

Coherence limits in interference-based $\cos(2\varphi)$ qubitsS. Messelot^{1,*}, A. Leblanc^{2,†}, J.-S. Tettekpoe¹, F. Lefloch², Q. Ficheux^{1,‡}, J. Renard¹ and É. Dumur^{2,§}¹Univ. Grenoble Alpes, CNRS, Grenoble INP, Institut Néel, 38000, Grenoble, France²Univ. Grenoble Alpes Grenoble, CEA, Grenoble INP, IRIG, PHELIQS, 38000, Grenoble, France

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We investigate the coherence properties of parity-protected $\cos(2\varphi)$ qubits based on interference between two Josephson elements in a superconducting loop. We show that qubit implementations of a $\cos(2\varphi)$ potential using a single loop, such as those employing semiconducting junctions, rhombus circuits, Flowermon, and KITE structures, can be described by the same Hamiltonian as two multiharmonic Josephson junctions in a SQUID geometry. We find that despite the parity protection arising from the suppression of single Cooper-pair tunneling, there exists a fundamental trade-off between charge and flux-noise dephasing channels. Using numerical simulations, we examine how relaxation and dephasing rates depend on external flux and circuit parameters, and we identify the best compromise for maximum coherence. With currently existing circuit parameters, the qubit lifetime T_1 can exceed milliseconds, while the dephasing time T_φ remains limited to only a few microseconds due to either flux or charge noise. Our findings establish practical limits on the coherence of this class of qubits and raise questions about the long-term potential of this approach.

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The transmon qubit [1] is arguably the most widely used superconducting qubit due to its simplicity and long coherence times, with state-of-the-art decay and dephasing times reaching $T_1, T_\varphi \sim 1$ ms [2,3]. Interestingly, the transmon qubit was originally introduced as a “charge-insensitive qubit” extending the charge noise protection of the quantronium [4] from first to all orders. As the ratio of Josephson energy to charging energy, E_J/E_C , increases, the sensitivity to charge noise decreases exponentially, providing excellent protection against dephasing. However, the qubit remains susceptible to energy decay via electrical coupling to environmental degrees of freedom, owing to its large charge matrix element on the order of unity.

Noise protection against dephasing can be extended to create qubits that are also resistant to energy relaxation [5–9]. The key concept behind this protection is to leverage the internal symmetry of the system, encoding the ground and excited states with wave functions that have different parities [10–15]. For example, the

Hamiltonian of the circuit can be engineered to preserve the parity of the number of Cooper pairs by only allowing the coherent tunneling of pairs of Cooper pairs through an operator $|N\rangle\langle N+2| + |N+2\rangle\langle N|$, which couples the charge states of the same parity. This is known as the $\cos(2\varphi)$ qubit. In practice, it resembles a transmon, but instead of a Josephson junction with a 2π -periodic potential $-E_J \cos(\varphi)$, we have a π -periodic circuit element. This element allows only pairs of Cooper pairs to be transmitted, resulting in a potential described by $E_{J_2} \cos(2\varphi)$. The ground- and excited-state wavefunctions have different parities in the charge basis, leading to disjoint wavefunction supports and, therefore, suppressed matrix elements.

One method to engineer a π -periodic potential is to leverage phase interference between two circuit elements with nonsinusoidal current-phase relations embedded in a loop. When the loop is biased at half a magnetic flux quantum, the odd harmonics of the current interfere destructively, while the even harmonics remain.

Several interference-based parity-protected circuits have been proposed, such as the rhombus [6], the bifluxon [15], the kinetic interference cotunneling element (KITE) [16,17], and the $0-\pi$ [18]. The aforementioned circuits rely on standard aluminum-aluminum oxide (Al/AlOx) Josephson junctions, and current-phase relation (CPR) higher harmonics stem from the detailed architecture of the $\cos(2\varphi)$ element. Alternatively, intrinsically multiharmonic Josephson junctions based on semiconductor weak links have been demonstrated, using material platforms such as InAs nano-wires [19], InAs [20,21] and

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Ge/SiGe [22] two-dimensional electron gas, graphene [23,24], and other 2D materials [25]. Other intrinsically multiharmonic junctions include pinhole Al junctions [26] and twisted cuprate van der Waals heterostructures [27]. However, only a handful of experiments [9,28] have successfully demonstrated effective protection against energy relaxation using $\cos(2\varphi)$ elements. To date, none have managed to combine this protection with protection against dephasing. This naturally raises the question of potential decoherence mechanisms specific to such an implementation.

In this article, we conduct a comprehensive investigation of the coherence properties of a general model of $\cos(2\varphi)$ qubits, where Cooper-pair pairing is achieved through interference between two bi-harmonic Josephson elements. We study the resilience of the energy relaxation protection as a function of circuit parameters, imperfections, and finite temperature effects. Furthermore, we show that the introduction of a loop in the circuit brings an unavoidable trade-off between charge and flux-noise protection for the dephasing time. We show that biasing the circuit loop away from the frustration point can be advantageous, as it helps to balance the circuit's susceptibility to these two noise sources and allows to reach coherence times of several microseconds with currently experimentally accessible circuit parameters.

We start with a detailed description of the circuit, its Hamiltonian, and the various platforms that implement it. This is followed by a discussion of the effects of magnetic flux and charge offset. In the second part, we focus on energy relaxation protection mechanisms and their robustness to variations in circuit parameters. In the third part, we describe the specific coherence issue faced by the $\cos(2\varphi)$ qubit, and in the fourth part, we optimize the circuit parameters to achieve maximum coherence.

I. INTERFERENCE-BASED $\cos(2\varphi)$

A. Hamiltonian

The circuit investigated in this paper is composed of two bi-harmonic Josephson junctions, assembled in a superconducting quantum interference device (SQUID) configuration and shunted by a single capacitance. It is shown in Fig. 1. We use the definition of the junction's Josephson potential $\hat{U} = E_{J_1} \cos(\hat{\varphi}) + E_{J_2} \cos(2\hat{\varphi})$ [29]. With this definition, in typical multiharmonic junctions, $E_{J_1} < 0$ and $E_{J_2} > 0$. From standard circuit quantum electrodynamics (cQED) formalism [30], we find the Hamiltonian

$$\hat{H} = 4E_C(\hat{n} - n_g)^2 + E_{J_{A1}} \cos(\hat{\varphi}_A) + E_{J_{B1}} \cos(\hat{\varphi}_B) + E_{J_{A2}} \cos(2\hat{\varphi}_A) + E_{J_{B2}} \cos(2\hat{\varphi}_B) \quad (1)$$

with the electron charging energy $E_C = e^2/(2C)$, the Josephson energy $E_{J_{ij}} = \Phi_0 I_{c_{ij}}/(2\pi \times j)$ of junction i and

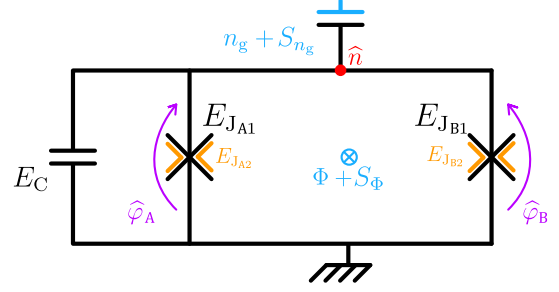


FIG. 1. Electrical circuit. The circuit is made of two junctions A and B in parallel. Each junction has its own Josephson energy (E_{J_i}), where $i = A, B$ denotes the junction and $j = 1, 2$ is the order of the harmonic of the Josephson junction. A large shunt capacitance creates the charging energy E_C . The circuit has three internal quantum operators: the charge number of the common metallic island $\hat{n}$, in red, and the two phases of the junctions $\hat{\varphi}_{A,B}$, in purple. It can be controlled via two classical external knobs shown in blue with the applied voltage, n_g , and the applied magnetic flux, Φ . The same control knobs couple to external noise densities denoted S_{n_g} and S_Φ , electrical and magnetic, respectively.

j -th harmonics, $\hat{n}$ is the quantum charge operator of the superconducting island, and n_g the classical charge offset. Similarly, $\hat{\varphi}_i$ is the quantum phase operator of junction i . Compared with a “pure” $\cos(2\varphi)$ Hamiltonian $\hat{H} = 4E_C(\hat{n} - n_g)^2 + E_{J_2} \cos(2\hat{\varphi})$, the “interference-based” $\cos(2\varphi)$ Hamiltonian encompasses the first-harmonic terms of each junction that come with any practical realization. The quantum phase operator $\hat{\varphi}_i$ of the two junctions obeys the flux quantization of the superconducting loop

$$\varphi_A - \varphi_B = 2\pi n + 2\pi \frac{\Phi}{\Phi_0}, \quad \text{with } n \in \mathbb{Z} \quad (2)$$

where Φ is the classical flux offset that can be externally controlled and Φ_0 is the magnetic flux quantum. We choose $\hat{\varphi} = (\hat{\varphi}_A + \hat{\varphi}_B)/2$ as the gauge for the phase degree of freedom, which satisfies the irrotational constraint [31] provided the two junctions have equal self capacitances (that we omitted in Fig. 1, for clarity). This condition is verified in the following as we consider a close to symmetric SQUID, which allows the direct calculation of coherence times [31]. Using Eq. (2), we can write the Hamiltonian of Eq. (1) as

$$\begin{aligned} \hat{H} = & 4E_C(\hat{n} - n_g)^2 \\ & + E_{J_{\Sigma 1}} \left[\cos(2\hat{\varphi}) \cos\left(2\pi\left(\delta\Phi + \frac{1}{2}\right)\right) \right. \\ & \left. + d_1 \sin(2\hat{\varphi}) \sin\left(2\pi\left(\delta\Phi + \frac{1}{2}\right)\right) \right] \\ & + E_{J_{\Sigma 2}} \left[\cos(2\hat{\varphi}) \cos\left(2\pi\left(\delta\Phi + \frac{1}{2}\right)\right) \right. \\ & \left. + d_2 \sin(2\hat{\varphi}) \sin\left(2\pi\left(\delta\Phi + \frac{1}{2}\right)\right) \right] \end{aligned} \quad (3)$$

with the SQUID Josephson energy $E_{J\Sigma_i} = E_{J_{Ai}} + E_{J_{Bi}}$ and the junction asymmetries $d_i = (E_{J_{Bi}} - E_{J_{Ai}})/(E_{J_{Bi}} + E_{J_{Ai}})$. As we will focus on variations of flux close to $\Phi = \Phi_0/2$ in the following discussion, we also introduce $\delta\Phi = \Phi/\Phi_0 - 1/2$, i.e., the offset flux with respect to the frustration point of the SQUID. We note that with the first two terms of Eq. (3), we recover the “split” transmon [1] while the last term is solely due to the second harmonic of the junctions. For simplicity, we will consider $d_1 = d_2 = d$ in the following, which amounts to assuming qualitatively similar Josephson elements A and B. In practice, only the first Josephson harmonics need precise symmetry to cancel out.

There are six circuit parameters, either set by design or controlled *in situ*: $E_{J\Sigma_2}/E_{J\Sigma_1}$, $E_{J\Sigma_2}/E_C$, $E_{J\Sigma_2} \times E_C$, $\delta\Phi$, n_g , and d . We will explore the effect of each of these parameters on the coherence properties of the $\cos(2\varphi)$ qubit, calculated using the scQubits [32] and QuTiP [33] Python packages.

B. Implementations

The circuit in Fig. 1 and its corresponding Hamiltonian in Eq. (3), while simple, encompass numerous experimental attempts to implement a $\cos(2\varphi)$ qubit, see Table I. We can see that the means used to implement a $\cos(2\varphi)$ are diverse, including superconducting circuits, but also semiconducting Josephson junctions and even heterostructures.

For each of the proposed implementations, we reported the $E_{J\Sigma_2}/E_{J\Sigma_1}$ ratio, see details in Appendix A [36]. It is interesting to note that, while very different in nature, all current efforts show a similar $E_{J\Sigma_2}/E_{J\Sigma_1}$ ratio of -0.1 , motivating the use of this value for the remainder of the paper. An exception to this, which we discuss at the end of the paper, is the Flowermon [27,35] which, if successfully implemented, could reach a ratio on the order of -15 .

TABLE I. Interference-based $\cos(2\varphi)$ implementations.

Circuit	$E_{J\Sigma_2}/E_{J\Sigma_1}$
Superconducting circuit based	
Rhombus [6,9]	-0.2
Pinhole JJ [26]	-0.1
KITE (low inductance) [17]	-0.025
KITE (high inductance) [16,34]	-0.6
Semiconductor	
Germanium [22]	-0.1
Graphene [24]	-0.1
InAs [19]	$-0.1 - -0.2$
Heterostructures	
^{a,b} Flowermon [27,35]	$\pm 0.1 - \pm 15$

^aOnly theoretical proposal at the time of this report.

^bThe sign of $E_{J\Sigma_1}$ may be positive or negative as opposed to other implementations.

C. Interferences

In Fig. 2, we show the magnetic flux dependence of the Josephson terms in Eq. (3). We see that as a function of the magnetic flux, the dominant Josephson harmonics, in blue and orange solid lines for the 1st and 2nd ones, are swapped. As a direct consequence, two regimes can be considered: near $\delta\Phi \sim \pm 0.5$ ($\Phi/\Phi_0 \sim 0$), the circuit is similar to a transmon, and near $\delta\Phi \sim 0$, the $\cos(2\varphi)$ behavior is the relevant one. Because of the inevitable junction asymmetry, out-of-phase supplemental terms arise, shown as dashed green and gold lines for the 1st and 2nd harmonics. The first and most important effect is that the 1st harmonic does not exactly cancel out at $\delta\Phi = 0$. Hence, it is of crucial importance to have highly symmetric junctions to keep the 2nd harmonic dominant in this regime. In the case of hybrid semiconductor Josephson junctions, the Josephson energy can be tuned *in situ* allowing asymmetry $d \lesssim 1\%$ to be achieved routinely [37], while exhibiting a significant second-order Josephson energy $E_{J\Sigma_2}$. Standard aluminum Josephson junctions reach asymmetry $d \approx 1\%$ on wafer scale [38] making them also suitable. This makes the interference-based $\cos(2\varphi)$ regime experimentally accessible in these platforms.

D. Wavefunctions

The interest of the $\cos(2\varphi)$ qubit is rooted in its expected long lifetime [6,16,28,29]. The tunneling of single Cooper

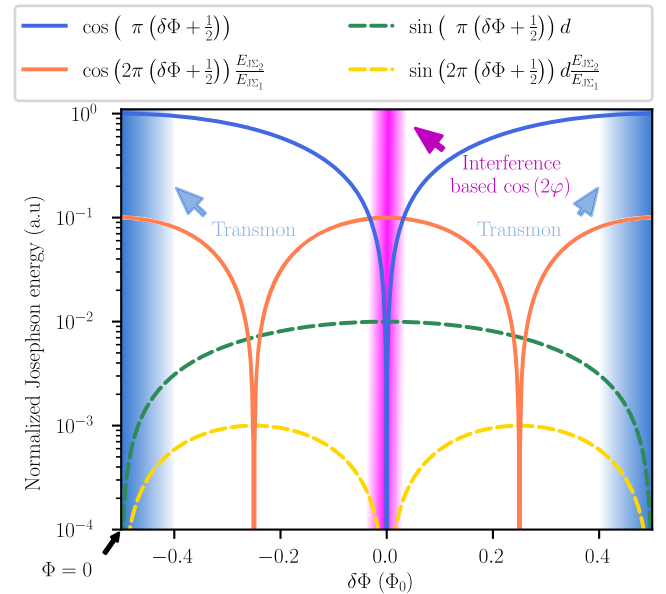


FIG. 2. Magnetic flux dependence of the junctions' harmonic energy. Magnetic flux dependence of the Josephson harmonics, see Eq. (3). With a small junction asymmetry, two regimes can be reached at $\delta\Phi \sim \pm 1/2$ ($\Phi/\Phi_0 \sim 0$), where the physics is similar to a transmon, and at $\delta\Phi \sim 0$, where the $\cos(2\varphi)$ is dominant. We took $E_{J\Sigma_2}/E_{J\Sigma_1} = -0.1$ and $d = 1\%$.

pairs is in principle forbidden in such a circuit, ensuring that the transition matrix element $\langle 1|\hat{n}|0\rangle$ between ground state $|0\rangle$ and excited state $|1\rangle$ vanishes. Only the tunneling of pairs of Cooper pairs is allowed. However, due to junction asymmetry or imperfect magnetic flux parking, the Hamiltonian has a residual $\cos(1\varphi)$ term, see Fig. 2. In the following, we will discuss protection against relaxation in these conditions.

One way to address this question is to look at the eigenstates' wavefunctions represented in Fig. 3. A pure $\cos(2\varphi)$ system exhibits a characteristic two-well potential. The two lowest-lying states are symmetric and anti-symmetric superpositions of states localized in each well. The amplitude of the matrix element $\langle 0|\hat{n}|1\rangle$ is exactly zero. Considering now an interference-based $\cos(2\varphi)$ qubit, if $\delta\Phi$ as well as d are precisely equal to zero, the eigenfunctions are identical to that of a pure $\cos(2\varphi)$ system.

Nonzero flux offset $\delta\Phi$ and asymmetry d result in significant modification of this picture. In Fig. 3(b), left panel, at $\delta\Phi = 0.03$, the circuit is close to the $\cos(2\varphi)$ regime as we can observe two potential wells. However, the first two wavefunctions are in the same well due to the remaining $\cos(\varphi)$ term and this regime offers no additional

protection compared with the transmon. Now in the right panel, we set a smaller $\delta\Phi = 10^{-3}$, which is compatible with experimentally achievable flux resolution [39]. The $\cos(\varphi)$ term is sufficiently suppressed and consequently, both wavefunctions are localized in different wells. These two states can be described by opposite persistent current in the SQUID loop. They remain, however, nondegenerate, since the offset flux creates an imbalance between the two potential wells greater than the interwell coupling energy κ .

In Fig. 3(c), the magnetic flux is kept close to half a flux quanta $\delta\Phi = 10^{-3}$ and the junction asymmetry d is set to 10% (left panel) and 1% (right panel). We observe that a 10% asymmetry results in a substantial overlap between the ground- and first-excited-state wavefunctions within the same potential well. This effectively breaks the protection from energy relaxation. However at a smaller asymmetry of 1%, an experimentally reachable parameter, both wavefunctions stay localized in different wells.

The conclusions are threefold. First, despite the fact that they both re-introduce a $\cos(1\varphi)$ term in the Josephson potential, offset flux and asymmetry have different effects: offset flux lifts the degeneracy between the two wells whereas junction asymmetry weakens the potential barrier between them. Second, since the degeneracy lifting at $\delta\Phi = 0$ results from interwell coupling through the phase potential barrier, the energy splitting is exponentially suppressed with $E_{J\Sigma_2}/E_C$, potentially resulting in MHz or even lower qubit transitions. Finally, while the circuit is not in a pure $\cos(2\varphi)$ regime due to finite precision in the junctions' symmetry and the magnetic flux, it can still localize the two wavefunctions in different wells. The consequence is a long lifetime as the matrix element $\langle 1|\hat{n}|0\rangle = -i\langle 1|\partial/\partial\hat{\varphi}|0\rangle$ is exponentially suppressed with $E_{J\Sigma_2}/E_C$.

E. Charge dispersion

In Fig. 4, we show the three lowest eigenenergies as a function of the island offset charge n_g for several values of the $E_{J\Sigma_{(1,2)}}/E_C$ ratio, increasing from top to bottom. In Fig. 4(a), at exactly $\delta\Phi = 0$ and with $d = 0\%$, we explore the pure $\cos(2\varphi)$ regime, which, despite being experimentally unachievable, has striking features. Its charge dispersion is equal to the transition frequency f_{01} at $n_g = 0$, which prevents the charge dispersion from flattening. Indeed, since $n_g = 0.5$ is a symmetry point between even and odd charge states, the two lowest-lying states have exactly the same energy for this particular value of n_g . This degeneracy is not lifted because of the absence of single Cooper-pair tunneling in the pure $\cos(2\varphi)$ regime.

In Fig. 4(b), at $\delta\Phi = 10^{-3}$ and with $d = 1\%$, we now explore the interference-based $\cos(2\varphi)$ case. The most

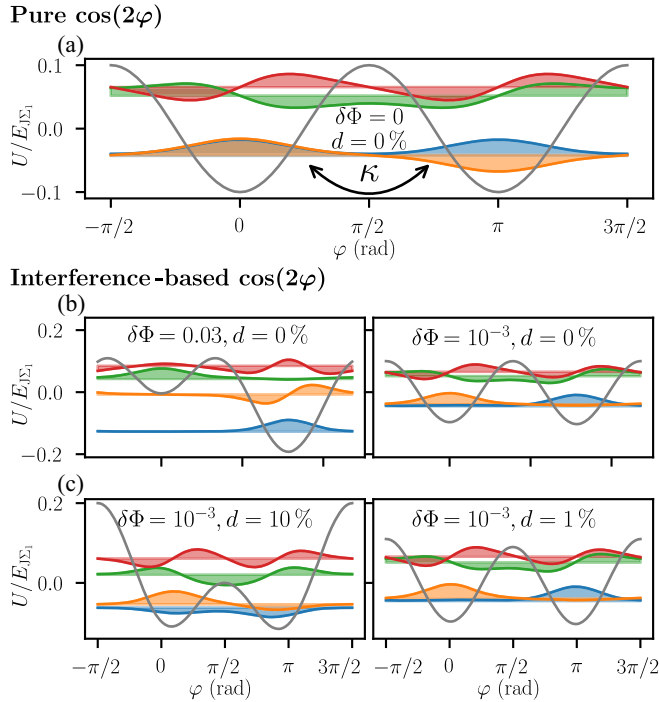


FIG. 3. Impact of magnetic flux offset and junction asymmetry on the wavefunctions. (a) Pure $\cos(2\varphi)$. The lowest eigenstates are delocalized in the two wells due to the interwell interaction energy κ . (b) Influence of the offset magnetic flux $\delta\Phi$ with asymmetry set to zero. (c) Influence of the junction asymmetry with the offset magnetic flux $\delta\Phi = 10^{-3}$. For all figures, we have $E_{J\Sigma_2}/E_{J\Sigma_1} = -0.1$ and $E_{J\Sigma_2}/E_C = 20$.

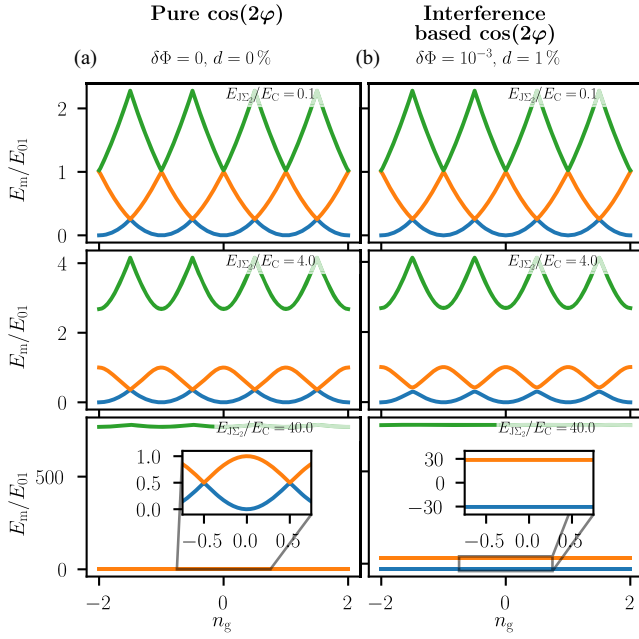


FIG. 4. Eigenenergies E_m , $m \in \{0, 1, 2\}$, of the circuit Hamiltonian, see Eq. (3), as a function of the offset charge n_g for different ratios $E_{J\Sigma_2}/E_C$, linewise, and offset flux Φ , columnwise. Energies are given in units of the transition energy E_{01} of the pure $\cos(2\varphi)$, evaluated at $n_g = 0$. The zero point of energy is chosen as the bottom of the $m = 0$ level. The left column shows the pure $\cos(2\varphi)$ regime, at exactly $\delta\Phi = 0$, where only the second harmonics are present. The right column shows the interference-based $\cos(2\varphi)$ case, where $\delta\Phi = 10^{-3}$. For all figures, we have $E_{J\Sigma_2}/E_{J\Sigma_1} = -0.1$.

noticeable difference with the pure regime is the recovery of a flat charge dispersion at large $E_{J\Sigma_2}/E_C$ ratio due to the reintroduction of a small $\cos(1\varphi)$ component. We see that at large $E_{J\Sigma_2}/E_C$ ratio, the third energy level stays extremely far from the second one, indicating a very large qubit anharmonicity, which is lacking in the transmon. This effect is due to the localization of the wavefunctions in the left or right well of the two-well potential as seen in Fig. 3. Indeed, there are two types of transitions in the $\cos(2\varphi)$ regime: intrawell transitions (plasmons), here from ground state to 2nd excited state, that are highly overlapping, and interwell transitions (fluxons) from ground state to 1st excited state, exhibiting a vanishing overlap. This spectrum classification is reminiscent of the fluxonium qubit [40], which can also show a large anharmonicity.

As a summary, in contrast to the transmon, there is no need for a compromise between charge insensitivity and high anharmonicity in the interference-based $\cos(2\varphi)$ qubit when choosing the Josephson energy. Both features become larger as $E_{J\Sigma_2}/E_C$ increases, motivating the exploration of larger than usual $E_{J\Sigma_2}/E_C$ ratio.

II. T_1 PROTECTION

A. How to estimate the decay?

So far, we have only described the circuit protection against energy relaxation in a qualitative manner. We will now move on to a quantitative discussion. The circuit is described by two conjugate quantum operators, $\hat{n}$ and $\hat{\varphi}$ (see Fig. 1), for which the main associated relaxation mechanisms are dielectric losses and coupling to the magnetic flux in the SQUID loop, respectively. The dominant source of decay due to flux is expected to be $1/f$ flux noise picked up by the loop [41,42]. To evaluate the relaxation rates and to be able to compare them to other quantum circuits, we first calculate the relevant matrix elements [1,43].

Dielectric loss couples via the charge operator

$$\hat{\mathcal{O}}_{\hat{n}} = 2e\hat{n}. \quad (4)$$

The charge matrix element $\langle 0|\hat{n}|1\rangle = M_n$ is thus the relevant figure of merit to discuss relaxation via the electric field.

For relaxation induced by magnetic flux noise, we derive the coupling operator

$$\hat{\mathcal{O}}_{\hat{\varphi}} = \frac{\partial \hat{H}}{\partial \Phi}, \quad (5)$$

which using Eq. (3) gives

$$\begin{aligned} \hat{\mathcal{O}}_{\hat{\varphi}} = & -E_{J\Sigma_1} \frac{\pi}{\Phi_0} [\cos(\hat{\varphi}) \cos(\pi\delta\Phi) + d \sin(\hat{\varphi}) \sin(\pi\delta\Phi)] \\ & - E_{J\Sigma_2} \frac{2\pi}{\Phi_0} [-\cos(2\hat{\varphi}) \sin(2\pi\delta\Phi) \\ & + d \sin(2\hat{\varphi}) \cos(2\pi\delta\Phi)]. \end{aligned} \quad (6)$$

Several operators are present involving either $1\hat{\varphi}$ or $2\hat{\varphi}$ harmonic terms. We consequently have two figures of merit

$$\begin{aligned} M_{1\varphi} = & \langle 0|\cos(\hat{\varphi})|1\rangle \cos(\pi\delta\Phi) \\ & + \langle 0|\sin(\hat{\varphi})|1\rangle d \sin(\pi\delta\Phi) \\ M_{2\varphi} = & -\langle 0|\cos(2\hat{\varphi})|1\rangle \sin(2\pi\delta\Phi) \\ & + \langle 0|\sin(2\hat{\varphi})|1\rangle d \cos(2\pi\delta\Phi). \end{aligned}$$

Notice their significantly different external magnetic field dependence, which may strongly influence the final relaxation rate. Furthermore, the junction asymmetry, even as low as $d = 1\%$ promotes interactions via the $\sin(n\hat{\varphi})$ operators and influences the final relaxation rate. In the following sections, we will discuss these figures of merit quantitatively.

B. Junction asymmetry

In this section, we examine how asymmetry in the junctions affects the matrix elements involved in qubit relaxation. In Fig. 5, we plot the qubit frequency and the relaxation figures of merit as function of $E_{J\Sigma_2}/E_C$ and the junction asymmetry d . From panels (b) to (d), we see that to reach low matrix element values, i.e., long relaxation times, both low asymmetry and large $E_{J\Sigma_2}/E_C$ are required. Indeed, symmetry is required for the wavefunctions to localize in different wells and a large $E_{J\Sigma_2}/E_C$ ratio increases the barrier height. Quantitatively, the figure shows that the interference-based $\cos(2\varphi)$ qubit can reach matrix element values several orders of magnitude lower than transmons [1] that can also be found in fluxonium qubits [12,13,44,45], ensuring an efficient protection from energy relaxation.

C. Flux Φ operating point

We now turn to the effect of the flux operating point on matrix element values. Any realistic experiment will

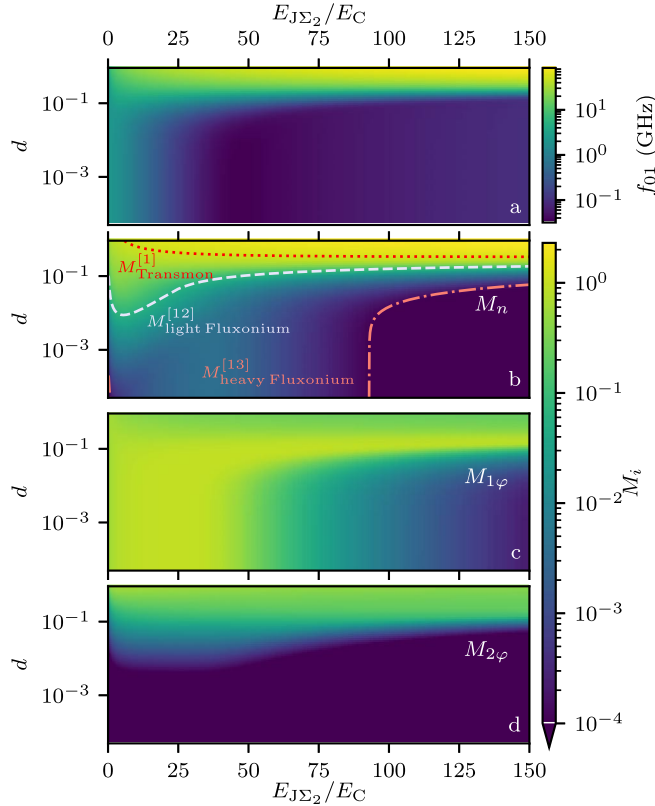


FIG. 5. Effect of the junction asymmetry. We show the effect of the junction asymmetry d and $E_{J\Sigma_2}/E_C$ ratio on the qubit frequency, (a), and its relevant relaxation figure of merits, (b), (c), and (d). In (b), we show in dotted red, dashed lavender, and dash-dotted salmon, the typical matrix elements for the transmon [1] and fluxonium [12,13] qubit, respectively. The bottom colorscales of (b), (c), and (d) were truncated to 1×10^{-4} for the sake of visibility. For all figures, we have $\delta\Phi = 10^{-5}$ and $n_g = 0.25$.

feature a small yet nonvanishing additional offset flux as well as flux noise so that $\delta\Phi \neq 0$, introducing again a $\cos(1\varphi)$ term in the Josephson potential. We show in Fig. 6 the resonance frequency and the figures of merit for energy relaxation in the different channels. All plots separate into three regions. In the left and bottom, $E_{J\Sigma_2}/E_C$ is small resulting in significant interwell coupling and hybridization of wavefunctions [see Fig. 3(a)]. In the bottom right, $E_{J\Sigma_2}/E_C$ is large, dramatically reducing interwell coupling and promoting in-well localization of the wavefunctions [see Fig. 3(b), right panel]. In the top part of the figures, for large offset flux, the circuit leaves the protected regime as the first transition becomes intrawell, with a large wavefunction overlap [see Fig. 3(b), left panel].

In Fig. 6(a), at low $E_{J\Sigma_2}/E_C$, the resonance frequency is dominated by the interwell coupling interaction and is mostly independent of $\delta\Phi$. At high $E_{J\Sigma_2}/E_C$, the resonance frequency is given by the imbalance between the two potential wells. The discontinuity observed in panels (b)–(d) at $\delta\Phi \sim 10^{-2}$ comes from a level crossing between states $|1\rangle$ and $|2\rangle$.

Focusing on small offset flux, i.e., the bottom of the graphs, we observe in (b) a striking property: the charge matrix element is almost independent of $\delta\Phi$ and is small in both the hybridized and the localized regimes. This means that in an interference-based $\cos(2\varphi)$ qubits the energy relaxation due to charge matrix elements is resilient to significant offset flux $\delta\Phi$. This is an unexpected feature. The vanishing energy relaxation is usually believed to be associated, in the $\cos(2\varphi)$ regime, to charge parity protection, i.e., that $|0\rangle$ and $|1\rangle$ states are built on disjoint supports (odd and even charge states). This picture only holds for small offset flux, in the interwell hybridized regime, i.e., the bottom left of the graph [see also Fig. 3(a), bottom panel]. This figure shows that the protection mechanism extends beyond this disjoint wavefunction picture: at larger offset flux, $|0\rangle$ and $|1\rangle$ both have contributions on even and odd charge states. Nevertheless, they are still localized in different wells, and their symmetry in charge space is such that the charge matrix element still vanishes. In Appendix B [36], we provide details about wavefunction symmetries in charge representation. This explains the robustness of the protection mechanism.

In Figs. 6(c) and 6(d), the figures of merit of flux-induced relaxation show, overall, an enhanced protection at large $E_{J\Sigma_2}/E_C$ and increasing $\delta\Phi$. Looking into details of the different figures of merit, the dominant operator for relaxation is $\cos(\hat{\varphi})$, whose symmetry enables a large overlap between interwell hybridized wavefunctions. This is unlike a typical transmon, in which the dominant term is $\sin(\hat{\varphi})$ due to intrawell transitions. Shifting away from $\delta\Phi = 0$ reduces the phase matrix elements by promoting in-well localization of wavefunctions, i.e., transitioning

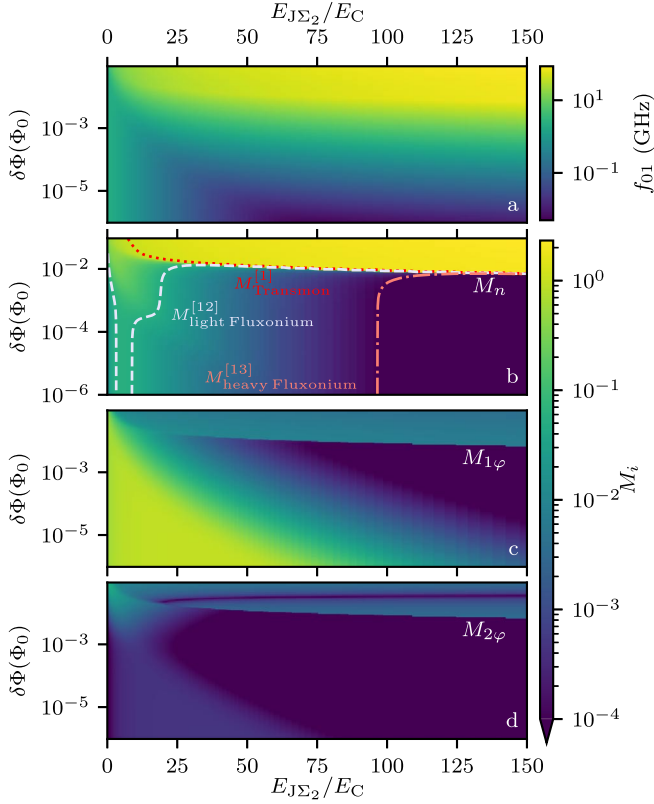


FIG. 6. Effect of magnetic flux offset. (a) Transition energy f_{01} for varying magnetic flux and $E_{J\Sigma_2}/E_C$ ratio. (b) Charge matrix element M_n for varying magnetic flux and $E_{J\Sigma_2}/E_C$ ratio. (c) $M_{1\varphi}$ figure of merit flux matrix element for varying magnetic flux and $E_{J\Sigma_2}/E_C$ ratio. (d) $M_{2\varphi}$ figure of merit for flux matrix element for varying magnetic flux and $E_{J\Sigma_2}/E_C$ ratio. We show in dotted red, dashed lavender, and dash-dotted salmon, the typical matrix elements for the transmon [1] and fluxonium [12,13] qubit, respectively. The bottom colorscales of (b), (c), and (d) were truncated to 1×10^{-4} for the sake of visibility. For all figures, we have $E_{J\Sigma_2}/E_{J\Sigma_1} = -0.1$, $d = 1\%$, and $n_g = 0.25$.

from a hybridized regime to a localized one. “ 2φ ” matrix elements follow similar variations to “ 1φ ” matrix elements, with a decrease amplitude due to favorable weighting, i.e., vanishing $\sin(2\pi\delta\Phi)$ and small asymmetry factor d . In a general perspective, the in-well wavefunction localization enables the suppression of the matrix element associated to all decay mechanisms that couple through the phase degree of freedom.

To conclude, interference-based $\cos(2\varphi)$ circuits show vanishing charge and phase matrix element amplitudes even with finite offset flux $\delta\Phi$ and junction asymmetry d . Larger $E_{J\Sigma_2}/E_C$ leads to lower matrix element amplitudes, motivating the exploration of larger than usual $E_{J\Sigma_2}/E_C$ ratio, as already seen in Sec. IE. Matrix elements smaller than approximately 10^{-3} are achievable for both flux and charge, which is orders of magnitude better than transmons.

III. T_φ PROTECTION

A. How to estimate the dephasing?

From the previous section, it is clear that our circuit holds protection against energy relaxation, even when junction asymmetry and finite magnetic flux offset are taken into account. We now quantitatively discuss the dephasing of the qubit transition due to charge and flux noise. To evaluate such dephasing and compare them to other quantum circuits, we compute [46] $\langle 1|\partial_\lambda H|1\rangle - \langle 0|\partial_\lambda H|0\rangle \sim \partial_\lambda f_{01}$ for $\lambda \in \{n_g, \Phi\}$.

B. Trade-off between charge noise and flux noise

A direct observation in Fig. 7 is the incompatibility between regions of insensitivity to charge noise, i.e., with large $E_{J\Sigma_2}/E_C$ ratio, and to flux noise, requiring small $E_{J\Sigma_2}/E_C$ ratio. This is one of the main conclusions of this work: it is not possible to protect the interference-based $\cos(2\varphi)$ qubits from both sources of dephasing.

In more detail, a low $E_{J\Sigma_2}/E_C$ ratio leads to significant charge dispersion, as in a transmon. The effect is, however, significantly enhanced in $\cos(2\varphi)$ qubits as the exponential coefficient of the charge dispersion suppression is twice lower [16], which is a direct consequence of preventing single Cooper-pair tunneling. On the other hand, in the transmon regime at $E_{J\Sigma_2} \gg E_C$, the suppression of the first-order Josephson energy $E_{J\Sigma_1}$ is controlled by interference between the SQUID’s arms and is very sensitive to

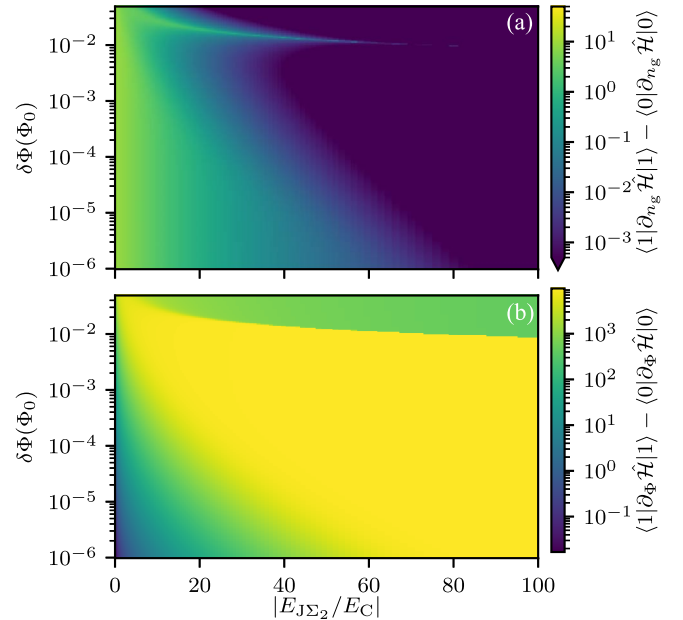


FIG. 7. Susceptibility to dephasing due to charge and flux noise. Sensitivity of eigenstates energies to (a) charge noise and (b) flux noise. The lower bound of the colorscale of (a) was truncated to 5×10^{-4} for the sake of visibility. For all figures, we have $E_{J\Sigma_2}/E_{J\Sigma_1} = -0.1$, $d = 1\%$, and $n_g = 0.25$.

flux noise. An $|E_{J\Sigma_2}/E_{J\Sigma_1}|$ ratio as large as possible reduces this flux-noise sensitivity and is hence desirable. This matter is discussed quantitatively in Sec. IV D.

C. T_φ : Interwell coupling interpretation

We can understand these features by examining the consequences of interwell coupling on the spectrum. Figure 8 shows the ground-state and first-excited-state energies as function of offset flux $\delta\Phi$ for $n_g = 0$ and $n_g = 0.5$, solid and dashed line, respectively. Close to a pure $\cos(2\varphi)$ regime, $\delta\Phi \sim 0$, at $n_g = 0$, the parabolic dispersion results from the interwell interaction with interaction strength given by [16]

$$\kappa(n_g) = 8E_C \sqrt{\frac{2}{\pi}} \left(\frac{2E_{J\Sigma_2}}{E_C} \right)^{\frac{3}{4}} e^{-\sqrt{\frac{2E_{J\Sigma_2}}{E_C}}} \cos(\pi n_g). \quad (7)$$

The interwell interaction is modulated by the charge offset n_g . When the interaction is finite the flux dispersion vanishes at $\delta\Phi \sim 0$ corresponding to a sweet spot in flux. However, we observe a very different behavior at $n_g = 0.5$, which is a high-symmetry point for charge. Here, the interaction vanishes, resulting in the absence of a flux sweet spot. Consequently, the charge dispersion is large, as large as the transition frequency $f_{01}(n_g = 0)$ itself.

The robustness of the sweet spot regarding flux fluctuations relates to the second derivative versus $\delta\Phi$ of the qubit fundamental frequency. In a simplified picture, the qubit frequency reads $f_{01} = 2\sqrt{\kappa^2 + (\alpha\delta\Phi)^2}$, where $\alpha = \pi|E_{J\Sigma_1}|$ is the slope of the flux-induced degeneracy lifting between the two potential wells (without coupling). We define here the sweetness $\delta\Phi_{\max}$, which quantifies the sweet spot robustness

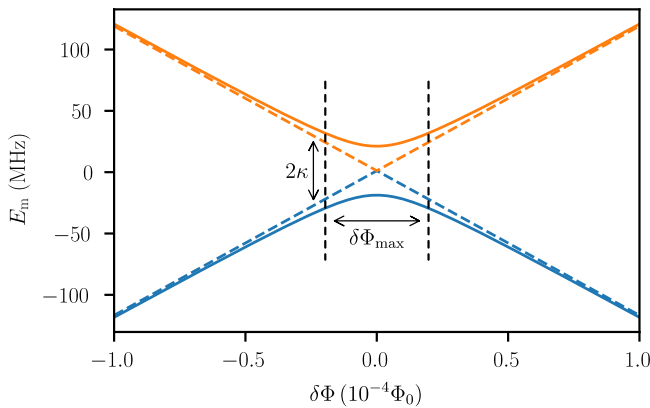


FIG. 8. Flux dispersion. Ground-state and first-excited-state energies versus offset flux for $n_g = 0$ (solid line) and $n_g = 0.5$ (dashed line). For $|\delta\Phi| > \delta\Phi_{\max}$, the flux dispersion is linear due to in-well localization of wavefunctions. For $|\delta\Phi| < \delta\Phi_{\max}$ in the center part, the two wells' wavefunctions hybridize for $n_g = 0$. $E_{J\Sigma_2}/E_{J\Sigma_1} = -0.1$, $E_{J\Sigma_2}/E_C = 40$, and $d = 0$.

$$\delta\Phi_{\max} = \frac{\alpha}{\frac{\partial^2 f_{01}}{\partial \delta\Phi^2}(\delta\Phi = 0)} = \frac{\kappa}{2\pi|E_{J\Sigma_1}|}, \quad (8)$$

where $\delta\Phi_{\max}$ identifies to the range of offset flux $\delta\Phi$ for which flux dispersion is significantly suppressed. The value of $\delta\Phi_{\max}$ sets the boundary between two different regimes visible in Fig. 8. For $\delta\Phi < \delta\Phi_{\max}$, the two wells' wavefunctions hybridize due to interwell coupling, and for $\delta\Phi > \delta\Phi_{\max}$ the wells degeneracy lifting is large enough to localize both wavefunctions.

Efficient transmon-like suppression of charge dispersion can be achieved by shifting away from the sweet spot with $\delta\Phi > \delta\Phi_{\max}$ (see Fig. 4), which remain compatible with the T_1 protection, but here the linear flux dispersion would make the qubit highly sensitive to flux noise, resulting in a small T_φ . Next, we explore the possibility to find a suitable trade-off between the two regimes.

IV. OPTIMIZING CIRCUIT PARAMETERS

A. Realistic experimental parameters

In this section, we explore the $E_{J\Sigma_2}-E_C-\delta\Phi$ parameter space. We limit the study to experimentally relevant parameters for interference-based circuits.

The smallest acceptable value of $E_{J\Sigma_2}$ is obtained by comparing the thermal energy $k_B T$ with the depth of the Josephson potential $2 \times E_{J\Sigma_2}$. In practice, a circuit with a photonic temperature $T \approx 50$ mK ≈ 1 GHz [12] greater than $2 \times E_{J\Sigma_2}/k_B$ will be thermally excited to states in the continuum above the potential, and not usable as a qubit. This effect is absent in some other low-frequency qubits, such as the fluxonium qubits, with unbounded potential, for which confinement is always ensured by the inductive energy.

Very large values of $E_{J\Sigma_2}$, up to several hundred GHz, are accessible as critical current value of several μ A has been demonstrated in graphene-based Josephson junctions for instance [47]. On the other hand, achieving very large shunt capacitance is a fabrication challenge, which limits the lowest possible value of E_C to several MHz [48], using typically aluminum-based planar superconducting circuits. Parallel plate capacitors could provide a way to overcome this limitation but remain limited due to the difficulty to deposit reproducibly thin films of lossless dielectric materials [49]. On the large E_C side, capacitances can be reduced to few fF in hybrid semiconductor Josephson junctions, enabling E_C in the tens of GHz range.

Realistically, small and well-defined offset flux $\delta\Phi$ is constrained by the minimum magnetic flux noise value $\sim 10^{-6} \Phi_0$ in state-of-the-art experiments [46], so we choose $\delta\Phi = 10^{-5}$ as a minimum value in the following.

In Secs. IV B and IV C, we set the harmonic Josephson energy ratio to $E_{J\Sigma_2}/E_{J\Sigma_1} = -0.1$ and the junction asymmetry to $d = 1\%$.

B. Moderate $E_{J\Sigma_2}/E_C$ for minimum dephasing

Figure 9 shows estimations of pure dephasing T_ϕ (left) and decay times T_1 (right) as a function of flux offset $\delta\Phi$, for different $E_{J\Sigma_2}/E_C$ ratio. T_ϕ estimations were obtained using typical $1/f$ noises as implemented in scQubits [32] with noise amplitudes $10^{-6}\Phi_0$ for flux noise and $10^{-4}e$ for charge noise [16]. T_1 due to dielectric losses is estimated using a capacitance quality factor $Q_{\text{capa}} = 10^6$ [40], and T_1 due to magnetic field relaxation is estimated using a noise amplitude of $10^{-6}\Phi_0$ [40].

As a general trend, we observe that the dephasing time T_ϕ is more limiting than the decay time T_1 , with T_ϕ values never exceeding a few μs . Focusing first on dephasing, we observe in panel (a) at low $E_{J\Sigma_2}/E_C$ ratio that charge noise dominates the decoherence as in a Cooper-pair-box-like regime, and T_ϕ cannot exceed a few tens of nano-seconds. At large $E_{J\Sigma_2}/E_C$ ratio, in panel (c), the linear dispersion versus flux dominates and T_ϕ is limited to only a few

nano-seconds due to flux noise. In the intermediate regime of $E_{J\Sigma_2}/E_C \sim 20$, panel (b), charge and flux noises both have effects of similar magnitude. The effect of flux noise is reduced by lowering $\delta\Phi$ until charge noise becomes the limiting factor. We see that T_ϕ has an upper limit on the order of $1\mu\text{s}$. Hence, the most favorable configuration is to balance the effect of charge and flux noise, which happens for moderate $E_{J\Sigma_2}/E_C$, see panel (b).

Considering now T_1 , we observe that $1/f$ flux noise is the dominant decay mechanism in the protected regime ($\delta\Phi \ll 10^{-2}$), which is expected since the charge matrix element is vanishing by symmetry.

The plateaus of the T_1 versus flux are reminiscent of the robustness of the T_1 protection away from the flux sweet spot discussed in Sec. II C.

C. Zero-frequency limits

Here, we discuss the zero frequency limit regimes of operation and their practical applicability. Low frequency qubits are generally appealing to mitigate dephasing issues because dephasing rates often scale with the fundamental qubit frequency [50]. There are two ways to obtain vanishingly low fundamental frequencies in $\cos(2\varphi)$ qubit.

First, we discuss the regime $\delta\Phi = 0$ ($\Phi/\Phi_0 = 0.5$) and large $E_{J\Sigma_2}/E_C \gg 50$. At $\delta\Phi = 0$, the fundamental qubit frequency is given by $f_{01} = 2\kappa$ and the charge dispersion is also equal to f_{01} (see Fig. 8). Similarly to the transmon, increasing $E_{J\Sigma_2}/E_C$ can be proposed to flatten the charge dispersion by reducing κ exponentially [see Eq. (7)], resulting in a charge-insensitive qubit with vanishingly small fundamental frequency. Additionally, the sweet spot at $\delta\Phi = 0$ can be expected to ensure flux noise insensitivity. However, the sweetness $\delta\Phi_{\text{max}}$ is reduced exponentially with $E_{J\Sigma_2}/E_C$ [see Eqs. (8) and (7)], making the circuit highly sensitive to flux noise at the second order. In practice, with a realistic $E_{J\Sigma_2}/E_C = 70$, the charge dispersion remains as large as 4 MHz and $\delta\Phi_{\text{max}} \lesssim 10^{-6}\Phi_0$, i.e., smaller than the experimentally achievable flux noise amplitude. The magnetic flux sweet spot is then in practice nonexistent in the large $E_{J\Sigma_2}/E_C$ regime. This situation is illustrated in Fig. 9(c): we do not observe any increase in T_ϕ due to flux noise when $\delta\Phi$ decreases because the sweet spot is never reached, even at $\delta\Phi = 10^{-6}$. For these reasons, relying on a degenerate computational space ($f_{01} \sim 0$) in the large $E_{J\Sigma_2}/E_C$ limit is not realistic using the interference-based $\cos(2\varphi)$ qubit architecture. More complex architecture based on an array of SQUIDs have been proposed to address this issue, providing enhanced extension of the sweet spot, see Ref. [51].

A second regime of low-frequency $\cos(2\varphi)$ qubits can be reached by reducing both $E_{J\Sigma_2}$ and E_C at constant, moderate values of the $E_{J\Sigma_2}/E_C$ ratio, hence conserving an optimum regime regarding T_ϕ . This strategy is, however,

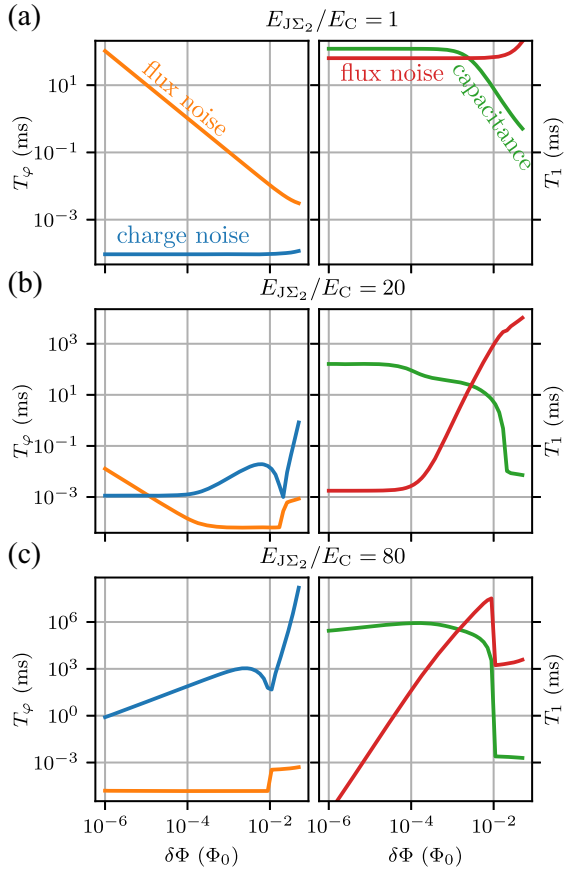


FIG. 9. Dephasing T_ϕ and decay T_1 times dependence on $\delta\Phi$ for different $E_{J\Sigma_2}/E_C$ ratios. Junctions' asymmetry is fixed to $d = 1\%$, the charging energy is set to $E_C = 500$ MHz, $E_{J\Sigma_2}/E_{J\Sigma_1} = -0.1$, and the ratio $E_{J\Sigma_2}/E_C$ varies. It is equal to 1, 20, 80, from top to bottom. The discontinuity observed for $\delta\Phi \geq 10^{-2}$ indicates the transition between the protected and unprotected regimes.

limited as well, because of temperature since $E_{J\Sigma_2}$ cannot be smaller than $k_B T/2h \sim 0.5$ GHz [12]. We conclude that a very low-frequency qubit, which was proposed as a means to suppress dephasing [50], cannot increase the coherence time of $\cos(2\varphi)$ qubits to arbitrarily large values.

D. Comprehensive study of circuit parameters

We perform in this section a comprehensive numerical investigation of the circuit coherence as function of $E_{J\Sigma_2}$, E_C , and $\delta\Phi$. We present in Supplemental Material Sec. C [36] the analytical method used to calculate coherence times in the low-frequency limit, that accounts for specific contributions of higher excited states due to thermal excitation. Detailed contributions of the different decoherence mechanisms are presented in Supplemental Material Sec. D [36].

To start the discussion, we focus on the case $E_{J\Sigma_2}/E_{J\Sigma_1} = -0.1$, which corresponds to realistic circuit parameters. We summarize in Table II the longest coherence times T_2 for different qubit frequencies f_{01} . For a high qubit frequency $f_{01} = 5$ GHz, the longest coherence is achieved for a moderate ratio $E_{J\Sigma_2}/E_C \sim 9$, the limiting factor being the dephasing time T_φ with similar contributions from charge and flux noise. As f_{01} is reduced, the optimal $E_{J\Sigma_2}/E_C$ ratio shows little variation. The dephasing time T_φ increases as $1/f$, while T_1 decreases but remains much larger, leading to an overall longer T_2 . We conclude that interference-based $\cos(2\varphi)$ qubit with $E_{J\Sigma_2}/E_{J\Sigma_1} = -0.1$ leads, at best, to a coherence time T_2 in the order of a few microseconds. This qubit is intrinsically limited by charge noise at low $E_{J\Sigma_2}/E_C$ and flux noise at large $E_{J\Sigma_2}/E_C$, with temperature setting a lower boundary for $E_{J\Sigma_2}$.

To summarize this, in Fig. 10, we show the expected T_2 as a function of circuit parameters and for the optimal flux offset $\delta\Phi$ at each $E_{J\Sigma_2}$, E_C . Additionally, we represent by color-shaded areas the main limiting factor for T_2 : charge noise (blue), flux noise (salmon) or temperature (black). For $E_{J\Sigma_2}/E_{J\Sigma_1} = -0.1$ [see panel (a)], the overlap between

the different shaded regions illustrates that there is no satisfactory working point because of these three decoherence channels.

Finding acceptable circuit parameters requires extending the discussion to $|E_{J\Sigma_2}/E_{J\Sigma_1}|$ ratios larger than 1.

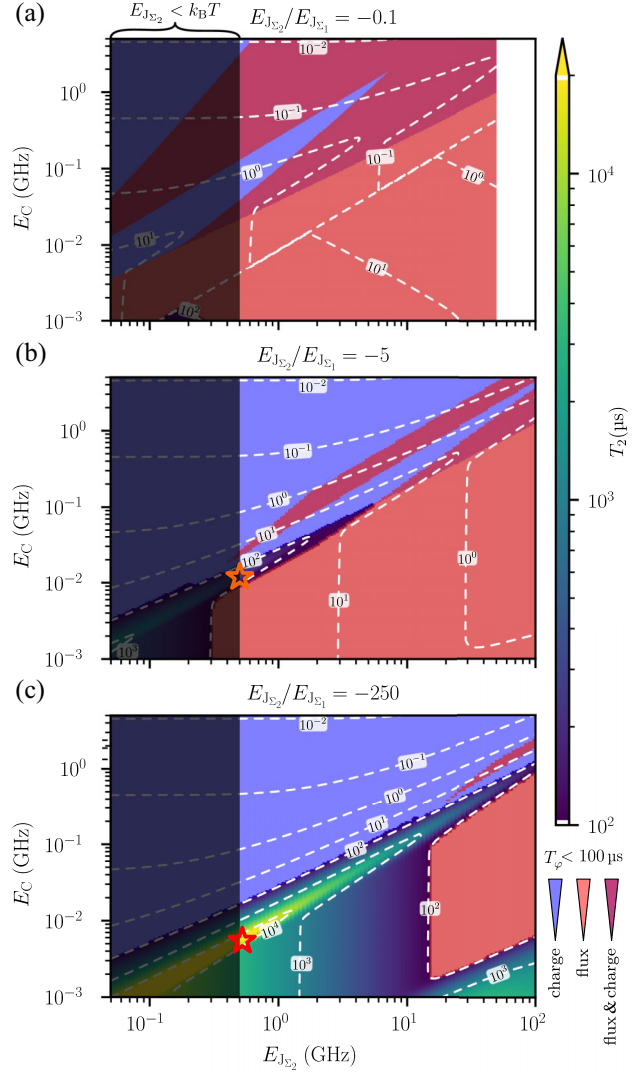


FIG. 10. Maximal coherence time T_2 achievable for given E_C and $E_{J\Sigma_2}$. The junction asymmetry is fixed at $d = 1\%$, and the charge offset is set to $n_g = 0.25$. The ratio $E_{J\Sigma_2}/E_{J\Sigma_1}$ takes the values (a) -0.1 , (b) -5 , and (c) -250 . For each point, the flux offset $\delta\Phi$ is varied in the range 10^{-5} to 9×10^{-3} to maximize T_2 (the corresponding optimal $\delta\Phi$ values are provided in Appendix D [36]). Colored regions indicate regimes of reduced qubit coherence due to different limiting mechanisms: temperature (black, where $2 \times E_{J\Sigma_2} < k_B T/h \sim 1$ GHz), charge noise (blue, where $T_\varphi^{\text{charge}} < 100 \mu\text{s}$), and flux noise (salmon, where $T_\varphi^{\text{flux}} < 100 \mu\text{s}$). For clarity, the colorscale is truncated between $T_2 = 100 \mu\text{s}$ and 20 ms. Stars indicate the operating points that yield the longest coherence times, namely $(E_C, E_{J\Sigma_2})$ pairs corresponding to $T_2 = 270 \mu\text{s}$ for $E_{J\Sigma_2}/E_{J\Sigma_1} = -5$ and $T_2 = 24$ ms for $E_{J\Sigma_2}/E_{J\Sigma_1} = -250$.

TABLE II. Optimal $E_{J\Sigma_2}$ and E_C to obtain the longest coherence time T_2 . For all simulations, we used $E_{J\Sigma_2}/E_{J\Sigma_1} = -0.1$, $d = 1\%$ and $n_g = 0.25$. $E_{J\Sigma_2}/E_C \sim 10$ is consistently found to be the optimal ratio to maximize T_2 . Similarly, the lowest allowed flux offset, $\delta\Phi = 10^{-5}\Phi_0$, always leads to the longest T_2 .

f_{01} (MHz)	α (MHz)	$\delta\Phi$ (Φ_0)	$E_{J\Sigma_2}$ (GHz)	E_C (GHz)	$E_{J\Sigma_2}/E_C$	T_1 (μs)	T_φ (μs)	T_2 (μs)
$5 \cdot 10^3$	$53 \cdot 10^3$	10^{-5}	48.5	5.6	9	1.10^6	0.02	0.02
$1 \cdot 10^3$	$16 \cdot 10^3$	10^{-5}	15.7	1.5	11	100	0.12	0.12
500	$9 \cdot 10^3$	10^{-5}	8.6	0.8	11	46	0.24	0.24
100	$1 \cdot 10^3$	10^{-5}	1.2	0.1	9	69	1	1

In panel (b), we present the results for a ratio of 5, that could be obtained in the Flowermon circuit for instance (see Table I). In panel (c), we explore the situation for a ratio as high as 250, which is currently unreachable in existing architectures, see Appendix A [36]. We observe that increasing the $|E_{J\Sigma_2}/E_{J\Sigma_1}|$ ratio is favorable in terms of flux noise induced decoherence because $\partial_\phi f_{01} \propto E_{J\Sigma_1}$ in the in-well localized regime. This opens a domain with coherence times higher than 100 μs , culminating to $T_2 = 270 \mu\text{s}$ [orange star of panel (b)] and $T_2 = 24 \text{ ms}$ [red star of panel (c)]. This is only a small improvement compared with state-of-the-art transmons [2,3,52] considering that it would require to increase $|E_{J\Sigma_2}/E_{J\Sigma_1}|$ by two orders of magnitude compared with what is currently achievable.

V. CONCLUSION

The concept of the $\cos(2\varphi)$ qubit offers an appealing pathway toward mitigating the energy relaxation mechanisms that limit all superconducting qubits. However, our analysis reveals that in interference-based implementations, this protection entails an unavoidable trade-off between flux and charge noise, which ultimately results in pure dephasing of the qubit. Interestingly, we have shown that these competing dephasing channels can be partially balanced by operating the qubit slightly away from its flux sweet spot.

By systematically exploring the parameter space of a generic circuit encompassing a broad range of experimental realizations, we find that with currently achievable circuit parameters, the dephasing time is limited to a few microseconds—substantially below the state-of-the-art coherence times demonstrated in transmons.

Looking ahead, we estimate that even for ratios as large as $E_{J\Sigma_2}/E_{J\Sigma_1} \sim -250$, corresponding to a two-order-of-magnitude improvement compared with any existing or proposed circuit, coherence times could reach approximately 20 ms. This projection underscores the fundamental constraints of interference-based $\cos(2\varphi)$ architectures and raises important questions about their suitability as building blocks for scalable superconducting quantum processors.

These findings highlight the need for alternative design strategies to achieve simultaneous protection against energy relaxation and dephasing. One promising direction is the recently proposed interference-based architecture featuring a finite junction asymmetry with $E_{J\Sigma_2} < 0$ [29]. This change in sign lifts the degeneracy between the bottoms of the two potential wells [like on Fig. 3(b), right]. The states now have the same charge dispersion as the transmon, while being able to operate at the magnetic sweet spot. Nonetheless, the experimental demonstration of a circuit element with $E_{J\Sigma_2} < 0$ that remains compatible

with high-coherence operation is an open and critical challenge for future research.

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DATA AVAILABILITY

The data that support the findings of this article are openly available [53].

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