

Competing phases in the kagome magnet FeGe from functional renormalization

Pietro M. Bonetti ^{1,2} Yi Jiang,³ Haoyu Hu,⁴ Dumitru Călugăru ⁴ Michael M. Scherer ⁵
B. Andrei Bernevig,^{3,4,6} and Laura Classen ^{2,7}

¹*Department of Physics, Harvard University, Cambridge Massachusetts 02138, USA*

²*Max Planck Institute for Solid State Research, Heisenbergstrasse 1, D-70569 Stuttgart, Germany*

³*Donostia International Physics Center (DIPC), Paseo Manuel de Lardizabal, 20018 San Sebastian, Spain*

⁴*Department of Physics, Princeton University, Princeton, New Jersey 08544, USA*

⁵*Theoretische Physik III, Ruhr-Universität Bochum, D-44801 Bochum, Germany*

⁶*IKERBASQUE, Basque Foundation for Science, 48013 Bilbao, Spain*

⁷*Department of Physics, Technical University of Munich, D-85748 Garching, Germany*



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The discovery of a charge density wave in FeGe extends the discussion of the nature of charge order in kagome metals to a magnetic compound. Motivated by this observation, we combine density functional theory (DFT) and functional-renormalization-group calculations to study interaction-induced Fermi-surface instabilities of the magnetic state of FeGe. We argue that the leading intra-band contribution to electronic correlations are approximately two-dimensional (2D) and come from Van Hove points at the projected M points. By varying parameters around DFT values, we determine a phase diagram for the quasi-2D scenario as function of on-site and nearest-neighbor interactions. We discuss universal aspects in the electronic mechanisms for the resulting phases, as well as the role of SU(2) symmetry breaking. We find FeGe to be in a regime of strong competition between p -wave charge density wave, f -wave pairing, and d -wave spin Pomeranchuk instabilities. This interplay can be influenced in favor of superconducting pairing for slightly increased nearest-neighbor interaction, suggesting a potential to induce superconductivity in FeGe.

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Introduction. Kagome metals constitute an important platform for studying electronic correlations. In recent years a rich phase structure was revealed in different such compounds [1–4]. It is based on an interplay of different degrees of freedom which continues to challenge our understanding of the emergent many-body behavior, with multiple possible competing phases. In particular, the type of charge orders and the mechanism behind their formation is being intensely discussed, as well as their connection to magnetic or superconducting phases.

In the prominent superconducting kagome family AV_3Sb_5 (AV_3Sb_5 $A = \text{K, Rb, Cs}$) [5], an exotic charge density wave (CDW) arises with wave vector connecting electronic Van Hove points (VHPs) [6–17]. It precedes a superconducting transition, while long-range magnetic order remains absent [18–24]. An unconventional CDW was also discovered in the magnetic kagome metal FeGe [25–34], which shows both similarities and differences to AV_3Sb_5 . Remarkably, in contrast to AV_3Sb_5 , the CDW in FeGe arises out of a collinear layer antiferromagnet (AFM) which has a (anti-)ferromagnetic alignment (between) within Fe kagome layers.

Furthermore, the CDW precedes an incommensurate density wave with canted spins [35–38]. Yet, similar to AV_3Sb_5 , the CDW vector connects VHPs, which, however, are now placed near the Fermi level only after the nearly flat bands of FeGe are split in the AFM phase [25,26,30]. Simultaneously, a structural transition occurs and magnetic moments are enhanced [29,39]. Generally, experimental observations demonstrate that FeGe realizes a strong coupling of spin, charge, orbital, and lattice degrees of freedom in a single system [27–34,39–42]. Naturally, the primary order parameter of the CDW remains debated, with no softening of phonons in the FeGe observed [34,43] or computed.

Here, we perform functional renormalization group (FRG) calculations for a *ab initio* band structure of FeGe to elucidate the role of electronic contributions in the CDW formation. The advantage of the FRG approach is an unbiased description of electronic charge, spin, and pairing correlations taking their mutual feedback into account. We do not include electron-phonon coupling. FRG was successfully applied to the single-orbital kagome lattice, pointing out the key role of sublattice interference [44] for charge ordering [45–48]. Different charge orders were also studied via Hartree-Fock [49], or via phenomenological Landau analyses, sometimes combined with DFT for AV_3Sb_5 [50–54]. Whether universal behavior among kagome metals extends to FeGe remains an open question. This motivates us to go beyond modeling via a single-orbital kagome lattice and connect to a realistic band

structure with five orbitals per kagome layer [55]. Our approach elucidates the role of breaking spin-SU(2) symmetry by the AFM parent phase on secondary phases in the electronic phase diagram.

In our instability analysis of FeGe electron bands, we uncover universal aspects connected to the kagome lattice, in addition to qualitatively different effects due to the AFM background. Based on the full three-dimensional (3D) model, we argue that bare band susceptibilities can be approximated by a 2D projection with VHPs at the M points of almost pure sublattice type despite the multiorbital nature. These features can only be brought into play because the AFM shifts the bands. On top of a resulting sublattice interference mechanism, we also find that an on-site Hubbard interaction is also ineffective in coupling opposite spins at (spin-degenerate) VHPs because of the FM layer polarization. As a consequence, transversal spin orders are suppressed in comparison to the SU(2) symmetric phase diagram. Furthermore, we find a strong tendency towards spin-triplet f -wave pairing, whose seed lies in quantum-geometric contributions of the projected interaction further enhanced by particle-hole fluctuations. Our analysis places FeGe at the boundary of a p -wave CDW, f -wave pairing, and intersublattice spin density wave instability. In the phase diagram, we also identify a canted antiferromagnetic order which is consistent with experimental observations [56] and a ferrimagnet.

Band structure and susceptibilities. We perform an *ab initio* calculation of the band structure of FeGe in the AFM phase as detailed in Ref. [55]. In the paramagnetic phase, the band structure exhibits quasiflat bands near the Fermi level. In the AFM state bands are reconstructed. They remain doubly degenerate due to a generalization of Kramers' theorem based on the combined operation of time-reversal and one-site translation along the c axis $T_z \otimes \mathcal{T}$. Consequently, spin and layer are locked. There are now five bands $E_{\mathbf{k}}^{\ell}$ crossing the Fermi level $\ell \in \{\alpha, \dots, \epsilon\}$, out of which some have 2D and some have 3D character, as shown in Figs. 1(a)–1(c) and in the Supplemental Material Ref. [57]. We find that the dominant contribution at the Fermi level comes from d_{xz} , d_{yz} , and $d_{x^2-y^2}$ Fe orbitals, which strongly contribute to the orbital composition of approximately 2D α , β , and γ Fermi surfaces.

In order to identify the Fermi surface sheets where intra-band electronic correlation effects are the most important, we compute the band susceptibility $\chi_{3D}^{\ell}(\mathbf{q}) = T \sum_{\omega} \int_{\text{BZ}} d^3\mathbf{k} G^{\ell}(\omega, \mathbf{k} + \mathbf{q}) G^{\ell}(\omega, \mathbf{k})$ for each band crossing the Fermi level, where $G^{\ell}(k) = (i\omega - E_{\mathbf{k}}^{\ell})^{-1}$ [57]. In Fig. 1(f) we show that the Fermi surface α sheet yields the largest susceptibility with a maximum at the L point and at related points M , L' with the same planar coordinates. Furthermore, we observe a quasi-2D form of the largest band susceptibilities $\chi_{3D}^{\ell}(\mathbf{q})$ in that their qualitative behavior is independent of q_z . We observe the analogous 2D behavior in the largest orbital susceptibilities (see SM [57]). To find an appropriate 2D approximation, we determine a representative Fermi surface from a cut at fixed $k_z = k_{z,0}$ [Fig. 1(e)] whose in-plane susceptibility $\chi_{2D}^{\alpha}(\mathbf{q}) = T \sum_{\omega} \int_{2D-\text{BZ}} d^2\mathbf{k} G^{\alpha}(\omega, \mathbf{k} + \mathbf{q}, k_{z,0}) G^{\alpha}(\omega, \mathbf{k}, k_{z,0})$ reproduces the largest band susceptibility χ_{3D}^{α} , Fig. 1(f). The peaks at M arise due to scattering processes that connect Van Hove points of the 2D approximation.

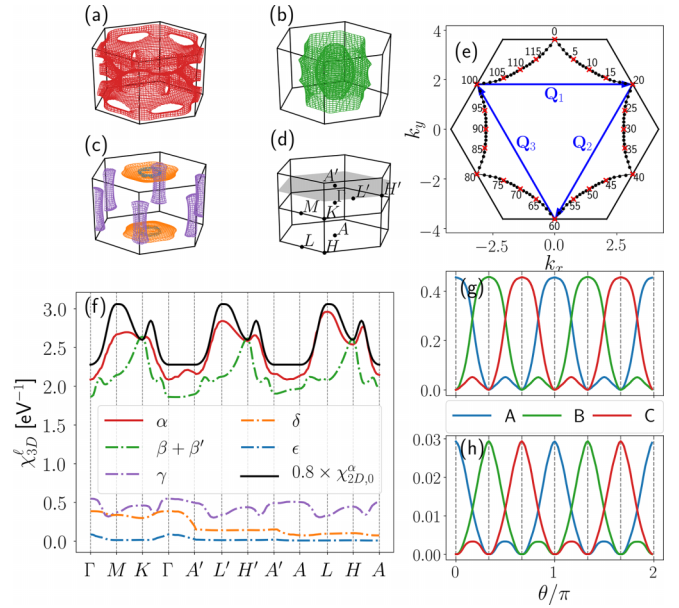


FIG. 1. Fermi surfaces of the (a) α band (b) β and β' bands, (c) γ (purple), δ (orange), and ϵ (blue) bands. (d) Points in the 3D Brillouin zone. The shaded plane is the $k_z = \pi/2$ plane. (e) Fermi surface and patching scheme used for the FRG calculations for the cut of the α band at $k_z = k_{z,0} \simeq 1.425$ with quasi-2D van Hove singularity. Blue arrows indicate approximate nesting vectors. (f) 3D band susceptibility $\chi_{3D}^{\ell}(\mathbf{q})$ for the bands crossing the Fermi level and 2D susceptibility χ_{2D}^{α} at $k_{z,0} \simeq 1.425$ (black solid line, same value repeated in different k_z planes for easier comparison) at temperature $T = 30$ meV. (g) and (h): Spin-up Fermi surface orbital weights of the Fe d_{xz} orbitals on the three kagome sublattices on layer 1 (g) and layer 2 (h) for the cut at $k_{z,0} = 1.425$. The angle θ swipes around the Fermi surface. Van Hove points (gray vertical lines) are mainly made of a single sublattice and layer contribution.

Ab-initio-derived 2D model. Based on the analysis of band susceptibilities, we consider an effective model that contains the α band and its projected interactions in the following FRG calculation. We start from the extended Hubbard-Kanamori Hamiltonian that contains spin-dependent intra- and interorbital density-density $U_{\sigma\sigma'}^{(i)}$ and exchange $J_{\sigma\sigma'}$ couplings for the five Fe d orbitals on a given site, as well as an orbital-independent nearest-neighbor interaction. To project the interaction onto the α band, we approximate the electron operator for sublattice b , orbital m , and spin-layer-locked σ as $c_{\mathbf{k},b,m,\sigma} \approx \phi_{\mathbf{k},b,m,\sigma} a_{\mathbf{k},\sigma}$, where $a_{\mathbf{k},\sigma}$ annihilates an electron in the α band with momentum $\mathbf{k}$ and $\phi_{\mathbf{k},b,m,\sigma}$ is the eigenvector of the *ab initio* Hamiltonian. We then obtain the projected interaction

$$\mathcal{H}_{\text{int}} = \int_{\mathbf{k}_1, \mathbf{k}_2} \sum_{\substack{\sigma_1, \sigma_2 \\ \sigma_3, \sigma_4}} V_{\sigma_1 \sigma_2 \sigma_3 \sigma_4}(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3) \times \delta_{\mathbf{k}_1 + \mathbf{k}_2 - \mathbf{k}_3 - \mathbf{k}_4, 0} a_{\mathbf{k}_1, \sigma_1}^{\dagger} a_{\mathbf{k}_2, \sigma_2}^{\dagger} a_{\mathbf{k}_4, \sigma_4} a_{\mathbf{k}_3, \sigma_3}, \quad (1)$$

with $\int_{\mathbf{k}} = \int_{2D-\text{BZ}} d^2k / (2\pi)^2$. Exploiting the discrete symmetry of the AFM, as well as its residual U(1) symmetry, we can reduce $V_{\sigma_1 \sigma_2 \sigma_3 \sigma_4}(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3)$ to two independent components: $V_{\uparrow\uparrow}, V_{\uparrow\downarrow}$ (SM [57]).

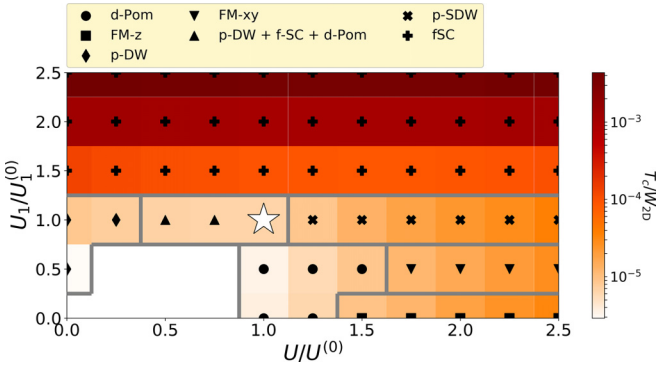


FIG. 2. Phase diagram and FRG critical temperatures T_c of *ab-initio*-derived, quasi-2D approximation for FeGe as a function of the onsite Hubbard U and nearest-neighbor interaction U_1 in units of their cRPA values $U^{(0)} = 5.72$ eV and $U_1^{(0)} = 2.30$ eV for fixed cRPA $U'_{\sigma,\sigma'}$ and J_σ . The band width is $W_{2D} \sim 450$ meV ~ 5000 K. Abbreviations mean the following. d-Pom: *d*-wave spin Pomeranchuk, FM-*z*: ferrimagnetism along the *z* axis, p-DW: *p*-wave charge and spin density wave order, FM-*xy*: canted antiferromagnetism, p-SDW: *p*-wave spin density wave order, and f-SC: *f*-wave superconductivity. The white star marks the point in the phase diagram corresponding to cRPA values of the interactions.

Analyzing the layer, and sublattice content of the Fermi surface cut at $k_z = k_{z,0}$, we find that each 2D-like van Hove singularity of the Fermi surface cut at $k_{z,0}$ has weight on only one of the three kagome sublattices, Figs. 1(g) and 1(h), as it happens in simpler tight binding models [44]. In our case, the Fermi surface contribution for spin up (down) mostly comes from the first (second) layer [cf. scales of Figs. 1(g) and 1(h)].

Instability analysis. We employ FRG to calculate the dressed interaction of the many-body system which takes into account all one-loop particle-particle and particle-hole fluctuations induced by the α -band electrons (see SM [57]). Because the band-projected bare interaction in Eq. (1) exhibits an intricate dependence on all three momenta and we consider an effective low-energy model valid near the Fermi surface, we resort to a patching scheme of the Fermi surface with 120 patch points [58] [Fig. 1(e)] [59,60]. An instability occurs when the renormalized interaction develops a characteristic singularity at a mean-field-like critical temperature T_c [e.g., Fig. 3(g)] and SM [57]. From the structure of the singularity on the Fermi surface, we infer the order parameter corresponding to the instability and the form factors that specify the angular harmonic of the particle-hole or Cooper pair. To diagnose if the leading instability occurs in charge (*c*), transverse (*x, y*), and longitudinal (*z*) spin channel, we define $V_c(\theta_1, \theta_2, \theta_3) = V_{\uparrow\uparrow}(\theta_1, \theta_2, \theta_3) + V_{\uparrow\downarrow}(\theta_1, \theta_2, \theta_3)$, $V_x(\theta_1, \theta_2, \theta_3) = V_y(\theta_1, \theta_2, \theta_3) = -V_{\uparrow\downarrow}(\theta_2, \theta_1, \theta_3)$, and $V_z(\theta_1, \theta_2, \theta_3) = V_{\uparrow\uparrow}(\theta_1, \theta_2, \theta_3) - V_{\uparrow\downarrow}(\theta_1, \theta_2, \theta_3)$. As in the experiments, the *z* axis is chosen along the alignment of the interplane antiferromagnetic order parameter.

Phase diagram. Based on the instability analysis, we determine the phase diagram as function of on-site U and nearest-neighbor U_1 interaction shown in Fig. 2. For small values of U and U_1 , we find no instability down to the lowest numerically resolved temperature, $T \sim 10$ mK. For large enough values of $U \gtrsim U^{(0)} = 5.72$ eV or $U_1 \gtrsim U_1^{(0)} =$

2.30 eV, we obtain different types of instabilities discussed below. They evolve from $q = 0$ spin/charge instabilities (ferrimagnetism, canted antiferromagnetism, or *d*-wave spin Pomeranchuk) over $q = M$ density waves to *f*-wave pairing instabilities upon increasing U_1 . Throughout the phase diagram, we obtain small critical temperatures T_c as compared to the 2D bandwidth, which signals strongly competing orders. We find that T_c depends only mildly on the interaction U and increases with U_1 .

The $q = 0$ instabilities spontaneously break the combined $T_z \otimes \mathcal{T}$ of the AFM. Among them, we find longitudinal ($i = z$) and transversal ($i = x, y$) ferrimagnetic (FM) orders for $U \gtrsim 1.5U^{(0)}$, $U_1 \lesssim U_1^{(0)}$ with order parameter

$$S^i = \int_{\mathbf{k} \in \text{FS}} \langle a_{\mathbf{k}}^\dagger \sigma^i a_{\mathbf{k}} \rangle. \quad (2)$$

The transversal FM also breaks the residual $U(1)$, the longitudinal does not. The longitudinal FM describes a magnetic phase in which the spins in neighboring layers point in opposite directions but with different magnitudes. Moreover, the two layers have different average charge densities. In the transversal FM, the spins get a canting pointing in the same direction in both layers. For $U \lesssim 1.5U^{(0)}$, we find a *d*-wave spin Pomeranchuk order

$$S_n^z = \int_{\mathbf{k} \in \text{FS}} d_{\mathbf{k}}^n \langle a_{\mathbf{k}}^\dagger \sigma^z a_{\mathbf{k}} \rangle, \quad (3)$$

in close competition with the corresponding charge instability. Due to the lattice D_{6h} symmetry, there are two degenerate form factors $d_{\mathbf{k}}^{x^2-y^2} = (2/\sqrt{3})[\cos(k_x) - \cos(k_x/2)\cos(\sqrt{3}k_y/2)]$ and $d_{\mathbf{k}}^{xy} = 2\sin(k_x/2)\sin(\sqrt{3}k_y/2)$, Fig. 3(a). The order additionally breaks D_{6h} spontaneously when a specific linear combination of them, develops below the critical temperature, i.e., it is a spin-polarized nematic [61–65]. The two independent d_{xy} and $d_{x^2-y^2}$ representatives have the same critical temperature but can have different energies in the symmetry-broken state. After a Fourier transform, the spin-Pomeranchuk phase manifests as an intersublattice spin-density wave in real space. The two corresponding patterns in Figs. 3(d)–3(e) represent an additional modulation of the charge density in each layer, which is proportional to the local spin amplitude. It has to be added (subtracted) from the spin magnitude in the top (bottom) layer. In order to determine which one of the two configurations in Figs. 3(d)–3(e) is favored below the critical temperature, one needs to microscopically evaluate a sixth-order coefficient of an effective Landau theory (see SM [57]).

For $U_1 \approx U_1^{(0)}$ and a range of U , we find that the system is unstable towards a *p*-wave density wave with $q = \mathbf{Q}_1 = (2\pi, 0)$, $q = \mathbf{Q}_2 = (-\pi, -\sqrt{3}\pi)$, and $q = \mathbf{Q}_3 = (-\pi, \sqrt{3}\pi)$ [Fig. 1(e)], where $\mathbf{Q} = M + G$ with G a reciprocal lattice vector. The order parameter reads

$$\Phi_n = \int_{\mathbf{k} \in \text{FS}} f_{\mathbf{k}}^{(n)} \langle a_{\mathbf{k}}^\dagger \eta a_{\mathbf{k}+\mathbf{Q}_n} \rangle, \quad (4)$$

where $\eta = 1$ or σ^z , and $f_{\mathbf{k}}^{(n)} = \sin(\mathbf{k} \cdot \mathbf{Q}_n/(4\pi))$ [66]. Such a phase corresponds to a charge or spin bond density wave and

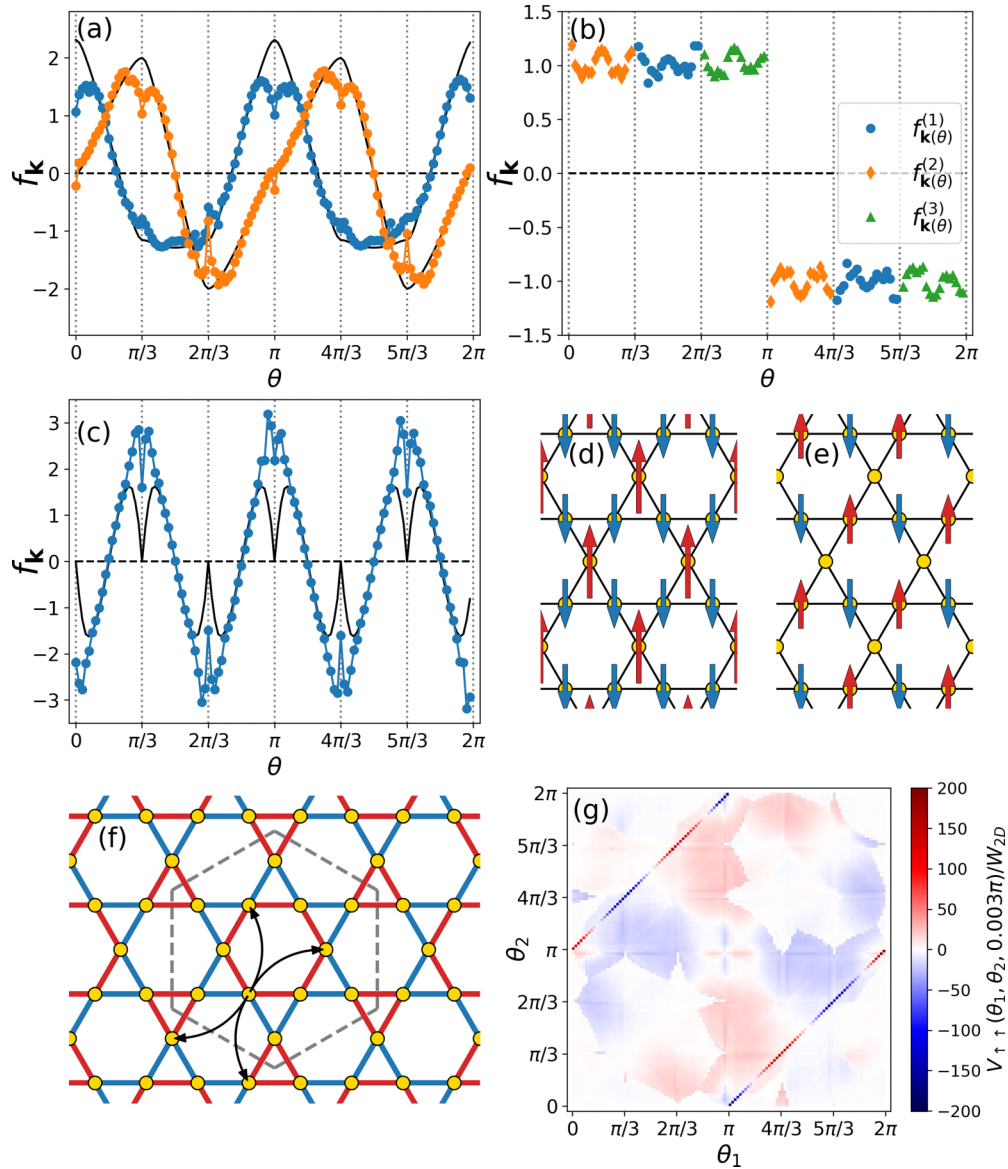


FIG. 3. (a) Leading (degenerate) eigenfunctions $f_{\mathbf{k}}$ of $V_z(\theta_1, \theta_2, \theta_1)$ in the spin-Pomeranchuk phase. Their angular dependence is well approximated by the form factors $d_{\mathbf{k}}^{x^2-y^2}$ and $d_{\mathbf{k}}^{xy}$ (solid black lines) with form factors defined in the main text. (b) Angular dependence of the leading vertex eigenfunction $f_{\mathbf{k}}$ in the p -CDW/SDW phase for $\mathbf{Q} = \mathbf{Q}_1$ (blue dots), $\mathbf{Q} = \mathbf{Q}_2$ (orange diamonds), and $\mathbf{Q} = \mathbf{Q}_3$ (green triangles). (c) Angular dependence of the leading eigenfunction $f_{\mathbf{k}}$ of $V_{\uparrow\uparrow}(\theta_1, (\theta_1 + \pi) \bmod(2\pi), \theta_2)$ in the f -wave superconducting phase. The function is well approximated by $(\sin(\sqrt{3}k_y) - 2\sin(3k_x/2))\cos(\sqrt{3}k_y/2)$ (solid black line). (d) $d_{x^2-y^2}$ and (e) d_{xy} representative spin patterns of the spin-Pomeranchuk phase. Spins pointing upwards in (d) are twice as long as those pointing downwards. Note that these patterns only depict the deviation from the underlying interlayer antiferromagnetic background. (f) Charge/spin bond density patterns in the p -CDW/SDW phase. A red (blue) bond represents an enhanced (suppressed) hopping along that bond. The enlarged unit cell contains 12 kagome sites (gray dashed line). Black arrows connect next-nearest-neighbor sites coupled by f -wave pairing. (g) Typical interaction vertex $V_{\uparrow\uparrow}(\theta_1, \theta_2, \theta_3 \simeq 0)$ between equal spin electrons for the f -wave pairing instability.

breaks translation symmetry. It also breaks $T_z \otimes \mathcal{T}$ symmetry if $\eta = \sigma_z$. It is analogous to instabilities found in paramagnetic Hubbard models on a kagome lattice [45–47]. In Fig. 3(b) we show the computed momentum dependence of the functions $f_{\mathbf{k}}^{(n)}$ for those combinations $\mathbf{k}$ and $\mathbf{k} + \mathbf{Q}_n$ that are close to the Fermi surface. In the ordered state, one or more of the order parameters in Eq. (4) can condense. To determine the ground

state configuration, we consider the Landau free energy

$$V[\Phi_1, \Phi_2, \Phi_3] = r \sum_{n=1}^3 \Phi_n^2 + u \left(\sum_{n=1}^3 \Phi_n^2 \right)^2 + \lambda (\Phi_1^2 \Phi_2^2 + \Phi_2^2 \Phi_3^2 + \Phi_3^2 \Phi_1^2). \quad (5)$$

In SM [57], we calculate the parameter λ from the band dispersion $E^\alpha(\mathbf{k})$, obtaining $\lambda < 0$, which means that a state with all Φ_n nonzero and equal in magnitude minimizes the energy. Note that the stability of the potential in Eq. (5) requires $u > \max(0, -\lambda)$, which is obeyed in our numerical calculation. We show the corresponding real-space charge or spin bond density patterns in Fig. 3(f).

Finally, for $U_1 \geq 1.5U_1^{(0)}$ independent of the value of U , we find a superconducting pairing instability in $V_{\uparrow\uparrow}$ indicating spin-triplet pairing of equal spins. We infer an order parameter

$$\Delta = \int_{\mathbf{k} \in \text{FS}} f_{\mathbf{k}}^f \langle a_{\mathbf{k}} (\cos \theta \sigma_x + e^{i\varphi} \sin \theta \sigma_y) i \sigma_y a_{-\mathbf{k}} \rangle, \quad (6)$$

parameterized by θ and φ with f -wave symmetry $f_{\mathbf{k}}^f \propto \sin(\sqrt{3}k_y) - 2 \cos(3k_x/2) \sin(\sqrt{3}k_y/2)$ in the B_{2u} irreducible representation of D_{6h} . To understand the pairing mechanism better, we analyze the bare pairing vertex, which includes quantum-geometric contributions from the full *ab initio* model due to the projection. We find that there is a small attractive f -wave component after projection into the α band. It induces a pairing instability through enhancement by fluctuations in the particle-hole channels. This pairing state couples electrons of equal spin on next-nearest-neighbor sites [see arrows in Fig. 3(f)]. In the AFM phase of FeGe equal spin pairing from electronic interactions is expected to be favored due to an almost complete layer polarization, which strongly suppresses interactions between unequal spin electrons. More generally, almost all the phases we found in our computations are described by order parameters that couple equal-spin electrons, the only exception being the xy canted antiferromagnetic phase.

Conclusion. We studied ordering tendencies of the magnetic kagome metal FeGe driven by electronic correlations. Our analysis of the *ab initio* band structure and quantum geometry revealed that Van Hove points and approximate sublattice polarization arise near the Fermi level in the AFM phase, establishing a connection between magnetic and nonmagnetic kagome metals. We also employed FRG as an unbiased many-body technique starting from a 2D approximation for realistic DFT band structures. We observe a strong competition of orders due to the interplay of spin, charge, and pairing correlations. The competing p -wave CDW with wave vector M and the canted AFM phase we found imply a consistency with experimental observations of the CDW [25–34] and canted phase [35–38,56,67], where they appear as consecutive transitions.

From a qualitative point of view, we find clear signatures of electronic behavior characteristic for the kagome lattice. We reveal sublattice interference in an effective, *ab initio* inspired model for FeGe, which we expect to be the reason behind similarities of our phase diagram with that of the pure kagome lattice Hubbard model [44–48]. Qualitative similarities include $q=0$ magnetic instabilities at small, charge/spin-bond order for intermediate, and pairing instabilities for large nearest-neighbor interaction. This constitutes a nontrivial outcome given the different starting points and exposes universal aspects between magnetic and nonmagnetic kagome metals. Taking a recent comparison [47] between different FRG schemes for the momentum resolution into

account, we expect this conclusion to remain qualitatively correct, even if exact phase boundaries may shift.

However, there are also qualitative differences which we mainly attribute to the broken SU(2) of the AFM parent state in FeGe. We find that this promotes longitudinal over transversal spin orders because the on-site interaction cannot couple electrons with opposite spins even though they have degenerate energies (the “Kramers doublet” is nonlocal due to $T_z \otimes T$ symmetry). Furthermore, we find a strong tendency towards f -wave triplet superconductivity. While f -wave pairing was discussed for the paramagnetic kagome lattice in the context of phonons, disorder, or doping [45,68,69], here it becomes a very robust instability for a slight increase of the nearest-neighbor interaction. This opens the intriguing possibility of inducing superconductivity also in magnetic kagome metals if the ratio of nearest-neighbor over on-site interaction can be tuned, e.g., via pressure or doping [43]. We caution, however, that we do not expect our approximation of the FRG equations to be valid in the strongly interacting regime, where local-moment formation may affect some of the phases, in particular superconductivity. So, while it is conceivable that a superconducting phase appears as the p -wave density waves get destabilized, further studies for strong coupling are needed.

Our results also indicate that a purely electronic mechanism has difficulty to explain the occurrence of the charge density wave at relatively high temperatures of 100 K. Thus, it would be very interesting to include the cooperation with phonons, [39,70,71] in particular, because our analysis of 3D susceptibilities in the *ab initio* model does not suggest another contribution with wave vector M or L that is not captured by the effective low-energy model. At the same time, we emphasize in this context that the transition temperature for long-range CDW order is significantly reduced in annealed samples [31,42,72]. Generally, it will be crucial to include other bands or interactions [73] when studying orders that compete with a CDW and further work out universal aspects of magnetic kagome metals. This can be done, e.g., by analyzing AFM kagome Hubbard models or by determining the CDW landscape of phenomenological Landau models for DFT results.

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