

Generation and amplification of microwave signals via planar waveguides with embedded paramagnetic color centers

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A concept for microwave (MW) amplification and maser generation in compact planar waveguides embedding a thin layer of paramagnetic color centers is presented. The ground state of the color centers can be optically or electrically spin-polarized, while population inversion is attained by Zeeman shifting of one spin sublevel below the populated state. A signal traveling along the waveguide, if resonant with the spin transition, can be amplified, or a self-sustained MW oscillation can be generated. A general analysis of the conditions for amplification and self-oscillation is developed for nitrogen-vacancy-rich diamond films, together with practical examples of impedance matching and resonator design. The proposed architecture combines near-unity filling factors with relatively low electromagnetic quality factors, enabling operation in strongly coupled configurations and broad frequency tunability through the external magnetic field. The same formalism naturally extends to compact lumped-element resonators operating at a few gigahertz while preserving millimeter-scale dimensions. Compared with the present state of the art of diamond masers, the approach offers reduced size, simplified integration with external electronics, and relaxed requirements on cavity quality factor.

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I. INTRODUCTION

Maser amplification has long represented a key technology in applications requiring low-noise amplification of extremely faint microwave (MW) signals: for instance, in space communication [1], radio astronomy [2], and radar systems. In all these cases, the maser's employment required bulky resonant cavities and involved cryogenics, due to the necessity of maintaining inversion populations in quantum systems whose energy difference spans in the μeV range. This circumstance limited the employment of maser technology due to the complexity of the system and motivated research on low-noise semiconductor amplifiers.

Nowadays, amplification techniques directly operating on electric signals at room temperature—e.g., low-noise

amplifiers (LNAs) based on pseudomorphic high-electron-mobility transistors and parametric amplifiers—have become widely used [3]. However, Josephson and kinetic-inductance parametric amplifiers reach the quantum limit only when cryogenically cooled [4,5], while solid-state LNAs at room temperature [6–8] typically exhibit noise figures in the order of approximately 0.5–2 dB (equivalent to tens-hundreds of Kelvin). For these reasons, and considering that room-temperature maser devices based on solid-state technology could mitigate the requirements in terms of bulk and services, since 2012 a concept for a solid-state maser using an organic molecular crystal has been proposed [9].

More recently, the use of negatively charged nitrogen-vacancy (NV^-) centers in diamond as a gain medium for solid-state masers has been studied, operating from room temperature to cryogenic temperatures [10–14]. The spin polarization of the system is ensured by optical excitation via intersystem crossing (ISC), and the population inversion is obtained by shifting one of the $m_s = \pm 1$ spin sublevels of the ground state below the $m_s = 0$ sublevel via application of a suitable magnetic field (>100 mT). A similar polarization mechanism is also offered by other paramagnetic defects, like silicon vacancies (VSi)

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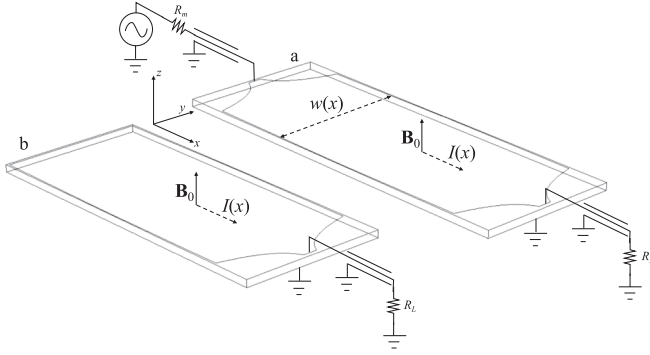


FIG. 1. Geometrical arrangement of two devices, each consisting of a thin rectangular diamond slab sandwiched between two metallic layers. Embodiment (a) represents an amplifier, in which the electric signal is magnified while traveling along the waveguide. Embodiment (b) is a generator, in which the signal is collected by a tapered section. In these embodiments, the phase electrode is deposited on a paramagnetic thin layer, while the ground electrode is uniformly deposited on the backside. The polarizing magnetic field $\mathbf{B}_0$ is perpendicular to the electrodes and the current flow and parallel to the $\hat{z}$ axis, while the current is parallel to the $\hat{x}$ axis.

in 6H- or 4H-SiC [15], also allowing continuous-wave maser generation due to the suitable dissipation characteristics. In the case of V3 defects in 4H-SiC, maser amplification or generation at about 30 MHz is also possible at zero magnetic field, since the preferentially populated sublevel, in this case, is the higher $m_s = \pm 3/2$ -state. This kind of device, though operating at relatively high temperatures, has maintained the necessity of a quite bulky and complex high-quality resonant cavity, along with waveguides for input and output signals.

In this article, we develop a concept based on a resonant planar waveguide encompassing a paramagnetic thin layer. The waveguide works as a resonant cavity for the oscillation of the magnetic moment of the paramagnetic centers and, simultaneously, extracts a fraction of the energy as an electric signal. Alternatively, an electric signal is injected into the line, and it is amplified during propagation in the waveguide to be extracted from the second end of the line. The taper of the waveguide is designed in a way not only to resonate at the proper frequency but also to assure the impedance matching with the external lines.

Figures 1(a) and 1(b) represent the geometric arrangement of two devices that work as an MW amplifier and an MW generator, respectively. From their inspection, the advantages of using a compact, planar device that operates directly with electric signals should be apparent, as it could pave the way for the implementation of the maser concept in full on-chip architectures. Compared with the current state of the art in solid-state diamond masers, the proposed arrangement provides much higher filling factors (approaching unity, versus typically below 0.1) and can therefore operate with substantially lower electromagnetic

quality factors (below 10^3). This relaxes the usual requirement for thick low-loss metallization and opens the possibility of employing partially transparent thin metal layers. Besides providing optical access for pumping, such layers may be incorporated into planar Fabry-Pérot-like structures that enhance the absorption of the pump radiation [16]. In this architecture, a reduction of the electromagnetic quality factor may therefore be partially or even fully compensated by an increase in the achievable inversion density. The radiative and, especially, resistive losses of the device are not negligible at room temperature. Nevertheless, the near-unity filling factor allows the maser effect to remain viable with NV^- concentrations of only several parts per million and spin-coherence times (T_2) of the order of microseconds or less.

Despite the implementation of the concept described here requiring only standard thin-film fabrication techniques, critical aspects emerge regarding the shaping and doping of suitable diamond samples. Preparation of millimeter-sized single-crystal samples, at a thickness level of tens of microns, requires careful attention and has high failure rates, while NV-doping density at the tens of ppm is still an open research subject (see Ref. [17] and related references). For these reasons, our theoretical study can be useful for the dimensioning of devices that are challenging the state of the art of diamond films.

In the following, in Sec. II, we first write the equations for the evolution of the electric current, for the magnetic moment of the material, and for the inversion population density. In Sec. III, we calculate the conditions for the amplification of a signal along a waveguide of any given shape, and we develop an example with convenient matching conditions with external electronics. In Sec. IV, the Maxwell-Bloch equations for the self-generation of an MW signal in a planar waveguide are developed, and two practical models are introduced, the first based on a nearly uniform transmission-line resonator, while the second adopts a lumped-element configuration. The latter enables operation at frequencies approximately one order of magnitude lower while preserving the compact dimensions of the device.

II. ELECTRIC CURRENT, MAGNETIC MOMENT, AND POPULATION INVERSION ALONG A PLANAR WAVEGUIDE

We consider a planar waveguide deposited on a thin diamond single-crystal, embedding an ensemble of NV^- centers, but this approach is valid for other paramagnetic color centers, such as V_{Si}^- or VV in silicon carbide [18,19]. If a polarizing magnetic induction field B_0 is applied, the $m_s = \pm 1$ sublevels of the ground state of each center will be shifted apart by $2\Delta E_Z$, with $\Delta E_Z = -2\mu_B B_0 \cos \theta$, where θ is the angle formed by the particular $\langle 111 \rangle$ symmetry axis of the NV^- center with the external magnetic field. Fixing the magnetic field intensity, the energy difference between the $m_s = 0$ sublevel and one of

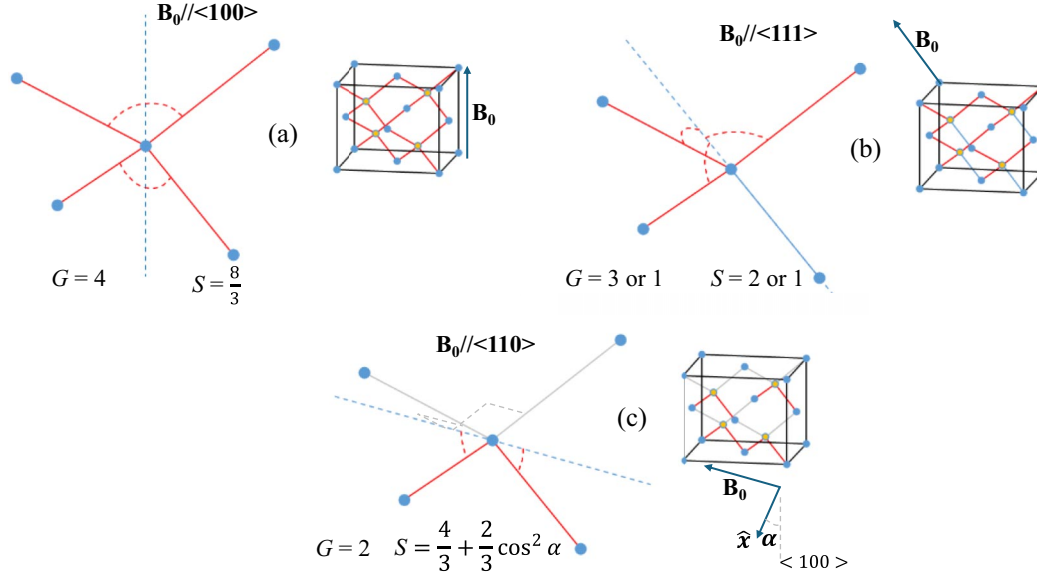


FIG. 2. Scheme of the possible orientations of the external field $\mathbf{B}_0$ with the corresponding number G of the NV^- orientations simultaneously tunable to a given frequency. The factor S in Eq. (17) measures the efficacy of the coupling between a waveguide perpendicular to $\mathbf{B}_0$ and a set of NV^- centers all forming the same angle with the field. The coupling directions simultaneously tunable to the same frequency are marked with the same color (red or blue, respectively).

the others (say $m_s = -1$) can be tuned to the waveguide resonant frequency. In this case, the color centers, previously spin-polarized (for example, through optical pumping via ISC), can relax to the lower $m_s = -1$ sublevel through stimulated emission.

There are two relevant conditions to consider when coupling the magnetic moment of the NV^- centers with the current flowing in the waveguide: the first is the orientation of the external magnetic field, which can be chosen to tune from one to four quarters of the available NV^- centers to the waveguide frequency. Particularly, the number of the NV^- orientations tunable to the same frequency can be $G = 4, 3, 2$, or 1 if the magnetic field is parallel to one of the $\langle 100 \rangle$, $\langle 111 \rangle$, and $\langle 110 \rangle$ directions, or none of them, respectively (see panels (a), (b), or (c) in Fig. 2). The second relevant direction is the orientation of the waveguide itself, whose magnetic coupling with NV^- centers depends on the projection of the NV^- axis onto the plane normal to the waveguide and containing the direction of propagation of the signal along the waveguide.

In the following, we shall assume an external magnetic field $\mathbf{B}_0 = B_0 \hat{\mathbf{z}}$ directed along a line perpendicular to the diamond layer, while the crystal lattice could have several orientations with respect to the crystal surfaces, given that in commercial samples the most common are the $\langle 100 \rangle$ (by far), the $\langle 111 \rangle$ and the $\langle 110 \rangle$.

In this work, the evolution of the electric current along the propagation direction $\hat{\mathbf{x}}$ of a planar waveguide is studied, considering it as a one-dimensional distributed-constant circuit. In this way, we deliberately neglect the radiative losses due to boundary effects and the nonuniformity along the transversal direction $\hat{\mathbf{y}}$ emerging at high frequencies.

Nevertheless, we verified in a separate study [20] by 3D Comsol[®] simulations that, for devices whose size and shape resemble those employed in this work, the one-dimensional approach is fairly accurate up to the tens of GHz relevant for this study. Moreover, it provides an intuitive tool for the design phase of this kind of device, useful for a fast exploration in the configurations space of the system.

We will consider (referring to Fig. 1) a current flowing in the waveguide whose intensity $I(x, t)$ varies in time and along the $\hat{\mathbf{x}}$ direction. The width of the waveguide will be given by $w(x)$, and it will be made by two faced metal strips of conductivity σ and thicknesses th_{ground} and th_{phase} , respectively. The two strips are deposited on a diamond layer of thickness d , they can be specularly reflected or, alternatively, one of them can be diffused on the entire back surface of the diamond layer, also if the field confinement is not as good as in the first case at low frequencies (much below resonance), as stated also in [20].

With these assumptions, a lossy telegrapher's-like equation [21] for a tapered transmission line coupled to the magnetized medium can be derived, emerging from the coupled differential equations for the current I and for the potential difference V between two faced points of the electrodes:

$$\frac{\partial V}{\partial x} = -\frac{1}{\sigma \cdot th \cdot w} I - \frac{\mu_0}{w} \frac{\partial I}{\partial t} - \mu_0 \frac{\partial M_y}{\partial t}, \quad (1a)$$

$$\frac{\partial V}{\partial t} = -\frac{d}{\epsilon_0 \epsilon_r w} I. \quad (1b)$$

The coupling with the medium has been taken into account by calculating the electromotive force given by the time derivative of the magnetic flux, bound to the magnetization density according to $\mathbf{B} = \mu_0(\mathbf{H} + \mathbf{M}) = \mu_0(\frac{I(x,t)}{w(x)}\hat{\mathbf{y}} + \mathbf{M}(x,t))$. By derivation of Eqs. (1a) and (1b), with respect to time and space, respectively, and by subtraction member to member, after some algebra, we find the resulting equation for the current along the line:

$$\frac{\partial^2 I}{\partial x^2} - \frac{1}{\Lambda} \frac{\partial I}{\partial x} - \frac{1}{v^2} \frac{\partial^2 I}{\partial t^2} - \frac{1}{D} \frac{\partial I}{\partial t} = \frac{w}{v^2} \frac{\partial^2 M_y}{\partial t^2}, \quad (2)$$

where v is the speed of an electromagnetic wave in the diamond layer, $D = (d \cdot th \cdot \sigma) / (\epsilon_0 \epsilon_r)$ is a diffusion constant, and $\Lambda(x) = \frac{w}{\partial w / \partial x}$. This equation differs from a conventional lossy telegrapher's equation (see Ref. [21]) in two respects. First, the space derivation of Eq. (1b) has produced a space-dependent term $\frac{1}{\Lambda} \frac{\partial I}{\partial x}$ taking properly into account the variation of the line width $w(x)$ along the line, with no assumptions about variation adiabaticity. Second, there is a forcing term $\frac{w}{v^2} \frac{\partial^2 M_y}{\partial t^2}$ taking into account the coupling with the magnetized medium.

The spatially dependent parameter $\Lambda(x)$ may be exploited not only for impedance matching and boundary-condition engineering but also as a design tool for tailoring the electromagnetic mode profile (see chapter 7 of Ref. [22]). In particular, periodic modulations of the waveguide width lead to telegrapher equations with periodic coefficients, for which Bloch-type solutions are expected, opening the way to slow-wave implementations. Alternatively, suitable variations of the cross section may be used to realize compact lumped-element resonators whose physical dimensions are substantially smaller than the wavelength.

The latter approach becomes particularly attractive at frequencies of a few gigahertz, where a conventional uniform resonator would be impractically large. In the present work, however, our primary interest is the investigation of near-field planar waveguides operating at frequencies of several tens of gigahertz, for which the cross section remains nearly uniform over most of the device. For this reason, slow-wave implementations are not analyzed in detail here. Nevertheless, in the final part of the paper, we present a first lumped-element slow-wave resonator, demonstrating how the operating frequency can be reduced by approximately one order of magnitude while maintaining millimeter-scale dimensions.

The amplification or the generation of an electrical signal along the waveguide, in Eq. (2), depends entirely on the oscillating magnetization of the material $\mathbf{M}$. To study this quantity, we will consider the evolution of the density matrix,

$$\rho^{(j)} = \begin{pmatrix} \rho_{gg}^{(j)} & \rho_{ge}^{(j)} \\ \rho_{eg}^{(j)} & \rho_{ee}^{(j)} \end{pmatrix},$$

of an ensemble of NV^- centers all having the symmetry direction (j) , among those (for $j = 1 \dots G$) tuned to a specific frequency ω_0 , according to: $\hbar\omega_0 = E_e - E_g$. Here, E_e is the energy of the $m_s = 0$ sublevel and E_g that of the $m_s = -1$ sublevel of the ground state of the NV^- center. For convenience, we fix $\text{Tr}(\rho^{(j)}) = N^{(j)}$, the volume density of the centers having the symmetry direction (j) . With this convention, $\Delta^{(j)} = \rho_{ee}^{(j)} - \rho_{gg}^{(j)}$ is the population inversion density related to the direction (j) , and the magnetization of the system is $\mathbf{M}^{(j)} = -2\mu_B \text{Tr}(\hat{\mathbf{S}}^{(j)} \rho^{(j)})$, where $\hat{\mathbf{S}}$ is the vector operator corresponding to the angular momentum, divided by $\hbar$, calculated between the spin states $|g\rangle$ and $|e\rangle$, and μ_B is the Bohr magneton.

In the following, we will adopt the rotating wave approximation (RWA), thinking of the off-diagonal terms $\rho_{eg}^{(j)}$ as rapidly rotating vectors in the complex plane with angular velocity ω , with only a slow drift of the phase and the intensity. In this approximation, we define $\mathcal{M}^{(j)}(t) = -2i\mu_B \sqrt{2} \rho_{eg}^{(j)} e^{i\omega t}$ and admit that $\dot{\mathcal{M}}^{(j)} \ll \omega \mathcal{M}^{(j)}$. With this convention, it turns out that, in a $x'y'z'$ reference frame with $\hat{\mathbf{x}}'$ parallel to the particular (j) direction of the NV^- axis, we have

$$\mathbf{M}^{(j)} = \begin{pmatrix} -\mu_B(\Delta^{(j)} - N^{(j)}) \\ \frac{1}{2}\mathcal{M}^{(j)}(t)e^{-i(\omega t + \pi/2)} + \text{c.c.} \\ \frac{1}{2}\mathcal{M}^{(j)}(t)e^{-i\omega t} + \text{c.c.} \end{pmatrix}. \quad (3)$$

Equation (3) is immediately interpreted as a nearly constant magnetization parallel to the symmetry axis of the NV^- center, which is maximum when $\Delta^{(j)} = -N^{(j)}$ (that is, when all the centers are in the ground state $m_s = -1$), and it is zero when they are all in the excited state $m_s = 0$; plus a component laying on the plane perpendicular to $\hat{\mathbf{x}}'$, only depending on the off-diagonal terms of the density matrix. In the RWA, we can think of the $y'z'$ component of $\mathbf{M}^{(j)}$ as a (rapidly) rotating vector whose modulus and phase change slowly compared with the period $2\pi/\omega$. The rotating wave approximation is verified *a posteriori*, determining the natural pulsation ω of the system and assessing the slow variation of $\mathcal{M}^{(j)}(t)$.

To use the magnetization in Eq. (2), we must express its y component (perpendicular to the direction of the propagation along the waveguide but parallel to the diamond surface) in terms of the $x'y'z'$ components of Eq. (3). The nearly constant term dependent on $\Delta^{(j)}$ will be neglected in the second time derivative with respect to the components dependent on $e^{i\omega t}$. Figure 3 shows that the x' axis is parallel to the direction of the (j) -th symmetry axis of one of the NV^- centers, which are tuned to the resonance by an

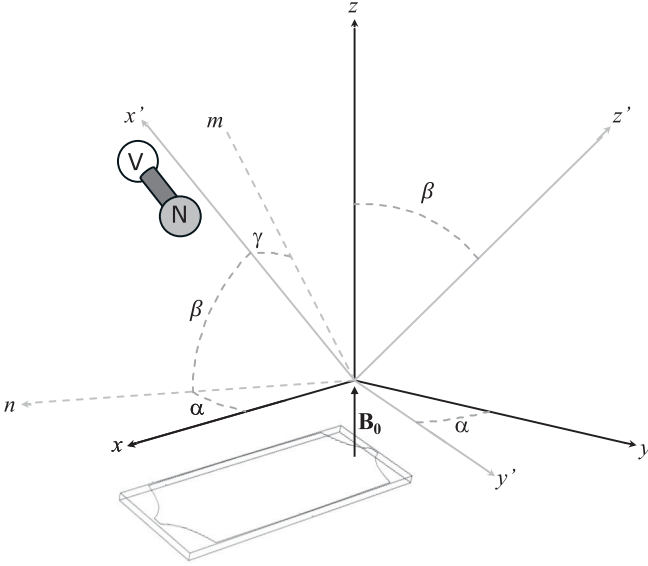


FIG. 3. The two reference systems relevant to the present discussion: xyz has the x axis parallel to the propagation direction along the waveguide and z perpendicular to the waveguide itself and parallel to the polarizing magnetic field $\mathbf{B}_0$. The x' axis is parallel to the axis of the NV^- center under consideration, and y' lies along the xy -plane. The $x'y'z'$ system can be obtained from the xyz by rotation of an angle α around the z axis (here positive in the clockwise direction) and another rotation of an angle β around the y' axis. The angle γ in the figure is formed by the x' axis with its projection m on the xz -plane.

external magnetic field, and that a convenient choice for the orientation of y' and z' is such that y' lies on the xy -plane, forming an angle $\alpha^{(j)}$ with the y axis, and z' forms an angle $\beta^{(j)}$ with the z axis. With these conventions, it turns out that the M_y component of the magnetization in Eq. (3), coming from the rapidly rotating terms, is given, summing up all the relevant directions of the NV^- centers, by:

$$M_y = \sum_{j=1}^G \left[-\frac{i}{2} \mathcal{M}^{(j)} \Gamma^{(j)} e^{-i\omega t} + \text{c.c.} \right], \quad (4)$$

where

$$\Gamma^{(j)} = \cos \alpha^{(j)} + i \sin \alpha^{(j)} \sin \beta^{(j)}.$$

The evolution in time of the magnetization $\mathcal{M}^{(j)}$ depends on the current flowing in the waveguide and on the population inversion density $\Delta^{(j)}$ according to the Schrödinger equation:

$$i\hbar \dot{\rho}^{(j)} = [H^{(j)}, \rho^{(j)}], \quad (5)$$

where

$$H^{(j)} = \begin{pmatrix} E_g & \sqrt{2} \mu_B B_+^{(j)} \\ \sqrt{2} \mu_B B_-^{(j)} & E_e \end{pmatrix},$$

with $B_{\pm}^{(j)} = B_{y'} \pm iB_{z'}$. Expanding Eq. (5), and adding two decay terms to account for the open nature of the system and for its interaction with the thermal bath and with the pumping process, the evolution equations for $\Delta^{(j)}$ and $\mathcal{M}^{(j)}$ read:

$$\begin{aligned} \dot{\Delta}^{(j)} = & -\frac{1}{\hbar} [(\mathcal{M}^{(j)})^* B_-^{(j)} e^{i\omega t} + \mathcal{M}^{(j)} (B_-^{(j)})^* e^{-i\omega t}] \\ & - \gamma_{\parallel} (\Delta^{(j)} - \Delta_0^{(j)}), \end{aligned} \quad (6a)$$

$$\dot{\mathcal{M}}^{(j)} = i(\omega - \omega_0) \mathcal{M}^{(j)} + \frac{4\mu_B^2}{\hbar} \Delta^{(j)} B_-^{(j)} e^{i\omega t} - \gamma_{\perp} \mathcal{M}^{(j)}. \quad (6b)$$

Here, the constant $\Delta_0^{(j)}$ depends on the pumping intensity and represents the equilibrium population inversion in the absence of coupling with the magnetic field. The recovery rate $\gamma_{\parallel}$, in absence of any population inversion mechanism, is determined by thermal processes and typically spans the interval from 1 to 10^3 Hz [23], but under effective optical illumination, well below saturation, it is given by $\gamma_{\parallel} = I_{\text{pump}} \sigma_{\text{abs}} p_{\text{flip}}$, where I_{pump} is the photon flux of the pumping radiation, σ_{abs} is the photon absorption cross section of the color centers (ranging from 3 to $10 \times 10^{-17} \text{ cm}^2$, depending on the environmental conditions [24,25]). The factor p_{flip} is the spin-flip probability per excitation event, typically evaluated between 0.05 and 0.4 depending on the sample type [26]. Saturation occurs only when $I_{\text{pump}} \sigma_{\text{abs}} p_{\text{flip}}$ approaches the slowest rate involved in the pumping process, which is of the order of several tens of MHz and can be reached only for unrealistically high photon fluxes. Finally, in Eq. (6b), the parameter $\gamma_{\perp}$ depends on the dephasing time affecting the off-diagonal terms of the density matrix.

Equations (6a) and (6b) are coupled to the current I through the magnetic field $B_-^{(j)}$. To make this dependence explicit, it is convenient to express the current flowing along the waveguide as:

$$I(x, t) = \frac{1}{2} \mathcal{I}(t) u(x) e^{-i\omega t} + \text{c.c.} \quad (7)$$

This representation decouples the spatial and temporal dependences of the current and expresses the time dependence in a form suitable for the rotating wave approximation, consistently with the magnetization dynamics.

With these assumptions, the potential difference between two facing points of the waveguide at position x is

$$V(x, t) = \frac{1}{2} \frac{\mathcal{I}(t)}{i\omega \frac{\varepsilon_0 \varepsilon_r w}{d}} \frac{\partial u}{\partial x} e^{-i\omega t} + \text{c.c.} \quad (8)$$

Equation (2), combined with Eqs. (4) and (7), and within the rotating wave approximation ($\dot{\mathcal{I}} \ll \omega I$, $\ddot{\mathcal{I}} \ll \omega \dot{\mathcal{I}}$, $\dot{\mathcal{M}}^{(j)} \ll \omega \mathcal{M}^{(j)}$, $\ddot{\mathcal{M}}^{(j)} \ll \omega \dot{\mathcal{M}}^{(j)}$), reduces to

$$\frac{2i\omega}{v^2} \dot{\mathcal{I}} u + \left(u'' - \frac{1}{\Lambda} u' + \left(\frac{\omega^2}{v^2} - \frac{i\omega}{D} \right) u \right) \mathcal{I} = \mathcal{F}, \quad (9)$$

where the forcing term $\mathcal{F}$ depends on the magnetization $\mathcal{M}^{(j)}$ according to

$$\mathcal{F} = i w \frac{\omega^2}{v^2} \sum_{j=1}^G \mathcal{M}^{(j)} \Gamma^{(j)}. \quad (10)$$

To study the conditions for amplification or self-generation of an electrical signal, the dependence of the magnetic field $B_{\perp}^{(j)}$ on the current I in Eqs. (6a) and (6b) must also be made explicit. Starting from $\mathbf{B} = \mu_0(\mathbf{H} + \mathbf{M}) = \mu_0(I/w \hat{\mathbf{y}} + \mathbf{M})$ and expressing the field in the reference frame associated with the j -th $\langle 111 \rangle$ -axis, one obtains

$$B_{\perp}^{(j)} = \mu_0 \left[\left(\frac{1}{2w} \mathcal{I} u \Gamma^{(j)*} - i \mathcal{M}^{(j)} \right) e^{-i\omega t} + \frac{1}{2w} \mathcal{I}^* u^* \Gamma^{(j)*} e^{i\omega t} \right], \quad (11)$$

while $B_{\perp}^{(j)} = (B_{\perp}^{(j)})^*$.

Substituting Eq. (11) into Eqs. (6a) and (6b) and averaging out the rapidly oscillating terms, one finally obtains, keeping $\kappa = \frac{2\mu_0 \mu_B^2}{\hbar}$, the rate per unit population inversion density:

$$\dot{\Delta}^{(j)} = -\frac{\mu_0}{2\hbar w} (\mathcal{M}^{(j)} \mathcal{I}^* u^* \Gamma^{(j)} + \text{c.c.}) - \gamma_{\parallel} (\Delta^{(j)} - \Delta_0^{(j)}), \quad (12)$$

$$\dot{\mathcal{M}}^{(j)} = i(\omega - \bar{\omega}_0^{(j)}) \mathcal{M}^{(j)} + \frac{\kappa}{w} \Delta^{(j)} \mathcal{I} u \Gamma^{(j)*} - \gamma_{\perp} \mathcal{M}^{(j)}, \quad (13)$$

where $\bar{\omega}_0^{(j)} = \omega_0 + 2\kappa \Delta^{(j)}$. For realistic values of $\Delta^{(j)}$ (tens of ppm or less), $\bar{\omega}_0^{(j)}$ differs from ω_0 by no more than a few tens of MHz.

Equations (12) and (13), together with Eq. (9) and the appropriate boundary conditions, are all we need to solve both the problem of amplification of an externally introduced signal, and the problem of self-generation of electromagnetic MW oscillations in a planar waveguide, which are addressed in the following two sections.

III. AMPLIFICATION OF A CURRENT SIGNAL ALONG A LINEAR WAVEGUIDE

A faint signal introduced at one end of a suitably designed waveguide can be amplified by stimulated emission during its travel along the line. Here we study the conditions for the onset of the amplification, assuming to operate off-resonance, avoiding spontaneous oscillations along the line whose conditions will be studied in the next paragraph.

Suppose to connect the two ends of a linear waveguide of length L with a source of signal and with a load with real or complex impedance Z_{out} . Moreover, suppose the signal at the first end to be described by

$$I(x=0, t) = \frac{1}{2} \mathcal{I}_0 e^{-i\omega t} + \text{c.c.}$$

The implication is that in Eq. (7), we can replace $\mathcal{I}(t) = \mathcal{I}_0$ and $u(0) = 1$. Moreover, in Eq. (9), the term depending on $\dot{\mathcal{I}}$ vanishes. Now, we can write Eq. (9) in terms of the stationary solution $u(x)$ only by replacing $\mathcal{M}^{(j)}$ with the expression to be found from Eqs. (12) and (13).

In the limit of faint signals, Eq. (12) has the stationary solution $\Delta^{(j)} = \Delta_0^{(j)}$. Replacing in Eq. (13), we find the equilibrium solution for $\mathcal{M}^{(j)}$, which is

$$\mathcal{M}^{(j)} = \frac{\kappa}{\gamma_{\perp} - i(\omega - \bar{\omega}_0^{(j)})} \Delta_0^{(j)} \mathcal{I}_0 u \Gamma^{(j)*}. \quad (14)$$

By substitution of Eq. (14) in Eq. (9) with Eq. (10), we have the following equation for the stationary wave function $u(x)$:

$$u'' - \frac{1}{\Lambda} u' + \frac{\omega^2}{v^2} \left[1 + i \left(\frac{1}{Q_c} - \frac{1}{Q_m} \right) \right] u = 0. \quad (15)$$

In Eq. (15), $Q_c = \omega \frac{D}{v^2}$ is the cavity quality factor, while Q_m is the magnetic quality factor of the gain medium [12], which is connected to the geometry of the device and to the medium characteristics by:

$$\frac{1}{Q_m} = \sum_{j=1}^G \frac{\kappa \Delta_0^{(j)} \cos^2 \gamma^{(j)}}{\gamma_{\perp}} \cdot \frac{1}{1 - i \frac{\omega - \bar{\omega}_0^{(j)}}{\gamma_{\perp}}}. \quad (16)$$

In Eq. (16), the constant

$$\cos^2 \gamma^{(j)} = \|\Gamma^{(j)}\|^2 = \cos^2 \alpha^{(j)} + \sin^2 \alpha^{(j)} \sin^2 \beta^{(j)}$$

is the cosine of the angle formed by the direction of the NV^- axis with the xz -plane (see Fig. 3).

If the pumping conditions can be considered practically isotropic, all the terms $\Delta_0^{(j)}$ are equal (say to Δ_0), as well as the values of $\bar{\omega}_0^{(j)}$ (say $\bar{\omega}_0$). In this case, Eq. (16) assumes a simpler form according to the value of

$$S = \sum_{j=1}^G \cos^2 \gamma^{(j)}, \quad (17)$$

depending on the orientation of the crystal with respect to the polarizing magnetic field. In this case, the inverse magnetic quality factor of the gain medium is

$$\frac{1}{Q_m} = \frac{\kappa S \Delta_0}{\gamma_{\perp}} \cdot \frac{1}{1 - i\eta}, \quad (18)$$

where $\eta = \frac{\omega - \omega_0}{\gamma_{\perp}}$ is the detuning normalized to the linewidth.

The sign of the real part of the difference between the quality factors in Eq. (15) determines the nature of the propagation of the signal along the waveguide: if it is negative, the intensity increases along the propagation direction; otherwise, it decreases. This term can be expressed as:

$$\Re\left(\frac{1}{Q_c} - \frac{1}{Q_m}\right) = \frac{1 - C}{Q_c}, \quad (19)$$

where C is the so-called cooperativity of the system [14], defined by:

$$C = \frac{\kappa S \Delta_0}{\frac{v^2}{\omega D} \gamma_{\perp}} \cdot (1 + \eta^2)^{-1}. \quad (20)$$

If $C > 1$, the signal is amplified along the waveguide.

It is apparent that the general information on the coupling between the magnetic moment of the color centers and the external magnetic field is included in the factor S of Eq. (17), depending on the reciprocal orientation of the field and of the lattice axes. This factor can be calculated in the most significant cases as follows.

We have assumed $\mathbf{B}_0$ perpendicular to the plane of the waveguide, thus oriented along the z axis. If the main faces of the diamond layer are perpendicular to the $\langle 100 \rangle$ direction [panel (a) in Fig. 2], we have $G = 4$, and proper calculations show S in Eq. (17) to be $S = 8/3$, irrespective of the orientation of the waveguide along the diamond crystal. If the direction of z is $\langle 111 \rangle$ [panel (b) in the same figure], then $G = 1$ or $G = 3$, depending on which of the NV^- centers are tuned to the waveguide frequency: if it is the NV^- parallel to the z axis or the others, respectively. In these cases, if $G = 1$, then $S = 1$, while if $G = 3$, then $S = 2$. Finally, if the direction is $\langle 110 \rangle$ [panel (c) in Fig. 2]), then $G = 2$, and S , differently from the other cases, depends on the angle α between the x axis and the $\langle 001 \rangle$ direction, being

$$S = \frac{4}{3} + \frac{2}{3} \cos^2 \alpha.$$

Now we can set the solution to the equation for the stationary wave function [Eq. (15)]. The boundary

conditions for $u(x)$ are to be fixed at the two ends of the waveguide. They are the already mentioned condition $u(0) = 1$ and $I(L, t)Z_{\text{out}} = V(L, t)$, that is [according to Eqs. (7) and (8)]

$$\frac{1}{i\omega \frac{\epsilon_0 \epsilon_r w}{d}} \frac{\partial u}{\partial x}(L) = Z_{\text{out}} u(L).$$

In fact, we assume the waveguide to be fed by a fixed input current and to be terminated by the impedance Z_{out} . The solution of the equation gives the input impedance in terms of

$$Z_{\text{in}} = \frac{1}{i\omega \frac{\epsilon_0 \epsilon_r w}{d}} \frac{\partial u}{\partial x}(0).$$

It could be useful, both to consistently match the device to external electronics and in view of the possible employment of two or more such amplifiers in series, to fix the output impedance such that $Z_{\text{in}} = Z_{\text{out}}$. The calculation of the suitable Z_{out} can be performed by replacing Z_{in} to Z_{out} in the boundary conditions, solving the equation, obtaining a new value of Z_{in} , and iterating such a procedure, eventually obtaining the same value of the input and the output impedance. The taper can also be designed in a way that for a wide range of frequencies, the impedance of the device is real and has a convenient value.

The stationary wave function $u(x)$, solving Eq. (15), with the appropriate boundary conditions, gives the current amplification factor of the waveguide as:

$$A_I = \left| \frac{u(L)}{u(0)} \right| = |u(L)|,$$

and the gain (in decibels) is

$$G_{\text{dB}} = \frac{20}{\ln 10} \ln A_I.$$

To treat a simple example (but renouncing real impedance matching), we consider a rectangular guide ($1/\Lambda = 0$) in resonant conditions ($\omega = \bar{\omega}_0$). With this configuration, the gain is positive if the imaginary term in Eq. (15) is negative, that is, if the cooperativity C defined in Eq. (20) is greater than one. This happens, in particular, if Δ_0 surpasses the threshold $\bar{\Delta}_0$, with

$$\bar{\Delta}_0 = \frac{Q_c \gamma_{\perp}}{\kappa S} = \frac{v^2 \gamma_{\perp}}{\kappa \bar{\omega}_0 D S}. \quad (21)$$

Note that the quality factor, here only dependent on the internal losses, will also be dependent on the external load in the case of self-oscillating cavities, treated in the next paragraph.

If the (complex) impedance is matched with the external lines so that only the progressive wave must be considered in the solution of Eq. (15), $u(x)$ takes the form

$$u(x) = \exp\left(i\frac{\bar{\omega}_0}{v}\lambda_+x\right) \exp\left(\frac{\bar{\omega}_0}{v}\lambda_-x\right),$$

with

$$\lambda_{\pm} = \sqrt{\frac{1}{2} \sqrt{\left(\frac{C-1}{Q_c}\right)^2 + 1} \pm \frac{1}{2}}.$$

The gain (in decibels) is

$$G_{\text{dB}} = \frac{20}{\ln 10} \frac{\bar{\omega}_0}{v} \lambda_- L.$$

The bandpass $\Delta\omega$ at -3 dB can also be calculated by Eq. (15), and provided that $\gamma_{\perp} \ll \bar{\omega}_0$, it is

$$\frac{\Delta\omega}{\gamma_{\perp}} = 2 \sqrt{\frac{\frac{C-1}{2Q_c}}{\sqrt{\left[2\left(\frac{G_{\text{dB}}-3}{\frac{20}{\ln 10} \frac{\bar{\omega}_0}{v} L}\right)^2 - 1\right]^2 - 1 - 1}}}.$$

It can be easily verified that $\Delta\omega$ is of the order of the off-diagonal terms' decay rate $\gamma_{\perp}$, slightly decreasing for increasing gain.

The input and the output impedance are both equal to

$$Z = \frac{1}{i\omega(\epsilon_0\epsilon_r w)/d} \frac{\partial u}{\partial x} = Z_0(\lambda_+ - i\lambda_-),$$

where

$$Z_0 = \frac{d}{w} \sqrt{\frac{\mu_0}{\epsilon_0\epsilon_r}}$$

is the line impedance at zero gain. For high gain, voltage and current tend to differ in phase by $1/8$ of the period.

Although the rectangular configuration is not convenient for matching the external lines, due to the relevant imaginary part of the impedance, it provides useful rule-of-thumb formulas to dimension the device. For instance, given the diffusion constant

$$D = \frac{d \cdot th \cdot \sigma}{\epsilon_0\epsilon_r},$$

with gold electrodes, the threshold for the onset of the amplification is

$$\bar{\Delta}_0 \text{ (ppm)} = 3200 \frac{\gamma_{\perp} \text{ (MHz)}}{f \text{ (GHz)} th \text{ (nm)} d \text{ (}\mu\text{m)} S},$$

while the gain is

$$G_{\text{dB}} = 0.19 f \text{ (GHz)} L \text{ (mm)} \sqrt{\frac{(\Delta_0 - \bar{\Delta}_0) \text{ (ppm)} S}{\gamma_{\perp} \text{ (MHz)}}},$$

and the absolute value of the impedance is

$$\|Z\| = 95 \Omega \frac{d \text{ (}\mu\text{m)}}{w \text{ (}\mu\text{m)}} \sqrt[4]{\left[\frac{(\Delta_0 - \bar{\Delta}_0) \text{ (ppm)} S}{\gamma_{\perp} \text{ (MHz)}}\right]^2 + 7.6}.$$

According to these formulas, with $\gamma_{\perp}$ of the order of 1 MHz, assuming $\Delta_0 \simeq N/4$, being N the overall density of the NV^- centers, one can achieve a positive gain at 10 GHz or more with an NV^- density above some ppm, provided that the thickness of the diamond layer is some tens of micrometers and the thickness of the electrodes is a few tens of nanometers (enough to exploit the semitransparency of gold to customarily pump the sample by illumination from above). Above this threshold, high gains (tens of dB) can be achieved if the NV^- density reaches some tens of ppm, with a waveguide length of a few mm. Within this range, the width of the guide can be fixed in a way to obtain about 50Ω impedance.

On the other hand, a convenient impedance matching can be obtained with a tapered waveguide. In this case, a numeric solution of Eq. (15) is obtainable for a general profile. Nevertheless, an advantageous way to arrange the taper is to assume the scale Λ to be constant on each of the disjoint intervals of a connected union, so that on each interval the taper has the analytical form

$$w(x) = w_0 e^{x/\Lambda},$$

(Λ can also be negative), and the function $u(x)$ is of the type

$$u(x) = u_+ e^{l_+ x} e^{ikx} + u_- e^{l_- x} e^{-ikx},$$

where $l_{\pm} \pm ik$ are the two complex roots of the equation:

$$l^2 - \frac{1}{\Lambda} l + \frac{\omega^2}{v^2} \left[1 + i\left(\frac{1}{Q_c} - \frac{1}{Q_m}\right)\right] = 0.$$

The solution of Eq. (15) over the entire waveguide requires the constants $u_{\pm}$ to satisfy the boundary condition for each interval, which is the continuity of the function and its derivative at the boundary between adjacent intervals and the conditions $u(0) = 1$ and $\frac{1}{i\omega(\epsilon_0\epsilon_r w)/d} \frac{\partial u}{\partial x}(L) = Z_{\text{out}} u(L)$.

As a matter of example, we consider here the taper whose shape is described in Fig. 4, made of four parts, whose geometrical parameters are specified in the caption. We assumed gold electrodes with an equivalent thickness of 30 nm, deposited on a 200 μm -thick diamond layer. Such thin electrodes are suitable for optical pumping by a

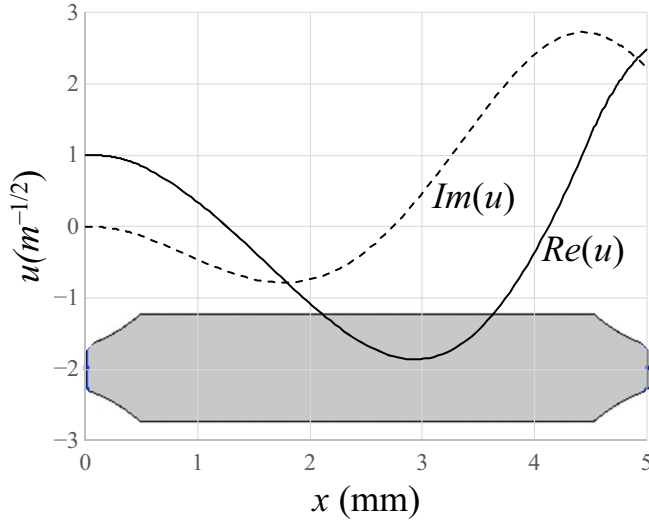


FIG. 4. The profile of a waveguide 5 mm long, along with the behavior of the real and the imaginary part of the spatial function $u(x)$ describing the amplification of the system at the frequency of 20 GHz. The vertical and the horizontal scale of the taper are the same, and the scale Λ are 0.56 and -0.56 mm in the first and the last section, while it is ± 1000 mm in the central part, at the left and at the right of the center (cross section practically constant). The input and the output impedance of the system are 51Ω at this frequency.

laser or LED source above the planar device, with no complication by lateral illumination. With these parameters, and for frequencies in the interval 1–100 GHz, the threshold value of the population inversion for positive amplification is low ($\bar{\Delta}_0 S = 0.017$ ppm at 21 GHz). With an effective population inversion such that $(\Delta_0 - \bar{\Delta}_0)S = 10$ ppm, at 21 GHz of frequency, the real and the imaginary part of $u(x)$ are also plotted in Fig. 4, assessing the amplification of the signal along the waveguide. The inversion population density has been fixed in a

way not to overtake the threshold for spontaneous oscillation whose behavior in dependence of the parameters is studied in the next paragraph. In these conditions, the impedance is almost purely real and equal to 46Ω . In Fig. 5 (left panel), for the same value of $(\Delta_0 - \bar{\Delta}_0)S$, the real and the imaginary parts of the impedance are plotted in ohms as a function of frequency, along with the power amplification in dB. For the frequency of 21 GHz, in Fig. 5 (right panel), the same quantities are shown as a function of the overall length of the waveguide.

Until now, it has been shown that a planar device, in the millimeter length range, can be coupled to external electronics by means of standard connectors and nearly 50Ω transmission lines, allowing the amplification of electric signals in the tens-of-decibels regime by exploiting the maser effect in an optically pumped paramagnetic medium, specifically an NV-rich diamond layer. The exploitation of NV⁻ centers as recombination centers in doped diamond junctions could also allow for electrical pumping of the population inversion, superimposing a continuous voltage supply onto the microwave signal. The next paragraph will show how the same effect can be exploited by overcoming a more stringent threshold for the generation of microwave signals in a waveguide connected to an external load.

IV. SIGNAL GENERATION IN A WAVEGUIDE TERMINATING ON A RESISTIVE LOAD

Suppose to terminate a tapered waveguide on a resistive load, or equivalently on a passive transmission line. We intend to write the equations for the evolution of the current $I(t)$ in Eq. (7) with the aim to find the conditions to trigger the self-oscillations and to calculate the output power of the device.

As to the spatial part $u(x)$ of the current I , we will assume that it satisfies the equation:

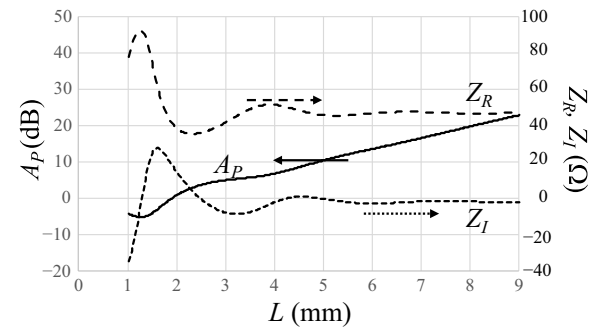
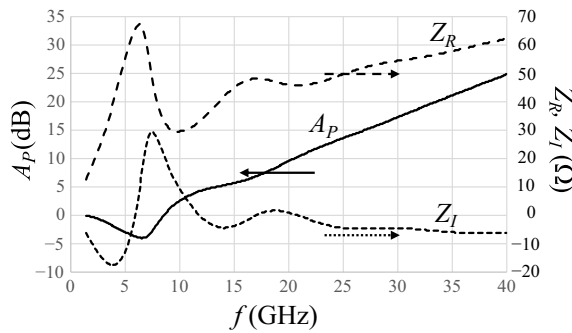


FIG. 5. (Left panel) For the device whose characteristics are specified in Fig. 4, the behavior in terms of power amplification (in dB) and impedance (real and imaginary parts, in ohm) is plotted against the frequency. On a very large bandwidth, the impedance is almost purely real, and at the center of such bandpass, the gain is about 10 dB. (Right panel) At 20 GHz, the gain and the impedance are plotted against the length of a waveguide with the initial and the final tapered sections shaped in the same way described in Fig. 4. The impedance is sensibly equal to 46Ω (real), and the amplification is proportional to the length of the waveguide for lengths greater than about 2 mm.

$$u'' - \frac{1}{\Lambda} u' - \left(\frac{\nu_R + i\omega_c}{v} \right)^2 u = 0, \quad (22)$$

where the constants ν_R and ω_c have to be fixed in a way to fit the boundary conditions $u(0) = 0$ (or $u'(0) = 0$ for a short-circuited line) and

$$\frac{1}{i\omega \frac{\epsilon_0 \epsilon_r w}{d}} \frac{\partial u}{\partial x}(L) = R u(L),$$

being R the resistive load connected at the end $x = L$. With these conditions, the pulsation ω_c is one of the natural oscillation frequencies of the cavities, while ν_R gives account of the damping due to the loss through the external impedance R .

With Eqs. (22) and (9) for the evolution of $I(t)$ becomes, assuming $\omega_c + \omega \simeq 2\omega$ and defining $\bar{\omega}_c^2 = \omega_c^2 - \nu_R^2$:

$$\begin{aligned} \dot{I} u &= i(\omega - \bar{\omega}_c) \mathcal{I} u - \left(\frac{v^2}{2D} + \nu_R \right) \mathcal{I} u \\ &+ \frac{\omega}{2} \sum_{j=1}^G \mathcal{M}^{(j)} w \Gamma^{(j)}. \end{aligned} \quad (23)$$

Considering the normalization condition $\int_0^L \|u\|^2 dx = 1$, and by multiplication of Eq. (23) by the complex conjugate u^* before integration in dx , if we define the *integrated magnetization*

$$m^{(j)}(t) = \int_0^L \mathcal{M}^{(j)}(x, t) w(x) u^*(x) dx \quad (24)$$

and the average diffusion constant

$$\bar{D} = \left(\int_0^L \frac{\|u\|^2}{D} dx \right)^{-1}, \quad (25)$$

then Eq. (23) can be written in terms of purely time-dependent quantities:

$$\dot{I} = i(\omega - \bar{\omega}_c) \mathcal{I} - \bar{\nu}_R \mathcal{I} + \frac{\omega}{2} \sum_{j=1}^G m^{(j)} \Gamma^{(j)}, \quad (26)$$

where $\bar{\nu}_R = \frac{v^2}{2D} + \nu_R$.

By multiplication of both members of Eq. (13) by $u^* w$, after integration in dx , defining the time-only dependent variable

$$\delta^{(j)}(t) = \int_0^L \Delta^{(j)}(x, t) \|u(x)\|^2 dx \quad (27)$$

(*integrated inversion population density*), one also finds an equation for the evolution of the integrated magnetization $m^{(j)}$:

$$\dot{m}^{(j)} = i(\omega - \bar{\omega}_0^{(j)}) m^{(j)} + k \delta^{(j)} \mathcal{I} \Gamma^{(j)*} - \gamma_{\perp} m^{(j)}. \quad (28)$$

We are close to finding a system of first-order equations in the time-dependent quantities $\mathcal{I}$, $m^{(j)}$, and $\delta^{(j)}$, describing a proper average value of all the fields of interest. To write an equation for $\delta^{(j)}$ from Eq. (12), we need a supplementary assumption, supposing the integral

$$\int_0^L \Delta^{(j)}(x, t) w^2(x) dx \simeq \psi \Theta \delta^{(j)}(t);$$

that is, the integral to be proportional to the time-only dependent variable $\delta^{(j)}$, according to $\Theta = \int_0^L w^2(x) dx$, a constant dependent on the geometry with a dimension of a volume and a dimensionless factor ψ equaling 1 if the population inversion $\Delta^{(j)}$ is constant everywhere.

With this assumption,

$$\dot{\delta}^{(j)} = -\frac{\mu_0}{2\hbar\psi\Theta} [m^{(j)} \mathcal{I}^* \Gamma^{(j)} + \text{c.c.}] - \gamma_{\parallel} (\delta^{(j)} - \delta_0^{(j)}). \quad (29)$$

Equations (26), (28), and (29) are the Maxwell–Bloch equations for a loaded planar waveguide embedding a paramagnetic medium.

We are looking for nontrivial stationary solutions of Eqs. (26)–(29). Posing $\dot{m}^{(j)} = 0$ in Eq. (28), solving for $m^{(j)}$ and replacing in Eq. (26) with $\dot{I} = 0$, we have a condition for the integrated equilibrium inversion population densities $\bar{\delta}^{(j)}$:

$$\sum_{j=1}^G \frac{\frac{k}{2} \omega \bar{\delta}^{(j)} \cos^2 \gamma^{(j)}}{[\gamma_{\perp} - i(\omega - \bar{\omega}_0^{(j)})][\bar{\nu}_R - i(\omega - \bar{\omega}_c)]} = 1. \quad (30)$$

Being $\bar{\delta}^{(j)}$ real, Eq. (30) is satisfied only for the specific value of ω making the imaginary part of the expression vanishing.

From here on, we assume the simplifying condition, reasonable in case of isotropic excitation, that $\bar{\delta}^{(j)}$ and $\bar{\omega}_0^{(j)}$ are independent of the orientation (j) of the color centers, as done in Sec. III. Thus, Eq. (30) becomes

$$\frac{\frac{k}{2} \omega \bar{\delta} S}{[\gamma_{\perp} - i(\omega - \bar{\omega}_0)][\bar{\nu}_R - i(\omega - \bar{\omega}_c)]} = 1, \quad (31)$$

where $S = \sum_{j=1}^G \cos^2 \gamma^{(j)}$ is the constant, depending only on crystal orientation, discussed in Sec. III. The pulsation ω , in order for $\bar{\delta}$ to be real, is given by

$$\omega = \frac{\gamma_{\perp} \bar{\omega}_c + \bar{\nu}_R \bar{\omega}_0}{\gamma_{\perp} + \bar{\nu}_R}. \quad (32)$$

With this frequency, and denoting now by $\eta = \frac{\bar{\omega}_0 - \bar{\omega}_c}{\gamma_{\perp} + \bar{\nu}_R}$ the detuning normalized to the overall linewidth, the

(integrated) equilibrium inversion population density $\bar{\delta}$ is written as:

$$\bar{\delta} = \frac{2\gamma_{\perp}\bar{\nu}_R}{\omega\kappa S} \left(1 + \frac{(\bar{\omega}_0 - \bar{\omega}_c)^2}{(\gamma_{\perp} + \bar{\nu}_R)^2} \right) = \frac{\gamma_{\perp}}{\kappa S} \frac{1 + \eta^2}{Q_c}, \quad (33)$$

where the cavity quality factor is now defined on the sum of external and internal loss rate: $Q_c = \frac{\omega_c}{2\bar{\nu}_R}$.

It is now possible to calculate the equilibrium value of the current $\bar{I}$. In Eq. (29), let us sum over the G different orientations (j) of the color centers posing $\delta = 0$. Then, we express the sum of Eq. (26) as a function of $\bar{I}$ (having assumed $\dot{I} = 0$). Finally, by substitution in Eq. (29) summed over j , one finds

$$\|\bar{I}\|^2 = \frac{G\hbar\psi\Theta\omega\gamma_{\parallel}}{2\mu_0\bar{\nu}_R} (\delta_0 - \bar{\delta}). \quad (34)$$

By inspection of Eq. (34), we note that self-generation of MW oscillation is possible only if δ_0 overtakes a threshold given by $\bar{\delta}$ in Eq. (33). This fact defines the condition for the onset of self-oscillation, $C > 1$, stated on the cooperativity C defined in Eq. (20) [provided that in Eq. (20), the constant value Δ_0 is replaced with the averaged value δ_0 calculated as in Eq. (27)].

It is worth noting that, given the relatively high losses implied by the concept presented here, the linewidth $\bar{\nu}_R$ is sufficiently broad to accommodate large detunings. As a result, the self-oscillation frequency can be tuned over a wide frequency range by varying the external magnetic field while preserving maser action.

In the case $\delta_0 \gg \bar{\delta}$, Eq. (34) can be expressed in a more convenient form assuming that the pumping level $\Delta_0(x)$ depends on the position x according to $\Delta_0 = \Delta_0^M f(x)$, where $f(x)$ is a positive function (without optical or electrical pumping the two sublevels are equally populated and $\Delta = 0$) whose maximum equals one. If we define a filling factor

$$\phi = \frac{1}{\Theta} \int_0^L f(x) w^2(x) dx \leq 1, \quad (35)$$

(the equality holds only for $f(x) = \text{constant} = 1$), Eq. (34) reduces to

$$\|\bar{I}\|^2 = \frac{G\hbar\phi\Theta\omega\gamma_{\parallel}}{2\mu_0\bar{\nu}_R} \Delta_0^M. \quad (36)$$

The significance of Eq. (36) derives from the factor Δ_0^M , the peak pumping intensity in the waveguide, and from the filling factor ϕ , describing the fraction of the cavity involved in the pumping process. The main advantage of the device introduced in this article is the filling factor easily approaching 1, one or two orders of magnitude higher than the state of the art of a solid-state maser

employing NV^- centers, and permitting a workable electromagnetic quality factor below 10^3 .

From knowledge of $\|\bar{I}\|^2$, the electric power dissipated in the external load (or, equivalently, injected in the external line) is $P = \frac{1}{2} \|\bar{I}\|^2 \|u(L)\|^2 R$, which is linear in the pump δ_0 above the threshold $\bar{\delta}$, as stated by Eq. (34) [or linear in Δ_0^M , according to Eq. (36)].

The calculation of the output power P requires the explicit calculation of $u(x)$, the space-dependent part of the current I . Here, we are especially interested in the case in which internal losses are much lower than the power transmitted through the external line (i.e., $v^2/(2\bar{D}) \ll \nu_R$, that is $\bar{\nu}_R \simeq \nu_R$), but the load is high enough to assure $\nu_R \ll \omega_c$.

In this case, we can assume that the function $u(x)$, solution of Eq. (22), is expressed at the first order in $\nu_R/\bar{\omega}_c$ by a sum of the kind

$$u(x) = u_0(x) + i \frac{\nu_R}{\bar{\omega}_c} u_1(x). \quad (37)$$

Replacing u in Eq. (22) and neglecting higher-order terms, we have the two equations for the real quantities u_0 and u_1 :

$$u_0'' - \frac{1}{\Lambda} u_0' + \frac{\bar{\omega}_c^2}{v^2} u_0 = 0, \quad (38)$$

$$u_1'' - \frac{1}{\Lambda} u_1' + \frac{\bar{\omega}_c^2}{v^2} u_1 = 2 \frac{\bar{\omega}_c^2}{v^2} u_0. \quad (39)$$

The boundary conditions for u_0 are $u_0(0) = 0$ (or $u_0'(0) = 0$, in the case of a short-circuited line) and $u_0(L) = 0$. The last condition is consequence of the fact that, when $\nu_R = 0$, the line is open at the end $x = L$, and the output current is null. With these conditions, Eq. (38) is an eigenvalue equation satisfied for a discrete set of values (equaling $\bar{\omega}_c = \pi v n / L$ if the waveguide is rectangular). Here we will assume that the threshold $\bar{\delta}$ is trespassed only for one (especially the lowest) of these frequencies.

The first-order term u_1 , on the other hand, satisfies the same condition of u_0 in $x = 0$, but its value in $x = L$ has to be chosen in a way that the physical boundary condition $\frac{1}{i\omega} \frac{\partial u}{\partial x}(L) = R u(L)$ is valid for ν_R real. Replacing the first-order expression for $u(x)$, we have

$$\frac{1}{i\omega} \frac{\varepsilon_0 \varepsilon_r w}{d} \left[u_0'(L) + i \frac{\nu_R}{\bar{\omega}_c} u_1'(L) \right] = R \left[u_0(L) + i \frac{\nu_R}{\bar{\omega}_c} u_1(L) \right].$$

The reality condition for ν_R implies $u_1'(L) = 0$, which, along with the already mentioned $u_0(L) = 0$, at the resonant frequency $\omega = \bar{\omega}_c$, results in

$$\nu_R = \frac{1}{RC_{\text{eq}}}. \quad \text{With} \quad C_{\text{eq}} = \frac{\varepsilon_0 \varepsilon_r w}{d} \left(-\frac{u_1(L)}{u'_0(L)} \right). \quad (40)$$

The two conditions $u_1(0) = 0$ (or $u'_1(0) = 0$) and $u'_1(L) = 0$ are satisfied for suitable values of the integration constants in Eq. (39), and the right choice of these constants allows for the solution of the problem and for the calculation of the output power, given a level of population inversion δ_0 .

In Eq. (40), C_{eq} is an equivalent capacity of the line as seen by the end at $x = L$. The damping rate of the system, in other words, is determined by the time constant of the capacity of the line connected to the output impedance.

The simpler analytic solution for the system of Eqs. (38) and (39) is obtained for a rectangular guide of constant width w . In this case, $u(x)$ could be calculated exactly by Eq. (22) in terms of the sum of complex exponentials. At the first order in ν_R , this solution coincides with the solution of Eqs. (38) and (39) and can be written as in Eq. (37) with

$$u_0(x) = c \cdot \sin\left(\frac{\bar{\omega}_c}{v} x\right) \quad \text{and} \\ u_1(x) = c \cdot \left(\sin\left(\frac{\bar{\omega}_c}{v} x\right) - \frac{\bar{\omega}_c}{v} x \cos\left(\frac{\bar{\omega}_c}{v} x\right) \right).$$

Here c is the normalization constant, equaling $\sqrt{2/L}$ at the lowest order in ν_R . With these values, the capacity, stated by Eq. (40), is $C_{\text{eq}} = \frac{\varepsilon_0 \varepsilon_r w}{d} L$, which is simply the low-frequency capacity of the line. Notably, if the line is oscillating in its n -th resonant mode, the equivalent capacity is $C_{\text{eq}} = \frac{\varepsilon_0 \varepsilon_r w}{d} \frac{L}{n}$, and ν_R results to be multiplied by the order of the mode, as well as $\bar{\omega}_c$.

The output power for such a rectangular guide, in the limit of $\delta_0 \gg \bar{\delta}$ [and of Eq. (36)] is

$$P = \frac{1}{2} \|\tilde{I}\|^2 \|u(L)\|^2 R = \frac{1}{2} \hbar \omega \gamma_{\parallel} \phi V G \Delta_0^M. \quad (41)$$

In Eq. (41), $\phi V = \phi w L d$ is the volume embedded by the waveguide multiplied by the filling factor ϕ . This volume, times $G \Delta_0^M$, is the number of color centers actively interested in the self-generation of the MW oscillation. The energy $\hbar \omega$ is the energy of each emitted photon, while $\gamma_{\parallel}$, below saturation, at say 1 W/mm² of power density, is around some kHz for spin-flip probabilities around 0.05–0.4. For an MW frequency of 16 GHz, we assume $L = 4$ mm, $w = 1$ mm, and $d = 0.1$ mm, so that $V = 0.4$ mm³. Thus, with $G \Delta_0^M$ of 10 ppm and a nearly uniform pumping intensity, we can count on an injected power reaching 20 μ W [consider that the threshold for the onset of amplification $\bar{\delta}$, from Eq. (25), with our assumptions, is only about 0.1 ppm].

This is a quite high-power level compared with the current state of the art [7,27]. Undoubtedly, this favorable prevision, despite the low quality factor $Q = \omega_c / \nu_R \simeq 200$ obtained with $R = 50 \Omega$, depends on the high filling factor, approaching unity, distinguishing this system with respect to maser geometries based on diamond crystals housed in high-quality-factor cavities of some cm³ of volume (filling factors at most of the order of some percent). The low quality factor also allows a higher frequency stability (not depending on cavity thermal noise at very high Q) and permits the use of smaller magnets and more compact geometries.

Moreover, these favorable conditions allow the tuning of the cavity also for frequencies quite different from $\bar{\omega}_c$. In fact, being $\nu_R \simeq 1.25$ GHz vs $\gamma_{\perp}$ of the order of some MHz or less, the oscillation frequency $\omega = \frac{\gamma_{\perp} \bar{\omega}_c + \bar{\nu}_R \bar{\omega}_0}{\gamma_{\perp} + \bar{\nu}_R}$ is almost entirely determined by the external polarizing magnetic field, and the condition $\delta_0 - \bar{\delta} > 0$, according to Eq. (33), can be satisfied also for high values of the detuning (up to several ν_R units, that is several GHz). In other words, the frequency of this kind of oscillator is easily adjustable, also off-resonance, by variation of the polarizing magnetic field.

Equations (21) and (33), along with Eq. (40), allow to fix a set of thresholds to distinguish in what conditions of population inversion attenuation, amplification or self-oscillation occurs, respectively, according to the frequency of oscillation, determined by the external polarizing magnetic field. In a Cartesian ω - Δ plane, one can identify regions where attenuation, amplification, or single or multiple mode generation occurs. According to the size and shape of the waveguide, a different plot must be drawn.

Figure 6, for instance, reports the case of a 4-mm-long, 1-mm-wide rectangular waveguide, with an equivalent thickness of the metal layer of 30 nm and a 100 μ m-thick diamond layer, connected to a 50 Ω load. It results, for instance, that a population inversion of 10 ppm is enough to assure self-oscillation with a bandwidth of about 7 GHz around the resonant frequency of 15.7 GHz. Another interval about 16 GHz wide is found around the second harmonic at 31.4 GHz, partially overlapped with the third harmonic interval. Between the first two intervals, a rectangular cavity can work as an amplifier at frequencies between 20 and 25 GHz, approximately. The rectangular shape is not the most suitable to match external electronics. In Fig. 7, the profile of a waveguide open on one of its sides and tapered on the other is shown, along with the behavior of the functions $u_0(x)$ and $u_1(x)$. The equivalent capacity C_{eq} of the device (with a diamond thickness of 50 μ m) is 1.28 pF, to be compared with the 1.44 pF of geometrical capacity $\varepsilon_0 \varepsilon_r A/d$. The resonant frequency of the device is nearly equal to that of the rectangular waveguide of equivalent length (16.4 vs 15.7 GHz).

For frequencies above 10 GHz, a rectangular waveguide can support stationary oscillations in diamond single

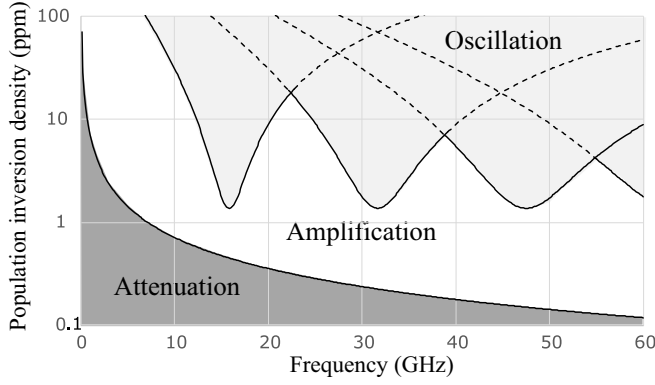


FIG. 6. $\omega\Delta$ diagram for attenuation, amplification, or oscillation conditions in a rectangular guide of 4×2 mm² size, connected to a 50 Ω load.

crystals of realistic size, but for lower frequencies of the order of a few GHz, the size of a rectangular cavity would be unpractically large, e.g., about 2 cm at 3 GHz. At these frequencies, some form of slow-wave engineering [22] becomes necessary to implement the maser concept in mm-sized diamond membranes. In the following, we follow the lumped-element slow-wave resonator approach by shaping the taper in a way that two sections of the waveguide work as inductance and capacitance, respectively. This principle has been envisaged in the fabrication of near-field planar antennas specifically designed for optically detected magnetic resonance (ODMR) [20]. Here we apply the same concept in reception instead of transmission mode.

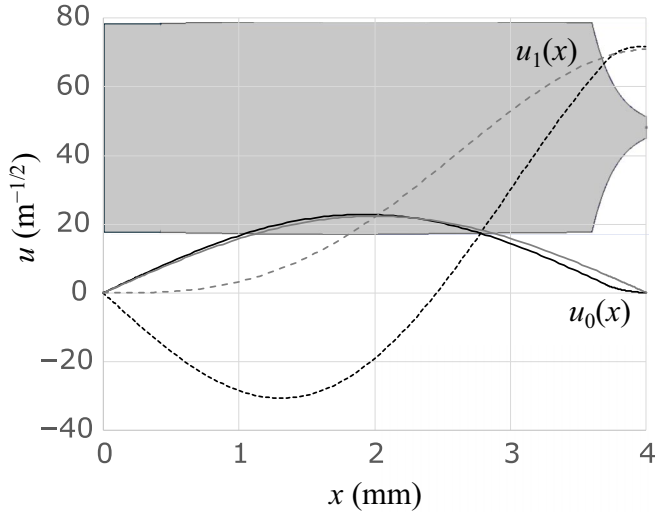


FIG. 7. Gray area: Profile of a resonant waveguide oscillating at 16.4 GHz (the scale is stated by the horizontal axis), tapered on the right side to match an external connection [the same profile is also shown in Fig. 1(b)]. The black curves represent the functions $u_0(x)$ (solid line) and $u_1(x)$ (dashed line), solutions of Eqs. (38) and (39). For comparison, the same functions calculated for a perfectly rectangular waveguide are shown in dark gray.

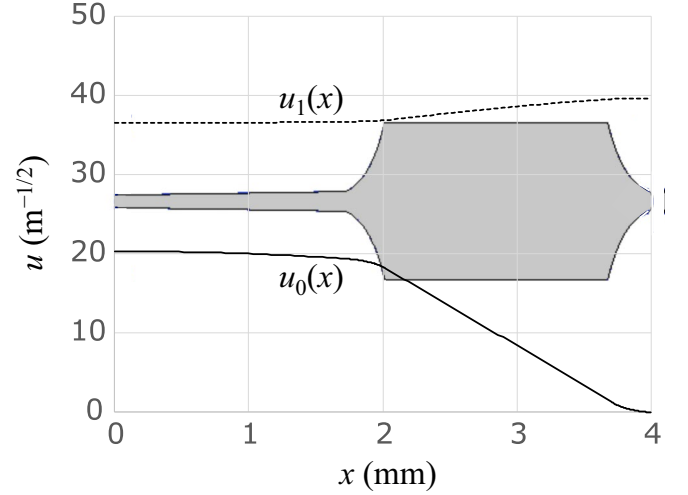


FIG. 8. Gray area: Profile of a resonant inductance-capacitance-like (LC-like) waveguide oscillating at 3.2 GHz, tapered on the right side to match with an external connection and short-circuited on the other side. In the plot, the functions $u_0(x)$ (solid line) and $u_1(x)$ (dashed line) are solutions of Eqs. (38) and (39).

In Fig. 8, the profile of a 4-mm-long antenna oscillating at 3.2 GHz is shown, consisting of an inductive and a capacitive section, with the inductive one short-circuited at one end. The functions $u_0(x)$ and $u_1(x)$ are plotted along the waveguide. Also, with only the capacitive section subject to pumping, a filling factor of about 90% is realistically obtained. The effective capacity is about 50% higher than the geometrical one (4.3 vs 2.6 pF), but even so, the corresponding ν_R at $R = 50$ Ω is about 4.7 GHz, about one quarter of the resonant $\bar{\omega}_c$. To apply the lower-order approximation in $\nu_R/\bar{\omega}_c$, we had better match the impedance of a conventional 50 Ω line by means of an external tapered line, for the oscillator to see a higher impedance load. With a value of $\gamma_{||}$ of a few kHz, the output power of such a device would be several μ W.

V. CONCLUSION

We have shown that a suitably designed, compact planar waveguide embedding a paramagnetic medium can serve both as a microwave signal amplifier and generator, operating with standard transmission lines for connection to input or output electronics. Compared with the present state of the art of high-quality-factor resonant cavities employing diamond or silicon carbide paramagnetic defects, the high filling factors (approaching unity) of such an arrangement assure high gain despite relatively poor quality factors (below 10^3). This favors the exploitation of thin metal layers partially transparent to light, allowing uniform and effective illumination from an external source for population inversion. Moreover, the two metal layers of the waveguide can be employed to obtain continuous-current electrical pumping across a

bipolar junction, exploiting the properties of some paramagnetic defects as recombination centers.

The low quality factor also favors frequency stability and makes the oscillation or amplification frequency of such devices easily tunable over an interval of the order of a few GHz around the resonance frequency of the waveguide by variation of an external magnetic field in the range of hundreds of mT. The small dimensions of the available samples, especially diamond, limit the employment of rectangular waveguides to high-frequency generation (typically tens of GHz), but the waveguide can be shaped in a way to resonate at low frequencies, and we have shown that the GHz range is easily reached also with samples of a few millimeters in size.

Moreover, the whole concept of a maser device operating in a planar configuration favors integration within electronic systems and next-generation quantum platforms, offering a promising pathway toward scalable on-chip maser devices.

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DATA AVAILABILITY

The data that support the findings of this article are not publicly available. The data are available from the authors upon reasonable request.

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