

Erratum: Extracting the spin excitation spectrum of a fermionic system using a quantum processor **[Phys. Rev. B **112**, 045143 (2025)]**

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In the original paper, we used the identity $(\hat{S}_j^x)^2 |\Psi\rangle = |\Psi\rangle$ in the derivation of Eq. (8). However, the condition we stated for this identity to hold (namely, a state $|\Psi\rangle$ symmetric under spin reflection) is not sufficient. In particular, it does not hold for the states we considered in our work. Therefore, Eqs. (8) and (9) of the original paper should be replaced with the following, more general identities:

$$|\Psi_\theta^j\rangle := e^{i\theta\hat{S}_j^x} |\Psi\rangle = [1 + (\hat{S}_j^x)^2 (\cos\theta - 1) + i \sin\theta \hat{S}_j^x] |\Psi\rangle \quad (8)$$

and

$$\begin{aligned} \langle \Psi_\theta^j | \hat{S}_k^x(t) | \Psi_\theta^j \rangle &= i \sin\theta [\langle \Psi | [\hat{S}_j^x, \hat{S}_k^x(t)] | \Psi \rangle - (\cos\theta - 1) \langle \tilde{\Psi}_j^x | [\hat{S}_j^x, \hat{S}_k^x(t)] | \tilde{\Psi}_j^x \rangle] \\ &= \sin\theta [G_{jk}^x(t) - (\cos\theta - 1) \tilde{G}_{jk}^x(t)], \end{aligned} \quad (9)$$

respectively. In the last equation, we defined $|\tilde{\Psi}_j^x\rangle := \hat{S}_j^x |\Psi\rangle$ and $\tilde{G}_{jk}^x(t) := -i \langle \tilde{\Psi}_j^x | [\hat{S}_k^x(t), \hat{S}_j^x] | \tilde{\Psi}_j^x \rangle$, for $t > 0$. In deriving Eq. (9), we used that for any state $|\Psi\rangle$ that is an eigenstate of the total spin projection operator $\hat{S}_{\text{tot}}^z = \sum_j \hat{S}_j^z$ and a Hamiltonian H such that $[H, \hat{S}_{\text{tot}}^z] = 0$, the expectation value of a product of an odd number of $\hat{S}_j^x$ operators is zero, as each of them raise or lower the total spin projection by 1. Note that all the states we considered in our work [namely, the ground state of the one-dimensional (1D) Fermi-Hubbard model, the ground state of the free fermion model, and the dense Givens approximate state] and the Hamiltonian of the Fermi-Hubbard model satisfy both these conditions.

Since, in general, $\tilde{G}_{jk}^x(t) \neq 0$ and $\tilde{G}_{jk}^x(t) \not\propto G_{jk}^x(t)$, $\langle \Psi_\theta^j | \hat{S}_k^x(t) | \Psi_\theta^j \rangle$ does not coincide with the exact retarded spin Green's function $G_{jk}^x(t)$, in contrast with our claim in the original paper. In the language of local quench spectroscopy introduced in Ref. [1], the space-time Fourier transform $\langle \Psi_\theta^j | \hat{S}_k^x(t) | \Psi_\theta^j \rangle$ is a proper quench spectral function (QSF), and its imaginary part only represents an approximation of the exact dynamical spin structure factor.

However, as can be seen from Fig. 1, the agreement between the exact dynamical spin structure factors [panels (a)–(e)] and the imaginary part of the QSF obtained from $\langle \Psi_{\pi/4}^j | \hat{S}_k^x(t) | \Psi_{\pi/4}^j \rangle$ [panels (f)–(j)] is excellent over a large range of values of U . Therefore, the errors in Eqs. (8) and (9) of the original paper do not affect the discussion of the results we obtained therein.

The excellent agreement between the QSF and the dynamical spin structure can be explained with the following argument. For a fermionic system, the quenched state $|\Psi_\theta^j\rangle$ of Eq. (8) is a low-energy state with support only in the sectors of the system's Hilbert space with total spin projection eigenvalues S_{tot}^z and $S_{\text{tot}}^z \pm 1$, with $\hat{S}_{\text{tot}}^z |\Psi\rangle = S_{\text{tot}}^z |\Psi\rangle$. Indeed, since $(\hat{S}_j^x)^2 = n_{j,\uparrow} + n_{j,\downarrow} - 2n_{j,\uparrow}n_{j,\downarrow}$, with $n_{i,\sigma} = c_{i,\sigma}^\dagger c_{i,\sigma}$ the density operator for the spin $\sigma \in \{\uparrow, \downarrow\}$ on site i , the action of $(\hat{S}_j^x)^2$ on the state $|\Psi\rangle$ does not change its total spin projection. Therefore, by virtue of the spin-charge separation of the one-dimensional (1D) Fermi-Hubbard model [2], the dynamics of $\langle \Psi_\theta^j | \hat{S}_k^x(t) | \Psi_\theta^j \rangle$ is dominated by two-spinon excitations and we expect the QSF to probe almost exactly the same two-spinon continuum as the dynamical spin structure factor (although their spectral weight may differ slightly) [1].

Finally, we show how the claim that our approach gives access to the exact retarded spin-correlation function and dynamical spin structure factor can be also restored. First, this occurs for any state $|\Psi\rangle$ such that $(\hat{S}_j^x)^2 |\Psi\rangle = \lambda |\Psi\rangle$, $\forall j$. In this case, one has $\tilde{G}_{jk}^x(t) = -\lambda G_{jk}^x(t)$ and Eq. (9) becomes

$$\langle \Psi_\theta^j | \hat{S}_k^x(t) | \Psi_\theta^j \rangle = \sin\theta [1 + \lambda(\cos\theta - 1)] G_{jk}^x(t).$$

Since $(\hat{S}_j^x)^2 = n_{j,\uparrow} + n_{j,\downarrow} - 2n_{j,\uparrow}n_{j,\downarrow}$, the condition above is satisfied only for states $|\Psi\rangle$ in which every site j has a well-defined occupation (such as, for instance, the Néel state [3]).

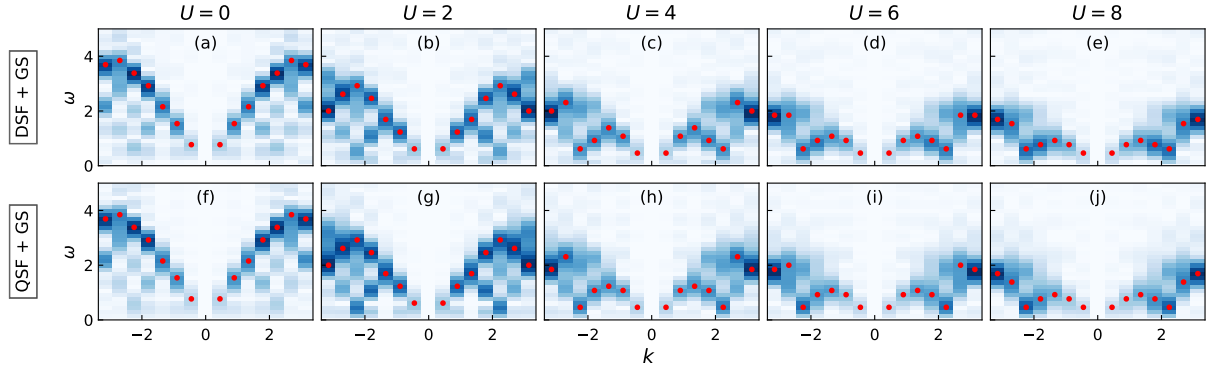


FIG. 1. Dynamical (spin) structure factors (DSFs) and QSFs for various protocols and values of U . Panels (a)–(e) show the exact dynamical spin structure factor for the 1D Fermi-Hubbard model. Panels (f)–(j) display the (imaginary part of the) QSF obtained by applying the unitary quench spectroscopy protocol of Eq. (9) (with $\theta = \pi/4$) to the ground state (GS) of the 1D Fermi-Hubbard model. Data have been obtained using the time-dependent variational principle algorithm with maximum bond dimension $\chi = 128$.

For more general states, the exact retarded spin Green's function of a fermionic system can always be obtained from Eq. (9) by combining two quench experiments from the states $|\Psi_\theta^j\rangle$ and $|\Psi_\phi^j\rangle$, with θ and ϕ such that

$$\sin \theta + \sin \phi \neq 0, \quad (1a)$$

$$\sin \theta (\cos \theta - 1) + \sin \phi (\cos \phi - 1) = 0. \quad (1b)$$

As one can see from Fig. 2(a), there exist infinitely many pairs (θ, ϕ) satisfying the conditions above. For each one of such pairs, the exact retarded spin Green's function can be obtained via

$$\langle \Psi_\theta^j | \hat{S}_k^x(t) | \Psi_\theta^j \rangle + \langle \Psi_\phi^j | \hat{S}_k^x(t) | \Psi_\phi^j \rangle = (\sin \theta + \sin \phi) G_{jk}^x(t). \quad (2)$$

This protocol can be viewed as a generalization to fermionic systems of the many-body Ramsey interferometry technique introduced for spin systems [4,5]. To get a good signal, it is important to choose a pair (θ, ϕ) that maximizes the coefficient of $G_{jk}^x(t)$ in Eq. (2), $\sin \theta + \sin \phi$. An example of such a pair is represented by the yellow dot in Fig. 2(a), corresponding to $(\theta, \phi) \approx (1.07\pi, \pi/3)$, for which one finds $\sin \theta + \sin \phi \approx 0.65$.

Additional material: exact charge response. While in the original paper we focused on the retarded spin Green's function, here we show that the double quench protocol described above can be used to probe the exact charge response of a fermionic system (and, therefore, its dynamical charge structure factor) [5]. We first define the retarded charge Green's function as

$$G_{jk}^n(t) := -i \langle \Psi | [n_k(t), n_j] | \Psi \rangle, \quad \text{for } t > 0. \quad (3)$$

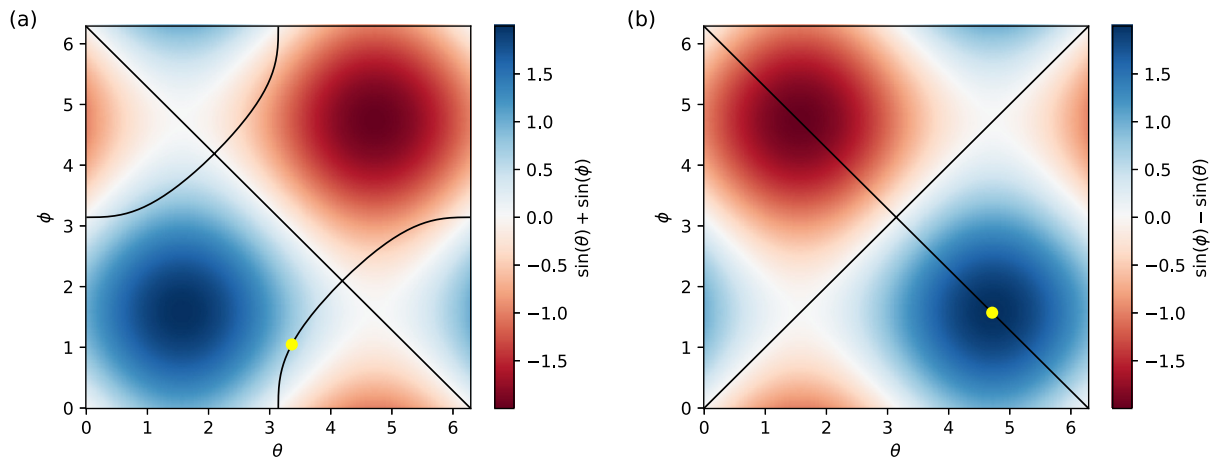


FIG. 2. Density plot of the coefficient of (a) $G_{jk}^x(t)$, $\sin \theta + \sin \phi$, and (b) $G_{jk}^n(t)$, $\sin \phi - \sin \theta$, in the double quench spectroscopy protocols of Eqs. (2) and (5), respectively. The black contours denote the region of the θ - ϕ plane in which (a) Eq. (1b) and (b) Eq. (4b) are satisfied. In both panels, the yellow dot, corresponding to (a) $(\theta, \phi) \approx (1.07\pi, \pi/3)$ and (b) $(\theta, \phi) = (3\pi/2, \pi/2)$, represent an (almost) optimal pair of angles for the double quench protocol.

Then, we consider the following quench operator:

$$e^{i\theta n_j} = 1 + (e^{i\theta} - 1)n_j,$$

where we have used that $(n_j)^k = n_j$. The dynamics of $n_k(t)$ on the quenched state

$$|\overline{\Psi}_\theta^j\rangle = e^{i\theta n_j} |\Psi\rangle = [1 + (e^{i\theta} - 1)n_j] |\Psi\rangle$$

is given by

$$\begin{aligned} \langle \overline{\Psi}_\theta^j | n_k(t) | \overline{\Psi}_\theta^j \rangle &= \langle \Psi | n_k(t) | \Psi \rangle + (\cos \theta - 1) \langle \Psi | \{n_k(t), n_j\} | \Psi \rangle \\ &\quad + i \sin \theta \langle \Psi | [n_k(t), n_j] | \Psi \rangle + 2(1 - \cos \theta) \langle \Psi | n_j n_k(t) n_j | \Psi \rangle, \end{aligned}$$

where $\{A, B\} := AB + BA$ is the anticommutator. In the third row, one can recognize the retarded charge Green's function of Eq. (3). To extract it, we consider two quenches with angles θ and ϕ such that

$$\sin \theta - \sin \phi \neq 0, \tag{4a}$$

$$\cos \theta - \cos \phi = 0. \tag{4b}$$

As in the previous case, there exist infinitely many pairs (θ, ϕ) satisfying these conditions [see Fig. 2(b)]. By picking one of them, the exact retarded charge Green's function can then be obtained as

$$\langle \overline{\Psi}_\theta^j | n_k(t) | \overline{\Psi}_\theta^j \rangle - \langle \overline{\Psi}_\phi^j | n_k(t) | \overline{\Psi}_\phi^j \rangle = (\sin \phi - \sin \theta) G_{jk}^n(t). \tag{5}$$

The yellow dot in Fig. 2(b), corresponding to $(\theta, \phi) = (3\pi/2, \pi/2)$, shows an optimal choice for the pair (θ, ϕ) , which maximizes the value of the coefficient of G_{jk}^n in Eq. (5), i.e., $\sin \phi - \sin \theta = 2$.

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