Coherent resonant transport of magnons through a ferromagnetic chain: A possible way to establish quantum communication between nanomagnets embedded in microwave cavities

Elmar G. Petrov* and Sergii M. Tunyk
Bogolyubov Institute for Theoretical Physics, National Academy of Sciences of Ukraine, Metrologichna Street 14-B, UA-03143 Kyiv, Ukraine

Victor V. Gorbach
Nanotechcenter LLC, Kyiv 03142, Ukraine

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In hybrid microwave structures based on magnonics (so-called magnon cavities), the energy of the electromagnetic field can be transduced into the energy of magnetostatic spin excitations of a nanomagnet (Kittel magnon). This paper proposes a mechanism for transfer of coherent long-range spin excitation between nanomagnets, showing how Kittel magnons can be used for communication between magnon cavities. Analytical expressions for the magnon transport rate are obtained, and key parameters controlling off-resonant and resonant magnon tunneling regimes are indicated. Physical conditions are determined under which peak values of the rate characterizing the resonant transmission of the Kittel magnon arise either at a fixed tunneling energy with an arbitrary number of units in the ferromagnetic chain or at a fixed number of chain units with a varying tunneling energy. The features of resonant tunneling of the Kittel magnon formed in a magnon cavity at frequencies of about 10 GHz are analyzed. It is also shown that due to the strong photon-magnon coupling in the magnon cavity, at a certain magnetic field, the tunneling flow of magnons can reach an additional peak value, which reflects the process of resonant photon-to-magnon conversion.

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I. INTRODUCTION

In quantum technologies, control of physical processes is carried out directly at the atomic-molecular level. In recent years, special attention has been paid to one of the specific areas in nanophysics: so-called cavity magnonics. This field studies resonance processes in structures that are microresonators with macrospin nanomagnets embedded in them [1–3]. Due to the large spins of such nanomagnets, the coupling between the quanta of the electromagnetic field (photons) of the cavity and the quanta of spin excitations (magnons) of the nanomagnet is so strong [4,5] that a magnon cavity can be used to create quantum transducers [6].

Magnonics, as a special section of spintronics [7,8], is capable of revealing its potential in nanotechnologies related to magnetophotonics, thermal nanoconversion, magnonic capacitors, magnonic transistors, etc.[9–11]. In spintronics, charged particles (electrons or holes) act as carriers of energy and information in structures containing magnetically ordered units. The transfer process is controlled by both electric and magnetic fields; however, due to the Joule heat generated by moving charges, using them to transmit information over long distances requires special cooling of the corresponding communication devices. At the same time, magnons, as uncharged carriers, are very good candidates for use in quantum communication. This is because the transport of magnons can also be controlled by magnetic and electric fields, but the movement of these quasiparticles occurs without the release of Joule heat. It has also been found that in magnetically ordered dielectrics, magnons can propagate over fairly large distances. For example, in yttrium garnet crystals, these distances are of the order of several centimeters [3,12]. Therefore, it can be assumed that nonmetallic magnetic wires may prove to be convenient structures for transmitting information between magnetically ordered centers using magnons [7,13].

In molecular electronics, it has been shown both experimentally [14–16] and theoretically [17–20] that the rate of off-resonant coherent electron transfer decreases

*Contact author: epetrov@bitp.kiev.ua

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exponentially with the increasing number of identical units in the chain connecting electron localization centers (which are donor and acceptor or source and drain). In molecular spintronics, a similar rate is additionally controlled by a magnetic field [21–23]. The exponential decrease in rate is interpreted as tunneling, where the chain or wire acts as a mediator facilitating the transfer of electrons. The role of the mediator is that the electronic states of the chain participate in the electron tunneling process as virtual states. A similar property of the chain as a mediator is also manifested in the transfer of magnons [24]. As for chain-mediated resonant electron tunneling, the corresponding coherent transport regime has only recently been achieved experimentally [25]. It is shown that in the resonant regime, the conductivity of the donor-chain-acceptor structure is close in magnitude to the conductivity quantum of an ideal conductor, whereas in off-resonant tunneling, the conductivity is several orders of magnitude lower. Thus, specially synthesized chains are capable of ensuring long-range electron transfer (in molecular electronics, over tens of nanometers). Theoretical estimates show [26] that coherent resonant electron transfer over long distances, carried out along a mediator chain, is possible if a number of conditions are met. The most important necessary conditions include the rigidity of the molecular chain, low static and dynamic disorder of identical units in the chain, and good alignment of the energy levels of the electron when it is located in the centers and in the chain itself. Clearly, similar conditions must be met during long-range transfer of magnons.

In this work, we propose a physical mechanism by which magnons generated in one magnon cavity can be resonantly transferred to another magnon cavity through a ferromagnetic chain. We show that long-range spin-excitation transfer can be achieved via coherent one-step tunneling of magnons between nanomagnets embedded in cavities. This opens a possible way to implement quantum communication using magnons as uncharged information carriers.

II. MODEL AND BASIC EXPRESSIONS

To the best of the authors' knowledge, there are currently no experimental data on the transfer of spin excitation (magnon) through chain bridges over long distances from one nanomagnet to another. Therefore, we will limit ourselves to a semiphenomenological approach to modeling magnon transport in the system where a bridge ferromagnetic dielectric chain F connects nanomagnets embedded in the corresponding magnon cavities A and B (AFB system; Fig. 1). The key parameters of the system are the energies of spin excitations on the structural units of the system, as well as the couplings between neighboring units.

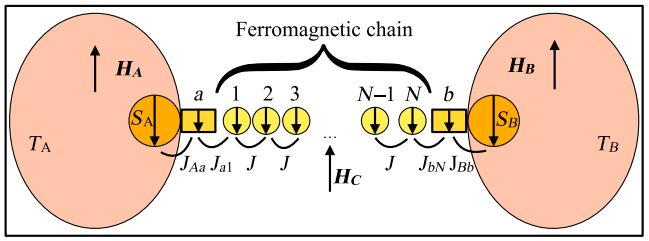


FIG. 1. Sketch of the AFB system, where macrospin nanomagnets A and B, inserted into the corresponding cavities, are connected by a ferromagnetic chain of N units through terminal units "a" and "b"; adjacent spins are coupled by the Heisenberg interaction.

The spin Hamiltonian of the entire AFB system can be represented as

$$\mathcal{H}_{\text{entire}} = \mathcal{H}_{\text{nmgnt}} + \mathcal{H}_{\text{bridge}} + \mathcal{H}_{\text{nmgnt-bridge}}, \tag{1}$$

where the first term,

$$\mathcal{H}_{\text{nmgnt}} = \mu_{\text{B}} \sum_{j=A,B} g_j \hat{\mathbf{S}}_j \mathbf{H}_j, \tag{2}$$

is the Hamiltonian of nanomagnets, while the second term,

$$\mathcal{H}_{\text{bridge}} = \sum_{l} \Big[\mu_{\text{B}} g_l \mathbf{H}_l \hat{\mathbf{S}}_l - D_l (\hat{S}_l^z)^2 - \frac{1}{2} \sum_{l'(\neq l)} J_{ll'} \hat{\mathbf{S}}_l \hat{\mathbf{S}}_{l'} \Big], \tag{3}$$

refers to the Hamiltonian of the bridge chain. In Eqs. (2) and (3), μ_{B} is the Bohr magneton and $\mathbf{H}_n$ is the static magnetic field acting on the nanomagnets ($n = j = A, B$) and paramagnetic units of the bridge ($n = l = a, 1, 2, \ldots, N, b$). The quantities g_n, $\hat{\mathbf{S}}_n$, and $D_l (> 0)$ are the g factors, spin operators, and easy-axis anisotropy parameters, respectively. The exchange interaction between spins located in bridge units l and l' is characterized by the value $J_{ll'} (> 0)$. Similar values also determine the Hamiltonian

$$\mathcal{H}_{\text{nmgnt-bridge}} = -\sum_{j=A,B} \sum_{l=a,b} J_{jl} \hat{\mathbf{S}}_j \hat{\mathbf{S}}_l \tag{4}$$

of the exchange interaction between the nanomagnets and the adjacent terminal units of the bridge chain. When choosing the form of the nanomagnet Hamiltonian (2), we took into account the fact that in a magnon cavity, electromagnetic field quanta can be transduced into magnetostatic excitation quanta (Kittel magnons) [2], the frequencies of which depend on the shape of the magnet sample and the magnetic field [27,28]. In the case under consideration, Eq. (2) is applicable to spherical or disc

nanomagnets. Equation (3) has a standard form suitable for estimating magnon frequencies in various types of ferromagnetic chains. (The structures of such chains are given, for example, in Refs. [29–33].)

It is assumed that magnon transport occurs at temperatures such that $\overline{n}_{\rm mgn} \ll 1$, where $\overline{n}_{\rm mgn}$ is the average number of magnons in the entire system. As for the magnon cavity, where the transduction of a photon into a magnetostatic magnon is possible, it has been shown experimentally [34] that the strongest coupling between the microwave photon mode of the cavity and the magnetostatic mode of the macrospin nanomagnet is achieved under purely quantum conditions, where no more than one photon and one magnon are involved in the transduction process. Taking this fact into account, in the entire Hamiltonian (1), we retain only those terms that refer to its ground state $|O\rangle = \prod_n |O_n\rangle$ and single-particle excited spin states $|f_n\rangle = |n\rangle \prod_{n' \neq n} |O_{n'}\rangle$, so that $\mathcal{H}_{\rm entire} \simeq \mathcal{E}_0 |0\rangle\langle 0| + \mathcal{H}_{\rm magn}$. To specify the energy $\mathcal{E}_0 = \langle O|\mathcal{H}_{\rm entire}|O\rangle$ of the ground state of the system, as well as the matrix elements of the Hamiltonian $\mathcal{H}_{\rm magn} \simeq \sum_{nn'} \langle f_{n'}|\mathcal{H}_{\rm entire}|f_n\rangle |f_{n'}\rangle\langle f_n|$ associated with the spin excitation in the same system, we note that $|0_n\rangle = |S_n, S_n^z = -S_n\rangle$ and $|n\rangle = |S_n, S_n^z = -(S_n - 1)\rangle$ are the eigenstates of the operators $\hat{\mathbf{S}}_n^2$ and $\hat{S}_n^z$ for the nth paramagnetic unit. Taking this fact into account and counting the energies of excited spin states from $\mathcal{E}_0$, we obtain

$$\mathcal{H}_{\rm magn} = \mathcal{H}_{\rm magn}^{\rm (static)} + \mathcal{H}_{\rm magn}^{\rm (bridge)} + \mathcal{H}', \tag{5}$$

where $\mathcal{H}_{\rm magn}^{\rm (static)} = \sum_{j=A,B} E_j |j\rangle\langle j|$ and $\mathcal{H}_{\rm magn}^{\rm (bridge)} = \sum_{l=a,b} E_l |l\rangle\langle l| + \sum_{\nu=1}^{N} E_\nu |\nu\rangle\langle\nu|$ are the Hamiltonians of Kittel magnons in nanomagnets and of magnons in the ferromagnetic bridge, respectively.

To specify the energies E_j and the states $|j\rangle$ of the spin excitation in the Hamiltonian $\mathcal{H}_{\rm magn}^{\rm (static)}$ of the jth magnon cavity, we note that due to the magnetic dipole interaction between the magnetostatic spin mode of the nanomagnet and the microwave mode of the resonator, the magnon frequency ω_m and the cavity photon frequency ω_c are transformed into hybrid frequencies [2,34]. The latter can be represented as $\omega_\pm = (\omega_c + \omega_m \pm \Omega)/2$. The gap $\Omega = |\Delta|\sqrt{1+\delta^2}$ between these frequencies is controlled by the parameter $\delta = 2g_{m\text{-}c}/|\Delta|$ through the frequency difference $\Delta = \omega_c - \omega_m$ and the photon-magnon coupling $g_{m\text{-}c}$. Physically, the parameter δ determines the "mixture" of the cavity photon state $|c\rangle$ and the Kittel magnon state $|m\rangle$ in their hybrid states $|+\rangle = \cos\chi|1\rangle + \sin\chi|2\rangle$ and $|-\rangle = -\sin\chi|1\rangle + \cos\chi|2\rangle$. (Note: $|1\rangle = |c\rangle$ and $|2\rangle = |m\rangle$ if $\Delta > 0$, while $|1\rangle = |m\rangle$ and $|2\rangle = |c\rangle$ if $\Delta < 0$.) The quantities $\cos\chi = \sqrt{(1 + 1/\sqrt{1+\delta^2})/2}$ and $\sin\chi = \sqrt{(1 - 1/\sqrt{1+\delta^2})/2}$ characterize the degree of mixing between the states $|c\rangle$ and $|m\rangle$. Maximum hybridization occurs at $\omega_c = \omega_m \equiv \omega^*$, i.e., under the resonance condition $\Delta = 0$. In this case, $\sin\chi = \cos\chi = 1/\sqrt{2}$ and $E_\pm = \hbar(\omega^* \pm g_{m\text{-}c})$, so that the magnon and photon states form hybrid states in equal proportions. Thus, in the general case, in a nanomagnet located in the jth magnon cavity, the energies of the excited states $\hbar\omega_{jc}$ and $\hbar\omega_{jm}$ are transformed into $E_j = \hbar\omega_{j\pm}$. For weak hybridization, which occurs outside the resonance region, we have $\omega_+ \approx \omega_c$ and $\omega_- \approx \omega_m$ if $\Delta > 0$, and $\omega_+ \approx \omega_m$ and $\omega_- \approx \omega_c$ if $\Delta < 0$. Now the frequency of spin excitation corresponds to the frequency of the Kittel magnon ω_m, and thus the energy of a magnetostatic spin excitation in the jth cavity is $E_j = \hbar\omega_{jm}$, with $\omega_{jm} \equiv \omega_j = \mu_{\rm B} g_j H_j$.

Below, to analyze the theoretical results, we will use experimental data concerning the quantum coupling between the magnetostatic Kittel mode and the microwave mode of the cavity [1,2,34]. For example, according to the data presented in the review in Ref. [2], for one of the samples, the coupling strength $g_{m\text{-}c}/2\pi \simeq 22.9$ MHz significantly exceeds the line widths $\kappa_c/2\pi \simeq 2.1$ MHz and $\gamma_m/2\pi \simeq 1.4$ MHz of the microwave resonator mode and the Kittel mode, respectively. Therefore, resonance is well fixed at the probe frequency $\omega^*/2\pi = 8.45$ GHz. As for the hybrid frequencies $\omega_+/2\pi$ and $\omega_-/2\pi$, outside the resonance window (8.50–8.40) GHz, these frequencies become close to the corresponding original frequencies $\omega_c/2\pi$ and $\omega_m/2\pi$. Theoretically, the width of the resonance window $\Delta_{\rm res}$ can be estimated from the condition $\delta^2 \ll 1$, at which $(\omega_+ - \omega_-)/2\pi \approx \Delta_{\rm res}/2\pi = (\mu_{\rm B} g/2\pi\hbar)|\Delta H|$, where ΔH is the change in the magnetic field inside the window. Having chosen the accuracy at which the relation $\delta^2/2 = 10^{-2}$ is valid, we obtain $\Delta_{\rm res} = 10\sqrt{2} g_{m\text{-}c}$. Thus, $\Delta_{\rm res}/2\pi \simeq 140$ MHz, which is in good agreement with the experimental value for the resonant window (8.50–8.40) GHz. An important consequence is that, since a window of about 0.1 GHz width is formed by changing the magnetic field ΔH by a value of the order of 10^{-3} T, the features of long-range tunneling transport of the magnon on the hybridization of photonic and magnonic states can manifest itself on a scale $\Delta H \sim 10^{-3}$ T. From the expressions $P_c(\tau) = 1 - [\delta^2/(1+\delta^2)]\sin^2(\Omega\tau/2)$ and $P_m(\tau) = [\delta^2/(1+\delta^2)]\sin^2(\Omega\tau/2)$, it is also obvious that the probabilities $P_c(\tau)$ and $P_m(\tau)$ of populating the corresponding quantum states $|c\rangle$ and $|m\rangle$ oscillate with frequency $\Omega = \sqrt{(\omega_c - \omega_m)^2 + 4g_{m\text{-}c}^2}$ around their mean values $\overline{P}_c = (1/2)(2+\delta^2)/(1+\delta^2)$ and $\overline{P}_m = (1/2)\delta^2/(1+\delta^2)$. In the resonant window, oscillations in time τ occur on a time scale $\Delta\tau_{\rm osc} \sim (\Omega/2\pi)^{-1} \sim (10^{-7}$–$10^{-8})$ s. Far from resonance, where $\delta \ll 1$, the conversion of a photon into a magnon is insignificant [$P_c(\tau) \approx 1$, $P_m(\tau) \approx 0$], whereas under resonance conditions it occurs with maximum efficiency ($\overline{P}_c = \overline{P}_m = 1/2$). As will be shown, this circumstance manifests itself in the dependence of the magnon flow on the magnitude of the magnetic field.

In the Hamiltonian $\mathcal{H}_{\rm magn}^{\rm (bridge)}$, the energy of a magnon on the bridge is taken to be $E_l = \mu_{\rm B} g_l H_l + D_l(2S_l - 1)$ if the magnon is localized at the lth terminal unit, or $E_\nu = E_c + 2t\cos(\pi\nu/N+1)$ $(\nu = 1,2,\ldots,N)$ if the magnon in the νth state is delocalized throughout the chain of N identical units. The spin excitation in each chain unit is determined by the expression $E_c = \mu_{\rm B} g H_c + D(2S-1)$, which depends on the magnitude of the easy-axis anisotropy (parameter D) and the external magnetic field H_c (the symbol g denotes the g factor for the unit). The coupling parameter $t = JS$, which characterizes the efficiency of spin-excitation jumps along the chain, is determined by the magnitude of the spin S for each chain unit and the exchange interaction between adjacent units (parameter J). The Hamiltonian $\mathcal{H}' = \sum_l \left[\sum_j t_{jl}|j\rangle\langle l| + \sum_\nu t_{\nu l}|\nu\rangle\langle l| + \text{c.c.}\right]$ characterizes two types of spin-excitation transitions from the $|l\rangle$th state of the lth terminal unit (the symbol c.c. denotes complex conjugate terms). One of them is associated with the transition to the hybrid excited state $|\pm\rangle_j$, formed in the neighboring jth cavity. The corresponding coupling $t_{jl} = J_{jl}\sqrt{S_j S_l}\cos\chi_j$, where $\cos\chi_j$ is the value of $\cos\chi$ for the jth magnon cavity, characterizes the efficiency of the magnon jump into the jth nanomagnet. The coupling parameter $t_{\nu l} = t_{ml}(\delta_{l,a}\delta_{m,1} + \delta_{l,b}\delta_{m,N})u_{m\nu}$, where $t_{ml} = J_{ml}\sqrt{SS_l}$ and $u_{m\nu} = (2/N+1)^{1/2}\sin(\pi m\nu/N+1)$, characterizes the magnon jump to the adjacent end unit of the ferromagnetic chain.

During the coherent transfer of a magnon from nanomagnet j to nanomagnet j', the bridge structure F can be considered as a scattering center. Thus, in accordance with the quantum theory of scattering [35], the rate of the corresponding $(j \to j')$-magnon transport is proportional to the square of the modulus of the matrix element $T_{j'j} = \langle j', 0_j|\hat{T}_{tr}(E)|j, 0_{j'}\rangle$, where $\hat{T}_{\rm tr}(E) = \mathcal{H}' + \mathcal{H}'\hat{G}(E)\mathcal{H}'$ is the transition operator. The latter contains the Green's operator $\hat{G}(E) = [E - (\mathcal{H}_{\rm magn}^{\rm (bridge)} + \mathcal{H}')]^{-1}$ defined at the energy $E = E_j \simeq E_{j'}$ of the Kittel magnon leaving the nanomagnet j. Since the operator $\hat{M}(E) = E\hat{I} - \mathcal{H}_{\rm magn}^{\rm (bridge)} - \mathcal{H}'$ is defined by a trigonal matrix in the basis of localized states, the desired matrix element can be represented as $T_{j'j} = t_{j'l'}t_{l'm'}(-t)^{N-1}t_{ml}t_{lj}/\text{Det}\,\hat{M}(E)$. Here, $\text{Det}\,\hat{M}(E) = \prod_r(E - \mathcal{E}_r)$ is the determinant of the matrix $\hat{M}(E)$, written in the form where $\mathcal{E}_r$ is one of the $N+2$ proper energies of the magnon on the bridge. In this paper, we consider the coherent long-range transport of the Kittel magnon under conditions where the energy of this magnon and the magnon delocalized on the chain is significantly less than the energy $E_{a(b)}$ of the magnon localized on the $a(b)$th terminal unit of the chain (see the diagram in Fig. 2). For example, such a situation may arise when the terminal units have a large easy-axis anisotropy (in particular, this applies to molecular magnets

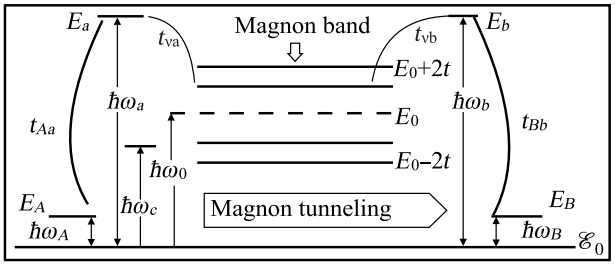


FIG. 2. Energies of the Kittel (E_A, E_B), localized (E_a, E_b), and delocalized (E_ν) magnons. The quantities t_{jl} and $t_{\nu l}$ are matrix elements characterizing the quantum jumps of the magnon, and $\omega_{A(B)}$, $\omega_{a(b)}$, and ω_0 are the frequencies of the magnon localized on the corresponding paramagnetic units of the AFB system; ω_c is the frequency of the microwave mode of the resonator.

[23,36]). Thus, the interaction associated with the Hamiltonian $\mathcal{H}'$ may have little effect on the energy of the localized magnon. As a result, the two proper energies can be identified with E_a and E_b, and the remaining N energies E_ν with those corresponding to the delocalized magnon in the chain. This gives $T_{j'j} = t_{j'm'}t_{mj}\sum_\nu \frac{u_{m'\nu}u_{\nu m}}{E-E_\nu}$. Due to the condition $E_l - E_j \gg t_{jl}, t_{lm}$, each of the quantities $t_{mj} = t_{ml}t_{lj}/(E - E_l)$ can be interpreted as an indirect spin-spin coupling between the nanomagnet $j\,(= A, B)$ and the corresponding nearest edge unit $m(= 1, N)$ of the chain. To clarify the dependence of the matrix element $T_{j'j}$ on N, we note that in the general case, various types of stochastic disturbances can affect the coherent transmission of a magnon along the chain. In the simplest version, these disturbances can be taken into account by adding the imaginary contribution to the energy [37]. Doing this by changing the energy E_0 to $E_0 - (i/2)\Gamma_0$, in the case of long-range transmission of the Kittel magnon with fixed energy $E = E_j \simeq E_{j'}$, we obtain $T_{j'j} = \left(t_{j'm'}t_{mj}/t\right)\Lambda(N)$, where $\Lambda(N) = \sinh\lambda/\sinh[(N+1)\lambda]$, $\lambda = \ln(z + \sqrt{z^2-1})$, and $z = (|E - E_0| + i\Gamma_0/2)/2t$.

Since the magnon transport rate $k_{j\to j'}$ is proportional to $|T_{j'j}(N)|^2$, it can be represented as

$$k_{j\to j'} = k^{(0)}_{j\to j'}R(N), \quad (N \geq 2), \tag{6}$$

where the N dependence is determined by the function $R = |\Lambda(N)|^2$, while $k^{(0)}_{j\to j'} = (2\pi/\hbar)|t_{BN}t_{1A}/t|^2\mathcal{F}_{j'j}$ is an N-independent quantity. The factor $\mathcal{F}_{j'j}$ characterizes transitions within the nanomagnets themselves. Due to the lack of a direct connection between the nanomagnet and the chain, a possible small dependence of $\mathcal{F}_{j'j}$ on N is ignored. A similar simplification is used in the study of long-range electron transport along molecular wires, where a quantity like $\mathcal{F}_{j'j}$ is related to the Franck-Condon factor.

Coherent transfer is possible if the width parameter Γ_c is significantly smaller than the energy difference $|E_\nu - E_{\nu\pm1}|$ for the nearest states of the delocalized

magnon. Therefore, if the energy $E \simeq E_A \simeq E_B$ does not exactly coincide with $E_0 \pm 2t$, so that $(\Gamma_0/2)^2/[(E - E_0)^2 + 4t^2] \ll 1$, then for the function $R(N)$, one can use the approximate analytical form

$$R = \frac{\sinh^2(\zeta/2) + \sin^2\eta}{\sinh^2[(N+1)\zeta/2] + \sin^2[(N+1)\eta]}. \quad (7)$$

This form is useful for understanding the physics of long-range transport of a magnetostatic magnon as a function of the length of the internal (regular) part of the ferromagnetic bridge. The transport is controlled by the quantities

$$\begin{aligned} \zeta/2 &= \ln(|\alpha| + \sqrt{\alpha^2 - 1}), \\ \eta &= (\xi/2)/\sqrt{\alpha^2 - 1}, \\ \alpha &= (E - E_0)/2t, \end{aligned} \quad (8)$$

both in the absence ($\xi = 0$) and in the presence ($\xi = \Gamma_0/2t \neq 0$) of stochastic influence, provided that $\eta \ll 1$.

The dependence $k_{j\to j'} = k_{j\to j'}(N)$ is well determined if the rates $k_{A\to B}$ and $k_{B\to A}$ characterize the slowest stage of magnon transfer inside the AFB structure. To demonstrate this, we considered the formation of a steady-state flow of magnons $\Pi_{A\to B}$ from nanomagnet A to nanomagnet B. Kinetics shows that in the presence of a bridge, it can be described by the expression $\Pi_{A\to B} = \left(k_A^{(c)} k_{A\to B} k_B^{(d)} - k_B^{(c)} k_{B\to A} k_A^{(d)}\right)/\mathcal{D}$, where $\mathcal{D} = k_A^{(d)}\left(k_B^{(d)}/2 + k_B^{(c)} + k_{B\to A}\right) + k_B^{(d)}\left(k_A^{(d)}/2 + k_A^{(c)} + k_{A\to B}\right) + (k_{A\to B} + k_{B\to A})\left(k_A^{(c)} + k_B^{(c)}\right)$. Here, $k_j^{(c)}$ and $k_j^{(d)}$ denote the corresponding rates of appearance and disappearance of the spin excitation in the j th nanomagnet in the absence of a connection between the nanomagnets. If the transfer of a magnon over a long distance is the slowest kinetic step, which is true when the conditions $k_{A\to B} \ll k_A^{(d)} + k_A^{(c)}$ and $k_{B\to A} \ll k_B^{(d)} + k_B^{(c)}$ are satisfied, then

$$\Pi_{A\to B} \simeq \frac{k_{A\to B} q_A (1 - q_B) - k_{B\to A} q_B (1 - q_A)}{1 - q_A q_B}. \quad (9)$$

Here, the quantity $q_j = k_j^{(c)}/\left(k_j^{(c)} + k_j^{(d)}\right)$ denotes the weight with which the magnetostatic spin excitation is formed on the j th nanomagnet in the absence of a bridge connection with another nanomagnet.

III. RESULTS AND DISCUSSION

The main objective of this work is to clarify the features of coherent one-step isoenergetic transfer of a magnetostatic excitation between nanomagnets connected by a quasidimensional ferromagnetic structure. Special attention is paid to the dependence of the rate of long-range magnon transfer on the distance between the nanomagnets, which, within the framework of superexchange coupling between the nanomagnets, is contained in the function R. Its expression has been obtained in a compact form $R_{\text{exact}} = |\sinh\lambda/\sinh[(N+1)\lambda]|^2$; however, since λ is a complex quantity, this form is inconvenient for analyzing the physics of long-range transport of the Kittel magnon. For this purpose, an analytical expression, Eq. (7), was used, obtained under the condition of weak stochastic influence on spin excitations in the bridge chain. From expressions (7) and (8), it follows that α is a key parameter controlling the dependence of the function $R = R(\alpha, N)$, and therefore of k_{tr}, on N. This is illustrated in Fig. 3. For $\alpha = 1.05$, the value of R decreases rapidly with increasing number of units N of the bridge chain. Indeed, according to the definition (8), we have $\zeta \approx 0.63$ and, consequently, the inequality $\exp[(N+1)\zeta] \gg 1$ is already satisfied for $N \geq 3$. As a result, we come to the dependence

$$R \approx 4\sinh^2(\zeta/2)\exp[-\zeta(N+1)], \quad (10)$$

which demonstrates an exponential decrease in the rate of magnon transfer k_{tr} with increasing chain length. This is why such a process can be considered as off-resonant tunneling, relating ζ to the attenuation coefficient. The situation changes significantly at $|\alpha| < 1$. The graph presented in the inset of Fig. 3 shows that in this case, the value of R becomes several orders of magnitude greater than the value of R realized for the same N during the off-resonant regime of magnon transfer. In addition, for each fixed α, peak values of the function $R = R(\alpha, N)$ are observed.

To understand the physical reason for the different behavior of the function $R(\alpha, N)$ depending on N, we note that according to the definition (8), the value ζ has a completely different meaning for $|\alpha| > 1$ and $|\alpha| < 1$. In the first case, the energy $E = E_A$ of a magnetostatic excitation is below (or above) the boundary of the magnon band $E_0 - 2t$ or $E_0 + 2t$, and thus ζ is a real quantity characterizing the attenuation of the transfer rate. At $|\alpha| < 1$, the energy $E = \hbar\omega_A$ of the Kittel magnon falls into the band region $E_0 - 2t < E < E_0 + 2t$ of the delocalized magnon. In this case, ζ becomes an imaginary quantity, $\zeta = i\phi$, where $\sin\phi = \sqrt{1 - \alpha^2}$. As a result, the function (7) can be rewritten as

$$R = \frac{\sin^2\phi + \sinh^2|\eta|}{\sin^2[(N+1)\phi] + \sinh^2[(N+1)|\eta|]}. \quad (11)$$

Thus, α is a key parameter controlling the process of coherent long-range transport of magnons. Physically, the parameter α characterizes (in units of the half-width of the magnon band $2t$) the difference between the energy $E = E_A$ of the Kittel magnon and the energy E_0 related to the center of the magnon band (Fig. 2). If $|\alpha| > 1$, then

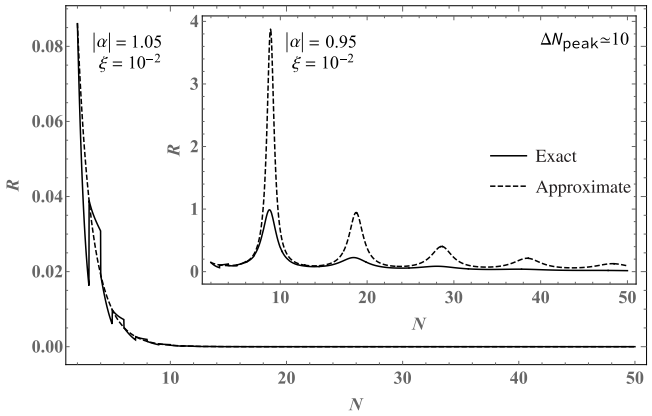


FIG. 3. An example of the dependence of the function R and, consequently, the rate k_{tr} of long-range transfer of magnetostatic excitation (Kittel magnon) between nanomagnets A and B in off-resonant ($\alpha > 1$) and resonant ($\alpha < 1$) transfer regimes. The analytical expressions for R, presented in the form (7) and (11), agree well with the results of calculations performed using the exact function R_{exact}, and are thus suitable for analyzing the physics of long-range magnon transport. Energy parameters of the bridge: $E_a = E_b = 4.3$ cm^{-1}, $E_c = 0.33$ cm^{-1}, $t = 0.03$ cm^{-1}.

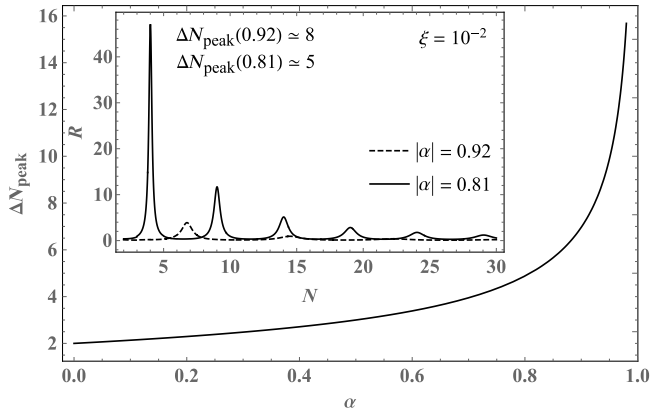


FIG. 4. Dependence of the distance ΔN_{peak} between neighboring peak values of the transfer rate k_{tr} on the key parameter α in the resonant regime of magnon transmission. The positions of the peaks at a given α value are shown in the inset. The energy parameters of the bridge are the same as in Fig. 3.

magnon transfer occurs in an off-resonant regime. This means that the energy of the Kittel magnon $E = E_A$ cannot coincide with any energy of the delocalized magnon E_v in the window $[E_0 - 2t, E_0 + 2t]$; however, if $|\alpha| < 1$, then the energy E can be close to the energy of the delocalized magnon in the window or even coincide with it.

The representation of the function $R = R(\alpha, N)$ in the form of Eq. (11) allows one, using the condition $\sin[(N+1)\phi] = 0$, to obtain the positions of the peak values of the rate k_{tr} at fixed α or N. If α is fixed, the distance between the peaks is determined by the expression

$$\Delta N_{\text{peak}}(\alpha) = \frac{\pi}{\arcsin\sqrt{1-\alpha^2}}, \tag{12}$$

in which $\arcsin\phi$ is given in radians. In the example presented in Fig. 3, we used $\alpha = 0.95$ and thus, $\Delta N_{\text{peak}}(0.95) \approx 10$. The dependence $\Delta N_{\text{peak}} = \Delta N_{\text{peak}}(\alpha)$, presented in Fig. 4, allows us to predict which value of α and, consequently, the frequency $\omega_A = E_A/\hbar$ of the magnetostatic excitation are suitable for achieving peak transmission of Kittel magnons through a bridge chain of N units. For a given stochastic parameter ξ, the height of the peaks decreases with increasing peak number proportionally to the factor $\sinh^{-2}[(N_{\text{peak}}+1)|\eta|] \simeq (N_{\text{peak}}+1)^{-2}|\eta|^{-2}$. Therefore, although the width of a particular peak changes, the position of the peak does not depend on the value of the stochastic parameter ξ. As for the peak height itself, it becomes 2 orders of magnitude larger if the value of the stochastic parameter ξ decreases by 1 order of magnitude. This conclusion is supported by the results presented in Fig. 5. For a chain of N fixed units, the positions of the peaks $\alpha = \alpha^{(r)}_{\text{peak}}(N)$ in the region $-1 < \alpha < 1$) are given by

$$\alpha^{(r)}_{\text{peak}}(N) = -\cos\left(\frac{\pi r}{N+1}\right), \quad (r = 1, 2, \ldots, N). \tag{13}$$

The dependence of the parameter α on the magnetic field H_A allows us to understand how this field can control the rate of coherent resonant transfer of the Kittel magnon. To do this, we consider the transfer between identical nanomagnets placed in identical magnon cavities (Fig. 1). Within the framework of the model under consideration, the key parameter α [Eq. (8)] depends linearly on the energy $E = E_A$ of the Kittel magnon and, therefore, is

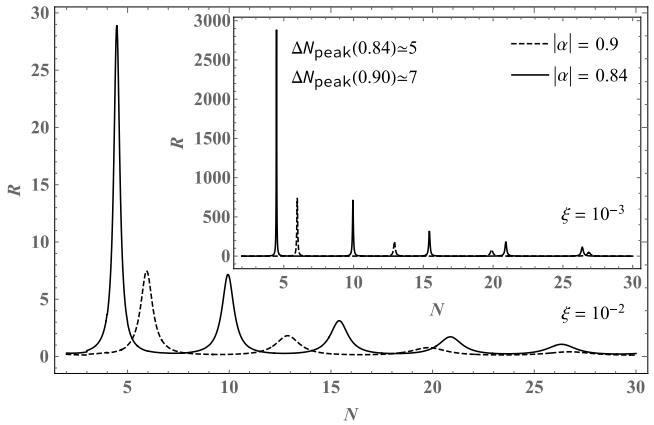


FIG. 5. Examples of the positions of the peak values of the magnon transfer rate k_{tr} and the relative intensities of the peaks in the resonant transmission regime for different values of the stochastic parameter ξ. The energy parameters of the bridge are the same as in Fig. 3.

proportional to the magnetic field H_A:

$$\alpha = \frac{1}{2t}(\mu_B g_A H_A - E_0). \tag{14}$$

Experimental data show that in copper microwave cavities, electromagnetic resonator modes can excite magnetostatic Kittel modes with a frequency of about 10 GHz [2]. For the purposes of the estimates, we will assume that the linear frequency $\omega_c/2\pi$ corresponds to the energy $E_c \approx 0.33$ cm^{-1} required to excite a magnon in the center of the band window $[E_0 - 2t, E_0 + 2t]$ and, therefore, is also equal to 10 GHz. Let us take 8 GHz as the lowest frequency of the delocalized magnon, corresponding to energy $E_0 - 2t \approx 0.27$ cm^{-1}. This gives $t \approx 0.03$ cm^{-1}. Note that the function R, which describes the N dependence of the magnon flow [Eq. (15)], has been derived under the condition that the energy $E_{a(b)}$ of a magnon localized at the terminal unit $a(b)$ strongly exceeds the energy of the magnetostatic excitation $E = E_A \simeq E_B$. This condition is fulfilled, for example, in a molecular magnet $[Mn_3]_2$ with a spin value of $S = 6$. In this magnet, the energy DS^2 required to deflect the spin from the easy axis to a perpendicular axis is about 1.5 meV [36]. Therefore, even at a weak magnetic field, the energy of the first spin excitation $\hbar\omega = D(2S + 1)$ is approximately 0.5 meV ≈ 4.3 cm^{-1}. Thus, the corresponding frequency of the localized magnon, 130 GHz, is an order of magnitude higher than the frequency of the Kittel magnon transmitted between the nanomagnets. These estimates are approximate. More precise estimates of the parameters of the molecular magnets involved in the transfer of magnons between nanomagnets can be obtained by comparing the analytical results with direct calculations for specific AFB structures. (As far as the authors know, there is currently no data on the creation of such a hybrid structure.)

Figure 6 shows how the magnetic field controls the rate of isoenergetic long-range transfer of magnons between identical nanomagnets. The effect of magnetic fields is visible in the appearance of peak values of the transfer rate. Relation (14) allows us to establish the number $\Delta N_{\text{peak}}(H_A)$ of chain units between the nearest peak values of the rate k_{tr} at a fixed magnetic field H_A [Fig. 6(a)], as well as to indicate the values of magnetic fields at which peaks are formed for a fixed number N of chain units [Fig. 6(b)]. Figures 3–6 show the main features of the behavior of the coherent transfer rate of a localized Kittel magnon through a ferromagnetic chain.

Analytical expressions (7) and (11) allow us to understand the physical reason for the exponential decrease in rate in the nonresonant regime of magnon transport, as well as the position and relative magnitude of the tunneling rate peaks depending on the chain length and the magnitude of the magnetic field in the resonant regime of magnon tunneling. It is shown that the key role in

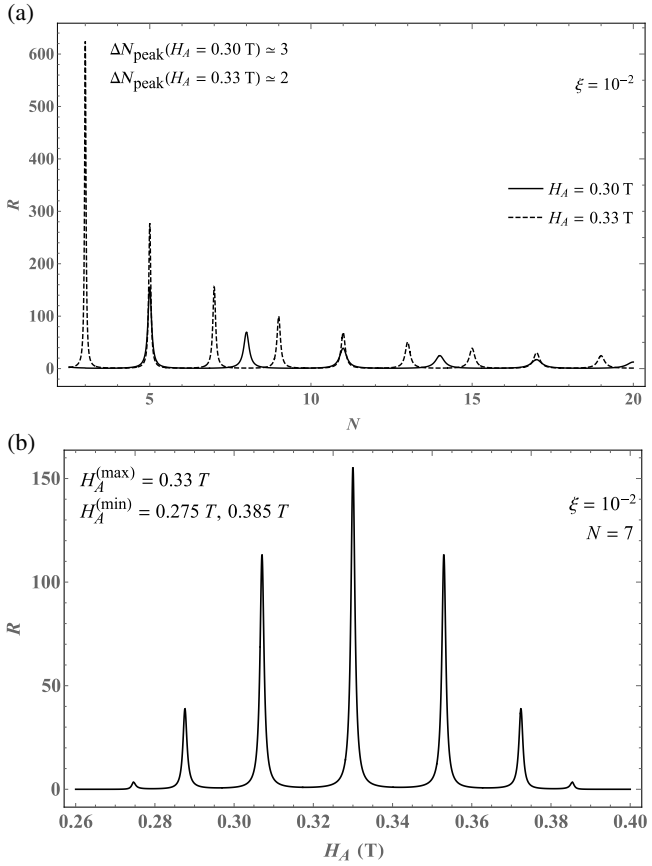


FIG. 6. Dependence of the Kittel resonant tunneling rate on the magnetic field $H_A(\simeq H_B)$ during coherent long-range isoenergetic transfer through a ferromagnetic bridge. (a) At a fixed magnetic field strength $H_A = 0.30(0.33)$ T, the distance $\Delta N_{\text{peak}}(H_A)$ between the resonant peaks is 3(2) chain units. (b) For a fixed N, the positions and magnitudes of the resonant peaks are controlled by the magnetic field: the number of peaks exactly coincides with N. The energy parameters of the bridge are the same as in Fig. 3.

regulating the tunneling regime is played by the position of the magnon tunneling energy E relative to the energy E_ν of the delocalized magnon.

As for the tunneling flow of magnons, represented in the form (9), here, an important role is played not only by the magnon tunneling rates but also by the factor $Q^{(\text{cont})}_{A\to B}$, which characterizes the efficiency of magnon formation in a nanomagnet through temperature activation or/and the transformation of a cavity photon into a nanomagnet magnon. In the case of quasi-isoenergetic transfer of magnon we have $k_{A\to B} = k_{B\to A} \equiv k_{\text{tr}} = k^{(0)}_{\text{tr}} R$, so the flow is given by the simple form

$$\Pi_{A\to B} = Q^{(\text{cont})}_{A\to B} k^{(0)}_{\text{tr}} R. \tag{15}$$

Here, $Q^{(\text{cont})}_{A\to B} = (q_A - q_B)/(1 - q_A q_B)$ is the factor that determines the direction of magnon transfer depending on the value of the weighting coefficients q_A and

q_B. For example, let identical nanomagnets be at such temperatures T_A and T_B that $q_j = \exp(-\hbar\omega_m/k_\mathrm{B}T) \ll 1$, (in which k_B is the Boltzmann constant). Outside the resonance window, where the mixture of Kittel and microwave modes is small, the spin excitations corresponds to the magnetostatic modes with frequencies $\omega_A = \omega_B \simeq \omega_m$. Therefore, at $T_A = T$, $T_B = T - \Delta T < T_A$, in a linear approximation in $\Delta T/T$ we obtain $Q^{(\mathrm{cont})}_{A\to B} \approx (\Delta T/T)\,(\hbar\omega_m/k_\mathrm{B}T)\exp(-\hbar\omega_m/k_\mathrm{B}T)$. This expression reflects the specific Seebeck effect, in which a ferromagnetic bridge ensures the transfer of a single temperature-activated magnon with energy $\hbar\omega_m$ between identical nanomagnets.

In the model under consideration, the formation of peaks in the tunneling rate k_tr does not depend on the mechanism of formation of the Kittel magnon in the nanomagnet. The number of peaks, the energy distance between peaks, and their intensity are completely determined by the position of the magnon energy $E = E_A$ relative to the energy E_ν of the delocalized magnon inside the chain. As for the magnon flow, its value depends significantly on the mechanism of magnon formation in the cavity, since it is this mechanism that determines the weight W with which the Kittel magnon is present in the nanomagnet. If the tunneling of magnons is the slowest stage of the transport process, then the characteristic time of this stage $\tau_\mathrm{tr} \sim k_\mathrm{tr}^{-1}$ significantly exceeds the characteristic time τ_osc of oscillations between the states $|c\rangle$ and $|m\rangle$ of the photon and magnon in the resonator, respectively. Therefore, to describe the magnon transport process on the time scale $\Delta\tau \gg \tau_\mathrm{osc}$, we can assume that in the hybrid "photon + magnon" state, the probabilities of detecting a photon and a magnon are determined by the average values $\overline{P}_c$ and $\overline{P}_m$, respectively. To find out how the probability of Kittel magnon formation determines the magnon flow, we consider the directed transport of a magnon from nanomagnet A to nanomagnet B, assuming that the interaction of microwave and magnetostatic modes leads to the creation of a spin excitation only in nanomagnet A. In addition, it is assumed that the thermal activation of a magnon in nanomagnet B is less efficient compared to that for nanomagnet A. This gives $q_B \simeq 0$, so the magnon flow [Eq. (15)] can be rewritten as $\Pi_{A\to B} = k^{(0)}_\mathrm{tr} WR$. Here, $W = \exp(-E_A/k_\mathrm{B}T)$ and $W = \overline{P}_A$ are the weights with which the magnetostatic (Kittel) magnon is formed in nanomagnet A due to thermal activation or interaction with a microwave photon, respectively.

Figure 7 shows the role of temperature activation of the Kittel magnon and photon-magnon interaction in the intensity of the magnon flow through the dependence of the factor WR on the magnetic field strength H_A. From a comparison of Fig. 7 with Fig. 6, it follows that the weighting factor W reduces the intensity of the peak values of the tunneling rate k_tr, characterized by the function R. Moreover, compared with the R function, a noticeable

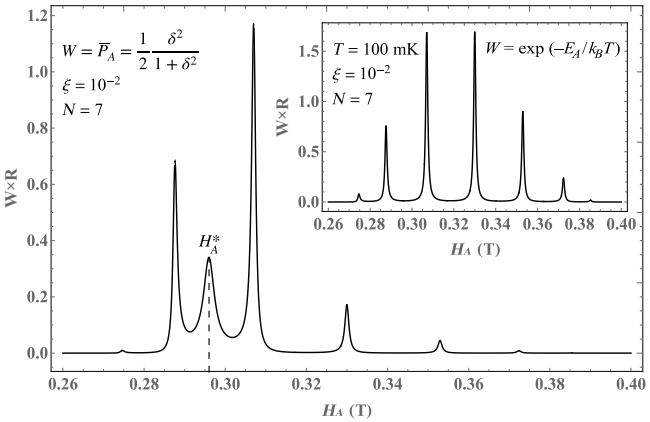


FIG. 7. The role of photon-magnon interaction and temperature in controlling the flow of magnons through a magnetically ordered bridge F from nanomagnet A to nanomagnet B. The appearance of an additional peak at $H^* = 0.296$ T is due to the hybridization of the resonator's microwave mode with the Kittel mode of the nanomagnet. The energy parameters of the bridge are the same as in Fig. 3. Cavity parameters: $\omega_c = 8.45$ GHz, $g_{m\text{-}c} = 0.023$ GHz.

asymmetry appears in the dependence of the WR factor on the magnetic field. This follows directly from the fact that the symmetric positions of the rate peaks relative to the magnetic field $H^{(0)}_A = E_0/\mu_\mathrm{B}g_A$ are determined by the condition $E_A = E_\nu$ for $\nu = 1, 2, \ldots, N$; however, upon thermal activation of the Kittel magnon, the probability of the appearance of the νth rate peak is proportional to $\exp\{-[E_0 - 2t\cos(\pi\nu/(N+1))]/k_\mathrm{B}T\}$, and thus the intensity of the peaks decreases asymmetrically with increasing peak number ν.

For example, comparing the position of the peaks shown in Fig. 6(b) and the inset in Fig. 7, it should be noted that at a temperature of 100 mK, the peaks at $H_A \simeq 0.31$ T and $H_A \simeq 0.35$ T become unequal in intensity relative to the central peak at $H^{(0)}_A = 0.33$ T. If the excitation of the magnetostatic mode is caused by photon-magnon coupling, then the realization of the Kittel magnon is proportional to the average population $\overline{P}_A$ of the magnon state. Since $\overline{P}_A$ is determined by the parameter $\delta = 2g_{m\text{-}c}/|\Delta_A|$, where $\Delta_A = \omega_c - \mu_\mathrm{B}g_A H_A/\hbar$, the difference $|\Delta_A|$ between the photon and magnon frequencies is the quantity that controls the intensity of the flow peaks. Under resonance conditions, when $\overline{P}_A = 1/2$, the probability $W = \overline{P}_A$ of realizing the Kittel magnon state in a hybrid photon-magnon state reaches its greatest value. This is more than an order of magnitude larger than the value of $\overline{P}_A$ outside the resonance window of width $\Delta H \sim 0.01$ T. Therefore, the flow of magnons can also increase by orders of magnitude.

In Fig. 7, the resonant increase in the magnon flow is manifested as the appearance of an additional peak at a magnetic field $H^* \simeq 0.269$ T, at which, on the $\Delta H \sim 0.01$ T scale, the Kittel magnon frequency coincides with the frequency $\omega_c = 8.45$ GHz of the microwave mode of the

resonator. It is important to note that such a resonance peak is absent during thermal activation of the Kittel magnon.

IV. CONCLUSION

The present study showed that magnons can carry out long-range transfer of information and energy through magnetically ordered bridge structures from one nanomagnet to another. A mechanism of coherent transfer is proposed based on the fact that the spin states of the bridge with magnetically ordered paramagnetic units play a virtual role, providing a specific quantum exchange of magnons both inside the bridge and between the terminal units of the bridge and neighboring nanomagnets. As a result, a long-range superexchange coupling arises between magnetostatic spin excitations, which ensures a tunnel regime of magnon transport between nanomagnets. This has a certain analogy with the mechanism of formation of a long-range superexchange coupling responsible for the tunneling of an electron through a molecular wire connecting metallic electrodes. The analogy is based on the fact that, in both cases, a tunnel transition of a particle (electron) or quasiparticle (magnon) from a single-particle state to a single-particle state occurs. Thus, in the case of electron tunneling between the metallic electrodes, the tunneling electron leaves one of the single-particle states of a conductive band of one of the electrodes and arrives in one of the empty single-electron states of another electrode. The magnon tunneling process we are studying occurs under the given temperature conditions and microwave field density of the resonator, where the lowest excited spin state of the nanomagnet is limited to a single Kittel magnon. In this case, a single magnon leaves the spin-excited nanomagnet and appears on another nanomagnet, which was previously in the ground spin state. In such a single-particle transport process, long-range superexchange coupling for both a fermion (electron) and a boson (magnon) can be realized according to the same scheme by involving single-particle virtual states in the formation of the coupling.

The fundamental difference between fermion and boson tunneling is that a fermion (in particular an electron) tunnels from a state occupied by only one fermion. In contrast, for bosons, there is no restriction on occupying the same single-particle state. Therefore, in most cases, a boson tunnels under conditions of interaction with other bosons in the same single-particle state. This circumstance manifests itself, for example, in the tunneling of a small number of bosons [38,39], as well as in tunneling under Bose-Einstein condensation conditions [40]. Thus, although fermions can tunnel individually, boson tunneling, controlled by interactions between them, is a much more complex cooperative phenomenon.

We investigated the transport of a single Kittel magnon between ferromagnetic nanomagnets with very large ("macroscopic") spins. The specificity of such a transfer is that the spin states of a macrospin nanomagnet are determined only by the projections S_z of its macroscopic spin S. Therefore, in a magnetic field directed along the z axis, the spin projection of a nanomagnet in its ground state is $S_z^{(\mathrm{ground})} = -S$, while in the single lowest excited spin state, it is $S_z^{(\mathrm{excited})} = -S + 1$. As a result, the appearance of a magnetostatic spin excitation (Kittel magnon) is associated with a single transition $(S_z = -S) \rightarrow (S'_z = -S + 1)$.

In contrast, in a standard ferromagnet, although the lowest excited spin state of its nth paramagnetic unit is formed by the analogous transition $(S_n^z = -S_n) \rightarrow (S_n^{'z} = -S_n + 1)$, the actual lowest spin state of the ferromagnet depends on the number of excited paramagnetic units. Due to the exchange interaction between the spins of paramagnetic units, each lowest localized excited spin state (the so-called localized magnon) is transformed into a delocalized magnon, the number of which coincides with the number of localized magnons. The quantum state of the delocalized magnon is characterized by the wave vector k. Since the lowest excited spin states can be simultaneously present in different paramagnetic units, a large number of delocalized magnons can arise in a ferromagnet even at low temperature. If the energy of a delocalized magnon with $k = 0$ is lower than the energy of a localized spin excitation, then magnons, being bosons, can occupy the state with $k = 0$, thereby opening the way for the formation of a Bose-Einstein condensation. Thus, the fundamental difference between a standard ferromagnet and a macrospin nanomagnet is that in a macrospin nanomagnet, the lowest excited spin state is identical to one localized magnon, whereas in a standard ferromagnet, the lowest excited state is associated with a number of delocalized magnons in the collective state with $k = 0$.

Our calculations have shown that even at a very low temperature of the order of 100 mK, there is a sufficient probability W of the formation of a single Kittel magnon with an energy corresponding to the energy of a delocalized magnon in a bridge ferromagnetic chain. Thus, the Kittel magnon can be formed either by converting a photon into a magnon or by thermal activation. The effectiveness of the macrospin ferromagnetic nanomagnet model is confirmed by the mechanism for the formation of a strong coupling $g_{c\text{-}m}$ between the resonator photon and the localized magnetostatic spin excitation (the Kittel magnon). This is experimentally manifested in the splitting of the energies of hybrid photon-magnon states. Therefore, when describing the tunneling of a single magnon between macrospin nanomagnets, it is quite sufficient to restrict ourselves to the lowest terms of the expansion of the Heisenberg Hamiltonian in single-particle excited states both in the nanomagnets and in the ferromagnetic bridge. This made it possible to obtain an analytical expression for the nanomagnet-nanomagnet coupling $T_{j'j}(N)$, which

determines the dependence of the rate k_{tr} of long-range transport of the Kittel magnon on the number of units N of the bridge ferromagnetic chain. Among the main results of the work, we note the following.

(i) It is shown that when the energy of a Kittel magnon is less than or greater than the energy of a delocalized magnon in a ferromagnetic chain, the rate of coherent long-range transport of the Kittel magnon decreases exponentially with increasing number of chain units. This type of transfer can be attributed to off-resonance tunneling of magnons. If the energy of a static magnon falls into a window where delocalized magnon energy levels are present, then the magnon transfer rate increases by orders of magnitude, especially at its peak values. This regime of facilitated coherent transport can be interpreted as resonant tunneling of the magnon.

(ii) Analytical expressions (7) and (11) were found for the function R, which characterizes the behavior of the magnon transfer rate k_{tr} during off-resonant and resonant tunneling. This made it possible to determine the main physical parameters [Eq. (8)] responsible for controlling the rate of magnon transfer between nanomagnets in both tunneling regimes.

(iii) Particular attention was paid to the analysis of resonant tunneling, which may prove to be a convenient way to implement quantum communication. In the resonant transfer of Kittel magnons, two types of rate peak formation were revealed. One of them is formed at the fixed energy E of the magnon. Accordingly, the number of chain units ΔN_{peak} between the nearest peaks depends on the position of the energy E relative to the energy E_0 of the center of the band of the delocalized magnon [Eq. (12); Figs. 4 and 5]. The second type appears when the number of chain units N is fixed. The positions of the peaks are given by Eq. (13), reflecting the fact that for a given N, each rate peak in the resonant transfer of the Kittel magnon occurs under the condition that the energy of this magnon coincides with the energy E_ν of the νth state of the delocalized magnon.

(iv) In the presence of a weak stochastic influence on the spin-excitation energies, the positions of the rate peaks do not depend on the value of the stochastic parameter ξ. At the same time, since the heights of the peaks are proportional to $[(N+1)\xi]^{-2}$, for a given number of chain units the peaks, narrowing, increase their height (by 2 orders of magnitude with a decrease in ξ by 1 order of magnitude; Fig. 5).

(v) The theoretical results were applied to demonstrate how a static magnetic field $H_A(=H_B)$ can control the rate of resonant tunneling of Kittel magnons formed in nanomagnets (Fig. 6). Based on the fact that the key parameter α depends linearly on the magnitude of the static magnetic field [Eq. (14)], it is possible to reveal the controlling role of the magnetic field in establishing the positions and heights of the resonance peaks for different chain lengths at a fixed magnetic field strength [Fig. 6(a)] or for a fixed chain length at different magnetic field strengths [Fig. 6(b)]. It was also shown that due to the strong photon-magnon coupling in the magnon cavity, at a certain magnetic field H^*, the tunneling flow of magnons can reach an additional peak value (Fig. 7), which reflects the process of resonant conversion of a photon into a magnon.

The results presented in this paper may contribute to the development of quantum technologies related to communication in magnonic systems.

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E.G.P. conceptualized the study, performed formal analysis and investigation, developed the methodology, wrote the manuscript, and supervised the project. S.M.T. developed software, performed formal analysis, and prepared visualizations. V.V.G. performed formal analysis and prepared visualizations.

There are no conflicts to declare.

DATA AVAILABILITY

The data that support the findings of this article are not publicly available upon publication because it is not technically feasible and/or the cost of preparing, depositing, and hosting the data would be prohibitive within the terms of this research project. The data are available from the authors upon reasonable request.

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