

Work statistics and quantum trajectories: No-click limit and non-Hermitian Hamiltonians

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We investigate quantum work statistics within the standard two-point measurement scheme in continuously monitored quantum systems, including the effects of generalized unitary evolution, possibly controlled by quantum circuit models, and multiple generalized measurements as well as postselection of *no-click* trajectories. We derive an explicit expression for the work generating function that naturally incorporates non-Hermitian dynamics arising from quantum jump processes and reveals deviations from the standard Jarzynski equality due to measurement-induced asymmetries. We illustrate our theoretical framework by analyzing a one-dimensional transverse-field Ising model under local spin monitoring. In this model, increased measurement strength projects the system onto the no-click state, leading to a suppression of energy fluctuations and measurement-induced energy saturation, reminiscent of the quantum Zeno effect. Moreover, we find signatures of the measurement-induced transition observed in the no-click limit in the moments of the work distribution.

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I. INTRODUCTION

Quantum thermodynamics is a rapidly developing interdisciplinary field, bridging quantum mechanics, statistical physics, and information theory, investigating how thermodynamic concepts extend to quantum systems operating far from equilibrium [1–5]. A central aspect of quantum thermodynamics involves the extension of classical stochastic thermodynamics [6] to account for quantum fluctuations, i.e., fluctuations of entropy, heat, and work at the quantum level, where quantum coherence, entanglement, and measurement back-action become fundamentally relevant [7–10]. Both classical fluctuation theorems and constraints such as the Jarzynski equalities can be extended to the quantum regime, provided that the definitions of heat, entropy, and work are appropriately refined in the quantum context. For example, focusing on quantum work statistics, one typically employs the two-point measurement scheme, which relies on initial and final energy measurements after a Hamiltonian evolution [4,7,11,12]. Starting with the formulation above, one can extend it to account for other generalized measurements during the evolution [13–15], to evolutions generated by unital and nonunital channels [16–19], and to special classes of non-Hermitian systems [20–23].

Focusing on non-Hermitian systems, previous research on the subject was devoted to the classification of the conditions under which the Jarzynski equalities [20,23], thermodynamic bounds [21], and fluctuation theorems [22,23] are still valid. Pseudo-Hermiticity and $\mathcal{PT}$ symmetry [24] appear to be the key to maintain the validity of quantum thermodynamics in the standard formulation. It is, however, important to note that, frequently, non-Hermitian Hamiltonians describe the evolution of open systems subject to continuous measurement [25]

once we postselect a special, *no-click* trajectory [26,27]. In this setting, instead of being guided by the consistency of the definition with fluctuation theorems, one may start directly from the definition of quantum trajectories [28] and the two-point measurement scheme to construct a notion of work statistics for non-Hermitian systems from scratch.

The purpose of this work is to investigate quantum work statistics (or, more appropriately, two-point energy statistics) for continuously monitored quantum systems, focusing in particular on no-click trajectories and their naturally arising non-Hermitian Hamiltonians. Our goal is to derive compact expressions for the work generating function in the non-Hermitian case arising from quantum jump processes and subsequent postselection. Introducing trajectories in a way similar to Ref. [28], we first clarify how to formulate appropriate work statistics in the presence of quantum trajectories generated by a generic unitary dynamics (including quantum circuits [29]) and interspersed generalized measurement. Once a no-click trajectory is singled out, we discuss what the expected modifications on the Jarzynski equality are [30]. Finally, we demonstrate our formalism by analyzing the work statistics for a one-dimensional transverse-field quantum Ising model subject to local spin monitoring, a model that was shown to display a measurement-induced phase transition (MIPT) also in the no-click limit [26]. On the one hand, our study reveals phenomena such as measurement-induced energy saturation and reduction in fluctuations, reflecting the effective confinement of the system to specific quantum states. These effects, reminiscent of the quantum Zeno effect [31], highlight how continuous observation can effectively freeze the system dynamics, drastically altering the energy exchange processes and associated fluctuations. On the other hand, we observe a signature of the MIPT occurring in the steady-state entanglement as singularities of the average work done at the phase boundary, revealing the potential of this quantity to reveal interesting many-body effects.

The rest of this paper is organized as follows. In Sec. II, we introduce the notion of quantum work statistics for quantum trajectories, subsequently focusing on quantum jumps and

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the no-click trajectory described by non-Hermitian quantum dynamics. In Sec. III, we present our analysis and results for the quantum work statistics in a monitored transverse-field Ising chain. Finally, we summarize our findings and conclude in Sec. IV.

II. WORK STATISTICS, TRAJECTORIES, AND THE NO-CLICK LIMIT

Our first goal is to formulate two-point measurement work statistics [32,33] for monitored dynamics with intermediate generalized measurements and then to specialize the resulting expressions to the postselected no-click trajectory, for which the conditioned evolution is generated by an effective non-Hermitian Hamiltonian [34]. In principle, for systems subject to Hamiltonian dynamics and generalized measurements, the quantity we compute should be called two-point energy statistics, and particular attention should be paid to defining work and heat appropriately along the trajectories [28]. Below we will, however, be dealing with generalized unitary dynamics as well, possibly generated by a unitary circuit of gates acting on a qubit register; therefore, in order to make connections with previous literature on the subject, in particular on non-Hermitian problems [20,21,23,35], we will use the term “work statistics” throughout the paper.

While previous attempts to define this quantity focused either on very specific non-Hermitian Hamiltonians ($\mathcal{PT}$ symmetric with the real spectrum) [20,21,23] or on the modification of the standard two-measurement scheme [35], the path that we will follow will be different. Indeed, one way to realize systematically the evolution with a non-Hermitian Hamiltonian is to consider a system subject to quantum jumps and postselect the so-called no-click trajectory [25,26], the exponentially rare trajectory that corresponds to a null result at every measurement attempt. Defining work statistics for such a trajectory requires extending the notion of work statistics to systems subject to multiple generalized measurements [13,36]. Once this is done, the work statistics for a non-Hermitian evolution is obtained by just postselecting the contribution of the no-click trajectory. This is the overall direction that we will follow in this section, where we also discuss in detail step by step the validity and significance of the constraint given by the Jarzynski equality [30,33] and why we should not expect, in general, the work statistics associated with a non-Hermitian evolution to satisfy such a constraint.

A. Work statistics and generalized measurements

The standard protocol to define work statistics in thermally isolated quantum systems consists of taking the system along a quantum trajectory defined by initial and final energy measurements (relative to initial, H_i , and final, H_f , Hamiltonians). If $p_i(n)$ and $p_f(m|n)$ are the probabilities of measuring the initial energy $E_i(n)$ and the final one $E_f(m)$ (conditional to the initial energy measurement), the work probability distribution is defined as [11,30,32]

$$P(W) = \sum_{n,m} p_i(n) p_f(m|n) \delta\{W - [E_f(m) - E_i(n)]\}. \quad (1)$$

The initial measurement is usually taken with respect to an equilibrium distribution at inverse temperature β , i.e., $p_i(n) =$

$\exp[-\beta E_i(n)]/Z_i$, where Z_i is the initial partition function. The conditional probability $p_f(m|n)$ in turn depends on the specifics of the evolution and of the processes occurring between the two measurements.

For a generic coherent evolution represented by the unitary operator U_{t_f, t_i} , we have [32]

$$p_f(m|n) = |\langle \psi_f(m) | U_{t_f, t_i} | \psi_i(n) \rangle|^2. \quad (2)$$

Coherent dynamics can be generated through either a unitary quantum circuit [29] or standard Hamiltonian dynamics. In the latter case, if a parameter $\lambda(t)$ is varied in time between an initial value $\lambda_i = \lambda(t_i)$ and a final value $\lambda_f = \lambda(t_f)$, we have

$$U_{t, t_0} = \mathcal{T} \exp\left(-i \int_{t_0}^t dt' H(\lambda(t'))\right), \quad (3)$$

where $\mathcal{T}$ represents the time-ordering operator. In the case of unitary circuits U will be defined by a sequence of unitary gates acting on a register of qubits. If during the evolution the system is also subject to a single measurement of some quantity with possible outcomes $\{r\}$, the expression above changes [13]. Let us first represent the *generalized* measurement by a set of measurement operators M_r subject to the constraint $\sum_r M_r^\dagger M_r = \mathbb{1}$. The conditional probability $p_f(m|r|n)$ to obtain $E_f(m)$ as the final energy measurement, given that $E_i(n)$ was obtained in the initial measurement and r was obtained in the intermediate one, is

$$p_f(m|r|n) = \frac{|\langle \psi_f(m) | U_{t_f, t_r} M_r U_{t_r, t_i} | \psi_i(n) \rangle|^2}{p(r|n)}, \quad (4)$$

where t_r is the time at which the intermediate measurement occurs and

$$p(r|n) = \langle \psi_i(n) | U_{t_r, t_i}^\dagger M_r^\dagger M_r U_{t_r, t_i} | \psi_i(n) \rangle \quad (5)$$

is the probability of obtaining outcome r given $E_i(n)$ as the initial energy measurement. Since the work statistics does not keep track of the result of the intermediate measurement, the quantity entering Eq. (1) is the unconditional probability distribution

$$\begin{aligned} p_f(m|n) &= \sum_r p(r|n) p_f(m|r|n) \\ &= \sum_r |\langle \psi_f(m) | T_{t_f, t_i}(r, t_r) | \psi_i(n) \rangle|^2, \end{aligned} \quad (6)$$

where we introduced the operator $T_{t_f, t_i}(r, t_r) = U_{t_f, t_r} M_r U_{t_r, t_i}$.

Generalizing the expressions above to the case where instead of a single measurement we have multiple measurements with possible outcomes $\{r_j\}$ at times $\{t_j\}$ ($j = 1, \dots, N$), Eq. (6) becomes

$$p_f(m|n) = \sum_{\{r_j\}} |\langle \psi_f(m) | T_{t_f, t_i}(\{r_j, t_j\}) | \psi_i(n) \rangle|^2, \quad (7)$$

where now

$$T_{t_f, t_i}(\{r_i, t_i\}) = U_{t_f, t_N} M_{r_N} U_{t_N, t_{N-1}} \cdots M_{r_1} U_{t_1, t_i}. \quad (8)$$

The combination of Eq. (1) with the expression for the conditional probability in Eq. (7), in terms of the operator T_{t_f, t_i} given by Eq. (8), constitutes the most general expression for work statistics for quantum trajectories generated by generalized measurements and generic unitary dynamics.

B. Generating function and Jarzynski equalities

In order to further extend our analysis and specialize it to the case of no-click trajectories, let us now consider the generating function of the work statistics, given by

$$\mathcal{G}(u) = \int dW P(W) e^{-iWu}. \quad (9)$$

Using Eqs. (1)–(8), one obtains

$$\mathcal{G}(u) = \sum_{\{r_j\}} \text{Tr} \left[T_{t_f, t_i}^\dagger(\{r_j, t_j\}) e^{-iH_f u} T_{t_f, t_i}(\{r_j, t_j\}) e^{iH_i u} \frac{e^{-\beta H_i}}{Z_i} \right], \quad (10)$$

which is the general expression for the generating function of the work statistics. While the identity

$$\sum_{\{r_j\}} T_{t_f, t_i}^\dagger(\{r_j, t_j\}) T_{t_f, t_i}(\{r_j, t_j\}) = \mathbb{1} \quad (11)$$

guarantees the normalization of the probability distribution $P(W)$, since $\mathcal{G}(u=0) = \int dW P(W) = 1$, the Jarzynski equality is obtained by setting

$$\mathcal{G}(-i\beta) = \frac{\text{Tr}[(\sum_{\{r_j\}} T_{t_f, t_i}(\{r_j, t_j\}) T_{t_f, t_i}^\dagger(\{r_j, t_j\})) e^{-\beta H_f}]}{Z_i},$$

from which we see that, provided [16]

$$\sum_{\{r_j\}} T_{t_f, t_i}(\{r, t_i\}) T_{t_f, t_i}^\dagger(\{r, t_i\}) = \mathbb{1}, \quad (12)$$

we obtain

$$\langle e^{-\beta W} \rangle = \mathcal{G}(-i\beta) = \frac{Z_f}{Z_i} = e^{-\beta \Delta F}. \quad (13)$$

Equation (12) is the condition of unitality of the quantum channel, representing the dynamics of our system [16–18]. Let us now discuss the physical meaning of the condition. Unitality is obvious for unitary evolutions, while the presence of generalized measurements implies that the constraint in Eq. (12) is a consequence of the requirement that measurement operators satisfy

$$\sum_r M_r M_r^\dagger = \mathbb{1}. \quad (14)$$

Since a measurement with outcome r modifies the density matrix ρ_0 according to the relation

$$\rho_r = \frac{M_r \rho_0 M_r^\dagger}{p_r}, \quad (15)$$

where $p_r = \langle M_r^\dagger M_r \rangle$, the condition in Eq. (12) implies that the mapping for *unconditional* evolution of a density matrix after a single measurement

$$\rho = \sum_r p_r \rho_r = \sum_r M_r \rho_0 M_r^\dagger \quad (16)$$

is itself *unital*.

While most of the measurement operators (projectors, quantum diffusion, quantum jumps, etc.) do satisfy these conditions, it does not come as a surprise that one can easily find a set that does not. Consider, for example, a set of measurement outcomes $r = 0, \dots, M$ and associate with them the states $|r\rangle$

in the Hilbert space ($\sum_r |r\rangle\langle r| = \mathbb{1}$). The operators

$$M_r = |0\rangle\langle r|, \quad (17)$$

describing a measurement after which the state is reset to $|0\rangle$ regardless of the outcome, satisfy the normalization condition of measurement operators but not Eq. (14).

C. No-click trajectories and non-Hermitian physics

We are now in the position to specialize Eq. (10) to the case of quantum jumps and, in particular, to no-click trajectories. Following Ref. [37], let us discretize the time span $[0, t]$ into N small steps of duration δt such that $\delta t = t/N$ and perform, in each interval, a measurement with the generalized operators

$$M_0(\delta t) = \mathbb{1} - \left(\frac{R}{2} + iH \right) \delta t, \quad (18)$$

$$M_1(\delta t) = \sqrt{\delta t} c, \quad (19)$$

where $R = c^\dagger c$ and H is a Hermitian operator representing the unitary evolution. Note that the condition in Eq. (14) is satisfied provided $c^\dagger c = c c^\dagger$, which is the case, for example, when $c = |1\rangle\langle 1|$ is a projector.

It is now rather straightforward to write down the generating function specialized to this set of operators. Our goal now is to specialize the work statistics to a single trajectory; that is, rather than computing the unconditional $P(W)$, we focus on the trajectory where the measurement gives systematically no result. In this case, we specialize Eq. (8) to the no-click record, i.e., $r_j = 0$ for all $j = 1, \dots, N$, and assume equally spaced monitoring times t_j . As the no-click Kraus operator $M_0(\delta t)$ in Eq. (18) already incorporates the system's Hermitian Hamiltonian H , the unitary segments $U_{t_j, t_{j+1}} = e^{-iH\delta t}$ appearing in Eq. (8) are effectively absorbed into $M_0(\delta t)$ [25,37]. With this convention and taking the continuum limit $N \rightarrow \infty$ ($\delta t \rightarrow 0$), the evolution operator for the no-click trajectory is given by

$$\begin{aligned} T_{\{0\}} &= \lim_{N \rightarrow \infty} [M_0(\delta t)]^N \\ &= \lim_{N \rightarrow \infty} \left[\mathbb{1} - \left(\frac{R}{2} + iH \right) \frac{t}{N} \right]^N \\ &= e^{-iH_{\text{eff}} t}, \end{aligned} \quad (20)$$

where $H_{\text{eff}} = (H - iR/2)$. The last equality follows from the standard limit definition of the matrix exponential.

Therefore, the generating function of the work statistics for the no-click trajectory is

$$\mathcal{G}_{\{0\}} = \frac{\text{Tr}[e^{iH_{\text{eff}}^\dagger t} e^{-iH_f u} e^{-iH_{\text{eff}} t} e^{iH_i u} \rho_i]}{\text{Tr}[e^{iH_{\text{eff}}^\dagger t} e^{-iH_{\text{eff}} t} \rho_i]}, \quad (21)$$

where we normalize by the probability of occurrence of the no-click trajectory. While we still have $\mathcal{G}_{\{0\}}(u=0) = 1$, the Jarzynski equality is not satisfied; instead, we have

$$\mathcal{G}_{\{0\}}(-i\beta) = \frac{\text{Tr}[e^{-iH_{\text{eff}} t} e^{iH_f^\dagger t} e^{-\beta H_f}]}{\text{Tr}[e^{iH_{\text{eff}}^\dagger t} e^{-iH_{\text{eff}} t} e^{-\beta H_i}]}, \quad (22)$$

which is formally similar but does not correspond to a thermodynamic equilibrium quantity. We can, nevertheless, cast it in a familiar form:

$$\langle e^{-\beta(W - \Delta F)} \rangle = \gamma_t, \quad (23)$$

where the *efficacy* [18] is

$$\gamma_t = \frac{\langle e^{-iH_{\text{eff}}t} e^{iH_{\text{eff}}^\dagger t} \rangle_f}{\langle e^{iH_{\text{eff}}^\dagger t} e^{-iH_{\text{eff}}t} \rangle_i}, \quad (24)$$

with $\langle \cdot \rangle_{f,i} = \text{Tr}[\cdot \rho_{f,i}]$. Notice that the efficacy characterizes the asymmetry between forward and backward evolution arising from the noncommutativity of the Hamiltonian and its adjoint, i.e., $[H_{\text{eff}}, H_{\text{eff}}^\dagger] \neq 0$.

Although projective energy measurements are impractical, even in many-body systems the modified Jarzynski equality can still be tested by measuring the work generating function with a Ramsey interferometer using an ancilla qubit, then Fourier transforming it to obtain the work distribution. This approach was proposed in single-qubit interferometry schemes [38,39] and demonstrated in NMR and trapped-ion experiments [40,41]. In our monitored setting the same interferometric protocol is run conditioned on a no-click record of the continuous measurement. Concretely, we take an ancilla qubit $\mathcal{A}$ prepared in $(|0\rangle + |1\rangle)/\sqrt{2}$ and the system in $|\Psi_0\rangle$, the ground state of H_i . Postselecting the no-click trajectories, the system evolves under the non-Hermitian propagator $e^{-iH_{\text{eff}}t}$ for time t , so the joined state becomes

$$|\Phi\rangle = \frac{|0\rangle \otimes e^{-iH_{\text{eff}}t} |\Psi_0\rangle + |1\rangle \otimes e^{-iH_{\text{eff}}t} |\Psi_0\rangle}{\sqrt{2\langle \Psi_0 | e^{iH_{\text{eff}}^\dagger t} e^{-iH_{\text{eff}}t} | \Psi_0 \rangle}},$$

where the no-click success probability $\langle \Psi_0 | e^{iH_{\text{eff}}^\dagger t} e^{-iH_{\text{eff}}t} | \Psi_0 \rangle$ in the denominator normalizes $|\Phi\rangle$. Now applying a controlled operation $e^{-iH_f u}$ on the system conditioned on ancilla $\mathcal{A} = |1\rangle$ gives

$$|\Phi_1\rangle = \frac{|0\rangle \otimes e^{-iH_{\text{eff}}t} |\Psi_0\rangle + |1\rangle \otimes e^{-iH_f u} e^{-iH_{\text{eff}}t} |\Psi_0\rangle}{\sqrt{2\langle \Psi_0 | e^{iH_{\text{eff}}^\dagger t} e^{-iH_{\text{eff}}t} | \Psi_0 \rangle}}.$$

A Ramsey readout on $\mathcal{A}$ then yields the postselected generating function

$$\mathcal{G}_{\{0\}}(u, t) = \frac{\langle \Psi_0 | e^{iH_{\text{eff}}^\dagger t} e^{-iH_f u} e^{-iH_{\text{eff}}t} | \Psi_0 \rangle}{\langle \Psi_0 | e^{iH_{\text{eff}}^\dagger t} e^{-iH_{\text{eff}}t} | \Psi_0 \rangle} e^{iE_0^i u}. \quad (25)$$

The overall phase $e^{iE_0^i u}$ reflects the known action of $e^{iH_f u}$ on the prepared eigenstate and can be incorporated by postprocessing.

III. RESULTS FOR A MONITORED QUANTUM ISING MODEL

We are now ready to study the work statistics for a one-dimensional transverse-field Ising model subject locally to the monitoring of the up component of the transverse spin. In the no-click limit, the effective Hamiltonian describing the dynamics is

$$H_{\text{eff}}[h, \gamma] = -J \sum_{i=1}^L \hat{\sigma}_i^z \hat{\sigma}_{i+1}^z - \left(h + i\frac{\gamma}{4}\right) \sum_{i=1}^L \hat{\sigma}_i^x, \quad (26)$$

where $\hat{\sigma}^\alpha$, with $\alpha \in \{x, y, z\}$, denote the Pauli spin matrices, h represents the transverse field, and γ is the measurement rate. Using the Jordan-Wigner transformation, Eq. (26) can be diagonalized exactly in terms of free fermions [26]. The model is known to display a MIPT at a critical value of

$\gamma_c = 4\sqrt{1-h^2}$ between a logarithmic entanglement phase ($\gamma < \gamma_c$) and an area law phase ($\gamma > \gamma_c$). Below we show that signatures of this criticality are also shown in work statistics.

In order to define work statistics, we must specify the *initial* and *final* Hamiltonians. In the following, we will, for simplicity, take $H_i = H_f = H_{\text{eff}}[h, \gamma = 0]$ and therefore study the work statistics originating solely from the introduction of the measurement. If the dynamics starts from the ground state $|\Psi_0\rangle$ of H_i , the characteristic function can be written as

$$\mathcal{G}_{\{0\}}(u, t) = \frac{\langle \Psi_0 | e^{iH_{\text{eff}}^\dagger t} e^{-iH_f u} e^{-iH_{\text{eff}}t} e^{iH_i u} | \Psi_0 \rangle}{\langle \Psi_0 | e^{iH_{\text{eff}}^\dagger t} e^{-iH_{\text{eff}}t} | \Psi_0 \rangle}. \quad (27)$$

Within the free fermionic framework, H_i can be expressed in terms of the Bogoliubov quasiparticle operator η_k , which diagonalizes it in k space, as

$$H_i = \sum_{k>0} \epsilon_k^i (\eta_k^\dagger \eta_k + \eta_{-k}^\dagger \eta_{-k}) + E_0^i, \quad (28)$$

where $E_0^i = -\sum_{k>0} \epsilon_k^i$ is the ground state energy and $\epsilon_k^i = 2\sqrt{(h - J \cos k)^2 + J^2 \sin^2 k}$ denotes the dispersion of quasiparticles, while the vacuum $|\Psi_0\rangle$ satisfies the relation $\eta_k |\Psi_0\rangle = 0$.

We can proceed in a similar way for both H_{eff} and $H_{\text{eff}}^\dagger$, which can be written as [26]

$$H_{\text{eff}} = \sum_{k>0} \epsilon_k^{\text{eff}} (\gamma_k^* \gamma_k + \gamma_{-k}^* \gamma_{-k}) + E_0^{\text{eff}}, \quad (29a)$$

$$H_{\text{eff}}^\dagger = \sum_{k>0} \epsilon_k^{\text{eff}*} (\tilde{\gamma}_k^* \tilde{\gamma}_k + \tilde{\gamma}_{-k}^* \tilde{\gamma}_{-k}) + (E_0^{\text{eff}})^*. \quad (29b)$$

The complex eigenvalues are given by $\epsilon_k^{\text{eff}} = 2\sqrt{(h - J \cos k + i\gamma/4)^2 + J^2 \sin^2 k} \equiv \lambda_k + i\Gamma_k$ with a nonpositive imaginary part $\Gamma_k \leq 0$, with $\epsilon_k^{\text{eff}*}$ being its complex conjugate. Here, γ_k and $\tilde{\gamma}_k$ represent the non-Hermitian quasiparticles satisfying $\gamma_k |\emptyset\rangle = 0$ and $\langle \emptyset | \tilde{\gamma}_k^* = 0$, where $|\emptyset\rangle$ and $\langle \emptyset |$ denote the right and left non-Hermitian vacuum states, respectively.

Our first aim is to express $|\Psi_0\rangle$ in terms of the non-Hermitian quasiparticle γ_k that diagonalizes H_{eff} . This can be achieved by rewriting η_k in terms of γ_k (see Appendix A for details) as follows:

$$\eta_k = X_k \gamma_k - iY_k \gamma_{-k}^*. \quad (30)$$

Detailed expressions for these terms are provided in Appendix A. Therefore, $|\Psi_0\rangle$ and $\langle \Psi_0 |$ can be expressed as

$$|\Psi_0^\gamma\rangle = \frac{1}{\mathcal{N}} \prod_{k>0} (|\emptyset\rangle + i\alpha_k |k, -k\rangle), \quad (31a)$$

$$\langle \Psi_0^{\tilde{\gamma}} | = \frac{1}{\mathcal{N}} \prod_{k>0} (\langle \emptyset | - i\alpha_k^* \langle k, -k |). \quad (31b)$$

Here, we defined $\alpha_k = Y_k/X_k$, with the state $|k, -k\rangle$ constructed as $\gamma_k^* \gamma_{-k}^* |\emptyset\rangle$ and the corresponding dual state $\langle k, -k | = \langle \emptyset | \tilde{\gamma}_{-k} \tilde{\gamma}_k$. The normalization constant $\mathcal{N}$ is determined by evaluating $\langle \emptyset | \emptyset \rangle$ and $\langle k, -k | k, -k \rangle$ based on the relationship between γ_k and $\tilde{\gamma}_k$. Now, by applying the evolution operators $e^{-iH_{\text{eff}}t}$ and $e^{iH_{\text{eff}}^\dagger t}$ as outlined in Eq. (27), we

obtain the characteristic function in the k basis as follows:

$$\mathcal{G}_{\{0\}} = \frac{\langle \Psi_0^\gamma(t) | e^{-iH_{\text{eff}}t} | \Psi_0^\gamma(t) \rangle e^{iE_0^u t}}{\langle \Psi_0^\gamma(t) | \Psi_0^\gamma(t) \rangle}, \quad (32)$$

where

$$|\Psi_0^\gamma(t)\rangle = \frac{e^{-iE_0^{\text{eff}}t}}{\mathcal{N}} \prod_{k>0} (|\emptyset\rangle + i\alpha_k e^{-2i\epsilon_k^{\text{eff}}t} |k, -k\rangle), \quad (33a)$$

$$\langle \Psi_0^\gamma(t) | = \frac{e^{iE_0^{\text{eff}*}t}}{\mathcal{N}} \prod_{k>0} (\langle \emptyset | - i\alpha_k^* e^{2i\epsilon_k^{\text{eff}*}t} \langle k, -k |). \quad (33b)$$

Below, for concreteness, we focus on determining both the mean and the fluctuations of the work, $\langle W \rangle = -i\partial_u \log(\mathcal{G}_{\{0\}})$ and $\langle \Delta W^2 \rangle = -\partial_u^2 \log(\mathcal{G}_{\{0\}})$, respectively, from the following relations:

$$\langle W \rangle = -E_0^i + \frac{\langle \Psi_0^\gamma(t) | H_f | \Psi_0^\gamma(t) \rangle}{\langle \Psi_0^\gamma(t) | \Psi_0^\gamma(t) \rangle}, \quad (34)$$

$$\langle \Delta W^2 \rangle = \frac{\langle \Psi_0^\gamma(t) | (H_f)^2 | \Psi_0^\gamma(t) \rangle}{\langle \Psi_0^\gamma(t) | \Psi_0^\gamma(t) \rangle} - \left(\frac{\langle \Psi_0^\gamma(t) | H_f | \Psi_0^\gamma(t) \rangle}{\langle \Psi_0^\gamma(t) | \Psi_0^\gamma(t) \rangle} \right)^2. \quad (35)$$

Since both Eqs. (33a) and (33b) are expressed in terms of the left-right quasiparticle states of the system, it is clear that we should reexpress H_f in terms of γ_k and $\tilde{\gamma}_k$ as well, paying particular attention to maintaining the explicit Hermiticity of H_f . While H_f is itself Hermitian, expressing it solely through γ_k or $\tilde{\gamma}_k$ introduces non-Hermiticity. To address this, we employ a symmetrized version: $H_f(\gamma_k, \tilde{\gamma}_k) = \{H_f(\gamma_k) + H_f^\dagger(\tilde{\gamma}_k)\}/2$, which is Hermitian by construction. Detailed expressions can be found in Appendix A.

For trajectories within the transient (finite t) Figs. 1(a) and 1(b) illustrate how the average work density $\langle w \rangle$ (with $w = W/L$) and its variance $\langle \Delta w^2 \rangle$ change as the measurement strength γ increases. When $\gamma = 0$, both $\langle w \rangle$ and $\langle \Delta w^2 \rangle$ are zero because the Hamiltonians H_f and H_{eff} coincide. However, as γ increases, the non-Hermitian dynamics gradually projects the system onto the no-click (spin-down) state. This projection leads to a reduction in fluctuations and the saturation of the work performed on the system since further measurements cannot add energy once the system is sufficiently aligned with the no-click state. This behavior is analogous to the quantum Zeno effect, where frequent “observations” effectively confine the system to a specific subspace.

If instead we focus on the time-independent steady-state analysis of this model [27], we observe signatures of the transition from the logarithmically entangled phase to the area law phase, which is set by the opening of an imaginary gap in the complex spectrum of the work statistics. Such a stationary state phase transition turns out to be accompanied by a change in the structure of both the stationary state and the non-Hermitian vacuum [42], with the corresponding spectrum evolving from a phase with gapless Γ_k to one with a finite gap [26]. Interestingly, as shown in Fig. 2, a feature in the average work $\langle w \rangle$, corresponding to a sharp kink near the critical $\gamma_c(h) = 4\sqrt{1 - h^2}$ for each value of h , is clearly observed at sufficiently long times ($\Gamma_k t \gg 1 \forall k$). For larger γ , all modes have sizable decay rates, coherent evolution is Zeno suppressed, and the available work slowly saturates.

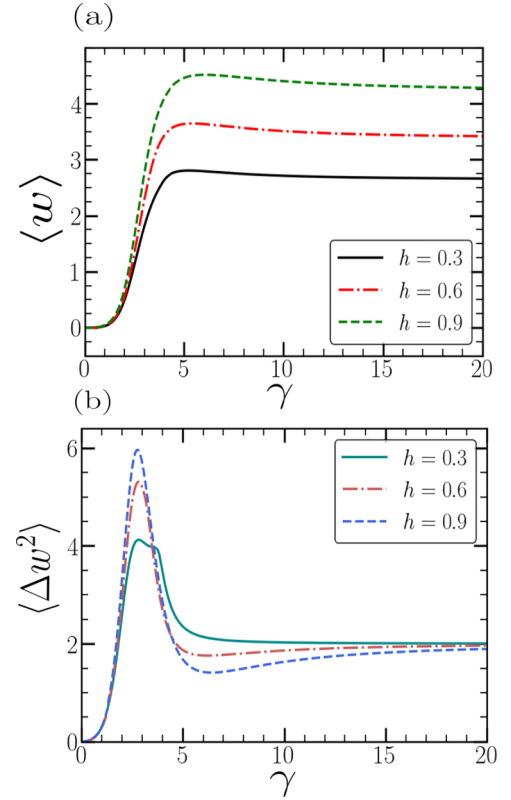


FIG. 1. (a) Average work density $\langle w \rangle$ (normalized by system size L) and (b) its variance $\langle \Delta w^2 \rangle$ plotted as a function of the measurement strength γ for the monitored one-dimensional transverse-field Ising model at different values of the transverse field h at time $t = 1.0$. The plots demonstrate that when $\gamma = 0$, both quantities are zero, reflecting the coincidence of the initial and effective non-Hermitian Hamiltonians. As γ increases, the non-Hermitian dynamics progressively project the system onto the no-click (spin-down) state, leading to a saturation of the work performed and a reduction in its fluctuations.

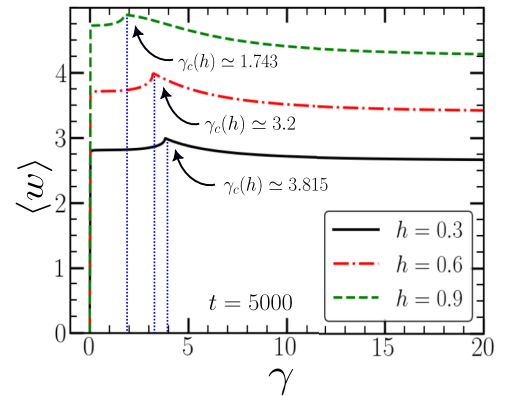


FIG. 2. Average work density $\langle w \rangle$ as a function of the measurement rate γ for $h = \{0.3, 0.6, 0.9\}$ at long evolution time $t = 5000$. A sharp rise appears at $\gamma \rightarrow 0^+$ when monitoring is first switched on. Each curve then shows a kink at a value of γ that coincides with the analytic no-click criticality $\gamma_c(h) = 4\sqrt{1 - h^2}$ (blue dotted guides and labels). For larger γ , $\langle w \rangle$ saturates due to Zeno suppression.

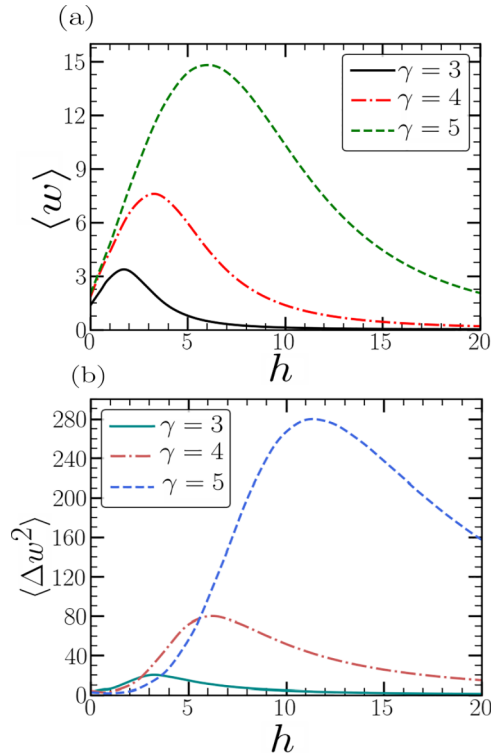


FIG. 3. (a) Average work density $\langle w \rangle$ and (b) its variance $\langle \Delta w^2 \rangle$ as a function of the transverse field h for various fixed measurement strengths γ at time $t = 1.0$. The plots illustrate that for a given γ , both quantities initially increase with h , reaching a maximum before decreasing as h is further increased, reflecting the interplay between unitary dynamics and measurement backaction. At higher γ values, a stronger transverse field is required to overcome the measurement-induced energy modifications.

On the other hand, for fixed values of γ , increasing the transverse field h leads to a regime in which the unitary dynamics dominate over the measurement-induced corrections. In this limit, both the final Hamiltonian H_f and the non-Hermitian Hamiltonian H_{eff} are essentially governed by the transverse field term, so that they become nearly identical. This leads to the asymptotic vanishing of the average work. As illustrated in Figs. 3(a) and 3(b), both $\langle w \rangle$ and $\langle \Delta w^2 \rangle$ initially increase with the transverse field h , reaching a maximum at a value that depends on the measurement strength γ . Beyond this point, as h is further increased, both quantities gradually decrease. Notably, for higher values of γ , a stronger transverse field is required to counterbalance the measurement. In Appendix B, we present a detailed calculation of the efficacy γ_t and plot its time dependence $\gamma_t(t)$ for different measurement rates γ .

IV. CONCLUSIONS

In this work, we presented a comprehensive study of quantum work statistics under continuous monitoring, revealing fundamental modifications to standard fluctuation theorems, particularly in the no-click limit of quantum trajectories. Using the notion of quantum trajectories in the context of quantum thermodynamics [28] in combination with the standard two-point measurement scheme, we discuss a general expression for the work generating function that is easily

specialized to a specific no-click record and its associated non-Hermitian evolution. Notably, we show that generalized measurements necessitate a modification of the Jarzynski equality to account for non-Hermitian dynamics.

Equipped with this formulation, we applied our theory to a one-dimensional transverse-field Ising model under local transverse spin monitoring to investigate the quantum work distribution arising from the interplay between coherent unitary evolution and non-Hermitian dynamics. We observed measurement-induced energy saturation and the suppression of work fluctuations, both originating from an effective confinement of quantum states reminiscent of the quantum Zeno effect. Specifically, we observed that increasing the measurement strength γ progressively projects the system onto the no-click eigenstate, thus altering energy-exchange processes. In contrast, increasing the transverse field h initially amplified energy fluctuations before eventually diminishing them, indicating a transition toward a regime dominated by unitary dynamics. Our results also show that the average work and its fluctuations are governed by the same decay rates that drive the steady-state entanglement transition in the no-click Ising chain, thus providing a practical and experimentally accessible way to pinpoint the critical measurement rate for a given field using energy-based protocols rather than full state tomography. On current hardware with mid-circuit readout, including superconducting-qubit processors and trapped-ion systems, these observables can be measured during continuous monitoring, making them natural tools for benchmarking monitored many-body dynamics [43–46]. Moreover, these underlying mechanisms also enable Zeno-based control and sensing, where tuning the measurement strength suppresses coherent dynamics in a controlled manner [47,48]. Taken together, this work advances both the theoretical framework for quantum work statistics in open systems undergoing continuous and generalized measurements and its concrete application to a many-body no-click setting. Future research can extend these ideas to broader classes of quantum systems and alternative measurement protocols to further clarify implications and expand practical applications.

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DATA AVAILABILITY

The data that support the findings of this article are not publicly available. The data are available from the authors upon reasonable request.

APPENDIX A: DETAILED DERIVATIONS FOR THE MONITORED QUANTUM ISING CHAIN

Here, we provide a comprehensive derivation of the expressions used in Sec. III. We begin by revisiting Eq. (26) for $\gamma = 0$, which, after Jordan-Wigner transformation, can be

expressed in terms of free fermions as follows:

$$H_i = -J \sum_i (\hat{c}_i^\dagger \hat{c}_{i+1} + \hat{c}_i^\dagger \hat{c}_{i+1}^\dagger + \text{H.c.}) - h(1 - 2\hat{c}_i^\dagger \hat{c}_i). \quad (\text{A1})$$

These operators satisfy the fermionic anticommutation relations $\{\hat{c}_i, \hat{c}_j^\dagger\} = \delta_{ij}$ and $\{\hat{c}_i, \hat{c}_j\} = \{\hat{c}_i^\dagger, \hat{c}_j^\dagger\} = 0$. Following the Fourier transformations of $\hat{c}_i$, $c_k = \sum_{R_i} e^{-ikR_i} \hat{c}_i / \sqrt{L}$, we obtain

$$H_i = \sum_k \begin{pmatrix} c_k^\dagger & c_{-k} \end{pmatrix} \begin{pmatrix} a_k & b_k^* \\ b_k & -a_k \end{pmatrix} \begin{pmatrix} c_k \\ c_{-k}^\dagger \end{pmatrix}, \quad (\text{A2})$$

where $a_k = 2(h - J \cos k)$ and $b_k = 2iJ \sin k$. Now, following the generalized Bogoliubov transformation, Eq. (A2) becomes

$$\begin{aligned} H_i &= \sum_k \begin{pmatrix} c_k^\dagger & c_{-k} \end{pmatrix} \hat{V}_k^i \hat{V}_k^{i-1} \begin{pmatrix} a_k^i & b_k^{i*} \\ b_k^i & -a_k^i \end{pmatrix} \hat{V}_k^i \hat{V}_k^{i-1} \begin{pmatrix} c_k \\ c_{-k}^\dagger \end{pmatrix} \\ &= \sum_k \begin{pmatrix} \eta_k^\dagger & \hat{\eta}_{-k} \end{pmatrix} \begin{pmatrix} \epsilon_k^i & 0 \\ 0 & -\epsilon_k^i \end{pmatrix} \begin{pmatrix} \eta_k \\ \eta_{-k}^\dagger \end{pmatrix}. \end{aligned} \quad (\text{A3})$$

As is clear, Eq. (A3) is nothing but Eq. (28). Here, the eigenvector $\hat{V}_k^i$ is given by

$$V_k^i = \begin{pmatrix} u_k^i & -iv_k^i \\ -iv_k^i & u_k^i \end{pmatrix}, \quad (\text{A4})$$

with $u_k^i = 1/\sqrt{1 + |b_k^i/\epsilon_k^i|^2}$ and $v_k^i = ib_k^i/(a_k^i + \epsilon_k^i)u_k^i$. The free fermions c_k are related to the quasiparticles η_k through the following Bogoliubov transformations:

$$\begin{aligned} c_k &= u_k^i \eta_k - iv_k^i \eta_{-k}^\dagger, \\ c_{-k} &= u_k^i \eta_{-k} + iv_k^i \eta_k^\dagger, \\ c_k^\dagger &= u_k^i \eta_k^\dagger + iv_k^i \eta_{-k}, \\ c_{-k}^\dagger &= u_k^i \eta_{-k}^\dagger - iv_k^i \eta_k. \end{aligned} \quad (\text{A5})$$

Similarly, Eq. (29a) [or Eq. (29b) for $H_{\text{eff}}^\dagger$] can be obtained from

$$\begin{aligned} H_{\text{eff}} &= \sum_k \begin{pmatrix} c_k^\dagger & c_{-k} \end{pmatrix} V_k^{\text{nc}} V_k^{\text{nc}-1} \begin{pmatrix} a_k^{\text{nc}} & b_k^{\text{nc}*} \\ b_k^{\text{nc}} & -a_k^{\text{nc}} \end{pmatrix} V_k^{\text{nc}} V_k^{\text{nc}-1} \begin{pmatrix} c_k \\ c_{-k}^\dagger \end{pmatrix} \\ &= \sum_k \begin{pmatrix} \gamma_k^* & \gamma_{-k} \end{pmatrix} \begin{pmatrix} \epsilon_k^{\text{eff}} & 0 \\ 0 & -\epsilon_k^{\text{eff}} \end{pmatrix} \begin{pmatrix} \gamma_k \\ \gamma_{-k}^* \end{pmatrix}, \end{aligned} \quad (\text{A6})$$

with the eigenvector V_k^{nc} taking the same form as in Eq. (A4), but with the superscript i replaced by nc. The elements of V_k^{nc} are $u_k^{\text{nc}} = 1/\sqrt{1 + |b_k^{\text{nc}}/\epsilon_k^{\text{nc}}|^2}$ and $v_k^{\text{nc}} = ib_k^{\text{nc}}/(a_k^{\text{nc}} + \epsilon_k^{\text{nc}})u_k^{\text{nc}}$, with $a_k^{\text{nc}} = 2(h - J \cos k) + i\gamma/2$ and $b_k^{\text{nc}} = b_k^i$. Both the Hermitian and non-Hermitian quasiparticles, η_k and γ_k , obey anticommutation relations similar to those for fermions. From Eq. (A6), the Bogoliubov transformation connecting the non-Hermitian quasiparticles γ_k and fermions c_k can be written as

$$c_k = u_k^{\text{nc}} \gamma_k - iv_k^{\text{nc}} \gamma_{-k}^*,$$

$$c_{-k} = (u_k^{\text{nc}} \gamma_{-k} + iv_k^{\text{nc}} \gamma_k^*) / \det(V_k^{\text{nc}}),$$

$$c_k^\dagger = (u_k^{\text{nc}} \gamma_k^* + iv_k^{\text{nc}} \gamma_{-k}) / \det(V_k^{\text{nc}}),$$

$$c_{-k}^\dagger = u_k^{\text{nc}} \gamma_{-k}^* - iv_k^{\text{nc}} \gamma_k. \quad (\text{A7})$$

Similarly, the mapping between c_k and the left non-Hermitian quasiparticles $\tilde{\gamma}_k$ is obtained by taking the complex conjugate of each Bogoliubov coefficient in Eq. (A7), that is, $u_k^{\text{nc}} \rightarrow u_k^{\text{nc}*}$ and $v_k^{\text{nc}} \rightarrow v_k^{\text{nc}*}$, which defines the corresponding rotation matrix $\tilde{V}_k^{\text{nc}}$. Then, by comparing Eqs. (A5) and (A7), we immediately derive the relation between η_k and γ_k , as given by Eq. (30) in the main text, where

$$\begin{aligned} X_k &= u_k^i u_k^{\text{nc}} + v_k^i v_k^{\text{nc}}, \\ Y_k &= u_k^i v_k^{\text{nc}} - v_k^i u_k^{\text{nc}}. \end{aligned} \quad (\text{A8})$$

Likewise, replacing X_k and Y_k with their complex conjugates in that same equation produces the equivalent mapping between η_k and the left non-Hermitian quasiparticles $\tilde{\gamma}_k$.

We now derive the analytical expression for the symmetrized Hamiltonian $H_f(\gamma_k, \tilde{\gamma}_k)$. We begin with the form of H_f from Eq. (28) and then rewrite η_k in terms of γ_k by using the relations provided in Eqs. (A5) and (A7). Consequently, the final Hamiltonian $H_f(\gamma_k)$ can be written as follows:

$$H_f(\gamma_k) = \sum_{k>0} \beta_k N_k - i\delta_k \Pi_k + E_k^{0\gamma}, \quad (\text{A9})$$

where the number operator is defined as $N_k = \gamma_k^* \gamma_k + \gamma_{-k}^* \gamma_{-k}$ and the pair operator can be defined as $\Pi_k = \gamma_k^* \gamma_{-k}^* + \gamma_k \gamma_{-k}$. The prefactors are given by

$$\beta_k = \frac{\epsilon_k^i (X_k^2 - Y_k^2)}{\det(V_k^{\text{nc}})}, \quad \delta_k = \frac{2\epsilon_k^i X_k Y_k}{\det(V_k^{\text{nc}})}, \quad (\text{A10})$$

and the ground state energy after transforming from η_k to γ_k is expressed as

$$E_k^{0\gamma} = -\sum_{k>0} \epsilon_k^i + \frac{2\epsilon_k^i Y_k^2}{\det(V_k^{\text{nc}})}. \quad (\text{A11})$$

Similarly, by applying an analogous procedure, we infer the expression for $H_f^\dagger(\tilde{\gamma}_k)$ as follows:

$$H_f^\dagger(\tilde{\gamma}_k) = \sum_{k>0} \beta_k^* \tilde{N}_k - i\delta_k^* \tilde{\Pi}_k + E_k^{0\tilde{\gamma}}. \quad (\text{A12})$$

Here, $\tilde{N}_k$ and $\tilde{\Pi}_k$ denote the number and pair operators for $\tilde{\gamma}_k$, respectively, and the prefactors as well as the ground state energy in Eq. (A12) are given by the complex conjugates of those in Eqs. (B3) and (A11).

Now that we have the expressions for $H_f(\gamma_k)$ and $H_f^\dagger(\tilde{\gamma}_k)$ in hand, we can proceed to formulate the average energy density $\langle w \rangle$ and its fluctuations $\langle \Delta w^2 \rangle$ by performing the appropriate averages over the initial states $|\Psi_0^\gamma(t)\rangle$ and $|\Psi_0^{\tilde{\gamma}}(t)\rangle$. After carrying out these calculations, Eq. (34) takes the following form:

$$\langle w \rangle = -E_0^i + \int_0^{2\pi} \frac{dk}{2\pi} \frac{2(\beta_k + \beta_k^*)|\alpha_k|^2 e^{-4\Gamma_k t} - (\delta_k + \delta_k^*)(\alpha_k^t + \alpha_k^{t*}) + (E_k^{0\gamma} + E_k^{0\tilde{\gamma}})(1 + |\alpha_k|^2 e^{-4\Gamma_k t})}{2(1 + |\alpha_k|^2 e^{-4\Gamma_k t})}, \quad (\text{A13})$$

where $\alpha_k^t = \alpha_k e^{-2i\epsilon_k^{\text{eff}} t}$ and α_k^{t*} denotes its complex conjugate. However, evaluating the first term in Eq. (35) is nontrivial because the square $[H_f(\gamma_k) + H_f^\dagger(\tilde{\gamma}_k)]^2$ expands into four different contributions: $[H_f(\gamma_k)]^2 + [H_f^\dagger(\tilde{\gamma}_k)]^2 + H_f(\gamma_k)H_f^\dagger(\tilde{\gamma}_k) + H_f^\dagger(\tilde{\gamma}_k)H_f(\gamma_k)$. Of these four contributions, the mixed term $H_f(\gamma_k)H_f^\dagger(\tilde{\gamma}_k)$ requires additional treatment. To evaluate the expectation value $\langle \Psi_0^\gamma(t) | H_f(\gamma_k) H_f^\dagger(\tilde{\gamma}_k) | \Psi_0^\gamma(t) \rangle$ properly, the operator $H_f(\gamma_k)$ acting on the left state must be rewritten in the $\tilde{\gamma}_k$ basis, while the operator $H_f^\dagger(\tilde{\gamma}_k)$ acting on the right state must be expressed in the γ_k basis. To accomplish this, we construct the following Bogoliubov-like rotation linking γ_k and $\tilde{\gamma}_k$ by combining the transformation between c_k and γ_k [Eq. (A7)] with the analogous relation between c_k

and $\tilde{\gamma}_k$:

$$\begin{aligned}\gamma_k &= (P_k \tilde{\gamma}_k + iQ_k \tilde{\gamma}_k^*) / \det(V_k^{\text{nc}}), \\ \gamma_{-k} &= (P_k \tilde{\gamma}_{-k} - iQ_k \tilde{\gamma}_{-k}^*) / \det(\tilde{V}_k^{\text{nc}}), \\ \gamma_k^* &= (P_k \tilde{\gamma}_k^* - iQ_k \tilde{\gamma}_{-k}) / \det(\tilde{V}_k^{\text{nc}}), \\ \gamma_{-k}^* &= (P_k \tilde{\gamma}_{-k}^* + iQ_k \tilde{\gamma}_k) / \det(V_k^{\text{nc}}),\end{aligned}\quad (\text{A14})$$

where $P_k = |u_k^{\text{nc}}|^2 + |v_k^{\text{nc}}|^2$ and $Q_k = u_k^{\text{nc}*} v_k^{\text{nc}} - u_k^{\text{nc}} v_k^{\text{nc}*}$.

We are now ready to evaluate the expectation value in the first term of Eq. (35). This term can be decomposed into three distinct contributions (the first, second, and third terms, respectively) as follows:

First term; $\langle \Psi_0^\gamma(t) | [H_f(\gamma_k)]^2 + [H_f^\dagger(\tilde{\gamma}_k)]^2 | \Psi_0^\gamma(t) \rangle$

$$\begin{aligned}& 4|\alpha_k|^2 e^{-4\Gamma_k t} (\beta_k^2 + \beta_k^{*2}) + (\delta_k^2 + \delta_k^{*2}) (1 + |\alpha_k|^2 e^{-4\Gamma_k t}) - 2(\alpha_k^t + \alpha_k^{t*}) [\delta_k (\beta_k + E_k^{0\gamma}) + \delta_k^* (\beta_k^* + E_k^{0\tilde{\gamma}})] \\ & + (E_k^{0\gamma} + E_k^{0\tilde{\gamma}}) + |\alpha_k|^2 e^{-4\Gamma_k t} [E_k^{0\gamma} (4\beta_k + E_k^{0\gamma}) + E_k^{0\tilde{\gamma}} (4\beta_k^* + E_k^{0\tilde{\gamma}})] \\ & = \frac{4(1 + |\alpha_k|^2 e^{-4\Gamma_k t})}{4(1 + |\alpha_k|^2 e^{-4\Gamma_k t})},\end{aligned}$$

Second term; $\langle \Psi_0^\gamma(t) | H_f^\dagger(\tilde{\gamma}_k) H_f(\gamma_k) | \Psi_0^\gamma(t) \rangle$

$$\begin{aligned}& 4|\alpha_k|^2 |\beta_k|^2 e^{-4\Gamma_k t} - 2(\alpha_k^{t*} \beta_k^* \delta_k + \alpha_k^t \beta_k \delta_k^*) + 2|\alpha_k|^2 e^{-4\Gamma_k t} (\beta_k E_k^{0\tilde{\gamma}} + \beta_k^* E_k^{0\gamma}) + |\delta_k|^2 (1 + e^{-4\Gamma_k t}) \\ & - (E_k^{0\tilde{\gamma}} \delta_k + E_k^{0\gamma} \delta_k^*) (\alpha_k^t + \alpha_k^{t*}) + E_k^{0\gamma} E_k^{0\tilde{\gamma}} (1 + |\alpha_k|^2 e^{-4\Gamma_k t}) \\ & = \frac{4(1 + |\alpha_k|^2 e^{-4\Gamma_k t})}{4(1 + |\alpha_k|^2 e^{-4\Gamma_k t})},\end{aligned}$$

Third term; $\langle \Psi_0^\gamma(t) | H_f(\gamma_k) H_f^\dagger(\tilde{\gamma}_k) | \Psi_0^\gamma(t) \rangle$

$$= \frac{A\tilde{A} + B\tilde{B}}{4(1 + |\alpha_k|^2 e^{-4\Gamma_k t})}, \quad \text{where}$$

$$A = (2i\alpha_k^t [(P_k^2 - Q_k^2)\beta_k^* - 2\delta_k^* P_k Q_k] - i\delta_k^* (P_k^2 - Q_k^2) - 2i\beta_k^* P_k Q_k + 2i\alpha_k^t [\beta_k^* Q_k^2 + \delta_k^* P_k Q_k] + i\alpha_k^t E_k^{0\tilde{\gamma}}) / D_k,$$

$$\tilde{A} = (-2i\alpha_k^{t*} [(P_k^2 - Q_k^2)\beta_k + 2\delta_k P_k Q_k] + i\delta_k (P_k^2 - Q_k^2) - 2i\beta_k P_k Q_k - 2i\alpha_k^{t*} [\beta_k Q_k^2 - \delta_k P_k Q_k] - i\alpha_k^{t*} E_k^{0\gamma}) / D_k,$$

$$B = E_k^{0\tilde{\gamma}} + \frac{2}{D_k} [Q_k^2 \beta_k^* + \delta_k^* P_k Q_k] - \frac{\alpha_k}{D_k} [\delta_k^* (P_k^2 - Q_k^2) + 2\beta_k^* P_k Q_k],$$

$$\tilde{B} = E_k^{0\gamma} + \frac{2}{D_k} [Q_k^2 \beta_k - \delta_k P_k Q_k] - \frac{\alpha_k^*}{D_k} [\delta_k (P_k^2 - Q_k^2) - 2\beta_k P_k Q_k], \quad (\text{A15})$$

where $D_k = \det(V_k^{\text{nc}} \tilde{V}_k^{\text{nc}})$. Now, by integrating the three terms in Eq. (A15) over dk as prescribed, we evaluate the first term in Eq. (35), while the second term is simply the square of the corresponding term in Eq. (A13).

APPENDIX B: CALCULATION OF THE EFFICACY FOR A MONITORED QUANTUM ISING CHAIN

Here, we explicitly compute the efficacy γ_t for the monitored quantum Ising chain. If we set $H_i = H_f$, Eq. (24)

reduces to

$$\gamma_t = \frac{\langle \Psi_0^\gamma | e^{-iH_{\text{eff}} t} e^{iH_{\text{eff}}^\dagger t} | \Psi_0^\gamma \rangle}{\langle \Psi_0^\gamma | e^{iH_{\text{eff}}^\dagger t} e^{-iH_{\text{eff}} t} | \Psi_0^\gamma \rangle}. \quad (\text{B1})$$

The denominator is just the biorthogonal overlap of the time-evolved left and right eigenstates in Eqs. (33a) and (33b). The numerator, in contrast, requires an explicit change in basis: Using the rotation $\gamma_k \leftrightarrow \tilde{\gamma}_k$ from Eq. (A15), we rewrite H_{eff} in terms of $\tilde{\gamma}_k$ and $H_{\text{eff}}^\dagger$ in terms of γ_k . With this choice, $H_{\text{eff}}^\dagger$ acts on the right sector $|\Psi_0^\gamma\rangle$ and H_{eff} acts on the left sector $\langle \Psi_0^\gamma |$ in their natural biorthogonal bases. If we apply the basis

transformation in Eqs. (29a) and (29b), the Hamiltonians takes the form

$$H_{\text{eff}}(\tilde{\gamma}) = \sum_{k>0} [\chi_k \tilde{N}_k - i \xi_k \tilde{\Pi}_k] + E_0, \quad (\text{B2a})$$

$$H_{\text{eff}}^\dagger(\gamma) = \sum_{k>0} [\bar{\chi}_k N_k - i \bar{\xi}_k \Pi_k] + E_0^*. \quad (\text{B2b})$$

Here, the coefficients are

$$\chi_k = \frac{\epsilon_k^{\text{eff}} (P_k^2 - Q_k^2)}{D_k}, \quad \xi_k = -\frac{2\epsilon_k^{\text{eff}} P_k Q_k}{D_k}, \quad (\text{B3})$$

and the ground state energy after the transformation is

$$E_0 = E_0^{\text{eff}} + \sum_{k>0} \frac{2\epsilon_k^{\text{eff}} Q_k^2}{D_k}. \quad (\text{B4})$$

Here, $D_k = \det(V_k^{\text{nc}}) \det(\tilde{V}_k^{\text{nc}})$, $\bar{\chi}_k = \chi_k^*$, and $\bar{\xi}_k = -\xi_k^*$.

Let us first calculate the term $e^{iH_{\text{eff}}^\dagger t} |\Psi_0^\gamma\rangle$ in the numerator. Since $H_{\text{eff}}^\dagger = \sum_{k>0} h_k^\dagger + E_0$ is a sum of independent k blocks with $[h_q^\dagger, h_k^\dagger] = 0$ for $q \neq k$, the evolution operator factorizes: $e^{iH_{\text{eff}}^\dagger t} = e^{iE_0 t} \prod_{k>0} e^{ih_k^\dagger t}$. For each k , we work in

the even-parity subspace spanned by $|0\rangle_k \equiv |\emptyset\rangle$ and $|1\rangle_k \equiv |k, -k\rangle$. Using the canonical anticommutation relations, we check that $N_k|0\rangle_k = 0$, $N_k|1\rangle_k = 2|1\rangle_k$, $\Pi_k|0\rangle_k = |1\rangle_k$, and $\Pi_k|1\rangle_k = -|0\rangle_k$. In the ordered basis $\{|0\rangle_k, |1\rangle_k\}$ they read

$$N_k = \begin{pmatrix} 0 & 0 \\ 0 & 2 \end{pmatrix} = \mathbb{1} - \sigma_z, \quad \Pi_k = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} = -i\sigma_y. \quad (\text{B5})$$

Here, $\sigma_{x,y,z}$ are the Pauli matrices acting in the $\{|0\rangle_k, |1\rangle_k\}$ space. The single-block Hamiltonian therefore can be written as $h_k^\dagger = \bar{\chi}_k \mathbb{1} - (\bar{\chi}_k \sigma_z + \bar{\xi}_k \sigma_y)$. Using the identity for Pauli-vector exponentials, $e^{i\theta \mathbf{b} \cdot \boldsymbol{\sigma}} = \cos(|\mathbf{b}|\theta) \cdot \mathbb{1} + i \frac{\sin(|\mathbf{b}|\theta)}{|\mathbf{b}|} (\mathbf{b} \cdot \boldsymbol{\sigma})$, with $\mathbf{b}_k = (0, -\bar{\chi}_k, -\bar{\xi}_k)$ and $\bar{\Omega}_k \equiv |\mathbf{b}_k| = \sqrt{\bar{\chi}_k^2 + \bar{\xi}_k^2}$, we obtain

$$e^{ih_k^\dagger t} = e^{i\bar{\chi}_k t} \left[\cos(\bar{\Omega}_k t) \cdot \mathbb{1} - i \frac{\sin(\bar{\Omega}_k t)}{\bar{\Omega}_k} (\bar{\chi}_k \sigma_z + \bar{\xi}_k \sigma_y) \right].$$

Equivalently, in the basis $\{|0\rangle_k, |1\rangle_k\}$, the above equation can be expressed as a 2×2 matrix:

$$e^{ih_k^\dagger t} = e^{i\bar{\chi}_k t} \begin{pmatrix} \cos(\bar{\Omega}_k t) - i(\bar{\chi}_k/\bar{\Omega}_k) \sin(\bar{\Omega}_k t) & -(\bar{\xi}_k/\bar{\Omega}_k) \sin(\bar{\Omega}_k t) \\ (\bar{\xi}_k/\bar{\Omega}_k) \sin(\bar{\Omega}_k t) & \cos(\bar{\Omega}_k t) + i(\bar{\chi}_k/\bar{\Omega}_k) \sin(\bar{\Omega}_k t) \end{pmatrix}. \quad (\text{B6})$$

Acting on the two-component amplitude $C_k(0) = (1, i\alpha_k)^T$ of the initial state $|\Psi_0^\gamma\rangle$ [Eq. (31a)] gives

$$\begin{pmatrix} C_{0,k}(t) \\ C_{1,k}(t) \end{pmatrix} = e^{ih_k^\dagger t} \begin{pmatrix} 1 \\ i\alpha_k \end{pmatrix}, \quad (\text{B7})$$

that is,

$$\begin{aligned} C_{0,k}(t) &= e^{i\bar{\chi}_k t} \left[\cos(\bar{\Omega}_k t) - i \frac{\bar{\chi}_k + \alpha_k \bar{\xi}_k}{\bar{\Omega}_k} \sin(\bar{\Omega}_k t) \right], \\ C_{1,k}(t) &= e^{i\bar{\chi}_k t} \left[i\alpha_k \cos(\bar{\Omega}_k t) + \frac{\bar{\xi}_k - \alpha_k \bar{\chi}_k}{\bar{\Omega}_k} \sin(\bar{\Omega}_k t) \right]. \end{aligned} \quad (\text{B8})$$

$$e^{-ih_k t} = e^{-i\chi_k t} \begin{pmatrix} \cos(\Omega_k t) + i(\chi_k/\Omega_k) \sin(\Omega_k t) & -(\xi_k/\Omega_k) \sin(\Omega_k t) \\ (\xi_k/\Omega_k) \sin(\Omega_k t) & \cos(\Omega_k t) - i(\chi_k/\Omega_k) \sin(\Omega_k t) \end{pmatrix}. \quad (\text{B10})$$

Now, acting on the initial amplitude row vector $D_k(0) = (1, -i\alpha_k^*)$ of the left initial state $\langle\Psi_0^\gamma|$ yields

$$(D_{0,k}(t) \quad D_{1,k}(t)) = (1 \quad -i\alpha_k^*) e^{-ih_k t}, \quad (\text{B11})$$

where

$$\begin{aligned} D_{0,k}(t) &= e^{-i\chi_k t} \left[\cos(\Omega_k t) + i \frac{\chi_k - \alpha_k^* \xi_k}{\Omega_k} \sin(\Omega_k t) \right], \\ D_{1,k}(t) &= e^{-i\chi_k t} \left[-i\alpha_k^* \cos(\Omega_k t) - \frac{\xi_k + \alpha_k^* \chi_k}{\Omega_k} \sin(\Omega_k t) \right]. \end{aligned} \quad (\text{B12})$$

Therefore, the final expression for $e^{iH_{\text{eff}}^\dagger t} |\Psi_0^\gamma\rangle$ is given by

$$e^{iH_{\text{eff}}^\dagger t} |\Psi_0^\gamma\rangle = \frac{e^{iE_0 t}}{\mathcal{N}} \prod_k [C_{0,k}(t)|0\rangle_k + C_{1,k}(t)|1\rangle_k]. \quad (\text{B9})$$

Let us now compute the backward evolution term $\langle\Psi_0^\gamma| e^{-iH_{\text{eff}} t}$. To evaluate this, we work in the biorthogonal basis $\{|\tilde{0}\rangle_k, |\tilde{1}\rangle_k\}$ defined by $\langle\tilde{0}|_k \equiv \langle\emptyset|$ and $\langle\tilde{1}|_k \equiv \langle k, -k| = \langle\emptyset| \tilde{\gamma}_{-k} \tilde{\gamma}_k$. In this basis, the dual number operator $\tilde{N}_k$ and the dual pair operator $\tilde{\Pi}_k$ act on the even sector as $\tilde{N}_k = \mathbb{1} - \sigma_z$ and $\tilde{\Pi}_k = i\sigma_y$. The $+i\sigma_y$ sign reflects the fact that we are acting on left (bra) states of the biorthogonal pair. With these 2×2 blocks in hand, each k sector can be exponentiated using the standard Pauli-matrix identity, and following the same steps as above, we obtain the explicit form of $e^{-ih_k t}$ in the dual basis:

Therefore, the final expression for the backward-evolved state is

$$\langle\Psi_0^\gamma| e^{-iH_{\text{eff}} t} = \frac{e^{-iE_0 t}}{\mathcal{N}} \prod_k [\langle\tilde{0}|_k D_{0,k}(t) + \langle\tilde{1}|_k D_{1,k}(t)]. \quad (\text{B13})$$

Since, by construction, one has $D_{0,k}(t) = C_{0,k}^*(t)$ and $D_{1,k}(t) = C_{1,k}^*(t)$, the mode-resolved contribution to the efficacy is

$$\gamma_t^{(k)} = e^{-4Q_k^2 \Gamma_k t / D_k} \left(\frac{|C_{0,k}|^2 + |C_{1,k}|^2}{1 + |\alpha_k|^2 e^{-4\Gamma_k t}} \right). \quad (\text{B14})$$

The many-body efficacy is then the product over positive momenta: $\gamma_t = \prod_{k>0} \gamma_t^{(k)}$. Figure 4 illustrates the time dependence of the efficacy γ_t at various measurement rates. As is evident from the plot, in the absence of monitoring ($\gamma = 0$), the evolution remains unitary, allowing the forward and backward segments to commute. Consequently, γ_t remains constant at 1 for all times. However, any nonzero measurement strength $\gamma > 0$ disrupts this symmetry, rendering the postselected no-click evolution effectively non-Hermitian. Each Bogoliubov mode then acquires a complex quasienergy ϵ_k^{eff} , where the real part induces coherent oscillations and the imaginary part drives the decay of the no-jump weight. For $\gamma = 2$, small oscillations appear at short times when the loss is moderate. As time progresses, accumulated decay rates across modes begin to dominate, causing γ_t to drop toward zero. For strong monitoring, at $\gamma = 5$, the no-jump weight decays more rapidly, and the coherent oscillations are strongly suppressed, resulting in a more rapid, nearly monotonic drop in γ_t .

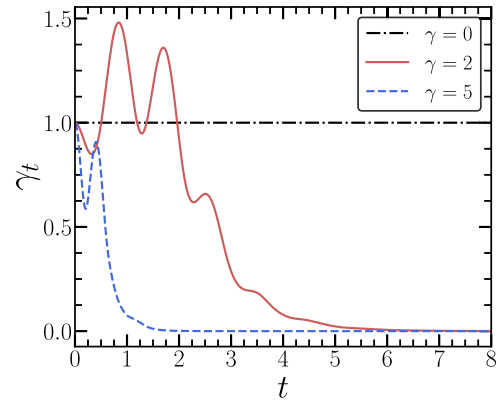


FIG. 4. Efficacy γ_t versus time t for the monitored Ising chain at $J = 1$ and $h = 0.5$, comparing three measurement rates. The unitary case $\gamma = 0$ yields $\gamma_t = 1$ for all times. For moderate monitoring at $\gamma = 2$ the curve shows a brief overshoot at initial times and then decays as the non-Hermitian loss builds up. For stronger monitoring, at $\gamma = 5$, the decay is faster, and the initial bump is much smaller.

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