

Tuning spin currents in collinear antiferromagnets and altermagnets

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Spin-current generation through nonrelativistic spin splittings, found in uncompensated magnets and *d*-wave altermagnets (AMs), is desirable for low-power spintronics. Such spin currents, however, are symmetry forbidden in conventional collinear antiferromagnets and higher-order AMs. Using spin-point-group analysis, we demonstrate that finite spin currents can be induced in these materials via magnetoelectric-, piezomagnetic-, and piezomagnetoelectric-like couplings. We utilize electric fields, strain, and their combinations to drive symmetry-lowering phase transitions into uncompensated magnetic or *d*-wave AM states, thereby enabling finite spin conductivity in a broader class of magnetic materials. We further substantiate this framework using density functional theory and Boltzmann transport calculations on representative magnetic materials—KV₂Se₂O, RuF₄, Cr₂O₃, FeS₂, and MnPSe₃—spanning these different cases. The charge-to-spin conversion and spin-splitter ratio reaches up to almost 100% via uncompensated magnetism and about 40% via *d*-wave AM under realistic conditions, highlighting the effectiveness of this approach for efficient spin-current generation.

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I. INTRODUCTION

The generation of pure spin currents is of fundamental interest in spintronics. In most materials, spin currents are typically achieved via the spin-Hall effect driven by spin-orbit interaction (SOI) [1,2]. However, SOI, which gives rise to spin currents, also results in spin-dephasing mechanisms, such as D'yakonov-Perel [3] and Elliott-Yafet processes [4,5]. Such dephasing mechanisms limit the coherence length of the spin currents, thereby constraining their practical applications. Recently, collinear compensated magnets [6–10] and non-collinear antiferromagnets (AFMs) [11–13] have been shown to generate highly spin-polarized currents through exchange fields. The charge-to-spin conversion efficiency and coherence length of such nonrelativistic spin currents can exceed those of SOI-driven mechanisms by orders of magnitude [14–16].

In the context of symmetry, collinear compensated magnets are classified as AFMs or altermagnets (AMs), depending on the symmetries that connect opposite spin sublattices [8]. In AFMs, the opposite spin sublattices are related by spatial inversion (*P*) and translation (*τ*), leading to spin degeneracy and zero spin conductivity. In contrast, AMs have mirror-rotation symmetries connecting opposite spin sublattices and are further categorized by their planar or bulk *d*-, *g*-, and *i*-wave

orders. This classification of AMs comes from the topology of the spin-polarized energy surfaces [8]. The *d*-, *g*-, and *i*-wave planar (bulk) AMs have two, four, and six symmetry-enforced spin-degenerate nodal lines (planes) passing through the Γ point, respectively. Spin-current generation has been theoretically predicted in *d*-wave AMs, such as RuO₂ [14]. However, the AM ground state of RuO₂ is still under debate [16]. Unfortunately, spin conductivity remains forbidden in higher-order AMs (*g*- and *i*-wave) due to additional spin-degenerate nodal surfaces. Nevertheless, one can use strain and electric fields to manipulate the AM order to allow for spin currents [17–19]. For instance, recent works used uniaxial strain to induce *g*-to-*d*-wave AM phase transitions in MnTe [17] and CrSb [18]. Other researchers showed that an electric field applied to the AFM MnPSe₃ can induce a transition to either a *d*-wave [20] or an *i*-wave AM phase [21]. While most such studies have focused on AM spin splittings, a general framework for generating nonrelativistic spin currents via magnetic phase transitions has remained unexplored.

In this article, we provide a general approach based on spin-group-symmetry formalism to induce spin currents through magnetoelectric-, piezomagnetic-, and piezomagnetoelectric-like coupling. We show that electric ($\mathcal{E}_i$) and strain (η_{ij}) fields and their combination ($\mathcal{E}_i\eta_{jk}$) can be utilized to induce spin currents in AFMs and higher-order AMs via reduction of their respective symmetries. By considering the lower effective spin-point groups (SPGs) under external stimuli for all possible AFMs and higher-order AMs, we determine the correlation between spin current and $\mathcal{E}_i$, η_{jk} , and $\mathcal{E}_i\eta_{jk}$. We compute the spin-polarized conductivity for several representative examples of collinear compensated magnets using DFT and Boltzmann transport simulations. We also determine the spin conductivity for the reduced SPGs

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when the materials are subjected to an external electric field along an arbitrary direction.

II. COMPUTATIONAL DETAILS

In the nonrelativistic case, conductivity can be described through the spin-up s_k and spin-down $-s_k$ channels with spin polarization along the k direction. Spin-polarized charge conductivity $\sigma_{ij}^{s_k}$ is defined as $J_i^{s_k} = \sigma_{ij}^{s_k} \mathcal{E}_j$, where $\mathcal{E}_j$ is the electric field along the j direction and $J_i^{s_k}$ is the current along the i direction. Within the Boltzmann transport theory, the conductivity $\sigma_{ij}^{s_k}$ (contributed by E_{nk}) is expressed as [22]

$$\sigma_{ij}^{s_k} = -\frac{e^2 \tau}{8\pi^3 \hbar^2} \sum_n \int \frac{\partial E_{nk}^{s_k}}{\partial k_i} \frac{\partial E_{nk}^{s_k}}{\partial k_j} \frac{\partial f^0}{\partial E_{nk}^{s_k}} d^3k. \quad (1)$$

Here, τ is the relaxation time, and f^0 is the Fermi-Dirac distribution function. Within Eq. (1), the net charge and spin conductivity, respectively, are defined as

$$\begin{aligned} \sigma_{ij} &= \sum_k \sigma_{ij}^{s_k} + \sigma_{ij}^{-s_k}, \\ \sigma_{ij}^k &= \frac{\hbar}{2e} (\sigma_{ij}^{s_k} - \sigma_{ij}^{-s_k}). \end{aligned} \quad (2)$$

Here, we would like to emphasize the difference between spin-polarized charge conductivity $\sigma_{ij}^{s_k}$ and spin conductivity σ_{ij}^k by considering the conduction due to charge carriers with spin polarization along the $\pm z$ direction. In this case, $\sigma_{ij}^{s_z}$ and $\sigma_{ij}^{-s_z}$ represent the charge conductivities carried by bands with spin polarization along the $+z$ and $-z$ directions, respectively. The spin conductivity is defined as the difference between these charge conductivities, with a multiplicative factor of $\hbar/2e$ ensuring that the spin conductivity has the units of spin angular momentum. If the charge conductivities carried by spin-up $\sigma_{ij}^{s_z}$ and spin-down $\sigma_{ij}^{-s_z}$ electrons are equal, the net spin conductivity along the z direction σ_{ij}^z is strictly zero.

Density functional theory (DFT) calculations were performed using the plane-wave ultrasoft pseudopotential [23] approach implemented in QUANTUM ESPRESSO [24]. The generalized gradient approximation with Hubbard correction (PBE+ U) was used to treat exchange correlations self consistently [25]. The Kubo formalism, combined with DFT, was used to determine the charge and spin conductivities. To do so, we constructed a tight-binding Hamiltonian in the atom-centered Wannier basis [26] using the WANNIER90 code [27]. In the single-particle energy eigenstate representation, the spin conductivity within the Kubo formalism is given by

$$\begin{aligned} \sigma_{ij}^k &= -\frac{e\hbar}{2\Gamma} \int \frac{d^3\mathbf{k}}{(2\pi)^3} \sum_n \langle \mathbf{n}\mathbf{k} | \hat{J}_i^k | \mathbf{n}\mathbf{k} \rangle \langle \mathbf{n}\mathbf{k} | \hat{v}_j | \mathbf{n}\mathbf{k} \rangle \\ &\quad \times \delta(E_{nk} - E_F), \end{aligned} \quad (3)$$

where the $\hat{J}_i^k = \frac{1}{2} \{\hat{v}_i, \hat{s}_k\}$ is the spin-current operator. Also, $\Gamma = \frac{\hbar}{2\tau}$ is a constant scattering rate and is inversely proportional to the relaxation time τ . Equation (3) represents the net spin conductivity $\frac{\hbar}{2e} (\sigma_{ij}^{s_k} - \sigma_{ij}^{-s_k})$, with $\sigma_{ij}^{s_k}$ and $\sigma_{ij}^{-s_k}$ obtained from Eq. (1). For simplicity, we have ignored temperature effects, and therefore, $\frac{\partial f^0}{\partial E_{nk}^{s_k}}$ is replaced with $\delta(E_{nk}^{s_k} - E_F)$. The

$\delta(x)$ function is approximated using $\delta(x) = \lim_{\epsilon \rightarrow 0} \frac{1}{\pi} \frac{\epsilon}{x^2 + \epsilon^2}$. The replacement of $\hat{J}_i^k$ with $\hat{v}_i^k$ in Eq. (3) gives the charge conductivity. Note that, in Eq. (3), only the intraband contributions are considered, while the interband contributions are neglected. This is a reasonable approximation since we find that the intraband contributions indeed dominate in the small- Γ regime (see Sec. S1 of the Supplemental Material (SM) [28]). The conductivity tensors were obtained by solving Eq. (3) as a postprocessing step of DFT calculations on a dense 350^3 k -mesh for three-dimensional materials and 350^2 k -mesh for two-dimensional (2D) materials, using the LINRES code [14]. We made use of the FINDSYM[29], Bilbao Crystallographic Server [30], FindSpinGroup [31], and TensorSymmetry [32] packages to obtain the nontrivial SPGs and the corresponding shape of σ_{ij}^z .

III. SYMMETRY ANALYSIS

Collinear magnets are more accurately described using spin-group-symmetry operations of the form $[R_1 || R_2]$, where R_1 and R_2 act on spin and real space, respectively [33]. These systems exhibit symmetries that fall into 32 SPGs associated with ferromagnetic (FM) order, 22 SPGs corresponding to conventional AFM order, and 36 SPGs belong to AM order [31].

The spin-polarized charge conductivity in Eq. (1) transforms under spin-group-symmetry operation $[R_1 || R_2]$ as

$$\sigma_{ij}^{s_k} = \sum_{i', j', k'} D(R_2)_{ii'} D(R_2)_{jj'} \sigma_{i'j'}^{D(R_1)_{kk'} s_{k'}}, \quad (4)$$

where $D(R)$ is the matrix representation of the symmetry operation R . Using Eq. (2), the transformed spin conductivity can be written as $\sigma_{ij}^{'k} = \frac{\hbar}{2e} (\sigma_{ij}^{s_k} - \sigma_{ij}^{-s_k})$. The spin-conductivity component σ_{ij}^k will be allowed if the SPG does not contain any symmetry operation enforcing $\sigma_{ij}^{s_k} = \sigma_{ij}^{-s_k}$ or equivalently $\sigma_{ij}^k = -\sigma_{ij}^k$. Note that, in accordance to Neumann's principle, we only need to consider point-group operations for the response tensors that represent macroscopic/bulk properties [34]. Hence, we have omitted the translation part from the symmetry operations. In addition, the spin-only group $\infty m 1$ contains the symmetry operation $[C_{z\infty} || I]$, $[M_x || I]$, and $[M_y || I]$, where $C_{z\infty}$ is an arbitrary rotation along the z axis, M_i is the mirror plane perpendicular to the i axis, and I is the identity. Under the symmetry operation $[M_y || I]$, $\sigma_{ij}^{s_k} = \sum_{i', j', k'} D(I)_{ii'} D(I)_{jj'} \sigma_{i'j'}^{D(M_y)_{kk'} s_{k'}}$. Since $D(I) = \text{diag}(1, 1, 1)$ and $D(M_y) = \text{diag}(1, -1, 1)$, under the symmetry operation $[M_y || I]$, we get $\sigma_{ij}^{s_x} = \sigma_{ij}^{s_x}$, $\sigma_{ij}^{s_y} = \sigma_{ij}^{-s_y}$, and $\sigma_{ij}^{s_z} = \sigma_{ij}^{s_z}$. Using Eq. (2), $[M_y || I]$ leads to $\sigma_{ij}^y = 0$. Similarly, $[M_x || I]$ leads to $\sigma_{ij}^x = 0$. This implies that, under nonrelativistic collinear magnetism, the only allowed nonzero spin-conductivity components are σ_{ij}^z . Hence, the spin-only symmetry only enforces conditions on the transverse spin-polarization response ($\sigma_{ij}^{x/y} = 0$) and imposes no conditions on σ_{ij}^z . Therefore, hereafter, we omit discussion of the spin-only group and trivially zero elements $\sigma_{ij}^{x/y}$. In addition, spin-orbit coupling (SOC) is expected to be negligible and is excluded from our simulations. Here, we choose the spin-quantization axis to be along the z direction [14]. In the

absence of SOC, there is no mechanism to induce spin canting, and consequently, $\sigma_{ij}^{x/y} = 0$. The spin-polarized conductivity σ_{ij}^{sk} transforms under time-reversal ($\mathcal{T}$) as $\mathcal{T}\sigma_{ij}^{sk} \rightarrow \sigma_{ij}^{-sk}$. Accordingly, the charge and spin-conductivity tensors are even and odd under $\mathcal{T}$, respectively.

The shape of σ_{ij}^z is further restricted by the nontrivial spin-group symmetry. For example, $[C_2||P]$ leads to $\sigma_{ij}^{sk} = \sum_{i',j',k'} D(P)_{ii'} D(P)_{jj'} \sigma_{i'j'}^{D(C_2)kk'}$. Using the fact that $D(P) = -\text{diag}(1, 1, 1)$ and $D(C_2) = \text{diag}(1, -1, -1)$ (considering the twofold axis about the x axis in spin space), $[C_2||P]$ imposes the conditions $\sigma_{ij}^{s_z} = \sigma_{ij}^{-s_z}$ and $\sigma_{ij}^z = -\sigma_{ij}^z$ on spin-polarized charge conductivity and spin conductivity, respectively. In other words, $[C_2||P]$ forbids σ_{ij}^z . Therefore, spin current is completely forbidden for all AFM SPGs. The $[C_2||P]$ symmetry can be broken by applying an electric field ($\mathcal{E}_i$) since $[C_2||P]\mathcal{E}_i \rightarrow -\mathcal{E}_i$. The applied electric field ($\mathcal{E}_i$) also breaks the $[E/C_2||M_i]$ and $[E/C_2||C_{nj}(j \neq i)]$ symmetries. Overall, the spin-conductivity component σ_{ij}^z becomes allowed if the electric field breaks all spin-group symmetries that enforce $\sigma_{ij}^{s_z} = \sigma_{ij}^{-s_z}$.

To understand the electric-field-induced magnetic phase transition, we construct minimal models starting from the generic four-band Hamiltonian [35]

$$H(\mathbf{k}) = \varepsilon_0(\mathbf{k}) \tau_0 s_0 + \sum_{a,b} f_{ab}(\mathbf{k}) \tau_a s_b. \quad (5)$$

This Hamiltonian is written in the sublattice-spin basis, where τ_a and s_b are Pauli matrices acting on the sublattice and spin degrees of freedom, respectively. The functions $f_{ab}(\mathbf{k})$ are fixed by the spin-group symmetry. For the AFM SPG $12_y/2m_y$, which contains $[C_2||P]$ and $[C_2||M_y]$, the lowest-order symmetry-invariant terms yield the AFM Hamiltonian

$$H_{\text{AFM}}(\mathbf{k}) = \varepsilon_0(\mathbf{k}) \tau_0 s_0 + t_0 \tau_x s_0 + \Delta \tau_z s_z. \quad (6)$$

The spin-polarized Fermi surface for the minimal model for AFM SPG $12_y/2m_y$ is shown in Fig. 1(a). Applying an electric field along the z direction $\mathcal{E}_z$ breaks P while preserving M_y . Hence, while $[C_2||P]$ is removed, $[C_2||M_y]$ remains preserved. The resulting minimal Hamiltonian is d -wave AM

$$H_{\mathcal{E}_z}(\mathbf{k}) = H_{\text{AFM}}(\mathbf{k}) + \lambda_z \mathcal{E}_z k_x k_y \tau_0 s_z. \quad (7)$$

Therefore, AFM SPG $12_y/2m_y$ reduces to the effective AM SPG of $2m_y$ under an electric field applied along $\mathcal{E}_z$ [see Fig. 1(b) for spin-polarized Fermi surfaces]. On the other hand, an electric field along the y axis breaks both $[C_2||P]$ and $[C_2||M_y]$, thereby allowing the minimal ferromagnetic correction

$$H_{\mathcal{E}_y}(\mathbf{k}) = H_{\text{AFM}}(\mathbf{k}) + \lambda_y \mathcal{E}_y \tau_0 s_z \quad (8)$$

Therefore, AFM SPG $12_y/2m_y$ reduces to an effective FM SPG of 12_y under electric field $\mathcal{E}_y$ [spin-polarized Fermi surfaces in Fig. 1(c)]. Extending this analysis to all other AFM SPGs, one obtains Table I, which lists electric field-induced transitions of AFM SPGs to either FM or AM SPGs. Note that collinear AFMs always contain $[C_2||P]$ symmetry. An applied electric field (in any direction) at least breaks inversion P symmetry. As a result, the applied electric field reduces the symmetry of every spin group of collinear AFMs

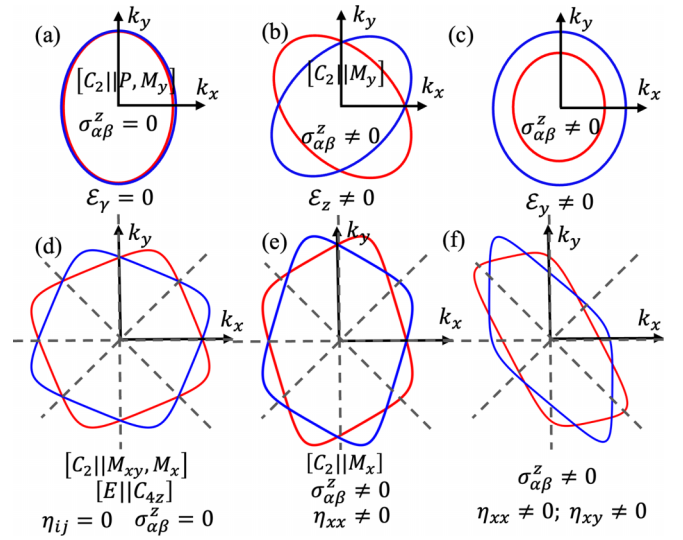


FIG. 1. Schematic of electric field and strain-induced spin conductivity. (a) Spin-polarized Fermi surface of an AFM with $[C_2||P]$ symmetry. The red and blue isosurfaces correspond to the opposite spin directions. Fermi surfaces of an AFM under an applied electric field when (b) the effective spin group of the material possesses a symmetry connecting opposite spin sublattices (e.g., d -wave AM with $[C_2||M_y]$) and (c) no such symmetry exists (ferromagnetic or compensated ferrimagnetic). (d) Fermi surface of a higher-order planar g -wave state. Fermi surfaces of a higher-order g -wave AM (d) without strain and under a strain field η_{ij} when (e) the effective spin group retains a symmetry connecting opposite spin sublattices (e.g., d -AM with $[C_2||M_{xy}]$) and (f) no such symmetry remains. Note that the effects of strain in (e) and (f) are small as the number of spin-degenerate nodal lines remain same under strain. However, the number of symmetry-enforced nodal lines (appearing on the dashed lines) reduces under strain from 4 in (d) to 2 in (e) and 0 in (f).

(Table I). If the reduced effective SPG is an FM or a d -wave AM, a net spin current becomes allowed. In such a case, spin-polarized conductivity can be considered bilinear in the electric field. This coupling between spin current and electric field is reminiscent of magnetoelectric coupling. Table I also lists the cases where the applied electric field reduces the AFM SPG to that of either a g - or an i -wave AM. For example, $\mathcal{E}_z$ applied to a $14/2m^2m^2m$ or $26/1m$ AFM and $\mathcal{E}_i$ ($i = x, y, z$) applied to a $2m^23^2m$ AFM result in g -wave AMs. Also, the $16/2m^2m^2m$ AFM reduces to an i -wave AM under the application of $\mathcal{E}_z$. In such cases, a higher-order (i.e., trilinear) electric field may be required to induce a finite spin conductivity [36].

Unlike the electric field that breaks the $[C_2||P]$ symmetry of AFMs, the strain field (η_{ij} , with $i, j = x, y, z$), which transforms like $k_i k_j$, remains invariant under $[C_2||P]\eta_{ij} \rightarrow \eta_{ij}$. Therefore, η_{ij} is not sufficient to induce spin currents in AFMs. However, strain can be used to induce phase transitions in g - and i -wave AMs to obtain either an FM or a d -wave AM. For example, AM SPG $14_2^2m_x^2m_{xy}$ is a g -wave AM [see Fig. 1(d)], with the minimal two-band model $H_g(\mathbf{k}) = \varepsilon_0(\mathbf{k}) s_0 + \alpha k_x k_y (k_x^2 - k_y^2) s_z$ [37]. The spin-split Fermi surface plotted in Fig. 1(d) for such a g -wave AM displays the four nodal lines along the $x, y, x + y$ and $x - y$ directions, protected

TABLE I. Electric field-induced transition of AFM SPG to FM (or uncompensated magnetic) and AM SPG. The first column provides the parent SPG, and the second, third, and fourth columns give the effective SPG under the application of electric fields $\mathcal{E}_x$, $\mathcal{E}_y$, and $\mathcal{E}_z$, respectively. The superscripts 1 and 2 preceding each symmetry operation indicate whether the spin sublattices are connected to the same or opposite spin sublattices. The SPG without any symmetry connecting opposite spin sublattices (SPGs with only superscript 1) are ferromagnetic. For the AM SPGs, the spin-orders (d , g , and i) are also indicated in parentheses. The xy , $x\bar{y}$, and xyz directions are along $\hat{x} + \hat{y}$, $\hat{x} - \hat{y}$, and $\hat{x} + \hat{y} + \hat{z}$, respectively.

SPGs	$\mathcal{E}_x$	$\mathcal{E}_y$	$\mathcal{E}_z$
$2\bar{1}$	1_1	1_1	1_1
$2_2y/1m_y$	$1m_y$	$2_2y (d)$	$1m_y$
$1_2y/2m_y$	$2m_y (d)$	1_2y	$2m_y (d)$
$1m_x^1m_y^1m_z$	$2_2x^1m_y^1m_z (d)$	$1m_x^12_1^1m_z (d)$	$1m_x^1m_y^12_z (d)$
$1m_x^1m_y^2m_z$	$2_2x^1m_y^2m_z (d)$	$1m_x^12_2^2m_z (d)$	$1m_x^1m_y^12_z (d)$
$2m_x^1m_y^1m_z$	$1_2x^1m_y^1m_z (d)$	$2m_x^12_2^1m_z (d)$	$2m_x^1m_y^12_z (d)$
$1_4z/2m_z$	$2m_z (d)$	$2m_z (d)$	1_4z
$2_4z/2m_z$	$2m_z (d)$	$2m_z (d)$	$2_4z (d)$
$1_4z/2m_z^1m_{xy}$	$2_2^1m_y^1m_z (d)$	$1m_x^12_2^1m_z (d)$	$1_4z^1m_{xy}$
$2_4z/2m_z^1m_{xy}$	$2_2^1m_y^1m_z (d)$	$1m_x^12_2^1m_z (d)$	$2_4z^1m_{xy} (g)$
$1_4z/2m_z^2m_{xy}$	$1_2x^1m_y^1m_z (d)$	$2m_x^12_2^1m_z (d)$	$1_4z^2m_{xy} (g)$
$2\bar{3}_z$	1_1	1_1	1_3z
$2\bar{3}_z^11m_y$	$1m_y$	$2_2y (d)$	$1_3z^11m_y$
$2\bar{3}_z^11m_y$	$2m_y (d)$	1_2y	$1_3z^11m_y (g)$
$2_6z/1m_z$	$1m_z$	$1m_z$	$2_6z (g)$
$1_6z/2m_z$	$2m_z (d)$	$2m_z (d)$	1_6z
$1_6z/2m_z^1m_{xy}$	$2_2^1m_y^1m_z (d)$	$1m_x^12_2^1m_z (d)$	$1_6z^1m_{xy}$
$2_6z/1m_z^1m_{xy}$	$2_2^1m_y^1m_z (d)$	$1m_x^12_2^1m_z$	$2_6z^1m_{xy} (g)$
$1_6z/2m_z^2m_{xy}$	$1_2x^1m_y^1m_z (d)$	$2m_x^12_2^1m_z (d)$	$1_6z^2m_{xy} (i)$
$2m_x^2\bar{3}_{xyz}$	$1_2x^1m_y^1m_z (d)$	$2m_x^12_2^1m_z (d)$	$2m_x^1m_y^12_z (d)$
$2m_x^2\bar{3}_{xyz}^1m_{xy}$	$2_4x^1m_{yz} (d)$	$2_4x^1m_{xz} (d)$	$2_4z^1m_{xy} (d)$
$2m_x^2\bar{3}_{xyz}^2m_{xy}$	$1_4x^1m_{yz} (g)$	$1_4x^1m_{xz} (g)$	$1_4z^2m_{xy} (g)$

by the $[C_2||M_x]$ and $[C_2||M_{xy}]$ symmetries. The $[E||C_{4z}]$, $[E||M_x]$, and $[E||M_{xy}]$ symmetries would forbid a spin current in such a material. However, an application of the strain η_{xx} breaks the $[C_2||M_{xy}]$ symmetry. This results in an effective SPG of $2m_x^1m_y^12_z$, with a spin-polarized Fermi surface that is similar to the Fermi surface of a d -wave AM. This can be seen in Fig. 1(e), which is plotted using the minimal model $H_g(\mathbf{k})$, with strain effects incorporated using $k_i \rightarrow (\delta_{ij} + \eta_{ij})k_j$. It should be added that, although the number of nodal lines in Fig. 1(e) has remained unchanged under strain, the number of symmetry-protected nodal lines is reduced due to the reduction of the SPG. In Fig. 1(e), the only symmetry-protected nodal lines are those along the x and y directions, while the nodes near $x + y$ and $x - y$ directions are accidental and not symmetry-enforced anymore. The positions of these two accidental nodes relative to the $x + y$ and $x - y$ directions depend on the strength of the strain. Note that, in our example of the g -wave AM, we can further reduce the symmetry of the structure with an additional strain field η_{xy} that breaks $[C_2||M_x]$ symmetry. Hence, a simultaneous application of η_{xx} and η_{xy} will

reduce the SPG $1_4z^2m_x^1m_{xy}$ to an effective ferromagnetic SPG 1_2z that has no symmetry connecting opposite spin sublattices. Therefore, in this case, the spin-polarized Fermi surface is similar to that for an FM, as shown in Fig. 1(f), where the areas enclosed by the spin-up and spin-down Fermi surfaces are no longer necessarily the same. We note that one needs to go beyond minimal two-band models to obtain a more complete description of the reduction of spin-degenerate nodal lines under strain. This can be achieved via tight-binding models and/or DFT calculations (see Fig. 5, for example).

Table II lists the effective SPGs of g - and i -wave AMs under linear strain (η_{ij}) by collecting spin-group-symmetry operations $[R_1||R_2]$ such that $R_2\eta_{ij} \rightarrow \eta_{ij}$. Depending on the type of strain and the SPG of AM, the strain may or may not induce symmetry breaking. For instance, strains along the z direction η_{zz} generally do not induce symmetry breaking since the principal axis (highest-symmetry axis in the crystal) is usually also the z direction (see Table II). However, this is not the case if the crystal has two or more principal axes. For instance, in the case of $2_4x^13_{xyz}^22_{xy}$ SPG (last entry in Table II) with three fourfold principal axes along the x , y , and z directions, a strain along the z axis will break the fourfold symmetries along the x and y collinear axes.

Our symmetry analysis shows how external stimuli allow spin conductivity in a larger group of materials by breaking spin-group symmetries that enforce zero spin conductivity. It should be further noted that the symmetry-lowering stimuli in Tables I and II can be applied simultaneously to obtain spin conductivities in conventional magnets through piezomagnetoelectric-like ($\mathcal{E}_i\eta_{jk}$) couplings, where electric and strain fields act in concert. For example, AFM SPG $1_4z/2m^2m$ allows spin conductivity under the application of $\mathcal{E}_z\eta_{ij}$ but not when $\mathcal{E}_z$ or η_{ij} are used separately.

IV. DFT ANALYSIS

In this section, we use DFT-based calculations to investigate spin-current generation in selected materials that are representative of the distinct symmetry cases summarized in Tables I and II. We have presented the band structures of the considered materials in the Sec. S2 of SM [28]. We begin by analyzing the spin conductivity in both bulk and 2D d -wave AMs, which intrinsically support finite spin currents. Subsequently, we consider prototypical materials to explore three key mechanisms for spin-current generation: (i) an electric field-driven transition from AFM to uncompensated magnetic order, (ii) a strain-induced symmetry transition from g - to d -wave AM, and (iii) a combined electric field and strain-mediated transition from i - to d -wave AM. In what follows, we define the charge-to-spin conversion ratio as $\frac{2e}{\hbar}\sigma_{ij}^z/\sigma_{ij}$ [17]. In the limiting case, when energy falls into a region without bands, the charge conductivity σ_{ij} approaches zero, and this ratio becomes mathematically indeterminate. To address this, we set the conversion ratio to zero whenever σ_{ij} falls below a defined threshold in the charge-to-spin conversion vs energy plots. A similar approach is also used for the spin-splitter ratio, where the directions of charge current and spin current are not the same. Note that, in semiconductors, hole or electron doping is required to observe spin transport. A low-level doping concentration ($\approx 2\%$) is sufficient to provide

TABLE II. Tuning AM by the application of strain. The first column gives the AM pattern, with P and B standing for planar and bulk, respectively, while g and i labels refer to the g - and i -wave spin-splitting patterns [8]. The second column gives the parent SPG for the unstrained, ideal materials. The rest of the columns provide the effective SPG of the parent space group after the application of the strain field η_{ij} .

	SPGs	η_{xx}	η_{yy}	η_{zz}	η_{xy}	η_{xz}	η_{yz}
P-g	$14_z/1m_z^2m_{xy}^2m_{xy}$	$2m_x^2m_y^1m_z (d)$	$2m_x^2m_y^1m_z (d)$	$14_z/1m_z^2m_x^2m_{xy} (g)$	$2m_{xy}^2m_{xy}^1m_z (d)$	$22_y/2m_y (d)$	$22_x/2m_x (d)$
P-g	$14_z^2m_z^2m_{xy}$	$2m_x^2m_y^12_z (d)$	$2m_x^2m_y^12_z (d)$	$14_z^2m_z^2m_{xy} (g)$	$2m_{xy}^2m_{xy}^12_z (d)$	$2m_y (d)$	$2m_x (d)$
P-g	$14_z^22_x^22_{xy}$	$22_x^22_y^12_z (d)$	$22_x^22_y^12_z (d)$	$14_z^22_x^22_{xy} (g)$	$22_{xy}^22_{xy}^12_z (d)$	$22_y (d)$	$22_x (d)$
P-g	$14_z^22_x^22_{xy}$	$22_x^22_y^12_z (d)$	$22_x^22_y^12_z (d)$	$14_z^22_x^22_{xy} (g)$	$2m_{xy}^2m_{xy}^12_z (d)$	$22_y (d)$	$22_x (d)$
B-g	$13_z^2m_x^11_y$	$22_x/2m_x (d)$	$22_x/2m_x (d)$	$13_z^2m_x^11_y (g)$	$1\bar{1}$	$1\bar{1}$	$22_x/2m_x (d)$
B-g	$13_z^2m_x^11_y$	$2m_x (d)$	$2m_x (d)$	$13_z^2m_x^11_y (g)$	11	11	$2m_x (d)$
B-g	$13_z^22_x^11_y$	$22_x (d)$	$22_x (d)$	$13_z^22_x^11_y (g)$	11	11	$22_x (d)$
B-g	$26_z/2m_z$	$22_z/2m_z (d)$	$22_z/2m_z (d)$	$26_z/2m_z (g)$	$22_z/2m_z (d)$	$1\bar{1}$	$1\bar{1}$
B-g	26_z	$2m_z (d)$	$2m_z (d)$	$26_z (g)$	$2m_z (d)$	11	11
B-g	26_z	$22_z (d)$	$22_z (d)$	$26_z (g)$	$22_z (d)$	11	11
B-g	$26_z/2m_z^2m_x^1m_y$	$2m_x^1m_y^2m_z (d)$	$2m_x^1m_y^2m_z (d)$	$26_z/2m_z^2m_x^1m_y (g)$	$22_z/2m_z (d)$	$12_y/1m_y$	$22_x/2m_x (d)$
B-g	$26_z^2m_x^1m_y$	$2m_x^1m_y^22_z (d)$	$2m_x^1m_y^22_z (d)$	$26_z^2m_x^1m_y (g)$	$22_z (d)$	$1m_y$	$2m_x (d)$
B-g	$26_z^22_x^12_y$	$22_x^22_y^22_z (d)$	$22_x^22_y^22_z (d)$	$26_z^22_x^12_y (g)$	$22_z (d)$	12_y	$22_x (d)$
B-g	$26_z^2m_x^12_y$	$2m_x^12_y^2m_z (d)$	$2m_x^12_y^2m_z (d)$	$26_z^2m_x^12_y (g)$	$2m_z (d)$	12_y	$2m_x (d)$
B-g	$26_z^22_x^1m_y$	$22_x^2m_y^2m_z (d)$	$22_x^2m_y^2m_z (d)$	$26_z^22_x^1m_y (g)$	$2m_z (d)$	$1m_y$	$22_x (d)$
P-i	$16_z/1m_z^2m_x^2m_y$	$2m_x^2m_y^1m_z (d)$	$2m_x^2m_y^1m_z (d)$	$16_z/1m_z^2m_x^2m_y (d)$	$12_z/1m_z$	$22_y/2m_y (d)$	$22_x/2m_x (d)$
P-i	$16_z^2m_z^2m_y$	$2m_x^2m_y^12_z (d)$	$2m_x^2m_y^12_z (d)$	$16_z^2m_z^2m_y (i)$	12_z	$2m_y (d)$	$2m_x (d)$
P-i	$16_z^22_x^22_y$	$22_x^22_y^12_z (d)$	$22_x^22_y^12_z (d)$	$16_z^22_x^22_y (i)$	12_z	$22_y (d)$	$22_x (d)$
P-i	$16_z^2m_x^22_y$	$2m_x^22_y^1m_z (d)$	$2m_x^22_y^1m_z (d)$	$16_z^2m_x^22_y (i)$	$1m_z$	$22_y (d)$	$2m_x (d)$
B-i	$1m_x^13_{xyz}^2m_{xy}$	$24_x/1m_x^1m_y^2m_{yz} (d)$	$24_y/1m_y^1m_x^2m_{xz} (d)$	$24_z/1m_z^1m_x^2m_{xy} (d)$	$2m_{xy}^2m_{xy}^1m_z (d)$	$2m_{xz}^2m_{xz}^1m_y (d)$	$2m_{yz}^2m_{yz}^1m_x (d)$
B-i	$24_x^13_{xyz}^2m_{xy}$	$24_x^12_y^2m_{yz} (d)$	$24_y^12_x^2m_{xz} (d)$	$24_z^12_x^2m_{xy} (d)$	$2m_{xy}^2m_{xy}^12_z (d)$	$2m_{xz}^2m_{xz}^12_y (d)$	$2m_{yz}^2m_{yz}^12_x (d)$
B-i	$24_x^13_{xyz}^22_{xy}$	$24_x^12_y^22_{yz} (d)$	$24_y^12_x^22_{xz} (d)$	$24_z^12_x^22_{xy} (d)$	$22_{xy}^22_{xy}^12_z (d)$	$22_{xz}^22_{xz}^12_y (d)$	$22_{yz}^22_{yz}^12_x (d)$

a stable carrier density while maintaining the compensated magnetic order [38].

A. d -wave AMs

KV₂Se₂O belongs to a broader class of materials crystallizing in the orthorhombic crystal structure with a space group of $P4/mmm$ [see Fig. 2(a)]. KV₂Se₂O is a room-temperature metallic AM with quasi-one-dimensional Fermi sheets extending along the k_x and k_y directions, and quasi-2D Fermi pockets centered around the X and Y points, as seen in Fig. 2(b) [39–41]. The SPG of KV₂Se₂O is $24/1m^1m^2m$, containing $[C_2||C_{4z}]$, $[E||P]$, $[E||M_z]$, $[E||M_x]$, and $[C_2||M_{xy}]$. Following Neumann's principle, the symmetries of the SPG of this material are adopted by its spin anisotropy. Using the transformation rules of Eqs. (1) and (2), $[C_2||C_{4z}]$ transforms $\sigma_{yy}^{s_z} = \sigma_{xx}^{-s_z}$, $\sigma_{xy}^{s_z} = -\sigma_{yx}^{-s_z}$, $\sigma_{zx}^{s_z} = \sigma_{zy}^{-s_z}$, and $\sigma_{zy}^{s_z} = -\sigma_{zx}^{-s_z}$. Therefore, $[C_2||C_{4z}]$ enforces $\sigma_{xz}^z = \sigma_{yz}^z = \sigma_{zx}^z = \sigma_{zy}^z = 0$. Furthermore, $[C_2||M_{xy}]$ transforms $\sigma_{xy}^z \rightarrow \sigma_{yx}^{-z}$, and hence, $\sigma_{xy}^z = \sigma_{yx}^{-z} = 0$ in Eq. (2). Following a similar procedure for all symmetry operations, the symmetry-allowed components of the spin-conductivity tensor are σ_{yy}^z and σ_{xx}^z , with $\sigma_{yy}^z = -\sigma_{xx}^z$. Figure 2(c) shows the spin conductivity σ_{xx}^z for the different scattering rates. The spin conductivity is inversely proportional to Γ within the considered range of 1 meV to 100 meV. The charge-to-spin conversion effi-

ciency ($\frac{2e}{h} \sigma_{ij}^k / \sigma_{ij}$) is 76%, which can reach almost 100% with electron doping of 0.5 eV [see Fig. 2(d)], where the Fermi pockets around the X and Y points vanish, and spin currents are entirely governed by the Fermi sheets.

RuF₄ is a well known 2D d -wave AM crystallizing in space group $P2_1/c$ [see Fig. 3(a)] [42,43]. The SPG of RuF₄ is $22/m$ with spin-symmetry operations $[C_2||M_y]$ and $[E||P]$ [see Fig. 3(b)]. Here, $[C_2||M_y]$ transforms $\sigma_{ii}^{s_z} = \sigma_{ii}^{-s_z}$; therefore, $\sigma_{ii}^z = 0$. Also, $[C_2||M_y]$ leads to $\sigma_{xy}^{s_z} = -\sigma_{xy}^{-s_z}$, $\sigma_{yx}^{s_z} = -\sigma_{yx}^{-s_z}$, $\sigma_{xz}^{s_z} = \sigma_{xz}^{-s_z}$, $\sigma_{zx}^{s_z} = \sigma_{zx}^{-s_z}$, $\sigma_{yz}^{s_z} = -\sigma_{yz}^{-s_z}$, and $\sigma_{zy}^{s_z} = -\sigma_{zy}^{-s_z}$. Therefore, the nonzero components of the spin-conductivity tensor are σ_{xy}^z , σ_{yx}^z , σ_{yz}^z , and σ_{zy}^z . Since RuF₄ is a semiconductor, the charge conductivity is zero at the Fermi energy. Figure 3(c) shows the spin conductivity σ_{xy}^z of the hole-doped RuF₄ as a function of the Fermi energy. The maximum spin-splitting efficiency is around 80% when the hole doping moves the Fermi level below the intrinsic Fermi level by ~ 0.1 eV [Fig. 3(d)].

Note that, although both KV₂Se₂O and RuF₄ are d -wave AMs, their respective responses to the applied electric field differ. This is owing to the different spin anisotropies of their Fermi surfaces. In the case of RuF₄, the bands are spin-degenerate along the x and y directions, while for KV₂Se₂O, the bands are spin degenerate along the $x + y$ and $x - y$ directions. Hence, if the applied electric field is in the x direction,

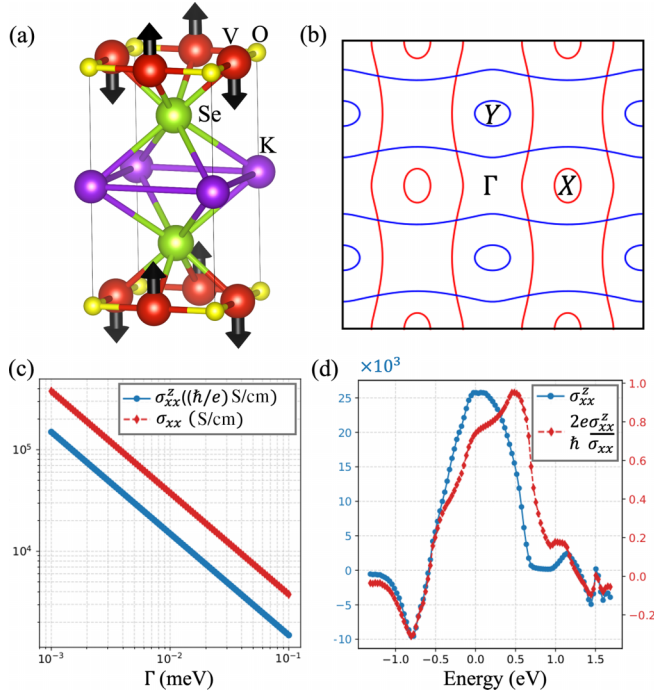


FIG. 2. (a) The crystal and magnetic structures of KV₂Se₂O. (b) The Fermi surface of KV₂Se₂O projected on the $k_z = 0$ plane, calculated without SOC. The red and blue curves denote spin-up and spin-down Fermi surfaces, respectively. (c) The charge and spin conductivity as a function of the scattering rate (Γ). (d) The spin conductivity and corresponding charge-to-spin conversion ratio as a function of the Fermi energy.

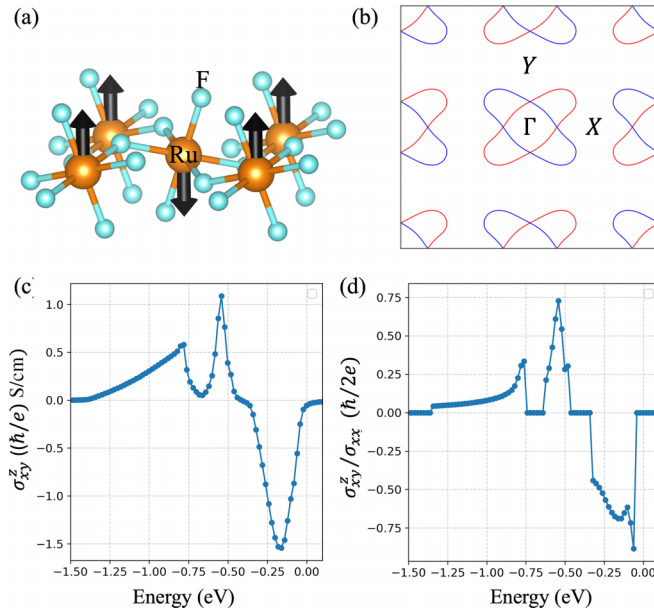


FIG. 3. (a) The crystal and magnetic structures of RuF₄. (b) The Fermi surface of RuF₄ projected on the $k_z = 0$ plane at the energy of $E_F - 0.20$ eV, calculated without SOC. (c) The spin conductivity and (d) corresponding spin-splitter ratio as a function of the Fermi energy.

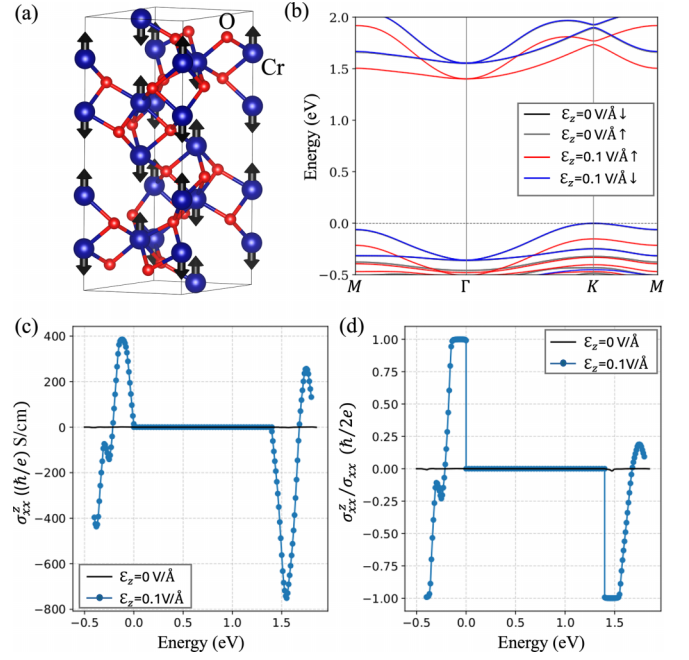


FIG. 4. (a) The crystal and magnetic structures of Cr₂O₃. The blue and red spheres are used for Cr and O atoms, respectively. (b) Spin-polarized band structure of [0001] film of Cr₂O₃ with and without the electric field $\mathcal{E}_z$. (c) The spin conductivity and (d) corresponding charge-to-spin conversion ratio as a function of the Fermi energy with and without $\mathcal{E}_z$.

KV₂Se₂O will show longitudinal spin conductivity, while RuF₄ will exhibit transverse spin conductivity. On the other hand, KV₂Se₂O will exhibit transverse spin conductivity, and RuF₄ will show longitudinal spin conductivity when an electric field is applied along the $x + y$ or $x - y$ direction. The longitudinal and transverse spin conductivity for an arbitrary direction of the electric field for KV₂Se₂O and RuF₄ is plotted in Sec. S3 of SM in the SM [28].

B. $\mathcal{E}_z$ -induced AFM to uncompensated magnetism

Bulk Cr₂O₃ is a well-known magnetoelectric AFM that crystallizes in the $R\bar{3}c$ space-group symmetry [44,45]. The SPG of Cr₂O₃ is $2\bar{3}11^1m$ with the following spin-symmetry operations: $[C_2||P]$, $[E||M_y]$, $[E||C_{3z}]$, and $[C_2||S_{3z}]$ [see Fig. 4(a)]. Since the $[C_2||P]$ symmetry operation transforms $\sigma_{ij}^{s_z}$ to $\sigma_{ij}^{-s_z}$, spin conductivity is forbidden in bulk Cr₂O₃. The application of $\mathcal{E}_z$ breaks $[C_2||S_{3z}]$ and $[C_2||P]$ while preserving $[E||M_y]$ and $[E||C_{3z}]$ symmetries. This reduces the original SPG to a FM $1\bar{3}11^1m$ SPG (see Table I). To apply the electric field, we used a [0001] film of Cr₂O₃ with a 20 Å vacuum separating the film from its image in the z direction (see Sec. S4 in the SM [28] for the structure). This structure was previously found to result in the most stable surface [46]. Since the electric field $\mathcal{E}_z$ breaks the $[C_2||P]$ symmetry, which is the only symmetry that connects the opposite spin sublattices, it leads to a Zeeman-like spin splitting of the conduction and valence bands, as seen in Fig. 4(b). Despite the presence of spin splitting, which is similar to that of a FM, our DFT calculations reveal a net zero

magnetic moment. This apparent contradiction can be attributed to the semiconducting nature of Cr_2O_3 , which gives rise to zero net magnetic moment according to Luttinger's criterion. In a stoichiometric compound, Luttinger's criterion requires that the net magnetization be equal to $N\mu_B$ ($N = \text{integer}$). When at least one spin channel is gapped (i.e., the material is either a semiconductor or a half-metal) and the orbital moment is quenched, this results in zero net magnetic moment for small perturbations. The $[E||M_y]$ and $[E||C_{3z}]$ symmetries allow spin-conductivity components— σ_{xx}^z , σ_{yy}^z , and σ_{zz}^z —to be nonzero, with $\sigma_{xx}^z = \sigma_{yy}^z$. It should be pointed out that, since $J_x^z = \sigma_{xx}^z \mathcal{E}_x$ and σ_{xx}^z itself depend on $\mathcal{E}_z$, we get that $J_x \propto \mathcal{E}_x \mathcal{E}_z$. In other words, the spin conductivity is at least second order in the electric field. Figure 4(c) plots the spin conductivity σ_{xx}^z for the electron- and hole-doped Cr_2O_3 as a function of Fermi energy with and without an external electric field ($\mathcal{E}_z = 0.1 \text{ V/\AA}$). The spin conductivity σ_{xx}^z switches signs on going from negative values of Fermi energy (corresponding to hole doping) to the positive values (electron doping). This is owing to the opposite spin magnetization of the topmost valence and lowest conduction bands. As seen in Fig. 4(c), the maximum value of σ_{xx}^z is $\sim 750(\hbar/e) \text{ S/cm}$ for the electron-doped material. Figure 4(d) plots the charge-to-spin conversion ratio as a function of Fermi energy. The charge current is fully spin polarized around the conduction band minimum (CBM) and valence band maximum (VBM), as only one type of spin-conduction channel contributes to the conductivity. On the other hand, away from the CBM and VBM, the charge-to-spin conversion ratio decreases as both spin-conduction channels start contributing.

C. η_{xy} -induced g - to d -wave transition

FeS_2 belongs to a large class of planar 2D materials that crystallize in the $P4/mbm$ space group [Fig. 5(a)] [47,48]. FeS_2 is a planar g -wave AM with nontrivial SPG $^{14/1}m^2m^2m$ containing $[C_2||M_x]$, $[C_2||M_y]$, $[E||C_{4z}]$, and $[E||M_z]$. As discussed previously, the symmetry operation $[C_2||M_x]$ suppresses the following spin-conductivity components: σ_{xx}^z , σ_{yy}^z , σ_{zz}^z , σ_{yz}^z , and σ_{zy}^z . Additionally, the mirror symmetry $[E||M_z]$ eliminates σ_{xz}^z and σ_{zx}^z , while $[C_2||M_y]$ forbids σ_{xy}^z and σ_{yx}^z . Therefore, spin current is completely forbidden in a planar g -wave AM. Consistent with the symmetry analysis, DFT calculations confirm a vanishing spin conductivity [see the solid black line in Fig. 5(e)]. The g -wave SPG $^{14/1}m^2m^2m$ of FeS_2 reduces to a d -wave SPG ($^2m^2m^1m$) through application of η_{xx} , η_{yy} , or η_{xy} strains, whereas it can be reduced to a d -wave SPG ($^{22/2}m$) through an application of η_{xz} or η_{yz} strains (see Table II). As a representative case, we use the shear strain η_{xy} of 2%. The structural changes produced by this strain can be seen in Fig. 5(b), where we have exaggerated the strain percentage to 20% for the sake of representation only. The SPG of FeS_2 under η_{xy} strain is $^2m^2m^1m$, with spin-symmetry operations $[C_2||M_x]$, $[C_2||M_{xy}]$, and $[E||M_z]$. Therefore, $^2m^2m^1m$ allows for nonzero σ_{xx}^z and σ_{yy}^z components, with $\sigma_{yy}^z = -\sigma_{xx}^z$. Figures 5(c) and 5(d) show the AM phase transition from planar g wave to planar d wave upon the application of the strain. The maximum spin-conductivity value is calculated to be up to $2800 \frac{\hbar}{e} \frac{\text{S}}{\text{cm}}$, as seen in Fig. 5(e). This occurs at an energy of -1.13 eV , where the Fermi surface is mostly

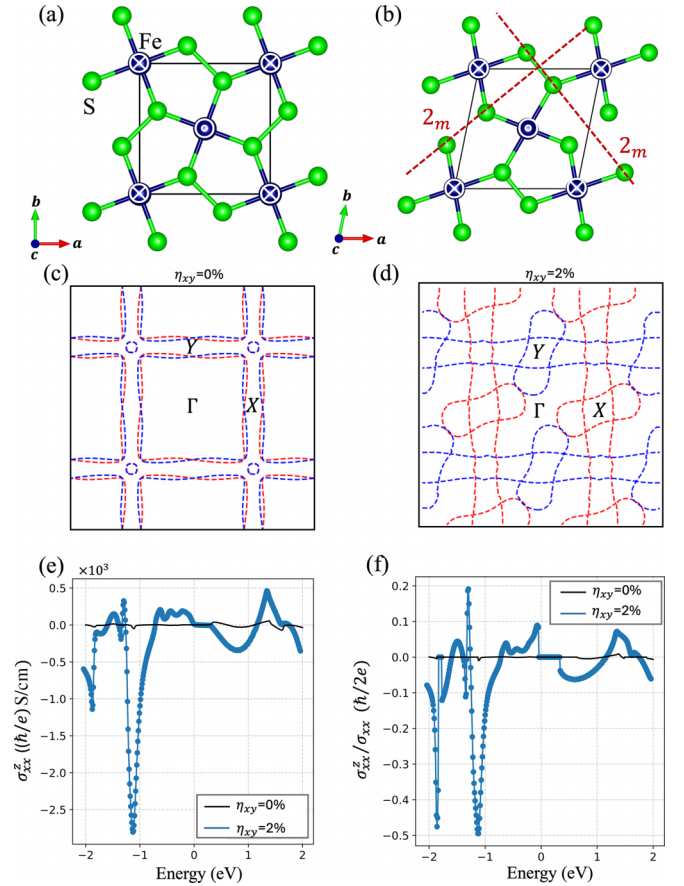


FIG. 5. (a) The crystal and magnetic structures of an FeS_2 monolayer (a) without and (b) with strain field η_{xy} . The magnetic structures in (a) and (b) are shown by superimposing the $\odot$ and $\otimes$ symbols on the Fe atoms, representing spins that point in $+z$ (i.e., out of the plane of paper) and $-z$ (into the plane of paper) directions, respectively. Also, to make the strain-induced distortions visible to the naked eye, we have exaggerated the strain to 20% in (b). The constant energy contours for (c) unstrained and (d) strained FeS_2 for $E_F - 1.13 \text{ eV}$ with maximum charge-to-spin conversion. The (e) spin conductivity and (f) charge-to-spin conversion ratio as a function of energy.

composed of spin-polarized sheets along the x and y directions [Figs. 5(d) and 5(e)]. These Fermi sheets are similar to those for the case of $\text{KV}_2\text{Se}_2\text{O}$ [Fig. 2(b)]. As seen in Fig. 5(f), the maximum spin-to-charge conversion ratio is 50%, which is smaller than that obtained for $\text{KV}_2\text{Se}_2\text{O}$ due to the Fermi pockets around X and Y , which are slighter bigger than those of $\text{KV}_2\text{Se}_2\text{O}$ [compare Figs. 2(b) and 5(d)]. Recall from Sec. IV A that transverse spin conductivity will be observed when the electric field is applied along the $x \pm y$ directions (see Sec. S3 of SM in the SM [28]).

D. $\mathcal{E}_z \eta_{xx}$ induced AFM to d -wave AM transition

MnPS_3 is a 2D AFM, which crystallizes in the $P\bar{3}1m$ space group [Fig. 6(a)] [9,49]. The SPG of MnPS_3 is $^{2\bar{3}2}m^11$, and spin current is forbidden due to spin-degenerate bands. MnPS_3 has been the subject of several reports that have

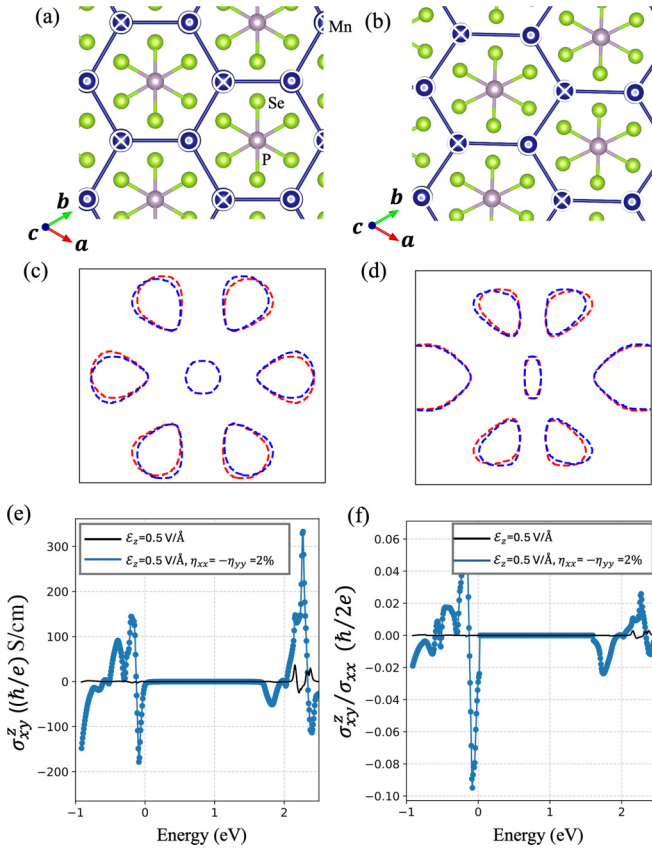


FIG. 6. (a) The crystal and magnetic structures of the monolayer MnPSe₃ (a) without and (b) with applied strain. In (b), the strain $\eta_{xx} + \eta_{yy}$ (with $\eta_{xx} = -\eta_{yy}$) is exaggerated to 10% to magnify the structural distortions, making them visible to the naked eye. Also, the magnetization in (a) and (b) are shown by superimposing the $\odot$ and $\otimes$ symbols on the Mn atoms, representing spins that point in $+z$ (i.e., out of the plane of paper) and $-z$ (into the plane of paper) directions, respectively. The constant energy contours (at an energy of $E_F - 0.16$ eV) for (c) $\mathcal{E}_z = 0.5$ V/Å and (d) $\mathcal{E}_z = 0.5$ V/Å, and $\eta_{xx} = -\eta_{yy} = 0.02$, showing the i - to d -wave transition. The (e) spin conductivity and (f) spin-splitter ratio as a function of energy.

studied its AFM to d -wave AM transition under the application of a vertical electric field [9,50]. Under the application of an electric field $\mathcal{E}_z$, the SPG of MnPSe₃ reduces from $2^3\bar{3}2m^11$ to 1^32m^11 , with the latter containing $[E||C_{3z}]$ and $[C_2||M_x]$ symmetries [see Table I and Fig. 6(b)]. Although the 1^31^2m SPG corresponds to the bulk g -wave AM, the 2D nature of MnPSe₃ ($k_z = 0$) makes it a planar i -wave AM [Fig. 6(c)]. The i -wave AM nature of monolayer MnPSe₃ can be confirmed from the sixfold symmetric constant energy contours in Fig. 6(c). The spin currents are forbidden with and without the electric field due to underlying spin-group symmetries $[E||C_{3z}]$ and $[C_2||M_x]$. This is due to the fact that $[E||C_{3z}]$ enforces $\sigma_{xx}^{s_z} = \sigma_{yy}^{s_z}$, $\sigma_{xy}^{s_z} = -\sigma_{yx}^{s_z}$, and $\sigma_{xz}^{s_z} = \sigma_{yz}^{s_z} = \sigma_{zx}^{s_z} = \sigma_{zy}^{s_z} = 0$, while the $[C_2||M_x]$ symmetry leads to $\sigma_{ii}^{s_z} = \sigma_{ii}^{-s_z}$, $\sigma_{yz}^{s_z} = \sigma_{yz}^{-s_z}$, $\sigma_{zy}^{s_z} = \sigma_{zy}^{-s_z}$, $\sigma_{xy}^{s_z} = -\sigma_{xy}^{-s_z}$, $\sigma_{yx}^{s_z} = -\sigma_{yx}^{-s_z}$, $\sigma_{xz}^{s_z} = -\sigma_{xz}^{-s_z}$, and $\sigma_{zx}^{s_z} = -\sigma_{zx}^{-s_z}$. However, the 1^32m^11 SPG can be further reduced to the 2^2m SPG by using η_{xx} , η_{yy} , and η_{yz} (Table II). On the other hand, an application of either η_{xy} or η_{xz} reduces the SPG to that of an uncompensated magnet, while the 1^32m^11

SPG remains unaltered by the uniaxial strain along the z direction (η_{zz}). Therefore, monolayer MnPSe₃ AFM will undergo a magnetic transition to a d -wave AM under $\mathcal{E}_z\eta_{xx}$, $\mathcal{E}_z\eta_{yy}$, and $\mathcal{E}_z\eta_{yz}$. For illustrative purposes, we consider $\mathcal{E}_z(\eta_{xx} + \eta_{yy})$ with $\eta_{xx} = -\eta_{yy} = 0.02$, resulting in the structure shown in Fig. 6(b), and the corresponding constant energy contour in Fig. 6(d). The new SPG of MnPSe₃ under this strain is 2^2m , containing the $[C_2||M_x]$ symmetry operation (see Tables I and II). As mentioned earlier, the only nonzero components of the spin-conductivity tensor are σ_{xy}^z , σ_{yx}^z , σ_{yz}^z , and σ_{zy}^z . The σ_{yz}^z and σ_{zy}^z components are forbidden due to the 2D nature of MnPSe₃. Figures 6(e) and 6(f) show the spin conductivity and spin-splitting ratio for MnPSe₃ with electric field $\mathcal{E}_z$ and $\mathcal{E}_z(\eta_{xx} + \eta_{yy})$. While there is no spin current in the absence of strain, the strain-induced spin-splitting ratio reaches up to $\sim 10\%$ around -0.10 eV, which corresponds to a spin-splitter angle of approximately 11° . Here, we have used the following expression for the spin-splitter angle: $\theta_{ij} = \arctan(2 \frac{2e}{\hbar} \frac{\sigma_{ij}^s}{\sigma_{ii}^c})$ [14,18], with σ_{ij}^s (σ_{ij}^c) being the transverse spin conductivity (longitudinal charge conductivity). This spin-splitter angle is experimentally measurable [18,51]. Note that the spin-conductivity components in the absence of strain may deviate slightly from the ideal zero values due to numerical inaccuracies and/or slight distortions from the ideal symmetric configuration during structural relaxation.

V. CONCLUSIONS

In this work, we have proposed a general framework to obtain nonrelativistic spin currents in collinear AFMs, and g - and i -wave AMs. Using symmetry analysis, we have demonstrated that spin conductivity can be induced through magnetoelectric-, piezomagnetic-, and piezomagnetoelectric-like couplings. The exact functional form of the spin-conductivity tensor is determined by the effective reduced SPG under the application of an external electric field and/or strain. By performing DFT calculations on realistic materials that were subjected to either electric fields, strains, or both, we have shown that the charge-to-spin conversion ratios in AFMs as well as g - and i -wave AMs can be as high as those seen for FMs, compensated ferrimagnets, and d -wave AMs. It should be pointed out that the external stimuli considered in our work are realistic and can be achieved experimentally. For example, the direction-dependent strain field can be obtained by transferring the exfoliating AM crystals onto a flexible substrate with prepatterned metal pads [52,53]. The feasibility of our proposed approaches is also confirmed by recent reports of in-plane strain and magnetic field-induced nematicity in AFM intercalated transition metal dichalcogenides [54]. In addition, recently, finite anomalous Hall conductivity was successfully measured in the semiconducting g -wave AM, α -Fe₂O₃, with 1% Ti doping [38]. Hence, doping to induce charge conductivity, along with electric fields and/or strain to drive the symmetry-lowering phase transitions, can be used to observe spin conductivity in intrinsically semiconducting AFMs and higher-order AMs. Our proposed design principles for inducing spin currents are general and will expand the range of materials available for ultrafast magnetic spintronics.

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DATA AVAILABILITY

The data that support the findings of this article are not publicly available. The data are available from the authors upon reasonable request.

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