

Impact of current-induced magnons on spin-orbit-torque analysis

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The second-harmonic Hall technique is a widely used, sensitive method for studying the spin-orbit torques generated by charge current. It exploits the dependence of the Hall resistance on the magnetization direction, although thermal phenomena also contribute. Historically, deviations from the expected magnetic field dependence have usually been ignored. On the basis of our studies on permalloy/platinum bilayers, we show that a counterpart to the magnon-related spin-flip unidirectional magnetoresistance—known to appear in the second-harmonic longitudinal resistance—appears in the Hall data, and that describing the results in a wide field range with these contributions is essential to accurately estimate the torques.

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I. INTRODUCTION

Two key pathways for advancing magnetic random-access memory are spin transfer torque (STT) and spin-orbit torque (SOT) [1,2]. In STT a spin current is created by driving a charge current through a ferromagnetic (FM) layer, which flips the state of the FM layer that constitutes the memory bit [3,4]. Spin currents can also be generated in a nonmagnetic material with strong spin-orbit coupling, via charge-to-spin conversion mechanisms such as the Rashba-Edelstein (also called the “inverse spin galvanic”) and the spin Hall effects [5]. The resulting spin-orbit torque can also be used to switch the magnetization of an adjacent FM layer [6–8].

For the analysis of torques, spin-torque ferromagnetic resonance [9,10] and second-harmonic Hall (2HH) measurements [11,12] are often used [13,14], from heavy metals [15–18] to van der Waals crystals [19–21] or their combinations [22–27]. The 2HH technique is sensitive to

spin-orbit fields generated by an alternating current applied to the structure. As these fields slightly alter the magnetization direction and thus the magnetoresistance (MR), voltage components appear that are quadratic with the current I , and thus can be detected at double frequency, 2ω [11,12,23,28].

Several other phenomena leading to a double-frequency voltage signal complicate the analysis of 2HH measurements. Various thermal effects from Joule heating (proportional to I^2), such as the anomalous Nernst effect, are prominent among them [29,30]. However, these are sometimes inadequate to fully describe the 2ω signals, which is generally circumvented by fitting to only part of the magnetic field range. Here we go beyond this approach: we discuss modifications to the original model and show that the unusual behavior of the Hall data is reminiscent of the magnon-related spin-flip (SF) unidirectional magnetoresistance (UMR) that appears in the longitudinal data [31–35]. We compare two approaches to describe our observations on permalloy (Py)/platinum bilayers, and for the second approach, we demonstrate satisfactory fits to the 2ω measurements over a broad range of magnetic field values. Our analysis leads to significant quantitative changes in the extracted SOT parameters, and highlights the importance of taking UMR-like contributions into account in future studies.

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II. EXPERIMENTAL RESULTS

We performed measurements on eight ferromagnet/normal metal Hall bar devices at room temperature, with qualitatively similar results. The devices were made of 6 nm Py capped by 3 nm Pt and contacted with Ti/Au electrodes on a SiO₂-covered wafer. The measurements presented in the main text were taken on the sample shown in Fig. 1(a); data on other samples can be found in Sec. 3 in Supplemental Material [36]. We applied an alternating current along the length of the Hall bar, and measured the longitudinal V_{xx} and transverse (Hall) V_{xy} voltages by a low-frequency lock-in technique, as shown in Fig. 1(a). Further measurement parameters and sample fabrication are detailed in Sec. V. We studied the first-harmonic and second-harmonic (2ω) longitudinal and Hall resistances $R_{xx,xy}$ and $R_{xx,xy}^{2\omega}$ as a function of the out-of-plane magnetic field as well as in-plane rotation in a constant magnetic field. We denote the polar angles of the dimensionless magnetization $\mathbf{M}$ and the magnetic field $\mathbf{H}$ relative to the out-of-plane direction z by θ and θ_H , respectively, and the azimuthal angles relative to the current direction x by φ and φ_H , respectively, as shown in Figs. 1(a) and 1(b). Since Py is an easy-plane ferromagnet, $\varphi \approx \varphi_H$ for all values of the magnetic field used in this work ($\mu_0 H > 15$ mT).

We plot measurements of R_{xy} as a function of out-of-plane field H_z in Fig. 1(c). Since this is the hard axis, the hysteresis was negligible. We observe a clear anomalous Hall effect (AHE) with saturation at $\mu_0 |H_z| > 0.7$ T, which reflects the behavior of the z component of the magnetization. M_z becomes finite as $\mathbf{M}$ is gradually tilted out of the easy (x - y) plane with increasing $|H_z|$, and then becomes parallel with $\mathbf{H}$. The shallow linear background originates in the ordinary Hall effect (OHE). The resistance offset and the parts that are even as a function of H (such as the dip at $H = 0$) are attributed to the mixing of R_{xx} .

In Fig. 1(d) we show R_{xx} and R_{xy} as a function of φ_H taken by our rotating the sample in a constant in-plane field. The π -periodic oscillations originate from the anisotropic magnetoresistance (AMR) and its sister phenomenon, the planar Hall effect (PHE) [37]. We summarize these effects in the Hall and longitudinal resistances at a given field, ignoring offsets and the OHE, as

$$R_{xy} = R_A \cos \theta + R_P \sin^2 \theta \sin 2\varphi, \quad (1a)$$

$$R_{xx} = R_0 + R_a \sin^2 \theta \cos^2 \varphi. \quad (1b)$$

In Eq. (1a) the first term is the AHE, and its magnitude is illustrated in Fig. 1(c). We correct for the small contribution of the OHE by extrapolating linear fits from the saturation region (nearly horizontal red lines) to zero field (blue line). The second terms in Eqs. (1a) and (1b) describe the PHE and AMR. The corresponding fits are shown in Fig. 1(d) in red and blue. Since the aspect ratio here is 1, their amplitudes are expected to be the same, $R_P = R_a/2$

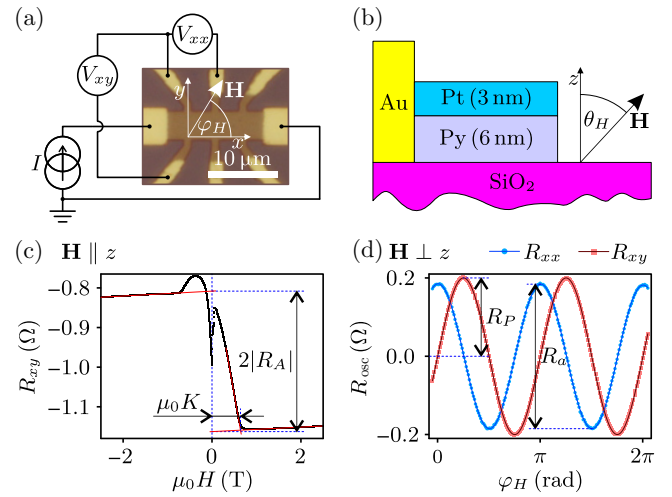


FIG. 1. Measurement geometry and first-harmonic data. (a) Optical image of the device and diagram of the measurement setup, also showing the in-plane magnetic field direction. (b) Device composition and the polar angle of the field $\mathbf{H}$. (c) Hall resistance R_{xy} as a function of out-of-plane field ($\theta_H = 0$). The red lines are fits to the saturation regions and to a part of the transition region. The anomalous Hall resistance ($R_A \approx -179$ mΩ) and the anisotropy field ($\mu_0 K \approx 0.685$ T) based on the lines' crossing points are highlighted. (d) Oscillating parts of R_{xx} and R_{xy} (blue and red symbols, respectively) for in-plane rotation ($\theta_H = \pi/2$) at $\mu_0 H = 0.186$ T. Solid lines are fits to Eqs. (1a) and (1b) after inclusion of small horizontal offsets. The constant backgrounds that were removed are 81.8 and -0.98 Ω, respectively.

[37]. We observe an approximately 10% deviation from this relation, which may be due to an imperfect geometry or inhomogeneities in the sample.

Additional MR effects may contribute to the terms labeled as the AHE, PHE, and AMR above, such as spin Hall MR [38], Hanle MR [39], anomalous Hall MR [40], and the geometrical size effect [41–43]. The isotropic offset R_0 is affected by magnon MR (MMR) [42,44]. However, they do not change the (θ, φ) dependence in Eq. (1a) and the φ dependence for in-plane rotation ($\theta = \pi/2$) in Eq. (1b), which are later exploited to determine SOT, and therefore can be collectively described by R_P and R_a . $\varphi \approx \varphi_H$ applies well for the fields studied, which is confirmed by the goodness of the fits to Eqs. (1a) and (1b) in Fig. 1(d), although there is a weak in-plane easy-axis magnetic anisotropy in our devices (see Sec. 5 in Supplemental Material [36] and also Ref. [45]).

In the following we discuss low-frequency 2ω measurements for quantifying SOT. The spin Hall effect in Pt produces spins with orientation σ in the $-y$ direction at the Py/Pt interface. They, along with the external $\mathbf{H}$, exert torques on the magnetization $\mathbf{M}$. On the timescale of the lock-in measurement, the precession of $\mathbf{M}$ is completely damped, and the torques can be described by effective

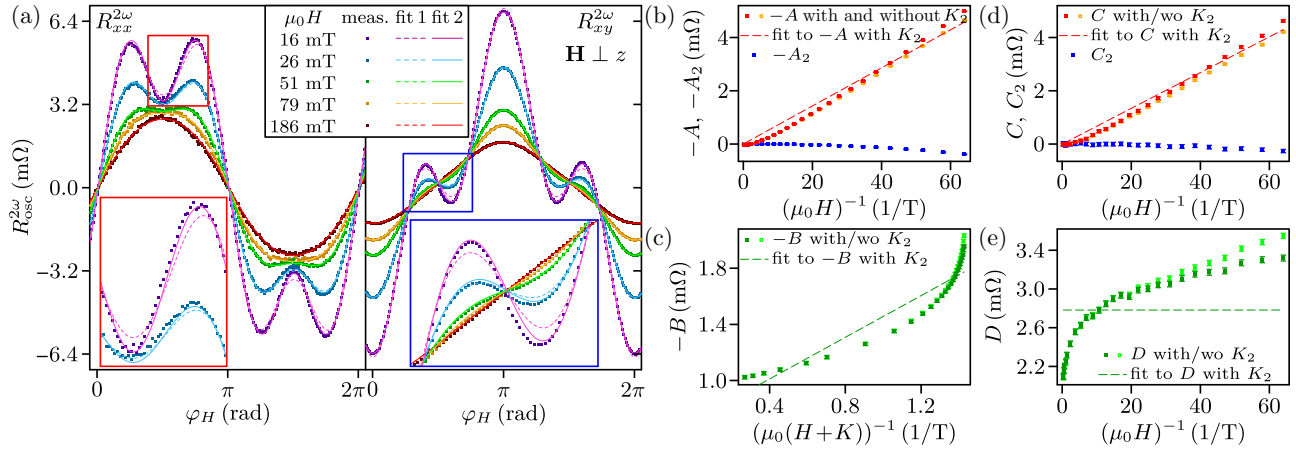


FIG. 2. Second-harmonic Hall measurements. (a) $R_{xx}^{2\omega}$ and $R_{xy}^{2\omega}$ with in-plane field rotation ($\theta = \theta_H = \pi/2$) for selected values of the absolute field $\mu_0 H$, after averaging clockwise and counterclockwise rotations and removal of constant offsets. The dashed lines (fit 1) are fits according to Eqs. (2a) and (2b) and $\varphi = \varphi_H$. The solid lines (fit 2) include the anisotropy K_2 as corrections to φ [Eq. (S8)] and include the terms of Eq. (S12). The insets show enlargements of the highlighted rectangles. (b)–(e) Coefficients $-A$ and $-A_2$, $-B$, C and C_2 , and D , respectively, as a function of $(\mu_0 H)^{-1}$ and $(\mu_0 (H + K))^{-1}$. The dashed lines are linear fits according to Eqs. (3a)–(3d).

fields. They slightly tilt the magnetization out of the equilibrium direction [see Eqs. (S2) and (S3) in Supplemental Material [36]] with the frequency ω of the current. The modulation $\Delta\theta, \Delta\varphi$ of the angles in Eqs. (1a) and (1b) thus leads to a 2ω out-of-phase component in the voltage signals [12]. In this high-symmetry system the second-harmonic resistances measured as a function of φ_H for $\theta_H = \pi/2$ and plotted in Fig. 2(a) can be fitted according to

$$R_{xy}^{2\omega}(\varphi) = A \cos 2\varphi \cos \varphi + B \cos \varphi + \text{const}, \quad (2a)$$

$$R_{xx}^{2\omega}(\varphi) = C \sin 2\varphi \cos \varphi + D \sin \varphi + \text{const}. \quad (2b)$$

As detailed in Sec. 2 in Supplemental Material [36] and Refs. [11, 23, 28], the terms with amplitudes A and C are attributed to the sum of the effective magnetic field H_{FL} of the fieldlike SOT and the Oersted field $H_{\text{Oe}} = I_{\text{pt}}/2W$ generated by the current in Pt [45]. H_{FL} and H_{Oe} always appear together and cannot be easily distinguished. For simplicity, we write H_{FL} instead of $H_{\text{FL}} + H_{\text{Oe}}$. The coefficient B originates from the antidamping SOT field H_{AD} and the anomalous Nernst effect R_{ANE} . The ANE Hall voltage is related to the Joule heating–induced thermal gradient ∇T in the sample by $V_{\text{ANE},y} \propto (\nabla T \times \mathbf{M})_y$, where $\nabla T \propto I^2(t)$ has 2ω frequency and an out-of-plane component. D is attributed to the ANE through $(\nabla T \times \mathbf{M})_x$. Finally, constant terms (independent of φ and H) are possible due to in-plane components of ∇T via the Seebeck effect. Other thermal effects with different φ dependence are possible [31, 46–48], but these are negligible in our data. The coefficients A , B , C , and D depend on the field as follows (see Refs. [11, 23, 28] and Secs. 1 and 2 in Supplemental

Material [36]):

$$A(H) = R_P \frac{H_{\text{FL}}}{H}, \quad (3a)$$

$$B(H) = \frac{R_A}{2} \frac{H_{\text{AD}}}{H + K} + R_{\text{ANE}}, \quad (3b)$$

$$C(H) = \frac{-R_a}{2} \frac{H_{\text{FL}}}{H}, \quad (3c)$$

$$D(H) = R_{\text{ANE}2}. \quad (3d)$$

We discuss further contributions, some of which can be built into the above terms, further below and in Sec. 4 in Supplemental Material [36] (see also Refs. [49–51]).

The φ dependence of $R_{xx}^{2\omega}$ and $R_{xy}^{2\omega}$ and fits corresponding to Eqs. (2a) and (2b) are shown in Fig. 2(a) at selected fields. Most rotations were performed between 15 and 100 mT, since the effect of SOT is strongest at low H . Just as when fitting the AMR and the PHE, we allowed for an offset in φ (on the order of 5°) to account for the misalignment of the sample and the magnet. The field dependence of the amplitudes A , B , C , and D is shown in Figs. 2(b)–2(e) by orange and light green markers. According to Eqs. (3a)–(3d), the coefficient D is expected to be constant, while A , C , and B are linear as a function of $(\mu_0 H)^{-1}$ and $(\mu_0 (H + K))^{-1}$, respectively. However, the observations do not match Eqs. (3a)–(3d) well. For A and C , the measured data depart from a linear behavior for low $1/H$ (high fields), while for B the trend is the opposite, and the data are nonlinear for high $(H + K)^{-1}$ (low fields). Therefore, it is unlikely that both deviations come from demagnetization and domain wall formation at low H . This conclusion is supported by the lack of sudden jumps from

domain switching, the overlap of the clockwise and counterclockwise (decreasing or increasing φ_H) rotations, and the goodness of the fits to Eqs. (2a) and (2b) in Fig. 2(a) (dashed lines). The difference between the latter and the data is relatively small even at 16 mT, and the error ranges of A , B , C , and D [see Figs. 2(b)–(e)] are comparable to the symbol size. We consistently observed such character in all our samples, as illustrated in Sec. 3 in Supplemental Material [36]. Therefore, Eqs. (3a)–(3d) do not describe our observations adequately. The systematic deviations give us an opportunity to reveal contributions from other, mainly magnon-related processes.

III. DISCUSSION

To account for the mismatch between the data and the fitting, we examined several extensions of the basic model. For example, we considered easy-axis shape anisotropy for the Hall bar. As detailed in Sec. 5 in Supplemental Material [36], from the first-harmonic resistances we estimate its strength $\mu_0 K_2$ to be on the order of 1 mT. It also introduces new oscillatory terms with amplitudes A_2 and C_2 and improves the fits to the second harmonics as shown by the solid lines (labeled “fit 2”) in Fig. 2(a). However, A , B , C , and D are weakly affected, as shown by red and dark green markers in Figs. 2(b)–2(e), and the differences from the field dependence governed by Eqs. (2a) and (2b) (dashed lines) remain present. Several other effects may appear in metallic bilayers, but those corrections were also deemed insignificant, as discussed in Sec. 4 in Supplemental Material [36].

We attribute the deviations in C and D to UMR [31–35], and we propose that a counterpart of similar origin also appears in $R_{xy}^{2\omega}$ measurements in the coefficients A and B . UMR can originate from (1) the dependence of the normal metal/ferromagnet interface resistance on the direction of σ relative to $\mathbf{M}$, (2) the spin dependence of the bulk conductivity of the ferromagnet, and (3) magnon-related effects [34]. Its contribution to the longitudinal resistance $R_{xx}^{2\omega}$ is proportional to $\sigma \cdot \mathbf{M} = -\sin \theta \sin \varphi$ [31]. Both (1) and (2) are independent of H , and are together called the “spin-dependent UMR.” In contrast, (3), which is SF UMR, depends on H . It is attributed to the generation and annihilation of magnons through angular momentum transfer from the spins generated by the current in the normal metal [32–35], which is also supported by its correlation with H_{AD} [52]. Through the modulation of electron-magnon scattering (MMR) or potentially of the length of the magnetization [53,54], this leads to a resistance contribution that depends on the current, and on the magnetization via $\sigma \cdot \mathbf{M}$. It decays with increasing field since H opens a gap in the magnon spectrum. Besides the term proportional to $\sin \varphi$, higher frequencies with φ have been observed [34,55]. Surprisingly, a corresponding correction to the Hall signal has not been reported in diffusive

systems to our knowledge, only in bilayers based on topological insulators [56,57]. In the following we present two possible explanations for why UMR can appear in 2HH measurements.

The first explanation involves symmetry considerations. In AMR the resistance depends on the direction of magnetization. Its twofold rotational symmetry leads to its transverse counterpart, the PHE [37]. In contrast, MMR is isotropic and leads to SF UMR through the modulation of the magnon population proportional to $\sin \varphi$, and a transverse part is not expected. However, higher frequencies in φ also exist and have been attributed to the combination of SF scattering and AMR [55], which implies a similar effect through the PHE. Therefore, we propose that $R_{xy}^{2\omega}$ can have components due to the effect that produces parts of SF UMR in $R_{xx}^{2\omega}$. If we include the phase shift in φ (like between AMR and the PHE) and account for higher-frequency terms with φ , all four coefficients in Eqs. (3a)–(3d) are extended:

$$A(H) = R_P \frac{H_{FL}}{H} + A_u(H), \quad (4a)$$

$$B(H) = \frac{R_A}{2} \frac{H_{AD}}{H + K} + B_u(H) + R_{ANE}, \quad (4b)$$

$$C(H) = \frac{-R_a}{2} \frac{H_{FL}}{H} + C_u(H), \quad (4c)$$

$$D(H) = D_u(H) + R_{ANE2}. \quad (4d)$$

A_u , B_u , C_u , and D_u are the field-dependent SF UMR contributions and their Hall counterparts. The specific form of their decay has been proposed to be of the form H^{-p} [34,55] or $(H + H_0)^{-1}$ [55], and was even calculated with the use of Boltzmann’s formalism in topological insulators [56,57] exploiting spin-momentum locking. For this (first) approach we use

$$X_u(H) = X_{u0} \left(\frac{K}{H} \right)^p, \quad (5)$$

where $X = A, B, C, D$ [58], since this H dependence describes our data considerably better than $(H + H_0)^{-1}$. The field-independent contributions of spin-dependent UMR and its counterpart to B and D are now included in R_{ANE} and R_{ANE2} .

In Fig. 3 we show the results of the fits to the experimentally determined values of A , B , C , and D using Eqs. (4a)–(4d) and (5) as solid lines. They are a significant improvement over the dashed lines that represent the fits lacking X_u [Eqs. (3a)–(3d)], especially in amplitude $D(H)$, and the residuals are at least an order of magnitude smaller for most fields. However, at high fields A and C do not converge to zero; the UMR contribution has been observed to deviate from Eq. (5) [59], but inclusion of H -linear terms as mentioned in Ref. [59] does not improve the results.

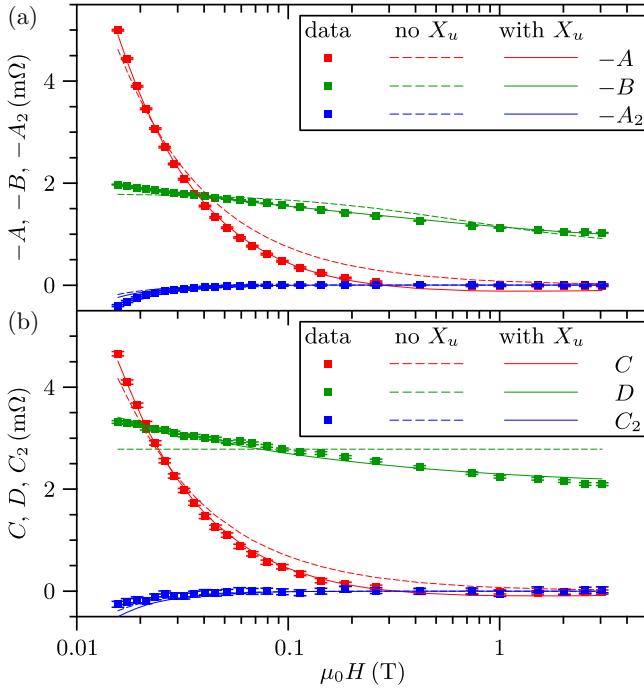


FIG. 3. Comparison of fits without and with H^{-p} -type X_u corrections. Amplitudes of (a) $R_{xy}^{2\omega}$ and (b) $R_{xx}^{2\omega}$ plotted as square symbols, determined from φ_H rotations with use of Eqs. (2a), (2b), and (S12). Fits to the field dependence without X_u are shown as dashed lines [Eqs. (3a)–(3d) and (S13)], and fits with X_u following Eqs. (4a)–(4d), (S13), and (5) are shown as solid lines. The field axis is logarithmic for visibility.

The fit parameters are summarized in Table I. Firstly, H_{FL} is negative, which means the effective field is parallel to $+y$ instead of σ being parallel to $-y$, and is likely dominated by H_{Oe} . The SOT fields are quite different after inclusion of X_u (compare the data in the second and fourth columns), especially H_{AD} , which is reduced from 8.4 mT to approximately 3 mT. The effect of the K_2 anisotropy is relatively small (see the third and fourth columns).

The exponent $p \approx 0.39$ is less than 1, similarly to that in Ref. [34]. For the role of the X_{u0} amplitudes, let us take A_{u0} in the fourth column (fifth row). Its 2HH component $A_{u0}K^pH^{-p} > 0$ has a sign opposite that of the corresponding SOT contribution $R_pH_{FL}H^{-1}$. Therefore, the total A plotted as a function of H^{-1} would exhibit a shallow start to the net curve, which becomes approximately linear later. The effect in C is equivalent but with opposite sign as $C_{u0} < 0$. This is indeed what we observe in Figs. 2(b) and 2(c).

The symmetry argument discussed above suggests that $|A_{u0}| = |C_{u0}|$ and $|B_{u0}| = |D_{u0}|$. However, since their origins may have different symmetries—consider that the AHE has no counterpart in R_{xx} —we set no restrictions on their values. From Table I, the values of $|A_{u0}|$ and $|C_{u0}|$ are comparable, while $|D_{u0}|$ is almost twice as large as $|B_{u0}|$.

The second explanation involves lengthwise modulation of the magnetization. This explanation is based on the fact that the transfer of angular momentum modifies the length of $\mathbf{M}$ [35,60] in addition to its direction (via antidamping torque). This concept has been expanded in recent publications [53,54], where it is used to describe deviations of the 2HH data from what is expected, similarly to what we presented above. Specifically, the length $M = |\mathbf{M}|$ is modified through current-induced magnon generation and annihilation in the form $-\Delta M(I)\sigma \cdot \mathbf{M}$. Like in Refs. [34,55], electron-magnon scattering (MMR) plays a role in UMR. In this picture, part of $R_{xx}^{2\omega}$ is attributed to the current modulation of MMR as ΔM oscillates with the current. However, AMR also contributes as its magnitude depends on M [61–64]. Moreover, similar terms appear in $R_{xy}^{2\omega}$ due to the PHE. Calculation of their contributions leads to the same φ dependences in the 2ω signal as in Eqs. (2a) and (2b) (see Sec. 6 in Supplemental Material [36]); therefore, they appear as the terms A_u , B_u , C_u , and D_u in Eqs. (4a)–(4d). The major difference is that a predefined field dependence is not imposed on ΔM ; rather, it is a free parameter at each H value.

TABLE I. Relevant fit parameters without X_u , and with H^{-p} -type X_u without and with K_2 anisotropy. All fits were based on H dependence only. H_{FL} was shared as a parameter between the fitting functions for A , A_2 , C , and C_2 . p was shared by the functions for A , B , C , and D .

	No X_u	With H^{-p} -type X_u without K_2 anisotropy	With H^{-p} -type X_u with K_2 anisotropy
$\mu_0 K_2$ (mT) from R_{xy}	0.63 ± 0.02	0	0.63 ± 0.02
$\mu_0 K_2$ (mT) from R_{xx}	1.45 ± 0.03	0	1.45 ± 0.03
$\mu_0 H_{FL}$ (mT)	-0.37 ± 0.01	-0.46 ± 0.01	-0.49 ± 0.02
$\mu_0 H_{AD}$ (mT)	8.4 ± 2.3	3.0 ± 0.07	3.0 ± 1.5
A_{u0} (mΩ)	0	0.21 ± 0.01	0.25 ± 0.02
B_{u0} (mΩ)	0	-0.16 ± 0.04	-0.18 ± 0.08
C_{u0} (mΩ)	0	-0.19 ± 0.01	-0.21 ± 0.02
D_{u0} (mΩ)	0	0.31 ± 0.05	0.32 ± 0.09
p	0	0.43 ± 0.03	0.39 ± 0.06
Equations	(3a)–(3d), (S13)	(4a)–(4d), (5)	(4a)–(4d), (S13), (5)
Figure	3	...	3

We use this second approach to achieve a better description of our measurements and more accurate values for H_{FL} and H_{AD} . The coefficients of the relative magnetization change ΔM are not arbitrary. As mentioned above, they originate from the magnetization dependence of the AMR, PHE, and MMR (see Sec. 6 in Supplemental Material [36]). As a result, the UMR and related components of Eqs. (4a)–(4d) are as follows:

$$A_u(H) = R_P \Delta M, \quad (6a)$$

$$B_u(H) = -R_P \Delta M, \quad (6b)$$

$$C_u(H) = -\frac{R_a}{2} \Delta M, \quad (6c)$$

$$D_u(H) = \frac{R_M}{2} \Delta M. \quad (6d)$$

R_M describes the MMR contribution; however, it is difficult to extract it from R_0 in Eq. (1b). Therefore, we treat it as a free parameter independent of H . Besides ΔM , R_P and R_a also depend on the field H and change by 10% in the range studied, as determined from fits to the first-harmonic data (plotted in Fig. S3).

The curve fits using Eqs. (6a)–(6d) are shown in Fig. 4. To achieve a small relative error in the most sensitive parameter, H_{AD} , we performed a global fit. This means φ and the H dependence of the second harmonics $R_{xx,xy}^{2\omega}$ were optimized simultaneously instead of “local” fitting to $X(H)$. In general, the fitted curves are better than with the first approach (Fig. 3), for example, at high fields, where the latter showed nonzero A and C . $\Delta M(H)$ is plotted in Fig. 4(c): as expected, ΔM decays fast to zero with increasing H . The log-log scale inset for $R_P \Delta M$ calculated from it illustrates that a power-law dependence of UMR-like corrections with H may work only in the few-hundred-millitesla range. The fit to D is also clearly better in the second approach.

The results are summarized in the last column in Table II. $H_{\text{AD}} \approx 0.49 \pm 0.11$ mT is significantly different from that estimated by the first approach (3.0 ± 1.5 mT; see the fourth column in Table I). The fact that the fits to A and C according to the first approach do not converge to zero, together with the difference between the two results, demonstrates that the magnonic contribution cannot be described adequately by Eq. (5), and that the second approach [Eqs. (6a)–(6d)] should be used to estimate SOT. The global fit greatly reduces the relative error of H_{AD} compared with the local fit (0.38 ± 0.96 mT, Table II, second column), but does not influence the result by a serious amount. Inclusion of the uniaxial anisotropy K_2 in the SOT analysis is crucial, as shown from comparison of the data in the fourth column in Table II with the data in the third column. Without it, H_{AD} is almost doubled. Similarly, we found that not taking the field dependence of R_a

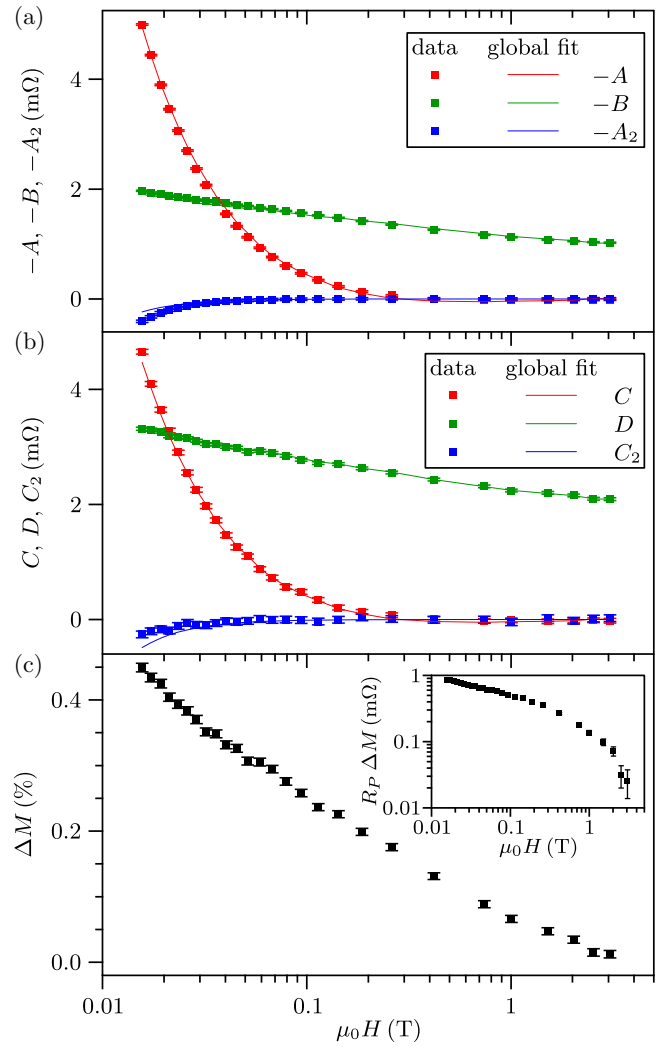


FIG. 4. Global fits with ΔM -type corrections. (a) Amplitudes of $R_{xy}^{2\omega}$ and (b) $R_{xx}^{2\omega}$ (symbols), following Eqs. (2a), (2b) and (S12). Solid lines are the results of global fits (to φ and H dependence together) following Eqs. (4a)–(4d), (S13), and (6a)–(6d). (c) Relative magnetization change ΔM induced by the current decays toward zero at high fields. The inset shows $R_P \Delta M$ on a log-log scale.

and R_P into account and treating them as constants would approximately double H_{AD} .

Let us compare the amplitudes A_{u0} , B_{u0} , C_{u0} , and D_{u0} in Table I with the coefficients of ΔM in Eqs. (6a)–(6d), which is the most strongly field-dependent quantity there. We find that the signs match. Moreover, $C_{u0} \approx -A_{u0}$, similarly to $C_u \approx -A_u$ in Eqs. (6a) and (6c). This is consistent with the symmetry argument since they come from the same effect (AMR and PHE). B_u and D_u stand out since they are affected by the AHE and MMR, which lack a symmetrical counterpart in R_{xx} and R_{xy} . This supports the validity of the second approach.

TABLE II. Relevant fit parameters with the second approach, i.e., ΔM -type X_u corrections. The second column shows the results of a local fit, i.e., based on H dependence only, similarly to Table I. In contrast, the third and fourth columns show results from global fits that take into account both φ and H dependence, without and with K_2 anisotropy, respectively.

	Local fit	Global fit without K2 anisotropy	Global fit with K2 anisotropy
$\mu_0 K_2$ (mT) from R_{xy}	0.63 ± 0.02	0	0.63 ± 0.02
$\mu_0 K_2$ (mT) from R_{xx}	1.45 ± 0.03	0	1.45 ± 0.03
$\mu_0 H_{FL}$ (mT)	-0.478 ± 0.008	-0.453 ± 0.001	-0.471 ± 0.001
$\mu_0 H_{AD}$ (mT)	0.38 ± 0.96	0.96 ± 0.13	0.49 ± 0.11
Equations	(4a)–(4d), (S13), (6a)–(6d)	(4a)–(4d), (6a)–(6d)	(4a)–(4d), (S13), (6a)–(6d)
Figure	...	...	4

The question arises of whether we can select a field range where we can ignore UMR and its Hall counterpart and Eqs. (3a)–(3d) can be applied, as has generally been done. It is not the low-field regime, as ΔM is strongest here. Moreover, because of the fast decay of A and C , a dense selection of H points is required for accuracy. For fitting B only at high fields, the contributions of the AD-SOT and B_u terms are both on the order of submilliohms and remain relevant, as is also evident in the still observable field dependence of D , which comes only from ΔM . Consequently, a wide range of H is required.

To compare our results with others in the literature, we analyzed the current-generated SOT fields. We calculated the spin-torque conductivity related to the AD torque, $\sigma_s = 2e\hbar^{-1}\mu_0 H_{AD} M_s t_{Py} / (I_0 \bar{R}_{xx} / L)$, where μ_0 is the vacuum permeability, t_{Py} is the thickness of the Py layer, and $M_s = K \approx 5.45 \times 10^5$ A/m is the saturation magnetization (since crystalline anisotropy is negligible [65]), which is comparable to the literature [66]. The denominator is the peak electric field calculated from the peak amplitude of the alternating current $I_0 = \sqrt{2} \times 2.5$ mA, the average longitudinal resistance $\bar{R}_{xx} = R_0 + R_a/2$, and the separation of the side contacts L . In other words, this is the current density of the angular momentum absorbed in Py in the tilting of $\mathbf{M}$ by $\Delta\theta$, normalized by the electric field and converted to the unit of siemens. The end result is $\sigma_s \approx (6.7 \pm 1.5) \times 10^4$ S/m, a value smaller but comparable to those in Ref. [67].

We also estimate the Oersted field $H_{Oe} = I_{Pt}/2W$. Since the current in Pt is the four-terminal voltage drop divided by the resistance of the Pt layer, $I_{Pt} = I_0 \bar{R}_{xx} W t_{Pt} / L \rho_{Pt} \approx \sqrt{2} \times 1.75$ mA, where we used $\rho_{Pt} \approx 35 \mu\Omega$ cm as the resistivity of a $t_{Pt} = 3$ -nm-thick Pt film [68], and as a result, we get $\mu_0 H_{Oe} = 0.389$ mT. Since this is comparable to $-\mu_0 H_{FL}$ in Table II, the actual effective field of the FL torque is likely much smaller.

IV. SUMMARY

We have studied several Py/Pt bilayer Hall bars with in-plane rotations in a wide range of magnetic fields. We have found that in the second-harmonic Hall resistance the amplitudes A and B contain unexpected field-dependent

components. We attribute them to a transverse counterpart of the magnonic unidirectional magnetoresistance and show that they are critical in accurately estimating SOT fields. We demonstrated two approaches: one based on symmetry arguments, with a H^{-p} -type dependence already used to describe UMR in $R_{xx}^{2\omega}$, and the other based on the concept that the angular momentum transfer leading to the antidamping torque and a perpendicular change in $\mathbf{M}$ also produces a lengthwise (parallel) change, ΔM . This approach was expanded in Refs. [53,54] and further developed here by inclusion of the shape anisotropy and the H dependence of R_a and R_P , which have a significant effect. The second approach yields a superior match to the observations as $\Delta M(H)$ is not described well by the phenomenological form H^{-p} . Fitting simultaneously to both φ and the H dependence greatly reduced the relative error of H_{AD} , the parameter that usually comes with a large error and appears the most sensitive to model corrections.

Our comparison of the two approaches has shown that the second one provides the correct way to evaluate SOT. We expect that this method will lead to an improved description of second-harmonic magnetoresistance effects and a more accurate determination of spin-orbit-torque fields. In turn, this will support research into novel materials for the development of SOT-based devices.

V. METHODS

Hall bars were fabricated on SiO₂-covered wafers from 6 nm Py (Ni₈₀Fe₂₀) and 3 nm Pt via standard electron-beam lithography and electron-beam evaporation techniques. They were contacted by the deposition of 5 nm Ti and 50 nm Au in a subsequent step. The width W of the Hall bars is 4 μ m. The longitudinal separation L of the side contacts is 4 μ m and their width w is 1 μ m. For SOT analysis of Hall data, there is a geometrical correction factor [69] due to the current density distribution at the Hall voltage contacts. In our case $w/W \ll 1$; therefore, we expect a less than 10% correction, which we ignore.

The electrical measurements were performed at room temperature with SR830 (Stanford Research Systems), MFLI (Zurich Instruments), and HF2LI (Zurich Instruments) lock-in amplifiers and Basel Precision Instruments

SP1004 low-noise, low-drift differential amplifiers. The current used for the resistance measurements was $I_{\text{rms}} \approx 2.5$ mA at $f = 132.43$ Hz. On the basis of measurements as a function of temperature on similar samples, we estimate that this induced a temperature increase of at most 50 K in the sample due to Joule heating. This is not expected to notably change the saturation magnetization [70,71]. The frequency f used is low enough for the second-harmonic technique. Because of the high thermal diffusivity of Pt and Py, $\alpha > 1.6 \times 10^{-6}$ m²/s [72–74], the thermal diffusion length $l = \sqrt{\alpha/f}$ is on the order of hundreds of micrometers. This is much larger than the dimensions L and W . Moreover, heat generation is distributed over the device; therefore, it can be considered to be in thermal equilibrium on the timescale of the measurement. This is confirmed by frequency-dependent measurements shown in Fig. S5 in Supplemental Material [36].

The samples were fixed on a rotating stage within a Janis 9TL-HRTB-30 superconducting magnet's bore. The actual field of the magnet was measured *in situ* by a Hall sensor, or corrected by a calibration curve taken by the same sensor. In-plane rotations were performed in both directions at all fields so that electrical signal drift or mechanical failures such as jumps or torsion may be noticed. All rotation measurements were performed as a series of decreasing fields to ensure a single magnetic domain. Raw data of measurements in the main text are available online [75].

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DATA AVAILABILITY

The data that support the findings of this article are openly available [75].

- [1] B. Dieny *et al.*, Opportunities and challenges for spintronics in the microelectronics industry, *Nat. Electron.* **3**, 446 (2020).
- [2] V. Krizakova, M. Perumkunnal, S. Couet, P. Gambardella, and K. Garello, Spin-orbit torque switching of magnetic tunnel junctions for memory applications, *J. Magn. Magn. Mater.* **562**, 169692 (2022).
- [3] S. Bhatti, R. Sbiaa, A. Hirohata, H. Ohno, S. Fukami, and S. N. Piramanayagam, Spintronics based random access memory: A review, *Mater. Today* **20**, 530 (2017).
- [4] Y. Liu and G. Yu, MRAM gets closer to the core, *Nat. Electron.* **2**, 555 (2019).
- [5] J. Sinova, S. O. Valenzuela, J. Wunderlich, C. H. Back, and T. Jungwirth, Spin Hall effects, *Rev. Mod. Phys.* **87**, 1213 (2015).
- [6] C. Bi, N. Sato, and S. X. Wang, *Spin-orbit torque magnetoresistive random-access memory (SOT-MRAM)* (Woodhead Publishing, Sawston, Cambridge (UK), 2019), p. 203.
- [7] A. Manchon, J. Železný, I. M. Miron, T. Jungwirth, J. Sinova, A. Thiaville, K. Garello, and P. Gambardella, Current-induced spin-orbit torques in ferromagnetic and antiferromagnetic systems, *Rev. Mod. Phys.* **91**, 35004 (2019).
- [8] V. D. Nguyen, S. Rao, K. Wostyn, and S. Couet, Recent progress in spin-orbit torque magnetic random-access memory, *npj Spintronics* **2**, 48 (2024).
- [9] L. Liu, T. Moriyama, D. C. Ralph, and R. A. Buhrman, Spin-torque ferromagnetic resonance induced by the spin Hall effect, *Phys. Rev. Lett.* **106**, 036601 (2011).
- [10] S. Karimeddiny, J. A. Mittelstaedt, R. A. Buhrman, and D. C. Ralph, Transverse and longitudinal spin-torque ferromagnetic resonance for improved measurement of spin-orbit torque, *Phys. Rev. Appl.* **14**, 024024 (2020).
- [11] K. Garello, I. M. Miron, C. O. Avci, F. Freimuth, Y. Mokrousov, S. Blügel, S. Auffret, O. Boulle, G. Gaudin, and P. Gambardella, Symmetry and magnitude of spin-orbit torques in ferromagnetic heterostructures, *Nat. Nanotechnol.* **8**, 587 (2013).
- [12] M. Hayashi, J. Kim, M. Yamanouchi, and H. Ohno, Quantitative characterization of the spin-orbit torque using harmonic Hall voltage measurements, *Phys. Rev. B* **89**, 144425 (2014).
- [13] X. Fan, J. Wu, Y. Chen, M. J. Jerry, H. Zhang, and J. Q. Xiao, Observation of the nonlocal spin-orbital effective field, *Nat. Commun.* **4**, 1799 (2013).
- [14] M. I. Dyakonov, *Spin Physics in Semiconductors* (Springer International Publishing, Berlin, 2017).
- [15] O. Mosendz, J. E. Pearson, F. Y. Fradin, G. E. W. Bauer, S. D. Bader, and A. Hoffmann, Quantifying spin Hall angles from spin pumping: Experiments and theory, *Phys. Rev. Lett.* **104**, 046601 (2010).
- [16] U. H. Pi, K. W. Kim, J. Y. Bae, S. C. Lee, Y. J. Cho, K. S. Kim, and S. Seo, Tilting of the spin orientation induced by Rashba effect in ferromagnetic metal layer, *Appl. Phys. Lett.* **97**, 162507 (2010).
- [17] K.-U. Demasius, T. Phung, W. Zhang, B. P. Hughes, S.-H. Yang, A. Kellock, W. Han, A. Pushp, and S. S. P. Parkin, Enhanced spin-orbit torques by oxygen incorporation in tungsten films, *Nat. Commun.* **7**, 10644 (2016).
- [18] Y. Wang, T. Taniguchi, P.-H. Lin, D. Zicchino, A. Nickl, J. Sahliger, C.-H. Lai, C. Song, H. Wu, Q. Dai, and C. H.

- Back, Time-resolved detection of spin-orbit torque switching of magnetization and exchange bias, *Nat. Electron.* **5**, 840 (2022).
- [19] H. Yang, S. O. Valenzuela, M. Chshiev, S. Couet, B. Dieny, B. Dlubak, A. Fert, K. Garello, M. Jamet, D.-E. Jeong, K. Lee, T. Lee, M.-B. Martin, G. S. Kar, P. S  n  r, H.-J. Shin, and S. Roche, Two-dimensional materials prospects for non-volatile spintronic memories, *Nature* **606**, 663 (2022).
- [20] T. Guillet, R. Galceran, J. F. Sierra, F. J. Belarre, B. Ballesteros, M. V. Costache, D. Dosenovic, H. Okuno, A. Marty, M. Jamet, F. Bonell, and S. O. Valenzuela, Spin-orbit torques and magnetization switching in $(\text{Bi}, \text{Sb})_2\text{Te}_3/\text{Fe}_3\text{GeTe}_2$ heterostructures grown by molecular beam epitaxy, *Nano Lett.* **24**, 822 (2024).
- [21] L. Pandey, B. Zhao, K. Tenzin, R. Ngaloy, V. Lamparsk  , H. Bangar, A. Ali, M. Abdel-Hafiez, G. Zhang, H. Wu, H. Chang, L. S  jstr  m, P. Rout, J. S  lwi  ska, and S. P. Dash, Tunable unconventional spin orbit torque magnetization dynamics in van der Waals heterostructures, *Nat. Commun.* **16**, 8722 (2025).
- [22] M. H. D. Guimar  es, G. M. Stiehl, D. MacNeill, N. D. Reynolds, and D. C. Ralph, Spin-orbit torques in NbSe_2 /permalloy bilayers, *Nano Lett.* **18**, 1311 (2018).
- [23] Q. Shao, G. Yu, Y. W. Lan, Y. Shi, M. Y. Li, C. Zheng, X. Zhu, L. J. Li, P. K. Amiri, and K. L. Wang, Strong Rashba-Edelstein effect-induced spin-orbit torques in monolayer transition metal dichalcogenide/ferromagnet bilayers, *Nano Lett.* **16**, 7514 (2016).
- [24] D. MacNeill, G. M. Stiehl, M. H. D. Guimaraes, R. A. Buhrman, J. Park, and D. C. Ralph, Control of spin-orbit torques through crystal symmetry in WTe_2 /ferromagnet bilayers, *Nat. Phys.* **13**, 300 (2017).
- [25] M. Alghamdi, M. Lohmann, J. Li, P. R. Jothi, Q. Shao, M. Aldosary, T. Su, B. P. T. Fokwa, and J. Shi, Highly efficient spin-orbit torque and switching of layered ferromagnet Fe_3GeTe_2 , *Nano Lett.* **19**, 4400 (2019).
- [26] J. Hidding and M. H. D. Guimar  es, Spin-orbit torques in transition metal dichalcogenide/ferromagnet heterostructures, *Front. Mater.* **7**, 594771 (2020).
- [27] L. Bainsla, B. Zhao, N. Behera, A. M. Hoque, L. S  jstr  m, A. Martinelli, M. Abdel-Hafiez, J.   kerman, and S. P. Dash, Large out-of-plane spin-orbit torque in topological Weyl semimetal TaIrTe_4 , *Nat. Commun.* **15**, 4649 (2024).
- [28] C. O. Avci, K. Garello, M. Gabureac, A. Ghosh, A. Fuhrer, S. F. Alvarado, and P. Gambardella, Interplay of spin-orbit torque and thermoelectric effects in ferromagnet/normal-metal bilayers, *Phys. Rev. B* **90**, 224427 (2014).
- [29] G. E. Bauer, E. Saitoh, and B. J. V. Wees, Spin caloritronics, *Nat. Mater.* **11**, 391 (2012).
- [30] B. L. Zink, Thermal effects in spintronic materials and devices: An experimentalist's guide, *J. Magn. Magn. Mater.* **564**, 170120 (2022).
- [31] C. O. Avci, K. Garello, A. Ghosh, M. Gabureac, S. F. Alvarado, and P. Gambardella, Unidirectional spin Hall magnetoresistance in ferromagnet/normal metal bilayers, *Nat. Phys.* **11**, 570 (2015).
- [32] S. Langenfeld, V. Tshitoyan, Z. Fang, A. Wells, T. A. Moore, and A. J. Ferguson, Exchange magnon induced resistance asymmetry in permalloy spin-Hall oscillators, *Appl. Phys. Lett.* **108**, 192402 (2016).
- [33] T. Li, S. Kim, S. J. Lee, S. W. Lee, T. Koyama, D. Chiba, T. Moriyama, K. J. Lee, K. J. Kim, and T. Ono, Origin of threshold current density for asymmetric magnetoresistance in Pt/Py bilayers, *Appl. Phys. Express* **10**, 073001 (2017).
- [34] C. O. Avci, J. Mendil, G. S. D. Beach, and P. Gambardella, Origins of the unidirectional spin Hall magnetoresistance in metallic bilayers, *Phys. Rev. Lett.* **121**, 087207 (2018).
- [35] T. Kodama, N. Kikuchi, T. Chiba, S. Okamoto, S. Ohno, and S. Tomita, Direct observation of current-induced nonlinear spin torque in Pt-Py bilayers, *Phys. Rev. B* **109**, 214419 (2024).
- [36] The Supplemental Material contains derivations of equations, as well as additional measurements, figures, and further discussion of details, and may be found at <http://link.aps.org/supplemental/10.1103/x78x-qt5z>.
- [37] P. Ritzinger and K. V  born  y, Anisotropic magnetoresistance: Materials, models and applications, *R. Soc. Open. Sci.* **10**, 230564 (2023).
- [38] J. Kim, P. Sheng, S. Takahashi, S. Mitani, and M. Hayashi, Spin Hall magnetoresistance in metallic bilayers, *Phys. Rev. Lett.* **116**, 097201 (2016).
- [39] S. V  lez, V. N. Golovach, A. Bedoya-Pinto, M. Isasa, E. Sagasta, M. Abadia, C. Rogero, L. E. Hueso, F. S. Bergeret, and F. Casanova, Hanle magnetoresistance in thin metal films with strong spin-orbit coupling, *Phys. Rev. Lett.* **116**, 016603 (2016).
- [40] Y. Yang, Z. Luo, H. Wu, Y. Xu, R. W. Li, S. J. Pennycook, S. Zhang, and Y. Wu, Anomalous Hall magnetoresistance in a ferromagnet, *Nat. Commun.* **9**, 2255 (2018).
- [41] T. G. S. M. Rijks, S. K. J. Lenczowski, R. Coehoorn, and W. J. M. de Jonge, In-plane and out-of-plane anisotropic magnetoresistance in $\text{Ni}_{80}\text{Fe}_{20}$ thin films, *Phys. Rev. B* **56**, 362 (1997).
- [42] A. Kobs, S. He  e, W. Kreuzpaintner, G. Winkler, D. Lott, P. Weinberger, A. Schreyer, and H. P. Oepen, Anisotropic interface magnetoresistance in Pt/Co/Pt sandwiches, *Phys. Rev. Lett.* **106**, 217207 (2011).
- [43] M. G. Kang, G. Go, K. W. Kim, J. G. Choi, B. G. Park, and K. J. Lee, Negative spin Hall magnetoresistance of normal metal/ferromagnet bilayers, *Nat. Commun.* **11**, 3619 (2020).
- [44] A. P. Mihai, J. P. Attan  , A. Marty, P. Warin, and Y. Samson, Electron-magnon diffusion and magnetization reversal detection in FePt thin films, *Phys. Rev. B* **77**, 060401(R) (2008).
- [45] D. MacNeill, G. M. Stiehl, M. H. D. Guimar  es, N. D. Reynolds, R. A. Buhrman, and D. C. Ralph, Thickness dependence of spin-orbit torques generated by WTe_2 , *Phys. Rev. B* **96**, 054450 (2017).
- [46] V. D. Ky, The planar Nernst effect in permalloy films, *Phys. Status Solidi (b)* **17**, K207 (1966).
- [47] P. B. Jayatilaka, D. D. Belyea, T. J. Fawcett, and C. W. Miller, Anisotropic magnetothermopower in ferromagnetic thin films grown on macroscopic substrates, *J. Magn. Magn. Mater.* **382**, 376 (2015).
- [48] O. Reimer, D. Meier, M. Bovender, L. Helmich, J.-O. Dreessen, J. Krieff, A. S. Shestakov, C. H. Back, J.-M.

- Schmalhorst, A. Hütten, G. Reiss, and T. Kuschel, Quantitative separation of the anisotropic magnetothermopower and planar Nernst effect by the rotation of an in-plane thermal gradient, *Sci. Rep.* **7**, 40586 (2017).
- [49] K. I. Uchida, H. Adachi, T. Ota, H. Nakayama, S. Maekawa, and E. Saitoh, Observation of longitudinal spin-Seebeck effect in magnetic insulators, *Appl. Phys. Lett.* **97**, 172505 (2010).
- [50] A. Slachter, F. L. Bakker, J. P. Adam, and B. J. V. Wees, Thermally driven spin injection from a ferromagnet into a non-magnetic metal, *Nat. Phys.* **6**, 879 (2010).
- [51] Y. Pu, E. Johnston-Halperin, D. D. Awschalom, and J. Shi, Anisotropic thermopower and planar Nernst effect in $\text{Ga}_{1-x}\text{Mn}_x\text{As}$ ferromagnetic semiconductors, *Phys. Rev. Lett.* **97**, 036601 (2006).
- [52] C. O. Avci, K. Garello, J. Mendil, A. Ghosh, N. Blasakis, M. Gabureac, M. Trassin, M. Fiebig, and P. Gambardella, Magnetoresistance of heavy and light metal/ferromagnet bilayers, *Appl. Phys. Lett.* **107**, 192405 (2015).
- [53] P. Noël, R. Schlitz, E. Karadža, C.-H. Lambert, L. Nesi, F. Binda, and P. Gambardella, Nonlinear longitudinal and transverse magnetoresistances due to current-induced magnon creation-annihilation processes, *Phys. Rev. Lett.* **134**, 146701 (2025).
- [54] P. Noel, E. Karadža, R. Schlitz, P. Welter, C.-H. Lambert, L. Nesi, F. Binda, C. L. Degen, and P. Gambardella, Estimation of spin-orbit torques in the presence of current-induced magnon creation and annihilation, *Phys. Rev. B* **111**, 144409 (2025).
- [55] Y. Xu, Y. Yang, Z. Luo, and Y. Wu, Disentangling magnon magnetoresistance from anisotropic and spin Hall magnetoresistance in NiFe/Pt bilayers, *Phys. Rev. B* **100**, 094413 (2019).
- [56] K. Yasuda, A. Tsukazaki, R. Yoshimi, K. S. Takahashi, M. Kawasaki, and Y. Tokura, Large unidirectional magnetoresistance in a magnetic topological insulator, *Phys. Rev. Lett.* **117**, 127202 (2016).
- [57] K. Yasuda, A. Tsukazaki, R. Yoshimi, K. Kondou, K. S. Takahashi, Y. Otani, M. Kawasaki, and Y. Tokura, Current-nonlinear Hall effect and spin-orbit torque magnetization switching in a magnetic topological insulator, *Phys. Rev. Lett.* **119**, 137204 (2017).
- [58] K in the numerator is added for the convenience of having a simple unit, Ω , for the coefficients.
- [59] L. Chen, K. Zollner, S. Parzefall, J. Schmitt, M. Kronseder, J. Fabian, D. Weiss, and C. H. Back, Connections between spin-orbit torques and unidirectional magnetoresistance in ferromagnetic-metal-heavy-metal heterostructures, *Phys. Rev. B* **105**, L020406 (2022).
- [60] V. E. Demidov, S. Urazhdin, E. R. J. Edwards, M. D. Stiles, R. D. McMichael, and S. O. Demokritov, Control of magnetic fluctuations by spin current, *Phys. Rev. Lett.* **107**, 107204 (2011).
- [61] W. Gerlach and K. Schneiderhan, Ferromagnetismus und elektrische Eigenschaften. I. Widerstand, magnetische Widerstandsänderung und wahre Magnetisierung beim Curie-Punkt, *Ann. Phys.* **398**, 772 (1930).
- [62] H. H. Potter, On the change of resistance of nickel in a magnetic field, *Proc. R. Soc. London Ser. A* **132**, 560 (1931).
- [63] N. F. Mott, The electrical conductivity of transition metals, *Proc. R. Soc. London Ser. A* **153**, 699 (1936).
- [64] A. Hamzic, S. Senoussi, I. A. Campbell, and A. Gert, The magnetoresistance of Pd-based dilute ferromagnetic alloys, *J. Phys. F: Metal Phys.* **8**, 1947 (1978).
- [65] L. F. Yin, D. H. Wei, N. Lei, L. H. Zhou, C. S. Tian, G. S. Dong, X. F. Jin, L. P. Guo, Q. J. Jia, and R. Q. Wu, Magnetocrystalline anisotropy in permalloy revisited, *Phys. Rev. Lett.* **97**, 067203 (2006).
- [66] K. Ounadjela, H. Lefakis, V. S. Speriosu, C. Hwang, and P. S. Alexopoulos, Thickness dependence of magnetization and magnetostriction of NiFe and NiFeRh films, *J. Phys. Colloques* **49**, 1709 (1988).
- [67] L. Liu, R. A. Buhrman, and D. C. Ralph, Review and analysis of measurements of the spin Hall effect in platinum, *ArXiv:1111.3702v3*.
- [68] S. Dutta, K. Sankaran, K. Moors, G. Pourtois, S. V. Elshocht, J. Bömmels, W. Vandervorst, Z. Tőkei, and C. Adelmann, Thickness dependence of the resistivity of platinum-group metal thin films, *J. Appl. Phys.* **122**, 25107 (2017).
- [69] L. Neumann and M. Meinert, Influence of the Hall-bar geometry on harmonic Hall voltage measurements of spin-orbit torques, *AIP Adv.* **8**, 95320 (2018).
- [70] X. Zhang, Y. Lao, J. Sklenar, N. S. Bingham, J. T. Batley, J. D. Watts, C. Nisoli, C. Leighton, and P. Schiffer, Understanding thermal annealing of artificial spin ice, *APL Mater.* **7**, 111112 (2019).
- [71] A. Devonport, A. Vishina, R. K. Singh, M. Edwards, K. Zheng, J. Domenico, N. D. Rizzo, C. Kopas, M. van Schilf-gaarde, and N. Newman, Magnetic properties of chromium-doped $\text{Ni}_{80}\text{Fe}_{20}$ thin films, *J. Magn. Magn. Mater.* **460**, 193 (2018).
- [72] Wolfram Research, Wolfram Research: Technical data for platinum, <https://periodictable.com/Elements/078/data.html>.
- [73] A. D. Avery, S. J. Mason, D. Bassett, D. Wesenberg, and B. L. Zink, Thermal and electrical conductivity of approximately 100-nm permalloy, Ni, Co, Al, and Cu films and examination of the Wiedemann-Franz law, *Phys. Rev. B* **92**, 214410 (2015).
- [74] ESPI Metals, ESPI Metals: Permalloy 80 properties, [https://www.espimetals.com/index.php/technical-data/175-Permalloy 80](https://www.espimetals.com/index.php/technical-data/175-Permalloy%2080).
- [75] E. Tóvári, Raw data for measurements in the main text, <https://doi.org/10.5281/zenodo.18310809> (2026).