

Light-induced transitions of valley Chern numbers and flat bands in a nontwisted moiré graphene–hexagonal boron nitride superlattice

Saud Alabdulal ¹, Miftah Hadi Syahputra Anfa ², Hocine Bahlouli,^{2,3} and Michael Vogl ^{2,4}

¹*PSE Division, King Abdullah University of Science and Technology KAUST, Thuwal 23955, Saudi Arabia*

²*Physics Department, King Fahd University of Petroleum & Minerals, Dhahran 31261, Saudi Arabia*

³*Interdisciplinary Research Center (IRC) for Advanced Materials, KFUPM, Dhahran, Saudi Arabia*

⁴*Interdisciplinary Research Center (IRC) for Advanced Quantum Computing (AQC), KFUPM, Dhahran, Saudi Arabia*



(Received 26 August 2025; revised 6 October 2025; accepted 17 October 2025; published 5 November 2025)

Motivated by the rich topology and interesting quasiband structure of twisted moiré materials subjected to light, we study a nontwisted moiré material under the influence of light. Our work is in part motivated by a desire to find an easier-to-synthesize platform that can help experimentally elucidate the interesting physics of moiré materials coupled to light. Similar to twisted moiré materials, we uncover rich topology and interesting band flattening effects, which we summarize in relevant plots such as a topological phase diagram. Our work demonstrates that much of the interesting phenomenology of twisted moiré materials under the influence of electromagnetic waves seems to be generically present even in more experimentally accessible untwisted moiré platforms, which remain highly tunable by light.

DOI: [10.1103/4zcv-2c1n](https://doi.org/10.1103/4zcv-2c1n)

I. INTRODUCTION

Recent advances in high-frequency lasers have paved the way for promising developments in condensed matter physics. This type of laser can be used for probing dynamics of, for example, electrons inside an atom [1,2], ferroelectrics [3–6], excitons in carbon nanotubes [7–9], and solid-state materials [10,11]. It also provides unprecedented control over solid-state systems through time-domain manipulation, such as controlling standing spin wave dynamics [12], magnetism [13,14], and electronic phases [15] in several solids. On the theoretical side, progress in this topic has been motivated by the realization that a system, when driven periodically, remains in a so-called prethermal regime for an exponentially long time rather than being completely disordered instantaneously, even if the system under consideration is interacting [16]. This observation enables us to study such a system employing methods similar to an equilibrium one, via an effective time-independent Hamiltonian [17–40].

Another reason why applying high-frequency lasers to solid-state systems is interesting is that it has been predicted to lead to exotic phenomena. Examples include superconducting states, such as those reported in Ref. [41–44], where the transition appears to be induced by a pulse of light. It has also been reported that light can induce ferroelectricity in

SrTiO₃ [45,46] and an effective magnetic field in antiferromagnetic ErFeO₃ [47].

Additional interesting results have also been obtained in the context of 2D materials. Among the most celebrated are the so-called Floquet topological insulating states. This effect has been predicted theoretically in monolayer graphene when driven with circularly polarized light [35] and subsequently observed experimentally in Ref. [48].

Even more interesting effects appear once one couples a laser to multilayer materials with moiré patterns. One way to construct such a pattern is via a relative twist between layers. In this context, studies have suggested that light can be used to induce flat energy bands in twisted bilayer graphene [36] and tune the topological properties of its electronic band structure [37,38]. Apart from twisted bilayer graphene, the interaction between light and twisted transition metal dichalcogenides (TMDs) has also been studied [39], leading to topological transitions in the energy bands. We note that similar effects also occur in the case of applied external pressure [49], although this work will focus on the effects of external laser fields.

Moiré patterns, however, are not restricted to twisted materials; instead, they also appear often when there are slight mismatches between lattice structures. For instance, they may also appear by stacking two or more layers that have a slight difference in lattice constants. A well-known example of this type is a heterobilayer of graphene and hexagonal boron nitride (G-hBN) [50]. The absence of twist angle as a tuning parameter that has to be matched between experiments can make sample preparation more reliable. As such, this type of moiré structure is expected to be easier to realize in experiments. It could therefore lead to various interesting studies that explore its physics at the interplay with a periodic drive. However, unlike the twisted counterpart, nontwisted moiré

Published by the American Physical Society under the terms of the Creative Commons Attribution 4.0 International license. Further distribution of this work must maintain attribution to the author(s) and the published article's title, journal citation, and DOI. Open access publication funded by King Abdullah University of Science and Technology (KAUST).

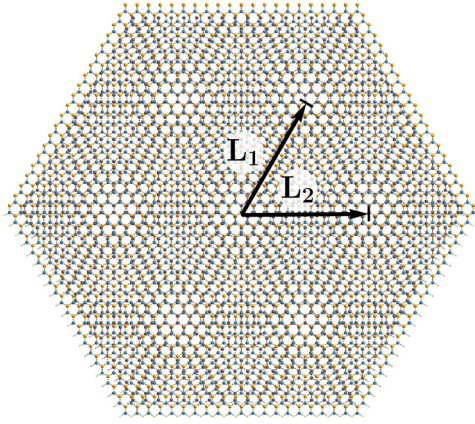


FIG. 1. A cartoon that demonstrates the moiré pattern of graphene (gray)/hBN (yellow and blue) and a moiré superlattice vector.

materials under the influence of light have not been exhaustively studied theoretically in the literature yet. Therefore, this will be the focus of our current work.

The remainder of the paper is structured as follows. In Sec. II, we will review the case of the G-hBN bilayer in equilibrium, i.e., without the influence of light. We will present the model developed in the literature and illustrate some well-known results to ensure our work is self-contained. Then in Sec. III, we prepare the model for the case when the G-hBN bilayer is under the influence of light. We then discuss results from our calculation in Sec. IV. In Sec. V, we conclude our work with a summary and discussion of future directions.

II. BRIEF REVIEW OF THE EQUILIBRIUM MODEL AND ITS RESULTS

Our study focuses on a bilayer moiré material consisting of hexagonal boron nitride (hBN) and graphene. A mismatch between lattice constants of this heterobilayer structure leads to the emergence of a moiré superlattice as depicted in Fig. 1(a). Like any periodic structure, it has an associated reciprocal lattice with the corresponding Brillouin zone depicted in Fig. 2.

Both graphene and hBN have hexagonal symmetry and a two-atom basis. We refer to the sublattices associated with each basis atom as A and B, respectively. In the case of graphene, both basis atoms are identical (carbon), implying a sublattice symmetry, and in the case of hBN, the two atoms differ (one is boron and the other nitrogen) also has a two-atom base hexagonal structure with boron and nitrogen atoms. We will consider the case where both lattices are oriented the same way, such that lattice vectors for both graphene $\mathbf{a}_i^G$ and hBN $\mathbf{a}_i^{\text{hBN}}$ can be defined simultaneously as

$$\mathbf{a}_1^{(\gamma)} = a_\gamma(1, 0), \quad \mathbf{a}_2^{(\gamma)} = a_\gamma(1/2, \sqrt{3}/2), \quad (1)$$

where $\gamma = (G, \text{hBN})$ stands for graphene and hBN, respectively. We stress that lattice constants differ for the two layers and are given as $a_G \approx 0.2460$ nm [51] and $a_{\text{hBN}} \approx 0.2504$ nm [52]. It is important to compute reciprocal lattice vectors of graphene and hBN to be able to find the moiré reciprocal superlattice vector. Using $\mathbf{a}_i^{(\gamma)} \cdot \mathbf{b}_j^{(\gamma)} = 2\pi\delta_{ij}$, the

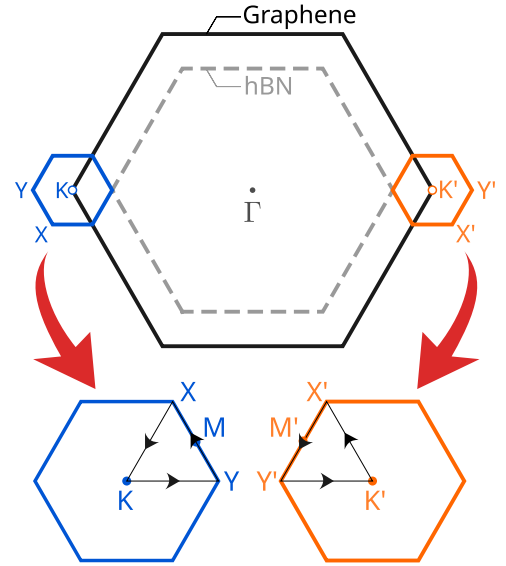


FIG. 2. An illustration of the moiré Brillouin zone (BZ) of both K and K' valley with the path used for the band structure calculation. Note that the ratio between graphene and hBN Brillouin zones has been greatly altered for illustrative purpose.

reciprocal lattice vectors of graphene and hBN are found to be $\mathbf{b}_1^{(\gamma)} = (1, -1/\sqrt{3})2\pi/a_\gamma$ and $\mathbf{b}_2^{(\gamma)} = (0, 2/\sqrt{3})2\pi/a_\gamma$. To find moiré superlattice vectors, we recognize that they can be defined as the shortest vectors connecting points where the graphene and hBN lattices are aligned. The number of times graphene's lattice vector can be repeated between such points is $a_{\text{hBN}}/(a_G - a_{\text{hBN}})$. Since the superlattice orientation in our case is the same as both underlying lattices, the superlattice vectors are then given as

$$\mathbf{L}_i = \frac{a_{\text{hBN}}}{a_G - a_{\text{hBN}}} \mathbf{a}_i. \quad (2)$$

Moiré reciprocal lattice vectors $\mathbf{G}_j$ can then be obtained by using the relation $\mathbf{L}_i \cdot \mathbf{G}_j = 2\pi\delta_{ij}$ and given below:

$$\mathbf{G}_i = \frac{a_G - a_{\text{hBN}}}{a_{\text{hBN}}} \mathbf{b}_i. \quad (3)$$

The Hamiltonian of the system, according to [50] and verified by a tight-binding description, is given below:

$$H_{G-\text{hBN}} = \begin{pmatrix} H_G & U^\dagger \\ U & H_{\text{hBN}} \end{pmatrix}. \quad (4)$$

Here, $H_G(\mathbf{k})$ is the approximate Hamiltonian of a monolayer graphene that is valid near the $\mathbf{K}$ and $\mathbf{K}'$ points and given as

$$H_G(\mathbf{k}) \approx -\hbar v \mathbf{k} \cdot \boldsymbol{\sigma}_\xi, \quad \boldsymbol{\sigma}_\xi = (\xi\sigma_x, \sigma_y), \quad (5)$$

where $\xi = +1$ (-1) indicates the K (K') valley and $v = 0.8 \times 10^6$ m/s is the so-called Fermi velocity of graphene. The second part of the Hamiltonian H_{hBN} is an approximate hBN Hamiltonian

$$H_{\text{hBN}} \approx \begin{pmatrix} V_N & 0 \\ 0 & V_B \end{pmatrix}, \quad (6)$$

where $V_N = -1.40$ eV and $V_B = 3.34$ eV are the potential due to the nitrogen and boron, respectively [53]. Lastly, U

represents the interlayer hopping term between the graphene and hBN layer and is given as

$$U = \begin{pmatrix} U_{A_{\text{hBN}}A_G} & U_{A_{\text{hBN}}B_G} \\ U_{B_{\text{hBN}}A_G} & U_{B_{\text{hBN}}B_G} \end{pmatrix} = u_0 \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} + \begin{pmatrix} 1 & \zeta_3^{-\xi} \\ \zeta_3^{\xi} & 1 \end{pmatrix} e^{i\xi \mathbf{G}_1 \cdot \mathbf{r}} + \begin{pmatrix} 1 & \zeta_3^{\xi} \\ \zeta_3^{-\xi} & 1 \end{pmatrix} e^{i\xi (\mathbf{G}_1 + \mathbf{G}_2) \cdot \mathbf{r}}, \quad (7)$$

where we used $\zeta_3 = \exp(2\pi i/3)$ as a shorthand notation and $u_0 \approx 0.152$ eV according to Ref. [50]. In detail, U_{ij} corresponds to hoppings between specific graphene sites A_G and B_G and hBN sites A_{hBN} (nitrogen), B_{hBN} (boron).

Since the hBN Hamiltonian H_{hBN} has a band gap at small energies, physics here is dominated by the graphene Hamiltonian H_G . Therefore, the Hamiltonian of the system using degenerate perturbation theory can be approximated as

$$H_{G-\text{hBN}} = H_G + V_{\text{hBN}}, \quad (8)$$

where

$$V_{\text{hBN}} = V_0 \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + \left\{ V_1 e^{i\xi \psi} \left[\begin{pmatrix} 1 & \zeta_3^{-\xi} \\ 1 & \zeta_3^{-\xi} \end{pmatrix} e^{i\xi \mathbf{G}_1 \cdot \mathbf{r}} + \begin{pmatrix} 1 & \zeta_3^{\xi} \\ \zeta_3^{\xi} & \zeta_3^{-\xi} \end{pmatrix} e^{i\xi \mathbf{G}_2 \cdot \mathbf{r}} + \begin{pmatrix} 1 & 1 \\ \zeta_3^{-\xi} & \zeta_3^{-\xi} \end{pmatrix} e^{-i\xi (\mathbf{G}_1 + \mathbf{G}_2) \cdot \mathbf{r}} \right] + \text{H.c.} \right\} \quad (9)$$

is a superpotential due to the hBN layer that electrons in the graphene layer experience. The parameters V_0 and V_1 are defined as [50]

$$V_0 = -3u_0^2 \left(\frac{1}{V_N} + \frac{1}{V_B} \right), \quad (10)$$

$$V_1 = -u_0^2 e^{-i\psi} \left(\frac{1}{V_N} + \zeta_3 \frac{1}{V_B} \right). \quad (11)$$

In accordance with Ref. [50], we set $V_0 \approx 0.0289$ eV, $V_1 \approx 0.0210$ eV, and $\psi \approx -0.29$ rad.

As can be seen from the expression of Eqs. (5) and (9), the effective Hamiltonian (8) can be used to study the K and K' valleys separately. Using Eq. (3), one can construct the moiré Brillouin zone (BZ) of this system, centered at the K and K' valleys. The illustration of the moiré BZ and its high-symmetry points are depicted in Fig. 2. We note that the location of K (K') valley is given by $\mathbf{K} = -\xi(2\mathbf{b}_1^{(G)} + \mathbf{b}_2^{(G)})/3$.

In order to calculate the eigenvalues of the Hamiltonian, we need to use Bloch's theorem [54] to write the Hamiltonian in a plane wave basis. We provide the full derivation in Appendix A. A computed band structure is shown in Fig. 3. We find that our result agrees with the results in Ref. [50]. We observe that the bands near the $\mathbf{K}$ and $\mathbf{K}'$ valleys share similar features, as they both exhibit a gapped region and Dirac-cone-like structures. We have included the band structure to ensure our analysis is self-contained and to provide a reference for the subsequent analysis of the Floquet system.

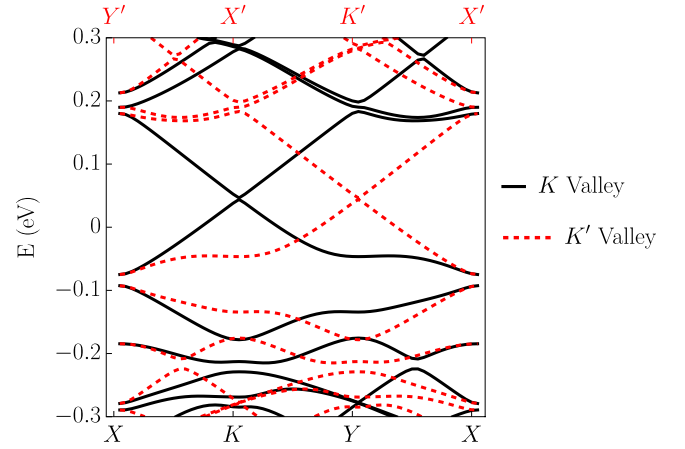


FIG. 3. The band structure for hBN/G moiré system for K , K' approximations in black and dashed red, respectively. We note that this is a reproduction of the results demonstrated in Ref. [50].

III. NON-EQUILIBRIUM MODEL

Now, we want to observe the effect of circularly polarized light on the graphene and hBN heterobilayer (see Fig. 4). We start by considering the general time-dependent Schrödinger equation

$$i\partial_t |\psi\rangle = H(t) |\psi\rangle, \quad (12)$$

for a time-periodic Hamiltonian $H(t) = H(t + T)$, where we may apply Floquet theory (analogous to Bloch theorem [54] but in time), which allows us to write any solutions in the following form:

$$|\psi_n(t)\rangle = e^{-i\varepsilon_n t} |u_n(t)\rangle, \quad (13)$$

$$|u_n(t + T)\rangle = |u_n(t)\rangle. \quad (14)$$

In the expression above, ε_n is called quasienergy and is a solution for the eigenvalue problem $Q(t)|u_n(t)\rangle = \varepsilon_n |u_n(t)\rangle$ where $Q(t) = H(t) - i\partial_t$ is the so-called quasienergy operator. This operator acts not in the original Hilbert space, but in an extended Hilbert space $\mathcal{F} = \mathcal{H} \otimes \mathcal{T}$, also known as Sambe

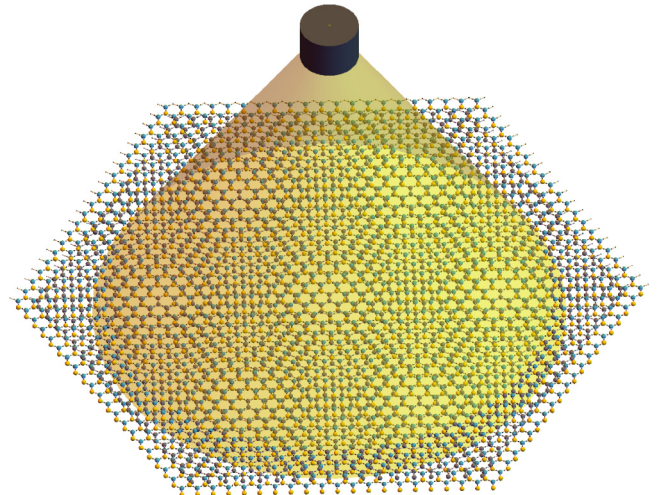


FIG. 4. A cartoon demonstration of irradiating a circularly polarized light on the graphene/hBN system.

space [55]. Here, $\mathcal{H}$ denotes the physical Hilbert space and $\mathcal{T}$ is a space of time-periodic functions with period T . For any kets $|u_n\rangle, |u_m\rangle \in \mathcal{F}$, the inner product between them is defined as

$$\langle u_n | u_m \rangle = \frac{1}{T} \int_0^T \langle u_n(t) | u_m(t) \rangle dt, \quad (15)$$

where $\langle \cdot | \cdot \rangle$ is the inner product of the original Hilbert space. This definition is suitable for this space since it takes into account the temporal variables in addition to the spatial variables.

We choose as our basis the so-called Floquet-Bloch states $|\alpha, m\rangle = |\alpha\rangle \exp(im\omega_0 t)$, where we use the same definition $|\alpha\rangle = \exp(-i\mathbf{G}_\alpha \cdot \mathbf{x})/\sqrt{\Omega}$ as in Sec. II. As one might have noticed, here we expanded the time-periodic functions in Fourier modes $e^{im\omega_0 t}$ where the integer m denotes the so-called Floquet copies. Each of them corresponds to an energy shift by $m\omega_0$. In this representation, the Floquet Hamiltonian acquires a block-structured form. Each block has a size of $d \times d$ where d is the dimension of the original Hilbert space $\mathcal{H}$ and corresponds to one Floquet copy. Formally, the number of blocks can reach infinity since m can take any values up to infinity. However, in practice, one would truncate the number of Floquet copies based on band convergence considerations.

From the preceding description, the matrix elements in our chosen basis are given as

$$\begin{aligned} Q_{\beta\alpha, nm} &= \langle \beta, n | H(t) - i\partial_t | \alpha, m \rangle \\ &= \langle \beta, n | H(t) | \alpha, m \rangle + m\omega_0 \delta_{\mathbf{G}_\beta, \mathbf{G}_\alpha} \delta_{n,m}. \end{aligned} \quad (16)$$

Similar to the case of plane wave expansion of a periodic Hamiltonian using Bloch's theorem, the expression in Eq. (16) is useful since it facilitates a numerical treatment.

To account for the effect of circularly polarized light, we make the following replacement $\mathbf{k} \rightarrow \mathbf{k}(t) = \mathbf{k} + \mathbf{A}(t)$ in Eq. (5) where

$$\mathbf{A}(t) = A(\sin(\omega_0 t)\hat{x} + \cos(\omega_0 t)\hat{y}). \quad (17)$$

Since circularly polarized light consists of transverse elements, the effects on interlayer hoppings are considered negligible for this study, and we therefore treat the replacement above as the only effect caused by light.

Matrix elements for Eq. (8), i.e., the matrix elements $\langle \beta, n | H(t) | \alpha, m \rangle$ can then be found if we decompose $H(t) = H_G(\mathbf{k}, t) + V_{hBN}$ to obtain

$$\langle \beta, n | H_G(\mathbf{k}, t) | \alpha, m \rangle = -\hbar v \left[(\mathbf{k} + \mathbf{G}_\alpha) \cdot \boldsymbol{\sigma}_\xi \delta_{n,m} - \frac{A}{2} (-i\xi \sigma_x (\delta_{n,m-1} - \delta_{n,m+1}) + \sigma_y (\delta_{n,m-1} + \delta_{n,m+1})) \right] \delta_{\mathbf{G}_\beta, \mathbf{G}_\alpha} \quad (18)$$

and

$$\begin{aligned} \langle \beta, n | V_{hBN} | \alpha, m \rangle &= \left\{ V_0 \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \delta_{\mathbf{G}_\alpha, \mathbf{G}_\beta} + \left[V_1 e^{i\xi\psi} \left[\begin{pmatrix} 1 & \zeta_3^{-\xi} \\ 1 & \zeta_3^{-\xi} \end{pmatrix} \delta_{\xi\mathbf{G}_1, \mathbf{G}_\alpha - \mathbf{G}_\beta} + \begin{pmatrix} 1 & \zeta_3^\xi \\ \zeta_3^\xi & \zeta_3^{-\xi} \end{pmatrix} \delta_{\xi\mathbf{G}_2, \mathbf{G}_\alpha - \mathbf{G}_\beta} \right. \right. \right. \\ &\quad \left. \left. + \begin{pmatrix} 1 & 1 \\ \zeta_3^{-\xi} & \zeta_3^{-\xi} \end{pmatrix} \delta_{\xi(\mathbf{G}_1 + \mathbf{G}_2), \mathbf{G}_\alpha - \mathbf{G}_\beta} \right] + \text{H.c.} \right\} \delta_{n,m}. \end{aligned} \quad (19)$$

From here, energy bands can be computed straightforwardly.

However, another primary objective of this study is also to calculate the valley Chern number of the graphene/hexagonal boron nitride (hBN) heterobilayer when subjected to the periodic drive and thereby explore its topological properties. Here, we focus on the high-frequency regime because, unlike for low-frequency drives, the Chern number captures the important physics [56] (eliminating the need for a more complicated winding number analysis). Moreover, at high frequencies, Floquet copies are sufficiently separated, preventing band crossings between different Floquet copies that, in a multiband analysis, can lead to complications—a chaotic mix of bands.

The Chern number of the n th band can be expressed in terms of Berry curvature $\mathbf{B}_n(\mathbf{k})$ as follows [57]:

$$C_n = \frac{1}{2\pi} \oint_{BZ} d\mathbf{k} \cdot \mathbf{B}_n(\mathbf{k}), \quad (20)$$

where

$$\mathbf{B}_n(\mathbf{k}) = \Delta_{\mathbf{k}} \times \mathcal{A}_n(\mathbf{k}), \quad (21)$$

$$\mathcal{A}_n(\mathbf{k}) = i \langle \psi_n(\mathbf{k}) | \Delta_{\mathbf{k}} | \psi_n(\mathbf{k}) \rangle, \quad (22)$$

and $|\psi_n(\mathbf{k})\rangle$ is the normalized eigenstate of the n th band.

Calculating the Chern number using obvious discretization approaches of Eq. (20) can be computationally expensive. To address this, we adopt an efficient method introduced by Fukui *et al.* [58], which discretizes the Brillouin zone and utilizes link variables to compute the Chern number more effectively. For the detailed procedure, one might refer to Appendix B.

With all pieces in place, we are now in the position to present results in the next section.

IV. NON-EQUILIBRIUM RESULTS

We begin with an investigation of the quasi-energy band structure of the G-hBN heterobilayer, which can be obtained by diagonalizing the Floquet Hamiltonian Eq. (16). We restrict our analysis to a relatively high-frequency regime, which allows us to truncate to three Floquet copies, i.e., indices run as $m, n = -1, 0, 1$. To validate the truncation of the Floquet Hamiltonian, we performed a convergence test with respect to the number of Floquet copies included in the basis.

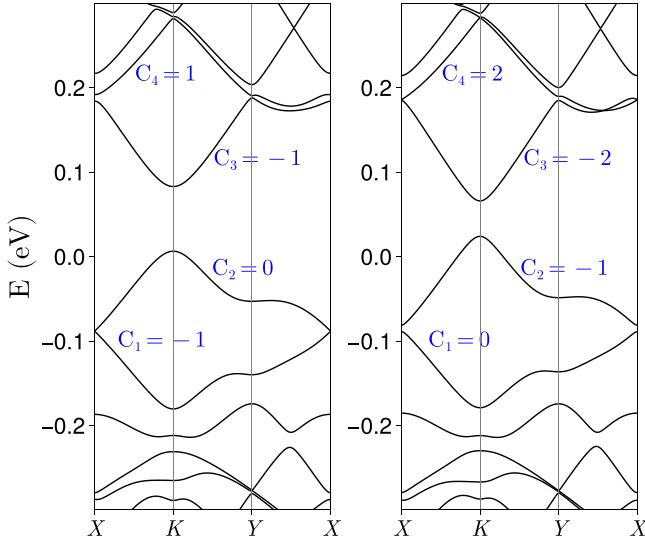


FIG. 5. The K -valley band structure of G-hBN under the influence of circularly polarized light for (left) $\omega_0 = 1.67t$ and (right) $3.15t$ ($t = 2.7$ eV is the nearest-neighbor hopping parameter of a monolayer graphene) at fixed $Aa_0 = 0.2$. The Chern numbers for the low-energy hole (electron) bands are also shown. We stress that, although due to resolution, it may appear that bands 3 and 4 overlap, they do not, which makes the computed Chern numbers well defined.

Specifically, we calculated the quasi-energy band structure using five ($m, n = -2, \dots, 2$) and seven ($m, n = -3, \dots, 3$) copies and found excellent agreement with the three-copy calculation used in our main analysis. The test confirms that truncating to three copies is enough to capture the physics in the high-frequency regime. Figure 5 shows the quasi-energy band structure of the G-hBN (for only the K -valley) system at a fixed driving strength $A = 0.2/a_0$ for driving frequency $\omega_0 = 1.67t$ and $3.15t$, which exhibits a similar behavior to the undriven system in Fig. 3. Here, t is the usual nearest-neighbor-hopping parameter of a monolayer graphene that we set as 2.7 eV following Ref. [50]. We restricted our analysis to the $\mathbf{K}$ valley to avoid overloading the figure; the $\mathbf{K}'$ valley experiences similar physics. Chern numbers are also labeled for four bands (labeled 1 through 4, as seen in the figure), and we observe that they can be modified by adjusting the driving frequencies. We note that although visually it is suggested that bands 3 and 4 are crossing for the case of $\omega_0 = 3.15t$ because of resolution, there is actually a gap of ≈ 0.34 meV between these two bands. Moreover, as shown in the plot, the application of circularly polarized light modifies various band gaps, such as the one between bands 2 and 3.

Changes in frequency ω_0 —even at fixed driving strength Aa_0 —can lead to band gap closings, which permit a change in Chern number. Such a change is interpreted as a topological phase transition. Figure 6 shows the Chern numbers of the four bands as marked in Fig. 5 (we consider the K valley only—results for the K' valley are similar). Here, our computations were done as a function of driving frequency ω_0 at fixed driving strength $A = 0.2/a_0$. The two lower plots show the energy gaps between the two pairs of bands 1(3) and 2(4), which were plotted to ensure we understand which bandgap closings lead to changes in Chern numbers.

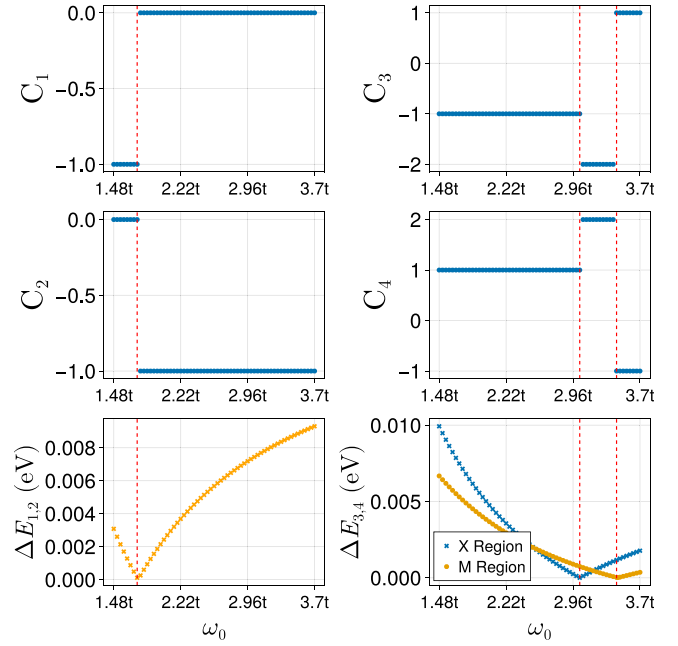


FIG. 6. Chern numbers of four low energy bands for a fixed driving strength $Aa_0 = 0.2$ and as a function of driving frequency ω_0 . The two lower plots show the energy gap between the two pairs of bands.

We find that the Chern numbers of the first and second bands in the lower-frequency range are given as $(C_1, C_2) = (-1, 0)$. The gap between these two bands shrinks as ω_0 increases and closes at $\omega_0 \approx 1.74t$, which is accompanied by a change of Chern numbers to $(C_1, C_2) = (0, -1)$. Both Chern numbers remain unchanged for the remainder of the investigated frequency interval. Chern number transitions also appear for bands 3 and 4. In this case, the Chern numbers start as $(C_3, C_4) = (-1, 1)$. Then the two bands approach

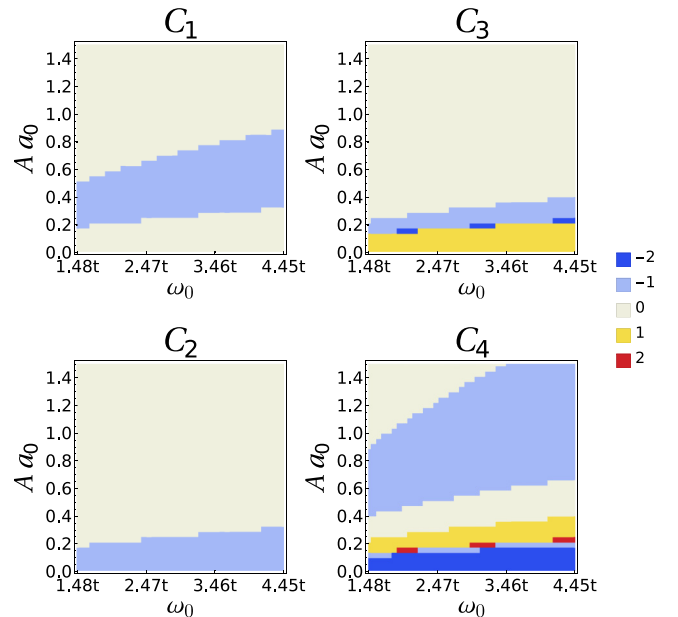


FIG. 7. The phase diagram showing the transitions of the Chern number of band 1 to 4.

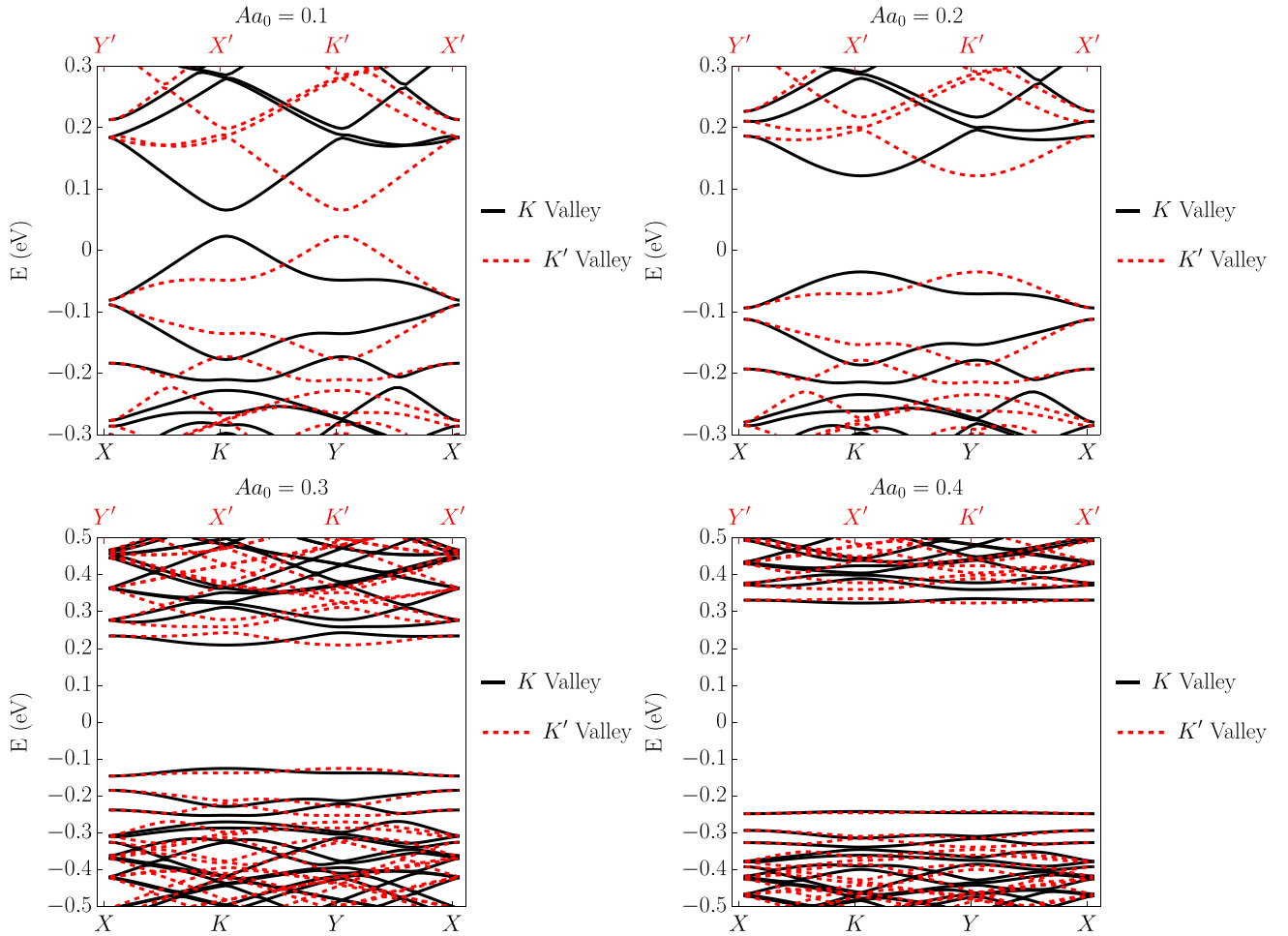


FIG. 8. The quasi-energy band structures of graphene/hBN system under the influence of circularly polarized light with different driving strength and frequency $\omega_0 = 0.78t$.

each other until the bands cross near the X point at driving frequency $\omega_0 \approx 3.07t$. This gap closing is accompanied by a change in Chern number as $(C_3, C_4) = (-2, 2)$. A final band crossing occurs at driving frequency $\omega_0 \approx 3.44t$ near the M point and Chern numbers change to $(C_3, C_4) = (1, -1)$.

To get a clearer picture of topology as a function of driving parameters, we also show a topological phase diagram in Fig. 7. The phase diagram shows how the Chern number of each energy band changes against the various driving strengths and frequencies. From this figure, we can observe a plethora of topological transitions due to the influence of light. The electron bands even obtain a Chern number of ± 2 at certain values of Aa_0 and ω_0 . We also note that band 1 (4) has more valley Chern number transitions than bands 2 (3) since they can also interact with the band below (above).

Lastly, we investigate how changes in driving strengths a_0A for a fixed driving frequency ω_0 impact the quasi-energy bands in Fig. 8.

Our result suggests that increasing the driving strength A (with a fixed ω_0) not only enlarges the gap between the hole and electron bands, but also flattens the bands. This feature is interesting since it indicates that circularly polarized light induces strong correlations between electrons in these bands.

V. CONCLUSIONS

In summary, we have studied a G-hBN heterobilayer under the influence of circularly polarized light. We began by reviewing the equilibrium case and reproducing literature results. To account for the effect of circularly polarized light, we then introduced a vector potential using the minimal substitution method. Subsequently, we employed Floquet theory, which led us to our Floquet-Bloch Hamiltonian for our system Eqs. (16), (18), and (19), which permitted a numerical treatment. We demonstrated that the application of circularly polarized light results in changes to the Chern numbers of four low-energy bands, yielding a topological phase diagram. As is widely known, a nonzero Chern number indicates the existence of an anomalous Hall effect that is characterized by a nonzero Hall conductivity σ_{xy} . The connection between Chern number and σ_{xy} is provided by the infamous TKNN (Thouless, Kohmoto, Nightingale, and Nijs) relation [59],

$$\sigma_{xy} = -\frac{e^2}{2\pi\hbar} \sum_{\alpha \in \text{filled}} C_{\alpha}. \quad (23)$$

This indicates that one could physically observe changes in Chern number via jumps in Hall current.

This result suggests that the application of circularly polarized light offers a promising approach to obtaining topological materials from the untwisted graphene-hBN system. Our observations may be beneficial for future electronic applications because they closely mirror results from more challenging-to-synthesize twisted moiré materials.

For future work, several directions can be suggested. For instance, one could study this material under different types of light, such as twisted light or any arbitrary electromagnetic fields. One might also want to introduce the effect of magnetic fields, pressure, defects, and impurities, and study their interplay with periodic driving. It might also be helpful to study the effect of light on the twisted version of this material, which is yet another type of moiré structure. Furthermore, the physical properties of this material could be further harnessed by developing simplified models in the interacting limit.

ACKNOWLEDGMENTS

M.V. gratefully acknowledges the support provided by the Interdisciplinary Research Center (IRC) for Intelligent Secure Systems (ISS) at King Fahd University of Petroleum & Minerals (KFUPM) for funding his contribution to this work through internal research Grant No. INSS2507. The Alfahd High-Performance Computing facility, provided by the College of Petroleum Engineering & Geosciences at KFUPM, is acknowledged for its essential role in supporting the computational calculations.

DATA AVAILABILITY

The code that supports the calculations in this paper is openly available [60].

APPENDIX A: EXPANSION OF THE POSITION DEPENDENT HAMILTONIAN IN TERMS OF BLOCH WAVES

Bloch's theorem allows us to express the wave function of a periodic system as

$$|\psi(\mathbf{x})\rangle = e^{i\mathbf{k}\cdot\mathbf{x}}|u(\mathbf{x})\rangle, \quad (\text{A1})$$

$$|u(\mathbf{x} + \mathbf{R})\rangle = |u(\mathbf{x})\rangle. \quad (\text{A2})$$

If we substitute Eq. (A1) into the Schrödinger equation, we get the Schrödinger equation of Bloch electrons

$$E|u(\mathbf{x})\rangle = H(\mathbf{p} + \mathbf{k}, \mathbf{x})|u(\mathbf{x})\rangle. \quad (\text{A3})$$

This equation can now be employed to compute the band structure by expanding the Hamiltonian in a plane-wave basis $|\alpha\rangle = \exp(-i\mathbf{G}_\alpha \cdot \mathbf{x})/\sqrt{\Omega}$ as

$$\langle\beta|H_{G\text{-hBN}}(k)|\alpha\rangle = \frac{1}{\Omega} \int e^{i\mathbf{G}_\beta \cdot \mathbf{x}} H(\mathbf{k}, \mathbf{p}, \mathbf{x}) e^{-i\mathbf{G}_\alpha \cdot \mathbf{x}} d\mathbf{x},$$

where Ω is the area of the real-space unit cell. This form of the Hamiltonian is useful because it allows us to compute the band structure of the Hamiltonian numerically.

APPENDIX B: FUKUI DISCRETIZATION METHOD OF CHERN NUMBER

We begin by discretizing the Brillouin zone into an $N_1 \times N_2$ grid of momentum points k_ℓ , where $\ell = (\ell_1, \ell_2)$ with $\ell_1 = 1, \dots, N_1$ and $\ell_2 = 1, \dots, N_2$. Discretized points k_ℓ are then defined via step sizes Δk_μ for each direction μ as

$$k_\ell = (\ell_1 \Delta k_1, \ell_2 \Delta k_2); \quad \Delta k_\mu = \frac{X_\mu}{N_\mu}, \quad (\text{B1})$$

where X_μ is the length of the Brillouin zone in the μ direction, and N_μ is the number of discretization points along that axis.

We may now introduce so-called link variables $U_\mu(k_\ell)$ as

$$U_\mu(k_\ell) = \frac{\langle n(k_\ell) | n(k_\ell + \hat{\mu}) \rangle}{|\langle n(k_\ell) | n(k_\ell + \hat{\mu}) \rangle|}, \quad (\text{B2})$$

where $|n(k_\ell)\rangle$ is the eigenstate of the n th band at the point k_ℓ . In our expression above, we used vectors $\hat{\mu} = \Delta k_\mu(\delta_{1\mu}, \delta_{2\mu})$ that point from one discretized site to another.

Next, following [58], we define the lattice field strength $F_{12}(k_\ell)$ on each point of the discretized Brillouin zone,

$$F_{12}(k_\ell) = \ln [U_1(k_\ell)U_2(k_\ell + \hat{1})U_1^{-1}(k_\ell + \hat{2})U_2^{-1}(k_\ell)]. \quad (\text{B3})$$

Finally, according to [58], the Chern number C_n for the n th band is calculated by summing the field strength over all points in the Brillouin zone

$$C_n = \frac{1}{2\pi i} \sum_\ell F_{12}(k_\ell). \quad (\text{B4})$$

-
- [1] M. Hentschel, R. Kienberger, Ch. Spielmann, G. A. Reider, N. Milosevic, T. Brabec, P. Corkum, U. Heinzmann, M. Drescher, and F. Krausz, Attosecond metrology, *Nature (London)* **414**, 509 (2001).
 - [2] S. X. Hu and L. A. Collins, Attosecond pump probe: Exploring ultrafast electron motion inside an atom, *Phys. Rev. Lett.* **96**, 073004 (2006).
 - [3] D. S. Rana, I. Kawayama, K. Mavani, K. Takahashi, H. Murakami, and M. Tonouchi, Understanding the nature of ultrafast polarization dynamics of ferroelectric memory in the multiferroic BiFeO₃, *Adv. Mater.* **21**, 2881 (2009).
 - [4] Y. M. Sheu, S. A. Trugman, L. Yan, Q. X. Jia, A. J. Taylor, and R. P. Prasankumar, Using ultrashort optical pulses to couple ferroelectric and ferromagnetic order in an oxide heterostructure, *Nat. Commun.* **5**, 5832 (2014).
 - [5] Y. Zhang, J. Dai, X. Zhong, D. Zhang, G. Zhong, and J. Li, Probing ultrafast dynamics of ferroelectrics by time-resolved pump-probe spectroscopy, *Adv. Sci.* **8**, 2102488 (2021).
 - [6] F. Chen, Y. Zhu, S. Liu, Y. Qi, H. Y. Hwang, N. C. Brandt, J. Lu, F. Quirin, H. Enquist, P. Zalden, T. Hu, J. Goodfellow, M.-J. Sher, M. C. Hoffmann, D. Zhu, H. Lemke, J. Glownia, M. Chollet, A. R. Damodaran, J. Park, and *et al.*, Ultrafast

- terahertz-field-driven ionic response in ferroelectric BaTiO₃, *Phys. Rev. B* **94**, 180104(R) (2016).
- [7] D. Stich, F. Späth, H. Kraus, A. Sperlich, V. Dyakonov, and T. Hertel, Triplet–triplet exciton dynamics in single-walled carbon nanotubes, *Nat. Photon.* **8**, 139 (2014).
- [8] Y. Bai, J.-H. Olivier, G. Bullard, C. Liu, and M. J. Therien, Dynamics of charged excitons in electronically and morphologically homogeneous single-walled carbon nanotubes, *Proc. Natl. Acad. Sci. USA* **115**, 674 (2018).
- [9] K. Birkmeier, T. Hertel, and A. Hartschuh, Probing the ultrafast dynamics of excitons in single semiconducting carbon nanotubes, *Nat. Commun.* **13**, 6290 (2022).
- [10] M. F. Jager, C. Ott, P. M. Kraus, C. J. Kaplan, W. Pouse, R. E. Marvel, R. F. Haglund, D. M. Neumark, and S. R. Leone, Tracking the insulator-to-metal phase transition in VO₂ with few-femtosecond extreme uv transient absorption spectroscopy, *Proc. Natl. Acad. Sci. USA* **114**, 9558 (2017).
- [11] M. R. Bionta, E. Haddad, A. Leblanc, V. Gruson, P. Lassonde, H. Ibrahim, J. Chaillou, N. Émond, M. R. Otto, Á. Jiménez-Galán, R. E. F. Silva, M. Ivanov, B. J. Siwick, M. Chaker, and F. Légaré, Tracking ultrafast solid-state dynamics using high harmonic spectroscopy, *Phys. Rev. Res.* **3**, 023250 (2021).
- [12] M. Deb, E. Popova, H.-Y. Jaffrès, N. Keller, and M. Bargheer, Controlling high-frequency spin-wave dynamics using double-pulse laser excitation, *Phys. Rev. Appl.* **18**, 044001 (2022).
- [13] C. E. Graves, A. H. Reid, T. Wang, B. Wu, S. de Jong, K. Vahaplar, I. Radu, D. P. Bernstein, M. Messerschmidt, L. Müller, R. Coffee *et al.*, Nanoscale spin reversal by non-local angular momentum transfer following ultrafast laser excitation in ferrimagnetic GdFeCo, *Nat. Mater.* **12**, 293 (2013).
- [14] C. D. Stanciu, F. Hansteen, A. V. Kimel, A. Kirilyuk, A. Tsukamoto, A. Itoh, and Th Rasing, All-optical magnetic recording with circularly polarized light, *Phys. Rev. Lett.* **99**, 047601 (2007).
- [15] M. Rini, R. Tobey, N. Dean, J. Itatani, Y. Tomioka, Y. Tokura, R. W. Schoenlein, and A. Cavalleri, Control of the electronic phase of a manganite by mode-selective vibrational excitation, *Nature (London)* **449**, 72 (2007).
- [16] S. A. Weidinger and M. Knap, Floquet prethermalization and regimes of heating in a periodically driven, interacting quantum system, *Sci. Rep.* **7**, 45382 (2017).
- [17] U. De Giovannini, and H. Hübener, Floquet analysis of excitations in materials, *J. Phys. Mater.* **3**, 012001 (2019).
- [18] T. Oka and S. Kitamura, Floquet engineering of quantum materials, *Annu. Rev. Condens. Matter Phys.* **10**, 387 (2019).
- [19] M. A. Sentef, M. Claassen, A. F. Kemper, B. Moritz, T. Oka, J. K. Freericks, and T. P. Devereaux, Theory of Floquet band formation and local pseudospin textures in pump-probe photoemission of graphene, *Nat. Commun.* **6**, 7047 (2015).
- [20] S. Ito, M. Schüler, M. Meierhofer, S. Schlauderer, J. Freudenstein, J. Reimann, D. Afanasiev, K. A. Kokh, O. E. Tereshchenko, J. Güdde *et al.*, Build-up and dephasing of Floquet–Bloch bands on subcycle timescales, *Nature (London)* **616**, 696 (2023).
- [21] M. A. Sentef, J. Li, F. Künzel, and M. Eckstein, Quantum to classical crossover of Floquet engineering in correlated quantum systems, *Phys. Rev. Res.* **2**, 033033 (2020).
- [22] D. M. Kennes, M. Claassen, M. A. Sentef, and C. Karrasch, Light-induced *d*-wave superconductivity through Floquet-engineered Fermi surfaces in cuprates, *Phys. Rev. B* **100**, 075115 (2019).
- [23] C. J. Eckhardt, G. Passeti, M. Othman, C. Karrasch, F. Cavaliere, M. A. Sentef, and D. M. Kennes, Quantum Floquet engineering with an exactly solvable tight-binding chain in a cavity, *Commun. Phys.* **5**, 122 (2022).
- [24] G. E. Topp, C. J. Eckhardt, D. M. Kennes, M. A. Sentef, and P. Törmä, Light-matter coupling and quantum geometry in moiré materials, *Phys. Rev. B* **104**, 064306 (2021).
- [25] E. V. Boström, M. Claassen, J. McIver, G. Jotzu, A. Rubio, and M. Sentef, Light-induced topological magnons in two-dimensional van der Waals magnets, *SciPost Phys.* **9**, 061 (2020).
- [26] O. V. Kibis, I. V. Iorsh, and I. A. Shelykh, Floquet engineering of 2D materials, *J. Phys.: Conf. Ser.* **1461**, 012064 (2020).
- [27] X. Kong, W. Luo, L. Li, M. Yoon, T. Berlijn, and L. Liang, Floquet band engineering and topological phase transitions in 1T' transition metal dichalcogenides, *2D Mater.* **9**, 025005 (2022).
- [28] M. S. Rudner, and N. H. Lindner, Band structure engineering and non-equilibrium dynamics in Floquet topological insulators, *Nat. Rev. Phys.* **2**, 229 (2020).
- [29] A. Castro, U. De Giovannini, S. A. Sato, H. Hübener, and A. Rubio, Floquet engineering the band structure of materials with optimal control theory, *Phys. Rev. Res.* **4**, 033213 (2022).
- [30] R. Fleury, A. B. Khanikaev, and A. Alù, Floquet topological insulators for sound, *Nat. Commun.* **7**, 11744 (2016).
- [31] F. Zhan, J. Zeng, Z. Chen, X. Jin, J. Fan, T. Chen, and R. Wang, Floquet engineering of nonequilibrium valley-polarized quantum anomalous Hall effect with tunable Chern number, *Nano Lett.* **23**, 2166 (2023).
- [32] D. Shin, H. Hübener, U. De Giovannini, H. Jin, A. Rubio, and N. Park, Phonon-driven spin-Floquet magneto-valleytronics in MoS₂, *Nat. Commun.* **9**, 638 (2018).
- [33] H. Cao, J.-T. Sun, and S. Meng, Floquet engineering of anomalous Hall effects in monolayer MoS₂, *npj Quantum Mater.* **9**, 90 (2024).
- [34] A. G. Grushin, Á. Gómez-León, and T. Neupert, Floquet fractional Chern insulators, *Phys. Rev. Lett.* **112**, 156801 (2014).
- [35] Y. Ashida, A. İmamoğlu, J. Faist, D. Jaksch, A. Cavalleri, and E. Demler, Quantum electrodynamic control of matter: Cavity-enhanced ferroelectric phase transition, *Phys. Rev. X* **10**, 041027 (2020).
- [36] Y. Li, H. A. Fertig, and B. Seradjeh, Floquet-engineered topological flat bands in irradiated twisted bilayer graphene, *Phys. Rev. Res.* **2**, 043275 (2020).
- [37] G. E. Topp, G. Jotzu, J. W. McIver, L. Xian, A. Rubio, and M. A. Sentef, Topological Floquet engineering of twisted bilayer graphene, *Phys. Rev. Res.* **1**, 023031 (2019).
- [38] M. Vogl, M. Rodríguez-Vega, and G. A. Fiete, Effective Floquet Hamiltonians for periodically driven twisted bilayer graphene, *Phys. Rev. B* **101**, 235411, (2020).
- [39] M. Vogl, M. Rodríguez-Vega, B. Flebus, A. H. MacDonald, and G. A. Fiete, Floquet engineering of topological transitions in a twisted transition metal dichalcogenide homobilayer, *Phys. Rev. B* **103**, 014310 (2021).
- [40] A. Dubey, R. Kundu, and A. Kundu, Time-resolved ARPES and optical transport properties of irradiated twisted bilayer graphene in a steady state, *Phys. Rev. B* **111**, 035431 (2025).
- [41] M. Mitrano, A. Cantaluppi, D. Nicoletti, S. Kaiser, A. Perucchi, S. Lupi, P. Di Pietro, D. Pontiroli, M. Riccò, S. R. Clark, D.

- Jaksch, and A. Cavalleri, Possible light-induced superconductivity in K_3C_{60} at high temperature, *Nature (London)* **530**, 461 (2016).
- [42] D. Fausti, R. I. Tobey, N. Dean, S. Kaiser, A. Dienst, M. C. Hoffmann, S. Pyon, T. Takayama, H. Takagi, and A. Cavalleri, Light-induced superconductivity in a stripe-ordered cuprate, *Science* **331**, 189 (2011).
- [43] M. Suda, R. Kato, and H. M. Yamamoto, Light-induced superconductivity using a photoactive electric double layer, *Science* **347**, 743 (2015).
- [44] S. Fava, G. De Vecchi, G. Jotzu, M. Buzzi, T. Gebert, Y. Liu, B. Keimer, and A. Cavalleri, Magnetic field expulsion in optically driven $YBa_2Cu_3O_{6.48}$, *Nature (London)* **632**, 75 (2024).
- [45] T. F. Nova, A. S. Disa, M. Fechner, and A. Cavalleri, Metastable ferroelectricity in optically strained $SrTiO_3$, *Science* **364**, 1075 (2019).
- [46] X. Li, T. Qiu, J. Zhang, E. Baldini, J. Lu, A. M. Rappe, and K. A. Nelson, Terahertz field-induced ferroelectricity in quantum paraelectric $SrTiO_3$, *Science* **364**, 1079 (2019).
- [47] T. F. Nova, A. Cartella, A. Cantaluppi, M. Först, D. Bossini, R. V. Mikhaylovskiy, A. V. Kimel, R. Merlin, and A. Cavalleri, An effective magnetic field from optically driven phonons, *Nat. Phys.* **13**, 132 (2017).
- [48] J. W. McIver, B. Schulte, F.-U. Stein, T. Matsuyama, G. Jotzu, G. Meier, and A. Cavalleri, Light-induced anomalous Hall effect in graphene, *Nat. Phys.* **16**, 38 (2020).
- [49] M. H. S. Anfa, S. Elatresh, H. Bahlouli, and M. Vogl, Effective k -valley Hamiltonian for transition metal dichalcogenide bilayers under pressure and application to twisted bilayers with pressure-induced topological phase transitions, *Phys. Rev. B* **111**, 245434 (2025).
- [50] P. Moon and M. Koshino, Electronic properties of graphene/hexagonal-boron-nitride moiré superlattice, *Phys. Rev. B* **90**, 155406 (2014).
- [51] M. Koshino and T. Ando, *Electronic Properties of Monolayer and Multilayer Graphene* (Springer International Publishing, Cham, 2013), pp. 173–211.
- [52] L. Liu, Y. P. Feng, and Z. X. Shen, Structural and electronic properties of h-BN, *Phys. Rev. B* **68**, 104102 (2003).
- [53] J. Sławińska, I. Zasada, and Z. Klusek, Energy gap tuning in graphene on hexagonal boron nitride bilayer system, *Phys. Rev. B* **81**, 155433 (2010).
- [54] F. Bloch, Über die quantenmechanik der elektronen in kristallgittern, *Z. Phys.* **52**, 555 (1929).
- [55] H. Sambe, Steady states and quasienergies of a quantum-mechanical system in an oscillating field, *Phys. Rev. A* **7**, 2203 (1973).
- [56] M. S. Rudner, N. H. Lindner, E. Berg, and M. Levin, Anomalous edge states and the bulk-edge correspondence for periodically driven two-dimensional systems, *Phys. Rev. X* **3**, 031005 (2013).
- [57] L. Wawer and M. Fleischhauer, Chern number and Berry curvature for Gaussian mixed states of fermions, *Phys. Rev. B* **104**, 094104 (2021).
- [58] T. Fukui, Y. Hatsugai, and H. Suzuki, Chern numbers in discretized brillouin zone: Efficient method of computing (spin) Hall conductances, *J. Phys. Soc. Jpn.* **74**, 1674 (2005).
- [59] D. J. Thouless, M. Kohmoto, M. P. Nightingale, and M. den Nijs, Quantized Hall conductance in a two-dimensional periodic potential, *Phys. Rev. Lett.* **49**, 405 (1982).
- [60] https://github.com/miftahhadi/driven_ghbn.