

Scalable suppression of XY crosstalk by pulse-level control in superconducting quantum processors

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(Received 21 January 2026; revised 17 June 2026; accepted 23 July 2026; published 1 September 2026)

As superconducting quantum processors continue to scale, high-performance quantum control becomes increasingly critical. In densely integrated architectures, unwanted interactions between nearby qubits give rise to crosstalk errors that limit operational performance. In particular, direct exchange-type (XY) interactions are typically minimized by designing large frequency detunings between neighboring qubits at the hardware level. However, frequency crowding in large-scale systems ultimately restricts the achievable frequency separation. While such XY coupling facilitates entangling gate operations, its residual presence poses a key challenge during single-qubit controls. Here, we propose a scalable pulse-level control framework, incorporating frequency modulation and dynamical decoupling to suppress XY crosstalk errors. This framework operates independently of coupling strengths, reducing calibration overhead and naturally supporting multiqubit connectivity. Numerical simulations show orders-of-magnitude reductions in infidelity for both idle and single-qubit gates in a two-qubit system. We further illustrate scalability in a five-qubit layout, where crosstalk between a central qubit and four neighbors is simultaneously suppressed. Our crosstalk suppression framework provides a practical route toward high-fidelity operation in dense superconducting architectures.

DOI: [10.1103/v8p3-qh5k](https://doi.org/10.1103/v8p3-qh5k)

I. INTRODUCTION

Enabled by advances in microwave-based control and flexible circuit architecture, superconducting qubit systems have become one of the leading platforms for quantum computation [1–4]. As superconducting quantum processors have reached the thousand-qubit regime [5], achieving high-fidelity operations within large-scale systems has become increasingly critical. Recent demonstrations of quantum error correction [6] further indicate that practical fault-tolerant operation may be within reach [7,8]. These developments highlight the significance of enhancing the performance of quantum hardware to reliably operate below the fault-tolerance thresholds for next-generation superconducting architectures [9].

Unintended effects associated with neighboring qubits have become an increasingly prominent source of error in large-scale quantum processors [10–13]. Such crosstalk

errors not only degrade gate and measurement performance but also generate nonlocal error patterns that are particularly detrimental to quantum error correction [14,15]. One major mechanism, commonly referred to as classical crosstalk [12], arises from microwave or flux-control pulses unintentionally driving neighboring qubits during active operations [10,16]. These control-induced errors can be reduced by improving hardware isolation [17,18] and further mitigated through pulse-shaping or compensation techniques [19,20]. Another significant mechanism, often termed quantum crosstalk [12], stems from static interactions between nearby qubits, which perturb qubit evolution during both active operations and idle periods.

We focus on the quantum crosstalk arising from XY interactions between neighboring qubits. Such interactions, typically originating from capacitive or inductive coupling, are well described by transverse exchange (XY -type) interactions [2,4], enabling i SWAP-type two-qubit entangling gates [21,22]. When neighboring qubits are strongly detuned, direct excitation exchange is suppressed, leaving effective ZZ -type interactions and frequency shifts as the dominant residual effects [23–25]. Consequently, mitigation strategies have mainly targeted ZZ crosstalk [22,26–36]. However, as processors scale and

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spectral crowding becomes increasingly severe, maintaining large detuning across all neighboring qubits is no longer feasible, and XY coupling emerges as a significant source of coherent crosstalk error in dense superconducting architectures.

In this work, we develop pulse-level control strategies to suppress residual XY coupling during single-qubit operations without additional hardware overhead. We first show that XY -induced errors can be passively reduced by choosing gate durations that average out the unwanted exchange interaction. Building on this idea, we propose active approaches based on engineering the instantaneous detuning between qubits, including continuous frequency modulation (FM) and discrete dynamical decoupling (DD) sequences, both of which further suppress XY crosstalk while remaining compatible with single-qubit gates. We further illustrate the scalability of these methods in a five-qubit system, highlighting their applicability to larger superconducting processors. Finally, we discuss practical hardware considerations and outline potential future directions.

II. CROSSTALK DYNAMICS

We consider two superconducting qubits coupled through an exchange-type interaction in a 2D lattice layout, as shown in Fig. 1. The coupling is represented by the XY Hamiltonian

$$\hat{H}_{XY} = J(\hat{\sigma}_1^+ \hat{\sigma}_2^- + \hat{\sigma}_1^- \hat{\sigma}_2^+) = \frac{J}{2}(\hat{\sigma}_1^x \hat{\sigma}_2^x + \hat{\sigma}_1^y \hat{\sigma}_2^y), \quad (1)$$

where J is the coupling strength and $\hat{\sigma}_j^\pm = (\hat{\sigma}_j^x \pm i\hat{\sigma}_j^y)/2$ are the raising and lowering operators of qubit j , with $\hat{\sigma}_j^{x,y,z}$ denoting the Pauli matrices acting on its computational subspace. We take $\hbar = 1$ throughout. This term describes coherent excitation exchange between adjacent qubits, which enables two-qubit entangling gates but also leads

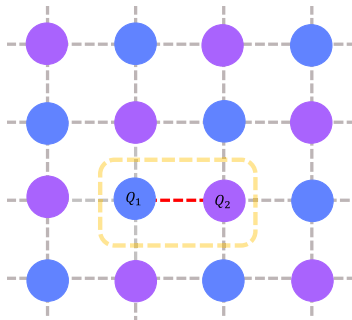


FIG. 1. Schematic of a nearest-neighbor qubit array with XY interactions, indicated by gray dashed lines. The pair Q_1 and Q_2 within the dashed orange outline defines a minimal two-qubit subsystem used to analyze XY -induced crosstalk under single-qubit control. The XY coupling between Q_1 and Q_2 is highlighted by red dashed lines.

to unwanted interactions during single-qubit operations. In particular, when controlling one qubit while the neighboring qubit remains idle, residual XY coupling can induce population leakage and phase errors, reducing gate fidelity. Suppressing these errors while preserving the intended interactions is therefore crucial for high-fidelity quantum computation.

Our goal is to develop an enhanced control scheme capable of suppressing the errors induced by residual XY coupling during single-qubit and idle operations. During gate operations, the system evolution under crosstalk dynamics (CD) is governed by the total Hamiltonian in the laboratory frame, given by

$$\hat{H}^{\text{CD}}(t) = \hat{H}_0 + \hat{H}_{XY} + \hat{H}_{\text{drive}}(t), \quad (2)$$

where $\hat{H}_0 = -\sum_{i=1}^2 \frac{\omega_i}{2} \hat{\sigma}_i^z$ is the bare qubit Hamiltonian and ω_i is the frequency of the qubit i , and $\hat{H}_{\text{drive}}(t)$ represents the driving field applied to the qubits to achieve the target operations.

To understand how the XY interaction influences the computational basis, we move to the rotating frame defined by the qubit frequencies, which corresponds to the operation frame used for qubit state control and measurement [2]. The transformation to this frame is defined by the unitary operator

$$\hat{U}_{H_0}(t) = \exp \left[i \left(\frac{\omega_1}{2} \hat{\sigma}_1^z + \frac{\omega_2}{2} \hat{\sigma}_2^z \right) t \right], \quad (3)$$

with the corresponding Hamiltonian in the operation frame given by

$$\begin{aligned} \tilde{H}^{\text{CD}}(t) &= \hat{U}_{H_0}^\dagger(t) \hat{H}^{\text{CD}}(t) \hat{U}_{H_0}(t) + i \dot{\hat{U}}_{H_0}^\dagger(t) \hat{U}_{H_0}(t) \\ &= \tilde{H}_{\text{drive}}(t) + \tilde{H}_{XY}(t), \end{aligned} \quad (4)$$

where $\tilde{O}(t) \equiv \hat{U}_{H_0}^\dagger(t) \hat{O}(t) \hat{U}_{H_0}(t)$ denotes the representation of an operator $\hat{O}(t)$ in this operation frame.

The gate unitary in the presence of XY crosstalk is

$$\hat{U}_{\text{gate}}^{\text{CD}}(T, 0) = \mathcal{T} \exp \left\{ -i \int_0^T [\tilde{H}_{\text{drive}}(t) + \tilde{H}_{XY}(t)] dt \right\}, \quad (5)$$

where $\mathcal{T}$ denotes the time-ordering operator. The unwanted XY term interferes with the intended control evolution, giving rise to crosstalk-induced gate errors.

Using the Magnus expansion [37], the evolution operator can be expressed as

$$\hat{U}_{\text{gate}}^{\text{CD,M}}(T, 0) = e^{-iT[\tilde{H}_{\text{gate}}^{\text{CD}(1)} + \tilde{H}_{\text{gate}}^{\text{CD}(2)} + \dots]}, \quad (6)$$

where $\tilde{H}_{\text{gate}}^{\text{CD}(n)}$ denotes the n th order term in the Magnus expansion, corresponding to the n th order contributions to the effective time-averaged Hamiltonian.

Considering only the leading-order contribution, the effective first-order Hamiltonian associated with the XY interaction is

$$\tilde{H}_{XY}^{\text{CD}(1)} = \frac{1}{T} \int_0^T \tilde{H}_{XY}(t) dt, \quad (7)$$

where $\tilde{H}_{XY}(t)$ is the XY interaction in the operation frame,

$$\begin{aligned} \tilde{H}_{XY}(t) &\equiv \hat{U}_{H_0}^\dagger(t) \hat{H}_{XY} \hat{U}_{H_0}(t) \\ &= J(e^{i\Delta t} \hat{\sigma}_1^+ \hat{\sigma}_2^- + e^{-i\Delta t} \hat{\sigma}_1^- \hat{\sigma}_2^+), \end{aligned} \quad (8)$$

and $\Delta = \omega_1 - \omega_2$ is the detuning between two qubits. The exchange term thus acquires an oscillatory phase factor dependent upon the detuning. This oscillatory behavior naturally suggests a timing-based mechanism for error suppression: by tailoring the phase accumulated through detuning, the effective exchange interaction can be averaged out over the gate duration.

A. Gate-time matching for XY suppression

In the simplest case of static qubit frequencies, XY error suppression can be achieved by choosing specific gate durations. Averaging the interaction over the gate duration yields

$$\begin{aligned} \tilde{H}_{XY}^{\text{CD}(1)} &= \frac{1}{T} \int_0^T J(e^{i\Delta t} \hat{\sigma}_1^+ \hat{\sigma}_2^- + e^{-i\Delta t} \hat{\sigma}_1^- \hat{\sigma}_2^+) dt \\ &= \frac{1}{T} \frac{J}{i\Delta} (e^{i\Delta T} \hat{\sigma}_1^+ \hat{\sigma}_2^- - e^{-i\Delta T} \hat{\sigma}_1^- \hat{\sigma}_2^+) \Big|_0^T. \end{aligned} \quad (9)$$

Because of this periodic structure, the oscillatory contribution can be averaged out by aligning operations at specific time points. To achieve first-order suppression, we choose a matched gate time T_M to satisfy $T_M = 2mT_\Delta$, $T_\Delta \equiv \pi/|\Delta|$, $m \in \mathbb{Z}^+$ to make the leading-order error term vanish, $\tilde{H}_{XY}^{\text{CD}(1)} = 0$. In the following analysis, we focus on the shortest matched gate time ($m = 1$) for concreteness, while the cancellation condition holds for arbitrary $m \in \mathbb{Z}^+$.

To quantify the performance of gate operations, we use the average gate fidelity as the metric [12,38]. For the unitary evolution considered here, it is given by

$$F = \frac{d + |\text{Tr}[(\hat{U}_{\text{gate}}^{\text{ideal}}(T, 0))^\dagger \hat{U}_{\text{gate}}(T, 0)]|^2}{d(d+1)}, \quad (10)$$

where d is the Hilbert-space dimension, $\hat{U}_{\text{gate}}(T, 0)$ denotes the actual gate evolution in the presence of XY crosstalk, and $\hat{U}_{\text{gate}}^{\text{ideal}}(T, 0)$ represents the target evolution of

the intended gate operation without the $\hat{H}_{XY}$ crosstalk term. This fidelity F quantifies how closely the actual evolution $\hat{U}_{\text{gate}}(T, 0)$ matches the ideal crosstalk-free evolution $\hat{U}_{\text{gate}}^{\text{ideal}}(T, 0)$ over the gate duration T , while the infidelity $1 - F$ characterizes deviations arising from unwanted interactions or imperfections.

Take the idle case as an example, where no active control is applied. The ideal idle gate operation is $\hat{U}_{I_1 I_2}^{\text{ideal}}(T, 0) = \hat{I}_1 \otimes \hat{I}_2$, where both qubits remain unchanged. The idle gate fidelity under CD, $F_{I_1 I_2}^{\text{CD}}$, is evaluated using Eq. (10) with $\hat{U}_{\text{gate}}(T, 0) = \hat{U}_{I_1 I_2}^{\text{CD}}(T, 0)$ and $\hat{U}_{\text{gate}}^{\text{ideal}}(T, 0) = \hat{U}_{I_1 I_2}^{\text{ideal}}(T, 0)$. Here, $\hat{U}_{I_1 I_2}^{\text{CD}}(T, 0)$ denotes the idle gate evolution in the presence of residual XY coupling without suppression control, obtained from Eq. (5) by setting $\tilde{H}_{\text{drive}}(t) = 0$. Throughout the manuscript, gate fidelities are defined in terms of the actual gate evolution, rather than the truncated Magnus-expanded approximations.

To examine the gate-time dependence of crosstalk errors, we numerically evaluate the idle gate infidelity as a function of the gate time for a fixed detuning $\Delta/2\pi = 50$ MHz, which is used in all numerical simulations. All average gate fidelities are evaluated numerically using QuTiP [39]. As shown in the simulated infidelity curve in Fig. 2, local minima of infidelity appear near even multiples of $T_\Delta = 10$ ns, corresponding to the matched gate-time condition $T_M = 2mT_\Delta$, while local maxima occur near odd multiples of T_Δ . Therefore, first-order suppression of the XY interaction can be achieved when the gate duration is properly synchronized with the detuning period, without requiring additional control fields.

To further reduce the influence of $\tilde{H}_{XY}(t)$, we introduce active control mechanisms in the following sections. If the qubit frequencies $\omega_{1,2}$ vary with time, the instantaneous detuning $\Delta + \delta(t) = \omega_1(t) - \omega_2(t)$ generates a time-dependent phase modulation that governs the excitation

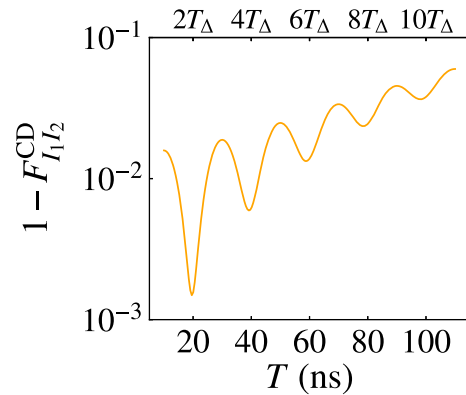


FIG. 2. Idle gate infidelity, $1 - F_{I_1 I_2}^{\text{CD}}$, as a function of the gate time T under the influence of crosstalk. Parameters are chosen as $\Delta/2\pi = 50$ MHz and $J/2\pi = 5$ MHz.

exchange. By intentionally engineering this detuning through Z -control pulses, one can tailor the accumulated phase to minimize the effect of the XY interaction over the gate duration, thereby suppressing crosstalk errors without altering the underlying hardware coupling. In the following, we extend this phase-accumulation mechanism to two active control schemes, frequency modulation and dynamical decoupling, to further suppress higher-order error contributions.

III. FREQUENCY MODULATION

We present an enhanced control method, termed frequency modulation (FM), which suppresses higher-order gate errors by continuously modulating the qubit frequency via an additional Z -control. For simplicity, we first consider the case where the drive is applied to Q_1 to perform single-qubit operations, while the Z -modulation is applied to the neighboring qubit Q_2 . This modulation applied to Q_2 is in the form of a sinusoidal flux control,

$$\hat{H}_{Z_2}^{\text{FM}}(t) = \gamma \sin\left(\frac{2\pi N}{T}t\right) \hat{\sigma}_2^z, \quad (11)$$

where γ denotes the modulation amplitude, T represents the total gate time, and N specifies the number of modulation cycles executed within the gate duration, such that the modulation period is defined as $\tau_N = T/N$. This Z -drive induces a time-dependent frequency shift on Q_2 , generating a periodic phase modulation that effectively averages out the crosstalk interaction.

A. Phase-averaging mechanism under FM

The total Hamiltonian under FM is

$$\begin{aligned} \hat{H}^{\text{FM}}(t) &= \hat{H}_0 + \hat{H}_{Z_2}^{\text{FM}}(t) + \hat{H}_{XY} + \hat{H}_{\text{drive}}(t) \\ &= -\frac{\omega_1}{2} \hat{\sigma}_1^z + \left[\gamma \sin\left(\frac{2\pi N}{T}t\right) - \frac{\omega_2}{2} \right] \hat{\sigma}_2^z \\ &\quad + J(\hat{\sigma}_1^+ \hat{\sigma}_2^- + \hat{\sigma}_1^- \hat{\sigma}_2^+) + \hat{H}_{\text{drive}}(t). \end{aligned} \quad (12)$$

To analyze the effect of this modulation, we move to a new rotating frame that incorporates the phase accumulated from both the static qubit frequencies and the frequency shift through the Z drive. We refer to this frame as the modulated frame. The unitary transformation associated with this modulated frame is

$$\begin{aligned} \hat{V}(t) &= \exp\left\{-i\left(-\frac{\omega_1}{2}t\hat{\sigma}_1^z + \left[\frac{\gamma T}{\pi N} \sin^2\left(\frac{\pi N}{T}t\right) - \frac{\omega_2}{2}t\right]\hat{\sigma}_2^z\right)\right\} \\ &= \hat{U}_{H_0}(t) \exp\left[-i\frac{\gamma A(t)}{2}\hat{\sigma}_2^z\right], \end{aligned} \quad (13)$$

where $\gamma A(t) = 2 \int_0^t \gamma \sin\left(\frac{2\pi N}{T}t'\right) dt'$ is the Z_2 rotation angle induced by the modulation, with $A(t) \equiv \frac{2T}{\pi N} \sin^2\left(\frac{\pi N}{T}t\right)$. The modulated frame differs from the operation frame generated by $\hat{U}_{H_0}(t)$ only through this additional Z_2 rotation.

The transformed Hamiltonian in the modulated frame is then given by

$$\begin{aligned} \tilde{H}^{\text{FM},V}(t) &= \hat{V}^\dagger(t) \hat{H}^{\text{FM}}(t) \hat{V}(t) + i\dot{\hat{V}}^\dagger(t) \hat{V}(t) \\ &= \tilde{H}_{\text{drive}}^V(t) + \tilde{H}_{XY}^V(t), \end{aligned} \quad (14)$$

where $\tilde{O}^V(t) \equiv \hat{V}^\dagger(t) \hat{O}(t) \hat{V}(t)$ denotes the representation of an operator $\hat{O}(t)$ in this modulated frame. The XY interaction in this modulated frame becomes

$$\begin{aligned} \tilde{H}_{XY}^V(t) &= \hat{V}^\dagger(t) \hat{H}_{XY} \hat{V}(t) \\ &= J(e^{i[\Delta t + \gamma A(t)]} \hat{\sigma}_1^+ \hat{\sigma}_2^- + e^{-i[\Delta t + \gamma A(t)]} \hat{\sigma}_1^- \hat{\sigma}_2^+), \end{aligned} \quad (15)$$

where the modulation appears as an additional time-dependent phase in the transformed XY interaction. The drive term remains identical to that in the operation frame, $\tilde{H}_{\text{drive}}^V(t) = \tilde{H}_{\text{drive}}(t)$, since it acts only on Q_1 and is therefore unaffected by the modulation on Q_2 .

The system evolution operator in the modulated frame, $\hat{U}^{\text{FM},V}(t_2, t_1)$, is related to that in the operation frame, $\hat{U}^{\text{FM}}(t_2, t_1)$, by $\hat{U}^{\text{FM},V}(t_2, t_1) = e^{i\gamma A(t_2)\hat{\sigma}_2^z/2} \hat{U}^{\text{FM}}(t_2, t_1) \times e^{-i\gamma A(t_1)\hat{\sigma}_2^z/2}$. Since $A(0) = A(T) = 0$, the two frames coincide at the gate boundaries, and therefore $\hat{U}_{\text{gate}}^{\text{FM},V}(T, 0) = \hat{U}_{\text{gate}}^{\text{FM}}(T, 0)$ for the full gate interval. The gate unitary under FM is thus

$$\hat{U}_{\text{gate}}^{\text{FM}}(T, 0) = \mathcal{T} \exp\left\{-i \int_0^T [\tilde{H}_{\text{drive}}(t) + \tilde{H}_{XY}^V(t)] dt\right\}. \quad (16)$$

To examine the effect of crosstalk under FM, we expand the gate unitary using the Magnus expansion

$$\hat{U}_{\text{gate}}^{\text{FM},M}(T, 0) = e^{-iT[\tilde{H}_{\text{gate}}^{\text{FM}(1)} + \tilde{H}_{\text{gate}}^{\text{FM}(2)} + \dots]}, \quad (17)$$

with the first-order contribution from the XY interaction given by

$$\tilde{H}_{XY}^{\text{FM}(1)} = \frac{1}{T} \int_0^T \tilde{H}_{XY}^V(t) dt = C_+^{\text{FM}(1)} \hat{\sigma}_1^+ \hat{\sigma}_2^- + C_-^{\text{FM}(1)} \hat{\sigma}_1^- \hat{\sigma}_2^+, \quad (18)$$

where

$$C_\pm^{\text{FM}(1)} \equiv \frac{J}{T} \int_0^T e^{\pm i[\Delta t + \gamma A(t)]} dt. \quad (19)$$

We define the first-order crosstalk error as the sum of the magnitudes of the coefficients preceding the first-order operators,

$$\epsilon^{\text{FM}(1)} \equiv |C_+^{\text{FM}(1)}| + |C_-^{\text{FM}(1)}| = \left| \frac{J}{T} \int_0^T e^{i[\Delta t + \gamma A(t)]} dt \right| + \left| \frac{J}{T} \int_0^T e^{-i[\Delta t + \gamma A(t)]} dt \right|. \quad (20)$$

Since a smaller $\epsilon^{\text{FM}(1)}$ corresponds to a weaker leading crosstalk-induced deviation from the ideal evolution, minimizing this quantity with respect to the modulation amplitude γ provides a criterion for reducing the expected gate infidelity.

For a given gate time T and detuning Δ , we can find the optimal modulation amplitude γ_{opt}^N at the assigned modulation cycle number N to minimize $\epsilon^{\text{FM}(1)}$. A useful first-order estimate of the optimized amplitude is $\gamma_{\text{opt}}^N/2\pi \simeq 2.4048 N/2T$, as discussed in Appendix A 5. When choosing a matched gate time $T_M = 2m\pi/|\Delta|$, $m \in \mathbb{Z}^+$, the first-order error can average to zero as in the CD case (see Appendix A 1 a). FM can further be used to suppress the second-order crosstalk error, which depends on the intended operation through $\hat{H}_{\text{drive}}(t)$.

We will show how FM suppresses second-order crosstalk errors and improves gate fidelity using the idle gate and the single-qubit X_1 gate as representative examples. While the present analysis assigns the gate operation to Q_1 and the modulation to its neighboring qubit Q_2 , similar control strategies can be implemented with both the drive and the modulation acting on a single qubit. Detailed derivations and extended analysis, covering the first-order optimization for nonmatched gate times, the single-qubit scheme of drive and modulation, and the case of parallel X_1X_2 gate operations, are provided in Appendix A.

B. Idle gate under frequency modulation

We consider the idle gate operation with target evolution $\hat{U}_{I_1I_2}^{\text{ideal}}(T, 0) = \hat{I}_1 \otimes \hat{I}_2$ under no drive, $\hat{H}_{\text{drive}}(t) = 0$. As detailed in Appendix A 1, the second-order error during idle gate is quantified as

$$\epsilon_{I_1I_2}^{\text{FM}(2)} = \frac{J^2}{T} \left| \int_0^T dt_1 \int_0^{t_1} dt_2 E_{I_1I_2}^{\text{FM}(2)}(t_1, t_2) \right|, \quad (21)$$

$$E_{I_1I_2}^{\text{FM}(2)}(t_1, t_2) \equiv \sin \{ \Delta(t_1 - t_2) + \gamma[A(t_1) - A(t_2)] \}.$$

For $\Delta/2\pi = 50$ MHz and a matched gate time $T_M = 2\pi/|\Delta| = 20$ ns, we obtain the optimized modulation amplitudes γ_{opt}^N that minimize the second-order crosstalk error $\epsilon_{I_1I_2}^{\text{FM}(2)}$ for each modulation cycle number N and simulate the gate dynamics to evaluate the resulting suppression of crosstalk errors. Unless otherwise stated, the numerical results presented in the main text are obtained using this choice of detuning and matched gate time.

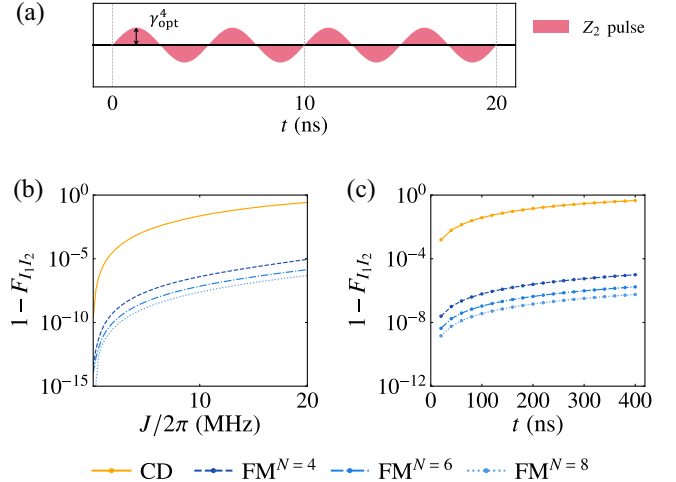


FIG. 3. Suppression of idle gate infidelity under FM. (a) Waveform of a sinusoidal Z_2 drive with $N = 4$ modulation cycles applied to Q_2 . (b) Infidelity of idle gate operation as a function of the coupling strength J for different numbers of modulation cycles N , evaluated at the matched gate time $T_M = 20$ ns. (c) Infidelity of consecutive idle gate operations as a function of time for different numbers of modulation cycles N , with $T_M = 20$ ns and $J/2\pi = 5$ MHz. Results under unsuppressed CD are shown for comparison.

To assess the performance of the scheme, we simulate the idle gate with FM at the optimized modulation amplitude γ_{opt}^N for each N . An example of the applied modulation, corresponding to the $N = 4$ case, is illustrated in Fig. 3(a). We first examine how the infidelity depends on the crosstalk coupling strength J . As shown in Fig. 3(b), FM largely suppresses the idle gate infidelity, yielding an improvement of more than 4 orders of magnitude compared with the unsuppressed CD case. The suppression becomes even stronger with increasing modulation cycles, reaching approximately 6 orders of magnitude at $N = 8$, reflecting more efficient averaging of the XY crosstalk, albeit at the expense of faster modulation and a stronger modulation amplitude (see Appendix A 1).

We further simulate a sequence of consecutive idle gates to evaluate the long-term stability of the suppression. As shown in Fig. 3(c), even after 20 consecutive operations, the idle gate infidelity shows an improvement of more than 4 orders of magnitude with FM compared with the unmodulated case, indicating consistent suppression of the XY crosstalk.

C. X_1 gate under frequency modulation

We next consider the single-qubit X_1 gate, whose target operation is $\hat{U}_{X_1I_2}^{\text{ideal}}(T, 0) = -i\hat{\sigma}_1^x \otimes \hat{I}_2$. The gate is generated by the drive Hamiltonian $\hat{H}_{\text{drive}}^V(t) = \Omega_1(t)\hat{\sigma}_1^x$ with a smoothly varying envelope $\Omega_1(t) = \Omega_{1x} \sin(\frac{\pi}{T}t)$, $\Omega_{1x} = \frac{\pi^2}{4T}$. The pulse satisfies $\int_0^T \Omega_1(t)dt = \frac{\pi}{2}$, yielding a total π rotation about the x axis on the Bloch sphere of Q_1 .

As detailed in Appendix A 2, the second-order X_1 gate error under FM is given by

$$\varepsilon_{X_1}^{\text{FM}(2)} = 2 \left| \frac{iJ}{2T} \int_0^T dt_1 \int_0^{t_1} dt_2 E_{X_1}^{\text{FM}(2)}(t_1, t_2) \right| + \varepsilon_{I_1 I_2}^{\text{FM}(2)},$$

$$E_{X_1}^{\text{FM}(2)}(t_1, t_2) \equiv \Omega_1(t_1) e^{i[\Delta t_2 + \gamma A(t_2)]} - \Omega_1(t_2) e^{i[\Delta t_1 + \gamma A(t_1)]}. \quad (22)$$

For $\Delta/2\pi = 50$ MHz and a matched gate time $T_M = 20$ ns, we obtain the optimized modulation amplitudes γ_{opt}^N that minimize the second-order crosstalk error $\varepsilon_{X_1}^{\text{FM}(2)}$ for each modulation cycle number N . Using these optimized values, we simulate the gate dynamics to evaluate the resulting improvement in fidelity, with Fig. 4(a) providing a concrete example of the Z-modulation and control-drive waveforms for the $N = 4$ case.

As shown in Fig. 4(b), FM reduces the X_1 gate infidelity by more than 2 orders of magnitude compared with the CD case, with further improvement as the number of modulation cycles N increases. To assess performance under extended operations, we simulate sequences of consecutive X_1 gates. Figure 4(c) shows that even after 21 consecutive gates, the infidelity remains more than 2 orders of magnitude lower than in the absence of FM, illustrating consistent suppression of XY crosstalk during repeated single-qubit operations.

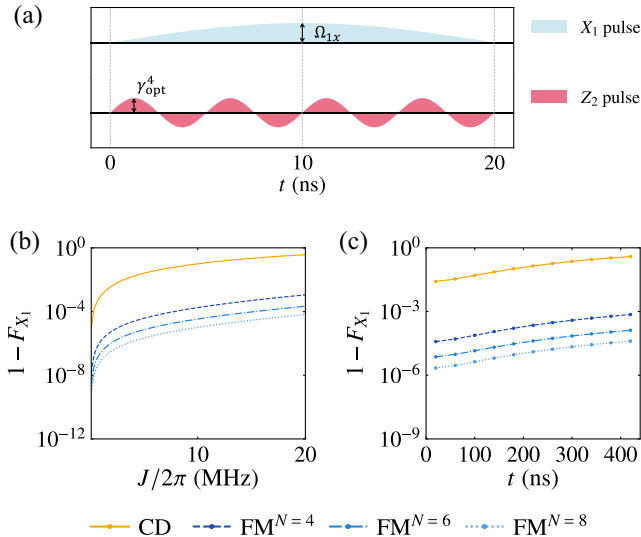


FIG. 4. Suppression of X_1 gate infidelity under FM. (a) Waveforms of the X_1 control pulses applied to Q_1 and the sinusoidal Z_2 drive with $N = 4$ modulation cycles applied to Q_2 . (b) Infidelity of X_1 gate operation as a function of the coupling strength J for different numbers of modulation cycles N . (c) Infidelity of consecutive X_1 gate operations as a function of time for different numbers of modulation cycles N , with a fixed $J/2\pi = 5$ MHz. Odd numbers of consecutive X_1 gates are applied to perform a net X_1 operation.

These results show that FM effectively suppresses cross-talk errors in both idle and active single-qubit X operations, providing enhanced single-qubit control. Since the modulation parameters are optimized for a given detuning, nearby qubits with similar frequency offsets can be simultaneously protected, enabling scalable suppression of XY crosstalk, as analyzed in a later section. The same principle also applies to the single-site protocol, where the drive term is modified accordingly (Appendix A 3), and can be further extended to parallel single-qubit operations ($X_1 X_2$) (Appendix A 4).

IV. DYNAMICAL DECOUPLING

Dynamical decoupling (DD) is a widely established technique for suppressing unwanted couplings in quantum systems through carefully designed pulse sequences [40–45]. By averaging out residual interaction terms over the evolution period, DD effectively mitigates coherent cross-talk. In this section, we apply DD strategies specifically to suppress errors arising from residual XY interactions.

To suppress the XY interaction, we apply Z_2 gates to Q_2 at regular intervals $\tau = T/S$, where S is the number of segments chosen to be an even integer and the total duration is set to the matched gate time $T = T_M = 2\pi/|\Delta|$. We model each Z_2 gate as a finite-width sinusoidal flux pulse centered at $t = s\tau$, with $w < \tau$,

$$\hat{H}_{Z_2}^{\text{DD}}(t) = \sum_{s=1}^S \frac{\pi}{2} \delta_w(t - s\tau) \hat{\sigma}_2^z, \quad (23)$$

where $\delta_w(t)$ is a nascent delta function of width w ,

$$\delta_w(t) \equiv \begin{cases} \frac{\pi}{2w} \cos\left(\frac{\pi t}{w}\right), & |t| \leq w/2, \\ 0, & \text{otherwise,} \end{cases} \quad (24)$$

which approaches the Dirac delta function in the zero-width limit, $\lim_{w \rightarrow 0} \delta_w(t) = \delta(t)$. Each Z_2 pulse imparts a π phase shift to Q_2 , periodically inverting the XY interaction and effectively averaging out the crosstalk.

A. Sign-alternating mechanism under DD

The total Hamiltonian under DD is given by

$$\begin{aligned} \hat{H}^{\text{DD}}(t) &= \hat{H}_0 + \hat{H}_{Z_2}^{\text{DD}}(t) + \hat{H}_{XY} + \hat{H}_{\text{drive}}(t) \\ &= -\frac{\omega_1}{2} \hat{\sigma}_1^z + \left[\sum_{s=1}^S \frac{\pi}{2} \delta_w(t - s\tau) - \frac{\omega_2}{2} \right] \hat{\sigma}_2^z \\ &\quad + J(\hat{\sigma}_1^+ \hat{\sigma}_2^- + \hat{\sigma}_1^- \hat{\sigma}_2^+) + \hat{H}_{\text{drive}}(t). \end{aligned} \quad (25)$$

To analyze the effect of DD, we move to the operation frame rotated by the static qubit frequencies as defined by Eqs. (3) and (4).

The Hamiltonian in the operation frame takes the form

$$\tilde{H}^{\text{DD}}(t) = \tilde{H}_{Z_2}^{\text{DD}}(t) + \tilde{H}_{XY}(t) + \tilde{H}_{\text{drive}}(t). \quad (26)$$

In the limit of short pulses, $w \rightarrow 0$, we approximate all Z_2 operations as instantaneous. The overall gate unitary under DD is then obtained by inserting a Z_2 gate into the crosstalk evolution at the end of each time segment,

$$\hat{U}_{\text{gate}}^{\text{DD}}(T, 0) = \hat{Z}_2 \hat{U}^{\text{CD}}[S\tau, (S-1)\tau] \cdots \hat{Z}_2 \hat{U}^{\text{CD}}[\tau, 0], \quad (27)$$

where the crosstalk evolution is

$$\hat{U}^{\text{CD}}(t_2, t_1) = \mathcal{T} \exp \left[-i \int_{t_1}^{t_2} [\tilde{H}_{XY}(t) + \tilde{H}_{\text{drive}}(t)] dt \right]. \quad (28)$$

We can absorb the instantaneous Z_2 gates into the even segments, such that the segment evolution operators take the form

$$\hat{U}_s = \begin{cases} \hat{U}^{\text{CD}}[s\tau, (s-1)\tau], & s \text{ odd}, \\ \hat{Z}_2 \hat{U}^{\text{CD}}[s\tau, (s-1)\tau] \hat{Z}_2, & s \text{ even}, \end{cases} \quad (29)$$

and the total DD evolution becomes

$$\hat{U}_{\text{gate}}^{\text{DD}}(T, 0) = \prod_{s=1}^S \hat{U}_s, \quad (30)$$

where the product is time ordered with earlier segments appearing on the right.

We define the segment Hamiltonian $\tilde{H}_s(t)$ through $\hat{U}_s = \mathcal{T} \exp [-i \int_{(s-1)\tau}^{s\tau} \tilde{H}_s(t) dt]$. Using the identity $Ue^M U^\dagger = e^{UMU^\dagger}$ for a unitary operator U , we have

$$\tilde{H}_s(t) = \begin{cases} \tilde{H}_{\text{drive}}(t) + \tilde{H}_{XY}(t), & s \text{ odd}, \\ \hat{Z}_2 [\tilde{H}_{\text{drive}}(t) + \tilde{H}_{XY}(t)] \hat{Z}_2, & s \text{ even}. \end{cases} \quad (31)$$

In even segments, the XY interaction acquires a minus sign because $\hat{Z}_2$ anticommutes with $\hat{\sigma}_2^\pm$. Drive components involving $\hat{\sigma}_2^x$ or $\hat{\sigma}_2^y$ would also flip sign under the same transformation. The resulting Hamiltonian can be written in an effective form that incorporates the alternating sign induced by the DD sequence,

$$\tilde{H}_{\text{eff}}^{\text{DD}}(t) = f(t) [\tilde{H}_{XY}(t) + \tilde{H}_{\text{drive}}^{Q_2}(t)] + \tilde{H}_{\text{drive}}^{Q_1}(t), \quad (32)$$

where we have introduced a sign function $f(t) = (-1)^{s-1}$ for $t \in [(s-1)\tau, s\tau]$. Here, $\tilde{H}_{\text{drive}}^{Q_1}(t)$ denotes the transverse (XY) control drive term acting on Q_1 , while $\tilde{H}_{\text{drive}}^{Q_2}(t)$ represents the transverse control drive term acting on Q_2 , which acquires the sign-flipping factor $f(t)$ due to the applied DD sequence. The alternating sign leads to an

effective averaging of the XY interaction, forming the basis for crosstalk suppression achieved by DD.

Using the Magnus expansion, the gate evolution under DD can be expressed as

$$\hat{U}_{\text{gate}}^{\text{DD},M}(T, 0) = e^{-iT[\tilde{H}_{\text{gate}}^{\text{DD}(1)} + \tilde{H}_{\text{gate}}^{\text{DD}(2)} + \cdots]}, \quad (33)$$

with the first-order contribution from the XY interaction given by

$$\tilde{H}_{XY}^{\text{DD}(1)} = \frac{1}{T} \sum_{s=1}^S \int_{(s-1)\tau}^{s\tau} (-1)^{s-1} \tilde{H}_{XY}(t) dt, \quad (34)$$

which is the sum of integrals over the XY crosstalk Hamiltonian in each segment.

The corresponding first-order crosstalk error is (see Appendix B 1)

$$\epsilon^{\text{DD}(1)} \equiv 2 \left| \frac{J}{T} \sum_{s=1}^S \frac{(-1)^{s-1}}{i\Delta} [e^{i\Delta s\tau} - e^{i\Delta(s-1)\tau}] \right|. \quad (35)$$

The factor $(-1)^{s-1}$ reflects the sign inversion of $\tilde{H}_{XY}(t)$ in every even segment, as seen directly from Eq. (34). Because of the periodicity of $e^{\pm i\Delta t}$, the above expression vanishes for any even number of segments $S \geq 4$, reproducing the same first-order cancellation that occurs at the matched gate time.

We will use the idle gate and the single-qubit X_1 gate as representative examples to illustrate how DD suppresses second-order XY crosstalk and improves gate fidelity. In the main analysis, the control drive is applied to Q_1 while the DD Z_2 pulses are applied to its neighbor Q_2 . An equivalent single-site protocol, in which both the drive and the decoupling pulses act on the same qubit, is also possible. Detailed derivations and further extensions, including the single-site version and parallel $X_1 X_2$ operations, are provided in Appendix B.

B. Idle gate under dynamical decoupling

We consider the specific case of $S = 4$ segments, corresponding to a DD sequence consisting of four equally spaced Z_2 pulses over the full gate duration T_M , with a segment interval $\tau = T_M/4$. This sequence will be referred to as DD Z-4. The second-order errors of the idle gate under CD and under DD are computed as (see Appendix B 2)

$$\begin{aligned} \epsilon_{I_1 I_2}^{\text{CD}(2)} &= 2 \left| \frac{J^2}{2\Delta} \right|, \\ \epsilon_{I_1 I_2}^{\text{DD}(2)} &= 2 \left| \frac{\pi - 4}{2\pi} \frac{J^2}{\Delta} \right|. \end{aligned} \quad (36)$$

Since $|\frac{\pi-4}{\pi}| < 1$, the second-order error is reduced under DD, which is expected to improve the resulting gate fidelity.

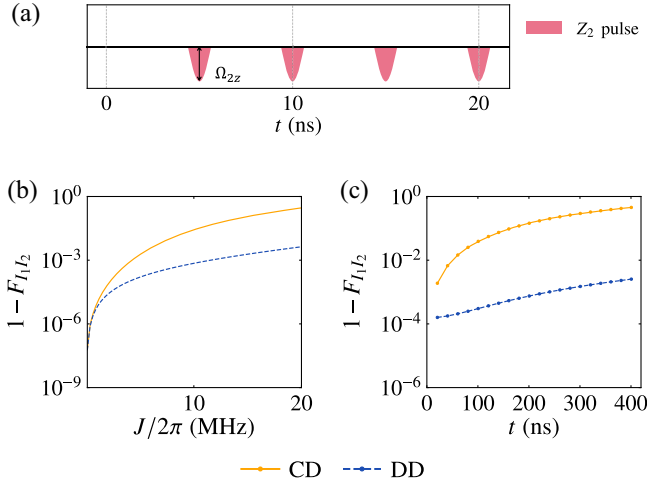


FIG. 5. Suppression of idle gate infidelity under DD. (a) Waveform of the DD Z-4 sequence containing four Z_2 gates applied to Q_2 . (b) Infidelity of idle gate operation as a function of the coupling strength J under DD Z-4. (c) Infidelity of consecutive idle gate operations as a function of time under DD Z-4, with $J/2\pi = 5$ MHz.

While the analytical calculations above assume instantaneous Z_2 operations, in numerical simulations we include their finite duration to reflect realistic pulse shapes. For the DD Z-4 sequence with $\tau = T_M/4$, we model each Z_2 pulse with a finite width $w = \tau/4$ and a drive amplitude $\Omega_{2z} = \pi^2/4w$. The total evolution time therefore includes this pulse duration, and for sequences of consecutive idle gates the fidelity is evaluated at $nT_M + \frac{w}{2}$, where n is the number of idle gates. The Z_2 pulses lower the frequency of Q_2 , increasing its detuning from Q_1 for better suppression of the XY interaction. The corresponding DD Z-4 pulse waveform for a single idle gate is shown in Fig. 5(a).

As shown in Fig. 5(b), for a detuning of $\Delta/2\pi = 50$ MHz and the matched gate time $T_M = 20$ ns, the DD Z-4 sequence reduces the idle gate infidelity by approximately 1 order of magnitude compared with the CD case for $J/2\pi \geq 5$ MHz. To assess performance over extended idle durations, we simulate sequences of consecutive idle gates. Figure 5(c) shows that even after 20 consecutive operations, the idle gate infidelity with DD is more than 2 orders of magnitude lower than without DD, indicating stable suppression of the XY interaction over repeated idling. These results show that DD Z-4 suppresses both the idle gate error and its accumulation over repeated operations, enabling more robust idling over extended durations.

C. X_1 gate under dynamical decoupling

To examine the effect of DD on active single-qubit operations, we analyze an X_1 gate applied within the DD Z-4 sequence, where the matched gate time T_M is divided

into four equal segments of duration $\tau = T_M/4$. The gate is decomposed into two $\sqrt{X_1}$ rotations applied in the odd segments between adjacent Z_2 pulses. For simplicity, we apply the control drive only during odd segments to ensure that the drive term remains unaffected by the Z_2 pulses in Eq. (31).

The corresponding drive Hamiltonian is given by

$$\begin{aligned} \tilde{H}_{\text{drive}}^{Q_1}(t) &= \Omega_{1x}(w) \cos\left(\frac{\pi}{\tau - w}\left[t - \left(s - \frac{1}{2}\right)\tau\right]\right) \hat{\sigma}_1^x, \\ t &\in \left[(s-1)\tau + \frac{w}{2}, s\tau - \frac{w}{2}\right], s \in \{1, 3\}, \end{aligned} \quad (37)$$

and is equal to zero otherwise. Here w is the pulse width of the Z_2 gate, and the drive amplitude is $\Omega_{1x}(w) = \frac{\pi^2}{8(\tau-w)}$. An example of waveforms with $w = \tau/4$ is illustrated in Fig. 6(a).

Under the short-pulse limit, $w \rightarrow 0$, the second-order errors with and without DD are (see Appendix B 3)

$$\begin{aligned} \epsilon_{X_1}^{\text{CD}(2)} &= 2\left|\frac{J^2}{2\Delta}\right| + 2\left|\frac{J}{4}\right|, \\ \epsilon_{X_1}^{\text{DD}(2)} &= 2\left|\frac{\pi - 4J^2}{2\pi\Delta}\right| + 2\left|\frac{J}{4}\right|, \end{aligned} \quad (38)$$

with $\epsilon_{X_1}^{\text{DD}(2)} < \epsilon_{X_1}^{\text{CD}(2)}$ indicating suppression of crosstalk errors during the X_1 gate under DD.

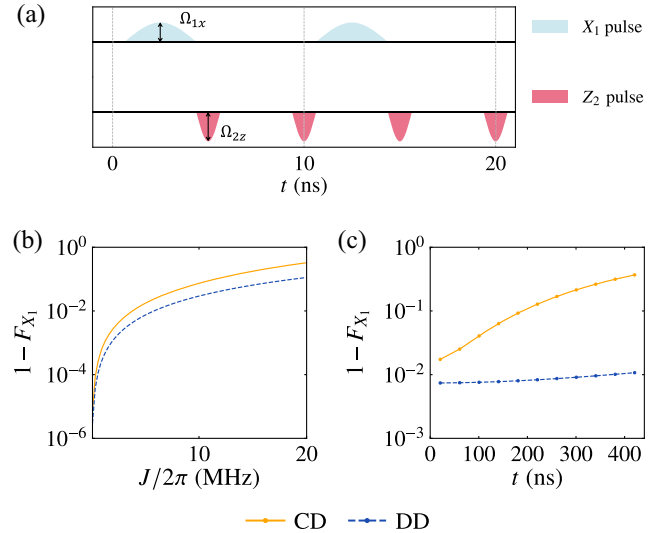


FIG. 6. Suppression of X_1 gate infidelity under DD. (a) Waveforms of the X_1 control pulses (generated by two $\sqrt{X_1}$ pulses) applied to Q_1 and the DD Z-4 sequence containing four Z_2 gates applied to Q_2 . (b) Infidelity of X_1 gate operation as a function of the coupling strength J under DD Z-4. (c) Infidelity of consecutive X_1 gate operations as a function of time under DD Z-4, with $J/2\pi = 5$ MHz.

We then consider finite-width Z_2 pulses for numerical simulations of the DD sequence, as shown in Fig. 6(a). The CD case without Z_2 pulses is simulated with $w = 0$, consistent with the analytical derivation. Figure 6(b) shows the resulting X_1 gate infidelity as a function of the crosstalk coupling strength J . The DD Z-4 sequence reduces the infidelity by approximately 0.37 orders of magnitude for $J/2\pi \geq 5$ MHz, with smaller improvements than in the idle gate case due to the presence of the drive during operation.

We also simulate sequences of consecutive X_1 gates, as shown in Fig. 6(c). Without DD, infidelity grows rapidly with the number of gates, while DD Z-4 slows this accumulation. After 21 consecutive gates, the DD-protected sequence achieves more than 1 order-of-magnitude reduction compared with the unprotected case, indicating sustained suppression of XY crosstalk during repeated single-qubit operations.

V. SCALABLE XY-CROSSTALK SUPPRESSION

The proposed XY-crosstalk suppression methods are naturally extensible to multiqubit settings, enabling scalable qubit architectures. As quantum processors expand to larger systems with denser connectivity, residual XY-type interactions increasingly give rise to coherent crosstalk, posing a growing challenge for high-fidelity control. To assess whether the suppression remains effective in large-scale architectures, we generalize the two-qubit model to a five-qubit configuration and evaluate the corresponding performance.

We study a five-qubit layout in which Q_2 couples to its four nearest neighbors (Fig. 7). The Hamiltonian of the five-qubit system is

$$\hat{H}_{5Q}(t) = -\sum_{i=1}^5 \frac{\omega_i}{2} \hat{\sigma}_i^z + \sum_{\substack{j=1 \\ j \neq 2}}^5 J(\hat{\sigma}_j^+ \hat{\sigma}_2^- + \hat{\sigma}_j^- \hat{\sigma}_2^+). \quad (39)$$

In the operation frame, i.e., the rotating frame at the static qubit frequencies, the five-qubit Hamiltonian becomes

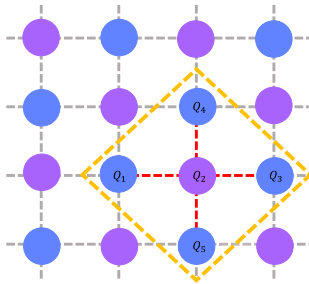


FIG. 7. A five-qubit subarray within a nearest-neighbor qubit array is indicated by the dashed orange outline. The XY interactions involving the central qubit Q_2 are highlighted by red dashed lines.

$$\tilde{H}_{XY5Q}(t) = \sum_{\substack{j=1 \\ j \neq 2}}^5 J(e^{i\Delta_j t} \hat{\sigma}_j^+ \hat{\sigma}_2^- + e^{-i\Delta_j t} \hat{\sigma}_j^- \hat{\sigma}_2^+), \quad (40)$$

where $\Delta_j = \omega_j - \omega_2$ denotes the frequency detuning between Q_2 and its neighboring qubit Q_j .

Since all parameters for performing FM and DD, including the optimal modulation amplitude γ_{opt}^N , the matched gate time T_M , and the length of the DD sequence, are determined by the magnitude of the detuning $|\Delta|$, we consider a symmetric case in which all neighbors of Q_2 share the same detuning Δ . The proposed uniform-detuning construction allows the pulse parameters optimized in the two-qubit setting to suppress the four neighboring couplings connected to Q_2 simultaneously. This construction can be further generalized to a two-dimensional lattice explicitly labeled by $Q_{i,j}$, where a uniform detuning pattern can be imposed such that $\omega_{i+1,j} - \omega_{i,j} = \omega_{i,j+1} - \omega_{i,j} = \Delta$ for all lattice indices (i, j) . Such a frequency arrangement establishes a systematic approach for suppressing XY crosstalk across large-scale qubit arrays.

A. Scalable crosstalk suppression under frequency modulation

We evaluate the performance and scalability of FM in suppressing XY crosstalk during both idle and single-qubit X_2 operations in the five-qubit configuration. For the idle gate, an example of the applied Z-modulation corresponding to the $N = 4$ case is shown in Fig. 8(a), which is identical to the two-qubit case shown in Fig. 3(a). For $\Delta/2\pi = 50$ MHz and the matched gate time $T_M = 20$ ns,

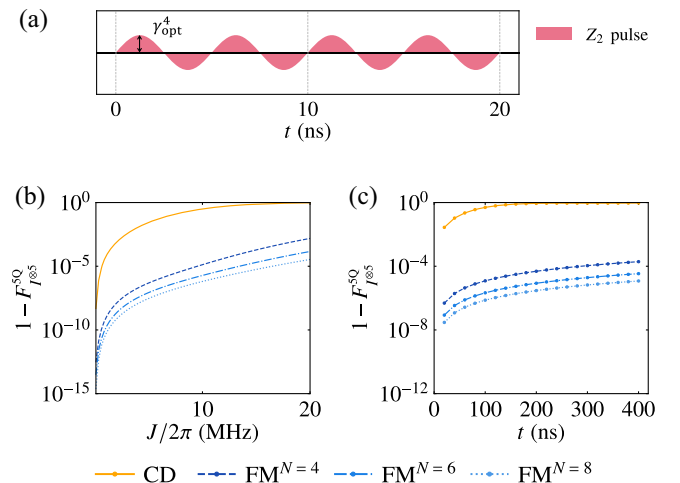


FIG. 8. Suppression of idle gate infidelity in a five-qubit subsystem under FM. (a) Waveform of a sinusoidal Z_2 drive with $N = 4$ modulation cycles applied to Q_2 . (b) Infidelity of idle gate operation as a function of the coupling strength J for different numbers of modulation cycles N . (c) Infidelity of consecutive idle gate operations as a function of time for different numbers of modulation cycles N , with $J/2\pi = 5$ MHz.

FM reduces the simulated idle gate infidelity by more than 4 orders of magnitude relative to CD for $J/2\pi \geq 5$ MHz, as shown in Fig. 8(b). Although the absolute infidelity is higher than in the two-qubit case due to the additional coupling paths, the suppression achieved by FM remains significant.

We further simulate repeated idle operations under FM. As shown in Fig. 8(c), after 20 consecutive idle gates, the FM-protected infidelity remains more than 3 orders of magnitude below that under CD. The suppression during repeated operations is comparable to the two-qubit case [Fig. 3(c)], indicating that FM maintains its effectiveness in larger qubit systems.

For active gate operation, we consider a single-qubit X_2 gate applied to Q_2 , the central qubit in the five-qubit layout. For scalable construction, we adopt the single-site scheme in which both the X_2 drive and the Z_2 modulation act on the same qubit. As a result, the pulse sequence differs from the two-qubit case, as detailed in Appendix A 3. An example of the frequency-modulated Z_2 pulse together with the X_2 and Y_2 control pulses used to generate the X_2 gate for the $N = 4$ case is shown in Fig. 9(a). As seen in Fig. 9(b), FM reduces the X_2 gate infidelity by more than 2 orders of magnitude for $J/2\pi \geq 5$ MHz. Although the reduction is slightly smaller than in the two-qubit model, the suppression trend remains consistent.

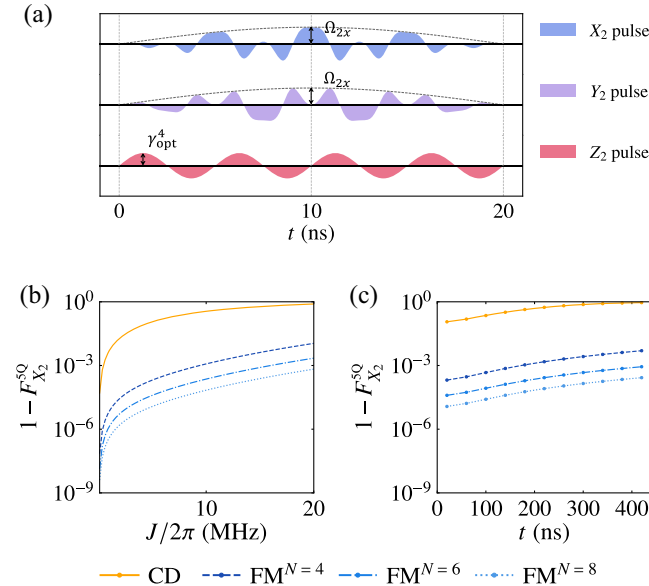


FIG. 9. Suppression of X_2 gate infidelity in a five-qubit subsystem under FM. (a) Waveforms of the X_2 and Y_2 control pulses for generating the X_2 gate, and the sinusoidal Z_2 drive with $N = 4$ modulation cycles applied to Q_2 . (b) Infidelity of X_2 gate operation as a function of the coupling strength J for different numbers of modulation cycles N . (c) Infidelity of consecutive X_2 gate operations as a function of time for different numbers of modulation cycles N , with $J/2\pi = 5$ MHz.

We then investigate the performance of FM during repeated gate operations by applying an odd number of X_2 gates, up to 21 consecutive operations with duration T_M . As shown in Fig. 9(c), the FM-protected gate infidelity remains more than 2 orders of magnitude lower than that under CD, indicating that FM effectively suppresses the accumulation of errors over extended sequences.

In the five-qubit system, the overall suppression achieved by FM is slightly reduced compared with the two-qubit case, likely due to the increased number of crosstalk coupling paths. Nevertheless, FM continues to suppress both idle and single-qubit X gate infidelities by several orders of magnitude, maintaining the same qualitative behavior observed in the two-qubit analysis. These results confirm that FM remains effective when extended to multiqubit systems.

B. Scalable crosstalk suppression under dynamical decoupling

Finally, we turn to the performance of DD in suppressing XY crosstalk within the five-qubit configuration. Using the same detuning of $\Delta/2\pi = 50$ MHz as in the previous analysis, we apply the DD Z-4 sequence to Q_2 during the idle gate, as illustrated in Fig. 10(a). As shown in Fig. 10(b), DD Z-4 reduces the idle gate infidelity by more than 1 order of magnitude at $J/2\pi = 5$ MHz. This reduction is greater than that observed in the two-qubit case [Fig. 5(b)], indicating enhanced suppression in the multiqubit configuration. However, the absolute idle gate infidelity remains higher due to the increased number of coupling paths in the five-qubit system. Under repeated idle operations, Fig. 10(c) shows that DD Z-4 continues to

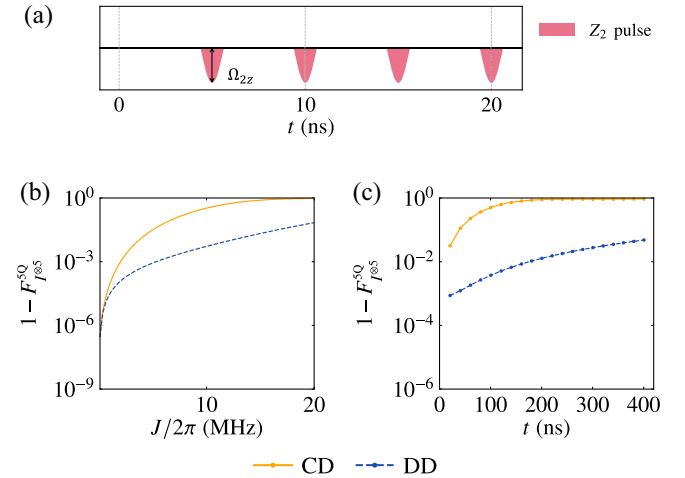


FIG. 10. Suppression of idle gate infidelity in a five-qubit subsystem under DD. (a) Waveform of the DD Z-4 sequence containing four Z_2 gates applied to Q_2 . (b) Infidelity of idle gate operation as a function of the coupling strength J under DD Z-4. (c) Infidelity of consecutive idle gate operations as a function of time under DD Z-4, with $J/2\pi = 5$ MHz.

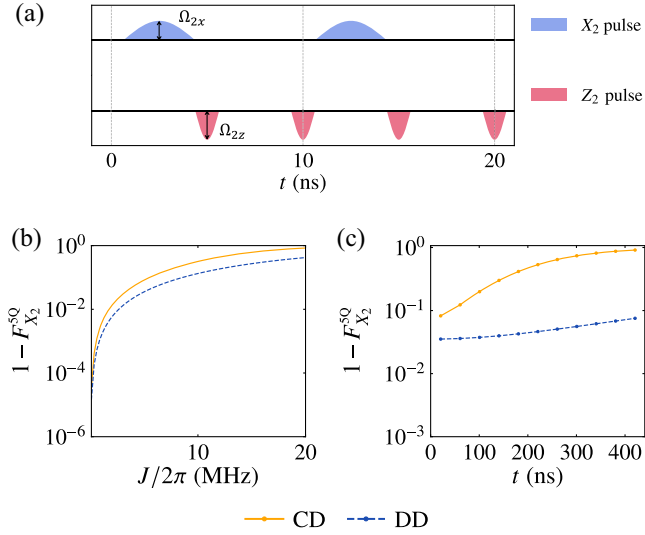


FIG. 11. Suppression of X_2 gate infidelity in a five-qubit subsystem under DD. (a) Waveforms of the X_2 control pulses (generated by two $\sqrt{X_2}$ pulses) and the DD Z_4 sequence containing four Z_2 gates applied to Q_2 . (b) Infidelity of X_2 gate operation as a function of the coupling strength J under DD Z_4 . (c) Infidelity of consecutive X_2 operations as a function of time under DD Z_4 , with $J/2\pi = 5$ MHz.

suppress the idle gate error by more than 1 order of magnitude after 20 consecutive gates.

For single-qubit gate operations, we apply the X_2 gate to Q_2 using the pulse sequence shown in Fig. 11(a), with details provided in Appendix B 4. As seen in Fig. 11(b), the DD Z_4 sequence reduces the X_2 gate infidelity by approximately 0.37 orders of magnitude for $J/2\pi \geq 5$ MHz, consistent with the two-qubit results. As in the idle-case analysis, the absolute infidelity is higher in the five-qubit system owing to additional crosstalk paths.

We further simulate repeated X_2 operations under DD protection. As shown in Fig. 11(c), applying up to 21 consecutive X_2 gates results in more than 1 order-of-magnitude reduction in infidelity relative to CD. This trend aligns with the two-qubit case and indicates that DD effectively slows the growth of crosstalk-induced errors during extended gate sequences.

Overall, DD remains effective in suppressing XY cross-talk in the five-qubit architecture. While the increased crosstalk connectivity leads to higher absolute infidelities compared with the two-qubit model, DD consistently reduces both idle and single-qubit gate errors and provides increasing relative suppression over extended operations. These results confirm that DD, like FM, can be reliably extended to multiqubit systems.

VI. PRACTICAL CONSIDERATIONS

In this section, we discuss practical considerations for implementing the proposed FM and DD crosstalk-

suppression protocols in realistic superconducting-qubit systems. We first include qubit relaxation and pure dephasing to examine how dissipative effects influence the performance of the protocols. We then consider detuning nonuniformity in multiqubit devices, which tests the sensitivity of the optimized pulse parameters to deviations from the designed qubit detuning. Finally, we discuss the residual ZZ interaction induced by higher excited levels, explain why it appears as a smaller higher-order correction to the XY exchange model, and comment on how it may be combined with existing ZZ-mitigation strategies.

A. Qubit decoherence

In realistic superconducting-qubit systems, gate dynamics are affected not only by coherent crosstalk but also by dissipative processes from the environment, most commonly energy relaxation and pure dephasing [4]. We model these dominant decoherence channels using a Lindblad master equation,

$$\partial_t \hat{\rho} = -i[\hat{H}_{\text{sys}}, \hat{\rho}] + \sum_i \gamma_{1,i} \mathcal{D}[\hat{\sigma}_i^-](\hat{\rho}) + \sum_i \gamma_{z,i} \mathcal{D}[\hat{\sigma}_i^z](\hat{\rho}), \quad (41)$$

where $\hat{\rho}$ is the density operator of the multiqubit system and $\hat{H}_{\text{sys}}$ is the corresponding Hamiltonian for each protocol. Specifically, $\hat{H}_{\text{sys}}(t) = \tilde{H}^{\text{CD}}(t)$ for CD, $\hat{H}_{\text{sys}}(t) = \tilde{H}^{\text{FM},V}(t)$ for FM, and $\hat{H}_{\text{sys}}(t) = \tilde{H}^{\text{DD}}(t)$ for DD.

Dissipative processes are incorporated through the Lindblad dissipator defined as

$$\mathcal{D}[\hat{c}](\hat{\rho}) \equiv \hat{c}\hat{\rho}\hat{c}^\dagger - \frac{1}{2}\{\hat{c}^\dagger\hat{c}, \hat{\rho}\} \quad (42)$$

for a jump operator $\hat{c}$. Here $\gamma_{1,i}$ is the relaxation rate of the i th qubit, associated with the jump operator $\hat{\sigma}_i^-$, and $\gamma_{z,i}$ is the pure-dephasing rate of the i th qubit, associated with the jump operator $\hat{\sigma}_i^z$. These dissipators retain the same form in the operation and modulated frames, since $\mathcal{D}[\hat{c}]$ is unchanged by an overall phase factor of the jump operator, $\hat{c} \rightarrow e^{i\varphi}\hat{c}$.

To examine the impact of decoherence on the proposed crosstalk-suppression schemes, we add these Lindblad terms to the FM and DD protocols considered in the previous sections and evaluate the gate infidelities. We use the same decoherence rates for all qubits, $\gamma_{1,i} = \gamma_{z,i} = 10$ kHz, corresponding to typical decoherence timescales on the order of 10–100 μs for superconducting qubits [2], and numerically simulate the resulting gate evolution using QuTiP [39]. The results are compared with both the unsuppressed CD evolution under the same decoherence rates and a decoherence-only reference case,

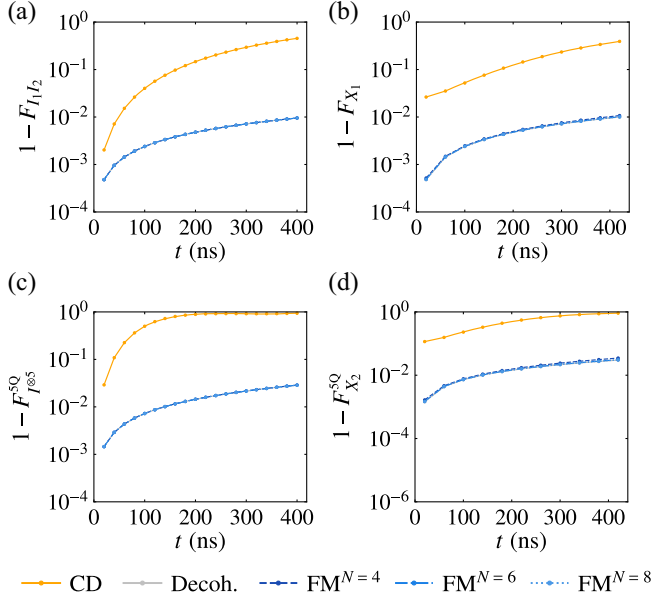


FIG. 12. Suppression of gate infidelity under FM in the presence of decoherence. Infidelity of consecutive gate operations as a function of time under XY crosstalk, relaxation, and pure dephasing for (a) idle gate, (b) X_1 gate, (c) five-qubit idle gate, and (d) five-qubit X_2 gate operations. The results are shown for different numbers of modulation cycles N , with $T_M = 20$ ns, $J/2\pi = 5$ MHz, $\Delta/2\pi = 50$ MHz, and $\gamma_{1,i} = \gamma_{z,i} = 10$ kHz for all qubits. The gray curve represents the decoherence-only reference case.

where relaxation and pure dephasing are included but the XY crosstalk term is removed from the Hamiltonian.

Figure 12 shows the gate infidelities obtained with the FM protocol in the presence of decoherence for consecutive idle and X gate operations in both two-qubit and five-qubit systems. Compared with the corresponding decoherence-free results shown in panel (c) of Figs. 3, 4, 8, and 9, the overall infidelity increases due to relaxation and pure dephasing. Nevertheless, FM still strongly suppresses the error relative to the unsuppressed CD case. All the FM curves nearly overlap with the decoherence-only reference curve (gray solid line), indicating that the coherent error induced by XY crosstalk is largely removed and the remaining fidelity degradation is mainly limited by qubit decoherence.

These results provide a practical criterion for choosing the FM modulation number. While larger N gives stronger coherent-error suppression in the decoherence-free analysis, it also increases the required modulation amplitude, which scales approximately as $\gamma_{\text{opt}}^N/2\pi \propto N/T$ in the first-order estimate of Appendix A 5, and requires faster flux control. For a 20-ns gate, $N = 4, 6$, and 8 correspond to modulation periods of approximately 5.0, 3.3, and 2.5 ns, respectively. Since $N = 4$ already approaches the decoherence-limited baseline while remaining compatible with superconducting hardware equipped with nanosecond-scale fast-flux control

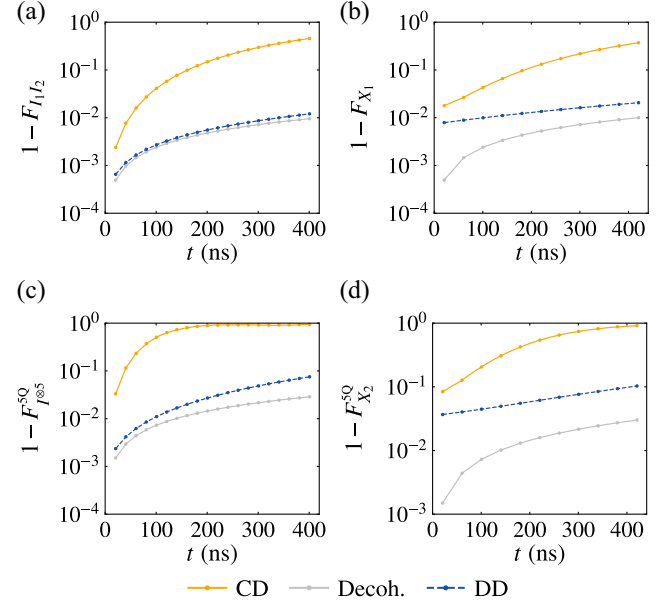


FIG. 13. Suppression of gate infidelity under DD in the presence of decoherence. Infidelity of consecutive gate operations as a function of time under XY crosstalk, relaxation, and pure dephasing for (a) idle gate, (b) X_1 gate, (c) five-qubit idle gate, and (d) five-qubit X_2 gate operations. The results under CD and DD Z-4 are shown with $T_M = 20$ ns, $J/2\pi = 5$ MHz, $\Delta/2\pi = 50$ MHz, and $\gamma_{1,i} = \gamma_{z,i} = 10$ kHz for all qubits. The gray curve represents the decoherence-only reference case.

and calibration methods [46,47], it serves as the practical operating condition for the FM protocol.

Figure 13 shows the simulated infidelities for the DD protocol in the presence of decoherence. Compared with the corresponding decoherence-free results shown in panel (c) of Figs. 5, 6, 10, and 11, the overall infidelity increases due to relaxation and pure dephasing, while DD still appreciably reduces the error relative to the unsuppressed CD case. For the two-qubit idle gate, the DD result approaches the decoherence-limited performance and shows an improvement of approximately 1.5 orders of magnitude after 20 consecutive gates, as shown in Fig. 13(a). In Figs. 13(b)–13(d), DD still provides about 1 order-of-magnitude reduction in infidelity relative to the CD case. Overall, these results show that both FM and DD remain effective in suppressing XY crosstalk in the presence of decoherence, with FM achieving stronger suppression and bringing the gate infidelity closer to the decoherence-limited baseline.

B. Sensitivity to detuning nonuniformity

In Sec. V, we assumed a uniform-detuning configuration for the five-qubit analysis, where the detunings between the central qubit and its neighboring qubits are identical. Under this condition, the FM and DD pulse parameters optimized for the two-qubit model can be directly applied to the five-qubit system. In realistic devices, however,

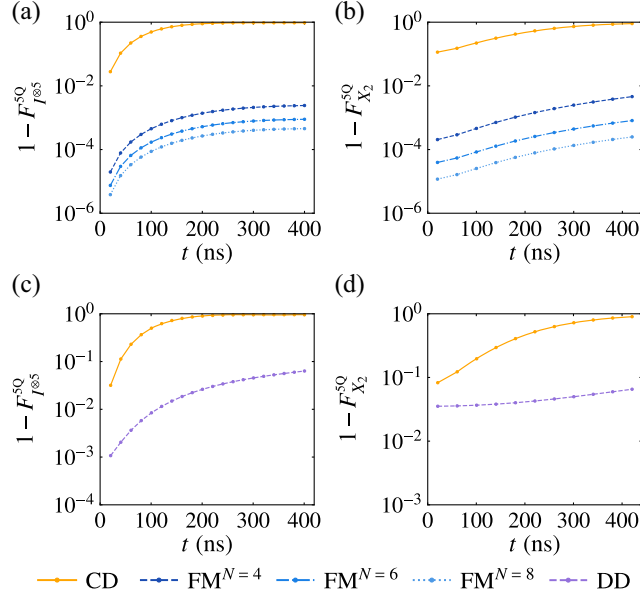


FIG. 14. Suppression of five-qubit gate infidelity in the presence of detuning nonuniformity. Infidelity of consecutive gate operations as a function of time under XY crosstalk with nonuniform neighboring-qubit detunings for (a) idle gate under FM, (b) X_2 gate under FM, (c) idle gate under DD, and (d) X_2 gate under DD. The FM results are shown for different numbers of modulation cycles N , while the DD results compare CD with DD Z-4. In all panels, $T_M = 20$ ns and $J/2\pi = 5$ MHz. For each curve, the infidelity is averaged over 100 random samples, with each neighboring-qubit detuning drawn from a Gaussian distribution centered at $\Delta/2\pi = 50$ MHz and with a standard deviation of $\sigma_\Delta/2\pi = 1$ MHz.

these detunings may not be perfectly identical because of device variations, calibration uncertainty, or residual frequency offsets. Therefore, detuning nonuniformity may reduce the effectiveness of a pulse design based on a single detuning value.

To evaluate the robustness of the proposed protocols in the five-qubit system, we include a finite detuning spread around the design value in the numerical simulations. Specifically, each neighboring-qubit detuning is sampled from a Gaussian distribution centered at the design value $\Delta/2\pi = 50$ MHz with a standard deviation of $\sigma_\Delta/2\pi = 1$ MHz. This corresponds to a detuning spread of about 2% relative to the design value and is used to test the sensitivity of the pulse design to imperfect detuning uniformity. The reported infidelity is averaged over 100 random samples.

Figure 14 shows the suppression performance of the FM and DD protocols under detuning nonuniformity in the five-qubit system. Compared with the corresponding uniform-detuning results shown in panel (c) of Figs. 8–11, the overall infidelity is slightly increased due to the detuning deviations from the design value. Nevertheless, both protocols still provide clear suppression relative to the unsuppressed CD case. These results indicate that the protocols are not sharply sensitive to small deviations from

the design detuning and remain effective for the detuning spread considered here.

C. ZZ crosstalk from higher excited levels

So far, we have focused on crosstalk arising from the XY exchange interaction by treating each superconducting qubit as an effective two-level system. In realistic superconducting qubits with finite anharmonicity, the higher excited states are not completely isolated from the computational states formed by the ground and first excited levels. As a result, exchange couplings that involve non-computational transition channels can also induce an additional ZZ interaction between qubits through virtual transitions. This effect appears as a second-order correction in the exchange coupling and gives the following effective contribution in the computational subspace,

$$\hat{H}_{\text{eff},Q}^{(2)}(t) = \frac{\zeta(t)}{4} (\hat{I}_1 \hat{I}_2 - \hat{\sigma}_1^z \hat{I}_2 - \hat{I}_1 \hat{\sigma}_2^z + \hat{\sigma}_1^z \hat{\sigma}_2^z), \quad (43)$$

where the last term represents the residual ZZ crosstalk. The corresponding coefficient is

$$\zeta(t) = \frac{2J^2}{\Delta(t) - \alpha_2} - \frac{2J^2}{\Delta(t) + \alpha_1}, \quad (44)$$

where $\Delta(t)$ is the instantaneous detuning and $\alpha_{1,2}$ are the qubit anharmonicities. For the FM protocol, $\Delta(t)$ includes the frequency shift induced by the continuous Z modulation. For the DD protocol, the detuning remains static between the Z pulses, so the same expression reduces to the static case with $\Delta(t) = \Delta$. A detailed derivation of Eq. (44) is provided in Appendix C.

This ZZ term is a second-order effect generated by virtual transitions associated with the exchange interaction. Its characteristic scale is set by $J^2/(\Delta \pm \alpha)$, making it typically smaller than the leading XY crosstalk. Nevertheless, after the leading XY crosstalk has been strongly suppressed by FM or DD, the additional ZZ crosstalk should also be considered as an independent error source and may limit the achievable gate fidelity. This indicates that practical implementations of the present XY -suppression protocols should incorporate appropriate ZZ-mitigation strategies when the residual ZZ interaction is non-negligible.

In particular, the proposed pulse-level XY -suppression protocols can be directly combined with the hardware-level ZZ-cancellation approach based on opposite-sign anharmonicities [27]. Choosing opposite anharmonicities for neighboring qubits such that $\alpha_1 = -\alpha_2$, the two denominators in Eq. (44) become identical and the residual ZZ coefficient vanishes, $\zeta(t) = 0$. Combining the present XY -suppression scheme with appropriate ZZ-mitigation strategies therefore provides a practical path toward

addressing coherent crosstalk in realistic superconducting qubit platforms.

VII. DISCUSSION

In this work, we develop pulse-level control strategies to mitigate residual XY crosstalk during single-qubit operations in superconducting qubit architectures. Through phase averaging at matched gate times and extending this principle through frequency modulation and dynamical decoupling, we show that crosstalk-induced errors can be suppressed for both idle periods and active gates. This framework improves the fidelity of standard single-qubit gates and naturally extends to suppress crosstalk from multiple nearby qubits, providing a scalable solution for densely connected architectures without additional hardware overhead.

The two proposed methods offer complementary trade-offs for experimental implementation. Continuous frequency modulation achieves robust error suppression with gate-time flexibility but comes at the expense of potential increased decoherence when operating away from flux sweet spots. In contrast, discrete dynamical decoupling preserves standard gate implementations but faces constraints in fitting sequences within short gate durations. Both methods also require consideration of practical hardware constraints, including fast-flux-control bandwidth, pulse distortions, and heating induced by flux control, making experimental calibration and platform-specific validation significant for assessing realistic performance.

These results open several avenues for future extension. Architectures with tunable couplers or indirect interactions offer a natural setting for further exploration. Extending crosstalk-suppression strategies beyond single-qubit operations to two-qubit gates remains a significant next step. These directions outline a viable pathway toward pulse-level crosstalk mitigation compatible with realistic and scalable superconducting quantum processors.

ACKNOWLEDGMENTS

We thank Chung-Ting Ke, Watson Kuo, and Li-Chieh Hsiao for helpful discussions. C.-H. W. acknowledges funding support from the National Science and Technology Council, Taiwan, under Grants No. 111-2112-M-002-049-MY3, No. 114-2119-M-007-013, No. 114-2124-M-002-003, No. 114-2112-M-002-021-MY3, No. 115-2124-M-002-014, and No. 115-2119-M-007-005, and from the National Taiwan University, under No. 114L895001 and No. 115L893701. C.-H. W. is also grateful for the NTU Eminence Scholar Fellowship funded by the Fubon Foundation and the Taiwan Semiconductor Manufacturing Company, as well as the support from the Physics Division, National Center for Theoretical Sciences, Taiwan.

DATA AVAILABILITY

The data that support the findings of this article are publicly available in Ref. [48].

APPENDIX A: DETAILED ANALYSIS OF FREQUENCY MODULATION

1. Optimization for the idle gate

a. First-order crosstalk error

For the FM scheme, the first-order Magnus Hamiltonian is obtained by averaging the full modulated-frame Hamiltonian,

$$\begin{aligned}\bar{H}_{\text{gate}}^{\text{FM}(1)} &= \frac{1}{T} \int_0^T [\tilde{H}_{\text{drive}}^V(t) + \tilde{H}_{XY}^V(t)] dt \\ &= \bar{H}_{\text{drive}}^{\text{FM}(1)} + \bar{H}_{XY}^{\text{FM}(1)}.\end{aligned}\quad (\text{A1})$$

The first term generates the intended control operation, while the second term represents the leading crosstalk-induced contribution. As defined in Eqs. (18)–(20), the XY contribution is written in terms of the coefficients $C_{\pm}^{\text{FM}(1)}$, and the corresponding scalar first-order crosstalk error is $\epsilon^{\text{FM}(1)} = |C_+^{\text{FM}(1)}| + |C_-^{\text{FM}(1)}|$.

Consider a matched gate time $T_M = 2m\pi/|\Delta|$, with $m \in \mathbb{Z}^+$. Since $A(t + \tau_N) = A(t)$, where $\tau_N = T_M/N$ is the period of the modulation, the coefficients $C_{\pm}^{\text{FM}(1)}$ can be decomposed into N modulation cycles as

$$\begin{aligned}C_{\pm}^{\text{FM}(1)} &= \frac{J}{T_M} \sum_{k=0}^{N-1} \int_{k\tau_N}^{(k+1)\tau_N} e^{\pm i[\Delta t + \gamma A(t)]} dt \\ &= \frac{J}{T_M} \left(\sum_{k=0}^{N-1} e^{\pm i\Delta k\tau_N} \right) \int_0^{\tau_N} e^{\pm i[\Delta t + \gamma A(t)]} dt.\end{aligned}\quad (\text{A2})$$

Using $\Delta\tau_N = \pm 2m\pi/N$, the prefactor becomes

$$\sum_{k=0}^{N-1} e^{\pm i\Delta k\tau_N} = \sum_{k=0}^{N-1} e^{\pm i2\pi mk/N}.\quad (\text{A3})$$

This geometric sum vanishes when m is not an integer multiple of N . Therefore, under this condition, $C_{\pm}^{\text{FM}(1)} = 0$, giving $\bar{H}_{XY}^{\text{FM}(1)} = 0$ and $\epsilon^{\text{FM}(1)} = 0$ at such a matched gate time. The optimization for nonmatched gate times, where this geometric cancellation no longer applies, is discussed separately in Appendix A 5.

b. Second-order crosstalk error

At a matched gate time, the first-order crosstalk error can vanish for both the FM and the unsuppressed CD case [Eq. (9)]. We therefore examine how FM further suppresses second-order error contributions and identify the

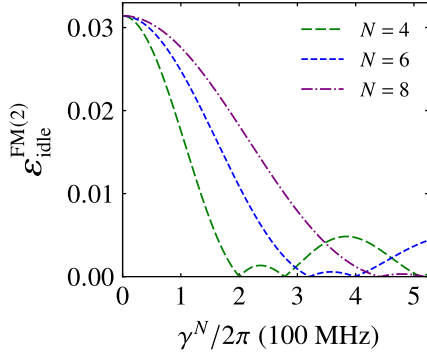


FIG. 15. Second-order idle gate crosstalk error under FM, $\varepsilon_{I_1 I_2}^{FM(2)}$, as a function of the modulation amplitude γ^N for different numbers of modulation cycles N . The chosen parameters are $T_M = 20$ ns, $\Delta/2\pi = 50$ MHz, and $J/2\pi = 5$ MHz.

optimal modulation amplitude γ_{opt}^N that achieves strong suppression.

$$\begin{aligned} \varepsilon_{I_1 I_2}^{FM(2)} &\equiv |C_{I_1 I_2}^{FM(2)}| + |-C_{I_1 I_2}^{FM(2)}| \\ &= \frac{J^2}{T} \left| \int_0^T dt_1 \int_0^{t_1} dt_2 \sin \{ \Delta(t_1 - t_2) + \gamma[A(t_1) - A(t_2)] \} \right|. \end{aligned} \quad (\text{A5})$$

For each modulation cycle number N , we numerically evaluate the second-order crosstalk error under different values of γ^N and identify the optimal modulation amplitude γ_{opt}^N for a matched gate time $T_M = 20$ ns, as shown in Fig. 15. We select the value of γ^N corresponding to the first local minimum of $\varepsilon_{I_1 I_2}^{FM(2)}$ as γ_{opt}^N , since a smaller modulation amplitude is more practical for hardware implementation.

We find that the numerically evaluated second-order crosstalk error for the idle gate $\varepsilon_{I_1 I_2}^{FM(2)}$ exhibits cornerlike minima. Therefore, in the simulations presented in Sec. III B of the main text, we do not directly use the numerically identified minimum at γ_{opt}^N . Instead, we define the idle gate infidelity using a numerically stable estimate evaluated by averaging the values at $\gamma_{\text{opt}}^N \pm \Delta\gamma$,

$$F_{I_1 I_2}^{FM} \equiv \frac{1}{2} [F_{I_1 I_2}^{FM}(\gamma_{\text{opt}}^N + \Delta\gamma) + F_{I_1 I_2}^{FM}(\gamma_{\text{opt}}^N - \Delta\gamma)], \quad (\text{A6})$$

where $\Delta\gamma$ denotes the sampling step used in the numerical scan of γ^N . For the parameters used in Fig. 15, the resulting optimal modulation amplitudes are $\gamma_{\text{opt}}^4/2\pi = 201$ MHz, $\gamma_{\text{opt}}^6/2\pi = 321$ MHz, and $\gamma_{\text{opt}}^8/2\pi = 442$ MHz, with $\Delta\gamma/2\pi = 1.59$ MHz.

2. Optimization for the X_1 gate

In contrast to the idle gate case, the second-order contribution for the X_1 gate includes additional

Using $\tilde{H}_{XY}^V(t)$ from Eq. (15), and noting that $\tilde{H}_{\text{drive}}^V(t) = 0$ for the idle gate, the corresponding second-order Magnus term for the idle gate under FM reads

$$\begin{aligned} \bar{H}_{I_1 I_2}^{FM(2)} &= -\frac{i}{2T} \int_0^T dt_1 \int_0^{t_1} dt_2 [\tilde{H}_{XY}^V(t_1), \tilde{H}_{XY}^V(t_2)] \\ &= \frac{J^2}{2T} \int_0^T dt_1 \int_0^{t_1} dt_2 E_{I_1 I_2}^{FM(2)}(t_1, t_2) (\hat{\sigma}_1^z \hat{I}_2 - \hat{I}_1 \hat{\sigma}_2^z) \\ &= C_{I_1 I_2}^{FM(2)} \hat{\sigma}_1^z \hat{I}_2 - C_{I_1 I_2}^{FM(2)} \hat{I}_1 \hat{\sigma}_2^z, \end{aligned} \quad (\text{A4})$$

where $C_{I_1 I_2}^{FM(2)} = \frac{J^2}{2T} \int_0^T dt_1 \int_0^{t_1} dt_2 E_{I_1 I_2}^{FM(2)}(t_1, t_2)$ and $E_{I_1 I_2}^{FM(2)}(t_1, t_2) \equiv \sin \{ \Delta(t_1 - t_2) + \gamma[A(t_1) - A(t_2)] \}$.

The second-order crosstalk error for the idle gate under FM is quantified as the sum of the magnitudes of the coefficients preceding the corresponding second-order operators,

commutators between the drive term and the XY crosstalk. Using $\tilde{H}_{XY}^V(t)$ from Eq. (15), and noting that the Z_2 modulation does not affect the drive acting on Q_1 , we have

$$\tilde{H}_{\text{drive}}^V(t) = \tilde{H}_{\text{drive}}(t) = \Omega_1(t) \hat{\sigma}_1^x, \quad (\text{A7})$$

where the driving pulse is chosen as $\Omega_1(t) = \Omega_{1x} \sin(\frac{\pi}{T}t)$ with $\Omega_{1x} = \frac{\pi^2}{4T}$, satisfying $\int_0^T \Omega_1(t) dt = \frac{\pi}{2}$. The Hamiltonian for generating a single-qubit X_1 gate in the presence of crosstalk, expressed in the frequency-modulated frame, is therefore given by

$$\begin{aligned} \tilde{H}^V(t) &= \tilde{H}_{XY}^V(t) + \tilde{H}_{\text{drive}}^V(t) \\ &= J(e^{i[\Delta t + \gamma A(t)]} \hat{\sigma}_1^+ \hat{\sigma}_2^- + e^{-i[\Delta t + \gamma A(t)]} \hat{\sigma}_1^- \hat{\sigma}_2^+) \\ &\quad + \Omega_1(t) \hat{\sigma}_1^x. \end{aligned} \quad (\text{A8})$$

The second-order Magnus term for the X_1 gate operation under FM is given by

$$\bar{H}_{X_1}^{FM(2)} = -\frac{i}{2T} \int_0^T dt_1 \int_0^{t_1} dt_2 [\tilde{H}^V(t_1), \tilde{H}^V(t_2)]. \quad (\text{A9})$$

The commutator can be decomposed into four components,

$$[\tilde{H}^V(t_1), \tilde{H}^V(t_2)] = [\tilde{H}_{XY}^V(t_1), \tilde{H}_{XY}^V(t_2)] + [\tilde{H}_{\text{drive}}^V(t_1), \tilde{H}_{\text{drive}}^V(t_2)] + [\tilde{H}_{\text{drive}}^V(t_1), \tilde{H}_{XY}^V(t_2)] + [\tilde{H}_{XY}^V(t_1), \tilde{H}_{\text{drive}}^V(t_2)]. \quad (\text{A10})$$

The first component arises solely from the XY crosstalk and is identical to that in Eq. (A4). The corresponding error has already been evaluated in the idle gate case [Eq. (A5)]. The second component corresponds to the ideal drive term and does not contribute to crosstalk-induced errors. We thus focus on the remaining two components, which involve the commutators between the drive and the XY crosstalk. The sum of these two components can be expressed as

$$\begin{aligned} [\tilde{H}_{\text{drive}}^V(t_1), \tilde{H}_{XY}^V(t_2)] + [\tilde{H}_{XY}^V(t_1), \tilde{H}_{\text{drive}}^V(t_2)] &= (-\Omega_1(t_1)J e^{i[\Delta t_2 + \gamma A(t_2)]} + \Omega_1(t_2)J e^{i[\Delta t_1 + \gamma A(t_1)]})\hat{\sigma}_1^z \hat{\sigma}_2^- \\ &\quad + (\Omega_1(t_1)J e^{-i[\Delta t_2 + \gamma A(t_2)]} - \Omega_1(t_2)J e^{-i[\Delta t_1 + \gamma A(t_1)]})\hat{\sigma}_1^z \hat{\sigma}_2^+ \\ &= -J E_{X_1}^{\text{FM}(2)}(t_1, t_2) \hat{\sigma}_1^z \hat{\sigma}_2^- + J (E_{X_1}^{\text{FM}(2)}(t_1, t_2))^* \hat{\sigma}_1^z \hat{\sigma}_2^+, \end{aligned} \quad (\text{A11})$$

where

$$E_{X_1}^{\text{FM}(2)}(t_1, t_2) \equiv \Omega_1(t_1) e^{i[\Delta t_2 + \gamma A(t_2)]} - \Omega_1(t_2) e^{i[\Delta t_1 + \gamma A(t_1)]}. \quad (\text{A12})$$

The two terms in Eq. (A11) have coefficients with the same magnitude. The total second-order error is therefore quantified as

$$\varepsilon_{X_1}^{\text{FM}(2)} = \left| \frac{iJ}{T} \int_0^T dt_1 \int_0^{t_1} dt_2 E_{X_1}^{\text{FM}(2)}(t_1, t_2) \right| + \varepsilon_{I_1 I_2}^{\text{FM}(2)}. \quad (\text{A13})$$

For a given matched gate time $T_M = 20$ ns, we numerically evaluate the magnitude of $\varepsilon_{X_1}^{\text{FM}(2)}$ for each modulation cycle number N , as shown in Fig. 16. We identify the optimal modulation amplitude γ_{opt}^N by selecting the value of γ^N corresponding to the first local minimum of $\varepsilon_{X_1}^{\text{FM}(2)}$ as γ_{opt}^N . The resulting optimal modulation amplitudes are $\gamma_{\text{opt}}^4/2\pi = 244$ MHz, $\gamma_{\text{opt}}^6/2\pi = 363$ MHz, and

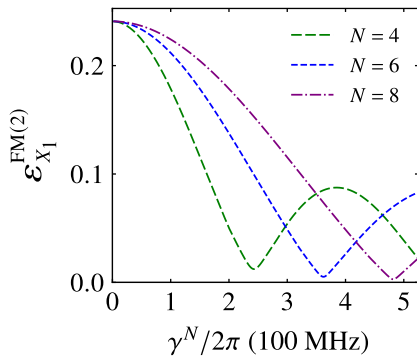


FIG. 16. Second-order X_1 gate crosstalk error under FM, $\varepsilon_{X_1}^{\text{FM}(2)}$, as a function of the modulation amplitude γ^N for different numbers of modulation cycles N . The chosen parameters are $T_M = 20$ ns, $\Delta/2\pi = 50$ MHz, and $J/2\pi = 5$ MHz.

$\gamma_{\text{opt}}^8/2\pi = 482$ MHz, which are used in the simulations presented in Sec. III C.

3. Single-site scheme of frequency modulation

In practical implementations, it is often desirable to apply both the control and modulation drives to the same qubit. We therefore consider the case where both the drive and Z -modulation are applied to the same qubit Q_2 to achieve single-qubit operations and crosstalk suppression. In the presence of Z -modulation, the action of the drive Hamiltonian is modified. To realize the desired single-qubit X_2 gate in the operation frame, the drive term must therefore be adjusted correspondingly. Specifically, the drive Hamiltonian in the frequency-modulated frame is related to that in the operation frame by

$$\tilde{H}_{\text{drive}}^V(t) = e^{i\gamma A(t)\hat{\sigma}_2^z/2} \tilde{H}_{\text{drive}}(t) e^{-i\gamma A(t)\hat{\sigma}_2^z/2}, \quad (\text{A14})$$

where $A(t) = \frac{2T}{\pi N} \sin^2(\frac{\pi N}{T} t)$.

To realize a single-qubit X_2 gate, the drive Hamiltonian in the frequency-modulated frame is chosen as $\tilde{H}_{\text{drive}}^V(t) = \Omega_2(t)\hat{\sigma}_2^x$ with a smooth envelope $\Omega_2(t) = \Omega_{2x} \sin(\frac{\pi}{T} t)$, where $\Omega_{2x} = \frac{\pi^2}{4T}$ is set such that $\int_0^T \Omega_2(t) dt = \frac{\pi}{2}$. The corresponding drive Hamiltonian in the operation frame is then given by

$$\tilde{H}_{\text{drive}}(t) = \Omega_2(t) [\cos(\gamma A(t))\hat{\sigma}_2^x + \sin(\gamma A(t))\hat{\sigma}_2^y], \quad (\text{A15})$$

obtained via the inverse transformation of Eq. (A14).

Following the derivation for the X_1 gate in Appendix A 2, we obtain the same second-order crosstalk error expression for the single-site X_2 gate. In particular, since the drive is chosen to have the same form in the frequency-modulated frame, the resulting error $\varepsilon_{X_2}^{\text{FM}(2)}$ is identical to $\varepsilon_{X_1}^{\text{FM}(2)}$ given in Eq. (A13). Consequently, the optimal modulation amplitudes γ_{opt}^N are the same as those obtained in Appendix A 2.

Figure 17(a) shows an example of the control waveforms for the single-site FM protocol with $N = 4$ at the

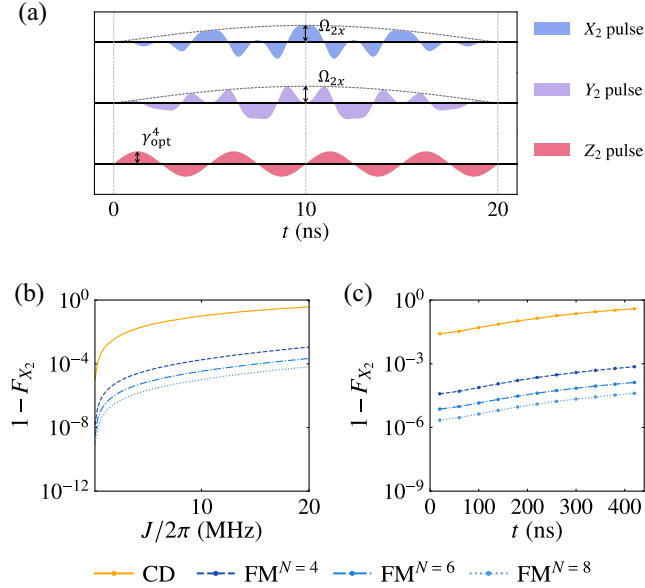


FIG. 17. Suppression of X_2 gate infidelity under single-site FM. (a) Waveforms of the X_2 and Y_2 control pulses for generating the X_2 gate, and the sinusoidal Z_2 drive with $N = 4$ modulation cycles applied to Q_2 . (b) Infidelity of X_2 gate operation as a function of the coupling strength J for different numbers of modulation cycles N . (c) Infidelity of consecutive X_2 gate operations as a function of time for different numbers of modulation cycles N , with $J/2\pi = 5$ MHz.

matched gate time $T_M = 20$ ns. In contrast to the two-site scheme, FM on the same qubit modifies the effective drive, such that the target X_2 gate is realized using combined X_2 and Y_2 drives, while the sinusoidal Z_2 modulation suppresses the residual XY interaction.

As shown in Fig. 17(b), single-site FM reduces the infidelity of X_2 gate by more than 2 orders of magnitude compared with the CD case. We further simulate sequences of consecutive X_2 gates to assess the long-term performance. As shown in Fig. 17(c), even after 21 consecutive operations, the infidelity remains more than 2 orders of magnitude lower than without FM. These results are consistent with those obtained for the X_1 gate (Fig. 4), confirming that both the control drive and the Z modulation can be applied to the same qubit to achieve robust suppression of XY crosstalk, provided that the drive waveform is appropriately modified.

4. Parallel X_1X_2 operations under frequency modulation

In the main text (Sec. III C), the Z_2 modulation is shown to suppress crosstalk errors for X_1 gate applied to Q_1 . Appendix A 3 further shows that the same Z_2 modulation can also protect an X_2 gate on Q_2 , provided that the drive waveform on Q_2 is correspondingly modified. We now consider parallel single-qubit operations, where X_1 and X_2 gates are applied simultaneously to Q_1 and Q_2 , respectively, under the same Z_2 modulation.

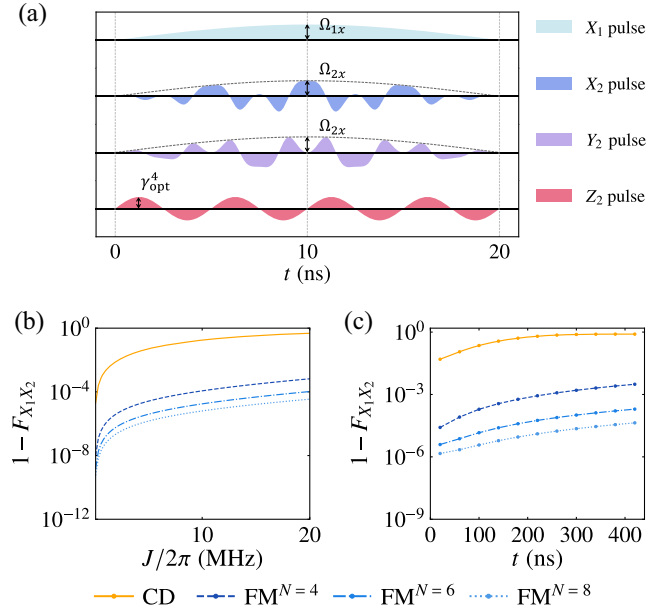


FIG. 18. Suppression of parallel X_1X_2 gate infidelity under FM. (a) Waveforms of the X_1 control pulses applied to Q_1 , the X_2 and Y_2 control pulses for generating the X_2 gate on Q_2 , and the sinusoidal Z_2 drive with $N = 4$ modulation cycles applied to Q_2 . (b) Infidelity of parallel X_1X_2 gate operation as a function of the coupling strength J for different numbers of modulation cycles N . (c) Infidelity of consecutive parallel X_1X_2 gate operations as a function of time for different numbers of modulation cycles N , with $J/2\pi = 5$ MHz.

We consider the same forms of the X_1 , X_2 , and Y_2 driving pulses as those used in Sec. III C and Appendix A 3. The corresponding frequency-modulated-frame Hamiltonian for the parallel X_1X_2 operation is

$$\begin{aligned} \tilde{H}_{X_1X_2}^V(t) &= \tilde{H}_{XY}^V(t) + \tilde{H}_{\text{drive}}^{V,Q_1}(t) + \tilde{H}_{\text{drive}}^{V,Q_2}(t) \\ &= \tilde{H}_{XY}^V(t) + \Omega_1(t)\hat{\sigma}_1^x + \Omega_2(t)\hat{\sigma}_2^x. \end{aligned} \quad (\text{A16})$$

Under this condition, the second-order crosstalk error for the parallel X_1X_2 operation is given by the sum of the corresponding single-gate errors, with the shared idle error contribution subtracted, $\epsilon_{X_1X_2}^{\text{FM}(2)} = \epsilon_{X_1}^{\text{FM}(2)} + \epsilon_{X_2}^{\text{FM}(2)} - \epsilon_{I_1I_2}^{\text{FM}(2)}$, and the optimal modulation amplitudes γ_{opt}^N remain identical to those determined previously for the single X gate case.

Figure 18(a) shows a representative example of the control waveforms for the parallel X_1X_2 operations with $N = 4$. The X_1 drive pulse is identical to that used in Sec. III C, while the X_2 and Y_2 drive components follow the single-site scheme described in Appendix A 3. These pulses are applied simultaneously under the same Z_2 modulation.

As shown in Fig. 18(b), for the matched gate time $T_M = 20$ ns, FM reduces the infidelity of parallel X_1X_2 operations by more than 3 orders of magnitude compared

with CD. The suppression remains robust under repeated operations: Fig. 18(c) shows that even after 21 consecutive parallel gates, the infidelity remains more than 2 orders of magnitude lower than in the absence of modulation.

5. Frequency modulation with nonmatched gate time

We have shown that FM can suppress XY crosstalk at the matched gate time T_M , where the first-order error vanishes and the remaining second-order contribution is further suppressed. More generally, for a given nonmatched gate time T , an optimal modulation amplitude γ can still be determined by minimizing the first-order crosstalk error,

$$\epsilon^{\text{FM}(1)} = \left| \frac{J}{T} \int_0^T e^{i[\Delta t + \gamma A(t)]} dt \right| + \left| \frac{J}{T} \int_0^T e^{-i[\Delta t + \gamma A(t)]} dt \right|. \quad (\text{A17})$$

We consider the same detuning $\Delta/2\pi = 50$ MHz as in the previous analysis and choose a nonmatched gate time $T_U = 3T_\Delta = 30$ ns. As shown in Fig. 19, $\epsilon^{\text{FM}(1)}$ exhibits local minima for different modulation cycle numbers N . For each N , we select the first local minimum to define the optimal modulation amplitudes $\gamma_{\text{opt}}^4/2\pi = 168.2$ MHz, $\gamma_{\text{opt}}^6/2\pi = 243.8$ MHz, and $\gamma_{\text{opt}}^8/2\pi = 322.9$ MHz.

These optimal amplitudes can be estimated from the phase factor in Eq. (A17). We rewrite the accumulated rotation angle as $\gamma A(t) = \frac{\gamma T}{\pi N} [1 - \cos(\frac{2\pi N}{T} t)]$ and define a dimensionless modulation strength $\beta \equiv \frac{\gamma T}{\pi N}$. The oscillatory factor in the first term of Eq. (A17) can then be written as

$$e^{i[\Delta t + \gamma A(t)]} = e^{i\beta} e^{i\Delta t} e^{-i\beta \cos(2\pi N t/T)}. \quad (\text{A18})$$

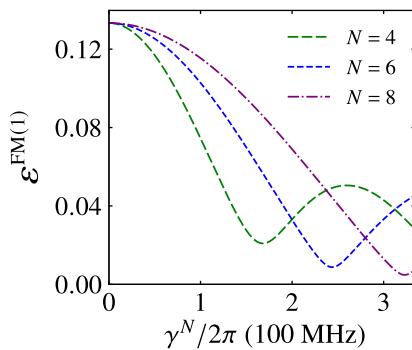


FIG. 19. First-order crosstalk error under FM, $\epsilon^{\text{FM}(1)}$, as a function of the modulation amplitude γ^N for different numbers of modulation cycles N at a nonmatched gate time. The chosen parameters are $T_U = 3T_\Delta = 30$ ns, $\Delta/2\pi = 50$ MHz, and $J/2\pi = 5$ MHz.

Using the Jacobi-Anger expansion,

$$e^{-i\beta \cos(2\pi N t/T)} = \sum_{\ell=-\infty}^{\infty} (-i)^\ell J_\ell(\beta) e^{i2\pi \ell N t/T}, \quad (\text{A19})$$

where $J_\ell(\beta)$ is the ℓ th Bessel function of the first kind.

The leading suppression is mainly governed by the $\ell = 0$ component, so the first strong suppression occurs when β is close to the first zero of $J_0(\beta)$, $\beta_0 \simeq 2.4048$. This gives the estimate

$$\frac{\gamma_{\text{opt}}^N}{2\pi} \simeq \frac{\beta_0 N}{2T}. \quad (\text{A20})$$

For $T_U = 30$ ns, Eq. (A20) gives $\gamma_{\text{opt}}^N/2\pi \simeq 160.3$, 240.5, and 320.6 MHz for $N = 4, 6$, and 8, respectively, in good agreement with the numerically optimized values 168.2, 243.8, and 322.9 MHz.

For the matched gate time $T_M = 20$ ns used in the main text, the same expression gives $\gamma_{\text{opt}}^N/2\pi \simeq 240.5$, 360.7, and 481.0 MHz for $N = 4, 6$, and 8, respectively. These values are not expected to exactly match those obtained by minimizing the second-order error quantities in Appendixes A 1–A 4, which involve more complicated time-ordered double integrals. Nevertheless, Eq. (A20) gives a similar order of magnitude and captures the increasing trend with N observed in the numerical

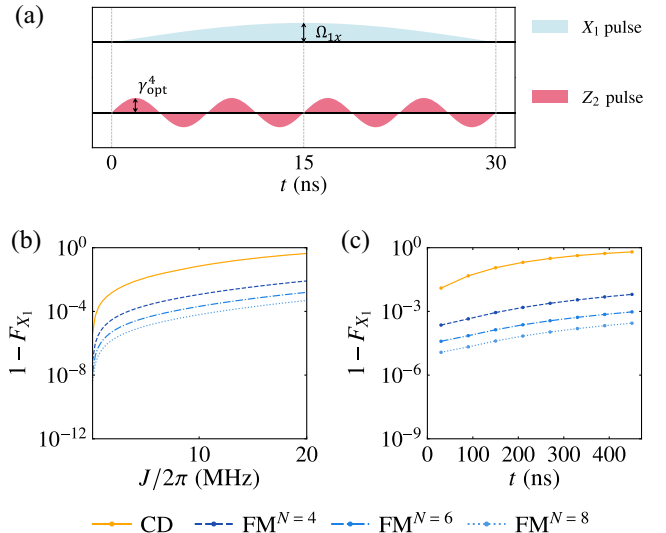


FIG. 20. Suppression of X_1 gate infidelity under FM at a nonmatched gate time. (a) Waveforms of the X_1 control pulses applied to Q_1 and the sinusoidal Z_2 drive with $N = 4$ modulation cycles applied to Q_2 . (b) Infidelity of X_1 gate operation as a function of the coupling strength J for different numbers of modulation cycles N . (c) Infidelity of consecutive X_1 gate operations as a function of time for different numbers of modulation cycles N . The chosen parameters are $T_U = 3T_\Delta = 30$ ns, $\Delta/2\pi = 50$ MHz, and $J/2\pi = 5$ MHz.

optimization, providing a useful estimate of the required hardware modulation amplitude.

Using the optimized modulation amplitudes γ_{opt}^N , we evaluate the performance of FM for a single-qubit X_1 gate at a nonmatched gate time in the presence of XY crosstalk. We consider the same form of drive used to generate the X_1 gate in the matched gate time case discussed in Sec. III C. The gate is generated by the drive Hamiltonian $\tilde{H}_{\text{drive}}^V(t) = \Omega_1(t)\hat{\sigma}_1^x$ with a sinusoidal envelope $\Omega_1(t) = \Omega_{1x} \sin(\frac{\pi}{T}t)$, where $\Omega_{1x} = \frac{\pi^2}{4T}$. An example of the corresponding control waveform for $N = 4$ is shown in Fig. 20(a).

The simulated gate infidelities as functions of the coupling strength are summarized in Fig. 20(b). At the nonmatched gate time $T_U = 30$ ns, FM reduces the X_1 gate infidelity by about 1.7 orders of magnitude compared with crosstalk dynamics for $J/2\pi \geq 5$ MHz, with larger modulation cycle numbers N leading to further suppression. For repeated operations, Fig. 20(c) shows that even after 15 consecutive X_1 gates, FM continues to suppress the accumulated infidelity by approximately 2 orders of magnitude relative to CD.

APPENDIX B: DETAILED ANALYSIS OF DYNAMICAL DECOUPLING

1. First-order XY crosstalk suppression condition

We derive the condition for suppressing the first-order XY crosstalk under a DD sequence. The first-order XY term $\tilde{H}_{XY}^{\text{DD}(1)}$, given in Eq. (34), is obtained from the Magnus expansion of the time evolution operator over the duration T as

$$\begin{aligned} \tilde{H}_{XY}^{\text{DD}(1)} &= \frac{1}{T} \sum_{s=1}^S \int_{(s-1)\tau}^{s\tau} (-1)^{s-1} \tilde{H}_{XY}(t) dt \\ &= \frac{1}{T} \sum_{s=1}^S \int_{(s-1)\tau}^{s\tau} (-1)^{s-1} J(e^{i\Delta t} \hat{\sigma}_1^+ \hat{\sigma}_2^- + \text{H.c.}) dt \\ &= \frac{J}{T} \sum_{s=1}^S (-1)^{s-1} \left(\frac{1}{i\Delta} e^{i\Delta t} \hat{\sigma}_1^+ \hat{\sigma}_2^- + \text{H.c.} \right) \Big|_{(s-1)\tau}^{s\tau}. \end{aligned} \quad (\text{B1})$$

The first-order crosstalk error is defined as the sum of the magnitudes of the coefficients preceding the first-order operators,

$$\epsilon^{\text{DD}(1)} \equiv 2 \left| \frac{J}{T} \sum_{s=1}^S \frac{1}{i\Delta} (-1)^{s-1} [e^{i\Delta s\tau} - e^{i\Delta(s-1)\tau}] \right|. \quad (\text{B2})$$

According to the periodicity of the phase factors $e^{\pm i\Delta t}$, the expression in Eq. (B2) vanishes for any even number of segments $S \geq 4$.

2. Second-order idle gate error

We first evaluate the second-order contribution induced by XY crosstalk during idle operations, which serves as a reference for understanding the suppression achieved by DD. Under CD, the second-order term in the Magnus expansion is

$$\tilde{H}_{I_1 I_2}^{\text{CD}(2)} = -\frac{i}{2T} \int_0^T dt_1 \int_0^{t_1} dt_2 [\tilde{H}_{XY}(t_1), \tilde{H}_{XY}(t_2)]. \quad (\text{B3})$$

The commutator of the XY interaction at two different times t_1 and t_2 reads

$$\begin{aligned} h_{I_1 I_2, \text{err}}^{\text{CD}(2)}(t_1, t_2) &\equiv [\tilde{H}_{XY}(t_1), \tilde{H}_{XY}(t_2)] \\ &= \frac{J^2}{2} [e^{i\Delta(t_1-t_2)} - e^{-i\Delta(t_1-t_2)}] (\hat{\sigma}_1^z \hat{I}_2 - \hat{I}_1 \hat{\sigma}_2^z). \end{aligned} \quad (\text{B4})$$

Substituting Eq. (B4) into Eq. (B3), we find that even at a matched gate time $T_M = 2T_\Delta = 2\pi/|\Delta|$, the second-order Magnus term under CD does not vanish and is given by

$$\tilde{H}_{I_1 I_2}^{\text{CD}(2)} = \frac{J^2}{2\Delta} (\hat{\sigma}_1^z \hat{I}_2 - \hat{I}_1 \hat{\sigma}_2^z). \quad (\text{B5})$$

We therefore identify Eq. (B5) as the residual second-order error induced by XY crosstalk under CD. The corresponding second-order idle gate error under CD is defined as the sum of the magnitudes of the coefficients of the second-order error terms,

$$\epsilon_{I_1 I_2}^{\text{CD}(2)} = 2 \left| \frac{J^2}{2\Delta} \right|. \quad (\text{B6})$$

We now turn to the evaluation of the second-order XY crosstalk error under DD in the idealized limit where the Z_2 pulses have zero width. During an idle period, the effective Hamiltonian defined in Eq. (32) reduces to

$$\tilde{H}_{\text{eff}}^{\text{DD}}(t) = f(t) \tilde{H}_{XY}(t), \quad (\text{B7})$$

where $f(t)$ is a sign function $f(t) = (-1)^{s-1}$ for $t \in [(s-1)\tau, s\tau]$ that captures the sign inversion of the XY interaction within even DD segments.

The second-order error integrand under DD is

$$h_{I_1 I_2, \text{err}}^{\text{DD}(2)}(t_1, t_2) \equiv f(t_1)f(t_2)[\tilde{H}_{XY}(t_1), \tilde{H}_{XY}(t_2)] = f(t_1)f(t_2)h_{I_1 I_2, \text{err}}^{\text{CD}(2)}(t_1, t_2) \\ = (-1)^{s_1+s_2} \frac{J^2}{2} (e^{i\Delta(t_1-t_2)} - e^{-i\Delta(t_1-t_2)}) (\hat{\sigma}_1^z \hat{I}_2 - \hat{I}_1 \hat{\sigma}_2^z). \quad (\text{B8})$$

For $t_1 \in [(s_1 - 1)\tau, s_1\tau]$ and $t_2 \in [(s_2 - 1)\tau, s_2\tau]$, the product $f(t_1)f(t_2) = (-1)^{s_1+s_2}$ encodes the relative sign of the XY interaction between the two DD segments. The second-order Magnus term under DD is then given by

$$\bar{H}_{I_1 I_2}^{\text{DD}(2)} = -\frac{i}{2T} \int_0^T dt_1 \int_0^{t_1} dt_2 h_{I_1 I_2, \text{err}}^{\text{DD}(2)}(t_1, t_2). \quad (\text{B9})$$

For the DD Z-4 sequence, the total gate time is divided into four equal segments. Evaluating the time-ordered double integral in Eq. (B9) by summing over all segment combinations, we obtain, at the matched gate time $T_M = 2T_\Delta$,

$$\bar{H}_{I_1 I_2}^{\text{DD}(2)} = \frac{\pi - 4}{2\pi} \frac{J^2}{\Delta} (\hat{\sigma}_1^z \hat{I}_2 - \hat{I}_1 \hat{\sigma}_2^z). \quad (\text{B10})$$

The corresponding second-order idle gate error under DD is therefore

$$\varepsilon_{I_1 I_2}^{\text{DD}(2)} = 2 \left| \frac{\pi - 4}{2\pi} \frac{J^2}{\Delta} \right|. \quad (\text{B11})$$

Although the second-order term under DD does not vanish, its magnitude is reduced compared with $\varepsilon_{I_1 I_2}^{\text{CD}(2)}$, indicating that the DD sequence suppresses the second-order contribution from XY crosstalk.

3. Second-order X_1 gate error

We now extend the second-order analysis to an active single-qubit operation by considering the X_1 gate under DD. As set up in Sec. IV C, the X_1 gate is decomposed into two $\sqrt{X_1}$ rotations applied in the odd intervals between adjacent Z_2 gates within the DD Z-4 sequence. For analytical calculations of errors, we consider the Z_2 gate as an ideal gate with zero width. According to Eq. (37) with $w = 0$ and $\tau = T_M/4$, the drive Hamiltonian is given by

$$\tilde{H}_{\text{drive}}^{Q_1}(t) = \Omega_1(t) \hat{\sigma}_1^x = \Omega_{1x} \cos \left(\frac{\pi}{\tau} \left[t - \left(s - \frac{1}{2} \right) \tau \right] \right) \hat{\sigma}_1^x, \\ t \in [(s-1)\tau, s\tau], \quad s \in \{1, 3\}, \quad (\text{B12})$$

and vanishes otherwise. The driving pulse amplitude is $\Omega_{1x} = \frac{\pi^2}{8\tau}$.

Under CD, the second-order term in the Magnus expansion of the X_1 gate evolution is given by

$$\bar{H}_{X_1}^{\text{CD}(2)} = -\frac{i}{2T} \int_0^T dt_1 \int_0^{t_1} dt_2 [\tilde{H}^{\text{CD}}(t_1), \tilde{H}^{\text{CD}}(t_2)], \quad (\text{B13})$$

where $\tilde{H}^{\text{CD}}(t) = \tilde{H}_{XY}(t) + \tilde{H}_{\text{drive}}^{Q_1}(t)$. The commutator can be expanded as

$$[\tilde{H}^{\text{CD}}(t_1), \tilde{H}^{\text{CD}}(t_2)] = [\tilde{H}_{XY}(t_1), \tilde{H}_{XY}(t_2)] \\ + [\tilde{H}_{\text{drive}}^{Q_1}(t_1), \tilde{H}_{\text{drive}}^{Q_1}(t_2)] \\ + [\tilde{H}_{XY}(t_1), \tilde{H}_{\text{drive}}^{Q_1}(t_2)] \\ + [\tilde{H}_{\text{drive}}^{Q_1}(t_1), \tilde{H}_{XY}(t_2)]. \quad (\text{B14})$$

The first term is the XY - XY commutator already evaluated in Eq. (B4). The second component corresponds to the ideal drive term and does not contribute to crosstalk-induced errors.

We therefore collect the XY - XY term and the mixed XY -drive terms into the crosstalk-induced second-order integrand,

$$h_{X_1, \text{err}}^{\text{CD}(2)}(t_1, t_2) \equiv h_{I_1 I_2, \text{err}}^{\text{CD}(2)}(t_1, t_2) + [\tilde{H}_{XY}(t_1), \tilde{H}_{\text{drive}}^{Q_1}(t_2)] \\ + [\tilde{H}_{\text{drive}}^{Q_1}(t_1), \tilde{H}_{XY}(t_2)]. \quad (\text{B15})$$

The mixed XY -drive commutators are

$$[\tilde{H}_{XY}(t_1), \tilde{H}_{\text{drive}}^{Q_1}(t_2)] + [\tilde{H}_{\text{drive}}^{Q_1}(t_1), \tilde{H}_{XY}(t_2)] \\ = (-\Omega_1(t_1) J e^{i\Delta t_2} + \Omega_1(t_2) J e^{i\Delta t_1}) \hat{\sigma}_1^z \hat{\sigma}_2^- \\ + (\Omega_1(t_1) J e^{-i\Delta t_2} - \Omega_1(t_2) J e^{-i\Delta t_1}) \hat{\sigma}_1^z \hat{\sigma}_2^+ \\ = -J E_{X_1}^{\text{CD}(2)}(t_1, t_2) \hat{\sigma}_1^z \hat{\sigma}_2^- + J (E_{X_1}^{\text{CD}(2)}(t_1, t_2))^* \hat{\sigma}_1^z \hat{\sigma}_2^+, \quad (\text{B16})$$

where

$$E_{X_1}^{\text{CD}(2)}(t_1, t_2) \equiv \Omega_1(t_1) e^{i\Delta t_2} - \Omega_1(t_2) e^{i\Delta t_1}. \quad (\text{B17})$$

The corresponding second-order error Hamiltonian from the Magnus expansion is

$$\begin{aligned}\bar{H}_{X_1, \text{err}}^{\text{CD}(2)} &= -\frac{i}{2T} \int_0^T dt_1 \int_0^{t_1} dt_2 h_{X_1, \text{err}}^{\text{CD}(2)}(t_1, t_2) \\ &= \frac{J^2}{2\Delta} (\hat{\sigma}_1^z \hat{I}_2 - \hat{I}_1 \hat{\sigma}_2^z) - \frac{J}{4} (\hat{\sigma}_1^z \hat{\sigma}_2^- + \hat{\sigma}_1^- \hat{\sigma}_2^+). \quad (\text{B18})\end{aligned}$$

The first term arises from the idle XY crosstalk contribution, as shown in Eq. (B5), while the second part originates from the commutator between the XY crosstalk and the drive. Accordingly, the second-order error $\epsilon_{X_1}^{\text{CD}(2)}$ of the CD case is quantified as the sum of the magnitudes of the coefficients of all second-order error terms,

$$\epsilon_{X_1}^{\text{CD}(2)} = 2 \left| \frac{J^2}{2\Delta} \right| + 2 \left| \frac{J}{4} \right|. \quad (\text{B19})$$

We now turn to the evaluation of the second-order contribution under DD. During the X_1 gate under DD Z-4 sequence, the effective Hamiltonian takes the form given in Eq. (32),

$$\tilde{H}_{\text{eff}}^{\text{DD}}(t) = f(t) \tilde{H}_{XY}(t) + \tilde{H}_{\text{drive}}^{Q_1}(t), \quad (\text{B20})$$

where $\tilde{H}_{\text{drive}}^{Q_1}(t)$ is defined in Eq. (B12) and vanishes outside the odd segments, with $f(t) = (-1)^{s-1}$ for $t \in [(s-1)\tau, s\tau]$.

Using the same separation as in the CD case, the crosstalk-induced second-order integrand under DD is

$$\begin{aligned}h_{X_1, \text{err}}^{\text{DD}(2)}(t_1, t_2) &\equiv h_{I_1 I_2, \text{err}}^{\text{DD}(2)}(t_1, t_2) \\ &+ f(t_1) [\tilde{H}_{XY}(t_1), \tilde{H}_{\text{drive}}^{Q_1}(t_2)] \\ &+ f(t_2) [\tilde{H}_{\text{drive}}^{Q_1}(t_1), \tilde{H}_{XY}(t_2)]. \quad (\text{B21})\end{aligned}$$

The first term is the DD-modified idle contribution already evaluated in Eq. (B11), while the remaining two terms are the mixed XY-drive commutators.

Evaluating the corresponding time-ordered integral at the matched gate time $T_M = 2T_\Delta = 2\pi/|\Delta|$, we obtain the crosstalk-induced second-order Magnus error Hamiltonian

$$\begin{aligned}\bar{H}_{X_1, \text{err}}^{\text{DD}(2)} &= -\frac{i}{2T} \int_0^T dt_1 \int_0^{t_1} dt_2 h_{X_1, \text{err}}^{\text{DD}(2)}(t_1, t_2) \\ &= \frac{\pi - 4J^2}{2\pi\Delta} (\hat{\sigma}_1^z \hat{I}_2 - \hat{I}_1 \hat{\sigma}_2^z) + \frac{iJ}{4} (\hat{\sigma}_1^z \hat{\sigma}_2^- - \hat{\sigma}_1^- \hat{\sigma}_2^+).\end{aligned}$$

The first term originates from the DD-modified idle crosstalk contribution, as shown in Eq. (B10), while the second term arises from the commutator between the XY interaction and the drive.

Accordingly, the second-order X_1 gate error under DD is defined as the sum of the magnitudes of the coefficients of all second-order error terms,

$$\epsilon_{X_1}^{\text{DD}(2)} = 2 \left| \frac{\pi - 4J^2}{2\pi\Delta} \right| + 2 \left| \frac{J}{4} \right|. \quad (\text{B22})$$

Although the second-order contribution does not vanish, DD reduces its magnitude, indicating suppression of XY-crosstalk-induced errors during the X_1 gate operation.

4. Single-site scheme of dynamical decoupling

We next consider the case where both the control drive and the DD Z-4 sequence are applied to the same qubit, Q_2 , enabling the simultaneous application of single-qubit operations and suppression of crosstalk on a single site. The single-qubit X_2 gate is realized through a drive Hamiltonian of the form

$$\begin{aligned}\tilde{H}_{\text{drive}}^{Q_2}(t) &= \Omega_{2x}(w) \cos\left(\frac{\pi}{\tau - w} \left[t - \left(s - \frac{1}{2}\right)\tau\right]\right) \hat{\sigma}_2^x, \\ t &\in \left[(s-1)\tau + \frac{w}{2}, s\tau - \frac{w}{2}\right], \quad s \in \text{odd}, \quad (\text{B23})\end{aligned}$$

where the driving pulse amplitude is $\Omega_{2x}(w) = \frac{\pi^2}{8(\tau-w)}$, and w denotes the pulse width of the Z_2 gate. The driving pulse shape is identical to that in Eq. (37). As a result, the second-order crosstalk error $\epsilon_{X_2}^{\text{DD}(2)}$ is identical to $\epsilon_{X_1}^{\text{DD}(2)}$, since the same DD-based driving protocol is applied to Q_2 . An example of the corresponding waveforms with $w = \tau/4$ is shown in Fig. 21(a).

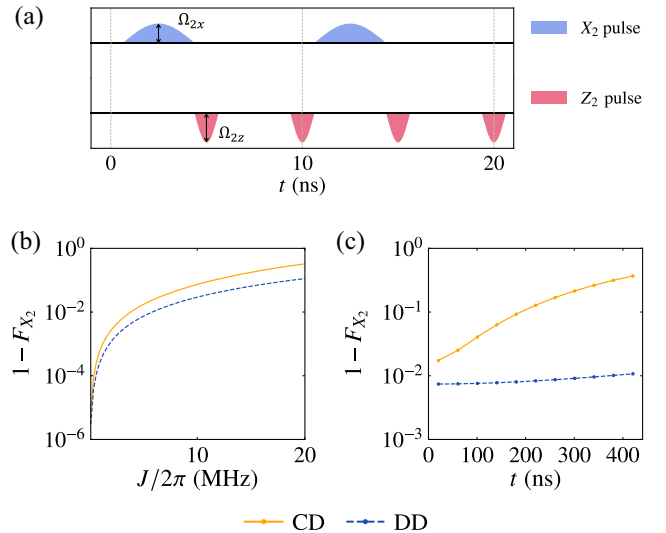


FIG. 21. Suppression of X_2 gate infidelity under single-site DD. (a) Waveforms of the X_2 control pulses (generated by two $\sqrt{X_2}$ pulses) and the DD Z-4 sequence containing four Z_2 gates applied to Q_2 . (b) Infidelity of X_2 gate operation as a function of the coupling strength J under DD Z-4. (c) Infidelity of consecutive X_2 gate operations as a function of time under DD Z-4, with $J/2\pi = 5$ MHz.

For $\Delta/2\pi = 50$ MHz and a matched gate time of $T_M = 20$ ns, as shown in Fig. 21(b), the DD sequence reduces the X_2 gate infidelity by approximately 0.37 orders of magnitude compared with the CD case. Under repeated gate operations, the suppression effect remains stable: after 21 consecutive X_2 gates, the infidelity remains more than 1 order of magnitude lower than that without DD, as shown in Fig. 21(c). These results are consistent with those presented in Fig. 6, confirming that the control drive and the DD Z-4 sequence can be applied to the same qubit. This establishes the feasibility of integrating active gate control and DD on a single qubit to suppress crosstalk-induced errors.

5. Parallel X_1X_2 operations under dynamical decoupling

Building upon the previous results on XY -crosstalk suppression during X_1 and X_2 gates under the DD Z-4 sequence, we extend our analysis to the case where parallel X_1X_2 gates are applied simultaneously to both qubits. In this configuration, the X_1 gate is applied to Q_1 , while the DD Z-4 sequence together with the X_2 gate is applied to Q_2 , as illustrated in Fig. 22(a). We use the same drive forms for the X_1 and X_2 gates as defined in Eqs. (B12) and (B23), respectively, with $w = 0$ and $\tau = T_M/4$.

Using the second-order errors obtained in Eqs. (B11) and (B22), together with $\epsilon_{X_2}^{\text{DD}(2)} = \epsilon_{X_1}^{\text{DD}(2)}$ from the previous

subsection, the second-order crosstalk error under DD during the parallel operation is evaluated as

$$\epsilon_{X_1X_2}^{\text{DD}(2)} = \epsilon_{X_1}^{\text{DD}(2)} + \epsilon_{X_2}^{\text{DD}(2)} - \epsilon_{I_1I_2}^{\text{DD}(2)} = 2 \left| \frac{\pi - 4J^2}{2\pi\Delta} \right| + 4 \left| \frac{J}{4} \right|. \quad (\text{B24})$$

Here, $\epsilon_{X_1}^{\text{DD}(2)}$ and $\epsilon_{X_2}^{\text{DD}(2)}$ each contain the common idle XY - XY contribution in addition to the corresponding mixed XY -drive contribution. We therefore subtract $\epsilon_{I_1I_2}^{\text{DD}(2)}$ once to avoid double counting.

Similarly, in the absence of DD, the corresponding second-order error under CD, $\epsilon_{X_1X_2}^{\text{CD}(2)}$, is given by

$$\epsilon_{X_1X_2}^{\text{CD}(2)} = \epsilon_{X_1}^{\text{CD}(2)} + \epsilon_{X_2}^{\text{CD}(2)} - \epsilon_{I_1I_2}^{\text{CD}(2)} = 2 \left| \frac{J^2}{2\Delta} \right| + 4 \left| \frac{J}{4} \right|. \quad (\text{B25})$$

A direct comparison shows that $\epsilon_{X_1X_2}^{\text{DD}(2)} < \epsilon_{X_1X_2}^{\text{CD}(2)}$, confirming that the DD sequence effectively suppresses XY crosstalk even during simultaneous parallel operations.

For the parallel X_1X_2 gate configuration, the dependence of gate infidelity on the coupling strength J is shown in Fig. 22(b). When $J/2\pi \geq 5$ MHz, the application of DD reduces the gate infidelity by approximately 1.2 orders of magnitude. This suppression is stronger than that observed for a single X_1 gate applied to Q_1 alone [see Fig. 6(b)].

We further examine the performance of DD under repeated parallel gate operations. As shown in Fig. 22(c), odd numbers of parallel X_1X_2 gate sets are applied. At early times, the infidelity is reduced by more than 1 order of magnitude. However, in contrast to the single- X -gate case [Fig. 6(c)], DD does not effectively suppress the subsequent growth of infidelity under repeated parallel X_1X_2 operations. After up to 21 gate sets, the net reduction in infidelity is limited to approximately 0.28 orders of magnitude.

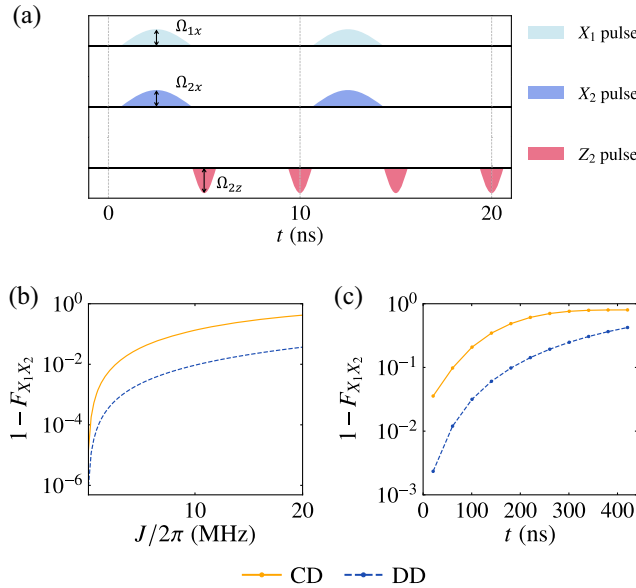


FIG. 22. Suppression of parallel X_1X_2 gate infidelity under DD. (a) Waveforms of the parallel X_1 and X_2 control pulses (each generated by two $\sqrt{X}$ pulses) and the DD Z-4 sequence containing four Z_2 gates applied to Q_2 . (b) Infidelity of parallel X_1X_2 gate operation as a function of the coupling strength J under DD Z-4. (c) Infidelity of consecutive parallel X_1X_2 gate operations as a function of time under DD Z-4, with $J/2\pi = 5$ MHz.

APPENDIX C: RESIDUAL ZZ INTERACTION FROM NONCOMPUTATIONAL STATES

1. Anharmonic-oscillator model

In the main text, we focus on the XY exchange interaction within the computational subspace. When higher excited levels are included, virtual transitions through the noncomputational states also induce an additional residual ZZ interaction. In this appendix, we show how this contribution arises from the same exchange coupling between two anharmonic oscillators.

Consider two superconducting qubits modeled as anharmonic oscillators with exchange coupling, without applying any control drive. The Hamiltonian is written as

$$\hat{H}_{\text{anh}} = \hat{H}_{0,\text{anh}} + \hat{H}_J, \quad (\text{C1})$$

where the bare two-qubit Hamiltonian is

$$\hat{H}_{0,\text{anh}} = \sum_{i=1}^2 \left(\omega_i \hat{a}_i^\dagger \hat{a}_i + \frac{\alpha_i}{2} \hat{a}_i^\dagger \hat{a}_i^\dagger \hat{a}_i \hat{a}_i \right), \quad (\text{C2})$$

and the exchange coupling term is given by

$$\hat{H}_J = J(\hat{a}_1^\dagger \hat{a}_2 + \hat{a}_1 \hat{a}_2^\dagger). \quad (\text{C3})$$

Here ω_i and α_i are the frequency and anharmonicity of qubit Q_i , and $\hat{a}_i$ is the annihilation operator of the corresponding anharmonic oscillator.

For the FM protocol, we add the Z_2 modulation term,

$$\hat{H}_{\text{anh}}^{\text{FM}}(t) = \hat{H}_{\text{anh}} + \hat{H}_{Z_2,\text{anh}}^{\text{FM}}(t), \quad (\text{C4})$$

where the Z_2 modulation Hamiltonian is

$$\hat{H}_{Z_2,\text{anh}}^{\text{FM}}(t) = -2\gamma \sin\left(\frac{2\pi N}{T}t\right) \hat{a}_2^\dagger \hat{a}_2. \quad (\text{C5})$$

Using a similar modulated-frame transformation as in Eq. (13), we define

$$\hat{\mathcal{V}}(t) = \exp\{-i(\omega_1 t \hat{a}_1^\dagger \hat{a}_1 + [\omega_2 t - \gamma A(t)] \hat{a}_2^\dagger \hat{a}_2)\}, \quad (\text{C6})$$

where $A(t) = \frac{2T}{\pi N} \sin^2\left(\frac{\pi N}{T}t\right)$ as defined before. The Hamiltonian in the modulated frame is then given by

$$\begin{aligned} \tilde{H}_{\text{anh}}^{\mathcal{V}}(t) &= \hat{\mathcal{V}}^\dagger(t) \hat{H}_{\text{anh}}^{\text{FM}}(t) \hat{\mathcal{V}}(t) + i\dot{\hat{\mathcal{V}}}^\dagger(t) \hat{\mathcal{V}}(t) \\ &= \hat{H}_{0,\text{anh}}^{\mathcal{V}} + \hat{H}_J^{\mathcal{V}}(t). \end{aligned} \quad (\text{C7})$$

The first term is diagonal in the number-state basis,

$$\begin{aligned} \hat{H}_{0,\text{anh}}^{\mathcal{V}} &= \frac{\alpha_1}{2} \hat{a}_1^\dagger \hat{a}_1^\dagger \hat{a}_1 \hat{a}_1 + \frac{\alpha_2}{2} \hat{a}_2^\dagger \hat{a}_2^\dagger \hat{a}_2 \hat{a}_2 \\ &= \sum_{n,m} E_{nm} |n, m\rangle \langle n, m|, \end{aligned} \quad (\text{C8})$$

where n and m denote the excitation numbers of Q_1 and Q_2 , respectively, and

$$E_{nm} = \frac{\alpha_1}{2} n(n-1) + \frac{\alpha_2}{2} m(m-1). \quad (\text{C9})$$

The exchange coupling term in the modulated frame is

$$\hat{H}_J^{\mathcal{V}}(t) = \hat{W}_{00}(t) + \sum_{n,m} \hat{W}_{nm}(t), \quad (\text{C10})$$

where the prime indicates that the computational exchange channel $(n, m) = (0, 0)$ is excluded, and

$$\phi(t) = \Delta t + \gamma A(t),$$

$$\begin{aligned} \hat{T}_{nm} &= |n+1\rangle \langle n| \otimes |m\rangle \langle m+1|, \\ \hat{W}_{nm}(t) &= J_{nm} [e^{i\phi(t)} \hat{T}_{nm} + e^{-i\phi(t)} \hat{T}_{nm}^\dagger], \\ J_{nm} &= J \sqrt{(n+1)(m+1)}. \end{aligned} \quad (\text{C11})$$

The first term in Eq. (C10) is the exchange coupling within the computational basis,

$$\hat{W}_{00}(t) = J[e^{i\phi(t)} |1\rangle \langle 0| \otimes |0\rangle \langle 1| + e^{-i\phi(t)} |0\rangle \langle 1| \otimes |1\rangle \langle 0|], \quad (\text{C12})$$

which corresponds to the XY interaction analyzed in the main text.

2. Time-dependent Schrieffer-Wolff transformation

Exchange couplings involving noncomputational states can be perturbatively eliminated by a Schrieffer-Wolff (SW) transformation [49] generated by

$$\hat{\mathcal{S}}(t) = \sum_{n,m} \beta_{nm}(t) e^{i\phi(t)} \hat{T}_{nm}, \quad (\text{C13})$$

where

$$\beta_{nm}(t) = \frac{-J_{nm}}{E_{n,m+1} - E_{n+1,m} - \dot{\phi}(t)}. \quad (\text{C14})$$

The channel $(n, m) = (0, 0)$ is not included in $\hat{\mathcal{S}}(t)$ because it belongs to the computational subspace and is kept as the XY interaction.

We now apply the time-dependent SW transformation to $\tilde{H}_{\text{anh}}^{\mathcal{V}}(t)$. With $\hat{G}(t) = \hat{\mathcal{S}}(t) - \hat{\mathcal{S}}^\dagger(t)$, the transformed Hamiltonian is defined as

$$\hat{H}'_{\text{anh}}(t) = e^{\hat{G}(t)} \tilde{H}_{\text{anh}}^{\mathcal{V}}(t) e^{-\hat{G}(t)} + i \frac{d}{dt} [e^{\hat{G}(t)}] e^{-\hat{G}(t)}. \quad (\text{C15})$$

Up to second order in the exchange coupling, this gives

$$\begin{aligned} \hat{H}'_{\text{anh}}(t) &= \tilde{H}_{\text{anh}}^{\mathcal{V}}(t) + [\hat{G}(t), \tilde{H}_{\text{anh}}^{\mathcal{V}}(t)] \\ &\quad + \frac{1}{2} [\hat{G}(t), [\hat{G}(t), \tilde{H}_{\text{anh}}^{\mathcal{V}}(t)]] + i\dot{\hat{G}}(t) \\ &\quad + \frac{i}{2} [\hat{G}(t), \dot{\hat{G}}(t)] + \dots \end{aligned} \quad (\text{C16})$$

Given $\tilde{H}_{\text{anh}}^{\mathcal{V}}(t) = \hat{H}_{0,\text{anh}}^{\mathcal{V}} + \hat{H}_J^{\mathcal{V}}(t)$, with $\hat{H}_{0,\text{anh}}^{\mathcal{V}} = O(1)$, $\hat{H}_J^{\mathcal{V}}(t) = O(J)$, and $\hat{G}(t) = O(J)$, the first-order contribution in Eq. (C16) is

$$\begin{aligned}
\hat{H}'_{\text{anh}}(t) &= \hat{H}_J^\nu(t) + [\hat{G}(t), \hat{H}_{0,\text{anh}}^\nu] + i\dot{\hat{G}}(t) \\
&= \hat{W}_{00}(t) + \sum_{n,m} \hat{W}_{nm}(t) + [\hat{G}(t), \hat{H}_{0,\text{anh}}^\nu] + i\dot{\hat{G}}(t) \\
&\simeq \sum_{n,m} \{J_{nm} e^{i\phi(t)} \hat{T}_{nm} + [\beta_{nm}(t) e^{i\phi(t)} \hat{T}_{nm}, \hat{H}_{0,\text{anh}}^\nu] \\
&\quad - \beta_{nm}(t) \dot{\phi}(t) e^{i\phi(t)} \hat{T}_{nm} + \text{H.c.}\} + \hat{W}_{00}(t).
\end{aligned} \tag{C17}$$

In the last approximation, we have dropped the terms proportional to $\beta_{nm}(t)$, which arise from the explicit time dependence of the SW coefficient. These terms give only a small correction to the leading SW result in the off-resonant regime considered here, where $|E_{n,m+1} - E_{n+1,m} - \dot{\phi}(t)|$ remains large compared with the associated exchange coupling strength J_{nm} .

Using the commutation relation

$$[\hat{T}_{nm}, \hat{H}_{0,\text{anh}}^\nu] = (E_{n,m+1} - E_{n+1,m}) \hat{T}_{nm}, \tag{C18}$$

the contribution from the channels included in $\hat{S}(t)$ is

$$\begin{aligned}
\hat{H}'_{\text{anh}}(t) - \hat{W}_{00}(t) \\
\simeq \sum_{n,m} [C_{nm}^{(1)}(t) e^{i\phi(t)} \hat{T}_{nm} + C_{nm}^{(1)*}(t) e^{-i\phi(t)} \hat{T}_{nm}^\dagger],
\end{aligned} \tag{C19}$$

where

$$C_{nm}^{(1)}(t) = J_{nm} + \beta_{nm}(t)(E_{n,m+1} - E_{n+1,m} - \dot{\phi}(t)). \tag{C20}$$

With the choice of $\beta_{nm}(t)$ in Eq. (C14), we have

$$C_{nm}^{(1)}(t) = 0, (n, m) \neq (0, 0), \tag{C21}$$

leading to the first-order cancellation condition for exchange couplings involving noncomputational states,

$$\sum_{n,m} \hat{W}_{nm}(t) + [\hat{G}(t), \hat{H}_{0,\text{anh}}^\nu] + i\dot{\hat{G}}(t) \simeq 0. \tag{C22}$$

Thus, the remaining first-order term is the residual XY interaction analyzed in the main text,

$$\hat{H}'_{\text{anh}}(t) = \hat{W}_{00}(t). \tag{C23}$$

The contribution from the channels included in $\hat{S}(t)$ next appears at second order in J . From Eq. (C16), the full second-order contribution is

$$\begin{aligned}
\hat{H}'_{\text{anh}}(2)(t) &= [\hat{G}(t), \hat{H}_J^\nu(t)] + \frac{1}{2} [\hat{G}(t), [\hat{G}(t), \hat{H}_{0,\text{anh}}^\nu]] \\
&\quad + \frac{i}{2} [\hat{G}(t), \dot{\hat{G}}(t)].
\end{aligned} \tag{C24}$$

Reordering the cancellation condition Eq. (C22),

$$[\hat{G}(t), \hat{H}_{0,\text{anh}}^\nu] + i\dot{\hat{G}}(t) \simeq - \sum_{n,m} \hat{W}_{nm}(t), \tag{C25}$$

the last two terms in Eq. (C24) become

$$\begin{aligned}
&\frac{1}{2} [\hat{G}(t), [\hat{G}(t), \hat{H}_{0,\text{anh}}^\nu]] + \frac{i}{2} [\hat{G}(t), \dot{\hat{G}}(t)] \\
&= \frac{1}{2} [\hat{G}(t), [\hat{G}(t), \hat{H}_{0,\text{anh}}^\nu] + i\dot{\hat{G}}(t)] \\
&= -\frac{1}{2} \left[\hat{G}(t), \sum_{n,m} \hat{W}_{nm}(t) \right].
\end{aligned} \tag{C26}$$

Substituting this into Eq. (C24), we obtain

$$\hat{H}'_{\text{anh}}(2)(t) = [\hat{G}(t), \hat{W}_{00}(t)] + \frac{1}{2} \left[\hat{G}(t), \sum_{n,m} \hat{W}_{nm}(t) \right]. \tag{C27}$$

3. ZZ interaction in the computational subspace

We now project the second-order Hamiltonian onto the computational subspace of the qubits using the projection operator,

$$\hat{P}_Q = \sum_{q_1, q_2=0,1} |q_1, q_2\rangle \langle q_1, q_2|. \tag{C28}$$

The commutator $[\hat{G}(t), \hat{W}_{00}(t)]$ vanishes after projection because $\hat{G}(t)$ connects the qubit subspace to noncomputational states, whereas $\hat{W}_{00}(t)$ acts only within the computational exchange channel $|01\rangle \leftrightarrow |10\rangle$. The projected second-order correction therefore comes from the primed-channel commutators, among which only $(n, m) = (1, 0)$ and $(n, m) = (0, 1)$ generate a $|11\rangle$ energy shift. Thus, we are left with

$$\begin{aligned}
&\hat{P}_Q \hat{H}'_{\text{anh}}(2)(t) \hat{P}_Q \\
&= \hat{P}_Q \{ \beta_{10}(t) J_{10} [\hat{T}_{10}, \hat{T}_{10}^\dagger] + \beta_{01}(t) J_{01} [\hat{T}_{01}, \hat{T}_{01}^\dagger] \} \hat{P}_Q.
\end{aligned} \tag{C29}$$

Using Eq. (C14), the two contributing terms are

$$\begin{aligned}
\beta_{10}(t) J_{10} [\hat{T}_{10}, \hat{T}_{10}^\dagger] &= \frac{2J^2}{\Delta + \gamma \dot{A}(t) + \alpha_1} (|20\rangle \langle 20| - |11\rangle \langle 11|), \\
\beta_{01}(t) J_{01} [\hat{T}_{01}, \hat{T}_{01}^\dagger] &= \frac{2J^2}{\Delta + \gamma \dot{A}(t) - \alpha_2} (|11\rangle \langle 11| - |02\rangle \langle 02|).
\end{aligned} \tag{C30}$$

After applying $\hat{P}_Q$, the states $|20\rangle$ and $|02\rangle$ are removed. The second-order effective correction in the qubit subspace is thus given by

$$\hat{H}_{\text{eff},Q}^{(2)}(t) \equiv \hat{P}_Q \hat{H}_{\text{anh}}^{(2)}(t) \hat{P}_Q = \zeta_{\text{FM}}(t) |11\rangle\langle 11|, \quad (\text{C31})$$

where

$$\zeta_{\text{FM}}(t) = \frac{2J^2}{\Delta + \gamma\dot{A}(t) - \alpha_2} - \frac{2J^2}{\Delta + \gamma\dot{A}(t) + \alpha_1}. \quad (\text{C32})$$

Expressing $|11\rangle\langle 11|$ in terms of Pauli matrices,

$$|11\rangle\langle 11| = \frac{1}{4}(\hat{I}_1\hat{I}_2 - \hat{\sigma}_1^z\hat{I}_2 - \hat{I}_1\hat{\sigma}_2^z + \hat{\sigma}_1^z\hat{\sigma}_2^z), \quad (\text{C33})$$

we obtain

$$\hat{H}_{\text{eff},Q}^{(2)}(t) = \frac{\zeta_{\text{FM}}(t)}{4}(\hat{I}_1\hat{I}_2 - \hat{\sigma}_1^z\hat{I}_2 - \hat{I}_1\hat{\sigma}_2^z + \hat{\sigma}_1^z\hat{\sigma}_2^z). \quad (\text{C34})$$

The first term is an overall energy shift, the middle two terms are single-qubit frequency shifts, and the last term is the residual ZZ crosstalk,

$$\hat{H}_{\text{ZZ,FM}}(t) = \frac{\zeta_{\text{FM}}(t)}{4}\hat{\sigma}_1^z\hat{\sigma}_2^z. \quad (\text{C35})$$

After applying the time-dependent SW transformation and projecting onto the computational subspace, the resulting Hamiltonian in the modulated frame is, up to second order in J ,

$$\begin{aligned} \hat{P}_Q \hat{H}_{\text{anh}}^{(2)}(t) \hat{P}_Q &= J(e^{i[\Delta t + \gamma A(t)]}\hat{\sigma}_1^+\hat{\sigma}_2^- + e^{-i[\Delta t + \gamma A(t)]}\hat{\sigma}_1^-\hat{\sigma}_2^+) \\ &= +\frac{\zeta_{\text{FM}}(t)}{4}(\hat{I}_1\hat{I}_2 - \hat{\sigma}_1^z\hat{I}_2 - \hat{I}_1\hat{\sigma}_2^z + \hat{\sigma}_1^z\hat{\sigma}_2^z) \\ &= \hat{H}_{\text{XY}}^V(t) + \hat{H}_{\text{eff},Q}^{(2)}(t). \end{aligned} \quad (\text{C36})$$

Here, the first term is the XY crosstalk analyzed in the main text [Eq. (15)], and the second term is the correction induced by virtual transitions through higher excited levels. We note that, in conventional ZZ analyses, $\hat{H}_{\text{XY}}$ is often eliminated perturbatively, producing local $\hat{\sigma}_1^z\hat{I}_2$ and $\hat{I}_1\hat{\sigma}_2^z$ frequency shifts of order J^2/Δ [50].

From Eq. (11), the applied Z_2 modulation shifts the transition frequency of Q_2 by

$$\delta\omega_2(t) = -2\gamma \sin\left(\frac{2\pi N}{T}t\right). \quad (\text{C37})$$

Since $\Delta = \omega_1 - \omega_2$, the instantaneous detuning is

$$\begin{aligned} \Delta(t) &= \omega_1 - [\omega_2 + \delta\omega_2(t)] \\ &= \Delta - \delta\omega_2(t) = \Delta + \gamma\dot{A}(t) = \dot{\phi}(t). \end{aligned} \quad (\text{C38})$$

The ZZ coupling strength can therefore be expressed in terms of the instantaneous detuning as

$$\zeta_{\text{FM}}(t) = \frac{2J^2}{\Delta(t) - \alpha_2} - \frac{2J^2}{\Delta(t) + \alpha_1}. \quad (\text{C39})$$

This expression has the same structure as the residual ZZ interaction generated by direct capacitive coupling between weakly anharmonic qubits [50], with the static detuning replaced by the instantaneous detuning $\Delta(t)$. For protocols without frequency modulation, such as DD, the detuning remains static and the expression reduces to the standard result with $\Delta(t) = \Delta$.

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