

Many-body spectral transitions through the lens of a variable-range quadratic Sachdev-Ye-Kitaev model

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The Sachdev-Ye-Kitaev (SYK) model is a cornerstone in the study of quantum chaos and holographic quantum matter. Real-world implementations, however, deviate from the idealized all-to-all connectivity, raising questions about the robustness of its chaotic properties. In this work, we investigate a quadratic SYK model with distance-dependent interactions governed by a power-law decay. By analytically and numerically studying the spectral form factor (SFF), we uncover how transitions present in the single-particle limit carry over to the many-body system. Nontrivial cancellations in the one-loop contributions lead to a robustness of the SFF under a considerable reduction of the interaction range. Further suppression leads to a breakdown of perturbation theory around the infinite-range path-integral saddle and the appearance of spectral regimes, marked by a higher dip and the emergence of a secondary plateau. Our results highlight the interplay between single-particle criticality and many-body dynamics, offering insights into the quantum chaos-to-localization transition and its reflection in spectral statistics.

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Introduction. The Sachdev-Ye-Kitaev (SYK) model [1,2] has emerged as a paradigmatic example in the study of quantum chaos and holographic quantum matter [3–6], thanks to its analytical tractability in the thermodynamic limit [7]. Additionally, it has inspired various proposals for experimental implementation [8–17], which could open the door to studying the theoretically challenging regime of intermediate system sizes, where quantum corrections to gravity become prominent. However, any realistic laboratory implementation would deviate from the idealized SYK model, which assumes all-to-all connectivity and perfectly uncorrelated random couplings. These deviations raise important questions about the robustness of the model’s properties when the interactions are constrained to a finite range. The introduction of distance-dependent interactions, as is typical for platforms such as trapped ions [18–21], cold atoms and molecules [22–24], solid state systems [25], etc., is expected to considerably impact the system’s chaotic behavior [26] and can potentially lead to significant many-body phenomena such as nonergodicity and localization [27–39], which are central to understanding

the quantum dynamics of strongly coupled but not chaotic systems.

In this work, we introduce and study the transitions of a system composed of N Majorana fermions with random hopping suppressed by a power-law decay $r^{-\alpha}$ [see Fig. 1(a)]—a version of the quadratic SYK that in the limit of a single particle becomes a power-law random banded matrix (PRBM) model [40]. The PRBM has become a popular model for studying Anderson criticality [41] and multifractality [42,43], as it is amenable to analytical studies using the supersymmetric sigma-model approach [44]. Here, we take a different route: we study the many-body spectral statistics of the SYK model using the average spectral form factor (SFF) [45]. In the thermodynamic limit, it can be treated analytically thanks to a path integration [46], which results in a different sigma model than the supersymmetric one.

The SFF, first introduced in the context of random matrix theory [45], reflects the distribution of energy levels in a system. In chaotic systems, the average SFF typically exhibits a universal structure characterized by an initial nonuniversal slope, a dip, a linear ramp, and a late-time plateau. However, in nonchaotic systems, the average SFF loses this universal behavior and is less well understood [47–62]. For example, in the quadratic SYK model, it displays an exponential ramp instead of a linear one [63,64], highlighting that the system is chaotic only at the single-particle level (see also [65]).

Our model allows us to take this analysis further by investigating how a variable interaction range impacts SYK physics. A central question is to what extent transitions known in the single-particle limit carry over to the many-body system, whose behavior can differ significantly due to fermionic statistics [66,67] and single-particle mobility edges [68]. In particular, we aim to determine whether the SFF can serve

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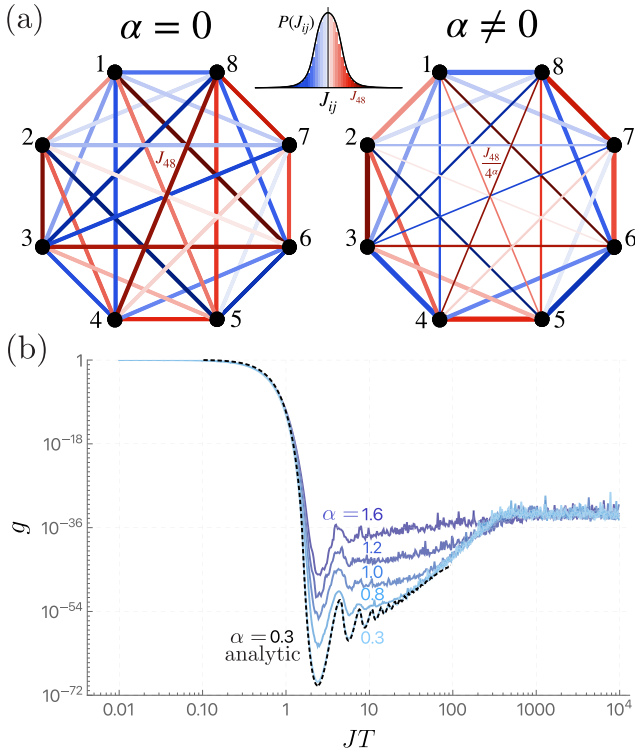


FIG. 1. (a) Illustration of the variable-range SYK₂ model given in Eq. (1) for $N = 8$ Majorana fermions on a circle, with all-to-all random couplings drawn from a Gaussian distribution. The line thickness represents the interaction strength, which decays with distance as $r^{-\alpha}$. (b) The SFF g for $N = 200$ fermions, for increasing α from light blue to violet. For $\alpha < 1/2$, the numerical results align well with the analytical prediction from Eq. (15) (black dashed line). For $\alpha > 1/2$, the system moves beyond the validity regime of the analytical approach, indicating a drastic change of the physics, as also suggested by a higher dip and the emergence of a secondary plateau, which merges with the late-time plateau for $\alpha \gtrsim 3/2$.

as a reliable marker for such transitions. As a saddle point calculation reveals, an initial decrease of the interaction range affects the SFF only subleadingly. This robustness derives from a nontrivial cancellation of one-loop contributions, demonstrating that the theory preserves ergodicity over an extended parameter range. However, once the interaction range is decreased beyond $\alpha \sim 1/2$, perturbation theory breaks down and a transition occurs marked by distinctive changes in the SFF [Fig. 1(b)]: a growth of the dip and the emergence of a secondary plateau, indicative of a prethermalization regime [48,58] where the Hilbert space is not fully explored [59]. The secondary-plateau height increases with α , exhibiting its steepest growth near $\alpha \sim 1$, until it reaches the late-time plateau at $\alpha \sim 3/2$, echoing the single-particle PRBM transitions. These features allow us to identify and characterize many-body level-statistics transitions, reveal their connection to single-particle localization transitions, and provide a robust diagnostic extendable to interacting systems.

Model. We introduce the variable-range SYK₂ model as a variant of the SYK₂ Hamiltonian where the random two-fermion term is weighted by a function of the distance

between the two sites involved,

$$\mathcal{H} = \frac{i}{N} \sum_{i < j}^N J_{ij} a(i-j) \psi_i \psi_j,$$

$$a(i-j) = (\min\{|i-j|, N-|i-j|\})^{-\alpha}. \quad (1)$$

The couplings J_{ij} are real Gaussian variables with variance $\langle J_{ij} J_{kl} \rangle = J^2 \delta_{ik} \delta_{jl}$ and the Majorana fermions are distributed on a line with periodic boundary conditions. For later convenience, we define the circular matrix A ,

$$A_{ij} = [a(i-j)]^2 / N^2, \quad A_{ii} = 0, \quad (2)$$

with $N^2 = \sum_i [a(i)]^2$.

The one-particle Hamiltonian $J_{ij} a(i-j)$, i.e., the coupling constant matrix, is an example of a PRBM model. It determines the excitations of the system in the single-particle sector [69] and exhibits different properties depending on the value of α [40]: for $\alpha \in [0, 0.5)$, the PRBM is ergodic and indistinguishable from the Gaussian orthogonal ensemble. For $\alpha > 1.5$, it becomes integrable [70]. At $\alpha = 1$, it has an Anderson critical point [40,41,71–74] separating the delocalized but nonergodic regime $\alpha \in (0.5, 1)$ [70,75,76], from the localized but superdiffusive regime $\alpha \in (1, 1.5)$. In this paper, we explore the transitions of the corresponding many-body system, the variable-range SYK₂ model.

Path integral for the average SFF. The main quantity we will use to analyze the many-body physics is the disorder-averaged SFF, defined as $g(T) = \langle |Z(iT)|^2 \rangle / |Z(0)|^2$. The angle brackets denote averaging over the random couplings, while $Z(0)$ is the Hilbert-space dimension. In what follows, we express $g(t)$ using collective-field variables, determine the dominant saddle-point solutions that spontaneously break some action symmetries, and identify the zero-mode contributions, which depend on the eigenvalues of A defined in Eq. (2).

First, we write the partition function as a path integral,

$$Z(iT) = \text{Tr} U = \int \mathcal{D}\psi \exp \left\{ \int_0^T \left(i\mathcal{H} - \sum_i \frac{1}{2} \psi_i \partial_t \psi_i \right) dt \right\}, \quad (3)$$

with $U = e^{-iT\mathcal{H}}$ the time-evolution operator, leading to

$$g(T) = \int \mathcal{D}\psi^L \mathcal{D}\psi^R \left\langle \exp \left\{ i \int_0^T dt (\mathcal{H}[\psi^R] - \mathcal{H}[\psi^L]) \right\} \right\rangle \times \exp \left\{ -\frac{1}{2} \sum_i \int_0^T dt (\psi_i^L \partial_t \psi_i^L + \psi_i^R \partial_t \psi_i^R) \right\}, \quad (4)$$

where the denominator has been absorbed in the measure and ψ_i^R, ψ_i^L are the Grassmannian variables over the forward and backward time evolutions, respectively [77].

After the Gaussian integration over the couplings, we can introduce collective fields in order to make the action quadratic in the fermions. In the usual SYK case, these are given by an averaged two-point function $G^{ab}(t, t') = \frac{1}{N} \sum_i \psi_i^a(t) \psi_i^b(t')$, with $a, b = L, R$. In contrast, to account for the spatial profile of the power-law coupling, we introduce a local two-point function $G_i^{ab}(t, t') = \psi_i^a(t) \psi_i^b(t')$, similarly to [78–80]. Imposing this definition at the path-integral level requires the introduction of a site-dependent auxiliary field $\Sigma_i^{ab}(t, t')$. Afterwards, we can perform two Gaussian

integrals, one over the fermions and the other one over G_i^{ab} . These manipulations, which are detailed in [81], lead to

$$g(T) = \int \mathcal{D}\Sigma_i e^{-I[\Sigma_i]},$$

$$I = -\frac{1}{2} \sum_i \text{Tr} \log(\partial - i\Sigma_i)$$

$$+ \sum_{abij} \frac{(-)^{a+b}}{4J^2} \iint_0^T A_{ij}^{-1} \Sigma_i^{ab} \Sigma_j^{ab}, \quad (5)$$

where the trace is over times and the L, R indices, and we defined $\partial_{at, bt'} = \delta(t - t')\delta_{ab}\partial_{t'}$.

Saddle point equation. Varying the effective action with respect to Σ and assuming time-translation invariance, we get the saddle-point equation in frequency space

$$\frac{(-)^{a+b}}{J^2} \sum_j A_{ij}^{-1} \Sigma_j^{ba}(-\omega_n) = -(\omega_n \delta^{ab} - \Sigma_i^{ab}(\omega_n))^{-1}, \quad (6)$$

where $\omega_n = (2n+1)\pi/T$ are Matsubara frequencies with $n \in \mathbb{Z}$. As discussed in [81], this equation has a SU(2) symmetry for each (positive) Matsubara frequency given by the adjoint action on the matrix indices of Σ , defining an infinite-dimensional symmetry group.

In principle, this equation may allow for several possible solutions. However, we will work in the ansatz where the dominating saddle does not depend on the site position. This assumption is consistent since averaging over the couplings restores space translation symmetry [82]. As we will see, an initial decrease of the interaction range just produces subleading effects on the SFF, due to nontrivial cancellations among the eigenvalues of A_{ij} in the one-loop corrections. Deviations from this behavior (see below) appear only for larger α , when the PRBM localizes and nonuniform saddle-point solutions can emerge. By imposing $\Sigma_i^{ab} = \tilde{\Sigma}^{ab}$, it is possible to obtain the same dominant saddles as in the $\alpha = 0$ case [81]:

$$\tilde{\Sigma}^{ab}(\omega_n) = \begin{cases} \frac{1}{2}(\omega_n - \sqrt{\omega_n^2 - 4J^2})\delta_{ab} & \omega_n > 2J, \\ \frac{1}{2}(\omega_n \delta_{ab} - i\sqrt{4J^2 - \omega_n^2}\sigma_3^{ab}) & 0 < \omega_n < 2J. \end{cases} \quad (7)$$

The solutions of Eq. (7) for negative frequencies can be recovered using $\tilde{\Sigma}^{ab}(-\omega_n) = -\tilde{\Sigma}^{ba}(\omega_n)$. One can check that the effective action evaluated on the homogeneous saddle point becomes linear in N as in the usual SYK model, making the semiclassical approximation reliable.

Notably, for $|\omega_n| > 2J$, $\tilde{\Sigma}$ is proportional to the identity and therefore invariant with respect to the adjoint action of the SU(2) symmetry discussed in [81], while this is not the case for $|\omega_n| < 2J$. In the latter regime, the classical solution spontaneously breaks SU(2) to U(1). The transformation in the coset space SU(2)/U(1) generates other equivalent solutions, leading to a contribution to the path integral proportional to the coset volume [83].

Quadratic fluctuations. The above symmetry-breaking pattern implies zero modes, which are responsible for the leading correction to the classical action and the development of the ramp in the SFF. As noted in [63], the ramp arises because, as T increases, more Matsubara modes enter the

symmetry-breaking regime $|\omega_n| < 2J$, each bringing a contribution proportional to the coset volume. However, unlike [63], the long-range decay can influence the number of zero modes.

To rigorously identify them, we analyze the Gaussian fluctuations around the saddle point solution

$$\Sigma_i^{ab}(t, t') = \tilde{\Sigma}^{ab}(t, t') + \delta\Sigma_i^{ab}(t, t'). \quad (8)$$

In contrast to the saddle itself, the fluctuations can have a spatial dependence. It is convenient to write $\delta\Sigma_i$ in frequency space and in a base that diagonalizes A . Denoting its eigenvalues as λ_k , the second-order action reads

$$\delta I^{(2)} = \sum_{\substack{k, a, b \\ \omega_1, \omega_2}} \delta\Sigma_k^{ab}(\omega_1, \omega_2) \frac{K_k^{ab}(\omega_1, \omega_2)}{4J^2 T^2} \delta\Sigma_k^{ba}(-\omega_2, -\omega_1), \quad (9)$$

where I_{cl} is the classical action contribution and

$$K_k^{ab}(\omega_1, \omega_2) = \frac{(-)^{a+b}}{\lambda_k} - \frac{1}{J^2} \tilde{\Sigma}^{aa}(\omega_1) \tilde{\Sigma}^{bb}(-\omega_2). \quad (10)$$

The zero modes arise from the condition $K_k^{ab}(\omega_1, \omega_2) = 0$ and appear in the symmetry-breaking saddle for $a \neq b$ and $\omega_1 = \omega_2$. In this case, Eq. (10) reduces to

$$K_k^{ab}(\omega, \omega) = -\lambda_k^{-1} + 1. \quad (11)$$

Therefore, the exponential ramp is generated by modes with eigenvalues $\lambda_k = 1$.

Eigenvalues of A . A can be diagonalized by going into Fourier space [81]. The constant vector is an eigenvector with eigenvalue $\lambda_{k=0} = 1$, so we always have at least one zero mode. The other eigenvalues are [84]

$$\lambda_k = \sum_{\Delta=1}^{N/2} \Delta^{-2\alpha} \cos\left(\frac{2\pi}{N} \Delta k\right) / \sum_{\Delta=1}^{N/2} \Delta^{-2\alpha}. \quad (12)$$

The limit $N \rightarrow \infty$ must be taken with care since both numerator and denominator can diverge. Analytic formulas can be derived for the cases when k is comparable to N or much smaller than N [81]. The eigenvalues corresponding to modes with $k \propto N$ are given by

$$\lambda(\tilde{k}) = \begin{cases} 0 & \alpha \leq 1/2, \\ \frac{\text{Re}(\text{Li}_{2\alpha}(e^{i\tilde{k}}))}{\zeta(2\alpha)} & \alpha > 1/2, \end{cases} \quad (13)$$

where Li is the polylogarithm, and we defined $\tilde{k} = \frac{2\pi}{N} k$, $-\pi < \tilde{k} < \pi$. The eigenvalues of the modes with $k = O(1)$, instead, are

$$\lambda_k = \begin{cases} (-1)^k + \frac{\pi^2 k^2}{3-2\alpha} {}_1F_2\left(\frac{3-2\alpha}{2}; \frac{3}{2}, \frac{5-2\alpha}{2}; -\frac{k^2 \pi^2}{4}\right) & \alpha \leq 1/2 \\ 1 & \alpha > 1/2, \end{cases} \quad (14)$$

where ${}_pF_q$ is the generalized hypergeometric function. A comparison between the analytic eigenvalues in the thermodynamic limit and the ones computed numerically at finite N can be found in Fig. 2.

A drastic change happens at $\alpha = 1/2$: as shown in Eq. (11), we have a zero mode whenever $\lambda_k = 1$. The above solutions imply that for $\alpha > 1/2$ an infinite number of zero modes

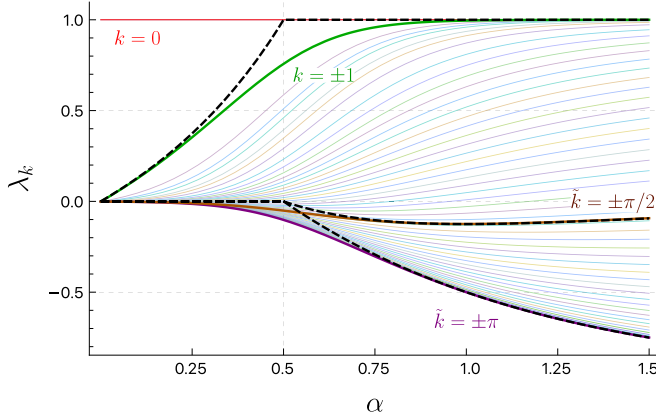


FIG. 2. Eigenvalues λ_k computed numerically for $N = 1001$ (colored) and analytically at $N \rightarrow \infty$ (black dashed). A transition is signaled by the nondifferentiable point at $\alpha = 1/2$.

should appear in the thermodynamic limit, each of them bringing a factor proportional to the coset volume. This should lead to a rapid increase of the ramp, if the saddle point we found

is still dominating. However, as we argue in [81], the same behavior of the eigenvalues breaks down perturbation theory around the uniform saddle (7), signaling that we are entering a different regime where either the physics is described by another saddle or it becomes nonperturbative. Both imply a transition at $\alpha = 1/2$; remarkably, this coincides with the critical point of the single-body PRBM. This transition has also been identified in other long-range models [85] and coincides with the value of α at which the normalization $\mathcal{N}(\alpha)^2$ ceases to diverge in the thermodynamic limit. However, the value of α is not universal, and quantum spin systems with similar power-law interaction [86–88] can exhibit dynamical transitions at significantly different values.

Analytic SFF. We now have all the ingredients to present the analytic expression for the SFF (see [81]). This is the product of three contributions: the classical action responsible for the slope, the time-symmetric fluctuations which contain the zero modes and are responsible for the exponential ramp, and the massive fluctuations that are not time symmetric and are responsible for the late-time corrections to the ramp. The first is spatially homogeneous, while the two one-loop contributions acquire a momentum dependence once $\alpha \neq 0$.

For JT large and for $\alpha < 1/2$, the SFF simplifies to

$$\log \frac{g(T)}{2^N} = N \sum_{j=1}^{\infty} \left[\frac{(-1)^{j+1}}{j} \frac{2J_1(2JjT)}{2JjT} \right] + \frac{JT}{\pi} \log \left(\frac{16}{e^2} \frac{N}{JT} \right) + \frac{JT}{\pi} \sum_{k \neq 0} \left[(1 + \lambda_k) \frac{\operatorname{arctanh} \sqrt{\lambda_k}}{\sqrt{\lambda_k}} - 1 + \log(1 - \lambda_k) \right], \quad (15)$$

where J_1 is a Bessel function of the first kind. Notably, for $\alpha \sim 0$ we have $\lambda_{k \neq 0} \sim N^{-1}$, the λ_k -dependent part cancels in a nontrivial way and becomes subleading for large N recovering the result in [63]. A comparison between Eq. (15) and numerical data for $\alpha = 0.3$ is presented in Fig. 1(b), demonstrating excellent agreement. The $\alpha = 0$ case would be indistinguishable from $\alpha = 0.3$. In general, for $\alpha < 1/2$, the eigenvalue contribution to (15) is subleading, as most eigenvalues vanish in the thermodynamic limit (13). The essentially unmodified form of the SFF highlights that, for $0 \leq \alpha < 1/2$, the variable-range SYK₂ model is robust under connectivity reduction, consistent with the ergodic phase of the PRBM model.

SFF and localization. In this section, we discuss the various regimes of the SFF for $\alpha \geq 1/2$. The analytical analysis displays a breakdown of perturbation theory around the uniform saddle point at $\alpha = 1/2$, at the same point where the single-particle PRBM displays an ergodic to nonergodic transition. For $\alpha > 1/2$, the single-particle PRBM becomes gradually less delocalized, encounters an Anderson critical point at $\alpha = 1$, and enters an integrable regime at $\alpha > 3/2$ [40]. From a path integral point of view, it is reasonable to expect that, as the system localizes, it will break the space translation symmetry we assumed for the saddle-point solution in the small- α region. This is supported by the fact that at early times the SFF takes larger values (in particular at the dip time) than what is predicted analytically, indicating that there may be another saddle that dominates the classical action. This makes the dip height a good candidate for identifying the ergodic/nonergodic transition. As it can be seen in Fig. 3(a),

the dip starts to rise considerably in the vicinity of $\alpha \approx 0.5$. A characteristic value of α where there is a qualitative change can be obtained similarly to the knee voltage in transistors. In this spirit, the intersection of a linear fit of the large- α part of the curve with the ordinate yields $\alpha = 0.501 \pm 0.006$.

In the regime $1/2 \leq \alpha \leq 3/2$, a secondary plateau emerges: it begins after the early-time oscillations and persists until it merges into the ramp characteristic of $\alpha < 1/2$ [see Fig. 1(b)]. It becomes more prominent at larger system sizes [81] and signals a prethermalization regime [59]. The height of this secondary plateau grows with α until it reaches the height of the late-time one around $\alpha \sim 3/2$, as expected for an integrable theory. To better quantify this transition, we define the logarithm of the ratio between the two plateau heights,

$$h = \left\langle \frac{1}{N} \log \frac{2^{-N/2}}{g(T)} \right\rangle_T. \quad (16)$$

Here, $2^{-N/2}$ is the theoretical value of the late-time plateau, while the time average is taken over the interval $JT \in [10, 15]$, where the secondary plateau has already developed. The remaining curvature in the SFF $g(T)$ is a small N effect due to the proximity of the ramp. The value of h at different N is displayed in Fig. 3(b).

By applying a constant vertical shift to h to align the values at $\alpha = 0$, the entire curves display an excellent collapse onto each other. A polynomial fourth-order fit of the collapsed curve above $\alpha = 0.8$ reveals a change in concavity at $\alpha = 1.02$, in the vicinity of the point where the PRBM has its Anderson transition. This suggests that transitions within

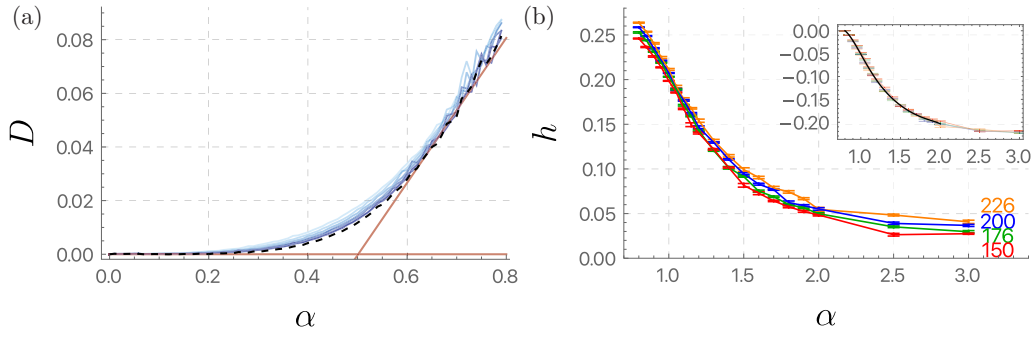


FIG. 3. (a) Dip position as a function of α for different system sizes from $N = 100$ (violet) to 400 (light blue) in steps of 50, averaged over $\sim 4 \cdot 10^6$ samples. The dip time is given by $T_{\text{Dip}} \sim 2.408$, and we define $D = \frac{1}{N} \log \frac{g(T_{\text{Dip}}, \alpha)}{g(T_{\text{Dip}}, \alpha=0)}$, effectively measuring the shift of the dip relative to the $\alpha = 0$ SFF. The black dashed line shows an extrapolation to $N \rightarrow \infty$. A linear fit to the curve at $\alpha \geq 0.6$ intersects $D = 0$ at a characteristic value of $\alpha = 0.501 \pm 0.006$. (b) Plateaus distance h , defined in Eq. (16), averaged over $\sim 4 \cdot 10^7$ samples for $N = 226, 200, 176, 150$. In the inset, the curves are vertically shifted to reveal a universal behavior. The black line represents a fourth-order polynomial fit.

SYK₂-type models can be identified through features of the SFF such as the point where its plateau height grows more quickly upon a change of the system parameters.

Within the regime accessible to our numerics, it is not possible to see a sharp transition at $\alpha = 3/2$, corresponding in the PRBM to the onset of the integrable regime. We do, however, observe a qualitative disappearance of the ramp [Fig. 1(b)]. To understand whether this PRBM transition is imprinted onto the SFF of our model, it would be desirable to access larger system sizes, where the secondary plateau is more prominent. However, the SFF for SYK₂ has fluctuations that grow with N^T [89], making it prohibitively expensive to gather sufficient statistics for convergence, which in our case reaches the order of 10^7 samples at $N = 226$. The implications of these large fluctuations are discussed in [81].

Conclusions. We have discussed how the SFF captures distinct regimes in the variable-range SYK₂ model. Key features include the dip height, as an indicator of ergodic-nonergodic transitions, and a secondary plateau, which appears at the onset of the nonergodic regime, starts to grow more rapidly at a point suggestive of the position of Anderson criticality, and merges into the late-time plateau as the system localizes.

These findings provide useful insights for the study of many-body localization (MBL), where numerics is often limited to small systems and where the existence of a transition in the thermodynamic limit is a matter of ongoing debate [90]. Our model offers a controlled setting in which many-body signatures of localization transitions can be directly connected to well-understood single-particle phenomena. For instance, the dip height has been used as a marker of transitions between holographic and nonholographic behavior in sparse SYK models [60]. Similarly, the emergence of a secondary

plateau has been observed and studied [48,58]. Since preramp features have been linked to nearly conserved charges [59], it would be compelling to investigate whether the secondary plateau can be understood in terms of localized integrals of motion [91,92], further bridging our framework with core questions in MBL.

Finally, our analysis can be extended to other models, such as sparse SYK, e.g., the adjacency matrix M introduced in [80] shares key properties with the matrix A in Eq. (2), such as a uniform eigenvector with eigenvalue 1. Exploiting the present framework may provide insights into the transitions appearing as the sparsification rate increases.

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Data availability. The data that support the findings of this article are openly available [94].

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