

Drop-drop coalescence: A simple crossover function between inertial and viscous dynamics

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The coalescence of liquid drops is a fundamental process that remains incompletely understood, particularly in the intermediate regimes where capillary, viscous, and inertial forces are comparable. Here, we experimentally investigate the dynamics of drop-to-drop coalescence during the transition between viscous and inertial regimes using high-speed imaging. Our results reveal that the liquid bridge between droplets shows power-law growth with exponents between $1/2$ and 1 during drop coalescence. We propose an alternative scaling approach using a dimensionless crossover function that smoothly transitions between viscous and inertial limits. This simple approach, inspired by previous work on drop impact, successfully collapses the experimental data for a wide range of liquid viscosities and coalescence times onto a single master curve. We further compare our results with recent theoretical models and demonstrate how our approach complements and extends current understanding in the crossover of drop coalescence. This study contributes to both the fundamental physics of drop coalescence and its practical applications in various industrial processes.

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I. INTRODUCTION

The coalescence of liquid drops is a fundamental process that plays a crucial role in many natural phenomena and industrial applications, ranging from drop formation in spray to ink-jet printing and emulsion stability [1–5]. Understanding and controlling coalescence dynamics is essential for optimizing processes that involve liquid-liquid interactions and for developing advanced materials and technologies [6,7]. For example, in spray for plant protection in agricultural applications, the balance between droplet coalescence and breakup during spraying directly determines deposition efficiency and environmental impact [8]. Smaller droplets (typically $< 100 \mu\text{m}$) provide better foliage coverage but exhibit increased wind-drift potential, leading to environmental contamination. One of the challenges lies in optimizing the droplet size distribution via controlling the drop generation and coalescence in spray. In the field of additive manufacturing, such as ink-jet and aerosol printing [2,3,9], the control of coalescence dynamics is critical for achieving high-resolution output and

preventing satellite droplet formation. The droplet impact and coalescence on substrates [10] determine the print quality and feature resolution. Emulsion stability, which is crucial in both food science and petroleum engineering, is intrinsically linked to coalescence processes [11,12]. In food products, controlled coalescence can create desirable textures and release flavors, while in enhanced oil recovery in petroleum engineering, understanding coalescence is useful for designing effective surfactant formulations to mobilize trapped oil droplets.

In a simple scenario, when two liquid drops come into contact, a thin liquid bridge—characterized by its radius R —forms between them and rapidly expands, driven by surface-tension forces (Fig. 1). The liquid bridge grows from microscopic to macroscopic scales, after which the two drops merge into one. The growth of the bridge has been shown to exhibit distinct regimes—viscous and inertial—depending on the relative significance of viscous and inertial forces [13–16]. At very early times or for viscous fluids, the drop coalescence dynamics is dominated by a balance between viscous and capillary forces, resulting in a linear growth in time t with a logarithmic correction [14], $R \sim t \ln t$. In the later stages or for low-viscosity fluids, inertial forces dominate the bridge's growth, leading to a second self-similar regime characterized by $R \sim t^{1/2}$. While the asymptotic behaviors of both viscous and inertial regimes are well established, the nature of the transition between them remains under debate, particularly in terms of the universality of the crossover regime. The presence of

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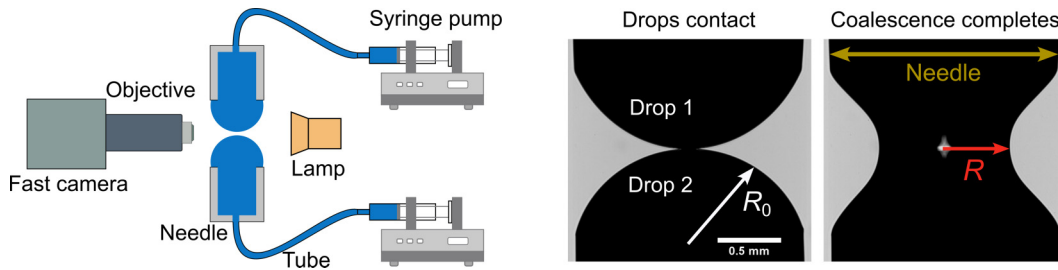


FIG. 1. Experimental setup. Two hemispherical liquid drops are formed at the ends of two vertically aligned needles connected to syringe pumps. A collimated light source and a high-speed camera are precisely aligned with the coalescing drops. High-speed recording is started when the two drops make contact and continues until coalescence is complete (see the example images). The drops have similar initial radii, R_0 and the radius of the liquid bridge is denoted as R .

logarithmic corrections to the viscous regime adds further complexity, obscuring how these corrections evolve during the viscous-to-inertial crossover, in which the capillary, inertial, and viscous forces all play significant roles. The complexity of the crossover currently hampers a comprehensive understanding of the coalescence process as well as the development of practical applications thereof, such as droplet size control [3,17]. In this paper, we focus experimentally on the viscous-to-inertial crossover dynamics in drop-drop coalescence.

From a practical standpoint, experimental observations of the crossover regimes in drop coalescence are challenging, especially for low-viscosity fluids. Whether viscous or inertial forces dominate the coalescence dynamics is determined by the local Reynolds number, $Re = \rho\gamma R^2 / (R_0\eta^2)$, which depends on the fluid's physicochemical properties: surface tension (γ), dynamic viscosity (η), and density (ρ), as well as the initial drop radius (R_0) [13]. For the coalescence of water drops ($\gamma \approx 70$ mN/m, $\rho = 1$ g/cm³, $\eta = 1$ mPa s, $R_0 = 1$ mm), the crossover occurs at a bridge radius $R \approx 3.7$ μ m, corresponding to a time scale of $R/(\gamma/\eta) \approx 10^{-8}$ s. These extremely small length and time scales for water and other low-viscosity fluids make it very difficult to probe the viscous-to-inertial transition regime. Increasing the fluid's viscosity or decreasing its surface tension is thus required to increase the crossover's length and time scales.

Previously, Thoroddsen *et al.* [18] studied the coalescence dynamics between pendent and sessile drops using various water-glycerol mixtures and alcohol drops, with fluid viscosities up to approximately 500 mPa s. Aarts *et al.* [15] used a similar geometry and extended the viscosities to 1000 mPa s using silicone oils. Both experimental studies show that the growth of the liquid bridge in the experiments agrees well with the inertial scaling in theory, $R \sim t^{1/2}$ while the bridge growth for viscous fluids follows a linear scaling $R \sim t$ without the logarithmic correction. Nevertheless, none of these studies addressed a scaling that predicts bridge growth for fluids with intermediate viscosities, where the crossover occurs.

Later on, Paulsen *et al.* [13,19] proposed a crossover function for fitting experimental data, $R/R_c = 2(1/t/t_c + 1/\sqrt{t/t_c})^{-1}$, based on experimental observations, where the crossover length scale R_c and timescale t_c are the free parameters for each fluid. This scaling gave the best collapse of the experimental data. Experimentally, they found $R_c \sim R_0 Oh$ and $t_c \sim t_v Oh$, where $t_v = \eta R_0 / \gamma$ is the viscous time, and $Oh = \eta / \sqrt{\rho\gamma R_0}$ is the Ohnesorge number, a nondimensional fluid viscosity. These crossover characteristics, R_c and t_c were further confirmed by Xia *et al.* [20] using an exact analytical solution for the flow in the vicinity of the liquid bridge, giving a more comprehensive physical description. Although experimental data can be rescaled well using their crossover functions, this collapse should be considered an approximation. A key point of contention, however, is their assertion that even minimal inertia eliminates logarithmic corrections to the bridge growth, a claim that contradicts established theory in drop coalescence [14,16]. The concept of a “definitive” crossover function is fundamentally problematic. The presence of logarithmic terms experimentally for the viscous regime means that the Ohnesorge number (Oh) cannot be strictly eliminated, and none of these crossover functions take the logarithmic terms into account.

In this paper, we propose a simple crossover function as an alternative to describe the viscous-to-inertial transition in drop-drop coalescence, confirmed by our experiments covering a large range of fluid viscosities, and by other experimental results in the literature. Our crossover function smoothly interpolates between the two limiting behaviors (viscous and inertial) and serves as an approximation rather than an exact representation. Our approach aims to strike a balance between simplicity and accuracy for describing the crossover regime, by employing a crossover function that is significantly simpler. Further, we aim to provide an alternative perspective on understanding coalescence dynamics, which may have implications for a wide range of applications, such as droplet size control [3,21–24].

TABLE I. Properties of drop fluids.

Fluids	η [mPa s]	ρ [g/ml]	γ [mN/m]
Water	1	0.99	72
Silicone oil 5cst	5	0.92	20
Silicone oil 10cst	10	0.93	20
Silicone oil 20cst	20	0.95	20
Silicone oil 50cst	50	0.96	20
Silicone oil 100cst	100	0.96	20
Silicone oil 350cst	350	0.97	20
Silicone oil 500cst	500	0.97	20
Silicone oil 1000cst	1000	0.97	20

II. METHODS

We used deionized water and silicone oils as the drop liquid to investigate the drop-drop coalescence dynamics. Table I shows the physicochemical properties of fluids used in this work. The averaged viscosity was measured using a rotational rheometer (Anton Paar, MCR 301) with shear rate $1 - 100 \text{ s}^{-1}$. The density was measured using a density meter (Anton Paar, DMA 35). Surface tension was taken from the literature [15].

The experimental setup is shown in Fig. 1. Two millimeter-scale needles are aligned vertically. Each needle is connected to an independent syringe pump (New Era NE-1010) via microtubes. The needles are initially positioned several millimeters apart for generating drops. To mitigate undesired electrostatic effects due to surface charges on the drops, an electrostatic ion gun is used to discharge the drop surfaces before triggering coalescence. The upper drop is then brought down while the bottom drop remains stationary. Once they are close enough, the upper drop is enlarged slowly by pumping the fluid at a flow rate of $100 \mu\text{l/h}$, which corresponds to an approach velocity of approximately $20 \mu\text{m/s}$. We have checked that the flow rate has no significant influence on the coalescence dynamics.

A high-speed camera (Phantom TMX7510) equipped with a microscopic objective ($20\times$, $\text{WD} = 20 \text{ mm}$, Edmund Optics) is well aligned with the drop pair and a collimated light source to capture the coalescence dynamics at 230 263 frames per second. The spatial resolution in imaging is $1.09 \mu\text{m/pix}$. Fast image recording starts once the drops are close enough, and continues until the coalescence process completes. After the drop merging, a stable meniscus forms between the two needles (images in Fig. 1). The growth of the liquid bridge with time is then obtained using a home-made Matlab algorithm. The contours of the drops in coalescence are obtained using an auto threshold method. Due to the optical opacity at the bridge [15,18], it is difficult to visually know the initial contact for coalescence. To determine the contact moment t_0 , we apply an extrapolation algorithm by fitting the experimental data using a power law $R = \beta(t - t_0)^\alpha$, where R is the liquid bridge radius, α and β are fitting parameters (see Fig. 2). A threshold is first applied to the raw experimental data [Fig. 2(a)] to determine the starting point of the fit. The intersection between the fitted curve and the threshold is considered as the initial contact time t_0 [Fig. 2(b)]. The plateau in the early stage in our experimental data is cut off in all analyses. The fitting parameters α and β are shown in Figs. 2(c), 2(d). For different fluid viscosities, the exponents α range between 0.5 and 1, covering the transition of viscous to inertial regimes in coalescence. For the drops ($\eta = 1000 \text{ mPa s}$), the exponent α is around 0.8 which is slightly smaller than the expected value of 1 in the viscous regime. We speculate that this may be because our simple fitting approach here cannot accurately describe the coalescence dynamics of very highly viscous fluids. Instead, the logarithmic correction might be considered in the fitting. Indeed, later we show that, the scaling with logarithmic correction appears to better describe the dynamics of the 1000 mPa s fluid in the early stage than the linear one. In the following discussion, we define time as $\tau = t - t_0$.

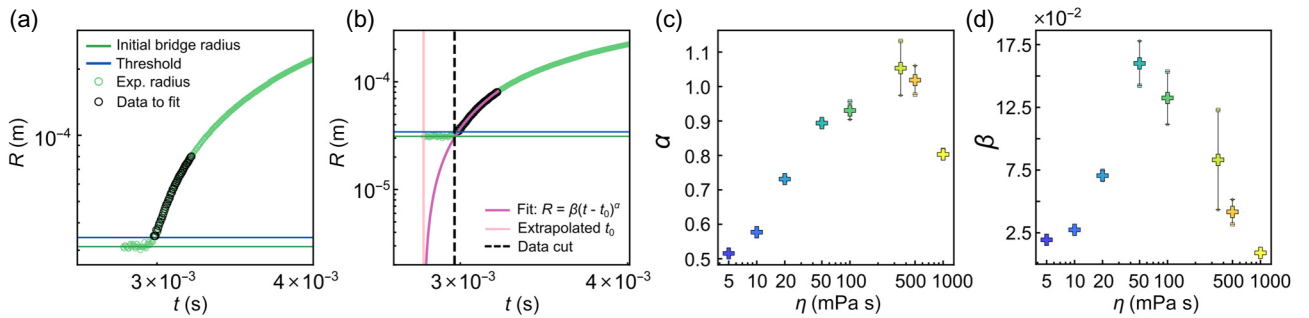


FIG. 2. Determination of initial contact in drop coalescence. (a) A threshold applied to the raw experimental data to determine the data range used for fitting. (b) Fitting and extrapolation of the experimental data. (c),(d) The exponents and prefactors from the power-law fitting for fluids with different viscosities.

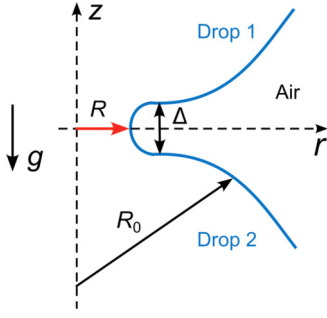


FIG. 3. Schematic representation of the liquid bridge between two drops during coalescence. R denotes the radius of the liquid bridge, R_0 the droplet's initial radius as shown in Fig. 1, and Δ the height of the bridge. During coalescence, the liquid bridge expands along the r direction. The z axis represents the axis of coalescing drops, aligning with the direction of gravity. Note that only half of the drop-and-bridge structure is shown here.

III. INERTIAL, INTERMEDIATE, AND VISCOUS REGIMES

The coalescence process, initiated when two droplets come into contact, is characterized by the formation and growth of a liquid bridge between the drops with a minimum radius R and height scale Δ (see the schematic in Fig. 3). The bridge radius R expands along the r axis driven by the surface tension. Depending on the relative dominance of inertial and viscous forces over the capillary forces, the size and shape of the bridge evolve differently, as exemplified in Figs. 4(a)–4(c). For low viscosities [e.g., water, 1 mPa s, Fig. 4(a)], the coalescence proceeds very rapidly, exhibiting a U -shaped contour (half of the liquid bridge). In this case, the separation of the meniscus $\Delta \sim R^2/R_0$. By balancing the capillary force with the inertial force, $\gamma/\Delta \sim \rho u^2$ (where $u \sim R/\tau$), one finds the nondimensional scaling for the bridge growth in the inertial regime,

$$\frac{R}{R_0} = C_0 \left(\frac{\tau}{\tau_i} \right)^{1/2}, \quad (1)$$

where $\tau_i = \sqrt{\rho R_0^3/\gamma}$ is the inertial time, and C_0 is a prefactor.

For higher fluid viscosities (e.g., silicone oil, 100 mPa s), the liquid bridge grows more slowly and exhibits a V -shaped contour [Fig. 4(c)]. In this case, the coalescence dynamics can take place over a wide range of scales (between R^3/R_0 and R) [14,16], leading to a more complex asymptotic solution that balances the capillary force with the viscous force. The nondimensional scaling for this viscous regime is expressed,

$$\frac{R}{R_0} = D_0 \frac{\tau}{\tau_v}, \quad (2)$$

with a prefactor D_0 and viscous time τ_v . The full theoretical prediction for three-dimensional drop coalescence suggests $D_0 \approx -1/\pi \ln(\tau/\tau_v)$ with a leading-order logarithmic term [14].

Equations (1) and (2) have been previously confirmed by various experimental and simulation works [13,15,18,25–31]. However, none of these scalings can describe the intermediate regime where the capillary, inertial, and viscous forces are all comparable during coalescence.

We study the effects of viscous and inertial forces on these power laws by systematically varying the fluid viscosities from 1 mPa s ($Oh = 0.0037$) to 500 mPa s ($Oh = 3.7$) [Fig. 5(a)]. This range allows us to explore the full coalescence dynamics as the Re number transitions from values smaller than unity to those exceeding it [Fig. 5(b)]. The Oh varies over 3 orders of magnitude ($Oh = 0.0037 - 3.7$). Figure 5(a) shows the evolution of the liquid bridge as a function of the fluid's viscosity, revealing a crossover between viscous ($R/R_0 \sim \tau$) and inertial ($R/R_0 \sim \tau^{1/2}$) coalescence regimes.

To further elucidate the distinct coalescence regimes, we compare our experimental results with the scaling laws in Eqs. (1) and (2), as shown in Fig. 6. In the inertial regime, typically observed for low-viscosity fluids or in the very late stages of coalescence, the bridge size follows the inertial scaling law [Eq. (1)], yielding a prefactor $C_0 = 1.4 \pm 0.2$. This value agrees well with the prefactors

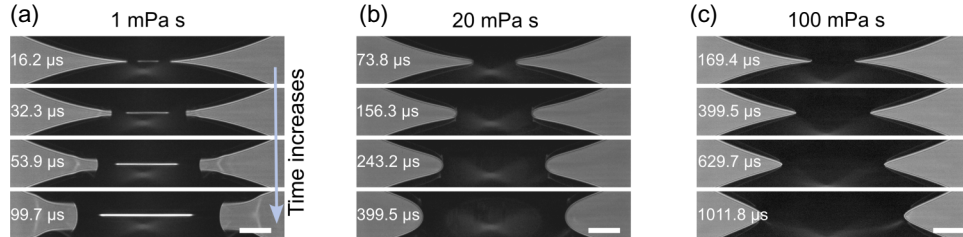


FIG. 4. Close-up image sequences of coalescing drops with viscosities of (a) 1 mPa s (water), (b) 20 mPa s, and (c) 100 mPa s, respectively. With increasing fluid viscosity, the coalescing drop shape changes from a U shape to V shape in the later stages. The dynamics is much slower for viscous drops than for water drops in achieving the same bridge size. The black regions represent the liquid drop and bridge areas. Scale bar: 100 μ m.

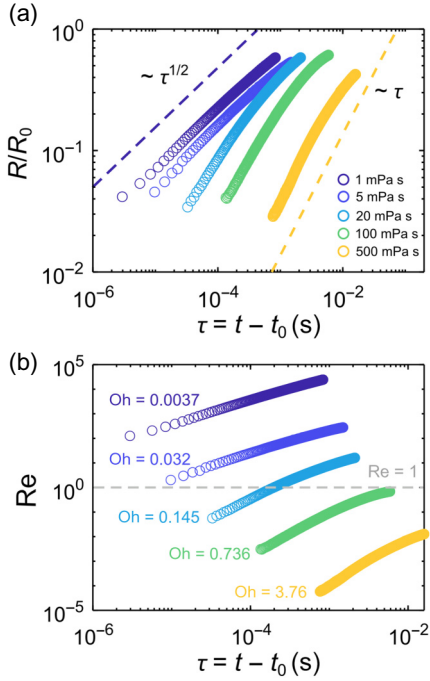


FIG. 5. The growth of the liquid bridge with time during coalescence. Normalized bridge size (a) and local Reynolds number (b) as a function time $\tau = t - t_0$ for different fluid viscosities (or Oh numbers). Here, t_0 is the moment of initial contact, which is determined using an extrapolation fitting with a power-law function (see Fig. 2 in Sec. II). In all experiments, the two coalescing drops have similar initial radius R_0 .

obtained by others [15,28,32], although it is lower than the value of 1.62 predicted numerically by Duchemin *et al.* [33] [Fig. 6(a)]. The prefactor of 1.4 is obtained by fitting the experimental data using Eq. (1) for R/R_0 in the range of 0.04–0.3. In the very late stage, the bridge size approaches the drop size, making comparison with the theory invalid. With the known prefactor, it allows precise prediction of the liquid bridge dimension at a given time during drop coalescence.

In contrast, in the viscous regime, prevalent in high-viscosity fluids or early stages of coalescence, our experimental data suggest linear growth with a prefactor $D_0 \approx 1$ in Eq. (2) rather than linear growth with a logarithmic correction [see Fig. 6(b)]. The discrepancy arises because the validity of the asymptotic theory with logarithmic correction [14] applies only to small bridge sizes, $R/R_0 < 0.03$, which is close to the bottom limit of the bridge radius in our experiments. The logarithmic correction may be present but overshadowed by azimuthal effects when $R/R_0 > 0.03$, and is therefore not observed in our or previous drop-drop coalescence experiments [15,18]. However, it has been borne out by recent numerical simulations [30,34]. Simulation results [30] show that the bridge growth of a Stokes fluid ($\text{Oh} \rightarrow \infty$) follows the theoretical prediction with the logarithmic correction [14] in the very early stage

(up to $\tau/\tau_v \approx 10^{-2}$ and $R/R_0 \approx 0.02$), but deviates from linear scaling when the logarithmic correction is omitted. However, in the later stage ($\tau/\tau_v > 10^{-2}$), there is no significant difference between the Stokes simulation and the linear scaling, which is the time range we probed in our experiments [Fig. 6(b)].

Of particular interest is the intermediate regime where we observe power-law growth of the bridge size with exponents between 1/2 and 1, bridging the gap between the inertial and viscous limits [Figs. 6(c), 6(d)]. The experimental results show that the intermediate regime starts at $\eta \leq 50$ mPa s ($\text{Oh} \leq 0.36$) and ends at $\eta \geq 10$ mPa s ($\text{Oh} \geq 0.07$). Clearly, for the fluid with a viscosity of 20 mPa s ($\text{Oh} = 0.14$), the bridge growth follows neither the inertial nor the viscous scaling laws. In this case, Re crosses unity throughout the entire coalescence process [Fig. 5(b)], further confirming the transition regime.

IV. CROSSOVER DYNAMICS

We now introduce a simple scaling approach to describe the bridge growth in coalescence across the different regimes. In our analysis, we employ the crossover rescaling method, which was proposed by Eggers *et al.* [35] for drop impact. Later, this approach was extended to describe the crossover between inertial-capillary and inertial-viscous drop impact regimes for both Newtonian and non-Newtonian fluids [36,37]. We follow this approach [36] by introducing a function $f(P)$ that serves to distinguish between the viscous and inertial regimes. Therefore, the nondimensional bridge growth should follow $R/R_0 = f(P)\tau/\tau_v$ for both regimes. The smooth crossover between these two regimes is then constructed by using a Padé approximant [38], $f(P) = \sqrt{P}/(A + \sqrt{P})$, where A is a constant. Then, the crossover scaling in drop coalescence can be expressed as,

$$\frac{R}{R_0} = \frac{\sqrt{P}}{A + \sqrt{P}} \frac{\tau}{\tau_v}, \quad (3)$$

with only one unknown dimensionless parameter P .

Now, let us consider the two limits—viscous and inertial—in drop coalescence and examine whether Eq. (3) can describe both regimes by using a proper function P and constant A . If $A \ll \sqrt{P}$, then $f(P) \approx 1$, and Eq. (3) simplifies to be $R/R_0 \approx \tau/\tau_v$. This matches the viscous scaling in Eq. (2) with a prefactor $D_0 = 1$, indicating that the crossover scaling is able to recover the viscous regime. In another limit, if $A \gg \sqrt{P}$, Eq. (3) becomes

$$\frac{R}{R_0} \approx \frac{\sqrt{P}}{A} \frac{\tau}{\tau_v}, \quad (4)$$

which is expected to recover the inertial regime. To obtain the same expression as in Eq. (1), we first need to determine the expression for $\sqrt{P}$. Since Eq. (1) contains the

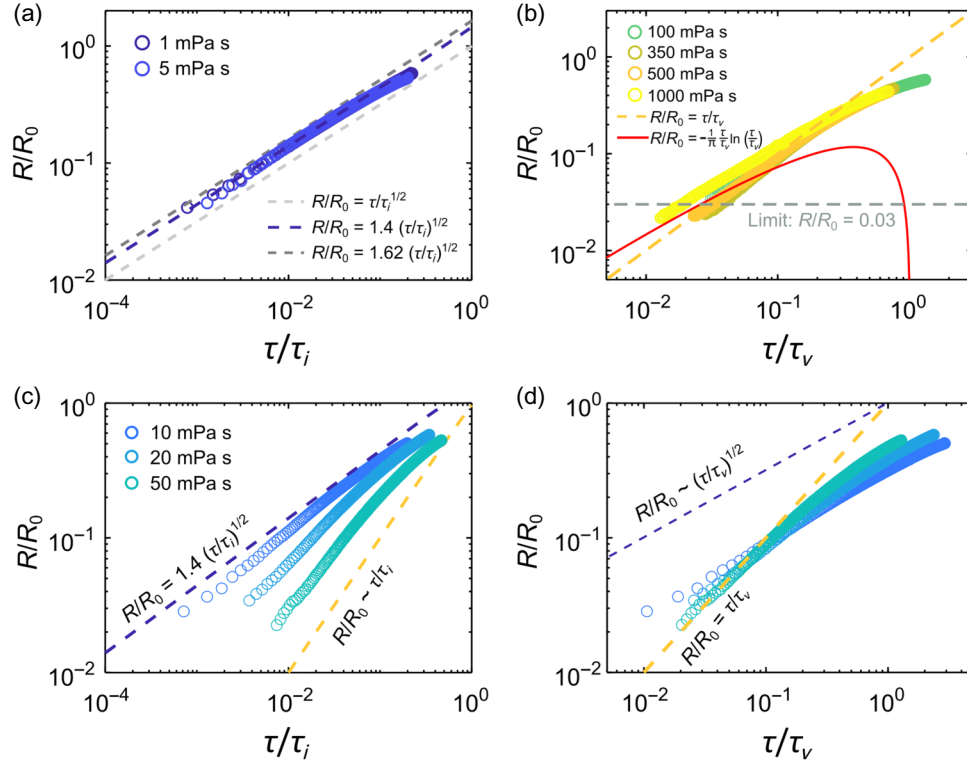


FIG. 6. Evolution of the liquid bridge radius during drop coalescence in different regimes. (a) Inertial regime. The bridge size is described well by the inertial scaling law [Eq. (1)]. Different prefactors are shown for comparison. The prefactor 1.4 is obtained by fitting the experimental data using the inertial scaling, while the prefactor 1.62 is the numerical result obtained by Duchemin *et al.* [33]. (b) Viscous regime. The liquid bridge size in experiment follows well the linear prediction [Eq. (2)] but deviates from the logarithmic scaling law with increasing dimensionless time. The highest-viscosity fluid (1000 mPa s) exhibits dynamics that more closely follow the viscous scaling with logarithmic correction in the very early stage of coalescence. The horizontal dashed line indicates the limit of validity for the theory with logarithmic correction. (c),(d) Intermediate regime. The bridge size grows with power laws between approximately t and approximately $t^{1/2}$ for the intermediate viscosities.

term τ/τ_i while Eq. (4) contains the term τ/τ_v , the role of $\sqrt{P}$ should be to eliminate τ/τ_v in Eq. (4) and introduce τ/τ_i instead. Therefore, we find the parameter $\sqrt{P}$ should satisfy $\sqrt{P} = \tau_v/\sqrt{\tau_i \tau}$. Note that the term $\sqrt{P}/(A + \sqrt{P})$ is not a constant prefactor but rather a part of the crossover function as it involves time. In this limit, Eq. (4) yields $R/R_0 \approx 1/A(\tau/\tau_i)^{1/2}$. Comparing this to Eq. (1), the prefactor is required to be $C_0 = 1/A$. From our experiments for inertial fluids, we can know $A \approx 0.71$, and we set this constant for all fluids. Although the constant A is determined only using the inertial scaling, it works well for other fluid viscosities [shown in Fig. 7(a)]. This is mainly because the constant A is eliminated in the limit of the viscous regime in our crossover scaling. We observe that the values of $\sqrt{P}$ at various times are much larger than $A = 0.71$ in the viscous regime, while they are much smaller than A in the inertial regime, consistent with our assumption above. In the intermediate regime, $\sqrt{P}$ crosses the value of A , clearly indicating the crossover dynamics in drop coalescence.

Next, we compare our crossover scaling to our own experimental data and other data from the literature in

Fig. 7(a). It shows that our crossover scaling agrees well with our experimental data for different fluids with viscosities ranging from 1 to 1000 mPa s, particularly in the late stage of coalescence. Coalescence data for alcohol drops [18] and water-glycerol drops [13] from the literature are also compared and show consistency with our scaling. We also plot the recent crossover theory proposed by Xia *et al.* [20] on top of our data in Fig. 7(a), showing that our simple scaling can largely capture their theoretical prediction. However, the deviation between our crossover scaling and Xia's theory becomes pronounced as the liquid bridge approaches the initial drop size (late stage), where our scaling aligns more closely with the experimental data. This is probably because the assumption of a small bridge size for geometric self-similarity in their theory does not accurately capture the large bridge in the late stage of coalescence. We further show the comparison between the recent simulation results by Anthony *et al.* [30] and our crossover scaling in Fig. 7(a). The simulation result for $Oh = 0.001$ shows a better consistency with our crossover scaling than $Oh = 20$ over a wide range of time scales. The

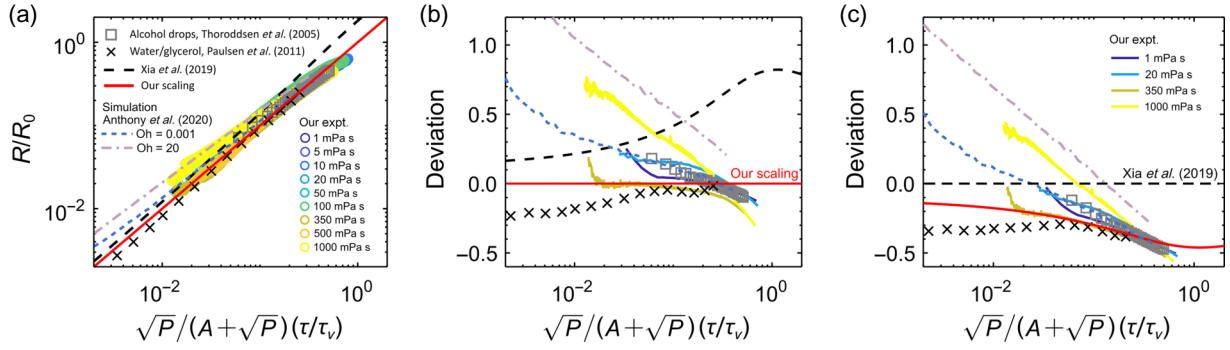


FIG. 7. Crossover in drop-drop coalescence. (a) Our experimental data, along with experimental [13,18], theoretical [20], and numerical [30] results from the literature, are compared with our crossover scaling law without the logarithmic corrections [Eq. (3)]. All of these data sets collapse well with our simple prediction. The viscosity for water-glycerol mixture is 11 mPa s, while the viscosity and surface tension of the alcohol drop are 1.2 mPa s and 27 mN/m, respectively. Two Oh numbers from the simulations of Anthony *et al.* [30] are plotted for comparison, representing inertial ($Oh = 0.001$) and viscous ($Oh = 20$) fluids. (b) The deviations of experimental, theoretical, and numerical data sets from our crossover scaling. The zero level represents our crossover scaling used as the baseline. (c) The same data used in (b), but plotted with respect to Xia *et al.* as the baseline (horizontal dashed line). The legend is the same as in (a). In (b),(c), our experimental data for inertial, intermediate, and viscous fluids are compared.

result for $Oh = 20$ (viscous regime) significantly deviates from our crossover scaling when $R/R_0 < 0.03$, where the logarithmic corrections are believed to exist [14].

To quantitatively illustrate the differences between our crossover scaling, our experimental data, and other data sets in the literature, we plot the deviations as a function of the dimensionless time scale in Figs. 7(b) and 7(c). The deviation is defined as $(\Sigma_i - \Sigma_i^s)/\Sigma_i^s$, where Σ_i represents the dimensionless bridge radius R/R_0 for each data set shown in Fig. 7(a), and Σ_i^s is the corresponding bridge size predicted by either our crossover scaling or Xia's theory. The zero levels in the deviation plots in Figs. 7(b) and 7(c) represent our crossover scaling and Xia's theory as the respective baselines. For simplicity, we present four fluids with different viscosities to compare deviations across the inertial, intermediate, and viscous regimes of the drop coalescence. The deviations of our experimental data from our crossover-scaling prediction remain within $\pm 20\%$, except for the 1000 mPa s fluid. The coalescence of alcohol drops and water-glycerol drops reported in the literature [13,18] is consistent with our experimental observations and falls within the same range of deviation [Fig. 7(b)]. In contrast, our experiments, along with the alcohol and water-glycerol drop coalescence data, show deviations ranging from -50% to 0 when using Xia's theory [20] as the baseline [Fig. 7(c)]. These comparisons demonstrate that our crossover scaling captures the experimental results more accurately, particularly in the late stage of coalescence. Nevertheless, for the highest tested viscosity of 1000 mPa s, both our crossover scaling and Xia's predictions exhibit large deviations in the early stages of drop-coalescence dynamics.

In Figs. 7(b) and 7(c), the simulation result of Anthony *et al.* [30] for $Oh = 0.001$ (inertial fluid) exhibits a trend

similar to our experimental data and the alcohol data. It shows an earlier coalescence dynamic, but with increasing deviations from both our crossover scaling and Xia's theory as the dimensionless time scale decreases. For the viscous fluid ($Oh = 20$), the simulation shows a significant discrepancy with both scaling predictions; it, however, exhibits a similar trend to our experiment with 1000 mPa s. The theoretical prediction of Xia *et al.* [20] exhibits a deviation of around 20% from our crossover scaling during the early coalescence; however, the deviation gradually increases to over 50% toward the end of the coalescence process.

A set of experimental data and the simulation results of Anthony *et al.* have illustrated that the drop-drop coalescence dynamics generally converge toward our crossover scaling as the dimensionless time increases [Fig. 7(b)]. However, in the early stages of coalescence, particularly for the highly viscous fluids (i.e., $\eta = 1000$ mPa s and $Oh = 20$), it exhibits significant deviations from our scaling. These discrepancies suggest that it will be useful to incorporate logarithmic corrections into our crossover scaling to achieve a more accurate description of the early-time dynamics.

V. DISCUSSION

The crossover scaling in Eq. (3) is derived asymptotically based on the linear growth of the bridge with a prefactor of $D_0 \approx 1$. This scaling works very well when the bridge radius exceeds $0.03 R_0$ for all viscosities, as shown in Fig. 7(a). However, it cannot precisely describe the crossover behavior in the early stages where the logarithmic corrections are present, as it exhibits significant

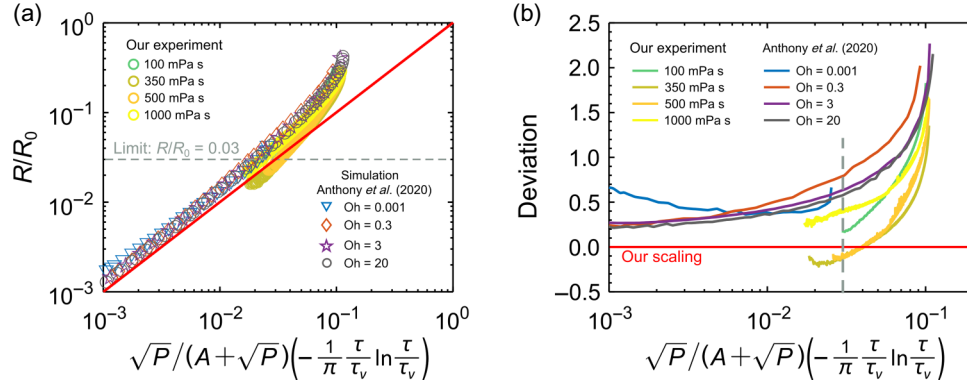


FIG. 8. Crossover scaling with logarithmic corrections. (a) Comparison between our crossover scaling with logarithmic corrections and simulation results. The simulation results by Anthony *et al.* [30] show a better alignment with our crossover by considering the logarithmic corrections in the early stage of coalescence. Over the limit ($R/R_0 \approx 0.03$), both the simulations and our experiments clearly diverge from our prediction (red line). Although our observations for the liquid bridge radius ($R/R_0 < 0.03$) are not well applicable, our experimental data exhibit the same trend as the simulation results. (b) The deviations of our experimental data and Anthony's simulation results from our logarithm-corrected crossover scaling (red horizontal line). The vertical dashed line indicates the dimensionless time corresponding to the bridge size of $R/R_0 = 0.03$ in (a).

deviations [shown in Fig. 7(b)]. To incorporate the logarithmic term, we take the prefactor of $D_0 \approx -\frac{1}{\pi} \ln \frac{\tau}{\tau_v}$ in Eq. (2). Using the viscous and inertial limit approximation, we thus modify the crossover parameter, $P = \left(-\frac{\tau}{\pi \tau_v} \ln \frac{\tau}{\tau_v} \right)^{-2} \left(\frac{\tau}{\tau_i} \right)$. Although our experimental data cannot clearly capture the early-stage behaviors due to the limitation in experimental observation, careful simulations enable us to verify our crossover scaling. In the numerical simulation work by Anthony *et al.* [30], drop-coalescence behaviors with different fluid viscosities (or Oh numbers), starting from the very early stages, are systematically discussed. Here, we replot their data along with our experimental data using the crossover scaling with logarithmic consideration in Fig. 8(a).

The simulation data extend our experimental observations of bridge size by an order of magnitude down, complementing the early-time coalescence dynamics that are difficult to capture experimentally. In the overlapping range, our experimental data exhibit the same trend as the simulation results. In the range of $R/R_0 < 0.03$ where the logarithmic corrections exist, the drop-drop coalescence dynamics for Oh values ranging from 0.001 to 20 agree well with our logarithm-corrected crossover scaling. However, above the limit ($R/R_0 = 0.03$), both Anthony's simulation and our experimental results show a gradual deviation from our crossover scaling. This is because the logarithmic correction fails, as shown in Fig. 6(b).

Figure 8(b) illustrates the deviations of our experimental and Anthony's simulation results from our logarithm-corrected crossover scaling. Unlike the deviations observed without the logarithmic corrections [shown in Fig. 7(b)], the deviations with the logarithm-corrected

crossover generally exhibit a gradual increase with increasing dimensionless time [Fig. 8(b)] except for the case of $Oh = 0.001$. The simulated drops with $Oh = 0.001$ exhibit a deviation of approximately +50%, which is slightly larger than that observed for other Oh values. This indicates that the logarithmic corrections in our crossover scaling are more applicable to viscous fluids. In contrast, the drops with $Oh = 0.3 - 20$ exhibit a deviation of approximately +20%, which gradually increases as the bridge radius expands, reaching a maximum deviation of around 50% at the limit of $R/R_0 = 0.03$ [the corresponding time is shown as the vertical dashed line in Fig. 8(b)]. Although limited data are available from our experimental observations before reaching the limit, the deviations roughly lie within $\pm 20\%$.

Comparing the results shown in Figs. 7(b) and 8(b), we find that the deviations from the logarithm-corrected crossover are much smaller during the early stages of coalescence than those without logarithmic corrections. However, in the later stages of coalescence, the crossover without logarithmic corrections provides a better description of the dynamics. Our findings thus demonstrate the necessity of incorporating logarithmic corrections into the crossover scaling in the early times of drop-drop coalescence ($R/R_0 < 0.03$).

In summary, our simple crossover function is versatile in describing drop-drop coalescence dynamics in both the very early and late stages. It can also be extended to account for logarithmic corrections in the viscous regime. This comprehensive study of drop-drop coalescence not only provides valuable experimental data across a wide range of fluid viscosities but also contributes to the theoretical framework for

understanding this fundamental fluid-dynamics process. The observed transitions between inertial and viscous regimes, along with the identification of intermediate behaviors, offer insights into the complex interplay of forces governing liquid-bridge formation and growth during coalescence.

Our crossover scaling potentially allows for a more direct and simple prediction for drop-coalescence behaviors and could be applicable in various liquid systems in applications. For example, since most chemicals used in industrial applications are liquids and gases, the technical relevance of coalescence in chemical and process engineering becomes evident [39]. A very common system in processing involves at least two phases with a miscibility gap, dispersed within one another through energy input—for example, to increase the interfacial area and enhance mass- and heat-transfer processes. This type of dispersed system is integral to many technical applications, including extraction, mixing, polymerization, emulsions, and multiphase reactors [40–43]. With the known physicochemical properties of the system fluids, our crossover scaling enables the measurement of the coalescence time scale and the prediction of the coalescence rate of the dispersed droplets, thereby facilitating droplet size control. For example, considering an oil phase dispersed in a continuous water-based phase, the oil drops are prone to coalesce in the absence of a surfactant. We thus compare experimental measurements from Yao *et al.* [44] with predictions from our crossover function and the theory proposed by Xia *et al.* [20]. The silicone oil droplets have an initial radius of 5 μm , a viscosity of 100 Pa s, and a density of 970 kg/m³. The interfacial tension between the oil and water phases is 9 mN/m. Since the viscosity of the droplet phase is much larger than that of the outer phase, the coalescence dynamics are dominated by the droplet properties. The inertial and viscous time scales are $\tau_i = 0.12$ s and $\tau_v = 53.9$ s, respectively, yielding an Ohnesorge number of $\text{Oh} = 464.3$. With these parameters, our crossover function [Eq. (3)] predicts that the time for the liquid bridge to reach $0.5R_0$ is 26 s, while Xia's theory predicts approximately 24 s. Both predictions are consistent with the experimental measurement of 20 s. However, our approach is simpler. For more complex fluid systems, such as non-Newtonian fluids [45,46], the crossover in coalescence remains relatively unexplored. For instance, it would be interesting to investigate how normal stresses in these fluids influence the crossover regime and whether the crossover scaling remains valid.

VI. CONCLUSION

In summary, we have shown that a Padé approximation can be used to obtain a crossover function that successfully captures the essential features of the viscous-to-inertial

transition in drop-drop coalescence. We compare our simple crossover function with our own experiments across a range of liquid viscosities as well as with experimental and numerical results from the literature, revealing excellent correspondence. We also compare our results with recent benchmark theoretical work on drop coalescence and demonstrate how our approach complements and extends the current understanding of the crossover regime in drop coalescence. Moreover, our crossover explicitly incorporates the dependence on the Oh number, while eliminating the need to determine the characteristic time and length scales at crossover. It also addresses and enables the description of dynamic behaviors with logarithmic corrections in the early stage of coalescence, which are not captured by current theoretical predictions.

Quantifying the dynamics of the crossover between viscous and inertial regimes in drop coalescence is not only of fundamental scientific interest but also has practical implications for a wide range of applications. This work may help shed light on such applications, including technologies to precisely control droplet size in microfluidics, aerosol generation, and emulsion.

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DATA AVAILABILITY

The data that support the findings of this article are openly available [47].

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