

Scattering and chirping at accelerated interfaces

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Space-time-varying media with moving interfaces unlock new ways to manipulate electromagnetic waves: however, analytical solutions have been limited mostly to interfaces moving at constant velocity or constant proper acceleration. Here we present exact scattering solutions for an arbitrarily accelerating interface, derived directly in the laboratory frame through a suitable change of variables. We show that acceleration introduces rich effects that do not occur with uniform motion, including transitions between multiple velocity regimes, multiple scattering events, and generalized frequency chirping. We also solve the inverse problem of designing an interface trajectory that produces a desired chirping profile, demonstrating how tailored acceleration can synthesize complex frequency modulations. These results provide a fundamental framework to understand and control wave interactions with accelerated boundaries, opening pathways for advanced applications in space-time signal processing and dynamic pulse shaping.

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I. INTRODUCTION

Space-time-varying media are engineered structures whose electromagnetic properties, such as the refractive index, are modulated in both space and time [1,2]. A characteristic parameter is their modulation velocity, which is the speed at which these parameters change in space and time. Depending on whether this velocity is infinite, constant, or time dependent, space-time-varying media exhibit distinct wave interactions and enable new physical effects [3].

Purely temporal interfaces have been studied extensively [4–6]. These unlock a broad range of phenomena unattainable by spatially static media, including time reversal [7], operation beyond fundamental bounds [8–10], photon cooling [11], inverse prism decomposition [12], temporal impedance matching [13], temporal aiming [14], parametric amplification [15], beam splitting [16], photon generation [17], temporal Brewster refraction [18], scattering at bianisotropic time interfaces [19], polarization conversion [20], and the temporal analogue of Faraday rotation [21,22].

Moving interfaces [23–27] have also attracted significant attention and support effects such as Doppler frequency shifting [26,28,29], magnetless nonreciprocity [30–32], space-time reversal [33], dynamic diffraction

[34], asymmetric band gap isolation [23,24,26,35], light deflection [36,37], shock-wave generation [38], Hawking radiation [39,40], space-time Fresnel prism scattering [41], light amplification [42,43], space-time wedge multiple scattering [44], and Doppler pulse amplification [45]. These effects extend the capabilities and applications of purely temporal interfaces.

The aforementioned studies on moving interfaces are characterized by a constant modulation velocity. Recently, *accelerated* interfaces have emerged, enabling complex temporal dynamics, enhanced spectral control, and advanced functionalities, such as space-time lensing [46], arbitrary pulse shaping [47], superluminal photon emission [48], frequency chirping [49,50], and gravity analogues [51]. However, a comprehensive analytical framework for such interfaces is still missing, except for the particular case of constant proper acceleration [49,51]. In this case, the problem can be addressed with use of the frame-hopping method [52] with Rindler transformations [53], but otherwise, when the acceleration has an arbitrary form, the frame-hopping approach is inapplicable because no global coordinate transformation exists. Consequently, the fundamental problem of wave scattering at an arbitrarily accelerated interface remains unsolved, leaving the associated physics mostly unexplored. This gap limits practical applications, which remain scarce and underdeveloped.

In this work, we present, to the best of our knowledge, the first exact analytical scattering solutions for an arbitrary accelerated interface and elucidate the underlying physics. Instead of relying on the frame-hopping approach, we solve the problem directly in the laboratory frame through a suitable change of variables, thereby avoiding the limitations of coordinate transformations.

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We show that acceleration introduces a time-dependent Doppler frequency shift, resulting in frequency chirping. We further address the synthesis problem, demonstrating how prescribed chirping profiles can be realized through tailored acceleration. These results open new possibilities for advanced wave manipulation and have potential applications in analog signal processing [54] and chirped pulse shaping [55].

This paper is structured as follows. Section II outlines the fundamental physics governing wave scattering at arbitrarily accelerated interfaces. In Sec. III the scattering solutions across all three velocity regimes are derived and the results are illustrated with four representative examples. Section IV addresses the synthesis problem and provides two examples. The paper is concluded in Sec. V, in which potential experimental realizations are discussed.

II. PHYSICS OF ACCELERATED INTERFACES

A single interface is the most fundamental building block of space-time-varying media. This elementary structure, which may be called a “space-time meta-atom” [56], governs the basic interactions between electromagnetic waves and space-time modulations. More complex configurations, such as slabs, stacks, crystals, and gradient structures, can be understood as a succession of such elementary interfaces. Even on their own, single interfaces enable practical functionalities such as space-time focusing [46] and pulse shaping [47].

We next describe the physics of wave scattering at accelerated interfaces with the help of Fig. 1. The interface separates two media with refractive indices n_1 and n_2 and impedances η_1 and η_2 . Its trajectory is given by $z[t]$ with normalized modulation velocity $\beta[t] = v_m[t]/c = dz[t]/d(ct)$, both parametrized with laboratory time, t . The interface parametrization must be differentiable, which is typically satisfied for physically realizable interfaces;

we can therefore consider that *arbitrary physical* motion is allowed. The interface is illuminated by an incoming electromagnetic wave that, upon interaction, scatters and generates multiple components. The characteristics of these scattered waves depend on the instantaneous velocity of the interface at the time of interaction. Our aim is to determine the scattered waveforms and their frequency content.

Locally—that is, over a sufficiently short interaction region—an accelerated interface (curved trajectory) behaves like a uniformly moving interface (straight trajectory), a case that has been extensively studied [26]. This is analogous to the principle that, locally, general relativity reduces to special relativity [53]. This local equivalence offers an intuitive and practical approximation: well-established solutions for uniformly moving interfaces can be used to approximate the scattering behavior of accelerated interfaces over a small region. This approximation holds when the curvature of the trajectory is negligible over the interaction region, meaning the interface velocity varies slowly compared with the wave dynamics. This equivalence is illustrated by comparing the curved accelerated trajectory with its locally tangent uniform-velocity counterpart in the middle configurations in Fig. 1.

Figure 1 summarizes the different possible local scattering scenarios. These corresponding configurations fall into one of three velocity regimes, each with distinct physical behavior and determined by the interface velocity at the time of scattering: subluminal (bottom part in Fig. 1), superluminal (top), and interluminal (middle), with $n_1 < n_2$, which is assumed throughout the text. In the *subluminal* regime ($|\beta[t]| < 1/n_2$), the interface velocity is less than the speed of light in both media. This case resembles that of a stationary interface: an incident wave generates a reflected wave in the incident medium, propagating in the opposite direction to the incident wave, and a transmitted wave in the other medium, propagating in

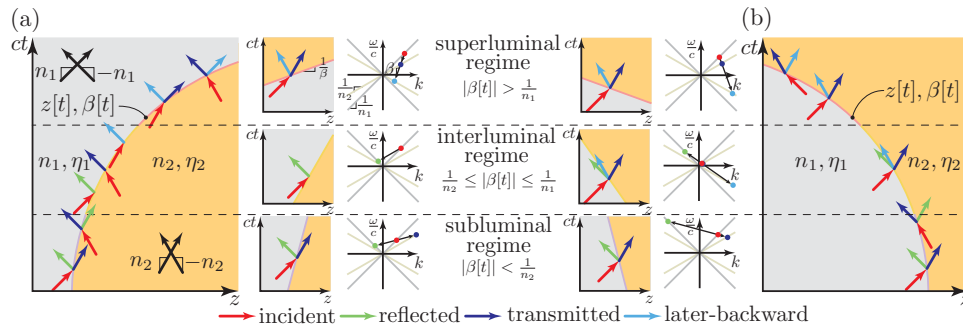


FIG. 1. Generalization of electromagnetic wave scattering from uniformly moving interfaces to accelerated interfaces, parametrized by a trajectory $z[t]$ and normalized modulation velocity $\beta[t] = dz[t]/d(ct)$. (a) Left: Space-time diagram of an accelerated interface, whose time-varying *positive* velocity sweeps through all three velocity regimes—subluminal ($|\beta[t]| < 1/n_2$), interluminal ($1/n_2 \leq |\beta[t]| \leq 1/n_1$), and superluminal ($|\beta[t]| > 1/n_1$), assuming $n_1 < n_2$. Right: Corresponding local constant-velocity reference cases for a forward-propagating wave in the first medium, each represented by the related space-time diagrams (left insets) and spectral transition diagrams (right insets). (b) Same as (a) but for a *negative* modulation velocity.

the same direction as the incident wave. In the *superluminal* regime ($|\beta[t]| > 1/n_1$), the interface velocity exceeds the speed of light in both media, and the behavior is analogous to that of a purely temporal interface. In contrast to the subluminal case, the “reflected” wave now appears in the opposite medium and is therefore often referred to as the “later-backward wave.” Between these regimes lies the *interluminal* regime ($1/n_2 \leq |\beta[t]| \leq 1/n_1$), the least understood so far, where the interface velocity is between the speeds of light in the two media. In this regime, the scattering behavior depends not only on whether the interface moves codirectionally or contradirectionally relative to the incident wave but also on the incident medium [57,58]. For instance, a forward codirectional wave incident from the first medium produces only a reflected wave [Fig. 1(a)], because although such a wave is fast enough to catch up with the interface ($1/n_1 > \beta[t]$), it is too slow to propagate in the second medium ($1/n_2 < \beta[t]$). In the contradirectional case [Fig. 1(b)], the same incident wave generates three scattered waves, since the interface now moves more slowly than light in the second medium ($\beta[t] < -1/n_2 < 1/n_2$), producing, in addition to the reflected wave, a transmitted ($1/n_2$)—or later-forward—wave and a later-backward ($-1/n_2$) wave. Finally, there is never interluminal scattering from the second medium into the first medium when the modulation velocity is negative [Fig. 1(b)], because any wave in the second medium—whether codirectional or contradirectional—cannot catch up with the interface.

While interfaces moving at constant velocity are well-understood, accelerating interfaces introduce complexities that cannot be captured by the local constant-velocity approximation alone. First, waves with some temporal duration may interact with the accelerating interface at different times and, therefore, at different velocities. When the wave encounters the interface at a point where the velocity regime changes, different portions of the wave undergo distinct interactions depending on the local velocity regime, resulting in more complex and diverse scattering dynamics. In contrast, a uniformly moving interface remains confined to a single velocity regime. Second, acceleration can lead to multiple scattering events. As the interface speeds up or down, it may overtake previously scattered waves, involving additional scattering. For example, in Fig. 1(a), an initially subluminal scattering can be followed by a second interaction when the accelerating interface catches up with the transmitted wave. Third, acceleration induces frequency chirping. Unlike a uniformly moving interface, which produces constant Doppler shifting, an accelerating interface generates time-varying frequency shifting. This is evident from the frequency transition diagrams in Fig. 1: for constant velocity, the transition slope is fixed, resulting in fixed Doppler shifting, but when the velocity changes due to acceleration, the slope varies over time. Consequently, different frequency transitions occur

at different times, producing frequency chirping in the scattered wave.

III. ANALYSIS

A. Scattering formulas

We consider two isotropic, linear, and nondispersive media separated by a moving interface in a one-dimensional configuration. The interface moves along the z direction and the electromagnetic fields depend solely on z and t . The electric field is polarized along the x direction and the magnetic field is polarized along the y direction. General forward-traveling and backward-traveling waveforms in each medium are expressed as

$$E_i^\pm = \psi_i^\pm[\phi_i^\pm], \quad H_i^\pm = \pm \frac{1}{\eta_i} \psi_i^\pm[\phi_i^\pm], \quad (1a)$$

$$D_i^\pm = \frac{1}{u_i \eta_i} \psi_i^\pm[\phi_i^\pm], \quad B_i^\pm = \pm \frac{1}{u_i} \psi_i^\pm[\phi_i^\pm], \quad (1b)$$

where square brackets indicate the arguments of the wave functions $\psi_i^\pm$, and $\phi_i^\pm = z/u_i \mp t$ denote the traveling wave variables. The subscript $i = 1, 2$ labels the medium, while the superscript $\pm$ denotes forward-propagating (+) or backward-propagating (−) waves. The wave speed in medium i is given by $u_i = c/n_i$. In the following, we analyze the scattering of an electromagnetic pulse propagating from medium 1 to medium 2 across the interface for all three velocity regimes (Fig. 1).

Conventionally, moving interfaces are treated by transforming the problem to the comoving frame, where the boundary appears stationary (see, e.g., Refs. [2,26,29,36,37,51]), an approach known as “frame hopping” [52]. In this approach, Lorentz transformations are applied in the case of a constant velocity [26] and Rindler transformations are applied in the case of a constant proper acceleration [51]. Here we solve the problem directly in the laboratory frame by enforcing *moving* boundary conditions at the interface. The solutions are extended to the entire space-time domain, through a suitable change of variables, without requiring frame hopping.

1. Subluminal regime

For an incident forward-propagating pulse in the first medium, ψ_1^+ , the reflected and transmitted waves are ψ_1^- and ψ_2^+ , respectively (Fig. 1). The boundary conditions require the continuity of $\mathbf{E} + c\boldsymbol{\beta} \times \mathbf{B}$ and $\mathbf{H} - c\boldsymbol{\beta} \times \mathbf{D}$ at the interface [2]. Feeding the general waveforms [Eqs. (1)] into these boundary conditions gives a system of equations for ψ_1^- and ψ_2^+ , whose solution reads (see Appendix A 1)

$$\psi_1^-[f_1^-[t^*]] = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} \frac{1 - n_1 \beta[t^*]}{1 + n_1 \beta[t^*]} \psi_1^+[f_1^+[t^*]], \quad (2a)$$

$$\psi_2^+[f_2^+[t^*]] = \frac{2\eta_2}{\eta_2 + \eta_1} \frac{1 - n_1\beta[t^*]}{1 - n_2\beta[t^*]} \psi_1^+[f_1^+[t^*]], \quad (2b)$$

where t^* denotes the time at which the incident pulse scatters at the interface, referred to as the “scattering time.” The auxiliary function $f_i^\pm$ is defined as

$$f_i^\pm[t^*] = \frac{z[t^*]}{u_i} \mp t^*. \quad (3)$$

Equations (2a) and (2b) define the scattered fields at the interface in terms of the scattering time t^* . To extend them across the entire space-time domain, the scattered waveforms must be re-expressed in terms of the traveling variables. This is accomplished by equating the arguments in Eqs. (2a) and (2b) to $\phi_i^\pm$, whose inverse is (see Appendix A 1)

$$t^* \mapsto (f_i^\pm)^{-1}[\phi_i^\pm]. \quad (4)$$

Equation (4) maps each space-time point (z, ct) —contained in the traveling wave variable $\phi_i^\pm$ —of the scattered wave in the entire space-time back to its scattering time t^* [59]. Insertion of Eq. (4) into Eqs. (2a) and (2b) yields the subluminal field solutions

$$\psi_1^-[\phi_1^-] = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} \frac{1 - n_1\beta[(f_1^-)^{-1}[\phi_1^-]]}{1 + n_1\beta[(f_1^-)^{-1}[\phi_1^-]]} \psi_1^+[f_1^+[(f_1^-)^{-1}[\phi_1^-]]], \quad (5a)$$

$$\psi_2^+[\phi_2^+] = \frac{2\eta_2}{\eta_2 + \eta_1} \frac{1 - n_1\beta[(f_2^+)^{-1}[\phi_2^+]]}{1 - n_2\beta[(f_2^+)^{-1}[\phi_2^+]]} \psi_1^+[f_1^+[(f_2^+)^{-1}[\phi_2^+]]]. \quad (5b)$$

Each solution consists of three components. The first factor corresponds to the stationary Fresnel scattering coefficients ($\beta[t] = 0$). The second factor accounts for the Doppler-related stretching or compression factor. The last term is the incident wave retimed according to Eq. (4).

The frequency shift $\omega_i^\pm$ may be obtained by differentiating the phase of each scattered waveform with respect to t (see Appendix A 1), viz.,

$$\omega_i^\pm[t^*] = \pm \frac{n_i\beta[t^*] - 1}{n_i\beta[t^*] \mp 1}. \quad (6)$$

For a nonconstant interface velocity $\beta[t]$, Eq. (6) yields a time-varying Doppler shift, resulting in *chirping* of the scattered wave. This expression recovers the well-known result for a constant-velocity interface [26] upon

removal of the t^* dependence, but generalizes it to arbitrary time-dependent motion through the scattering time dependency.

To illustrate Eqs. (5a) and (5b), we consider an interface moving at constant normalized velocity β_0 , parameterized as $z[t] = \beta_0 ct$. In this case, the auxiliary function $f_i^\pm[t^*] = (n_i\beta_0 \mp 1)t^*$ is linear due to the linearity of the equation of motion and can be easily inverted as $(f_i^\pm)^{-1}[\phi_i^\pm] = 1/(n_i\beta_0 \mp 1)\phi_i^\pm$. Substitution of this into Eqs. (5a) and (5b) yields

$$\psi_1^-[\phi_1^-] = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} \frac{1 - n_1\beta_0}{1 + n_1\beta_0} \psi_1^+ \left[-\frac{1 - n_1\beta_0}{1 + n_1\beta_0} \phi_1^- \right], \quad (7a)$$

$$\psi_2^+[\phi_2^+] = \frac{2\eta_2}{\eta_2 + \eta_1} \frac{1 - n_1\beta_0}{1 - n_2\beta_0} \psi_1^+ \left[\frac{1 - n_1\beta_0}{1 - n_2\beta_0} \phi_2^+ \right]. \quad (7b)$$

As expected, the reflected and the transmitted waves both undergo a constant Doppler shift (factor multiplying the traveling waveform) because the interface velocity is constant, $\beta[t^*] = \beta_0$, in agreement with Eq. (6). Consequently, the scattered waves do not exhibit chirping. Note that for more general trajectory profiles, Eq. (4) may not be analytically tractable, and typically requires a numerical solution.

2. Superluminal regime

The approach to the superluminal regime follows the same procedure as in the subluminal case. For a forward-propagating incident wave in the first medium, the scattered waves are the later-backward wave ψ_2^- and the transmitted wave ψ_2^+ . Once again, the boundary conditions enforce the continuity of $\mathbf{E} + c\boldsymbol{\beta} \times \mathbf{B}$ and $\mathbf{H} - c\boldsymbol{\beta} \times \mathbf{D}$ at the interface [2]. Inserting the general waveforms [Eqs. (1a) and (1b)] into the boundary conditions, solving the resulting system, and extending the solution throughout the space-time plane, we obtain (see Appendix A 2)

$$\psi_2^-[\phi_2^-] = -\frac{\eta_2 - \eta_1}{2\eta_1} \frac{1 - n_1\beta[(f_2^-)^{-1}[\phi_2^-]]}{1 + n_2\beta[(f_2^-)^{-1}[\phi_2^-]]} \psi_1^+[f_1^+[(f_2^-)^{-1}[\phi_2^-]]], \quad (8a)$$

$$\psi_2^+[\phi_2^+] = \frac{\eta_2 + \eta_1}{2\eta_1} \frac{1 - n_1\beta[(f_2^+)^{-1}[\phi_2^+]]}{1 - n_2\beta[(f_2^+)^{-1}[\phi_2^+]]} \psi_1^+[f_1^+[(f_2^+)^{-1}[\phi_2^+]]]. \quad (8b)$$

Notably, the later-backward wave [Eq. (8a)] depends on n_2 in the denominator rather than n_1 as in the subluminal regime [Eq. (5a)], reflecting that this wave propagates in the second medium.

3. Interluminal regime

The general boundary conditions for the interluminal regime have not yet been elucidated in the literature. Therefore, we do not derive the interluminal scattering coefficients directly; instead, we adopt the results from Ref. [58], which solves the problem for the particular case of nonmagnetic materials ($\mu_1 = \mu_2$), and we extend these solutions to cover the entire space-time plane (see Appendix A 3). The scattering behavior in the interluminal regime depends on whether the interface moves codirectionally or contradirectionally and on the incident medium, resulting in different numbers of scattered waves, as illustrated in Fig. 1. For a forward-propagating incident wave in the first medium and a codirectionally moving interface [Fig. 1(a)], the reflected wave is denoted by ψ_1^- and is given by

$$\psi_1^-[\phi_1^-] = - \left(\frac{1 - n_1 \beta [(f_1^-)^{-1} [\phi_1^-]]}{1 + n_1 \beta [(f_1^-)^{-1} [\phi_1^-]]} \right)^2 \psi_1^+ [f_1^+ [(f_1^-)^{-1} [\phi_1^-]]]. \quad (9)$$

In the contradirectional case [Fig. 1(b)], the scattering is more complex, involving a reflected wave ψ_1^- , a later-backward wave ψ_2^- , and a transmitted wave ψ_2^+ , which are given by

$$\psi_1^-[\phi_1^-] = -\psi_1^+ [f_1^+ [(f_1^-)^{-1} [\phi_1^-]]], \quad (10a)$$

$$\psi_2^-[\phi_2^-] = -\frac{n_1}{n_2} \psi_1^+ [f_1^+ [(f_2^-)^{-1} [\phi_2^-]]], \quad (10b)$$

$$\psi_2^+[\phi_2^+] = \frac{n_1}{n_2} \psi_1^+ [f_1^+ [(f_2^+)^{-1} [\phi_2^+]]]. \quad (10c)$$

B. Examples

Figure 2 shows two representative examples of the analysis problem [Eqs. (5a), (5b), (8a), (8b), (9),

and (10a)–(10c)], showcasing the space-time evolution, time-domain waveforms, and corresponding spectra. Figure 2(a) shows an S-shaped trajectory, while Fig. 2(b) demonstrates an example of a space-time cavity. Both examples use nonmagnetic materials to ensure the validity of the adopted interluminal solutions [Eqs. (9) and (10a)–(10c)].

In the S-shaped trajectory example [Fig. 2(a)], the interface smoothly oscillates through subluminal, interluminal, and superluminal velocity regimes, producing the scattering behavior described in Sec. II. The motion imprints a clear chirping pattern on the scattered waves, visible in the time-domain signal of the transmitted wave. Moreover, the interface causes multiple scattering events. For example, the center part of the incoming pulse scatters at a contramoving interluminal section of the interface, resulting in a back-scattered wave into the second medium. As the interface reverses direction, it interacts again with this previously scattered wave, producing additional scattering.

The solutions presented in this section apply not only to a single interface but also to configurations involving multiple interfaces, since each scattering event may be treated as independent of prior interactions. Figure 2(b) shows an example of a space-time cavity formed by two moving interfaces enclosing a region of different refractive index for a finite time. Such a cavity may act as a dynamic analogue of a Fabry-Pérot resonator: it opens and closes in both space and time, trapping waves through repeated reflections. As the wave encounters the first interface, part of it is reflected with a nonuniform Doppler shift, determined by Eq. (6). Inside the cavity, the wave undergoes multiple scattering events, each introducing additional nonuniform Doppler shifts.

To validate our analytical results, we have performed full-wave finite-difference time-domain (FDTD) simulations using a generalized Yee grid [49,60] that ensures the enforcement of proper moving boundary conditions. The simulation results are shown in Fig. 3. They include a constant, proper-accelerated interface [Fig. 3(a)] and an oscillatory backward-moving interface [Fig. 3(b)]. Excellent agreement with the analytical predictions is observed.

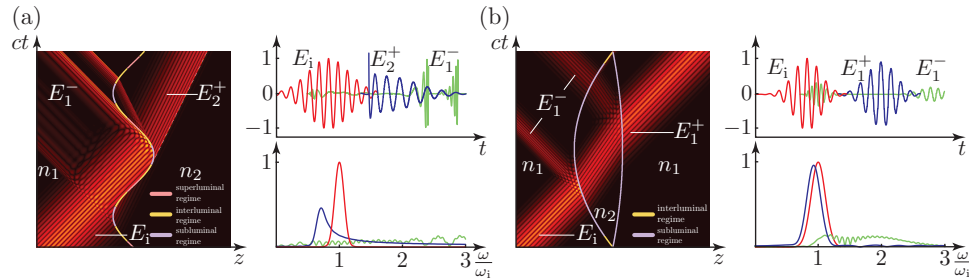


FIG. 2. Illustrative examples for the analysis problem [Eqs. (5a), (5b), (8a), (8b), (9), and (10a)–(10c)] of electromagnetic scattering at arbitrary accelerated interfaces (Fig. 1) in different velocity regimes, showing the space-time evolution, time-domain waveforms far from the interface, and corresponding spectra. (a) Single interface with an S-shape trajectory. (b) Space-time cavity.

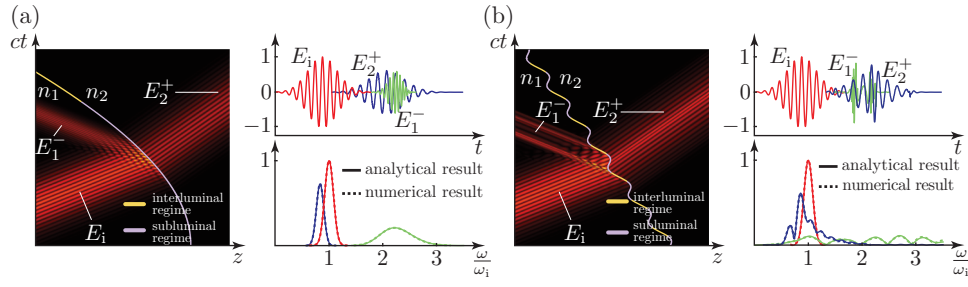


FIG. 3. Full-wave FDTD validation of the analysis problem [Eqs. (5a), (5b), (8a), (8b), (9), and (10a)–(10c)], showing the space-time evolution, time-domain waveforms far from the interface, and corresponding spectra. (a) Constant, proper-accelerated interface. (b) Backward oscillating trajectory.

Since the theory is exact, any discrepancies are attributable to numerical approximations, as we verified by increasing the computational mesh density within the memory limits of our computer system. These examples are restricted to subluminal and interluminal velocities, because the FDTD code used tends to be unstable in the superluminal regime.

IV. SYNTHESIS

A. Interface equation of motion

In Sec. III, we showed that the scattered waves exhibit a specific temporal chirping profile governed by the acceleration of the interface [Eq. (6)]. In this section, we address the inverse (synthesis) problem: given a desired chirping profile, denoted by $\varphi[t]$, for the *transmitted* wave, we seek the interface trajectory, $z[t]$, that produces it. Although the synthesis focuses on the transmitted component, the method is equally applicable to other scattered waves, including reflected and later-backward-propagating components.

Solving this inverse problem is nontrivial because the phase argument of the transmitted wave [Eq. (5b)] contains nested dependencies of the unknown trajectory, $z[t]$: f_1^+ is evaluated at the inverse of f_2^+ , and both depend on $z[t]$ [Eq. (3)]. Nevertheless, by introducing a suitable change of variables, we obtain an explicit expression for the interface equation of motion (see Appendix B 1), viz.,

$$z[t] = \frac{u_1 u_2}{u_1 - u_2} (x[t] - \varphi[x[t]]), \quad (11)$$

where the auxiliary function $x[t]$ satisfies

$$\frac{u_2 x - u_1 \varphi[x]}{u_1 - u_2} = t, \quad (12)$$

For complex chirping profiles, Eq. (12) may need to be solved numerically.

To illustrate Eqs. (11) and (12), consider the simple case of a uniform Doppler shift, viz., $\varphi[t] = at$. Solving Eq. (12) for $x[t]$, we obtain $x[t] = (u_1 - u_2) / (u_2 - au_1) t$, which

can then be substituted into Eq. (11), giving

$$z[t] = \frac{1 - a}{n_1 - an_2} ct. \quad (13)$$

Equation (13) describes an interface moving at a constant velocity $\beta_0 = (1 - a) / (n_1 - an_2)$, resulting in the Doppler shift $(1 - n_1 \beta_0) / (1 - n_2 \beta_0) = a$ [Eq. (6)], as expected.

Although Eqs. (11) and (12) permit *a priori* arbitrary chirping profiles, they include a fundamental constraint arising from the physics of the scattering process: not every configuration supports a transmitted wave in the second medium. Specifically, for a forward incident wave, codirectional interluminal and superluminal interfaces do not generate a transmitted component in the second medium (Fig. 1). Therefore, we must restrict the interface velocity to remain below the lower codirectional interluminal bound, viz., $\beta[t] < 1/n_2$. This requirement imposes the following condition on the admissible chirping profiles (see Appendix B 2):

$$\dot{\varphi}[x[t]] > \frac{n_1}{n_2}, \quad (14)$$

where the dot denotes differentiation with respect to time, indicating that the instantaneous frequency must exceed this threshold for consistent transmission.

B. Examples

Figure 4 demonstrates the synthesis problem [Eqs. (11) and (12)] for two chirping profiles—a linear chirp [Fig. 4(a)] and an oscillatory chirp [Fig. 4(b)], both satisfying the chirping constraint [Eq. (14)]. Each case is illustrated through a space-time diagram, time domain waveforms, and related spectrograms. The red curve in each spectrogram marks the analytically prescribed chirping profile. Both examples assume impedance matching ($\eta_1 = \eta_2$) to suppress spurious reflections and focus solely on the desired transmitted wave.

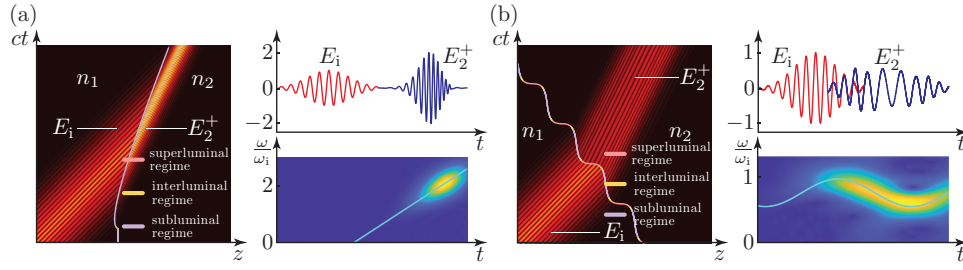


FIG. 4. Illustrative examples of the synthesis problem [Eqs. (11) and (12)] illustrated through space-time diagrams, time-domain scattered waveforms far from the interface, and corresponding spectrograms. (a) Linear chirp. (b) Oscillatory chirp.

Figure 4(a) shows the synthesis problem for a linear chirping profile, viz., $\varphi[t] = at + bt^2$, with a the transmitted Doppler shift and b the chirp rate. After a stationary period, the interface moves for a very short time at an extremely high negative velocity (flat segment) and starts accelerating toward the interluminal asymptote $\beta = 1/n_2$, where the local Doppler shift diverges [Eq. (6)]. Consequently, the transmitted frequency continuously upshifts over time, which is clearly visible in both the time-domain signal and its spectrogram.

Figure 4(b) presents a more complex situation, where the chirping profile combines a linear shift with a sinusoidal modulation, $\varphi[t] = at + A \sin(2\pi\omega_c t)$, where A is the amplitude and ω_c the oscillation frequency of the chirp profile. The synthesized interface trajectory consists of a backward drift with superimposed oscillations, causing it to traverse multiple velocity regimes. Physically, this motion can be viewed as the combination of a steady backward velocity (responsible for a) and an oscillatory component that modulates the instantaneous frequency, producing the characteristic sinusoidal chirp pattern evident in the spectrogram.

V. CONCLUSIONS AND DISCUSSION

We have presented exact field solutions for scattering at an arbitrarily accelerated interface, covering the three interface velocity regimes. Unlike previous approaches that rely on the frame-hopping method, we solved the problem directly in the laboratory frame through a suitable change of variables. Our results show that accelerating interfaces introduce phenomena that do not occur in uniformly moving interfaces, including transitions between multiple velocity regimes, multiple scattering events, and generalized frequency chirping. In addition, we have addressed the inverse problem, demonstrating how tailored acceleration can be used to realize prescribed chirping profiles. Solving this synthesis problem is a crucial step toward designing space-time interfaces capable of producing targeted frequency modulation, with potential applications in analog signal processing [54] and chirped pulse shaping [55].

Experimental realization of accelerated interfaces, while challenging, appears feasible on the basis of existing demonstrations of purely temporal interfaces in both microwave [61–63] and optical [64–66] systems. In the microwave regime, the effective impedance of artificial transmission lines can be modulated through high-speed electronic switches driven in a controlled, accelerated sequence via synchronized timing or corporate feed networks with optical delay lines. In the optical regime, pump-probe setups are able to generate temporal refractive-index fronts. By positioning of the pump beam obliquely with respect to the substrate and before a focusing element, the resulting curved pump wave front excites the substrate at differing incidence angles, thereby producing an effectively accelerated interface. These approaches indicate that accelerated space-time interfaces could be experimentally investigated in both the microwave domain and the optical domain.

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The authors declare that they have no competing interests.

DATA AVAILABILITY

The data that support the findings of this article are not publicly available upon publication because it is not technically feasible and/or the cost of preparing, depositing, and hosting the data would be prohibitive within the terms of this research project. The data are available from the authors upon reasonable request.

APPENDIX A: SCATTERING AT ACCELERATED INTERFACES

In this appendix we derive the electromagnetic scattering solutions at a nonuniformly (accelerating) moving

interface. The interface separates two isotropic, linear, and nondispersive media, characterized by refractive indices n_1 and n_2 and impedances η_1 and η_2 . The interface equation of motion has the explicit form $z[t]$, with normalized velocity $\beta[t] = dz/d(ct)$. We consider a one-dimensional geometry: the interface moves along the z direction, the electric field is polarized along the x direction and magnetic field is polarized along the y direction. A general traveling wave in each medium takes the form

$$E_i^\pm = \psi_i^\pm[\phi_i^\pm], \quad H_i^\pm = \pm \frac{1}{\eta_i} \psi_i^\pm[\phi_i^\pm], \quad (\text{A1a})$$

$$D_i^\pm = \frac{1}{u_i \eta_i} \psi_i^\pm[\phi_i^\pm], \quad B_i^\pm = \pm \frac{1}{u_i} \psi_i^\pm[\phi_i^\pm], \quad (\text{A1b})$$

where we have used square brackets to indicate the arguments of the wave functions $\psi_i^\pm$, and

$$\phi_i^\pm = z/u_i \mp t \quad (\text{A2})$$

are the traveling wave variables. The subscript $i = 1, 2$ denotes the medium, while the superscript $\pm$ indicates forward-propagating (+) or backward-propagating (−) waves. The wave speed in medium i is given by $u_i = c/n_i$.

1. Subluminal regime

In the subluminal velocity regime ($|\beta[t]| < 1/n_2$), scattering consists of a reflected wave, in the same medium but propagating in the opposite direction to the incoming wave, and a transmitted wave, in the opposite medium, propagating in the same direction as the incident wave. The boundary conditions are the continuity of $\mathbf{E} + c\boldsymbol{\beta} \times \mathbf{B}$ and $\mathbf{H} - c\boldsymbol{\beta} \times \mathbf{D}$ at the interface [2]. Considering a forward-propagating incident wave in the first medium, ψ_1^+ , the reflected and transmitted waves are ψ_1^- and ψ_2^+ , respectively. The boundary conditions then become

$$(E_1^+ - c\beta B_1^+) + (E_1^- - c\beta B_1^-)|_{z=z[t^*]} = (E_2^+ - c\beta B_2^+)|_{z=z[t^*]}, \quad (\text{A3a})$$

$$(H_1^+ - c\beta D_1^+) + (H_1^- - c\beta D_1^-)|_{z=z[t^*]} = (H_2^+ - c\beta D_2^+)|_{z=z[t^*]}, \quad (\text{A3b})$$

where t^* represents the *scattering time* at the interface. Inserting the general waveforms [Eqs. (A1a) and (A1b)] into these boundary conditions [Eqs. (A3a) and (A3b)] and evaluating the result at the interface, we obtain the following system of equations for ψ_1^- and ψ_2^+ :

$$(1 - n_1\beta[t^*]) \psi_1^+\left[\frac{z[t^*]}{u_1} - t^*\right] + (1 + n_1\beta[t^*]) \psi_1^-\left[\frac{z[t^*]}{u_1} + t^*\right] = (1 - n_2\beta[t^*]) \psi_2^+\left[\frac{z[t^*]}{u_2} - t^*\right], \quad (\text{A4a})$$

$$\frac{1}{\eta_1} (1 - n_1\beta[t^*]) \psi_1^+\left[\frac{z[t^*]}{u_1} - t^*\right] - \frac{1}{\eta_1} (1 + n_1\beta[t^*]) \psi_1^-\left[\frac{z[t^*]}{u_1} + t^*\right] = \frac{1}{\eta_2} (1 - n_2\beta[t^*]) \psi_2^+\left[\frac{z[t^*]}{u_2} - t^*\right]. \quad (\text{A4b})$$

Solving Eqs. (A4a) and (A4b) for ψ_1^- and ψ_2^+ , we obtain

$$\psi_1^-[f_1^-[t^*]] = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} \frac{1 - n_1\beta[t^*]}{1 + n_1\beta[t^*]} \psi_1^+[f_1^+[t^*]], \quad (\text{A5a})$$

$$\psi_2^+[f_2^+[t^*]] = \frac{2\eta_2}{\eta_2 + \eta_1} \frac{1 - n_1\beta[t^*]}{1 - n_2\beta[t^*]} \psi_1^+[f_1^+[t^*]], \quad (\text{A5b})$$

where we used the shortcut function $f_i^\pm$, defined as

$$f_i^\pm[t^*] = \frac{z[t^*]}{u_i} \mp t^*. \quad (\text{A6})$$

Equations (A5a) and (A5b) are valid only at the interface, since the time parameter they involve is the scattering time at the interface. To determine the scattered waves at *any space-time point*, we require that the arguments on the left-hand side of Eqs. (A5a) and (A5b) match the traveling

wave variables, ϕ_1^- and ϕ_2^+ :

$$f_1^-[t^*] = \phi_1^-, \quad (\text{A7a})$$

$$f_2^+[t^*] = \phi_2^+. \quad (\text{A7b})$$

Next, we invert both sides of Eqs. (A7a) and (A7b), yielding the changes of variables

$$t^* \mapsto (f_1^-)^{-1}[\phi_1^-], \quad (\text{A8a})$$

$$t^* \mapsto (f_2^+)^{-1}[\phi_2^+]. \quad (\text{A8b})$$

Substituting Eqs. (A8a) and (A8b) into Eqs. (A5a) and (A5b), we obtain the final result:

$$\psi_1^-[\phi_1^-] = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} \frac{1 - n_1\beta[(f_1^-)^{-1}[\phi_1^-]]}{1 + n_1\beta[(f_1^-)^{-1}[\phi_1^-]]} \psi_1^+[f_1^+[(f_1^-)^{-1}[\phi_1^-]]], \quad (\text{A9a})$$

$$\psi_2^+[\phi_2^+] = \frac{2\eta_2}{\eta_2 + \eta_1} \frac{1 - n_1\beta[(f_2^+)^{-1}[\phi_2^+]]}{1 - n_2\beta[(f_2^+)^{-1}[\phi_2^+]]} \psi_1^+[f_1^+[(f_2^+)^{-1}[\phi_2^+]]], \quad (\text{A9b})$$

where the t 's within $\phi_i^\pm$ [Eq. (A2)] represent now the *general laboratory time*, applying to both the scattering time and the propagation time.

The Doppler-induced frequency shift may be obtained by differentiating the phase argument of the scattered waves in Eqs. (A9a) and (A9b) with respect to time. Since both scattered waves have a phase of the form $f_1^+[t^*]$, with $t^* = (f_i^\pm)^{-1}[\phi_i^\pm]$ [Eqs. (A8a) and (A8b)], we find, using the definition of f_1^+ in Eq. (A6),

$$\begin{aligned} \omega_i^\pm[t^*] &= -\frac{\partial}{\partial t} (f_1^+[t^*]) \\ &= \left(1 - \frac{1}{u_1} \frac{dz[t^*]}{dt^*}\right) \frac{\partial t^*}{\partial t} \\ &= (1 - n_1\beta[t^*]) \frac{\partial t^*}{\partial t}, \end{aligned} \quad (\text{A10})$$

where $\beta[t^*] = dz[t^*]/d(ct^*)$ and $u_1 = c/n_1$ were used in the last equality. Using the identity $\partial_x(g^{-1}[x]) = 1/g'[g^{-1}[x]]$, we find for the last factor in Eq. (A10)

$$\begin{aligned} \frac{\partial t^*}{\partial t} &= \frac{\partial}{\partial t} \left((f_i^\pm)^{-1}[\phi_i^\pm] \right) \\ &= \mp \frac{1}{\partial_{t^*} (f_i^\pm[t^*])} \\ &= \mp \frac{1}{n_i\beta[t^*] \mp 1}. \end{aligned} \quad (\text{A11})$$

Inserting Eq. (A11) into Eq. (A10), we obtain the frequency shift

$$\omega_i^\pm[t^*] = \pm \frac{n_1\beta[t^*] - 1}{n_i\beta[t^*] \mp 1}, \quad (\text{A12})$$

where $t^* = (f_i^\pm)^{-1}[\phi_i^\pm]$ [Eqs. (A8a) and (A8b)]. If $\beta[t^*]$ is a nonconstant function, Eq. (A12) describes a time-varying Doppler shift—a chirp—whose profile is determined by the velocity trajectory.

2. Superluminal regime

In the superluminal regime ($|\beta[t]| > 1/n_1$), scattering consists of a later-backward wave, in the opposite medium and propagating in the opposite direction to the incoming wave, and a transmitted wave, in the opposite medium, propagating in the same direction as the incident wave. The boundary conditions are again the continuity of $\mathbf{E} + c\boldsymbol{\beta} \times \mathbf{B}$ and $\mathbf{H} - c\boldsymbol{\beta} \times \mathbf{D}$ at the interface [2]. Considering a forward-propagating incident wave in the first medium, ψ_1^+ , the later-backward and transmitted waves are ψ_2^- and ψ_2^+ , respectively. The boundary conditions then become

$$\begin{aligned} (E_1^+ - c\beta B_1^+)|_{z=z[t^*]} &= (E_2^- - c\beta B_2^-) \\ &\quad + (E_2^+ - c\beta B_2^+)|_{z=z[t^*]}, \end{aligned} \quad (\text{A13a})$$

$$\begin{aligned} (H_1^+ - c\beta D_1^+)|_{z=z[t^*]} &= (H_2^- - c\beta D_2^-) \\ &\quad + (H_2^+ - c\beta D_2^+)|_{z=z[t^*]}, \end{aligned} \quad (\text{A13b})$$

where once again t^* represents the scattering time at the interface. Inserting the general waveforms [Eqs. (A1a) and (A1b)] into these boundary conditions [Eqs. (A13a) and (A13b)] and evaluating the result at the interface, we obtain the following system of equations:

$$(1 - n_1\beta[t^*]) \psi_1^+\left[\frac{z[t^*]}{u_1} - t^*\right] = (1 + n_2\beta[t^*]) \psi_2^-\left[\frac{z[t^*]}{u_2} + t^*\right] + (1 - n_2\beta[t^*]) \psi_2^+\left[\frac{z[t^*]}{u_2} - t^*\right], \quad (\text{A14a})$$

$$\frac{1}{\eta_1} (1 - n_1\beta[t^*]) \psi_1^+\left[\frac{z[t^*]}{u_1} - t^*\right] = -\frac{1}{\eta_2} (1 + n_2\beta[t^*]) \psi_2^-\left[\frac{z[t^*]}{u_2} + t^*\right] + \frac{1}{\eta_2} (1 - n_2\beta[t^*]) \psi_2^+\left[\frac{z[t^*]}{u_2} - t^*\right]. \quad (\text{A14b})$$

Solving Eqs. (A14a) and (A14b) for the scattered waves, we obtain

$$\psi_2^-[f_2^-[t^*]] = -\frac{\eta_2 - \eta_1}{2\eta_1} \frac{1 - n_1\beta[t^*]}{1 + n_2\beta[t^*]} \psi_1^+[f_1^+[t^*]], \quad (\text{A15a})$$

$$\psi_2^+[f_2^+[t^*]] = \frac{\eta_2 + \eta_1}{2\eta_1} \frac{1 - n_1\beta[t^*]}{1 - n_2\beta[t^*]} \psi_1^+[f_1^+[t^*]], \quad (\text{A15b})$$

where the functions f_2^+ and f_2^- are defined in Eq. (A6). Again, Eqs. (A15a) and (A15b) are valid only at the interface, and to extend them across the space-time domain,

we enforce that the arguments on the left-hand side of Eqs. (A15a) and (A15b) take the traveling waveform variables, ϕ_2^- and ϕ_2^+ :

$$f_2^-[t^*] = \phi_2^-, \quad (\text{A16a})$$

$$f_2^+[t^*] = \phi_2^+. \quad (\text{A16b})$$

Inversion of both sides of Eqs. (A16a) and (A16b) yields the change of variables

$$t^* \mapsto (f_2^-)^{-1}[\phi_2^-], \quad (\text{A17a})$$

$$t^* \mapsto (f_2^+)^{-1}[\phi_2^+]. \quad (\text{A17b})$$

Substitution of Eqs. (A17a) and (A17b) into Eqs. (A15a) and (A15b) yields the final result

$$\psi_2^-[\phi_2^-] = -\frac{\eta_2 - \eta_1}{2\eta_1} \frac{1 - n_1\beta \left[(f_2^-)^{-1}[\phi_2^-] \right]}{1 + n_2\beta \left[(f_2^-)^{-1}[\phi_2^-] \right]} \psi_1^+ \left[f_1^+ \left[(f_2^-)^{-1}[\phi_2^-] \right] \right], \quad (\text{A18a})$$

$$\psi_2^+[\phi_2^+] = \frac{\eta_2 + \eta_1}{2\eta_1} \frac{1 - n_1\beta \left[(f_2^+)^{-1}[\phi_2^+] \right]}{1 - n_2\beta \left[(f_2^+)^{-1}[\phi_2^+] \right]} \psi_1^+ \left[f_1^+ \left[(f_2^+)^{-1}[\phi_2^+] \right] \right]. \quad (\text{A18b})$$

The Doppler frequency shift can be determined in a fashion similar to that in the previous section.

3. Interluminal regime

The boundary conditions for the interluminal regime ($1/n_2 \leq |\beta[t]| \leq 1/n_1$) are unknown and hence do not admit a general waveform solution, not even for uniformly moving interfaces. However, if the media are nonmagnetic ($\mu_1 = \mu_2$), the problem becomes solvable [57,58]. Under this assumption, we apply the solutions from Ref. [58] across space-time using a change of variables. As before, we consider a forward-propagating incident wave in the first medium, ψ_1^+ . In the *comoving* case [Fig. 1(a)], there is only reflection, ψ_1^- , with scattering coefficient at the interface $(1 - n_1\beta[t^*])^2 / (1 + n_1\beta[t^*])^2$. Applying the change of variables as in the subluminal case for reflection [Eq. (A8a)], we obtain

$$\psi_1^-[\phi_1^-] = - \left(\frac{1 - n_1\beta \left[(f_1^-)^{-1}[\phi_1^-] \right]}{1 + n_1\beta \left[(f_1^-)^{-1}[\phi_1^-] \right]} \right)^2 \psi_1^+ \left[f_1^+ \left[(f_1^-)^{-1}[\phi_1^-] \right] \right]. \quad (\text{A19})$$

For the *contramoving* case [Fig. 1(b)], there are three scattering coefficients: -1 for reflection (ψ_1^-), $-n_1/n_2$ for

later-backward (ψ_2^-), and n_1/n_2 for transmission (ψ_2^+). Application of the change of variables to these waves results in the following final solutions:

$$\psi_1^-[\phi_1^-] = -\psi_1^+ \left[f_1^+ \left[(f_1^-)^{-1}[\phi_1^-] \right] \right], \quad (\text{A20a})$$

$$\psi_2^-[\phi_2^-] = -\frac{n_1}{n_2} \psi_1^+ \left[f_1^+ \left[(f_2^-)^{-1}[\phi_2^-] \right] \right], \quad (\text{A20b})$$

$$\psi_2^+[\phi_2^+] = \frac{n_1}{n_2} \psi_1^+ \left[f_1^+ \left[(f_2^+)^{-1}[\phi_2^+] \right] \right]. \quad (\text{A20c})$$

APPENDIX B: ARBITRARY CHIRPING

Here we derive the interface trajectory, $z[t]$, corresponding to a prescribed chirping modulation profile, $\varphi[t]$, for the *transmitted* wave. Although the analysis focuses on the transmitted component, the method is equally applicable to other scattered waves.

1. Interface profile

To impose a prescribed chirping profile on the transmitted wave, we require that its phase argument [Eq. (A9b)] matches the chirping profile:

$$\frac{z[t^*]}{u_1} - t^* = \varphi[x], \quad (\text{B1})$$

where $t^* = (f_2^+)^{-1}[x]$ denotes the scattering time at $x = z/u_2 - t$ [Eq. (A8b)]. Directly solving Eq. (B1) for $z[t]$ is not straightforward, as both $z[t^*]$ and t^* depend on the unknown interface profile. However, Eq. (B1), together with the change of variables relating $z[t^*]$ and t^* to x [Eqs. (A7a) and (A7b)], forms a system of equations

$$\begin{cases} \frac{z[t^*]}{u_1} - t^* = \varphi[x], \\ \frac{z[t^*]}{u_2} - t^* = x, \end{cases} \quad (\text{B2})$$

which can be solved for $z[t^*]$ and t^* :

$$\begin{cases} z[t^*] = \frac{u_1 u_2}{u_1 - u_2} (x - \varphi[x]), \\ t^* = \frac{u_2 x - u_1 \varphi[x]}{u_1 - u_2}. \end{cases} \quad (\text{B3})$$

Note that Eq. (B3) provides $z[t^*]$, whereas we seek $z[t]$. To express the interface equation of motion in terms of the laboratory time, we substitute the second equation in Eq. (B3)

into the first, giving

$$z \left[\frac{u_2 x - u_1 \varphi[x]}{u_1 - u_2} \right] = \frac{u_1 u_2}{u_1 - u_2} (x - \varphi[x]). \quad (\text{B4})$$

Next, we introduce the change of variables:

$$\frac{u_2 x - u_1 \varphi[x]}{u_1 - u_2} \mapsto t, \quad (\text{B5})$$

which implicitly defines $x[t]$ for a given chirping profile $\varphi[t]$. Substituting Eq. (B5) into Eq. (B4), we obtain the final expression for the interface profile,

$$z[t] = \frac{u_1 u_2}{u_1 - u_2} (x[t] - \varphi[x[t]]), \quad (\text{B6})$$

where $x[t]$ solves Eq. (B5) and may require numerical methods for complex chirping profiles $\varphi[x]$.

2. Limitation

There is a physical constraint on the choice of the chirping profile $\varphi[t]$ as scattered transmitted waves exist only when $\beta[t] < 1/n_2$ (Fig. 1). This constraint translates into a condition on $\varphi[t]$ upon differentiation of Eq. (B6) with respect to time and normalization of the result with respect to the speed of light in vacuum, which yields

$$\beta[t] = \frac{1}{c} \frac{dz}{dt} = \frac{1}{c} \frac{u_1 u_2}{u_1 - u_2} (1 - \dot{\varphi}[x[t]]) \dot{x}[t], \quad (\text{B7})$$

where dots indicate time derivatives. A corresponding expression for $\dot{x}[t]$ may be found by differentiating Eq. (B5) with respect to t and solving for $\dot{x}[t]$, viz.,

$$\dot{x}[t] = \frac{u_1 - u_2}{u_2 - u_1 \dot{\varphi}[x[t]]}. \quad (\text{B8})$$

Substituting Eq. (B8) into Eq. (B7), we obtain

$$\beta[t] = \frac{1 - \dot{\varphi}[x[t]]}{n_1 - n_2 \dot{\varphi}[x[t]]}. \quad (\text{B9})$$

Imposing $\beta[t] < 1/n_2$, we find the following condition on the choice of the chirping profile:

$$\dot{\varphi}[x[t]] > \frac{n_1}{n_2}. \quad (\text{B10})$$

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