omega |\lambda| \lambda t^2 + (\hbar^2 \omega^2 + 4\Delta^2)(\lambda^4 + t^4)) \quad (23)$$

around the X point. The optical conductivity is shown as a function of ω in Fig. 2(a). The optical conductivity takes the maximum value (21) at the optical band edge $\hbar\omega = 2|\Delta|$ and monotonically decreases $\sigma^X(\omega; \vartheta^Y) \propto 1/\omega$ as the increase of ω .

Similarly, no electrons are excited at the X point if

$$g^X(\mathbf{0}; \vartheta^X) = 0 \quad (24)$$

with

$$\vartheta^X = \eta \arccos \frac{\lambda}{\sqrt{\lambda^2 + t^2}}, \quad (25)$$

where the optical conductivity at the optical band edge is given by

$$\sigma\left(\omega = \frac{2|\Delta|}{\hbar}; \vartheta^X\right) = \sigma_0 g^Y(\mathbf{0}; \vartheta^X) \quad (26)$$

with

$$g^Y(\mathbf{0}; \vartheta^X) = \pi \frac{(\lambda^2 + \eta \frac{\lambda\Delta}{|\Delta|} t^2)^2}{2t^2 |\Delta| (\lambda^2 + t^2)}. \quad (27)$$

The optical conductivity is shown as a function of ϑ at the optical band edge in Fig. 2(b).

It is possible to make $g^Y(\mathbf{0}; \vartheta^Y) = 0$ or $g^X(\mathbf{0}; \vartheta^X) = 0$ by tuning the ellipticity of irradiated light ϑ , where only electrons at the X point or the Y point are selectively excited. It is elliptic optical dichroism.

Jerk current under elliptically polarized light. We now show that these excited electrons are extracted by applying electric field additionally. The second-order photocurrents such as the injection current [16–25] and the shift current [16–20, 23–33] are zero due to inversion symmetry. On the other hand, the third-order photocurrent [34–38] is not prohibited by the presence of inversion symmetry. The higher-order injection current was derived for the linearly polarized light [38]. The injection current $J_{\text{inject}}^{x,x2}$ originates in the velocity difference $v_+^x - v_-^x$, while $J_{\text{jerk}}^{x,x3}$ originates in the contribution $\frac{dv_+^x}{dt} - \frac{dv_-^x}{dt}$. In the similar way, the ℓ -th order photocurrent originates in

$$\frac{d^{\ell-2} v_+^x}{dt^{\ell-2}} - \frac{d^{\ell-2} v_-^x}{dt^{\ell-2}} = \frac{\partial^{\ell-1} \omega_{+-}}{\partial k_x^{\ell-1}}. \quad (28)$$

We generalize it to elliptically polarized light, where the ℓ -th order photocurrent is given by

$$J^{x;x^\ell}(\omega) = \frac{\ell-1}{(i\omega_0+1/\tau)^{\ell-1}} \frac{2\pi e^{\ell+1}}{\hbar^\ell V} \int d\mathbf{k} g(\mathbf{k}; \vartheta) \delta(\omega_{+-} - \omega) \times \frac{\partial^{\ell-1} \omega_{+-}}{\partial k_x^{\ell-1}} E(\omega) E(-\omega) [E_x(0)]^{\ell-2}, \quad (29)$$

where ω_0 is the frequency of applied electric field and $\hbar\omega_{+-} \equiv (\varepsilon_+ - \varepsilon_-)$ with Eq. (5) is the difference of the energy between the conduction and valence bands. The detailed derivation is shown in the Supplemental Material [52].

We calculate the jerk current with $\ell = 3$ at the optical band edge, which is the leading photocurrent. It is obtained from Eq. (29) as

$$\frac{J^{x;x^3}(\Delta; \vartheta^Y)}{\frac{2}{(i\omega_0+1/\tau)^2} \frac{2\pi e^4}{\hbar^3 V} E(\omega) E(-\omega) E_x(0)} = \frac{\pi t(\lambda^2 - t^2)^2}{2\Delta^2 \lambda(\lambda^2 + t^2)} \quad (30)$$

at the X point. This result is derived as follows. It follows from Eq. (5) that

$$\frac{\partial^2 \omega_{+-}(\mathbf{k})}{\partial k_x^2} = \frac{2a^2 t^2}{\sqrt{(tak'_x)^2 + (\lambda ak_y)^2 + \Delta^2}} - \frac{2t^4 a^4 k_x^2}{((tak'_x)^2 + (\lambda ak_y)^2 + \Delta^2)^{3/2}}, \quad (31)$$

which leads to

$$\frac{\partial^2 \omega_{+-}(\mathbf{0})}{\partial k_x^2} = \frac{2a^2 t^2}{|\Delta|} \quad (32)$$

at the optical band edge. In addition, it follows from Eqs. (6)–(8) that

$$g_{xx}^X(\mathbf{0}) = \frac{a^2 t^2}{4\Delta^2}, \quad g_{yy}^X(\mathbf{0}) = \frac{a^2 \lambda^2}{4\Delta^2}, \quad \Omega_{xy}^X(\mathbf{0}) = \frac{a^2 t \lambda \Delta}{2\Delta^3}. \quad (33)$$

With the use of these and Eq. (16), we obtain

$$g(\mathbf{0}; \vartheta) = \frac{a^2 t^2}{4\Delta^2} \cos^2 \vartheta + \frac{a^2 \lambda^2}{4\Delta^2} \sin^2 \vartheta + \frac{a^2 t \lambda \Delta}{2\Delta^3} \sin \vartheta \cos \vartheta. \quad (34)$$

By inserting Eqs. (32) and (34) into Eq. (29), we obtain Eq. (30). It is interesting that the jerk current is zero for isotropic Dirac cone with $\lambda = t$ at the optical band edge as in Eq. (30).

In general, the ω dependence of the jerk current is explicitly given by

$$\frac{J^{x;x^3}(\omega; \vartheta^Y)}{\frac{2}{(i\omega_0+1/\tau)^2} \frac{2\pi e^4}{\hbar^3 V} E(\omega) E(-\omega) E_x(0)} = \frac{a^2 \pi t}{2(\hbar\omega)^6 (\lambda^2 + t^2)} [(\lambda^4 + 3t^4)((\hbar\omega)^4 + 16\Delta^4) + 8\Delta^2(3\lambda^4 + t^4)(\hbar\omega)^2 + 16\hbar\omega\Delta\lambda^2 t^2((\hbar\omega)^2 + 4\Delta^2)] \quad (35)$$

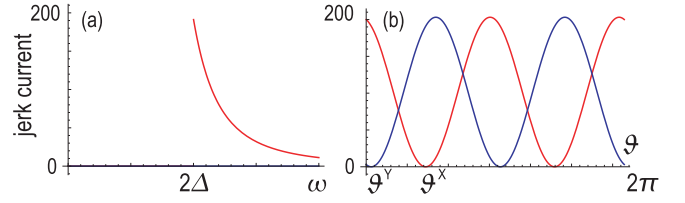


FIG. 3. (a) Jerk current as a function of ω dependence in units of $\frac{2}{(i\omega_0+1/\tau)^2} \frac{2\pi e^4}{\hbar^3 V} E(\omega) E(-\omega) E_x(0)$. Red (blue) curve corresponds to the spin up (down) polarization. We have set $\vartheta = \vartheta^Y$, where only electrons with up spin is excited. (b) Jerk current as a function of ϑ at the optical band edge $\hbar\omega = 2|\Delta|$.

at the X point, and

$$\frac{J^{x;x^3}(\omega; \vartheta^Y)}{\frac{2}{(i\omega_0+1/\tau)^2} \frac{2\pi e^4}{\hbar^3 V} E(\omega) E(-\omega) E_x(0)} = \frac{2\pi (a\lambda t)^2 (\hbar^2 \omega^2 + 4\Delta^2) (\hbar\omega + 2\Delta)^2}{(\hbar\omega)^6 (\lambda^2 + t^2)} \quad (36)$$

at the Y point.

The jerk current is shown as a function of ω in Fig. 3(a), where the jerk current takes the maximum value (30) at the optical band edge $\hbar\omega = 2|\Delta|$ and monotonically decreases $J_{\text{inject}}^{x;x^3}(\omega; \vartheta^Y) \propto 1/\omega^2$ as the increase of ω . The jerk current with up spins is induced by elliptically polarized light with $\vartheta = \vartheta^Y$, while there is no jerk current with down spin at the optical band edge $\hbar\omega = 2|\Delta|$. We show the jerk current as a function of ϑ at the optical band edge in Fig. 3(b). The portion of the spin polarization is controlled by tuning the ellipticity ϑ .

We note that, in the case of the isotropic Dirac cone with $\lambda = t$, the jerk current is identical at the X and Y points, and given by

$$\frac{J^{x;x^3}(\omega; \vartheta^Y)}{\frac{2}{(i\omega_0+1/\tau)^2} \frac{2\pi e^4}{\hbar^3 V} E(\omega) E(-\omega) E_x(0)} = \frac{a^2 \pi t^2 (\hbar^2 \omega^2 + 4\Delta^2) (\hbar\omega + 2\Delta)^2}{(\hbar\omega)^6}. \quad (37)$$

The jerk current is zero at the optical band edge for the isotropic Dirac cones,

$$J^{x;x^3}\left(\omega = \frac{2|\Delta|}{\hbar}; \vartheta^Y\right) = 0. \quad (38)$$

Hence, the jerk current at the optical band edge is present only for anisotropic Dirac cones.

Discussions. We have shown that the low-energy physics of d -wave altermagnets is governed by anisotropic Dirac cones at the X and Y points, where the electronic spins are perfectly polarized in opposite directions. This unique band structure enables complete spin selectivity under elliptically polarized light, giving rise to elliptic optical dichroism. When static electric field is applied simultaneously with elliptically polarized light, a perfectly spin-polarized jerk current is generated. Notably, this jerk current exhibits a pronounced enhancement at the optical band edge, a feature that appears only in systems with anisotropic Dirac cones. We have

demonstrated that insulating d -wave altermagnets host a spin-polarized third-order photocurrent as their leading electric current. These findings establish d -wave altermagnets as a promising platform for photoinduced spintronics.

The perfect spin selectivity identified here originates in the spin-split band structure with fully diagonal spin eigenstates. This stands in sharp contrast to Dirac cones arising from spin-orbit interactions such as those in Rashba systems or topological-insulator surfaces, where the spin texture is not diagonal and perfect selectivity is impossible. Altermagnets, by realizing large spin-split bands without spin-orbit coupling, provide an ideal setting in which opposite spin polarizations at the two valleys are protected by $C_{4z}\mathcal{T}$ symmetry. By the same reasoning, we expect analogous spin-selective optical phenomena in g -wave and i -wave altermagnets owing to $C_{8z}\mathcal{T}$ and $C_{12z}\mathcal{T}$ symmetries, respectively, as well.

Spin-polarized current is experimentally observable by means of the tunneling magneto-resistance (TMR) [53,54]. Especially, perfectly spin-polarized current was observed by using the TMR [55–57]. In addition, the spin-polarized scanning tunneling spectroscopy [58] and point-contact Andreev reflection [59] are used for the detection of spin-polarized current.

Acknowledgments. This work is supported by CREST, JST (Grants No. JPMJCR20T2) and Grants-in-Aid for Scientific Research from MEXT KAKENHI (Grant No. 23H00171).

Data availability. The data that support the findings of this article are not publicly available upon publication because it is not technically feasible and/or the cost of preparing, depositing, and hosting the data would be prohibitive within the terms of this research project. The data are available from the authors upon reasonable request.

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