

Krylov space dynamics of ergodic and dynamically frozen Floquet systems

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In isolated quantum many-body systems periodically driven in time, the asymptotic dynamics at late times can exhibit distinct behavior such as thermalization or dynamical freezing. Understanding the properties of and the convergence towards infinite-time (nonequilibrium) steady states, however, remains a challenging endeavor. We propose a physically motivated Krylov space perspective on Floquet thermalization which offers a natural framework to study rates of convergence towards steady states and, simultaneously, an efficient numerical algorithm to evaluate infinite-time averages of observables within the diagonal ensemble. The effectiveness of our algorithm is demonstrated by applying it to the periodically driven mixed-field Ising model, reaching system sizes of up to 30 spins. Our method successfully resolves the transition between the ergodic and dynamically frozen phases and provides insight into the nature of the Floquet eigenstates across the phase diagram. Furthermore, we show that the long-time behavior is encoded within the localization properties of the Ritz vectors under the Floquet evolution, providing an accurate diagnostic of ergodicity.

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I. INTRODUCTION

Understanding the late-time dynamics of isolated quantum systems remains a central problem at the forefront of modern physics. In periodically driven systems, such dynamics are generally expected to follow the prescriptions of the Floquet eigenstate thermalization hypothesis (Floquet ETH) [1–4], describing the drive-induced heating towards an infinite-temperature state [5–9]. Identifying regimes where such ergodicity is avoided opens access to rich and unconventional physics [10–16]. However, numerically probing and characterizing these regimes remains a formidable challenge. Distinguishing true ergodicity breaking from prethermalization, where ergodicity may only set in at large but finite time scales that mimic genuinely diverging ones, requires both large system sizes and late times (as exemplified in strongly disordered systems [17]).

In isolated, periodically driven systems, the time evolution over a single period is described by the unitary Floquet operator $\mathcal{U}$. Its eigendecomposition,

$$\mathcal{U}|\mu_i\rangle = \xi_i|\mu_i\rangle, \quad (1)$$

defines the Floquet eigenstates $|\mu_i\rangle$ and eigenvalues $\xi_i \in S^1$. These eigenstates play a central role in diagnosing ergodicity

breaking in larger systems within feasible computational effort. Ergodicity breaking manifests as freezing or localization around low-entanglement initial states, such as quantum scars [18–27], Hilbert-space fragmentation [16,28,29], many-body localization [30,31], or dynamically frozen (DF) [32–49] systems stabilized by emergent conserved quantities.

Given an initial state $|\psi_0\rangle$, the late-time nonoscillatory dynamics of an observable $\mathcal{O}$ with respect to the Floquet time evolution $\mathcal{U}^m|\psi_0\rangle$ for $m \rightarrow \infty$ is captured by the diagonal ensemble average (DEA) [4,50,51],

$$\langle \mathcal{O} \rangle_{\text{DEA}} = \sum_{i=1}^D |\langle \mu_i | \psi_0 \rangle|^2 \langle \mu_i | \mathcal{O} | \mu_i \rangle, \quad (2)$$

where D denotes the Hilbert space dimension. Deviations of this DEA from the ergodic value, $\text{Tr}[\mathcal{O}]/D$, directly indicate the breaking of ergodicity. For local operators $\mathcal{O}$, the DEA reveals the system's fate in the long-time limit, enabling conclusive statements about its ergodicity, thermalization, and final ensemble description, bridging quantum dynamics with statistical mechanics [2,4,52–54]. A direct computation of the DEA, however, requires all eigenstates of the relevant Hamiltonian, making it exponentially costly and limiting analyses to small systems. This bottleneck similarly appears in full time-evolution simulations, where capturing late-time quantities or linear entanglement growth rapidly becomes intractable.

Yet, the dynamics of local observables are often far simpler than this exponential complexity suggests. In ergodic systems, $\langle \mathcal{O}(t) \rangle$ typically relaxes to its thermal value after only modest evolution times. In nonergodic systems, emergent conservation laws can cause observables to remain effectively frozen [40–42,44,45,48,55]. In both limits, full diagonalization appears excessive. Motivated by these observations, we propose

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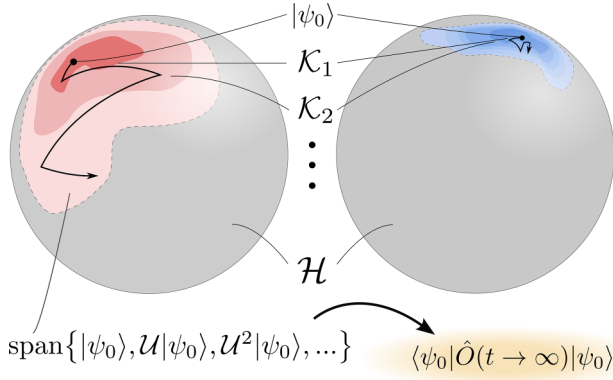


FIG. 1. We introduce a Krylov space approximation for computing the infinite-time average, specifically the diagonal ensemble average (DEA) of observables in periodically driven systems. The figure illustrates two contrasting scenarios: (left) an ergodic regime, where observables rapidly relax to equilibrium, and (right) a dynamically frozen regime, where emergent conservation laws keep observables locked near their initial values for long times. Our Krylov subspace approach yields accurate infinite-time averages in both cases.

a framework for tackling ergodicity breaking and the approach to the DEA in terms of a Krylov space approximation. Using this approach we introduce an efficient Arnoldi-based algorithm for evaluating the DEA. We demonstrate its effectiveness using a driven transverse field Ising model, exhibiting a transition from an ergodic to a dynamically frozen regime, depicted in Fig. 1. Finally, we demonstrate how Ritz vector localization can serve as a reliable method for studying ergodicity.

II. KRYLOV APPROXIMATION OF THE DEA

Within the first $m - 1$ cycles (discrete time steps) of the Floquet evolution, the system explores the space spanned by the states $\mathcal{U}^n |\psi_0\rangle$, $n = 0, \dots, m - 1$. These states define the m th Krylov space,

$$\mathcal{K}_m(\mathcal{U}, |\psi_0\rangle) = \text{span}\{|\psi_0\rangle, \mathcal{U}|\psi_0\rangle, \dots, \mathcal{U}^{(m-1)}|\psi_0\rangle\}. \quad (3)$$

In the following, we abbreviate $\mathcal{K}_m \equiv \mathcal{K}_m(\mathcal{U}, |\psi_0\rangle)$. We denote by $\mathcal{P}_m = V_m V_m^\dagger$ the projector onto $\mathcal{K}_m$, where V_m denotes a matrix whose columns form an orthonormal basis of $\mathcal{K}_m$. Moreover, we define the projected Floquet operator as,

$$\tilde{\mathcal{U}}_m = \mathcal{P}_m^\dagger \mathcal{U} \mathcal{P}_m. \quad (4)$$

We now introduce the Krylov-space projected DEA as

$$\langle \mathcal{O} \rangle_{\text{DEA},m} = \sum_{i=1}^m |\langle \tilde{\mu}_i | \psi_0 \rangle|^2 \langle \tilde{\mu}_i | \mathcal{O} | \tilde{\mu}_i \rangle / \mathcal{N}. \quad (5)$$

Here the $|\tilde{\mu}_i\rangle$ denote the eigenvectors of the projected Floquet operator (also referred to as Ritz vectors),

$$\tilde{\mathcal{U}}_m |\tilde{\mu}_i\rangle = \xi_i |\tilde{\mu}_i\rangle, \quad (6)$$

for which $\mathcal{P}_m |\tilde{\mu}_i\rangle \neq 0$. While the diagonalization of a unitary matrix gives rise to an orthonormal basis, the projected Floquet operator is not necessarily unitary, which can lead to a

loss of orthogonality of the generated Ritz vectors. We include a normalization factor given by

$$\mathcal{N} = \sum_{i=1}^m |\langle \tilde{\mu}_i | \psi_0 \rangle|^2 \quad (7)$$

in Eq. (5) to remedy this loss of orthogonality. We discuss other options such as the use of left and right eigenvectors or the isometric Arnoldi routine [56] in the Appendix. The projected DEA $\langle \mathcal{O} \rangle_{\text{DEA},m}$ in Eq. (5) describes the long-time limit of the Floquet dynamics restricted to the m th Krylov space $\mathcal{K}_m$. Thus, $\langle \mathcal{O} \rangle_{\text{DEA},m}$ is independent of the choice of basis of $\mathcal{K}_m$. The deviation from the exact result,

$$\epsilon_m = |\langle \mathcal{O} \rangle_{\text{DEA}} - \langle \mathcal{O} \rangle_{\text{DEA},m}|, \quad (8)$$

is a natural measure of the proximity of the system to the infinite-time (nonequilibrium) steady state after $m - 1$ Floquet cycles. In other words, ϵ_m determines the accuracy to which the Krylov space $\mathcal{K}_m$ captures the infinite-time dynamics of the operator $\mathcal{O}$ as illustrated in Fig. 1, where typically $m \ll D$. We propose an iterative Arnoldi-based algorithm for evaluating $\langle \mathcal{O} \rangle_{\text{DEA},m}$ in the appendix. Throughout the text, we refer to the first m iterations of the algorithm as corresponding to the Krylov subspace of dimension m .

III. NUMERICAL RESULTS

We demonstrate the effectiveness of the algorithm in the context of DF [32,39,57]. Dynamical freezing is a phenomenon where a periodically driven quantum many-body system evades ergodicity due to the emergence of conserved local operators under strong driving. These conservation laws are approximate, yet perpetual. DF has been studied in systems mappable to free fermions and hard-core bosons [32–38], as well as in interacting systems [39–48,58]. However, studies of DEAs for the latter have remained restricted to small system sizes.

We consider an interacting Floquet system with the following time-dependent Hamiltonian, describing a uniform chain of interacting Ising spins where the longitudinal field is periodically switched:

$$H(t) = - \sum_{i=1}^L (J \sigma_i^z \sigma_{i+1}^z + \kappa \sigma_i^z \sigma_{i+2}^z + h^z \sigma_i^z + h^x \sigma_i^x) + H_d \text{sgn}(\sin \omega t) \left(\sum_{i=1}^L \sigma_i^z \right), \quad (9)$$

where $\sigma_i^{x,y,z}$ are Pauli matrices. Following Ref. [49], we set $\kappa = -0.25J$, $h^z = -0.075J$, $h^x = 0.577J$, use periodic boundaries and fix the drive period $T = 4\pi/J$. Varying the drive amplitude H_d/J tunes the system between ergodic and dynamically frozen regimes. We take a fully polarized initial state $|\psi_0\rangle = |\uparrow \dots \uparrow\rangle_z$ and compute the long-time average of the total magnetization density, $m^z = \frac{1}{L} \sum_i \sigma_i^z$. A vanishing DEA, $\langle m^z \rangle_{\text{DEA}} \approx 0$, indicates ergodicity, whereas a nonzero DEA close to the initial (maximal) expectation value, $\langle m^z \rangle_{\text{DEA}} \approx 1$, is a signature of dynamical freezing.

Figure 2 shows the results of our Krylov algorithm for system sizes up to $L = 30$, exceeding the reach of exact

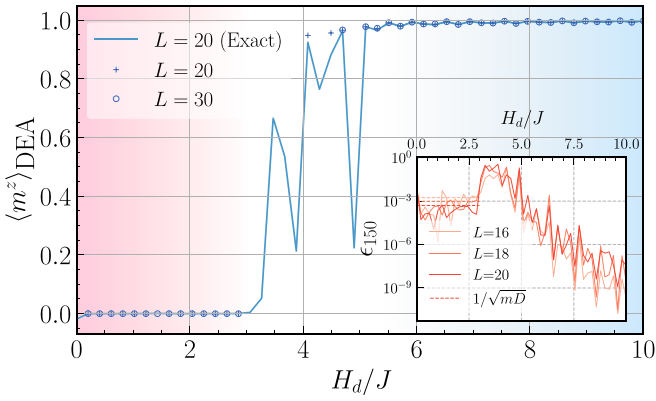


FIG. 2. Infinite-time average of the total magnetization density, m_z , showing the transition to a dynamically frozen regime at large H_d/J . The solid line shows the exact long-time average for a 20-site chain computed with exact diagonalization. Pluses and circles show where the algorithm has converged for 20 and 30 sites, respectively. The inset shows the error ϵ_{150} after 150 iterations of our Krylov algorithm with lighter to darker showing increasing system sizes. Dashed lines show a suggestive scaling of the error in the ergodic regime.

diagonalization, along with comparisons to exact results for $L = 20$. We use a convergence criteria where the absolute difference between the last few iterations is less than 10^{-4} and only show those data points which have converged within a computationally feasible time limit. We note the poor convergence near the transition. The inset shows how the exact error behaves after a fixed number of iterations (150) for several system sizes.

We analyze the DEA error ϵ_m at each iteration m in Fig. 3. In the frozen regime ϵ_m quickly achieves a small value, $\epsilon_m < 10^{-6}$ within a few iterations, and decreases exponentially with increasing drive strength. Deep in the frozen regime, the Floquet eigenstates are close to product states. As a result, the overlap between these eigenstates and the fully polarized initial state receives only exponentially small corrections and is quickly accounted for by diagonalizing the projected unitary

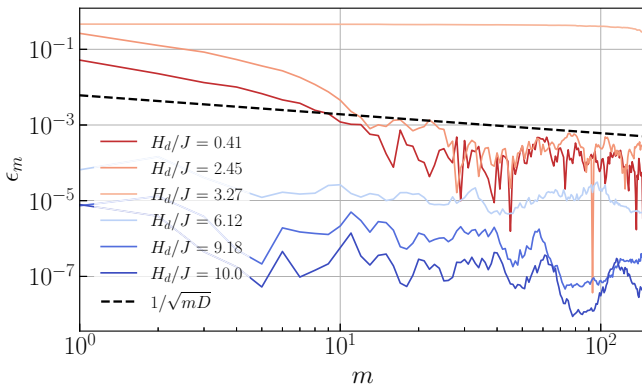


FIG. 3. The error ϵ_m of the DEA at each step during the first 150 iterations of the algorithm. Simulations were performed on a 20-site spin-chain, with red and blue lines indicating ergodic and dynamically frozen model parameters, respectively. The dashed line shows a suggestive scaling for the error in the ergodic regime.

after just a few iterations. This contrasts to the ergodic phase, where the error rapidly decays to a scaling $\epsilon_m \sim (mD)^{-1/2}$. This scaling arises due to the fact that the initial state becomes delocalized over the Ritz vectors (which we later show explicitly) and the DEA approximation becomes an average over m matrix elements each expected to obey ETH, which predicts fluctuations scaling as $D^{1/2}$ for each matrix element [4]. Remarkably, this indicates that the Ritz vectors obtained by diagonalizing the $m \times m$ projected Floquet unitary reproduce the fluctuations expected from the Floquet eigenstates obtained by diagonalizing the exact $D \times D$ Floquet unitary. We show in the Appendix that this behavior is specific to the presented algorithm and cannot be expected generically. Namely, alternative diagonalization procedures such as an isometric Arnoldi iteration lead to long tails and poor convergence in the ergodic phase. In the frozen regime, these results indicate that the dynamics explores an effective Hilbert space which does not grow with system size, and can be spanned to a very good approximation by the Ritz vectors. Note that irrespective of the regime, the Ritz vectors accurately describe the short-time dynamics *by construction*: the Ritz vectors in the Krylov subspace of dimension m span the time-evolved states for the first m cycles by definition, since they are iteratively generated from the initial state. As such, the first m values of $\langle O(t) \rangle$, $t = 0 \dots m - 1$, are accurately described within this Krylov space, with the only error appearing due to the loss of orthogonalization. We here establish that the Ritz vectors also accurately capture the late-time behavior.

Both the phenomena of dynamical freezing and its more generic counterpart, thermalization, are not restricted to a particular initial state or choice of observable. In the Appendix, we demonstrate that our algorithm can accurately reproduce the long-time averages for other observables of interest, such as nearest-neighbor spin correlations, and for different initial states, including mid-spectrum and Néel type states.

IV. LOCALIZATION OF RITZ VECTORS

We can make these statements more quantitative by analyzing the Ritz vectors and the corresponding Krylov subspace. To this end, we compute the inverse participation ratio (IPR) [59–63] in two complementary ways. The first is the IPR of the initial state with respect to the Ritz vectors for a Krylov subspace of dimension m . We define,

$$\text{IPR}^{(\text{kry})} = \sum_{i=0}^m |\langle \tilde{\mu}_i | \psi_0 \rangle|^4 / \mathcal{N}^2, \quad (10)$$

which is inversely proportional to the number of Ritz vectors over which the initial state is delocalized. A value close to one indicates a localized initial state in the Krylov subspace, whereas a value of $1/m$ indicates complete delocalization. $\text{IPR}^{(\text{kry})}$ is shown in Fig. 4(a) for different drive strengths as the Krylov space dimension increases. The two dynamical regimes are clearly distinguished by their scaling behaviors: in the ergodic phase, the initial state is delocalized over the basis of Ritz vectors, whereas in the dynamically frozen regime, only a small subset contributes significantly.

We also examine the localization of the individual Ritz vectors in the computational basis, i.e., the product basis of

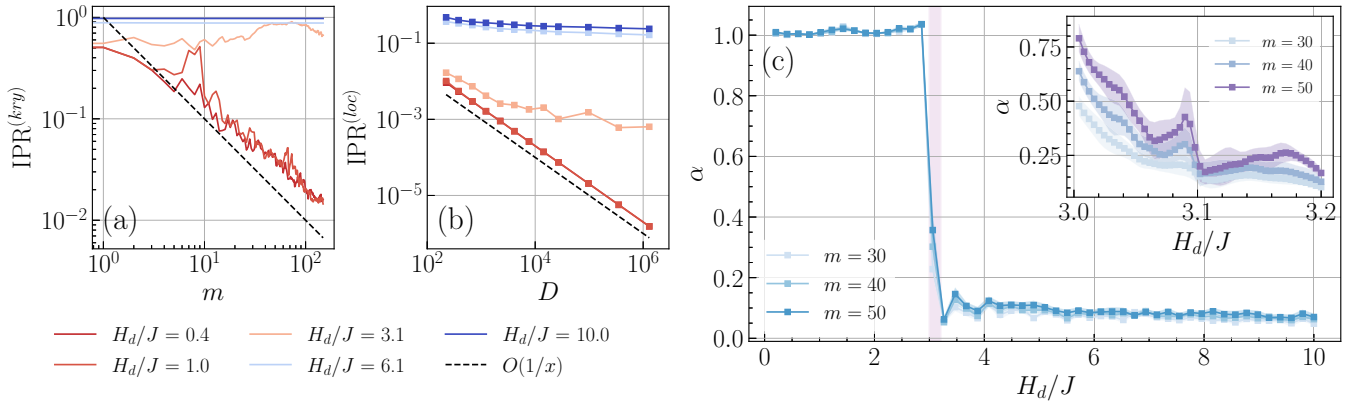


FIG. 4. Inverse participation ratios. (a) The inverse participation ratio of the initial state in the basis of Ritz vectors (approximate Floquet eigenvectors) for different Krylov space dimensions. Blue and red lines show frozen and ergodic model parameters, respectively; the initial state can be seen to go between localized to delocalized in the Krylov subspace as the drive strength is increased. (b) The inverse participation ratio of the Ritz vectors in the computational basis, averaged over all Ritz vectors in a Krylov subspace of dimension 50. The dashed lines correspond to $1/m$ and $1/D$ in (a) and (b), respectively. (c) We fit the lines from (b) to the form $\sim D^{-\alpha}$ and show the scaling exponent, with error bars, for different drive strengths H_d/J . Different lines correspond to different Krylov subspace dimensions, m . A sharp transition between localized and delocalized Ritz vectors in the full Hilbert space can be seen. The inset shows a more fine-grained parameter scan nearer the transition (purple shaded region of the main plot).

local σ^z eigenstates (the standard spin-configuration basis in which all σ_i^z are diagonal),

$$\text{IPR}_j^{(loc)} = \sum_{i=1}^D |\langle c_i | \tilde{\mu}_j \rangle|^4, \quad (11)$$

where $\{c_i\}$ denotes the set of computational basis vectors. This quantity measures how localized each Ritz vector is in the computational basis. Averaging this IPR across all of the Ritz vectors for a given K_m ,

$$\text{IPR}^{(loc)} = \frac{1}{m} \sum_{j=1}^m \text{IPR}_j^{(loc)} \quad (12)$$

gives us a measure of how localized or delocalized the Krylov subspace is as a whole. Figure 4(b) shows this average for increasing Hilbert space dimension with red and blue lines corresponding to ergodic and DF drive strength parameters, respectively. Again, the two phases demonstrate differing localization properties. We observe a delocalized nature with $O(1/D)$ scaling of $\text{IPR}^{(loc)}$ in the ergodic regime and a localized nature in the dynamically frozen regime. The delocalization in the ergodic regime corresponds to the scaling expected from exact Floquet eigenstates.

This scaling was obtained by diagonalizing a smaller $m \times m$ matrix in a Krylov basis that contains highly nonthermal states (i.e., the time-evolved state at short times). On the level of the IPR, the Ritz vectors are statistically indistinguishable from exact Floquet eigenstates. Fitting $\text{IPR}^{(loc)}$ to a power law reveals a sharp crossover between the delocalized and localized regimes, as shown in Fig. 4(c). Near the transition (inset), the scaling deviates from the simple localized/delocalized limits, indicating multifractality, and this behavior appears to be sensitive to the Krylov subspace dimension m ; we leave a detailed study of this dependence for future work.

This analysis confirms that in the ergodic regime, the initial state explores a large part of the Krylov subspace, whilst in

the case of DF it remains frozen and explores little of the generated subspace. The former is consistent with the interpretation of ergodicity as delocalization in Krylov space [64,65].

V. DISCUSSION AND CONCLUSION

We here illustrated the efficiency of the Krylov approach to the DEA in the ergodic and dynamically frozen regime of a driven mixed-field Ising model.

Beyond this specific application, we additionally show in the Appendix that our algorithm can be used to efficiently compute the DEA for a Floquet system in the many-body localized (MBL) regime, where the usual analysis with exact diagonalization again severely limits the size of the systems that can be studied. Taken together, these findings suggest the Ritz vectors constitute a powerful diagnostic for characterizing ergodicity-breaking transitions, while also providing a compact, reduced Hilbert space within which the system dynamics can be accurately described across all time scales. Our focus has been on the application to periodically driven systems, but the underlying Krylov-space projection scheme proposed here is not Floquet-specific. In particular, analogous constructions can be applied to time-independent (static) Hamiltonians.

The efficiency of the Krylov approach can be directly related to the physics of the different regimes. In the dynamically frozen regime, where one may expect the time-evolved state to remain close to the initial state, it is natural to expect that Krylov methods perform well. By applying the unitary Floquet operator to the initial state, one quickly finds the relevant few eigenstates accounting for a correction in the DEA on the trivial value given by the initial state. Here, we essentially build a subspace that is highly nonthermal and find that our initial state remains tightly localized within a small part of this subspace. In the ergodic regime, however, one may expect that the infinite-time average requires knowledge of all

the Floquet eigenstates of the system to accurately compute the DEA. Here, the Krylov subspace algorithm still performs well, but for different reasons. In this regime, ergodicity actually plays to our advantage. While the initial state indeed has overlap with many of the Floquet eigenstates, the ETH dictates that these eigenstates are themselves thermal, and quantum typicality implies that one can compute observables using a small subset of (approximate) eigenstates. We find that our algorithm is capable of accurately generating representative thermal states of the system within a number of iterations, and once the Krylov space becomes large enough to reflect these states, the DEA essentially averages over the expectation values of these thermal states; the Krylov subspace is highly thermal and the initial state rapidly explores this subspace, as seen by the respective IPRs.

These results motivate the use of the Ritz vectors as an appropriate set of dynamical eigenstates in both ergodic and nonergodic dynamics. Given a set of such states, it is now possible to apply further probes of quantum chaos through, e.g., studies of their entanglement [66–71] or fidelity susceptibility [72–75], or study the effect of conservation laws or kinetic constraints. This approach can also be contrasted with matrix product state (MPS)-based methods [76–79], which exhibit entanglement barriers before thermalizing and do not provide a complete eigenbasis in which the dynamics can be described. This work fits within the broader topic of studying quantum chaos and ergodicity using Krylov approaches [64,65,80–91]. Rather than focusing on the growth of and (de)localization within the Krylov subspace, we here consider *Floquet eigenstates* constructed in this Krylov subspace. We show how these are able to capture early- and late-time dynamics in both ergodic and frozen systems, and present a reduced basis through which quantum many-body dynamics can be efficiently and systematically studied, here exemplified through the DEA.

We have put forward an efficient Krylov subspace algorithm for computing the infinite-time average of observables in periodically driven systems capable of going beyond the size limitations of exact diagonalization. We have demonstrated the effectiveness of the algorithm in computing the DEA of the total magnetization density in a model exhibiting a transition to a dynamically frozen regime. The nature of the Krylov subspaces has been characterized and it was shown that in the ergodic phase, our subspace is highly thermal, while in the dynamically frozen phase it quickly generates the relevant corrections to the trivial DEA given by the initial state. Lastly, we have found that the algorithm succeeds in computing the DEA for a Floquet-MBL system, and we expect it to prove useful in a variety of other exciting settings relevant to the study of Floquet matter.

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DATA AVAILABILITY

The data that support the findings of this article are openly available [93].

APPENDIX: END MATTER

1. Arnoldi iteration algorithm

To evaluate the DEA approximation $\langle \mathcal{O} \rangle_{\text{DEA},m}$ numerically we propose an algorithm employing the Arnoldi iteration [94]. Given the initial vector $|\psi_0\rangle$ we iteratively construct an orthonormal basis $\{|v_k\rangle\}_{k=1,\dots,m}$ of the m th Krylov space $\mathcal{K}_m$ using the Arnoldi recursion,

$$\begin{aligned} |v_1\rangle &= |\psi_0\rangle / \|\psi_0\|, \\ |w_k\rangle &= \mathcal{U} |v_k\rangle, \\ |\hat{v}_{k+1}\rangle &= |w_k\rangle - \sum_{i=1}^k \langle v_i | w_k \rangle |v_i\rangle, \\ |v_{k+1}\rangle &= |\hat{v}_{k+1}\rangle / \|\hat{v}_{k+1}\|, \end{aligned} \quad (\text{A1})$$

which can be understood as the successive Gram-Schmidt orthogonalization of the spanning vectors $\mathcal{U}^k |\psi_0\rangle$. Combining the Arnoldi vectors $|v_k\rangle$ as columns of the isometric matrix $V_m = (v_1 | \dots | v_m)$ we write the recursion Eq. (A1) as,

$$\mathcal{U} V_m = V_{m+1} \hat{U}_m, \quad (\text{A2})$$

where we introduce a $(m+1) \times m$ matrix $\hat{U}_m$,

$$\hat{U}_m = \begin{pmatrix} u_{1,1} & u_{1,2} & u_{1,3} & \cdots & u_{1,m} \\ u_{2,1} & u_{2,2} & u_{2,3} & \cdots & \vdots \\ 0 & u_{3,2} & u_{3,3} & & \vdots \\ 0 & 0 & \ddots & \ddots & u_{m-1,m} \\ \vdots & & 0 & u_{m,m-1} & u_{m,m} \\ 0 & \cdots & 0 & 0 & u_{m+1,m} \end{pmatrix}, \quad (\text{A3})$$

where $u_{i,j} = \langle v_i | w_j \rangle$. The $m \times m$ submatrix U_m obtained from the first m rows of $\hat{U}_m$ has zero elements $u_{i,j}$ whenever $i > j+1$, and is commonly referred to as a Hessenberg matrix. The i -th eigenvalue of U_m is given by $\tilde{\xi}_i$ as in Eq. (6), whereas the corresponding Ritz vectors are given by $|\tilde{\mu}_i\rangle = V_m v_i$, where $v_i \in \mathbb{C}^m$ denotes the i th eigenvector of U_m . Finally, the approximate DEA $\langle \mathcal{O} \rangle_{\text{DEA},m}$ is computed by evaluating Eq. (5). Notice that the definition of $\langle \mathcal{O} \rangle_{\text{DEA},m}$ in Eq. (5) is independent of the choice of orthonormal basis of $\mathcal{K}_m$.

2. Algorithmic choices

In the above, we outlined an iterative algorithm for computing the DEA of local observables. However, in projecting the unitary time evolution onto the Krylov subspace, unitarity is lost. This means the corresponding matrix U_m is also no longer unitary and no longer shares the same eigenvectors as

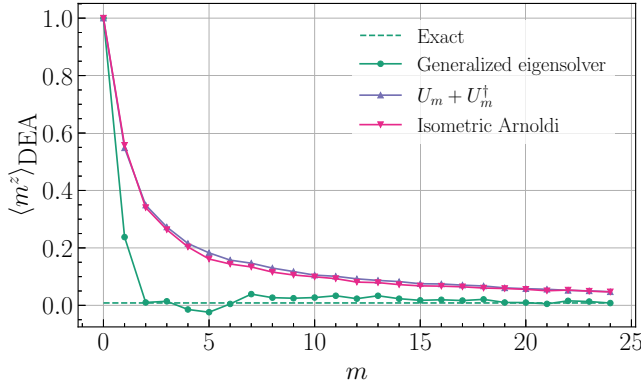


FIG. 5. We compare different numerical approaches to computing the DEA with our Krylov subspace algorithm. Dashed line denotes the exact DEA for the magnetization density on a spin-chain with 12 sites and $H_d/J = 0.5$. Generalized eigensolver (green line) denotes the algorithm choice used in the main text where we take the right eigenvectors of U_m . The purple line shows the DEA computed with the eigenvectors of $U_m + U_m^\dagger$ and the pink line shows the DEA computed using an isometric Arnoldi procedure.

$U_m + U_m^\dagger$. One way to remedy this is to rescale the final column of the matrix U_m in a procedure known as the isometric Arnoldi iteration [56] to obtain a canonical unitary approximation to U_m . Another option is to compute the eigenvectors of $U_m + U_m^\dagger$. Lastly, a generalized eigensolver routine can be used to find the left and right eigenvectors of U_m . The last approach will in general lead to a loss of orthonormality of the right eigenvectors. One can make use of the biorthogonality between the two sets of vectors, however, this does not guarantee a real value of the DEA. Instead we work with just the right eigenvectors and introduce a normalization to the DEA in Eq. (7). We compare each of these three approaches to numerically computing the eigenvectors of the projected Floquet operator in Fig. 5. We show the same magnetization computed in the main text for a spin-chain of 12 sites during the first 25 iterations of the algorithm. We find that both the isometric Arnoldi routine as well as the eigenvectors from $U_m + U_m^\dagger$

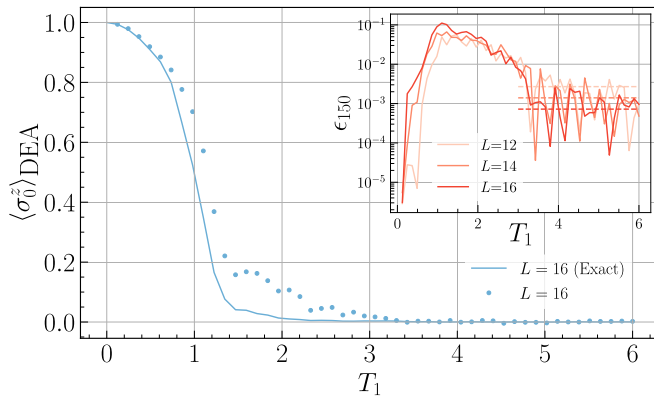


FIG. 6. Krylov subspace computation of DEA for MBL in a periodically driven system. We show the DEA for local magnetization, both exactly and following 150 iterations of our algorithm on a chain of 16 sites. The inset shows the corresponding error for several system sizes.

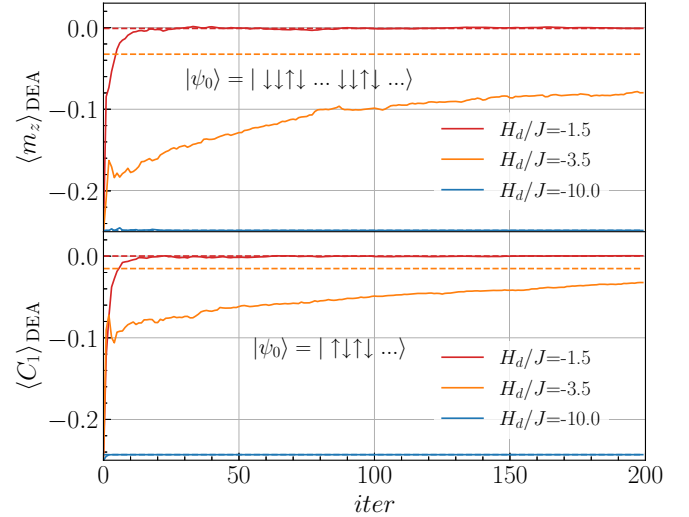


FIG. 7. Benchmarks of the DEA approximation for different initial states and observables. Left: magnetization DEA computed for a mid-spectrum product state for representative ergodic, frozen, and near-transition drive strengths, showing the convergence of the projected DEA during the first iterations of the algorithm. Right: nearest-neighbor correlator DEA (e.g., $\langle \sigma_1^z \sigma_2^z \rangle$) computed for an initial Néel state for the same drive strengths on a 16-site chain. These benchmarks demonstrate that the method remains effective across multiple initial conditions and observables.

lead to long tails and poor convergence for an ergodic choice of parameters, $H_d/J = 0.5$. We see a notably faster convergence to the ergodic DEA value (dashed line) when using the right eigenvectors of U_m and using the normalization in Eq. (7) for the DEA as used in the main text. We note that in the DF frozen regime there is no loss of orthonormality, the normalization $\mathcal{N}$ is unity and all approaches give consistent results.

3. Floquet-MBL

In the main text, we employed our Krylov subspace algorithm for a system with dynamical freezing. We now demonstrate its effectiveness for a periodically driven system in the presence of disorder and compute the DEA of local observables across an expected MBL transition. The following periodic drive is prescribed:

$$H(t) = \begin{cases} J_x \sum_i \sigma_i^x \sigma_{i+1}^x + \sigma_i^y \sigma_{i+1}^y & \text{for } 0 \leq t \leq T_1, \\ \sum_i h_i \sigma_i^z + J_z \sum_i \sigma_i^z \sigma_{i+1}^z & \text{for } T_1 < t < (T_1 + T_0), \end{cases} \quad (\text{A4})$$

with period $T = T_0 + T_1$. The h_i are chosen uniformly from an interval $[-W, W]$, $J_x = J_z = \frac{1}{4}$, $T_0 = 1$, $W = 2.5$, as set out in [54] with open boundaries. The former and latter parts of the drive serve delocalizing and localizing purposes, respectively. Starting from an initial Néel product state $|\psi_0\rangle = |\uparrow\downarrow\uparrow\downarrow \dots\rangle$ we compute the DEA of the local magnetization on a given site as the delocalizing time T_1 is varied. Figure 6 shows the long-time average of the local magnetization computed both exactly and after just 150 iterations of our algorithm for a chain of length 16 sites. The algorithm performs best in the

two limits of small T_1 , where the eigenstates are localized, and large T_1 , where the phase is ergodic. The inset shows the corresponding error for several system sizes. Similarly to the case of DF, we observe a large compression of the error down to 10^{-5} in the localized regime and an error that seems consistent with the scaling $\sqrt{mD}$ in the ergodic regime. While near the transition the convergence suffers, our algorithm is nonetheless able to successfully capture a localized to delocalized transition and allow for both the study of larger systems and further diagnostics of the transition based on the knowledge of the Ritz vectors.

4. Benchmarks: Additional initial states and observables

To further validate the robustness of the Krylov-space projected diagonal-ensemble approach, we benchmark the DEA approximation for additional initial states and observables beyond those shown in the main text, which we show in Fig. 7. In particular, we consider (i) a generic mid-spectrum product state and compute the DEA of the magnetization, and (ii) an initial Néel state and compute nearest-neighbor correlators, for the same representative drive strengths used in the main text (ergodic, dynamically frozen, and near the crossover).

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