

Continuous cloud position spectroscopy using a magneto-optical trap

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We demonstrate a continuous spectroscopy technique with frequency sensitivity well below the natural transition linewidth, while maintaining a locking range hundreds of times larger. The method exploits the position dependence of a continuous, broadband magneto-optical trap (MOT) operating on the 7.5-kHz-wide intercombination line of strontium. We show that the frequency sensitivity is fundamentally insensitive to the effective MOT laser linewidth. By applying active feedback on the MOT position to a dispersion-optimized frequency comb, which serves as the reference for stabilizing the MOT laser (Boughdachi *et al.*, arXiv:2507.23617), we achieve a frequency instability below 4.4×10^{-13} after 400 s of averaging in both the optical and radio-frequency domains, surpassing the performance of conventional hot-vapor modulation transfer spectroscopy. Our method is a broadly applicable alternative route to frequency references in the high 10^{-14} range around 100 s.

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Introduction. Continuously operating atomic clocks, such as hydrogen masers in the radio-frequency (rf) domain [1,2], and systems employing modulation transfer spectroscopy in the optical domain [3–6], have demonstrated frequency stability of the order of 10^{-14} after 100 s of averaging [3–7]. Recent advances have enabled the development of compact, field-deployable optical atomic clocks, including systems based on tetrahedral and pyramidal magneto-optical traps (MOTs) [8–10], microcell-based clocks [11,12], and vapor-based clocks [13–15]. While modulation transfer spectroscopy remains widely used in these systems, it is limited by short-term probe laser instability and a narrow frequency locking range, both typically of the order of the transition linewidth [16,17]. These limitations are especially pronounced for narrow-line optical transitions, such as those found in the alkaline-earth and alkaline-earth-like atoms. Additionally, modulation transfer spectroscopy does not provide direct access to the rf domain.

In this paper, we present a simple, continuous spectroscopy technique that exploits the dependence of the MOT position along the vertical axis (gravity) on the MOT laser frequency in a five-beam configuration. We demonstrate that this method improves both the frequency locking range and tolerance to short-term probe laser instability by two orders of magnitude compared to the natural linewidth, while simultaneously achieving a spectral resolution more than an order of magnitude below the 7.5-kHz natural linewidth of the ^{88}Sr strontium intercombination transition. By applying active feedback based on the MOT position to a dispersion-optimized frequency comb, used for stabilizing the MOT laser, we extract both an 800-MHz rf reference and an optical reference at 689 nm, reducing frequency instabilities below 4.4×10^{-13} after 400 s of averaging. Finally, we demonstrate that this technique can outperform modulation transfer spectroscopy with hot atomic vapors in terms of averaging times of around 100 s on the same transition.

Theory. To model the vertical dynamics of the five-beam MOT, where in comparison to a standard MOT the sixth beam propagating in the direction of gravity is removed [18], we consider the balance between gravitational force mg and optical force along the vertical axis for a single atom with mass m [19]. The resulting steady-state condition determines the MOT position as a function of laser detuning and magnetic field gradient [20,21]. For strontium, using the $^1S_0 \rightarrow ^3P_1$ intercombination line at

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689 nm with a narrow linewidth of $\gamma_0 = 2\pi \cdot 7.5$ kHz, power broadening alone is insufficient to achieve high capture velocities. Instead, frequency modulation is used to extend the effective capture range by more than an order of magnitude. While this “broadband (BB) MOT” technique increases the capture velocities, it cannot be modeled simply as a power-broadened single-frequency (SF) light force. In our implementation, the rf drive of the acousto-optic modulator (AOM) is triangularly modulated between δ_{start} and $\delta_{\text{stop}} = \delta_{\text{start}} - 2\pi f_{\text{bandwidth}}$, where both δ values denote detunings from resonance, and $f_{\text{bandwidth}}$ is the total modulation range [18,22]. In the low-saturation regime (see Supplemental Material SIV [23]), the total force in the vertical direction is

$$F_{\text{total}} = mg - \frac{\hbar k \gamma_0 I_{\text{total}}}{2I_0} \sum_{i=0}^n \frac{\alpha_i}{1 + (2\delta_i/\gamma_0)^2}, \quad (1)$$

where

$$\delta_i = \delta_{\text{start}} - i2\pi f_{\text{mod}} \mp kv - \frac{\mu_B g_J B(z)}{\hbar}, \quad (2)$$

where $n = \text{round}(f_{\text{bandwidth}}/f_{\text{mod}})$ is the total number of frequency components considered, f_{mod} is the modulation frequency, $\sum_{i=0}^n \alpha_i = 1$ are the relative intensities of the i th frequency component, $I_{\text{total}} = 2P/(\pi\omega_0^2)$ is the total intensity of the MOT beam (defined by the beam power P and Gaussian waist size ω_0) and I_0 the saturation intensity. For $m_J = \pm 1$ of this transition, the Zeeman shift is $\mu_B g_J/\hbar = 2101$ kHz/G [22], where μ_B is the Bohr magneton and g_J is the Landé g -factor. In equilibrium, $F_{\text{total}} = 0$, and the MOT position is directly related to the Zeeman shift. Since we operate close to the center of a quadrupole magnetic field with a linear gradient B' , a change $\delta_{\text{start}} \rightarrow \delta_{\text{start}} + 2\pi \Delta f$ results in a corresponding vertical displacement Δz , where the frequency-to-position conversion factor $\Delta f/\Delta z = \mu_B g_J B'/\hbar$ is independent of natural linewidth γ_0 and power broadening.

Experimental setup. We employ our previously developed architecture for generating a narrow-line ^{88}Sr MOT using a dispersion-optimized frequency comb-locked laser system, without the need for a free space high-finesse optical cavity [18]. In contrast to typical experiments that execute cooling stages time-sequentially, we execute them in a spatial sequence. Atoms effuse from an oven, are Zeeman slowed and then cooled in a blue two-dimensional MOT, out of which they continuously fall downwards, forming a stream of millikelvin-temperature atoms. These atoms are subsequently captured by a second-stage cooling system in a separate chamber: a five-beam, red BB MOT operating on the narrow $^1S_0 \rightarrow ^3P_1$ transition [see Fig. 1(a)]. The red MOT quadrupole field is generated by a coil pair of approximately 20 cm diameter, arranged in an anti-Helmholtz configuration with vertical axis. The

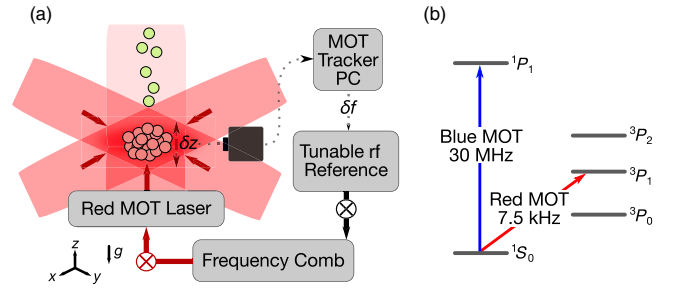


FIG. 1. Experimental schematic. (a) Atoms (green disks) are continuously loaded from an upper two-dimensional blue MOT [(b), operating on the $^1S_0 \rightarrow ^3P_1$ transition at 461 nm] into a five-beam broadband ^{88}Sr red MOT [(b), operating on the narrow $^1S_0 \rightarrow ^3P_1$ transition at 689 nm]. The red MOT is monitored nondestructively via fluorescence imaging, and the center position along the vertical (gravity) axis is extracted every 50 ms. The red MOT laser is locked to an optical frequency comb, which provides short-term frequency stability, while long-term stability is achieved via the MOT position where the vertical displacement δz serves as an error signal. Feedback is applied through a tunable rf oscillator used to synchronize the comb, which in turn transfers its stability to both the rf and optical domains.

MOT laser beams are frequency modulated by a single-pass AOM, which is driven by a modulated rf signal. This signal is generated by a direct digital synthesizer (Analog Devices AD9852) with a triangular frequency modulation with a total modulation range of $f_{\text{bandwidth}} = 1.5$ MHz and modulation frequency $f_{\text{mod}} = 40$ kHz (see Fig. 2, inset). Further details on the apparatus and methodology can be found in Ref. [18].

To characterize the MOT position dependence on the red MOT laser frequency, as well as shifts caused by other perturbations, we use absorption imaging, which is inherently destructive. For these initial calibration measurements, the red MOT laser source (a Toptica DL pro followed by an injection-locked diode laser) is frequency-stabilized in a conventional way: at short timescales (less than 1 s) to an optical cavity and at longer timescales to a modulation transfer spectroscopy signal from a hot Sr vapor [24,25]. For comb and conventional systems, the MOT laser linewidth is below the natural transition linewidth γ_0 .

Although our apparatus is designed for continuous operation, we can also generate a pulsed single-frequency (SF) MOT, increasing phase space density and reducing the cloud temperature well below a microkelvin [18]. In this procedure, we first load the BB MOT with several hundred million atoms, then transfer them to an SF MOT by switching to a single frequency tone while keeping the magnetic field unchanged. After a 100-ms equilibration period that allows the MOT to stabilize, we take an absorption image. To compare the frequency sensitivity (i.e., the frequency-to-position conversion factor) of the SF MOT to that of the BB MOT, the sum in Eq. (1) reduces to the $i = 0$ term for the SF MOT.

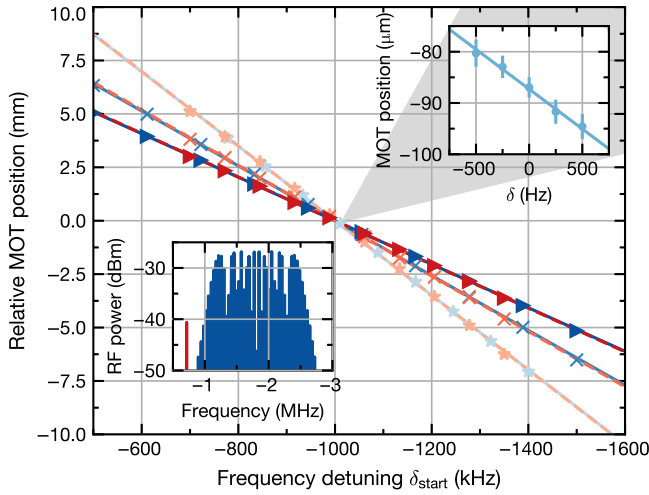


FIG. 2. Frequency sensitivity of the broadband (BB) and single-frequency (SF) MOTs, demonstrating MOT laser linewidth independence. Despite differing forces, both MOTs exhibit similar sensitivities within error bars. Triangles: vertical magnetic field gradient 0.47 G/cm, sensitivities 98.19(30) Hz/ μ m (BB, blue) and 98.76(50) Hz/ μ m (SF, red). Crosses: 0.37 G/cm, with 77.91(30) Hz/ μ m (light blue, BB) and 77.21(40) Hz/ μ m (light red, SF). Stars: 0.27 G/cm, with 57.37(24) Hz/ μ m (light orange, BB) and 57.25(40) Hz/ μ m (turquoise, SF). An offset magnetic field shifts the intersection point of the six curves to $\delta_{\text{start}} = 1.05$ kHz. (Bottom inset) MOT AOM rf spectra showing BB modulation (blue) and SF (red) tone. Top inset: high-resolution BB MOT position scan near the intersection point, with frequency resolution below 250 Hz at 0.31 G/cm.

When using the BB MOT position as a frequency reference, we lock the MOT laser source to the frequency comb (see Supplemental Material SI [23]). The comb repetition rate is synchronized to a tunable, low-noise 800-MHz oscillator (TOPTICA RF TUNE), which serves as a short-term reference, while long-term frequency stability is transferred via the MOT position to the tunable rf oscillator (see Supplemental Material SII [23]). To enable continuous feedback, we switch to nondestructive fluorescence imaging. The MOT position is extracted every 50 ms via live fitting of the fluorescence image. Using the known frequency-to-position conversion factor $\Delta f/\Delta z$, a Python-based proportional-integral loop continuously feeds back to the tunable rf oscillator. This stabilization scheme, utilizing the comb, effectively transfers frequency stability to both the rf and optical domains simultaneously. We monitor both using a K + K counter (FXE Phase + Frequency Meter); the rf is mixed down and measured directly, while the optical stability is measured via a beat note between the DL pro (locked to the MOT position) and our conventional laser system. The K + K counter is synchronized to a stable 10-MHz rf reference from the Dutch Metrology Institute (VSL), which has a fractional instability of approximately 10^{-14} after 100 s of averaging [7,18,26,27].

Sensitivity and linewidth independence. We first characterize the sensitivity of both MOT types' (BB and SF) position to changes in the laser frequency using the conventional laser system. Specifically, we measure the relationship between the MOT laser detuning δ_{start} and the MOT position along the vertical direction, shown in Fig. 2. We measure over the wide frequency range for which we can form a stable MOT, from approximately 600 kHz to 1.5 MHz red-detuned from the transition, at three different vertical magnetic field gradients ($B' < 1$ G/cm). For all magnetic field gradients, we observe a linear relationship between the MOT position and the frequency detuning δ_{start} for the BB MOT and the SF MOT. A linear fit to the data using the Zeeman shift allows us to extract the magnetic field gradient experienced by the atoms. For the smallest field gradient (0.27 G/cm), we measure a sensitivity, or frequency-to-position conversion factor $\Delta f/\Delta z$, for the BB MOT (orange stars) of 57.37(24) Hz/ μ m and for the SF MOT (light blue stars) of 57.25(40) Hz/ μ m. We are able to resolve frequency shifts as small as 250 Hz by restricting the scan range around the intersection point of the three sensitivity curves, where magnetic field gradient fluctuations have the least effect (see the inset at the top of Fig. 2). This sensitivity is more than 30 times smaller than the natural linewidth of the transition, indicating the potential to outperform conventional modulation transfer spectroscopy, specifically considering narrow intercombination lines as they can be found in alkaline-earth(-like) atoms. Notably, the usable locking range of our system corresponds to the frequency range over which a stable BB MOT can be maintained, spanning approximately 1 MHz. This is two orders of magnitude wider than the typical locking range achievable with conventional transfer modulation spectroscopy on the same transition, which is usually limited to a few γ_0 [16].

As determined in Section II, the frequency-to-position conversion factor is insensitive to the MOT laser linewidth, as long as a five-beam MOT can be created. The BB MOT laser has a linewidth that is broadened by more than two orders of magnitude, due to modulation and power broadening, whereas in the SF MOT, operated in the low-intensity regime ($I \approx I_{\text{sat}}$), the effective linewidth is expected to remain close to the natural linewidth (see the inset at the bottom of Fig. 2). We observe that the frequency-to-position conversion factor agrees within experimental uncertainties across all three magnetic field gradients for both MOT types (see Fig. 2). This supports the conclusion that MOT position spectroscopy reduces the requirements for the MOT/probing laser linewidth as long as it is constant over time.

Systematic errors. Before we can use MOT position spectroscopy as a long-term frequency reference, we must evaluate potential systematic errors that could limit its stability. We begin with an error budget analysis at a magnetic field gradient of 0.39 G/cm with

TABLE I. Summary of dominant noise sources limiting long-term MOT position stability at a magnetic field gradient of 0.39 G/cm around $\delta_{\text{start}} = 1.05$ kHz. The listed stability values represent the required control levels to maintain a position uncertainty of $\pm 1 \mu\text{m}$ averaged over 100 s, corresponding to a fractional frequency instability of $\sigma(100 \text{ s}) = 1.9 \times 10^{-13}$. Details of the individual noise characterizations are provided in Supplemental Material SV [23].

Noise source	Stability
Power (vertical direction)	$\pm 0.5\%$
Power (horizontal direction)	$\pm 0.1\%$
Atom number	$\pm 0.9\%$
Vertical magnetic field	$\pm 33.4 \mu\text{G}$
Horizontal magnetic field	$\pm 340 \mu\text{G}$
Vertical magnetic quadrupole field	$\pm 3.9 \text{ mG/cm}$

an observed frequency-to-position conversion factor of $82.5(10) \text{ Hz}/\mu\text{m}$, a good balance between frequency sensitivity and magnetic field gradient strength. The parameters that contribute most significantly to uncertainty in the MOT position are: fluctuations in the optical powers of the vertical and horizontal MOT beams; atom number fluctuations; drifts in the background magnetic field; and instabilities in the quadrupole magnetic field gradient. Magnetic field variations are the dominant contribution to the error budget, as they directly affect the local Zeeman shift and thus lead to displacements of the MOT's equilibrium position. The effects of atom number fluctuations and beam intensity variations are more subtle. A detailed characterization of the sensitivity of the MOT position to each of these parameters is provided in Supplemental Material SV [23]. The required stability levels for each of these parameters to limit the MOT position uncertainty to $\pm 1 \mu\text{m}$ after 100 s of averaging are summarized in Table I. This level of spatial control corresponds to a fractional frequency instability below 1.9×10^{-13} , which should be well within reach using a combination of active and passive stabilization [28,29]. Note that in the following implementation, we actively compensate only for power fluctuations; for all other effects, we rely on the passive stability of the system.

Response time. To evaluate the achievable feedback bandwidth for continuous measurement of the MOT position, it is essential to characterize the system's response time, that is, the time required for the MOT to reach a new equilibrium after a change in MOT laser frequency. Figure 3 shows the BB MOT position as a function of time following frequency jumps of 10 kHz (red disks), 50 kHz (light red triangles), 150 kHz (orange crosses), and 300 kHz (light orange stars), using the same parameters we later employ in the long-term stability measurement (averaging times of about 100 s). The MOT's motion can be described by an overdamped harmonic oscillator model, with the position evolving as $z(t) = z_{\text{start}} +$

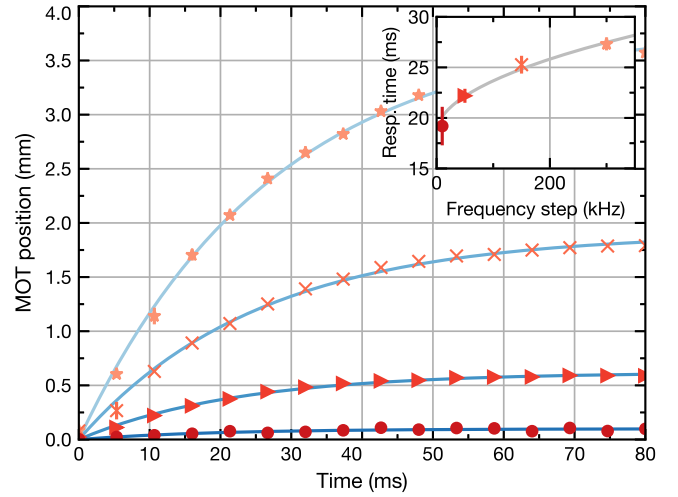


FIG. 3. Measurement of the MOT response time following a step change in the red MOT laser frequency. The MOT position relaxes to a new equilibrium according to an overdamped harmonic oscillator model. Red disks, 10 kHz jump; light red triangles, 50 kHz; orange crosses, 150 kHz; light orange stars, 300 kHz. (Inset) Extracted response time τ as a function of frequency jump, fit with a square-root dependence resulting in an amplitude $A = 0.529(40) \text{ s/kHz}^{1/2}$ and an offset $\tau_{\text{offset}} = 18.49(57) \text{ ms}$. This relationship sets an upper bound on the MOT response time for a given frequency change.

$(1 - e^{-t/\tau})(z_{\text{end}} - z_{\text{start}})$, where z_{start} and z_{end} denote the initial and final MOT positions, and τ is the characteristic response time. As shown in the inset of Fig. 3, τ increases with the magnitude of the frequency jump. We model this dependence empirically as $\tau(f) = A\sqrt{f} + \tau_{\text{offset}}$. For a frequency change of less than 100 kHz, the system reaches equilibrium in about 25 ms, corresponding to a maximum feedback bandwidth of roughly 40 Hz.

Continuous mot position spectroscopy. Finally, we implement continuous MOT position spectroscopy using fluorescence imaging of the BB MOT (10 ms exposure time, 20 Hz repetition rate). The red MOT laser is stabilized to a frequency comb, and the feedback loop is closed by transferring the MOT position stability to the tunable rf oscillator, which provides the frequency reference for the comb. We compare the overlapping Allan deviation (with 100 ms gate time) for a total measurement time of $T = 4600 \text{ s}$. Error bars indicate 1σ confidence intervals, calculated using a chi-squared distribution with conservative $T/\tau - 1$ degrees of freedom where τ is the cluster time [30]. Comparing the free-running rf oscillator to the MOT position stabilized case, we observe an improvement of almost two orders of magnitude after 400 s of averaging (see Fig. 4). The system reaches a minimum fractional frequency instability of 4.4×10^{-13} at 400 s, mainly limited by magnetic field fluctuations. Notably, we see that the MOT position stabilized tunable rf oscillator can outperform the beat signal between the

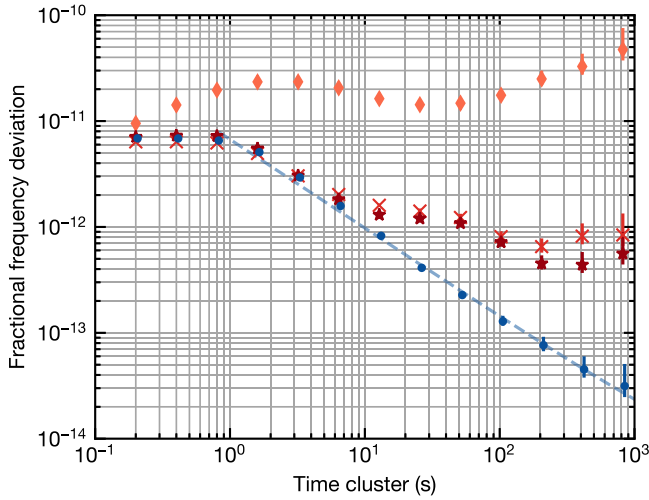


FIG. 4. Overlapping Allan deviation of the free-running tunable rf oscillator (orange diamonds) and the same oscillator stabilized to the MOT position (red stars). Beat signal between the MOT-referenced and hot-vapor-locked lasers (red crosses). Between 100 and 1000 s, the MOT position locked rf oscillator can outperform the optical reference based on the hot vapor modulation transfer spectroscopy, reaching a minimum instability of 4.4×10^{-13} at around 400 s, limited mainly by magnetic field fluctuations. All rf signals are compared to a stable 10 MHz rf reference from the VSL. The in-loop error signal (blue points) is fit by $\sigma(\tau) = (6.71 \pm 0.48) \times 10^{-12} / \tau^{0.84 \pm 0.03}$ (blue dashed line). Error bars indicate 1σ confidence intervals using a chi-squared distribution.

MOT laser (locked via the comb to the MOT position) and the conventional laser system in the range of 100 to 1000 s. This points to a limitation of the conventional laser system due to the hot vapor spectroscopy. These results demonstrate that MOT position spectroscopy can exceed the frequency stability of hot vapor modulation transfer spectroscopy for narrow intercombination lines in alkaline-earth(-like) atoms for averaging times of around 100 s with the potential for even longer time scales, even when relying solely on passive stability for magnetic field and atom number. With the addition of active stabilization, the system could approach the performance limit set by the in-loop error signal, calculated by converting MOT positions (recorded with feedback on) into frequency space via the frequency-to-position conversion factor. Therefore, the Allan deviation is expected to follow $\sigma(\tau) = (6.71 \pm 0.48) \times 10^{-12} / \tau^{0.84 \pm 0.03}$, potentially reaching the high 10^{-14} range after just over 100 s of averaging (see Supplemental Material SV [23]), where we also expect the noise floor to be limited by magnetic field fluctuations.

Summary and outlook. We have demonstrated a continuous spectroscopy based on the position of a narrow-line strontium MOT, achieving a frequency sensitivity 30 times narrower than the natural linewidth of the transition. This method provides frequency stability surpassing that of hot vapor modulation transfer spectroscopy for averaging

times of around 100 s with the potential for even longer time scales, while maintaining a locking range and effective laser linewidth two orders of magnitude broader than the transition's natural linewidth. If we actively stabilize the MOT position by feeding back to the repetition rate of a dispersion-optimized frequency comb [18], we reach a minimum fractional instability of less than 7×10^{-13} after 400 s of averaging in both the rf and optical domains, primarily limited by magnetic field fluctuations. With magnetic shielding [28] or active stabilization of stray environmental fields [29], it is possible to reduce the uncertainty even closer to the limit set by the in-loop error signal. Further improvement of the short-term stability could be achieved by synchronizing the tunable rf oscillator to a lower noise quartz oscillator [31] or a four-quadrant photodiode for higher-bandwidth MOT position measurements. On the other hand, one could also leverage the magnetic field sensitivity of this spectroscopy method and use such a system as a magnetometer in the direction of gravity [32].

This technique can be applied to any element for which a low-temperature five-beam MOT can be achieved (see Supplemental Material SIII [23]), such as ytterbium [33], erbium, and dysprosium [34], or for which the final MOT temperature is sufficiently low that the contribution of the sixth beam propagating in the direction of gravity can be neglected [21,35]. The spectroscopic sensitivity of the MOT position is only dependent on the magnetic field gradient required for the MOT and the transition-specific Landé g -factor, meaning that any cycling transition of any species for which these parameters are suitable can be used to convert MOT position to laser frequency with high resolution.

The use of the frequency comb makes this method convenient, robust, and versatile, while also enabling transfer of the stability into the rf domain, making this method well suited for applications that require long-term stable rf references, such as Global Navigation Satellite System redundancy [36,37]. In the future, the scheme could be simplified even more by using an injection-lock-amplified comb tooth, which is tuned directly to the target transition. Nevertheless, MOT position spectroscopy does not require a frequency comb or a continuous MOT. The stability of a pulsed MOT could also be transferred to a tunable cavity, which is commonly used in cold atom experiments. Here, the MOT cycle time needs to be fast enough to keep the MOT inside the locking range. This simple, widely applicable spectroscopy method opens up a host of possibilities for advancing quantum technologies.

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Data availability. The data that support the findings of this paper are openly available [38]. Embargo periods may apply.

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