

Flash temperature in sliding contacts

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The temperature increase in the contact regions between solids in sliding contact can easily reach several hundred Kelvin and thereby dramatically affect friction and wear. The classical theories by Jaeger, Archard, and Greenwood, commonly used to estimate flash temperature, ignore the multiscale nature of real surfaces and instead approximate the frictional heat sources with circular or square shapes. Here, we present an analytical theory for the flash temperature valid for randomly rough surfaces with roughness across arbitrarily many decades in length scale. The theory extends established methods for stress correlation functions and peak stresses to temperature. Numerical results for rubber sliding on concrete, and granite on granite, are presented as illustrations. We show that classical theories for flash temperature fail severely for surfaces with multiscale roughness.

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I. INTRODUCTION

Friction between surfaces generates heat, leading to temperature increases at the contact points. This phenomenon is known as flash temperature, which is the high, localized, and brief temperature spike that occurs at the true points of contact between two rubbing solids. The rapid generation of heat at these locations causes thermal spikes, resulting in intense flash temperatures as kinetic energy is converted into heat. These spikes can be extremely high, sometimes reaching over 1000 °C, but they are also incredibly brief, lasting only for the instant that the asperities are in contact. This process is so rapid that the generated heat has little time to conduct away into the bulk of the materials, trapping thermal energy and further elevating the temperature at the contact points.

In almost all cases, most of the dissipated energy in sliding friction ends up as thermal energy within the sliding solids. The temperature field in the solids can be written as $T(\mathbf{x}, t) = T_0(\mathbf{x}, t) + \Delta T(\mathbf{x}, t)$. The background temperature $T_0(\mathbf{x}, t)$ varies slowly in space and time while the flash temperature $\Delta T(\mathbf{x}, t)$ varies rapidly in space and time. $\Delta T(\mathbf{x}, t)$ is nonzero only close to the asperity contact regions and is highly localized in space.

The flash temperature is nearly independent of the external conditions, but this is not the case for the background temperature. If there was no heat transfer to the surrounding environment, no (time-independent) steady-state temperature

profile could form, even after a long time. We demonstrate that this holds true even for a semi-infinite system.

Consider a rectangular block sliding on a substrate. We assume that all the friction work is converted into heat. If $\sigma(\mathbf{x}, t)$ is the normal stress acting on the block, we assume that the heat source is $\dot{q}(\mathbf{x}, t) = \mu v \sigma(\mathbf{x}, t)$, where v is the sliding speed. The average $\dot{q}_0(t) = \langle \dot{q}(\mathbf{x}, t) \rangle$ (where $\langle \dots \rangle$ stands for averaging over the surface area) determines the background temperature. Assuming no heat transfer to the surroundings (isolated system) for a semi-infinite system ($z > 0$), if sliding starts at the time $t = 0$, the background temperature on the surface $z = 0$ at time t is easily obtained from the heat diffusion equation

$$T_0(t) = \frac{1}{\kappa} \left(\frac{D}{\pi} \right)^{1/2} \int_0^t dt' \frac{\dot{q}_0(t')}{(t - t')^{1/2}}, \quad (1)$$

where κ is the thermal conductivity and D is the heat diffusivity. If $\dot{q}_0$ is time independent for $t > 0$, this gives

$$T_0(t) = \frac{2\dot{q}_0}{\kappa} \left(\frac{D}{\pi} \right)^{1/2} \sqrt{t}, \quad (2)$$

which increases without limit as $t \rightarrow \infty$. Thus, a steady state (constant temperature distribution) can prevail only if heat energy is transferred to the surroundings. However, the temperature increase in the asperity contact regions, which we will refer to as the flash temperature, reaches a steady state after a very short sliding distance on the order of the diameter of the contact regions. For example, a stationary parabolic (Hertzian) heat source

$$\dot{q}(\mathbf{x}) = \dot{q}_1 \left[1 - \left(\frac{r}{R} \right)^2 \right]^{1/2}, \quad (3)$$

which is turned on at time $t = 0$ results in a time-independent temperature profile for $t \gg R^2/D$, where for $z = 0$ and $r < R$

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the temperature is

$$\Delta T(\mathbf{x}) = T_1 \left[1 - \frac{1}{2} \left(\frac{r}{R} \right)^2 \right], \quad (4)$$

with $T_1 = \dot{q}_1 \pi R / 4\kappa$.

Frictional heating is important in a wide range of applications, spanning from ice and rubber friction to the sliding of minerals like granite, which is central to earthquake dynamics. The flash temperature occurs in the asperity contact regions, where frictional energy is converted into heat. This localized heating can have a crucial influence on the resulting friction, usually reducing it. This is the case for sliding ice, where the flash temperature can melt the ice [1–3], or at least shift the temperature in the contact region toward the melting point [4,5], thereby reducing the frictional shear stress and the friction force. The same is true for granite sliding on granite, where the melting point of the mineral (primarily quartz) may be reached at the sliding speeds (on the order of ~ 1 m/s) involved in earthquakes [6–8]. Rubber friction depends exponentially on the temperature. An increase in thermal energy shifts the friction coefficient master curve to higher sliding speeds, which usually reduces the friction but sometimes increases it [9–11].

In Ref. [7], Rice studied the importance of the flash heating to earthquake dynamics. He considered a model in which the frictional shear stress is constant until the flash temperature reaches the order of the pseudotachylite (rather than quartz or silica) melting point, after which the shear stress was assumed to be negligible. However, the high stresses and temperatures in the mineral contact regions already weaken the interface during slip substantially, well before true melting occurs [12]. Minerals like quartz, which are crystalline, may become amorphous during slip in the contact region (where quartz may be locally converted to silica), and will soften continuously with increasing temperature and sliding speed [12]. This results in a frictional shear stress that could decrease significantly even before true melting occurs, as also observed for other crystalline materials, e.g., ice [2], which undergoes cold, displacement-driven amorphization [4].

One way the flash temperature can manifest itself in experiments is through sliding-induced phase transformations in materials. This was already observed in the classical studies by Bowden and Tabor, and more recently in many other studies [13,14]. However, in general, these transformations may be mechanochemical in nature, involving both the high contact stresses and the flash temperature. That is, the chemical or structural modifications involve stress-aided thermal excitation.

All surfaces possess roughness on many length scales, which complicates the determination of the flash temperature. One key complication is the thermal interaction between the hot spots, which is particularly important between the closely spaced contact regions within the macroasperity contact areas. Classical theories generally treat these heat sources as isolated, neglecting the collective heating effects inherent in the hierarchical asperity-on-asperity structure of real surfaces. Alternatively, if they lump many small asperities into larger ones, they do not resolve the local temperature spikes.

The thermal coupling between different scales is illustrated in Fig. 1, which shows the exact numerical solution of the

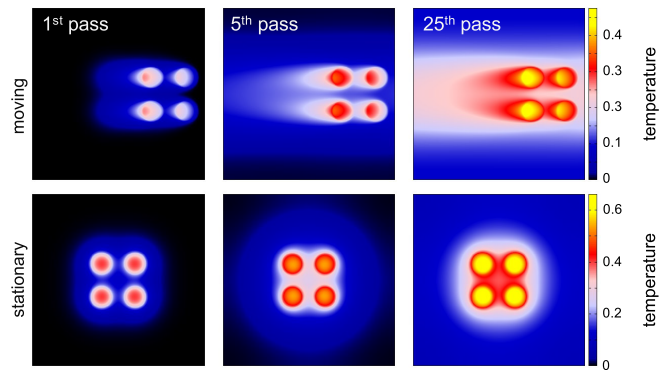


FIG. 1. The temperature on the surface of a semi-infinite solid with a periodic set of heat sources, where each unit cell contains four circular heat sources of constant intensity. Different panels show the heat distribution at various sliding stages. Top row is for $\nu R/D = 20$ and bottom row for $\nu R/D = 0$, where D is the thermal diffusivity.

heat diffusion equation, for four circular and uniform heat sources on a semi-infinite solid. The top row shows moving heat sources and the lower row shows lower stationary heat sources. The calculations use periodic boundary conditions so the system consists of a semi-infinite solid with a finite concentration of heat sources. For moving heat sources at short sliding distances, as in Fig. 1, top-left panel, there is negligible thermal overlap between the periodically extended heat sources. However, after a long enough sliding distance, as in Fig. 1, top-right panel, a steady state forms where the temperature at a heat source depends on the temperature increase induced by the heat sources in front of it.

In this article, we present a multiscale theory for the flash temperature that includes all relevant length scales. We assume steady sliding (constant sliding velocity), but the theory is easily extended to nonsteady sliding (see Ref. [15]), as is important, for example, in fretting contacts. We derive temperature-temperature and temperature-stress correlation functions, which contain information about the flash temperature. In the limiting case of roughness on a single length scale, the results are compared to the classical theories of Jaeger, Archard, and Greenwood [16–18] (see also [19–24]). These authors studied the temperature resulting from moving heat sources with circular or rectangular shapes.

A multiscale contact temperature model was developed by Choudhry *et al.* [21,25], and successfully applied to the experimental study in Ref. [26]. However, their study was purely numerical and was limited to surfaces with roughness restricted to a narrow range of wavelengths. The methodology presented here is analytical and results in equations that can be applied to systems with roughness across arbitrarily many decades of length scales. In Ref. [27], we have shown that the basic equations derived here can successfully explain the experimental observations presented by Sutter and Ranc [26]. Both the study of Choudhry *et al.* [21,25] and the study in Ref. [27] depend on the power spectrum of the surface roughness, which was not presented in Ref. [26] but modeled in Refs. [21,25] assuming plausible roughness parameters, and in Ref. [27] using the measured surface roughness of a similarly prepared steel surface with almost the same root-mean-square (rms) roughness.

Many experimental studies of the flash temperature have been presented [6,8,21,28]. However, we are not aware of any study where the surface roughness power spectra of the used materials was given, which is an input for the theory presented below. Hence our theory cannot be directly compared to measured flash temperatures. We note, however, that the theory is based on well-tested theories, namely, the Fourier heat diffusion equation and the Persson contact mechanics theory. It involves no other approximations or assumptions.

The theory for the flash temperature presented below assumes semi-infinite solids. Since the flash temperature is localized to small regions of the solids close to the asperity contact regions, this assumption is valid in almost all practical applications. There is at present a strong drive to produce micro or nanoscale machines, which sometimes involves high sliding velocities [29]. For such systems, the assumption of semi-infinite solids when calculating the flash temperature may break down.

In this study, we assume that the frictional energy ends up as thermal energy. This is a good approximation in most cases, as the energy involved in wear processes are almost always negligible. In some applications, the frictional heating may induce phase transformations in the asperity contact regions [30]. This could consume some fraction of the frictional energy and also change the local thermal properties, e.g., the thermal conductivity, which would influence the frictional heating.

Heat energy can be transferred from one solid to another, even without involving frictional heating. This is the case for stationary contact if one solid is at a higher temperature than another solid. In this case, the temperature in the solids depends on the contact heat resistance, which depends on the surface roughness. For solids with multiscale surface roughness, this topic has been studied in detail in earlier publications [31–34] and will not be addressed here.

In this study, from Sec. III onward, all temperatures refer to the increase in temperature above the background temperature T_0 . Thus, T_{flash} is a weighted average flash temperature, and the actual temperature in the contact regions is $T_0 + T_{\text{flash}}$. Similarly, temperature correlation functions like $\langle T(\mathbf{x})T(\mathbf{x}') \rangle$ are calculated with the background temperature T_0 subtracted from $T(\mathbf{x})$. Stated differently, all temperatures refer to actual temperatures only if the background temperature vanishes.

II. PRELIMINARY DISCUSSION

Before presenting the formal theory, it is helpful to discuss the different length and time scales entering the model shown in Fig. 1 as a conceptual framework for heating in a sliding multiscale contact. The smallest scale is defined by an individual circular heat source, which, in the given context, is referred to as a microasperity contact regions with radius r . In the model, the centers of mass of two adjacent microasperity contact regions are separated by $16r/5$, yielding an effective radius of $R = (1 + 8/5)r$ for the macroasperity. The periodically repeated cell is $L = 8r$ long in the in-plane directions and represents a measure of the separation between macroasperities. The height of the cell is $h \approx L$. At the bottom of the cell, the boundary condition $T(x, y, z = -h) = 0$ is imposed.

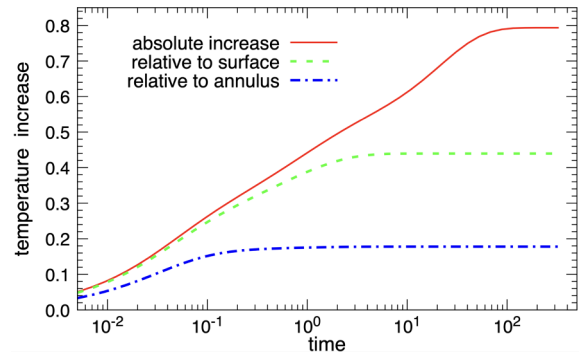


FIG. 2. Temperature increase at the center point of a microasperity: absolute increase (full, red line), relative to the mean surface temperature (dashed, green line), and relative to the asperity's annulus (dash-dotted, blue line).

We first consider the stationary case or, alternatively, a scenario where all heat is dissipated into the sliding body. In this case, four distinct relaxation times can be defined, arising from the structure of the homogeneous diffusion equation,

$$\frac{\partial}{\partial t} T(\mathbf{x}, t) = D \nabla^2 T(\mathbf{x}, t),$$

where D is the thermal diffusivity. The spatial Fourier transform of this equation,

$$\frac{\partial}{\partial t} T(\mathbf{q}, t) = -Dq^2 T(\mathbf{q}, t),$$

connects the relaxation time τ_q to a temperature undulation with wave vector q via $\tau_q = 1/Dq^2$.

For the microasperity, we associate the relaxation with a wave number $q_r = \pi/r$, yielding $\tau_r = (1/\pi)^2 \approx 0.10$ in units of $[\tau] = r^2/D$. Similarly, for the macroasperity, $q_R = \pi/R$ leads to $\tau_R = (R/\pi r)^2 \approx 0.69$. The slowest in-plane relaxation is related to the smallest in-plane wave number of the periodic cell, $q_L = 2\pi/L$, yielding $\tau_L = (L/2\pi r)^2 \approx 1.62$. Finally, for the cell height h , the Dirichlet boundary condition at $z = h$ requires a fundamental mode $q_h = \pi/2h$. This results in $\tau_h = (2h/\pi r)^2 \approx 25.9$. As detailed in the technical comment below, this requires approximately one million time steps to be done for each τ_h when choosing a mesh size of $a = r/32$.

The dash-dotted, blue line in Fig. 2 reveals that the local flash temperature, which could be defined as the temperature at the center of a microasperity relative to its annulus, plateaus shortly after the local relaxation time τ_r has elapsed. The flash temperature reaches a plateau at a later time when defined relative to the annulus of the macroasperity or relative to the entire surface. Since the associated relaxation times, τ_R and τ_L , are similar in the studied model, we focus on the latter (see the dashed, green line in Fig. 2). All results in later sections will refer to this definition of the flash temperature. Finally, as is important in practice when the relative contact area is not very small, the absolute flash temperature keeps rising until τ_h is surpassed.

When heat is dissipated into the substrate rather than into the slider, as illustrated by the thermal footprint shown in Fig. 1, new characteristic timescales emerge. Specifically, heat transport on length scales l satisfying $v/l > \tau_l^{-1}$ becomes convective rather than diffusive. This transition is evident in

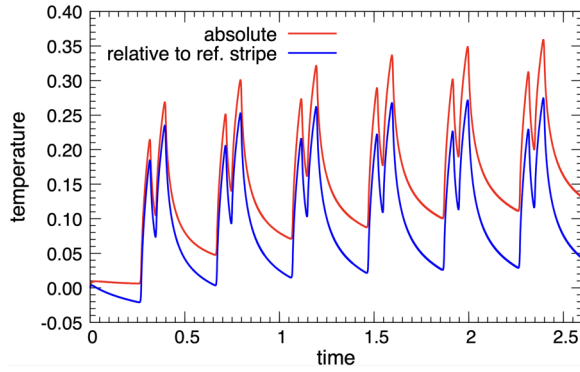


FIG. 3. Temperature increase at a surface point as a function of time: absolute increase (solid red line) and increase relative to a reference stripe containing the centers of two circular heat sources (dash-dotted blue line).

Fig. 3, where the excess temperature is measured relative to a reference stripe aligned with the centers of two consecutive heat sources. The temperature profile assumes nearly the same shape during the first passage of the macroasperity as it does during the second. Thus, the relative temperature profile along the sliding direction reaches a quasisteady state much faster than the absolute temperature. Each quasiperiod of the profile shown here effectively represents the (inverted) spatial temperature distribution along a path through the centers of the microasperities, translated into the time domain.

As a technical comment, we remark that the discretization at the interface was $a = L/n_x$, which for $n_x = 512$ corresponds to $a = r/32$. This yields a maximum wave number $q = \pi/a$ with an associated grid-level relaxation time of $\tau_a = (a/\pi)^2 \approx 2.5 \times 10^{-5}$ (in units of r^2/D). For the numerical integration, we employed a forward Euler scheme. The integration time step was set to $\Delta t = 0.1 \times (a^2/D)$, which corresponds to approximately $0.1\pi^2\tau_a$. This choice is well-suited for the explicit scheme, as it sits safely below the von Neumann stability limit ($\Delta t \leq a^2/6D$ for 3D) while providing sufficient temporal resolution to capture the transients of the microasperities.

Yet, $\tau_h/\tau_a \approx 10^6$ time steps must be performed for the largest relaxation time. While this is still manageable on a 512×512 large system using commodity computers, it illustrates how challenging (nonsteady-state) simulations become in a true multiscale system. Specifically, in our house-written hierarchical solver, where the mesh size is doubled every four layers into the bulk, doubling the L/a ratio increases the required numerical effort 16-fold. This is because the number of grid points in the outermost layer quadruples while the time step must be reduced by a factor of 4. In contrast, steady-state profiles can be obtained much more readily, as they require merely one fast Fourier transform (FFT) of the heat-flow profile into the Fourier representation, and an inverse FFT to harvest the temperature profile.

III. THEORY

The temperature distribution in a sliding contact is considered in the steady state after run-in. We assume that frictional energy is produced only in the real contact area

within a nanometer-thin surface layer, which we consider to be infinitesimally thin. We analyze the heat diffusion in the half-space $z > 0$ with a heat source (energy per unit area and unit time) $\dot{q}(\mathbf{x}, t) = \dot{q}(x, y, t)$ on the surface $z = 0$, and with the heat current J_z vanishing on the regions of the surface where $\dot{q}(\mathbf{x}, t) = 0$. This heat diffusion problem can be solved most easily by extending the problem to the heat diffusion in an infinite solid with a heat source on the surface $z = 0$, which is twice as strong, $2\dot{q}(\mathbf{x}, t)$. Thus, the temperature distribution in the sliding solid is determined by the heat diffusion equation

$$\rho c_p \frac{\partial T}{\partial t} - \kappa \nabla^2 T = 2\dot{q}(\mathbf{x}, t)\delta(z),$$

where ρ , c_p , and κ are the mass density, the specific heat capacity, and the thermal conductivity, respectively. Introducing the thermal diffusivity $D = \kappa/\rho c_p$, we can write

$$\frac{\partial T}{\partial t} - D \nabla^2 T = \frac{2D}{\kappa} \dot{q}(\mathbf{x}, t)\delta(z), \quad (5)$$

where $\dot{q}$ depends on the surface position $\mathbf{x} = (x, y)$ and time t . The heat source $\dot{q}$ is non-vanishing only in the area of real contact, which is the area where the normal stress is nonzero. In the simplest case, one can assume that $\dot{q}$ is proportional to the normal stress $\sigma(\mathbf{x})$, so $\dot{q}(\mathbf{x}, t) = \dot{q}(\mathbf{x} - \mathbf{v}t) = \mu v \sigma(\mathbf{x} - \mathbf{v}t)$. Writing

$$\dot{q}(\mathbf{x}, t) = \int d^2q \dot{q}(\mathbf{q}) e^{i\mathbf{q} \cdot (\mathbf{x} - \mathbf{v}t)}$$

and using the identity

$$\delta(z) = \frac{1}{2\pi} \int dk e^{ikz},$$

we obtain from (5):

$$T = \frac{1}{\pi\kappa} \int d^2q dk \frac{\dot{q}(\mathbf{q})}{q^2 + k^2 - i\mathbf{q} \cdot \mathbf{v}/D} e^{i[\mathbf{q} \cdot (\mathbf{x} - \mathbf{v}t) + kz]}. \quad (6)$$

We are interested in the temperature at the surface, $z = 0$. For this case, the integral in (6) can be performed using the identity

$$\int_{-\infty}^{\infty} dx \frac{1}{x^2 + a^2} = \frac{\pi}{a}.$$

It follows that

$$T(\mathbf{x}, t) = \frac{1}{\kappa} \int d^2q \frac{\dot{q}(\mathbf{q})}{(q^2 - i\mathbf{q} \cdot \mathbf{v}/D)^{1/2}} e^{i\mathbf{q} \cdot (\mathbf{x} - \mathbf{v}t)},$$

or, assuming Amontons' law holds locally,

$$T(\mathbf{x}, t) = \frac{\mu v}{\kappa} \int d^2q \frac{\sigma(\mathbf{q})}{(q^2 - i\mathbf{q} \cdot \mathbf{v}/D)^{1/2}} e^{i\mathbf{q} \cdot (\mathbf{x} - \mathbf{v}t)}. \quad (7)$$

Writing

$$T(\mathbf{x}, t) = \int d^2q d\omega T(\mathbf{q}, \omega) e^{i(\mathbf{q} \cdot \mathbf{x} - \omega t)},$$

we obtain

$$T(\mathbf{q}, \omega) = \frac{\mu v}{\kappa} \frac{\sigma(\mathbf{q})}{(q^2 - i\mathbf{q} \cdot \mathbf{v}/D)^{1/2}} \delta(\omega - \mathbf{v} \cdot \mathbf{q}). \quad (8)$$

In what follows, we will assume that the average temperature has been subtracted from T , so $\langle T \rangle = 0$. Similarly, we

assume that the average normal stress has been subtracted from σ , so $\langle \sigma \rangle = 0$. This implies that the correlation function $\langle T(\mathbf{x}, t)T(\mathbf{0}, 0) \rangle$, when averaged over the surface $z = 0$, vanishes, i.e., the correlation function takes both positive and negative values. The same holds for the stress-stress correlation function. Note that the average temperature is the $\mathbf{q} = \mathbf{0}$ component in (7), so subtracting the average temperature just means excluding the $\mathbf{q} = \mathbf{0}$ component in the Fourier expansion of the temperature. The same is true for the normal surface stress. Using that

$$\begin{aligned} \langle T(\mathbf{x}, t)T(\mathbf{0}, 0) \rangle \\ = \frac{(2\pi)^3}{A_0 t_0} \int d^2 q d\omega \langle T(\mathbf{q}, \omega)T(-\mathbf{q}, -\omega) \rangle e^{i(\mathbf{q} \cdot \mathbf{x} - \omega t)}, \end{aligned}$$

where A_0 is the nominal surface area and t_0 is the sliding time, and noting that

$$[\delta(\omega - \mathbf{v} \cdot \mathbf{q})]^2 = \frac{t_0}{2\pi} \delta(\omega - \mathbf{v} \cdot \mathbf{q}),$$

we find, using (8):

$$\begin{aligned} \langle T(\mathbf{x}, t)T(\mathbf{0}, 0) \rangle &= \frac{(2\pi)^2}{A_0} \left(\frac{\mu v}{\kappa} \right)^2 \int d^2 q \\ &\times \frac{\langle \sigma(\mathbf{q})\sigma(-\mathbf{q}) \rangle}{[q^4 + (\mathbf{q} \cdot \mathbf{v}/D)^2]^{1/2}} e^{i\mathbf{q} \cdot (\mathbf{x} - \mathbf{v}t)}. \end{aligned} \quad (9)$$

Furthermore, substituting the stress power spectrum [35]

$$\langle \sigma(\mathbf{q})\sigma(-\mathbf{q}) \rangle = \frac{A_0}{(4\pi)^2} (E^*)^2 q^2 C(q)W(q), \quad (10)$$

where $E^* = E/(1 - \nu^2)$ is the effective modulus, where $C(q)$ is the surface roughness power spectrum, and

$$W(q) = P(q)[\gamma + (1 - \gamma)P^2(q)],$$

where

$$P(q) = \text{erf}\left(\frac{\sigma_0}{2\sqrt{G}}\right),$$

$$G = \frac{\pi}{4} (E^*)^2 \int_{q_0}^q dq' q'^3 C(q'),$$

we obtain

$$\begin{aligned} \langle T(\mathbf{x}, t)T(\mathbf{0}, 0) \rangle \\ = \left(\frac{\mu v}{\kappa} \right)^2 \frac{(E^*)^2}{4} \int d^2 q \frac{q^2 C(q)W(q)}{[q^4 + (\mathbf{q} \cdot \mathbf{v}/D)^2]^{1/2}} e^{i\mathbf{q} \cdot (\mathbf{x} - \mathbf{v}t)} \end{aligned}$$

and the equal time correlation function

$$\begin{aligned} \langle T(\mathbf{x}, 0)T(\mathbf{0}, 0) \rangle \\ = \left(\frac{\mu v}{\kappa} \right)^2 \frac{(E^*)^2}{4} \int d^2 q \frac{q^2 C(q)W(q)}{[q^4 + (\mathbf{q} \cdot \mathbf{v}/D)^2]^{1/2}} e^{i\mathbf{q} \cdot \mathbf{x}}. \end{aligned} \quad (11)$$

The integral over $\mathbf{q} = (q_x, q_y)$ in this expression may be evaluated using a two-dimensional fast Fourier transform. Note that the correlation function depends not only on the distance $r = |\mathbf{x}|$ but also on the angle θ between the velocity vector $\mathbf{v}$ and $\mathbf{x}$. Here, we focus on the angular averaged (in the

xy-plane) temperature correlation function, which we denote by $g_T(r)$. Using the identity

$$\frac{1}{2\pi} \int_0^{2\pi} d\theta e^{iqr \cos \theta} = J_0(qr),$$

we obtain

$$g_T(r) = \left(\frac{\mu v}{\kappa} \right)^2 \frac{(E^*)^2}{4} \int d^2 q \frac{q^2 C(q)W(q)}{[q^4 + (\mathbf{q} \cdot \mathbf{v}/D)^2]^{1/2}} J_0(qr).$$

Transforming to polar coordinates, the $\mathbf{q}$ integration yields

$$\begin{aligned} g_T(r) &= \left(\frac{\mu v}{\kappa} \right)^2 \frac{(E^*)^2}{4} \int_{q_0}^{q_1} dq q^2 C(q)W(q) J_0(qr) \int_0^{2\pi} d\phi \\ &\times \frac{1}{[q^2 + (v/D)^2 \cos^2 \phi]^{1/2}} \end{aligned}$$

or

$$\begin{aligned} g_T(r) &= \left(\frac{\mu v}{\kappa} \right)^2 (E^*)^2 \int_{q_0}^{q_1} dq q C(q)W(q) J_0(qr) \int_0^{\pi/2} d\phi \\ &\times \frac{1}{[1 + (v/Dq)^2 \cos^2 \phi]^{1/2}}. \end{aligned} \quad (12)$$

The q integral involving the Bessel function $J_0(qr)$ may be evaluated using the method described in Appendix D in Ref. [36]. The ϕ integral is of the form

$$f(x) = \int_0^{\pi/2} d\phi \frac{1}{[1 + x \cos^2 \phi]^{1/2}}$$

and can be related to the complete elliptic integral of the first kind ($0 \leq m < 1$):

$$K(m) = \int_0^{\pi/2} d\phi \frac{1}{[1 - m \cos^2 \phi]^{1/2}},$$

using

$$f(x) = \frac{1}{(1+x)^{1/2}} K\left(\frac{x}{1+x}\right).$$

This representation is computationally advantageous, as highly efficient numerical routines are available for the evaluation of elliptic integrals.

We first consider the two limiting cases of very low and very high velocities. As $v \rightarrow 0$, Eq. (12) yields

$$g_T(r) \approx \left(\frac{\mu v}{\kappa} \right)^2 (E^*)^2 \int_{q_0}^{q_1} dq q C(q)W(q) J_0(qr) \left(\frac{\pi}{2} \right),$$

indicating that $g_T \sim v^2$ as $v \rightarrow 0$. To study the $v \rightarrow \infty$ limit, we note that for large v :

$$\begin{aligned} &\int_0^{\pi/2} d\phi \frac{1}{[1 + (v/Dq)^2 \sin^2 \phi]^{1/2}} \\ &\approx \int_0^{\pi/2} d\phi \frac{1}{[1 + (v/Dq)^2 \phi^2]^{1/2}} \\ &\approx \frac{Dq}{v} \int_0^{\pi v/2qD} dx \frac{1}{[1 + x^2]^{1/2}} \approx \frac{Dq}{v} \ln\left(\frac{2\pi v}{qD}\right). \end{aligned}$$

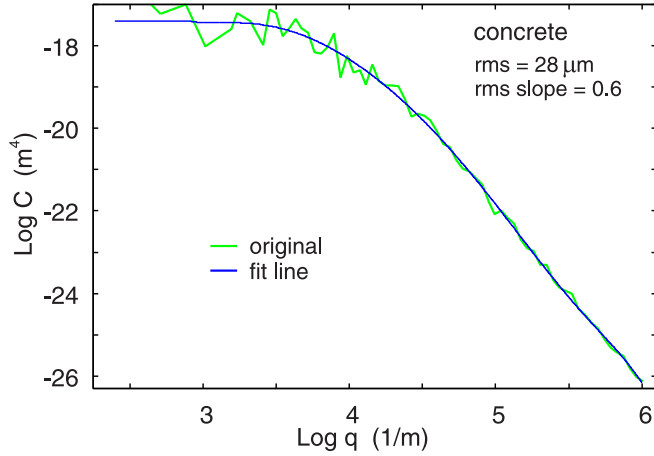


FIG. 4. The surface roughness power spectrum of the concrete surface used in the present study. The fitted spectrum was employed in the numerical calculations to eliminate nonmonotonic artifacts in the correlation functions that would otherwise be induced by stochastic scatter in the raw data.

Hence, as $v \rightarrow \infty$,

$$g_T(r) \approx \left(\frac{\mu v}{\kappa}\right)^2 (E^*)^2 \int_{q_0}^{q_1} dq q C(q) W(q) J_0(qr) \times \frac{Dq}{v} \ln\left(\frac{2\pi v}{qD}\right),$$

showing that $g_T \sim v \ln v$ in the high-velocity limit.

To illustrate the application of the theory, we present numerical results for a representative elastomer system. We consider a rubberlike solid with Young's modulus $E = 14$ MPa and Poisson ratio $\nu = 0.5$, sliding with a flat bottom surface on a rigid substrate. The substrate roughness is characterized by the power spectrum of a concrete surface, as shown in Fig. 4. The calculations assume a nominal contact pressure $\sigma_0 = 0.1$ MPa and a friction coefficient $\mu = 1$. The material's thermal properties are defined by a mass density $\rho = 1000$ kg/m³, a specific heat capacity $c_p = 1000$ J/(kgK), and a thermal conductivity $\kappa = 0.1$ W/(mK).

Figure 5 shows the mean square temperature fluctuation $\langle T^2 \rangle$ as a function of the sliding speed on a log-log scale. Note that $\langle T^2 \rangle \sim v^2$ for low sliding speeds and $\sim v \ln v$ for high sliding speeds.

Figure 6 shows the dependence of the normalized temperature-temperature correlation function $g_T(r)/g_T(0)$ on the distance $r = |\mathbf{x} - \mathbf{x}'|$ for several sliding speeds. The results are nearly independent of the sliding speed in the high-velocity region ($v > 0.01$ m/s) and the low-velocity region ($v < 10^{-5}$ m/s). These two velocity regimes correspond to the ranges where $\langle T^2 \rangle$ scales with the velocity as $\sim v \ln v$ and $\sim v^2$, respectively.

It is interesting to compare (12) with the stress-stress correlation function:

$$g_\sigma(r) = \langle \sigma(\mathbf{x}, 0) \sigma(\mathbf{0}, 0) \rangle.$$

Using (10), we obtain [36,37]

$$g_\sigma(r) = (E^*)^2 \frac{\pi}{2} \int_{q_0}^{q_1} dq q^3 C(q) W(q) J_0(qr). \quad (13)$$

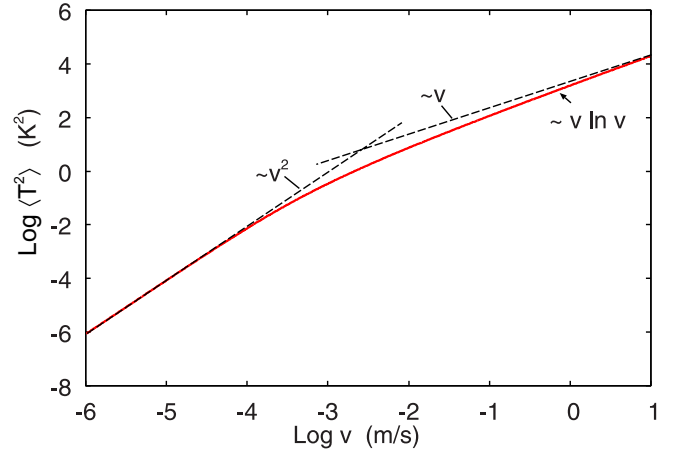


FIG. 5. The mean square temperature fluctuation $\langle T^2 \rangle$ as a function of the sliding speed (log-log scale). The results are for an elastic solid with Young's modulus $E = 14$ MPa and Poisson ratio $\nu = 0.5$ sliding on a rigid concrete surface. The following parameters were assumed: nominal contact pressure $\sigma_0 = 0.1$ MPa, friction coefficient $\mu = 1$, mass density $\rho = 1000$ kg/m³, specific heat capacity $c_p = 1000$ J/kgK, and thermal conductivity $\kappa = 0.1$ W/Km.

In Fig. 7, we compare the dependence of $g_T(r)/g_T(0)$ and $g_\sigma(r)/g_\sigma(0)$ on the distance $r = |\mathbf{x} - \mathbf{x}'|$. The temperature correlation function is calculated for a sliding speed of $v = 10$ m/s. The radius of the nominal contact region can be deduced from the stress-stress correlation function and is $r_0 < 1$ mm. At the sliding speed v , it takes a time $\tau = r_0/v$ to move a distance on the order of the contact region size. During this time, the heat diffuses a distance $d \approx (D\tau)^{1/2} \approx (Dr_0/v)^{1/2} \approx 3$ μm, which is much smaller than the size of the asperity contact region. Nevertheless, Fig. 7 shows that the effective width of the temperature distribution is much larger than that of the stress distribution. The reason for this is the hot

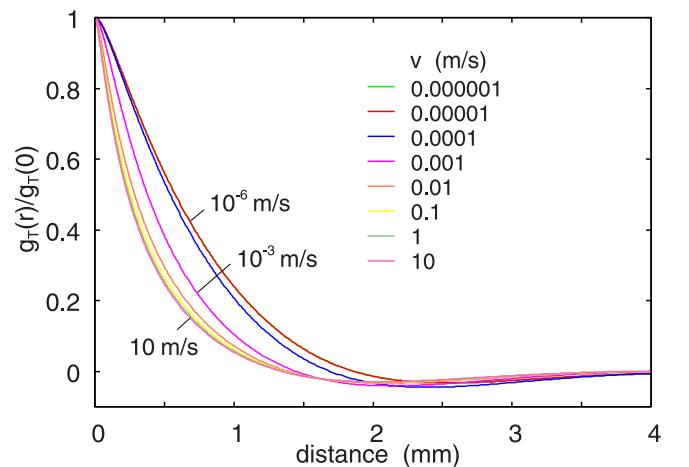


FIG. 6. The dependence of the normalized temperature-temperature correlation function $g_T(r)/g_T(0)$ on the distance $r = |\mathbf{x} - \mathbf{x}'|$ for several sliding speeds. The results are nearly independent of the sliding speed in the high-velocity region ($v > 0.01$ m/s) and the low-velocity region ($v < 10^{-5}$ m/s). These two regions correspond to the regimes where $\langle T^2 \rangle$ scales with the velocity as $\sim v \ln v$ and $\sim v^2$, respectively.

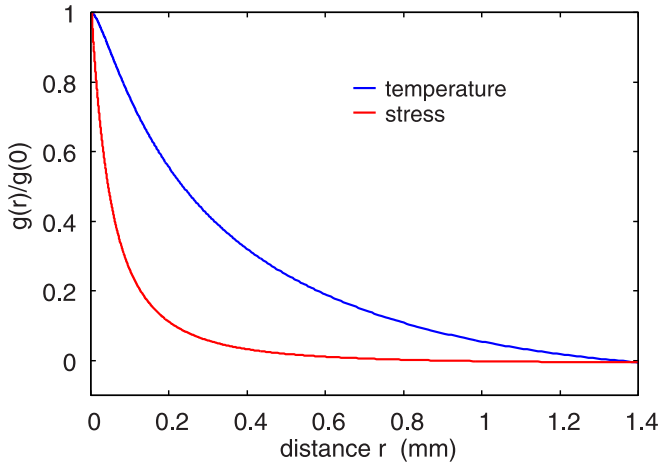


FIG. 7. The dependence of the normalized temperature and stress correlation functions on the distance $r = |\mathbf{x} - \mathbf{x}'|$. The parameters are the same as in Fig. 5. The temperature correlation function is calculated for a sliding speed of $v = 10$ m/s.

tracks left on the rubber surface behind the moving contact regions (see Fig. 8). At high sliding speeds, these hot tracks will only disappear (via heat diffusion) at a long distance behind the moving contact regions. At low sliding speeds, the hot tracks will widen by thermal diffusion and could also be reduced by heat radiation (emission of photons) or by heat transfer to the surrounding gas or fluid.

Since the temperature distribution extends far beyond the contact regions while the stress correlation is nonvanishing only within the contact area, the normalized temperature distribution is more affected than the stress distribution by a change in the nominal contact pressure. We will illustrate this with two examples.

As the contact pressure increases, the concentration of macroasperity contact regions increases. For low sliding speeds, heat diffusion broadens the temperature profile, and the profile approaches the average temperature faster with increasing r as the concentration of macroasperities grows. Similarly, for high sliding speeds, the tails of the hot tracks (see Fig. 8) overlap other contact regions more frequently; consequently, the temperature profile again approaches the average temperature faster as σ_0 increases. As a result, in both cases, the normalized temperature correlation will decay faster as the contact pressure increases. This is illustrated in

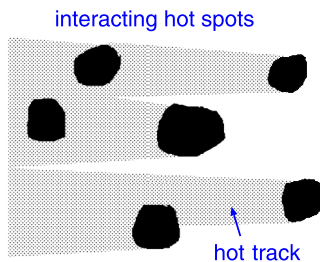


FIG. 8. At high sliding speeds, a hot track is formed behind each macroasperity contact. These tracks overlap with the hot spots and tracks of other asperity contact regions located further downstream.

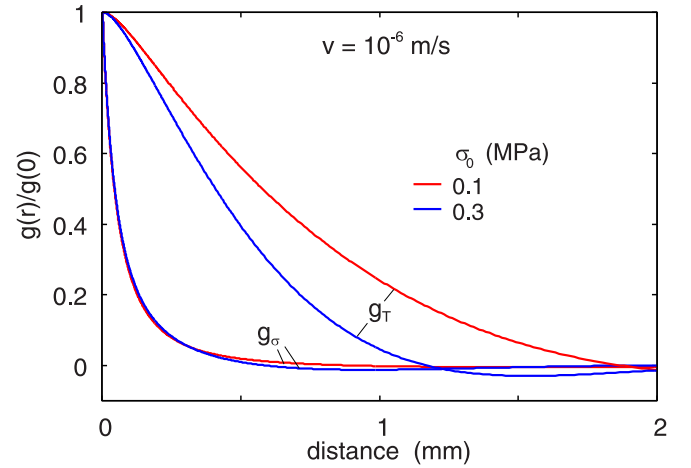


FIG. 9. The dependence of the normalized temperature and stress correlation functions on the distance $r = |\mathbf{x} - \mathbf{x}'|$ for the nominal pressures $\sigma_0 = 0.1$ MPa (red curves) and 0.3 MPa (blue curves). The parameters are the same as in Fig. 5. The temperature correlation function is calculated for a low sliding speed of $v = 10^{-6}$ m/s.

Fig. 9 for low sliding speeds and in Fig. 10 for high sliding speeds.

In applications to sliding friction, the temperature within the asperity contact regions is of primary importance rather than the temperature distribution across the entire surface. To isolate this, one can define an effective flash temperature by weighting the temperature by the local contact stress:

$$T_{\text{flash}} = \frac{\langle T(\mathbf{x})\sigma(\mathbf{x}) \rangle}{\langle \sigma(\mathbf{x}) \rangle}, \quad (14)$$

where $\langle \sigma(\mathbf{x}) \rangle = \sigma_0$ is the nominal contact pressure. Defined this way, the temperature is weighted only where the stresses are high and is ignored in the noncontact regions where

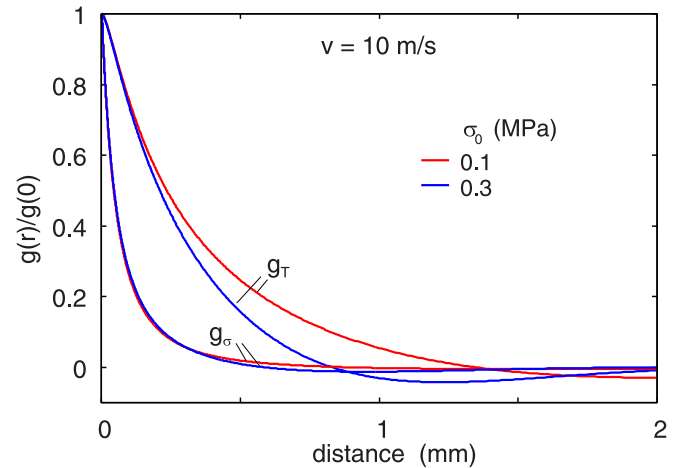


FIG. 10. The dependence of the normalized temperature and stress correlation functions on the distance $r = |\mathbf{x} - \mathbf{x}'|$ for nominal pressures $\sigma_0 = 0.1$ MPa (red curves) and 0.3 MPa (blue curves). The parameters are the same as in Fig. 5. The temperature correlation function is calculated for a high sliding speed of $v = 10$ m/s.

$\sigma(\mathbf{x}) = 0$. Using the relation

$$\langle T(\mathbf{x})\sigma(\mathbf{x}) \rangle = \frac{(2\pi)^2}{A_0} \int d^2q \langle T(\mathbf{q})\sigma(-\mathbf{q}) \rangle$$

and substituting (8), we obtain

$$\langle T(\mathbf{x})\sigma(\mathbf{x}) \rangle = \frac{\mu v}{\kappa} \frac{(2\pi)^2}{A_0} \int d^2q \frac{\langle \sigma(\mathbf{q})\sigma(-\mathbf{q}) \rangle}{(q^2 - i\mathbf{q} \cdot \mathbf{v}/D)^{1/2}}.$$

Finally, applying (10) yields the expression

$$T_{\text{flash}} = \frac{\mu v}{\kappa \sigma_0} (E^*)^2 \int_{q_0}^{q_1} dq q^2 C(q) W(q) \text{Re} \int_0^{\pi/2} d\phi \times \frac{1}{[1 - i(v/Dq) \cos \phi]^{1/2}}. \quad (15)$$

For Hertz contacts at low sliding speed, the flash temperature as defined by (14) is 1.068 times higher than the average flash temperature and a factor of 0.80 times smaller than the maximum of the flash temperature. At high sliding speeds, the corresponding factors are 1.090 and 0.597, respectively (see Ref. [27]). However, as shown below, for surfaces with multiscale roughness, the flash temperature defined by (14) [and given by (15)] differs a lot from the Hertz contact prediction.

The temperature distribution within the macroasperity contacts during sliding is anisotropic, with higher temperatures occurring at the trailing edge compared to the leading edge of the contact. This asymmetry arises because the material at the trailing edge has already been heated by the preceding contact area. The anisotropy of the average temperature distribution can be quantified using the approach described above. We consider the effective temperature gradient:

$$\nabla T_{\text{flash}} = \frac{\langle \nabla T(\mathbf{x})\sigma(\mathbf{x}) \rangle}{\langle \sigma(\mathbf{x}) \rangle}.$$

Using the relation

$$\nabla T(\mathbf{x}) = \nabla \int d^2q T(\mathbf{q}) e^{i\mathbf{q} \cdot \mathbf{x}} = \int d^2q (i\mathbf{q}) T(\mathbf{q}) e^{i\mathbf{q} \cdot \mathbf{x}},$$

we obtain

$$\begin{aligned} \langle \nabla T(\mathbf{x})\sigma(\mathbf{x}) \rangle &= \frac{(2\pi)^2}{A_0} \int d^2q (i\mathbf{q}) \langle T(\mathbf{q})\sigma(-\mathbf{q}) \rangle \\ &= \frac{\mu v}{\kappa} \frac{(2\pi)^2}{A_0} \int d^2q \frac{(i\mathbf{q}) \langle \sigma(\mathbf{q})\sigma(-\mathbf{q}) \rangle}{(q^2 - i\mathbf{q} \cdot \mathbf{v}/D)^{1/2}}. \end{aligned} \quad (16)$$

By choosing the x axis along the sliding direction, only the x component of (16) remains nonvanishing. Denoting this as T'_{flash} (where the prime indicates the derivative with respect to x), we find

$$T'_{\text{flash}} = \frac{\mu v}{\kappa \sigma_0} (E^*)^2 \int_{q_0}^{q_1} dq q^3 C(q) W(q) \text{Re} \times \int_0^{\pi/2} d\phi \frac{i \cos \phi}{[1 - i(v/Dq) \cos \phi]^{1/2}}. \quad (17)$$

Higher-order derivatives of $T(\mathbf{x})$ can be calculated in a similar manner. For instance, considering that a Laplacian in real space is just a multiplication by $-q^2$ in Fourier space, we

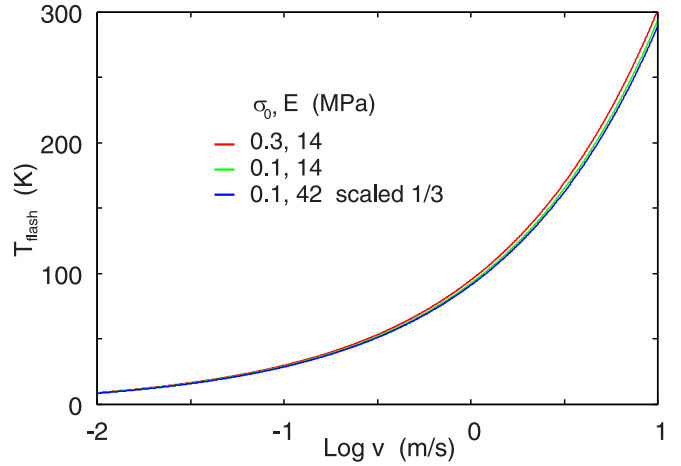


FIG. 11. The flash temperature T_{flash} as a function of the sliding speed for an elastic block sliding on a concrete surface. The red and green lines correspond to nominal contact pressures of $\sigma_0 = 0.3$ MPa and 0.1 MPa, respectively, with a Young's modulus of $E = 14$ MPa. The blue line is for $\sigma_0 = 0.1$ MPa and $E = 42$ MPa, but with the flash temperature scaled by $1/3$. In all calculations, the Poisson ratio $\nu = 0.5$, friction coefficient $\mu = 1$, mass density $\rho = 1000$ kg/m³, heat capacity $c_p = 1000$ J/kgK, and thermal conductivity $\kappa = 0.1$ W/mK.

obtain

$$\langle \nabla^2 T(\mathbf{x})\sigma(\mathbf{x}) \rangle = \frac{\mu v}{\kappa} \frac{(2\pi)^2}{A_0} \int d^2q \frac{(-q^2) \langle \sigma(\mathbf{q})\sigma(-\mathbf{q}) \rangle}{(q^2 - i\mathbf{q} \cdot \mathbf{v}/D)^{1/2}}.$$

Defining the effective flash temperature Laplacian as

$$\nabla^2 T_{\text{flash}} = \frac{\langle \nabla^2 T(\mathbf{x})\sigma(\mathbf{x}) \rangle}{\langle \sigma(\mathbf{x}) \rangle},$$

we find

$$\begin{aligned} \nabla^2 T_{\text{flash}} &= - \frac{\mu v}{\kappa \sigma_0} (E^*)^2 \int_{q_0}^{q_1} dq q^4 C(q) W(q) \text{Re} \\ &\times \int_0^{\pi/2} d\phi \frac{1}{[1 - i(v/Dq) \cos \phi]^{1/2}}. \end{aligned} \quad (18)$$

The way to numerically perform the ϕ integrals in (15) and (17) is discussed in the Appendix.

We now present numerical results for the flash temperature. Figure 11 shows T_{flash} , obtained from (15), as a function of sliding speed for an elastic block on the concrete surface. The red and green lines represent contact pressures of $\sigma_0 = 0.3$ MPa and $\sigma_0 = 0.1$ MPa, respectively, both with a Young's modulus of $E = 14$ MPa. The blue line corresponds to $\sigma_0 = 0.1$ MPa and $E = 42$ MPa, with the flash temperature scaled by a factor of $1/3$.

Note that the result is independent of the nominal contact pressure. This holds as long as the pressure is sufficiently small such that the relative contact area $A/A_0 \ll 1$. In this regime, increasing the pressure does not change the size or pressure distribution within individual contact regions, but merely increases the number of macroasperity contacts proportional to σ_0 . Furthermore, lateral thermal interaction between hot spots remains negligible if the separation between contact regions is large.

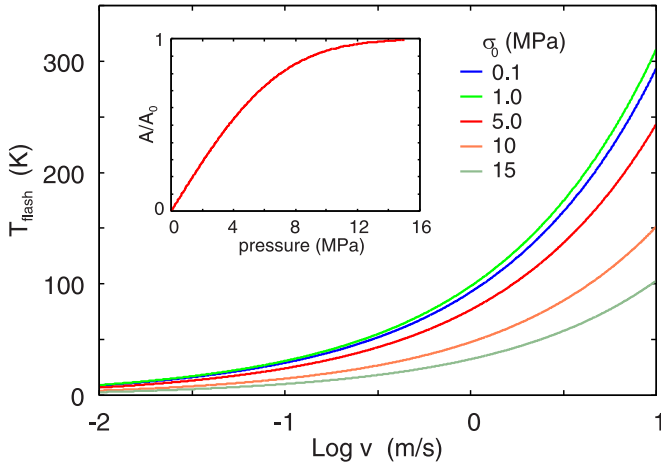


FIG. 12. The flash temperature for different nominal contact pressures indicated. The inset shows the contact area as a function of the applied pressure σ_0 .

Similarly, when $A/A_0 \ll 1$, increasing the Young's modulus does not significantly change the size of the macroasperity contact regions but decreases their number proportional to $1/E$. Consequently, the local pressure acting within those contact regions increases proportional to E , resulting in a flash temperature that is also proportional to E . This explains why scaling the $E = 42$ MPa curve by $1/3$ yields the same temperature-velocity dependence as the $E = 14$ MPa case.

When the applied pressure σ_0 increases, the flash temperature T_{flash} decreases; see Fig. 12. At the same time, the background temperature (not shown), which is the cumulative contribution of the flash temperature, increases in such a way that the surface temperature everywhere on the surface increases. The reason the flash temperature decreases is easy to understand in the special case where the frictional heat source $\dot{q}$ is constant in the contact regions. In this case, the surface temperature averaged over the whole surface will approach the temperature in the contact area as $A/A_0 \rightarrow 1$, so in that limit the flash temperature vanishes and the surface temperature will be constant on the whole surface, and given by (2).

IV. ONE LENGTH-SCALE ROUGHNESS: COMPARING THE THEORY WITH THE CLASSICAL RESULT

It is interesting to compare the expression (15) with the standard expression for the maximum flash temperature resulting from a circular uniform heat source (a disk with radius r_c) moving with velocity v on the surface of a semi-infinite solid. For this case, Greenwood presented an interpolation formula:

$$\Delta T \approx \frac{\sigma_f v \alpha r_c}{\kappa} \frac{1}{[1 + (\alpha/\beta)^2 \pi r_c v / 8D]^{1/2}}. \quad (19)$$

When $\alpha = \beta = 1$, this equation interpolates between the known maximum flash temperature for low and high sliding velocities, and with $\alpha \approx 0.849$ and $\beta \approx 0.610$ it interpolates between the known average temperature for low and high velocities. The expression (15) presents a weighted average of the contact temperature, so when compared to (19) we expect to find $\alpha < 1$ and $\beta < 1$.

Using the local friction stress, $\sigma_f = \mu \bar{\sigma}$, we obtain

$$\Delta T \approx \frac{\mu \bar{\sigma} v \alpha r_c}{\kappa} \frac{1}{[1 + (\alpha/\beta)^2 \pi r_c v / 8D]^{1/2}} \quad (20)$$

The classical approach assumes roughness only on a single length scale, where the contact regions are compact. This corresponds to using a power spectrum of the form

$$C(q) \approx C_0 \delta(q - q_m), \quad (21)$$

where C_0 is chosen such that the rms slope equals ξ_m :

$$\xi_m^2 = 2\pi \int_0^\infty dq q^3 C(q) = 2\pi q_m^3 C_0,$$

which yields

$$C_0 = \frac{\xi_m^2}{2\pi q_m^3}. \quad (22)$$

Substituting (21) and (22) into (15) gives

$$T_{\text{flash}} = \frac{\mu v}{\kappa \sigma_0} (E^*)^2 q_m^2 \frac{\xi_m^2}{2\pi q_m^3} W(q_m) \text{Re} \times \int_0^{\pi/2} d\phi \frac{1}{[1 - i(v/Dq_m) \cos \phi]^{1/2}}.$$

The classical approach assumes well-separated asperity contact regions, in which case the thermal interaction between the contact regions can be neglected. This corresponds to the limit where the relative contact area $A_m/A_0 \ll 1$. In this regime, $W(q_m) \approx \gamma A_m/A_0$ with $\gamma \approx 0.5$, so

$$T_{\text{flash}} \approx \frac{\mu v}{\kappa \sigma_0} (E^*)^2 \frac{\gamma \xi_m^2}{2\pi q_m} \frac{A_m}{A_0} \text{Re} \times \int_0^{\pi/2} d\phi \frac{1}{[1 - i(v/Dq_m) \cos \phi]^{1/2}}. \quad (23)$$

At small loads, when $A_m/A_0 \ll 1$, we have [38–40]

$$\frac{A_m}{A_0} \approx \frac{2\sigma_0}{\xi_m E^*},$$

so

$$T_{\text{flash}} \approx \frac{\mu v}{\kappa \sigma_0} \frac{\gamma}{2\pi q_m} 4\sigma_0^2 \frac{A_0}{A_m} \text{Re} \times \int_0^{\pi/2} d\phi \frac{1}{[1 - i(v/Dq_m) \cos \phi]^{1/2}}.$$

Using that $\bar{\sigma} = \sigma_0 A_0/A_m$ yields

$$T_{\text{flash}} \approx \frac{\mu \bar{\sigma} v}{\kappa} \frac{\gamma}{q_m} \frac{2}{\pi} \text{Re} \int_0^{\pi/2} d\phi \frac{1}{[1 - i(v/Dq_m) \cos \phi]^{1/2}}. \quad (24)$$

The function

$$f(x) = \frac{2}{\pi} \text{Re} \int_0^{\pi/2} d\phi \frac{1}{[1 - ix \cos \phi]^{1/2}}$$

has the limits $f \rightarrow 1$ as $x \rightarrow 0$ and $f \rightarrow 2/\sqrt{\pi x}$ as $x \rightarrow \infty$. Thus, as $v \rightarrow 0$, we get from (24):

$$T_{\text{flash}} \approx \frac{\mu \bar{\sigma} v}{\kappa} \frac{\gamma}{q_m},$$

TABLE I. The mass density ρ , the heat conductivity κ_{th} , the heat capacity per unit mass c_p , and the heat diffusivity D for different materials.

Solid	ρ (kg/m ³)	κ_{th} (W/Km)	c_p (J/kgK)	D (m ² /s)
Aluminum	2700	240	490	1.8×10^{-4}
Steel	7900	10–50	490	$(2.6 - 12.9) \times 10^{-6}$
Rubber	1200	0.2–0.6	1600	$(1.0 - 2.6) \times 10^{-7}$
Silica	2400	1.4	700	8.3×10^{-7}
Quartz	2400	11.7	700	7.0×10^{-6}

while in the same limit the classical expression (20) gives

$$\Delta T \approx \frac{\mu \bar{\sigma} v \alpha r_c}{\kappa}.$$

Thus, if we choose

$$r_c = \frac{\gamma}{\alpha q_m}, \quad (25)$$

the results agree. As $v \rightarrow \infty$, Eq. (24) gives

$$T_{\text{flash}} \approx \frac{\mu \bar{\sigma} v}{\kappa} \frac{\gamma}{q_m} \left(\frac{4Dq_m}{\pi v} \right)^{1/2},$$

while (20) gives

$$\Delta T \approx \frac{\mu \bar{\sigma} v \beta r_c}{\kappa} \left(\frac{8D}{\pi r_c v} \right)^{1/2}.$$

These two expressions agree if

$$r_c = \frac{\gamma^2}{2\beta^2 q_m}.$$

Using (25), this implies $\beta = (\alpha\gamma/2)^{1/2} \approx 0.5\alpha^{1/2}$.

V. QUALITATIVE DISCUSSION AND MORE NUMERICAL RESULTS

Heat diffusion is characterized by the heat diffusivity $D = \kappa/(\rho c_p)$, where κ , ρ , and c_p are the heat conductivity, mass density, and heat capacity, respectively. In Table I, we give the heat diffusivity at room temperature for some materials of interest. Most materials have a heat diffusivity D between 10^{-7} and 10^{-4} m²/s. Qualitatively, the heat diffusion equation shows that during the time τ , heat diffusion will broaden the temperature distribution by a characteristic distance $d \approx (D\tau)^{1/2}$.

A macroasperity contact region consists of many smaller, closely spaced microasperity contact regions. The microasperity contact regions consist, in turn, of even smaller asperity contact regions. We may say that the contact area in a macroasperity contact region exhibits granularity on many length scales, which could extend down to atomistic scales. The frictional heat source will show the same granularity as the contact area. The flash temperature in one microasperity contact region depends not only on the frictional energy dissipation in the contact area but the local flash temperature will be enhanced from the heat diffusion or convection from the other microasperity contact areas in the same macroasperity contact region.

The microasperity contact regions within a macroasperity contact area are densely distributed. This is the case even when the nominal contact pressure is (arbitrary) small, where

the separation between the macroasperity contact regions are (arbitrary) large. Hence, even if the thermal coupling between the macroasperity contact regions can be neglected, this is never the case for the microasperity contact regions within the macroasperity contact regions. At low sliding speed, thermal diffusion will couple the microasperity heat spots. For high sliding speeds, a microasperity contact close to the exit of the macroasperity contact region will experience the hot tracks from the microasperity contact regions in front of it.

The numerical results presented in Sec. II correspond roughly to those characteristic of rubber sliding on a concrete surface. Here, we will extend the study to quartz on quartz and silica on silica, which are minerals of interest in earthquake dynamics, as the continental parts of tectonic plates are often described as granitic in overall composition, and granite consists mainly of quartz. We will use the measured surface roughness power spectrum of a run-in granite block and substrate system shown in Fig. 13 (see Ref. [12]). After run-in, the contact regions deform mostly elastically at the length scales for which the power spectrum has been measured (down to the micrometer length scale, which is the resolution of the stylus topography-instrument used). Thus, we will neglect plastic flow in the model calculations and will use the thermal parameters for quartz and silica given in Table I.

Assume that a high asperity from one of the granite surfaces, say the sliding block, contacts the other granite surface. In that case, it is likely that during sliding, the asperity will remain in contact with the substrate for at least a distance of the order of a few times the diameter of the contact

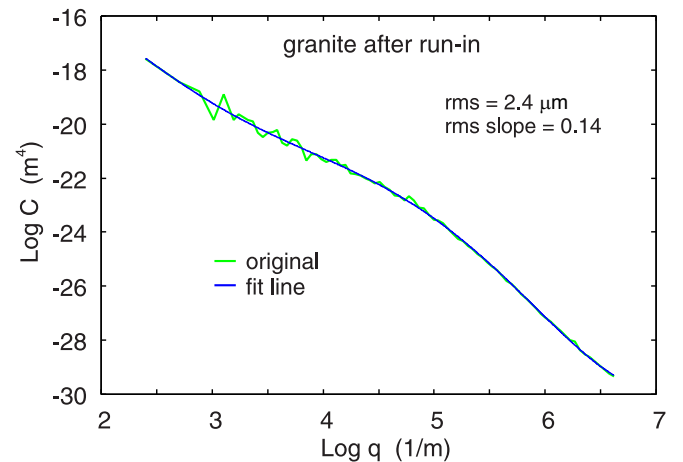


FIG. 13. The surface roughness power spectrum of the granite surface used in the present study. The spectrum represents the run-in state and is shown as a function of the wave number q .

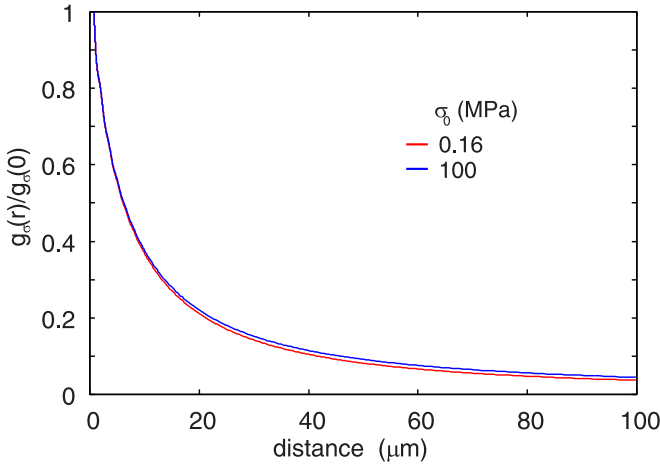


FIG. 14. The normalized stress correlation $g_\sigma(r)/g_\sigma(0)$ as a function of the separation $r = |\mathbf{x} - \mathbf{x}'|$ for the contact between two granite surfaces, using the elastic properties of quartz (which are nearly the same as those for silica). The red and blue lines were obtained assuming nominal contact pressures of 0.16 and 100 MPa, respectively. If the contact diameter r_0 were defined by $g_\sigma(r_0) = \alpha g_\sigma(0)$ with $\alpha = 0.2$, then we would get $r_0 \approx 20 \mu\text{m}$. However, the macroasperity contacts consist of a central compact region surrounded by disconnected islands (see Ref. [37]); these islands of contact are elastically correlated with the central part of the stress correlation function $g_\sigma(r)$, which results in a long tail of $g_\sigma(r)$ extending to larger distances r .

region. Hence, the frictional heat source will be stationary (not moving) with respect to the block but sliding on the substrate. Let $T_1(v)$ be the average temperature in the sliding contact, assuming that the entire frictional energy goes into the substrate. Similarly, let $T_2(v)$ be the average temperature in the stationary contact, assuming that the entire frictional energy goes into the substrate. In reality, a fraction s of the total energy will go into the sliding contact and a fraction $1 - s$ into the stationary contact, and the (average) temperature in the two contact regions will be $sT_1(v)$ and $(1 - s)T_2(v)$. Assuming that the (average) temperature is continuous gives

$$sT_1(v) = (1 - s)T_2(v)$$

or $s = T_2(v)/[T_1(v) + T_2(v)]$. Using this the average flash temperature in the contact will be

$$T_{\text{flash}} = \frac{T_1(v)T_2(v)}{T_1(v) + T_2(v)}.$$

Figure 14 shows the normalized stress correlation $g_\sigma(r)/g_\sigma(0)$ as a function of the separation $r = |\mathbf{x} - \mathbf{x}'|$ for the contact between two granite surfaces using the elastic properties of quartz (which are nearly the same as those for silica). If the contact radius were defined by $g_\sigma(r) = \alpha g_\sigma(0)$ with $\alpha = 0.5$, then we would get $r_0 \approx 6 \mu\text{m}$. However, the macroasperity contacts consist of a central compact region surrounded by disconnected islands (see Ref. [37]), and these islands of contact are elastically correlated with the central part of the stress correlation function $g_\sigma(r)$, which results in a long tail of $g_\sigma(r)$ extending to larger distances r .

The red and blue lines in Fig. 14 were obtained assuming nominal contact pressures of 0.16 and 100 MPa, respectively.

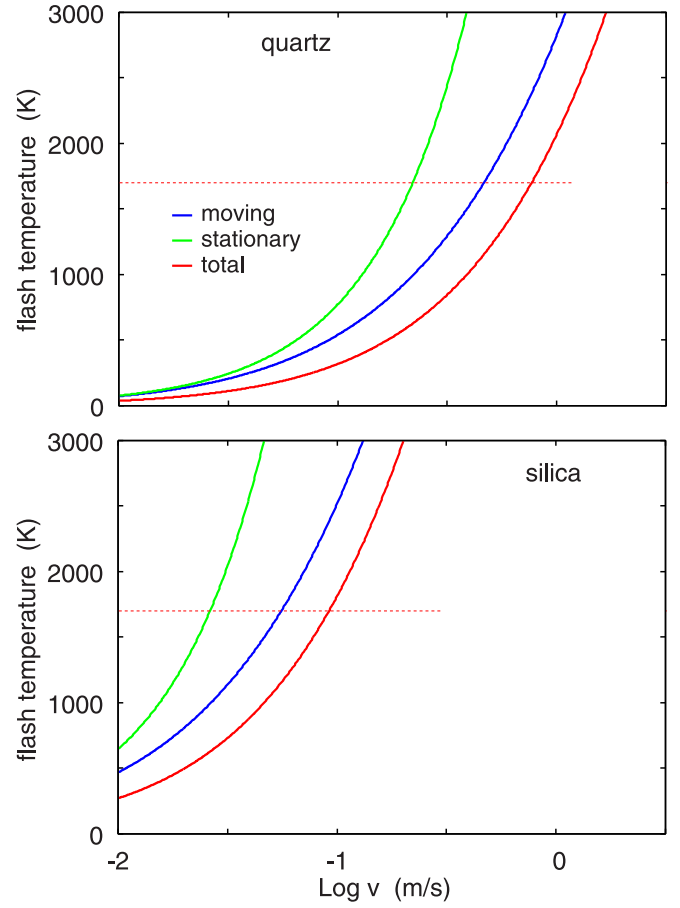


FIG. 15. The temperatures $T_1(v)$ and $T_2(v)$ in the sliding (blue line) and stationary (green line) contacts, assuming the entire frictional energy goes into the sliding and stationary contact, respectively. Thus, the red line is the result when part of the frictional energy goes into the sliding contact and part into the stationary contact in such a way that the temperature is continuous at the contacting interface. The red line is given by $T_1 T_2 / (T_1 + T_2)$. The melting temperature of quartz (about 1700 K, which is also the temperature where quartz and silica become identical high-viscosity fluids) is indicated by the dashed line. For the silica system, the quartz melting temperature is reached for a sliding $v \approx 0.1 \text{ m/s}$.

Note that $g_\sigma(r)/g_\sigma(0)$ is nearly independent of the applied pressure, which holds true as long as the relative contact area satisfies $A/A_0 \ll 1$. The reason is that increasing the nominal pressure σ_0 when $A/A_0 \ll 1$ increases the number of macroasperity contact regions in proportion to σ_0 , while the pressure distribution within the macroasperity contact regions remains nearly unchanged. As long as $A/A_0 \ll 1$, the elastic coupling between the macroasperity contact regions is small, and $g_\sigma(r)/g_\sigma(0)$ will nearly vanish for $r = |\mathbf{x} - \mathbf{x}'|$ larger than the diameter of the macroasperity contact regions. These statements require a large enough system with a surface roughness power spectrum that has a roll-off, which is always the case for surfaces of engineering interest.

Figure 15 shows the temperatures $T_1(v)$ and $T_2(v)$ in the sliding (blue line) and stationary (green line) contacts, assuming the entire frictional energy goes into the sliding or stationary contact, respectively. The red line is the result when

part of the frictional energy goes into the sliding contact and part into the stationary contact in such a way that the (average) temperature is continuous at the contacting interface. The red line is given by $T_1 T_2 / (T_1 + T_2)$. For the silica-silica contact, the melting temperature of quartz (about 1700 K, which is also the temperature where quartz and silica become identical high-viscosity fluids) is reached for a sliding velocity of ≈ 0.1 m/s. We note that at the high normal and tangential stresses (of order 10 GPa) acting in the contact regions between the quartz grains on the opposite surfaces for granite sliding on granite, the quartz is likely to locally convert to a silicalike material, which can have a large effect on the local temperature as the thermal conductivity of silica is a factor of $\sim 1/10$ smaller than that of quartz. This is due to the much shorter phonon mean free path in the disordered silica compared to quartz.

The melting temperature of quartz is ≈ 1700 °C, while the melting temperature of silica is not well defined, as it softens gradually with increasing temperature. If the friction coefficient were velocity-independent and equal to 1, as assumed above and found in experiments at room temperature, the temperature in the asperity contact regions for silica would reach ≈ 1700 °C at a sliding speed of ≈ 0.1 m/s. However, in reality, the friction coefficient, and hence the shear stress σ_f , drops strongly at high temperature, so silica would actually melt at much higher sliding speeds than predicted for a velocity-independent friction coefficient.

We now focus on T'_{flash} and $\nabla^2 T_{\text{flash}}$. We define the curvature-length D_{flash} and the slope-length d_{flash} by

$$D_{\text{flash}} = \left[\frac{-T_{\text{flash}}}{\nabla^2 T_{\text{flash}}} \right]^{1/2}, \quad (26)$$

$$d_{\text{flash}} = \frac{-T_{\text{flash}}}{T'_{\text{flash}}}. \quad (27)$$

Both $D_{\text{flash}}(v)$ and $d_{\text{flash}}(v)$ depend on the sliding speed v . The interpretation of these quantities are best illustrated in the classical limit of a single sliding Hertz contact (radius R) with the heat source (3). We can consider this as a macroasperity contact but where the roughness at shorter length scale is neglected, so resulting from a smooth macroasperity. For this one-length scale problem, in Ref. [27] we have show that

$$D_{\text{flash}} = \alpha R, \quad (28)$$

where $\alpha \approx 0.63$ for very low velocity (stationary contacts) and $\alpha \approx 0.45$ for very high velocities. Here high and low velocities, refer to $v \gg v^*$ and $v \ll v^*$, with $v^* = D/R$, respectively. Similarly, the slope-length for high sliding speeds

$$d_{\text{flash}} \approx 1.36R, \quad (29)$$

while $d_{\text{flash}} = \infty$ for $v = 0$. Note that for high sliding speed, $d_{\text{flash}}/D_{\text{flash}} \approx 3.0$.

In Ref. [27], we found that these equations are consistent with the measured flash temperature for steel sliding on steel [26]. The reason for this is that the steel deforms plastically already at relative long length scales, so effectively roughness occurs only on a relative narrow wavelength region, and, in particular, the short wavelength roughness on the original

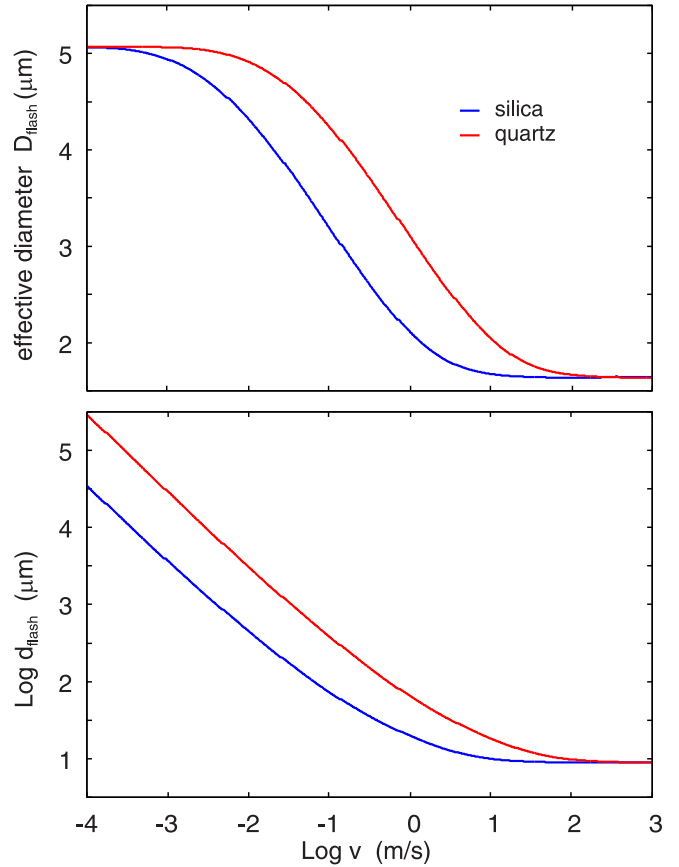


FIG. 16. (a) The curvature-length of the hot spots, $D_{\text{flash}} = [-T_{\text{flash}}/\nabla^2 T_{\text{flash}}]^{1/2}$, and (b) the logarithm of the temperature slope length, $d_{\text{flash}} = -T_{\text{flash}}/T'_{\text{flash}}$, as a function of the sliding speed (log-log scale). When the slope length d_{flash} is of the order of the effective radius of the macroasperity contact region (about 10 μm for the present system) the temperature at the exit of the temperature profile will be roughly twice the average temperature T_{flash} .

surfaces is irrelevant for the flash temperature. As we will now show, this is not the case for the granite studied above, where the classical approach fail because of the multiscale roughness.

In Fig. 16, we show (a) the curvature-length D_{flash} and (b) the slope length d_{flash} as a function of the sliding speed. When the slope-length d_{flash} is on the order of the effective radius of the macroasperity contact region (about 10 μm in the present case), the temperature at the trailing edge will be roughly twice the average temperature T_{flash} . In the present case for silica, this occurs at a sliding speed of $v \approx 10$ m/s. However, at this sliding speed, the silica (or quartz) has melted. At the melting temperature of quartz, the decay length is approximately 100 μm for both silica and quartz. Hence, the temperature profile exhibits only a relatively small upward tilt toward the trailing edge, even immediately before melting occurs.

Figure 16(a) shows that the curvature-length D_{flash} decreases by a factor of $1.649/5.056 \approx 0.33$ as the sliding speed increases. This differs from the result for a sliding Hertz contact, where D_{flash} decreases by a factor of $0.45/0.63 \approx 0.71$ as the sliding speed increases.

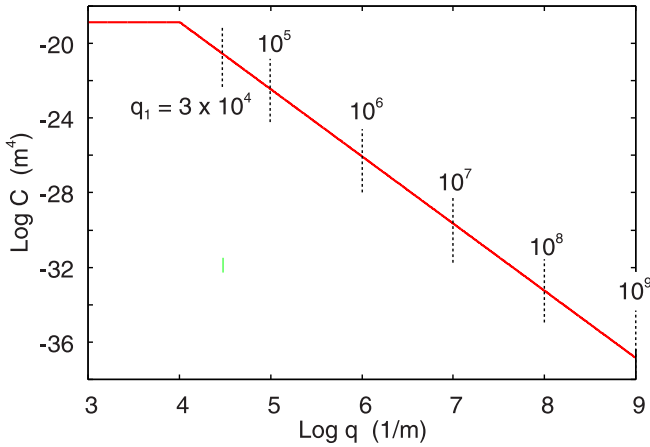


FIG. 17. Power spectra with the Hurst exponent $H = 0.8$ and different large wave number cutoff q_1 . For the largest cutoff $q_1 = 10^9 \text{ m}^{-1}$, the rms-roughness and rms-slope are $10 \text{ } \mu\text{m}$ and 1.49 , respectively.

Next let us focus on d_{flash} . As in the classical case, $1/d_{\text{flash}} \rightarrow 0$ as $v \rightarrow 0$. For large speeds, $d_{\text{flash}} \approx 9.00 \text{ } \mu\text{m}$ and $D_{\text{flash}} \approx 1.649 \text{ } \mu\text{m}$, giving $d_{\text{flash}}/D_{\text{flash}} \approx 5.45$ while the classical description of sliding Hertz contact predicts $d_{\text{flash}}/D_{\text{flash}} \approx 3.0$. It follows that the classical description fails to accurately describe the flash temperature.

To illustrate how the flash temperature depends on the large wave number cutoff q_1 , we have performed calculations for self-affine fractal surfaces with the Hurst exponent $H = 0.8$ with six different cutoff wave numbers q_1 as shown in Fig. 17. Figure 18 shows the stress-stress correlation function for the power spectrum with the highest cutoff $q_1 = 10^9 \text{ m}^{-1}$. The figure shows that the effective diameter of the macroasperity contact regions is $\approx 0.3 \text{ mm}$, which should be interpreted as the diameter of agglomerates of much smaller contact spots (see Fig. 19). The effective diameter of the macroasperity contact

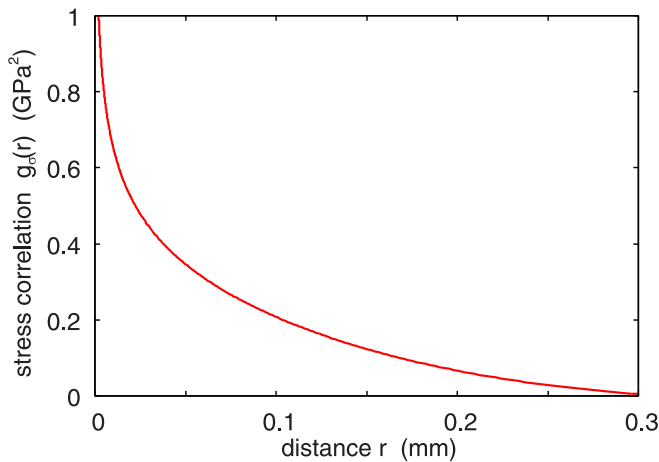


FIG. 18. The stress correlation $g_\sigma(r)/g_\sigma(0)$ as a function of the separation $r = |\mathbf{x} - \mathbf{x}'|$ for the contact between two surfaces with the effective power spectrum given in Fig. 17 with $q_1 = 10^9 \text{ m}^{-1}$, the elastic and thermal properties of quartz, and the nominal contact pressure 100 MPa .

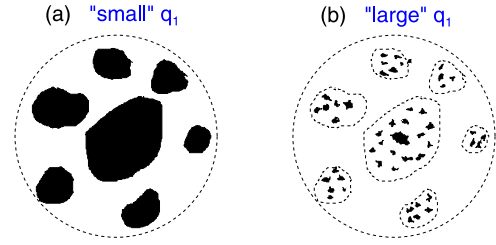


FIG. 19. Macroasperity contact regions for surface roughness power spectra with small and large cutoff wave number q_1 . The picture in (b) is obtained by adding shorter wavelength roughness to the surface in (a). This does not change the size of the macroasperity contact regions, as it is determined by the long wavelength roughness, but decreases the sizes of the microasperity contact regions. (Schematic.)

regions is determined by the long wavelength roughness and is hence the same for all six different surfaces with different cutoffs q_1 . However, the sizes of the actual microasperity contact regions within the macroasperities becomes smaller as q_1 increases, as illustrated in Fig. 19.

Figure 20(a) shows the flash temperature as a function of the logarithm of the sliding speed for the six different power spectra. As the cutoff wave number increases, the flash temperature curves converge toward a large q_1 -limiting curve as is clear from Fig. 20(b), which shows the ratio $T_{\text{flash}}/T_{\text{flash}}^o$ between the flash temperature for the large wave number cutoff q_1 and the smallest cutoff wave number q_1^o ($\text{Log} q_1^o = 4.48$).

For low sliding speeds, the flash temperature is nearly independent of the cutoff q_1 except for the two smallest q_1 , where roughly one decade of self-affine roughness occurs. We attribute this to thermal diffusion, which effectively broadens the temperature profile in such a way that the detailed distribution of the heat source within the macroasperity contact regions becomes unimportant. As the sliding speed increases, the thermal diffusion cannot smear out the temperature profile, and the results for the different q_1 start to differ and also the ratio $T_{\text{flash}}/T_{\text{flash}}^o$ of the flash temperatures increases. Note that for quartz, the thermal diffusivity $D \approx 10^{-5} \text{ m}^2/\text{s}$ and, assuming the typical diameter of the macroasperities $d \approx 0.1 \text{ mm}$ (see Fig. 18), we get $vd/D \approx 1$ for $v \approx 0.1 \text{ m/s}$, which indeed is the velocity where the ratio $T_{\text{flash}}/T_{\text{flash}}^o$ starts to increase. When $v \gg D/d$, the temperature profile cannot be smoothed by thermal diffusion at the size of the macroasperity but only at shorter length scale.

The breakup of contact regions into smaller contact regions as illustrated in Fig. 19 result in smaller contact-spots. This is illustrated in Fig. 21(a), which shows the curvature-length D_{flash} as a function of the sliding speed for all six different large wave number cutoff. As q_1 increases, the curvature-length decreases from $\sim 100 \text{ } \mu\text{m}$ for the lowest q_1 , which is similar to the size of the macroasperity contact regions, to $\approx 0.1 \text{ } \mu\text{m}$ for the largest q_1 cutoff surface. Nevertheless, the flash temperature for low sliding speed is nearly independent of q_1 when the q range of fractal roughness is larger than ~ 1 decade. This is possible only if the thermal diffusion broadens the temperature profile, so the amplitude of the fluctuations in the temperature profile within the macroasperity contact

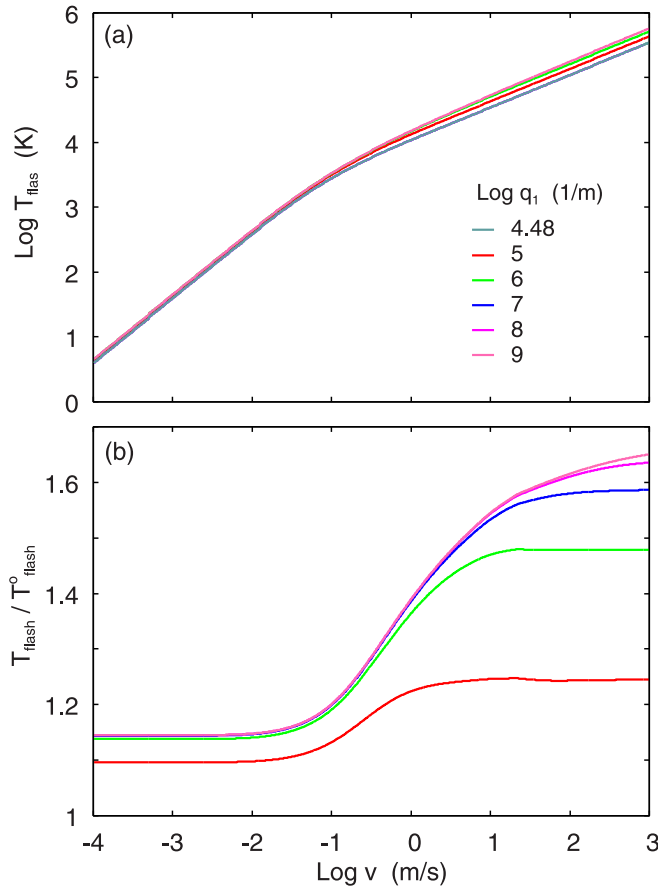


FIG. 20. The (a) logarithm of the flash temperature and (b) the ratio $T_{\text{flash}}/T_{\text{flash}}^o$ between the flash temperature for the large wave number cutoff q_1 and the smallest cutoff wave number q_1^o ($\text{Log } q_1^o = 4.48$) as a function of the logarithm of the sliding speed for the power spectra shown in Fig. 17. The thermal and mechanical parameters are for quartz, the nominal contact pressure is 100 MPa, and the friction coefficient $\mu = 1$.

regions is very small for low sliding speeds even when q_1 is large. That is, the temperature profile exhibits much weaker fine-scale granularity than the contact stress distribution, and the curvature length parameter D_{flash} reflects the curvature of the temperature profile within the microasperity contact regions as indicated in Fig. 22. It is important to note that in the calculations the total frictional energy is assumed the same, independent of the cutoff wave number q_1 .

Figure 21(b) shows the tilt length d_{flash} . For low sliding speed, $d_{\text{flash}}(v)$ is very large, which implies that the temperature distribution is uniform (on the average angularly independent) but, as the sliding speed increases, the tilt-length decreases, which shows that the temperature in the contact regions on the exit side is higher than at the front. This is due to the hot tracks left on the surface from the microasperity contact regions in front of a microasperity contact, as illustrated in Fig. 8.

To illustrate the failure for the classical theory to correctly describe the flash temperature for the systems studied above, we show in Fig. 23 the ratio $D_{\text{flash}}(0)/D_{\text{flash}}(\infty)$ and in Fig. 24 for the ratio $d_{\text{flash}}(\infty)/d_{\text{flash}}(0)$ as a function of the number

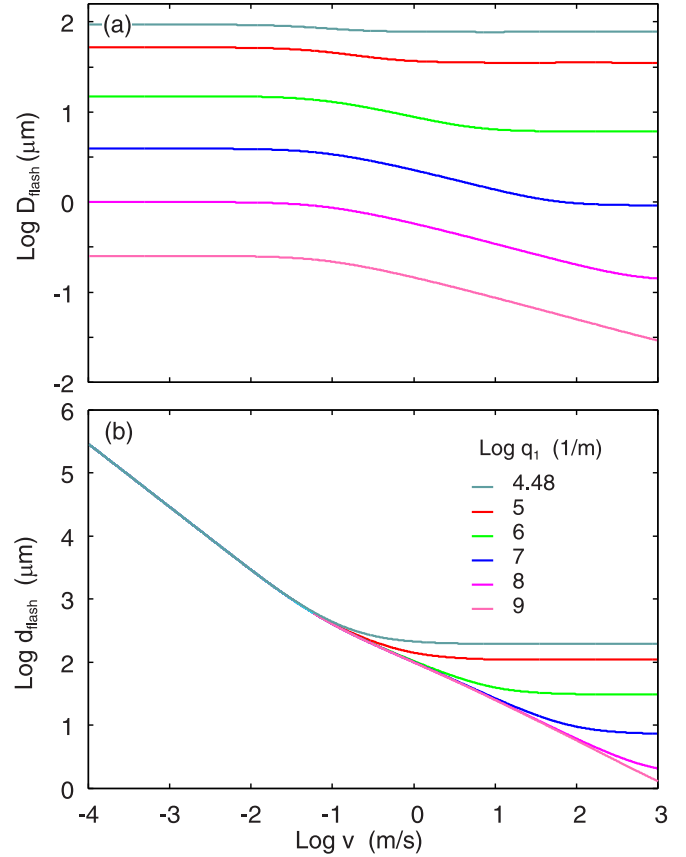


FIG. 21. The curvature-length $D_{\text{flash}}(v)$ and the slope parameter $d_{\text{flash}}(v)$ as a function of the logarithm of the sliding speed for the power spectra shown in Fig. 17.

of decades of self-affine fractal roughness with the power spectra shown in Fig. 17. The dashed lines gives the classical result for sliding Hertz contact. The figures show that the sliding Hertz theory gives approximately correct results for surfaces with roughness over one decade in length scale but fail severely when the roughness occur over two or more decades of roughness.

We will now comment on the assumption that the heat source $\dot{q}(\mathbf{x})$ is proportional to the local stress $\sigma(\mathbf{x})$. Assume

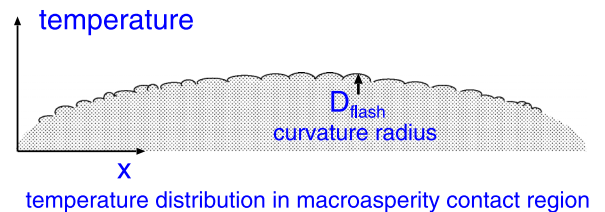


FIG. 22. When the cutoff q_1 is large, the temperature within the macroasperity contact regions exhibit fine-scale granularity, which result in a very small curvature-length D_{flash} . For low sliding speeds, this results in very small amplitude fluctuations in the flash temperature compared to the much larger background value resulting from the thermal diffusion (spreading) of the heat produced in all the microasperity contact regions.

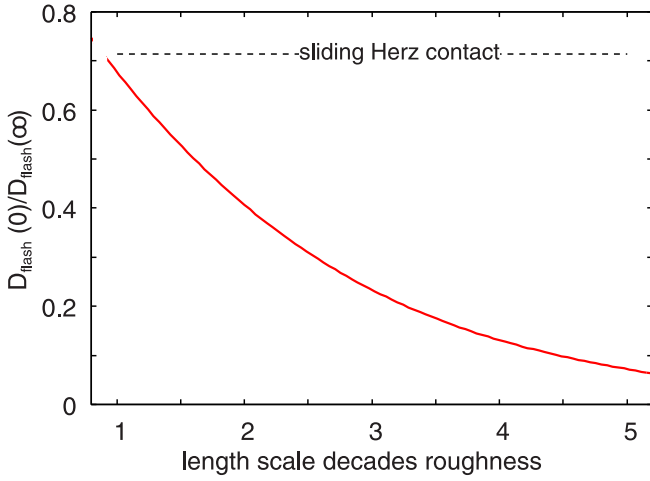


FIG. 23. The ratio $D_{\text{flash}}(0)/D_{\text{flash}}(\infty)$ between $D_{\text{flash}}(v)$ for $v = 0$ and ∞ , as a function of the number of decades of self-affine roughness.

that two solids are squeezed into contact with the nominal stress σ_0 . If we study the interface at the magnification ζ , we observe the normal stress $\sigma(\mathbf{x}, \zeta)$. At the highest magnification ζ_1 , where the atomic-scale roughness can be observed, the stress $\sigma(\mathbf{x}, \zeta_1) = \sigma_1(\mathbf{x})$ is the true normal stress acting at the interface, but this stress is in general not linearly related to the frictional shear stress. However, if we study the interface at a lower magnification where the apparent relative contact area $A(\mathbf{x})/A_0 \lesssim 0.3$, then contact mechanics theories for elastic and elastoplastic contact between randomly rough surfaces show that $A(\mathbf{x})/A_0$ depends linearly on $\sigma(\mathbf{x}, \zeta)$. Hence, at this lower magnification we expect the frictional shear stress $\sigma_f(\mathbf{x})$ to be proportional to $\sigma(\mathbf{x}, \zeta)$, and the relation $\sigma_f(\mathbf{x}) = \mu\sigma(\mathbf{x}, \zeta)$ to hold. In this case, $\dot{q}(\mathbf{x})$ is proportional to $\sigma(\mathbf{x}, \zeta)$. Since the flash temperature appears to converge to a large q_1 -limiting curve, it is in general not necessary to include all the roughness down to the atomic scale and the assumption that $\dot{q}(\mathbf{x})$ is proportional to $\sigma(\mathbf{x}, \zeta)$ may hold. And even if all roughness is included, it is still true

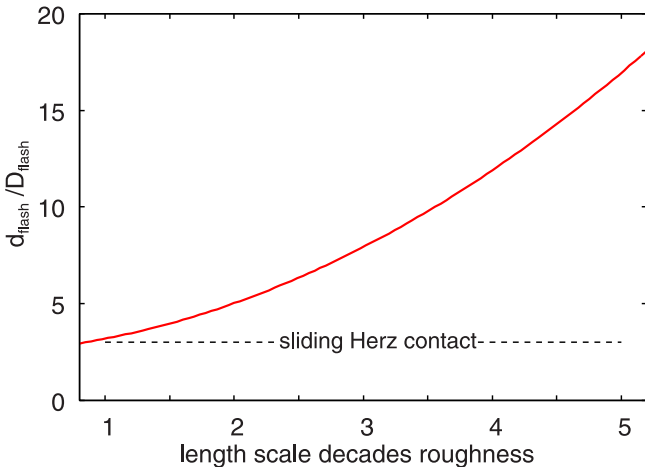


FIG. 24. The ratio $d_{\text{flash}}(\infty)/D_{\text{flash}}(\infty)$ as a function of the number of decades of self-affine roughness.

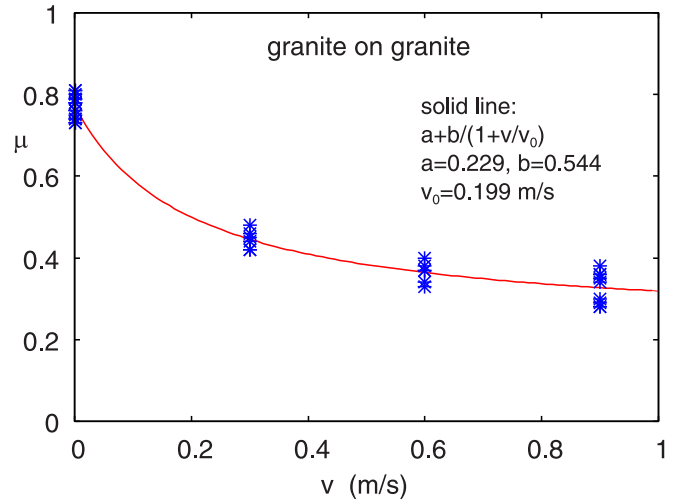


FIG. 25. The star symbols (*) are measured friction coefficients (from Ref. [6]), and the solid line represents a fit to the data of the form $\mu(v) = a + b/(1 + v/v_0)$, with $a = 0.229$, $b = 0.544$, and $v_0 = 0.199$ m/s.

in most cases that $\dot{q}(\mathbf{x})$ is nonvanishing only in the surface area where $\sigma(\mathbf{x})$ is nonvanishing. However, this is not the case for viscoelastic solids, for which the theory presented above can be generalized, as will be discussed elsewhere.

VI. APPLICATION TO EARTHQUAKE DYNAMICS

Geophysical observations often reveal large dynamic stress drops during earthquakes, indicating that the effective dynamic friction coefficient between the fault surfaces is substantially lower than the static or break-loose friction coefficient. During major seismic events, the slip velocities at the fault line are typically of the order of ~ 1 m/s. Laboratory friction studies of granite sliding on granite demonstrate that the friction coefficient drops from $\mu \approx 0.8$ for very low sliding speeds, to ≈ 0.35 at speeds of the order of ~ 1 m/s. It has been proposed that the origin of this drop in the friction coefficient is flash heating in the contact regions [7].

Barbery *et al.* [6] have measured the velocity dependence of the friction between Westerly granite under a normal pressure of ≈ 9 MPa. The sliding blocks were initially ground flat, but no information about the surface roughness was provided. The resulting friction coefficient measured for granite sliding on granite is shown in Fig. 25. The solid fit line is given by

$$\mu(v) = a + \frac{b}{1 + v/v_0}, \quad (30)$$

where $a = 0.229$, $b = 0.544$, and $v_0 = 0.199$ m/s. We assume that in the studied velocity interval, the velocity dependency of the friction coefficient is due to the increase in the flash temperature. Here we use a similar temperature dependency of the friction coefficients as found from studies of friction on ice:

$$\mu = \mu_0 \left(1 - \frac{T}{T_m}\right)^\alpha, \quad (31)$$

where T_m is the melting temperature of quartz, which we take to be $T = 1700$ K and where $\mu_0 = a + b = 0.843$ is the static

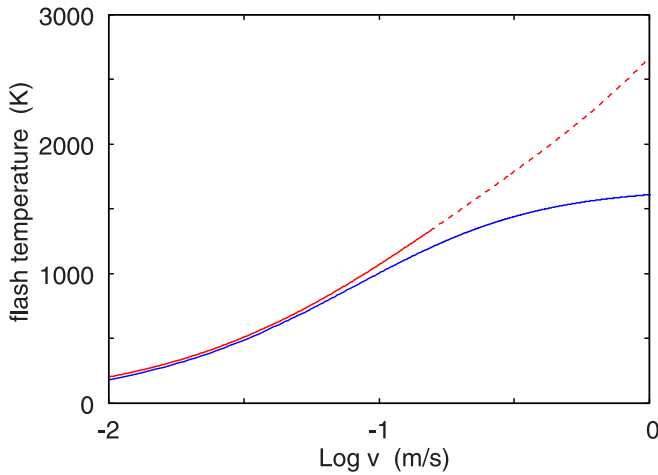


FIG. 26. Blue line: The flash temperature calculated from Eqs. (22) and (23). Red line: The theoretically calculated flash temperature using the measured friction coefficient.

or break-loose friction coefficient. Here, we use $\alpha = 0.3$. By combining (30) and (31), we obtain the velocity dependence of the flash temperature shown in Fig. 26 as the blue line. The red line is the calculated flash temperature using the theory presented above for silica, and is obtained from the red curve in Fig. 15(b) by scaling it with the $\mu(v)$ given by (30). Here, we have used that the flash temperature is proportional to $\mu(v)$ and that the curves in Fig. 15(b) were obtained with $\mu(v) = 1$. We note that at the high normal and shear stresses (and high

temperatures) occurring in the contact regions between quartz grains, quartz is most likely locally converted into silica [12].

The theory for the flash temperature is, of course, not valid when the temperature exceeds the melting temperature. In fact, even before reaching the melting temperature, the silica softens and the contact radius of the macroasperities increases, while the average normal stress $\bar{\sigma}$ decreases at least as fast as $1/r_0^2$. As a result, the flash temperature becomes smaller than it would be if this effect were neglected [see (20), where the prefactor $\bar{\sigma}_0 r_0 \sim 1/r_0$]. This can explain why the flash temperature deduced from the measured friction coefficient flattens out as the quartz melting temperature is approached.

VII. NUMERICAL TEST

The flash-temperature theory contains no independent description of the contact mechanics but derives the temperature field from the stress distribution predicted by Persson theory. Since the underlying stress correlations have already been thoroughly validated [40], also in the contact-mechanics challenge [41], it is sufficient to test whether the conversion from stress correlations to temperature correlations is correctly captured. To this end, we consider, as an example, a randomly rough surface characterized as follows: System size $L = 10$, roll-off wavelength $\lambda_r = 4$, and short-wavelength cutoff $\lambda_s = \lambda_r/128$. The square domain is discretized into 8196×8196 grid points. The height profile is scaled such that the root-mean-square gradient is unity, $\xi = 1$, which yields a root-mean-square height $h_{\text{rms}} \approx 0.17071$. For λ_s to coincide with atomic dimensions, one would have to choose a unit

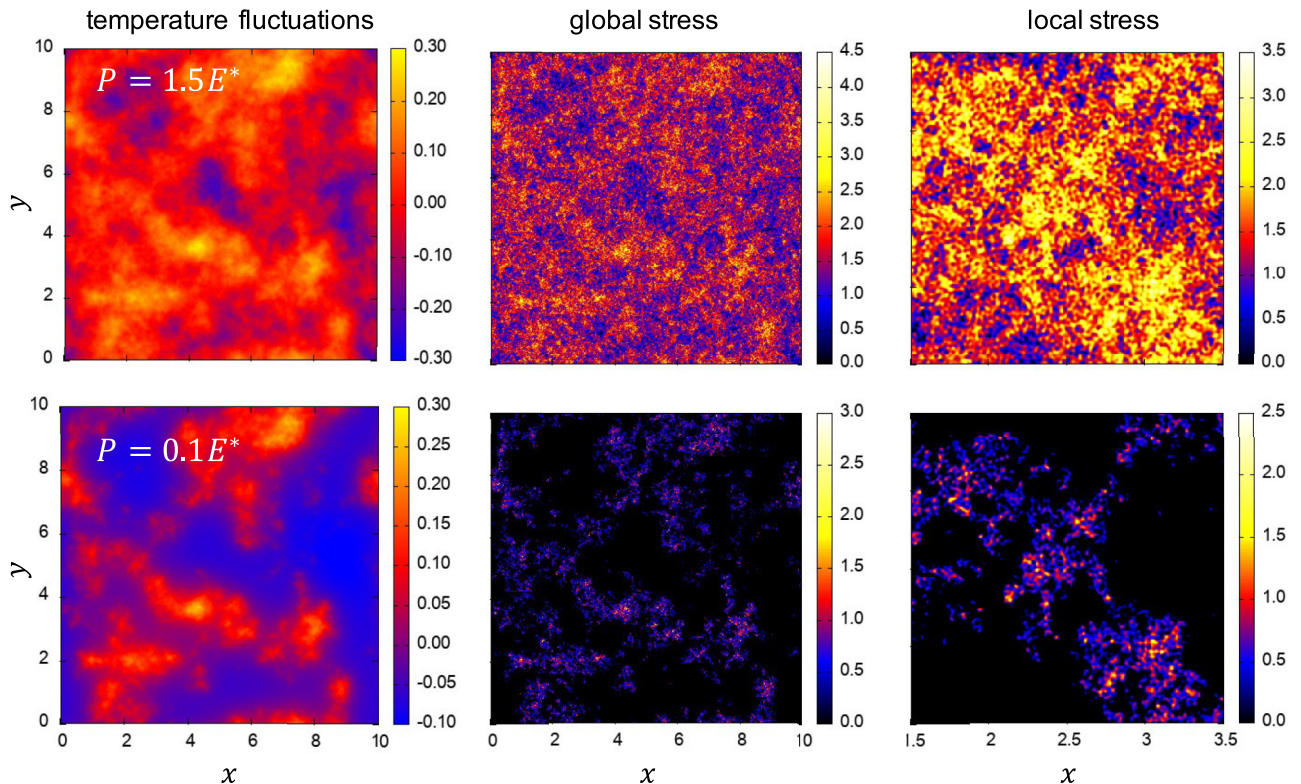


FIG. 27. Local temperature deviations from the mean temperature (left column) as well as global (center column) and local (right column) stresses in a randomly rough interface. The upper row pertains to a normal compressive stress of $\sigma = 1.5 E^*$, the lower to $\sigma = 0.1 E^*$.

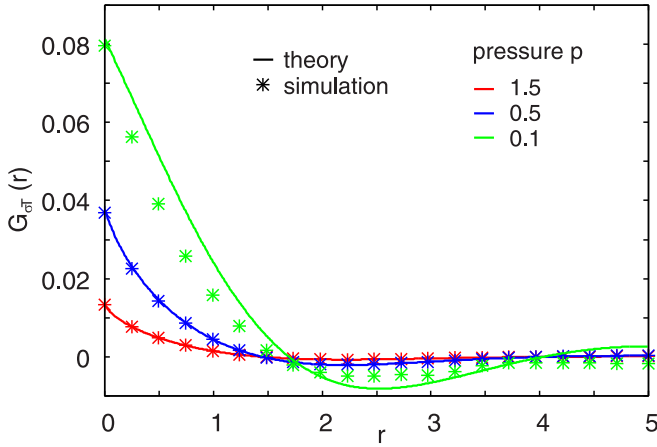


FIG. 28. The stress-temperature correlation function $G_{\sigma T}(r)$ at different nominal pressures. Full lines are the theory and the stars from Green's function molecular dynamics simulations.

of length $[l] = 10$ nm. If the discretization is assumed to be of atomic dimension, then $[l] = 250$ nm is a more appropriate choice. We use the Green's function molecular dynamics method [42] in the same code variant as used for a recent contact-mechanics study [43].

In the following, we consider the limit of a vanishing Péclet number, $Pe = vL/D \ll 1$, while setting $\mu v/\kappa = 1$. Figure 27 shows the temperature and stress fields obtained at pressures of $p = 0.1\xi E^*$ and $p = 1.5\xi E^*$. The lowest pressure yields approximately 20% contact area, whereas the highest pressure corresponds to nearly full contact. It is readily noticeable that temperature fluctuations occur on much coarser scales than spatial features in the stress. At lower pressure, the temperature heat map conveys the visual impression of large contact patches, which clearly exceed those based on defining them via connected domains of positive stress. Note also that the temperature data reflects deviations from the mean rather than absolute temperatures.

To test the theory, we define the stress-normalized stress-temperature cross-correlation function:

$$G_{\sigma T}(\mathbf{r}) = \frac{\langle \sigma(\mathbf{x})T(\mathbf{x} + \mathbf{r}) \rangle}{\langle \sigma(\mathbf{x}) \rangle}.$$

The spectrum of this cross-correlation function is $C_{\sigma T}(\mathbf{q}) \propto \langle |\sigma(\mathbf{q})|^2 \rangle / q$ as can be readily seen by comparing Eqs. (10) with the equation before (15). The value of the cross-correlation function at the origin, $G_{\sigma T}(\mathbf{0})$, corresponds to our flash-temperature [Eq. (14)], i.e., a stress-weighted mean contact temperature. Figure 28 shows excellent agreement between the theoretical predictions and the computed data.

VIII. SUMMARY AND CONCLUSION

We have presented an analytical theory for the flash temperature which includes the surface roughness on arbitrary many decades in length scale. The most important results is (15), which gives the weighted average temperature increase in the contact regions. We note that the weighting factor $\sigma(\mathbf{x})$ is proportional to the heat source, so the (15) is the optimum

average which can be defined. That is, if the flash temperature is characterized by a single number to be used in friction calculations, then the definition (14) we use is the optimal one. In addition, the theory gives information about the variation of the temperature within the macroasperity contact regions as contained in derivatives of the temperature profile [Eqs. (17) and (18)]. We have shown that for roughness on a single length scale, the theory reduces to the classical theory of the flash temperature by Jaeger, Archard, and Greenwood for circular moving heat sources. However, for surfaces with multiscale roughness, the classical theories for the flash temperature fail severely. We are not aware of any experimental studies of the flash temperature, where the power spectrum of the surface roughness was measured. We propose that such studies should be performed, so the theory developed here can be tested and a quantitative understanding of flash temperatures established.

DATA AVAILABILITY

The data that support the findings of this article are not publicly available. The data are available from the authors upon reasonable request.

APPENDIX

Consider the integral

$$f(x) = \frac{2}{\pi} \text{Re} \int_0^{\pi/2} d\phi \frac{1}{(1 - ix \sin\phi)^{1/2}}. \quad (\text{A1})$$

As $x \rightarrow 0$, we get

$$f(x) \approx 1 - \frac{3x^2}{16}. \quad (\text{A2})$$

We are also interested in the limit $x \rightarrow \infty$. Assume $\epsilon \ll 1$. For $\phi < \epsilon$, we have $\sin\phi \approx \phi$ and we can write

$$f(x) \approx \frac{2}{\pi} \text{Re} \left[\int_0^\epsilon d\phi \frac{1}{(1 - ix\phi)^{1/2}} + \int_\epsilon^{\pi/2} d\phi \frac{1}{(1 - ix \sin\phi)^{1/2}} \right].$$

For $x \gg 1/\epsilon$, in the second term in this expression we can replace $(1 - ix \sin\phi)^{1/2}$ with $(-ix \sin\phi)^{1/2}$ to get

$$f(x) \approx \frac{2}{\pi} \text{Re} \left[\frac{2}{ix} (1 - (1 - ix\epsilon)^{1/2}) + \frac{1}{(-ix)^{1/2}} \int_\epsilon^{\pi/2} d\phi \frac{1}{(\sin\phi)^{1/2}} \right]. \quad (\text{A3})$$

Next, consider the integral

$$g(x) = \frac{2}{\pi} \text{Re} \int_0^{\pi/2} d\phi \frac{i \sin\phi}{(1 - ix \sin\phi)^{1/2}}. \quad (\text{A4})$$

As $x \rightarrow 0$, we get

$$g(x) \approx -\frac{x}{4}. \quad (\text{A5})$$

We are also interested in the limit $x \rightarrow \infty$. For $x \gg 1/\epsilon$, we get

$$\begin{aligned}
 g(x) &\approx \frac{2}{\pi} \operatorname{Re} \left[\int_0^\epsilon d\phi \frac{i\phi}{(1-ix\phi)^{1/2}} \right. \\
 &\quad \left. + \frac{1}{(-ix)^{1/2}} \int_\epsilon^{\pi/2} d\phi \frac{i \sin\phi}{(\sin\phi)^{1/2}} \right] \\
 &= \frac{2}{\pi} \operatorname{Re} \left[\frac{2}{ix^2} \left(\frac{2}{3} + \frac{1}{3} (1-ix\epsilon)^{3/2} - (1-ix\epsilon)^{1/2} \right) \right. \\
 &\quad \left. + \frac{i}{(-ix)^{1/2}} \int_\epsilon^{\pi/2} d\phi (\sin\phi)^{1/2} \right]. \quad (\text{A6})
 \end{aligned}$$

Figure 29 shows the integrals (A1) and (A4). The region to the left of the first dashed vertical line is given by the asymptotic small- x results (A2) and (A5), and the region to the right of the second vertical line is given by the asymptotic large- x results (A3) and (A6) where we have used $\epsilon = 0.03$. The region in between are given by the integrals (A1) and (A4) by numerical integration.

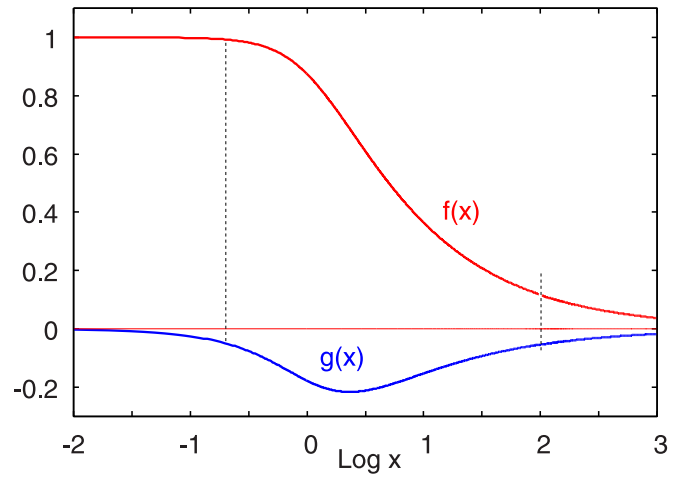


FIG. 29. The function $f(x)$ and $g(x)$ given by (A1) and (A4). The region to the left of the first dashed vertical line is given by the asymptotic small- x results (A2) and (A5), and the region to the right of the second vertical line is given by the asymptotic large- x results (A3) and (A6), where we have used $\epsilon = 0.03$. The regions in between are given by the integrals (A1) and (A4) by numerical integration.

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