

Exploring weak turbulence of phonon and magnon beams in magnetoacoustic ultrathin films

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This study presents a simple theoretical model describing narrow envelope surface acoustic waves (phonons) and spin waves (magnons) in an ultrathin ferromagnetic film. Based on the general principles of weak wave turbulence, the model considers interactions between beams of an ideal phonon gas and a weakly nonideal magnon gas, which represent magnetoacoustic oscillations in the system. Equations for the wave envelopes of phonons and magnons, along with their harmonics, are derived, incorporating nonlinear effects from three- and four-particle interactions. In the general nonresonant case, linear stationary envelope simulations are sufficient. These clarify the experimentally observed angular dependence of the transmitted acoustic signal with respect to the orientation of the magnetic field. The study highlights increased energy losses associated with enhanced magnetoacoustic coupling. Given the broad interdisciplinary interest in weak turbulence phenomena within condensed-matter physics and nonlinear wave dynamics, our model offers significant predictive capabilities and greatly simplifies calculations of quasiparticle beam interactions.

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I. INTRODUCTION

Acoustic-spintronic devices, enabled by magnetoelastic coupling between surface acoustic waves and spin waves, offer a compact, energy-efficient platform for advanced rf and logic applications [1–3]. These systems support highly sensitive magnetic and strain sensors, tunable signal processors, and spin-wave logic architectures with low power consumption and parallelism potential. Their integration into next-generation electronics promises significant reductions in size, weight, and power, while opening avenues for hybrid classical-quantum and neuromorphic computing.

The study of surface acoustic waves in ultrathin magnetic films is of interest for fundamental and applied science [4–9]. On the one hand, we have a linear wave system of Rayleigh surface acoustic waves (phonons). On the other hand, this system consists of spin waves (magnons), in which small deviations of magnetization lead to nonlinear effects. It is evident that the magnetoelastic interaction,

which connects magnetic deviations with elastic deformations, is most pronounced at and near the intersection of the phonon and magnon spectra. Linear excitation of spin waves by surface sound near the intersection of their spectra is usually called acoustically driven ferromagnetic resonance. It should be noted, however, that magnetoelastic effects are experimentally observed for surface acoustic waves when the frequencies of phonons and magnons at a given wave vector differ significantly. We clarify this effect in this paper. Thus, we have a rich nonlinear wave system that can be excited by both acoustic and radio-frequency fields and controlled by an external magnetic field.

The most common method for describing interacting elastic and spin waves is direct computer modeling. The sample is divided into many cells. Each cell utilizes the Landau-Lifshitz-Gilbert equation, which determines the local oscillations of magnetization in response to effective fields from both the magnetic and elastic systems. Local elastic oscillations of the film are described by oscillations of the strain tensor, considering the stresses from the magnetic system. This approach may seem quite comprehensive, but this is a misconception, since the problem necessarily considers phenomenological attenuation, in which the processes of energy redistribution in the thermostat are already averaged. Thus, the microscopic dynamics become truncated. With this approach, by fitting several phenomenological parameters to the problem,

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it is potentially possible to align the simulation results with the experimental data. However, the predictive part of such a theory is minimal. The disadvantages of such modeling include high requirements for computing power and computing time. The whole point is that the dynamics of all degrees of freedom of the system are calculated, most of which do not play a noticeable role in the process of interaction of phonons and magnons in a narrow region of the spectrum.

Since the most interesting physical processes occur in the vicinity of the intersection point of the elastic and magnetic oscillation spectra, the role of a vast number of degrees of freedom from other spectral regions in the problem can be neglected (their role is taken into account in the relaxation parameters). Here, we develop the theory of narrow wave packets (envelopes) describing beams of quasiparticles (phonons and magnons). This approach significantly simplifies theoretical calculations and has a high predictive potential. The concept of envelopes was developed in the theory of weak wave turbulence [10,11]. Our contribution to the development of this concept is that we study the weak turbulence of two interacting flows of waves of different types with different spectra. In this case, in the general case, the flow of phonons linearly nonresonantly excites the flow of magnons and also nonresonantly receives feedback.

In this paper, we derive a system of fundamental equations for describing wave packets of phonons and magnons moving in the same direction, taking into account three- and four-particle interactions, as well as for wave packets of the second and third elastic harmonics. This results in a rich nonlinear system with various solutions depending on many external and internal parameters. These equations can be used to analyze stationary flows (beams) of phonons and magnons in an ultrathin ferromagnetic film. In the nonresonant case, when the amplitudes of the excited magnons are noticeably weakened, nonlinear interactions can be neglected at first. We perform simulations of linear stationary flows of phonons and magnons depending on the magnitude of the magnetoelastic interaction, the magnetic field, the film size, and the frequency of the excited phonons. The paper concludes with a discussion. All of the numerous, but necessary for the calculations, details are included in the Supplemental Material [12].

II. MAGNONS AND PHONONS

We will describe the motions of the magnetization vector $\mathbf{M}$ using $\mathbf{k}$ space and classical Hamiltonian formalism (all details are given in the Supplemental Material [12]) using complex variables $c_{\mathbf{k}}^*$ and $c_{\mathbf{k}}$, which are analogs of the Bose creation and annihilation operators of magnons in quantum mechanics [11–15]. The Hamiltonian of magnons

can be written as

$$\begin{aligned} \mathcal{H}_m/\hbar = & \sum_{\mathbf{k}} \omega_{m\mathbf{k}} c_{\mathbf{k}}^* c_{\mathbf{k}} \\ & + \frac{1}{2} \sum_{\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3, \mathbf{k}_4} \Phi_{4m}(\mathbf{k}_1, \mathbf{k}_2; \mathbf{k}_3, \mathbf{k}_4) c_{\mathbf{k}_1}^* c_{\mathbf{k}_2}^* c_{\mathbf{k}_3} c_{\mathbf{k}_4} \\ & \times \Delta(\mathbf{k}_1 + \mathbf{k}_2 - \mathbf{k}_3 - \mathbf{k}_4), \end{aligned} \quad (1)$$

where the first term describes an ideal gas of magnons with the spectrum $\omega_{m\mathbf{k}}$ and the second term describes the nonlinear scattering with the amplitude $\Phi_{4m}(\mathbf{k}_1, \mathbf{k}_2; \mathbf{k}_3, \mathbf{k}_4)$, $\Delta(\mathbf{k})$ is the Kronecker delta function. Plank's constant $\hbar$ is used to match with quantum mechanics and as a dimensional parameter. The Hamiltonian (1) models the weak turbulence of magnons in a magnetic film.

The dynamic equation for Eq. (1) can be written in the following form:

$$\begin{aligned} i \left(\frac{\partial}{\partial t} + \eta_{m\mathbf{k}} \right) c_{\mathbf{k}} - \omega_{m\mathbf{k}} c_{\mathbf{k}} \\ = \sum_{\mathbf{k}_2, \mathbf{k}_3, \mathbf{k}_4} \Phi_{4m}(\mathbf{k}, \mathbf{k}_2; \mathbf{k}_3, \mathbf{k}_4) c_{\mathbf{k}_2}^* c_{\mathbf{k}_3} c_{\mathbf{k}_4} \Delta(\mathbf{k} + \mathbf{k}_2 - \mathbf{k}_3 - \mathbf{k}_4), \end{aligned} \quad (2)$$

where the energy-loss term with the phenomenological relaxation $\eta_{m\mathbf{k}}$ is included.

A. Magnon wave envelope

Consider a narrow wave packet $\mathbf{k} = \mathbf{k}_e + \mathbf{q}$ with the central beam wave vector $\mathbf{k}_e = \{k_e, 0\}$ and $|\mathbf{q}| \ll |\mathbf{k}_e|$. Then the frequency can be represented as an expansion

$$\omega_{m\mathbf{k}} \simeq \omega_{mk_e} + v_g q_z + \frac{v_g}{2k_e} q_x^2 + \frac{\omega''}{2} q_z^2. \quad (3)$$

Here $\mathbf{v}_g = (\partial\omega_{m\mathbf{k}}/\partial\mathbf{k})_{\mathbf{k}=\mathbf{k}_e}$ is the magnon group velocity and $\omega'' \equiv (\partial^2\omega_{m\mathbf{k}}/\partial k^2)_{\mathbf{k}=\mathbf{k}_e}$.

Let us assume that the magnon envelope is linearly excited by an external wave with wave vector $\mathbf{k}_e$ and frequency Ω_{k_e} . Then the complex magnon amplitude will oscillate with frequency Ω_{k_e} : $c_{k_e}(t) \propto \exp(-i\Omega_{k_e}t)$. We can introduce a slow variable $\psi_m(\mathbf{r}, t)$, which describes the local dynamics of the envelope:

$$c_{\mathbf{k}} = \frac{1}{L_x L_z} \int_0^{L_x} dx \int_0^{L_z} dz \psi_m(\mathbf{r}, t) \exp(-i\Omega_{k_e}t - i\mathbf{q}\mathbf{r}). \quad (4)$$

Having carried out a series of simple transformations, we obtain the magnon envelope equation

$$\begin{aligned} i \frac{\partial \psi_m(\mathbf{r}, t)}{\partial t} + (i\eta_{m\mathbf{k}_e} + \Delta\omega_{k_e}) \psi_m(\mathbf{r}, t) + iv_g \frac{\partial}{\partial z} \psi_m(\mathbf{r}, t) \\ + \frac{\omega''}{2} \frac{\partial^2}{\partial z^2} \psi_m(\mathbf{r}, t) + \frac{v_g}{2k_e} \frac{\partial^2}{\partial x^2} \psi_m(\mathbf{r}, t) \\ - \Phi_{4m,e} |\psi_m(\mathbf{r}, t)|^2 \psi_m(\mathbf{r}, t) = 0, \end{aligned} \quad (5)$$

where $\Delta\omega_{k_e} = \Omega_{k_e} - \omega_{mk_e}$ and $\Phi_{4m,e} \equiv \Phi_{4m}(\mathbf{k}_e, \mathbf{k}_e; \mathbf{k}_e, \mathbf{k}_e)$. This equation with $\Delta\omega_{k_e} = 0$, without taking into account the dissipative term, is encountered in many areas of nonlinear physics [16–18]. It is called the nonlinear Schrödinger equation or the Gross-Pitaevsky equation. The amplitude $\Phi_{4m,b}$ can be both positive (repulsive interaction) and negative (attraction). In the second case, the intense flow of waves narrows, and nonlinear self-focusing occurs.

In the case when $\Delta\omega_{k_e} \neq 0$ magnon envelope is excited nonresonantly, on the wing of the resonance lineshape. In the general case, we have this option when a surface acoustic wave with wave vector $\mathbf{k}_e$ passes through an ultrathin magnetic film and nonresonantly excites magnetic oscillations with the same wave vector but different frequencies.

B. Phonon and magnon wave envelopes

Let us now consider a phonon envelope. For simplicity, we will consider only longitudinal phonons with their polarization along the direction of motion. These waves model Rayleigh waves in the plane [5]. The phonon Hamiltonian contains the quadratic form only

$$\mathcal{H}_{ph}/\hbar = \sum_{\mathbf{k}} v_s k b_{\mathbf{k}}^* b_{\mathbf{k}}, \quad (6)$$

where v_s is the sound velocity and $b_{\mathbf{k}}^*$, $b_{\mathbf{k}}$ are the complex phonon amplitudes. The dynamic equation for $b_{\mathbf{k}}$ is similar to Eq. (2) without nonlinearity.

Magnetoelastic interaction between magnetization and elastic deformations at the point of the intersection of their spectra leads to processes of interaction of magnons and phonons. Let us derive equations for envelopes of phonons and magnons resonantly interacting. Only the envelopes moving along the z axis will be considered. In this case,

the following terms must be added to the model magnon Hamiltonian (1) and phonon Hamiltonian (6):

$$\begin{aligned} \mathcal{H}_{mph}/\hbar = & (F_{\mathbf{k}_e} c_{\mathbf{k}_e}^* b_{\mathbf{k}_e} + \text{c.c.}) \\ & + \Phi_{2m2ph}(\mathbf{k}_e, \mathbf{k}_e; \mathbf{k}_e, \mathbf{k}_e) c_{\mathbf{k}_e}^* c_{\mathbf{k}_e} b_{\mathbf{k}_e}^* b_{\mathbf{k}_e} \\ & + [\Phi_{\text{pump}}(\mathbf{k}_e, \mathbf{k}_e; \mathbf{k}_e, \mathbf{k}_e) c_{\mathbf{k}_e} c_{\mathbf{k}_e} b_{\mathbf{k}_e}^* b_{\mathbf{k}_e}^* + \text{c.c.}] \\ & + [\Phi_{3mph}(\mathbf{k}_e, \mathbf{k}_e, \mathbf{k}_e; \mathbf{k}_e) c_{\mathbf{k}_e}^* c_{\mathbf{k}_e} c_{\mathbf{k}_e} b_{\mathbf{k}_e}^* + \text{c.c.}] \\ & + [\Psi_{2mph2}(\mathbf{k}_e, \mathbf{k}_e; 2\mathbf{k}_e) c_{\mathbf{k}_e} c_{\mathbf{k}_e} b_{2\mathbf{k}_e}^* + \text{c.c.}] \\ & + [\Phi_{3mph3}(\mathbf{k}_e, \mathbf{k}_e, \mathbf{k}_e; 3\mathbf{k}_e) c_{\mathbf{k}_e} c_{\mathbf{k}_e} c_{\mathbf{k}_e} b_{3\mathbf{k}_e}^* + \text{c.c.}]. \end{aligned} \quad (7)$$

The first term with amplitude U_{k_e} describes the linear excitation of the magnon by a resonant phonon and the inverse process. The nonlinear addition to this process is written in the form of a second, third, and fourth terms with amplitude Φ_{2m2ph} , Φ_{pump} , and Φ_{3mph} , correspondingly. The fifth term with amplitude Ψ_{2mph2} describes the transformation of two magnons into a phonon with doubled frequency and doubled wave vector (and the reverse process). The last term with Φ_{3mph3} is the transformation of three magnons into a phonon with tripled frequency and tripled wave vector (and the reverse process).

The resulting picture can be described by four envelopes ψ_{ph} , ψ_m , ψ_{ph2} , and ψ_{ph3} as follows. If we initially excite an envelope ψ_{ph} of phonons along z axis with a frequency $v_s k_e$, then due to linear interaction, this phonon flow excites envelope ψ_m of magnons with frequency $v_s k_e$. Then magnon envelope excites second ψ_{ph2} and third ψ_{ph3} sound harmonics. The equation for the phonon envelope is

$$\begin{aligned} i \frac{\partial \psi_{ph}}{\partial t} + i\eta_{ph} \psi_{ph} + iv_s \frac{\partial}{\partial z} \psi_{ph} + \frac{v_s}{2k_e} \frac{\partial^2}{\partial x^2} \psi_{ph} - F_{\mathbf{k}_e}^* \psi_m \\ - \Phi_{2m2ph} |\psi_m|^2 \psi_{ph} - 2\Phi_{\text{pump}} \psi_m^2 \psi_{ph}^* \\ - \Phi_{3mph} |\psi_m|^2 \psi_m = 0. \end{aligned} \quad (8)$$

It contains a linear interaction with the magnon envelope with a coefficient $F_{\mathbf{k}_e}^*$, as well as nonlinear terms responsible for the scattering of magnons on phonons (Φ_{2m2ph}), the processes of converting a pair of phonons into a pair of magnons (Φ_{pump}), and the processes of interaction of three magnons with a phonon (Φ_{3mph}). The equation for the magnon envelope can be written as

$$\begin{aligned} i \frac{\partial \psi_m}{\partial t} + (i\eta_m + v_s k_e - \omega_{mk_e}) \psi_m + iv_g \frac{\partial}{\partial z} \psi_m + \frac{\omega''}{2} \frac{\partial^2}{\partial z^2} \psi_m + \frac{v_g}{2k_e} \frac{\partial^2}{\partial x^2} \psi_m - \Phi_{4m} |\psi_m|^2 \psi_m \\ - F_{\mathbf{k}_e} \psi_{ph} - \Phi_{2m2ph} |\psi_{ph}|^2 \psi_m - \Phi_{\text{pump}} \psi_{ph}^2 \psi_m^* - \Phi_{3mph} \psi_m^2 \psi_{ph}^* \\ - 2\Phi_{3mph}^* |\psi_m|^2 \psi_{ph} - 2\Psi_{2mph2} \psi_m^* \psi_{ph2} - 3\Phi_{3mph3} (\psi_m^*)^2 \psi_{ph3} = 0. \end{aligned} \quad (9)$$

Note that if $v_s k_e \neq \omega_{mk_e}$ the phonon flow nonresonantly excites envelope ψ_m of magnons on the wing of their resonance lineshape in the same direction with frequency $v_s k_e$. In addition to the previously mentioned terms, the equation for the magnon envelope includes magnon-magnon scattering (Φ_{4m}) and the processes of excitation of the envelopes of the second ($\Psi_{2m\text{ph}2}^*$) and third ($\Phi_{3m\text{ph}3}^*$) sound harmonics by magnons. The equations for the envelopes of the second and third sound harmonics are as follows:

$$i \frac{\partial \psi_{\text{ph}2}}{\partial t} + i \eta_{\text{ph}2} \psi_{\text{ph}2} + i v_s \frac{\partial}{\partial z} \psi_{\text{ph}2} + \frac{v_s}{4k_e} \frac{\partial^2}{\partial x^2} \psi_{\text{ph}2} - \Psi_{2m\text{ph}2} \psi_m^2 = 0. \quad (10)$$

$$i \frac{\partial \psi_{\text{ph}3}}{\partial t} + i \eta_{\text{ph}3} \psi_{\text{ph}3} + i v_s \frac{\partial}{\partial z} \psi_{\text{ph}3} + \frac{v_s}{6k_e} \frac{\partial^2}{\partial x^2} \psi_{\text{ph}3} - \Phi_{3m\text{ph}3} \psi_m^3 = 0. \quad (11)$$

Note that all the above equations have a general form, and the coefficients of these equations depend on the specific model of magnetic, elastic, and magnetoelastic properties

of the sample. The coefficients η_{ph} , η_m , $\eta_{\text{ph}2}$, and $\eta_{\text{ph}3}$ describe the relaxation parameters of phonons, magnons and the second and third sound harmonics, respectively.

III. MODEL OF ULTRATHIN FERROMAGNETIC FILM

Consider an ultrathin ferromagnetic film with a thickness of τ and dimensions of $0 \leq z \leq L_z$, $-L_x/2 \leq x \leq L_x/2$. The equilibrium is supposed to be a uniformly magnetized state in which the magnetization $\mathbf{M}(z, x)$ is oriented in the (z, x) plane along an equilibrium axis z_0 which is deviated from the z axis by the angle ϕ_0 . The magnetic energy of the film contains exchange energy, uniaxial anisotropy, Zeeman energy, and demagnetization energy. The changes in magnetization $\mathbf{M}(z, x)$ and other physical parameters across the film thickness are neglected. We provide all the details of the description of the oscillations of the magnetic and elastic subsystems in the Supplemental Material [12].

Let us consider the specifics of the magnon spectrum, which can be represented as

$$\omega_{m\mathbf{k}} = \gamma \left[\alpha_E k^2 + H \cos(\phi_H - \phi_0) + H_K \cos 2\phi_0 + 4\pi M_s G(k\tau) \right]^{1/2} \times \left[\alpha_E k^2 + H \cos(\phi_H - \phi_0) + H_K \cos^2 \phi_0 + 4\pi M_s [1 - G(k\tau)] \left(\frac{k_x \cos \phi_0 - k_z \sin \phi_0}{k} \right)^2 \right]^{1/2}. \quad (12)$$

Here α_E is the exchange constant, H is the external magnetic field rotated by an angle ϕ_H from the z axis, H_K is the uniaxial anisotropy field along z axis, $G(\xi) = [1 - \exp(-\xi)]/\xi$. We see that the spectrum of magnons, Eq. (12), has nontrivial dependences on the angles ϕ_H , ϕ_0 and on the wave vector $\mathbf{k} = (k_z, k_x)$. Below, we will consider the beam of phonons and magnons in the direction of the z axis. The anisotropy field for simplicity will be neglected ($H_K = 0$ Oe, $\phi_0 = \phi_H$).

Figures 1(a)–1(d) show the magnon and phonon spectra at $\phi_H = 0$. To calculate these spectra, the following Ni film parameters were used in Eq. (12): $\mathbf{k} = (k, 0)$, $\tau = 70$ nm, $v_s = 3.5 \times 10^5$ cm/s, $\alpha_E = 8 \times 10^{-7}$ erg/cm, $M_s = 484$ Gs, $\gamma = 3.05 \times 10^6$ Hz/Oe. We see that the magnon spectrum has a negative group velocity at small k [see Fig. 1(c)], and then it becomes zero at the point indicated in Fig. 1(a). At larger k we have $v_g > 0$. Figure 1(e) shows three-dimensional (3D) phonon and magnon spectra and their intersection. Figure 1(f) demonstrates the surface of the phonon and magnon spectra intersection.

From Figs. 1(e) and 1(f), it is evident that the intersection points of the phonon and magnon spectra at which the magnetoelastic interaction is maximal have a strong

angular dependence on the direction and magnitude of the magnetic field. This means that if, for a given magnetic-field vector, we have resonant excitation of magnons by phonons (their frequencies are approximately equal within the linewidth of the excited magnons), then changing the field direction or its magnitude generally moves the magnon frequency away from resonance, and the excitation of magnons becomes nonresonant. This situation, for example, occurs in Refs. [7–9].

IV. LINEAR BEAMS

Here, we consider the most common case in surface acoustic-wave experiments, when the frequency of excited phonons $v_s k_e$ noticeably different from the frequency ω_{mk_e} of magnons with the same wave vector $\mathbf{k}_e$. In this case, magnons are excited nonresonantly on their lineshape wing and magnon amplitudes are weakened by the parameter $\eta_m/|\omega_{mk_e} - v_s k_e| \ll 1$. Then, nonlinear interactions will be noticeably weakened and can be neglected for simplicity. Thus, we limit ourselves to the linear interaction of envelopes without nonlinear excitation of the envelopes of the second and third harmonics. Only stationary flows

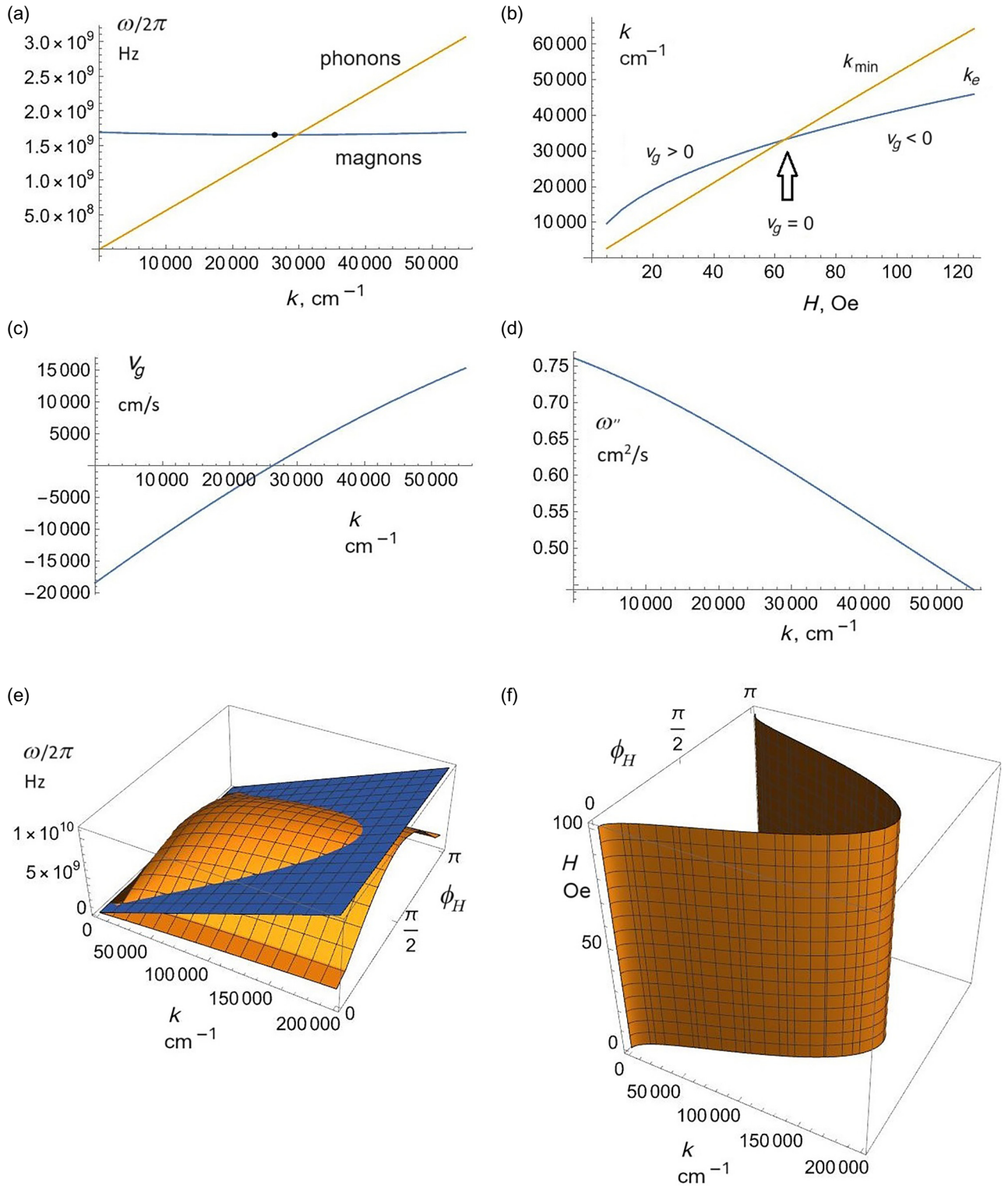


FIG. 1. Characteristics of phonon and magnon spectra. (a) Spectra of phonons $v_s k$ and magnons ω_{mk} at $\phi_H = 0$, $H = 50$ Oe. The black dot indicates the minimum frequency of magnons at $k_{\min}$ when their group velocity $v_g = 0$. (b) Magnetic-field dependence of $k_{\min}$ when $v_g = 0$ and of k_e , wave-vector magnitude of spectra intersection. (c) Wave-vector dependence of the magnon group velocity. (d) Wave-vector dependence of the second derivatives on k for the magnon spectrum. (e) 3D representation of magnon (yellow surface) and phonon (blue surface) spectra showing the spectral intersection curve. (f) The surface describes the intersection points of the spectra of phonons and magnons, depending on the magnitude and angle of rotation of the magnetic field and the wave vector.

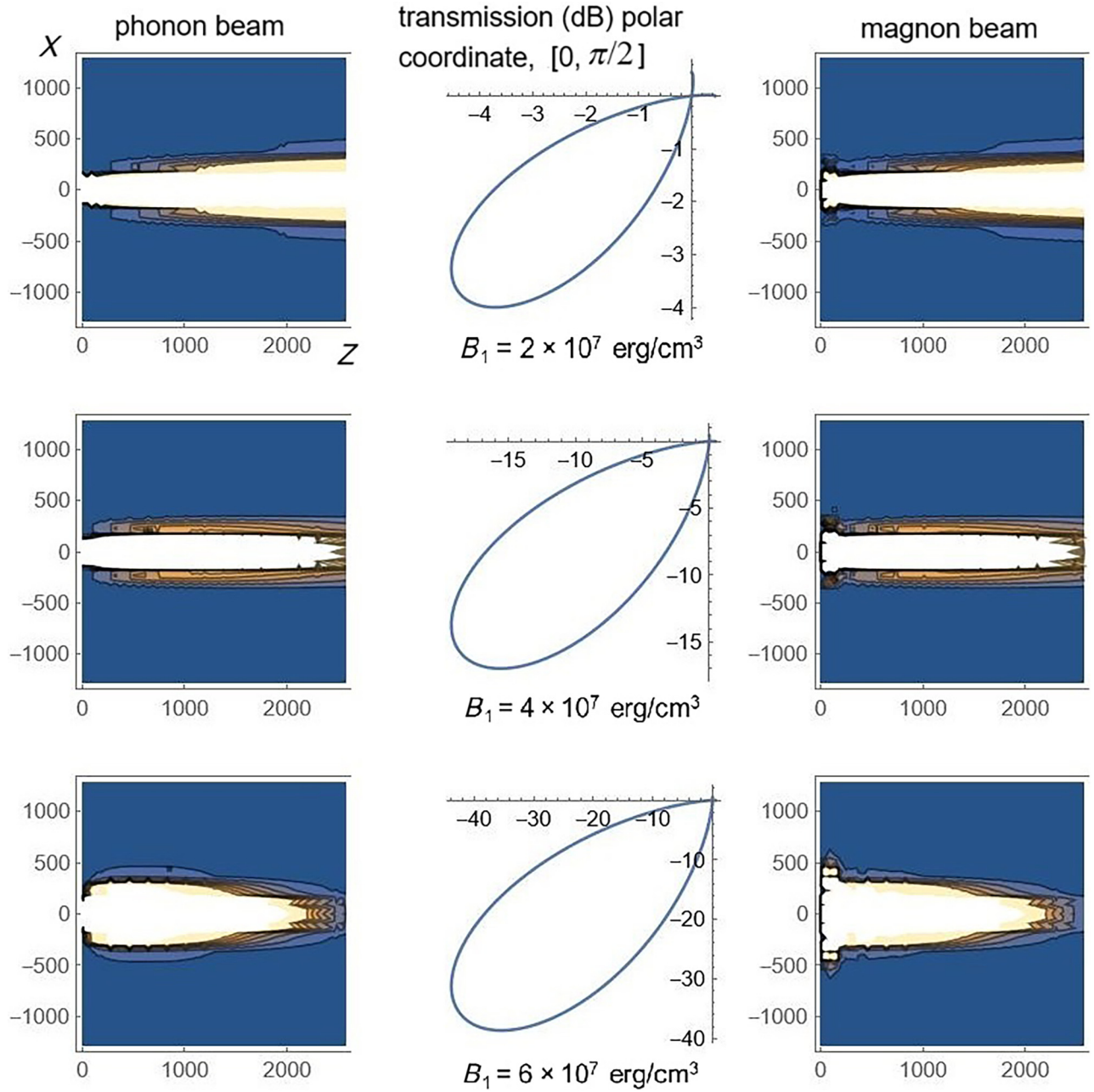


FIG. 2. Phonon (first column), magnon (third column) beams at $\phi_H = \pi/4$, and the transmission signal (second column) at different constants of magnetoelastic interaction (in one row) and $H = 50$ Oe, $v_s k/2\pi = 287$ MHz.

of quasiparticles are considered, which we call beams. The linear beams of the system (8) and (9) are reduced to the following equations, written for convenience in dimensionless coordinates $X = k_e x$, and $Z = k_e z$:

$$i \frac{\eta_{ph}}{v_s k_e} \psi_{ph} + i \frac{\partial}{\partial Z} \psi_{ph} + \frac{1}{2} \frac{\partial^2}{\partial X^2} \psi_{ph} - \frac{F_{\mathbf{k}_e}^*}{v_s k_e} \psi_m = 0, \quad (13)$$

$$\left(i \frac{\eta_m}{v_s k_e} + 1 - \frac{\omega_{mke}}{v_s k_e} \right) \psi_m + i \frac{v_g}{v_s} \frac{\partial}{\partial Z} \psi_m + \frac{\omega'' k_e}{2 v_s} \frac{\partial^2}{\partial Z^2} \psi_m + \frac{1}{2} \frac{v_g}{v_s} \frac{\partial^2}{\partial X^2} \psi_m - \frac{F_{\mathbf{k}_e}}{v_s k_e} \psi_{ph} = 0. \quad (14)$$

We have calculated the phonon (13) and magnon (14) beams in a reasonably wide parameter range using Wolfram Mathematica and the finite-element method. Without significant loss of generality, we present the Ni ultrathin

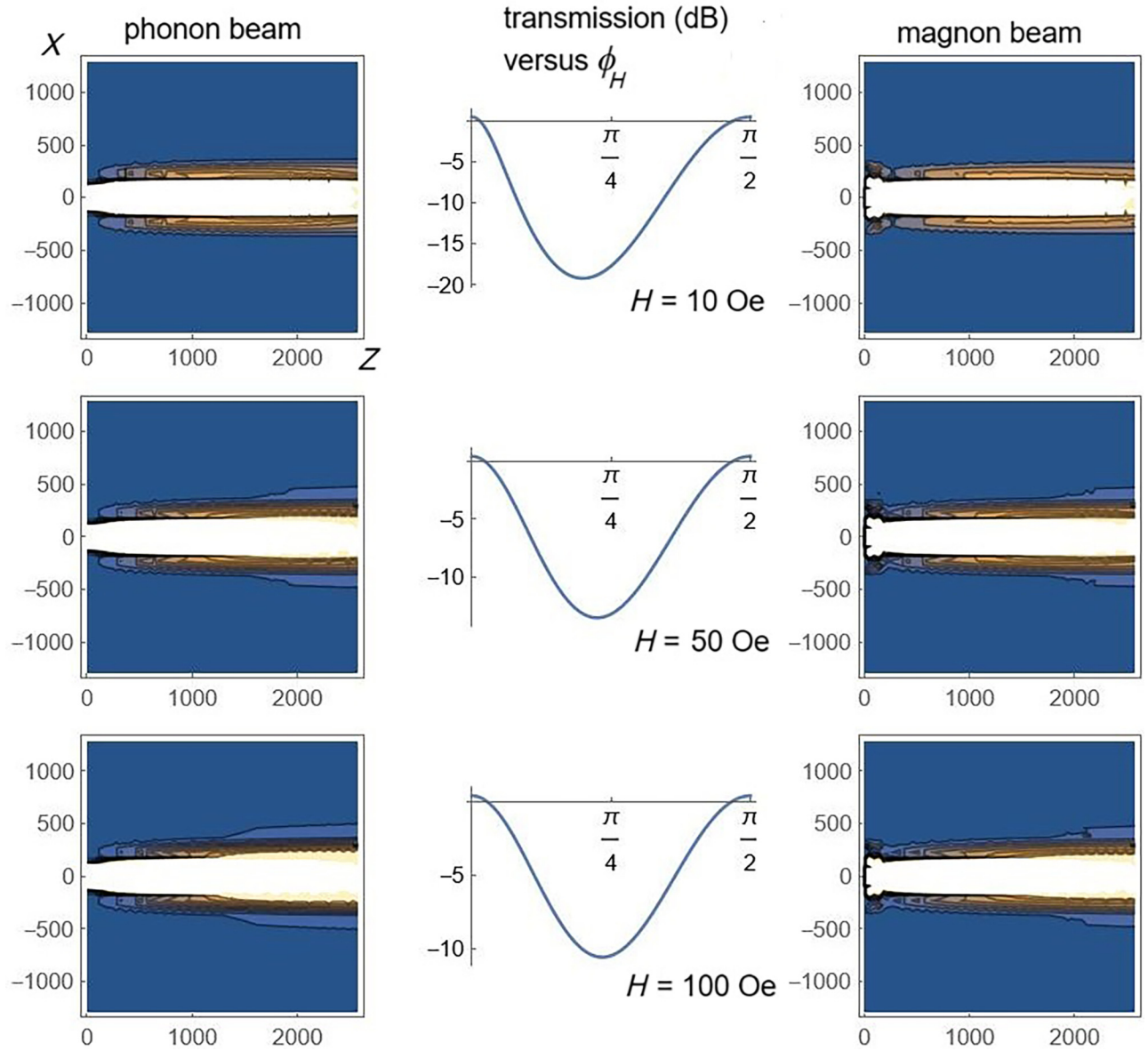


FIG. 3. Phonon beams (first column) and magnon beams (third column) at $\phi_H = \pi/4$, and the angular dependence of the transmission signal (second column) at different magnetic fields (in one row). $B_1 = 3 \times 10^7$ erg/cm³, $v_s k/2\pi = 287$ MHz.

magnetic-film calculation results. In addition, we consider (1) a linear emitter of acoustic waves with a vertical size of $10 \mu\text{m}$ installed on the left in the center at the edge of the film and (2) an acoustic receiver of the same size installed on the right edge. The amplitude of the excited beam of phonons was assumed to be $\psi_{\text{ph}} = 1$. The transmitted signal was calculated using the formula

$$S_{\text{trans}}(\phi_H) = 20 \log_{10} \int_{-X_R/2}^{X_R/2} |\psi_{\text{ph}}| dX. \quad (15)$$

The problem is subject to boundary conditions for the magnetic system: $\psi_m = 0$ at the left, lower, and upper edges of the magnetic film. The difference in the boundary conditions for beams of phonons and magnons is natural, as phonons leave the magnetic film with virtually no reflection and enter the substrate. Magnons, on the other hand, are completely reflected from the film edges. The absence of nonlinear terms in Eqs. (13) and (14) means that we are in the initial stage of weak turbulence. In this stage, the terms with second derivatives and the boundary conditions of the problem determine the smearing of beams. We assumed that $\eta_{\text{ph}}/v_s k_e = \eta_m/v_s k_e = 10^{-4}$. The quantities

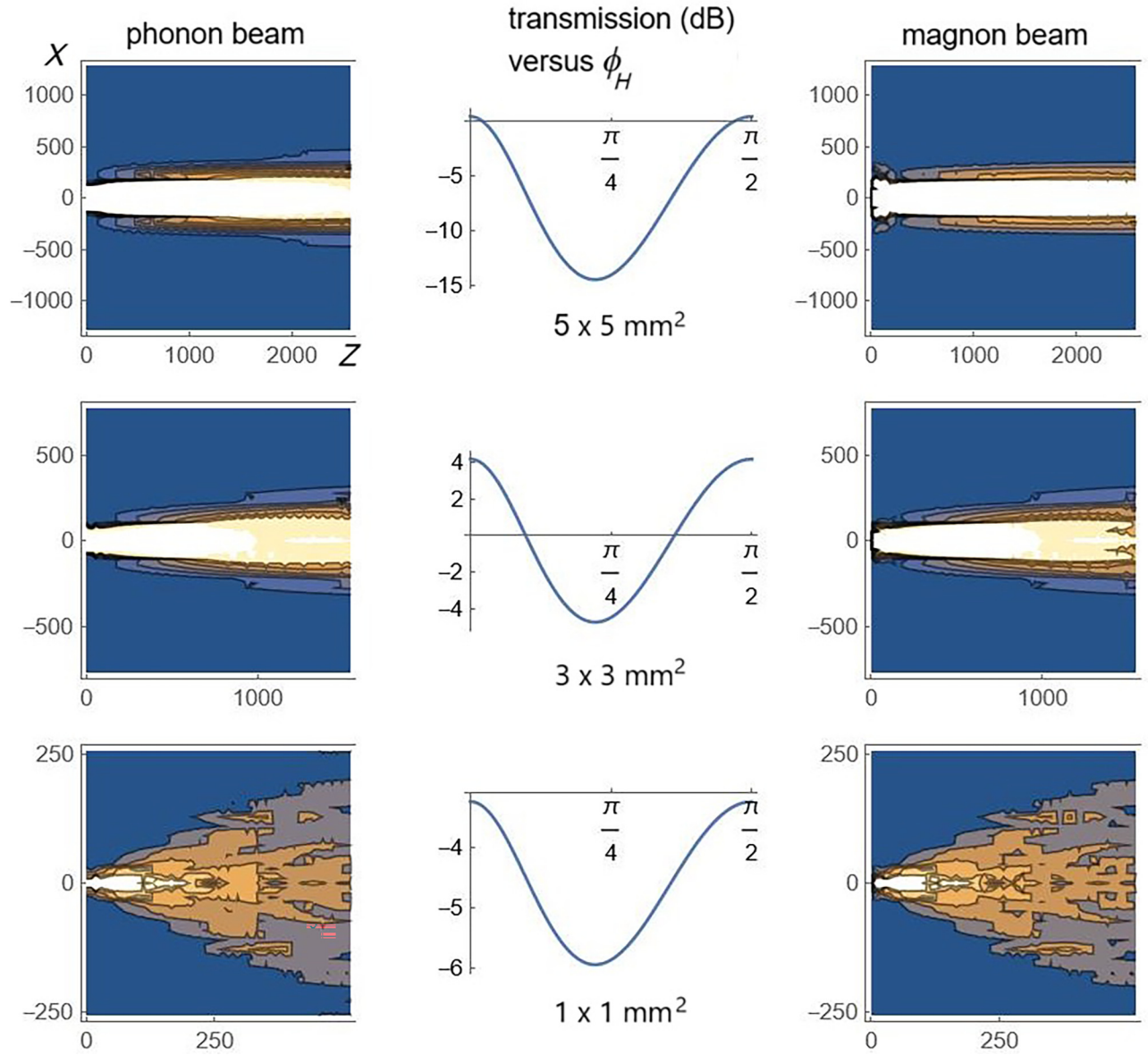


FIG. 4. Phonon (first column) and magnon (third column) beams at $\phi_H = \pi/4$, and the angular dependence of the transmission signal (second column) at different linear sizes of ultrathin magnetic film (in one row). $H = 50$ Oe, $B_1 = 3 \times 10^7$ erg/cm³, $v_s k/2\pi = 287$ MHz.

ω_{mke} , v_g , and ω'' were calculated from the spectrum (12). The amplitude of the linear magnetoelastic interaction (its derivation is given in the Supplemental Material [12]) is

$$F_k = -\frac{iB_1}{2} \sqrt{\frac{\gamma k}{\rho M_s v_s}} (U_k + V_k) \sin 2\phi_0, \quad (16)$$

where B_1 is the magnetoelastic constant (approximately equal to 10^8 erg/cm³ for Ni [19]), ρ is the density (8.9 g/cm³ for Ni), coefficients U_k and V_k are given in the Supplemental Material [12].

Let us first consider how the picture of the beams of phonons and magnons depends on the value of the magnetoelastic constant B_1 . Figure 2 shows the beams of phonons $|\psi_{ph}|$ and magnons $|\psi_m|$ and the angular dependence of the transmission signal in polar coordinates when the angle ϕ_H changes from zero to $\pi/2$. It is easy to see that with the growth of the value of B_1 of the magnetoelastic interaction, the turbulence in the beams of phonons and magnons increases. At the same time, the attenuation of the transmitted signal increases and it is maximum at an angle ϕ_H slightly less than $\pi/4$. This fact agrees well with the observed data [7–9].

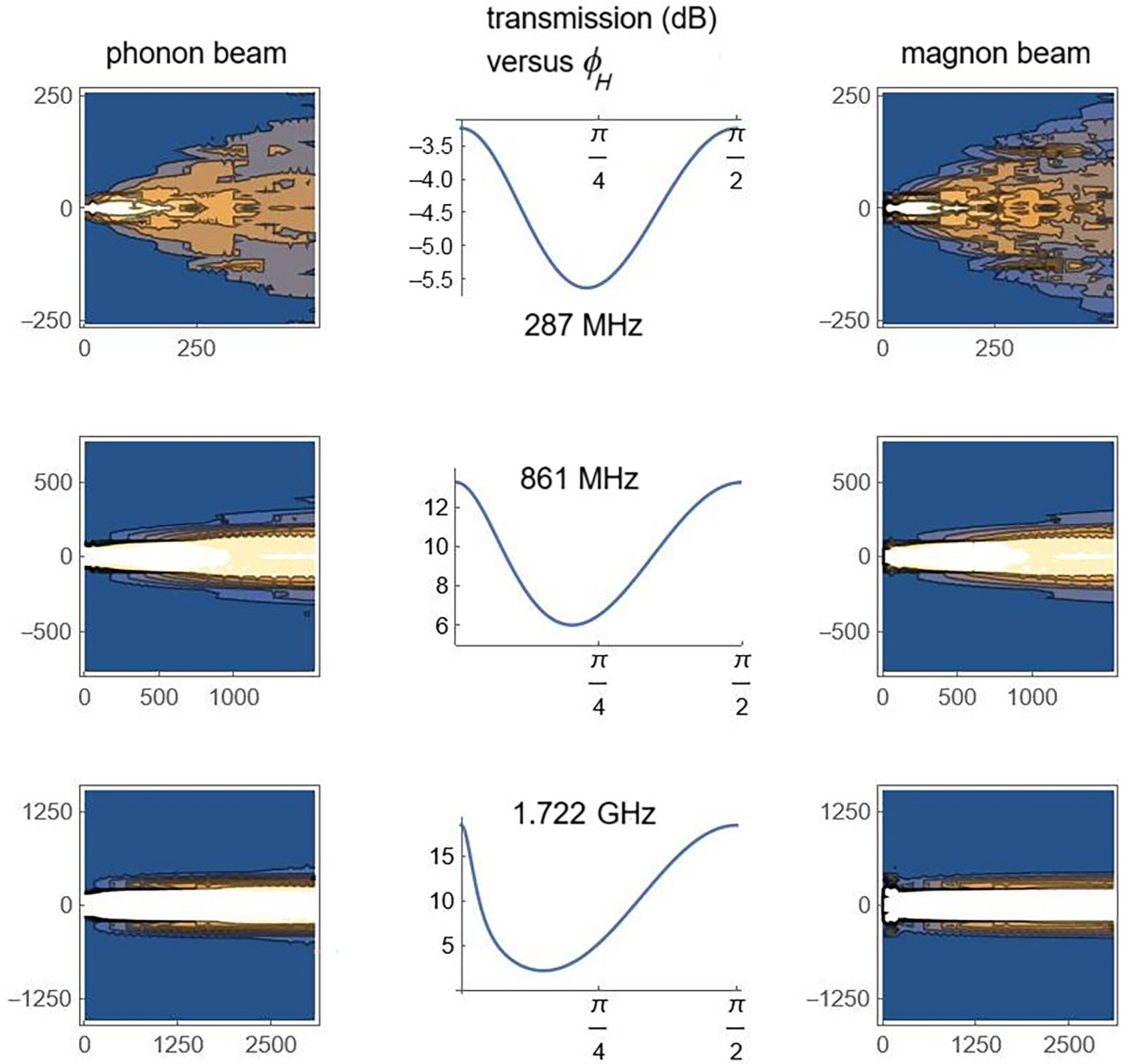


FIG. 5. Phonon (first column) and magnon (third column) beams at $\phi_H = \pi/4$, and the angular dependence of the transmission signal (second column) at different frequencies $\nu_s k/2\pi$ of an ultrathin magnetic film (in one row). $H = 60$ Oe, $B_1 = 3 \times 10^7$ erg/cm³, film 1×1 mm².

Let us now consider what the beams of phonons and magnons look like in different magnetic fields. Figure 3 demonstrates that with increasing magnetic-field strength, the turbulent picture of the beams becomes calmer, and the transmitted signal has less attenuation. This fact is understandable because the phonons and magnons' "non-resonance" effect increases, effectively reducing the magnetoelastic coupling. Let us now consider how the beam pattern changes when the linear dimensions of the ultrathin film change. Figure 4 shows the beams of phonons $|\psi_{ph}|$ and magnons $|\psi_m|$ and the angular dependence of the transmitted signal. On the one hand, the phonon beam

path becomes shorter, and thus, the attenuation becomes more pronounced. On the other hand, magnons reflected from the upper and lower film boundaries as their dimensions decrease can make an ever-increasing contribution to the turbulence of both flows, which is probably what is observed.

The change in the turbulence of phonon and magnon beams with increasing phonon-exciting frequency $\nu_s k_e/2\pi$ is shown in Fig. 5. We see that the turbulence of both beams decreases with increasing excitation frequency. This is due to the weakening of the role of magnons reflected from the film boundaries with increasing k_e (and the

corresponding decrease in wavelength). The increase in the deviation of the maximum absorption of the phonon beam from the angle $\pi/4$ with increasing excitation frequency is noteworthy.

The analysis of the dependence of phonon and magnon beams on their attenuation coefficients and on the film thickness showed that beam turbulence (with a change in these coefficients by $\pm 100\%$) has a weak dependence on them.

V. DISCUSSION

In this paper, we considered a simple approach to describing unidirectional phonon and magnon beams coupled by magnetoelastic interaction in an ultrathin ferromagnetic film. In the weak turbulence approximation, Eqs. (8)–(11) for the wave envelopes were obtained as a system of partial differential equations with nonlinear terms. Of course, this is a rich nonlinear system with various solutions depending on many external and internal parameters, possessing high predictive capabilities. Therefore, it makes perfect sense to conduct purposeful experiments in parallel with the proposed theory.

The development of weak turbulence is a scenario that depends on the intensity of the quasiparticle flux. We considered only the initial part of this scenario, when this intensity is not yet high enough for nonlinear terms in the equations to play a role. These terms are responsible for nonlinear frequency shifts, the second and third harmonics, and parametric processes, such as the conversion of two phonons into two magnons and back again when exceeding a certain excitation threshold. However, despite this simplification, we observe that the presence of second derivatives in Eqs. (13) and (14) causes transverse blurring of the beams and results in a fairly pronounced turbulence pattern when the parameter $k_e L$ is of the order of 500. Apparently, magnon reflection processes at the sample boundaries are at play here. As the above modeling showed, even in the linear approximation with nonresonant interaction of phonons and magnons with a given wave vector, we have nontrivial beam patterns that differ with changes in the magnetoelastic constant, magnetic field, film size, and phonon excitation frequency. Moreover, the obtained results qualitatively describe the experimental results.

It is easy to guess that the equations for beams can also be obtained for noncollinear cases when we have two or more sources of excitation of quasiparticles in different directions. Naturally, the linear interactions of phonons and magnons do not change the direction of the beams. New beams can arise when three-particle and four-particle processes associated with noncollinear excited beams are considered by momentum and energy conservation laws. For example, from two beams of phonons with the same frequency Ω and wave vectors $\mathbf{k}_1$ and $\mathbf{k}_2$, a beam of

magnons with the wave vector $\mathbf{k} = \mathbf{k}_1 + \mathbf{k}_2$ and a forced oscillation frequency 2Ω arises, which does not necessarily coincide with the natural frequency ω_{mk} of these magnons. This example can clarify the effect of phonon focusing observed experimentally [7–9].

We are confident that the simple theory of beams of acoustic surface waves (phonons) interacting with spin waves (magnons) that we have developed will be of great interest to specialists studying and describing magnetoelastic dynamics in ultrathin magnetic films. Our model offers significant predictive capabilities of the theory, significantly simplifying calculations. In addition, we assume a broad interdisciplinary interest of specialists studying weak turbulence phenomena in condensed-matter physics, plasma, and the physics of nonlinear wave processes in systems with two or more wave spectra.

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DATA AVAILABILITY

The data that support the findings of this article are not publicly available because of legal restrictions preventing unrestricted public distribution. The data are available from the authors upon reasonable request.

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- [1] J. Puebla, Y. Hwang, S. Maekawa, and Y. Otani, Perspectives on spintronics with surface acoustic waves, *Appl. Phys. Lett.* **120**, 220502 (2022).
 - [2] Huang Ming-Xian, Hu Wen-Bin, and Bai Fei-Ming, Surface acoustic wave-spin wave coupling and magnetoacoustic nonreciprocal devices, *Acta Phys. Sin.* **73**, 158501 (2024).
 - [3] L. Ushii, A. Slavin, and R. Verba, Nonreciprocity of surface acoustic waves coupled to spin waves in a ferromagnetic bilayer with noncollinear layer magnetizations, *Phys. Rev. Appl.* **22**, 034046 (2024).
 - [4] M. Weiler, L. Dreher, C. Heeg, H. Huebl, R. Gross, M. S. Brandt, and S. T. B. Goennenwein, Elastically driven ferromagnetic resonance in nickel thin films, *Phys. Rev. Lett.* **106**, 117601 (2011).
 - [5] L. Dreher, M. Weiler, M. Pernpeintner, H. Huebl, R. Gross, M. S. Brandt, and S. T. B. Goennenwein, Surface acoustic wave driven ferromagnetic resonance in nickel thin films: Theory and experiment, *Phys. Rev. B* **86**, 134415 (2012).
 - [6] X. Li, D. Labanowski, S. Salahuddin, and C. S. Lynch, Spin wave generation by surface acoustic waves, *J. Appl. Phys.* **122**, 043904 (2017).
 - [7] D. A. Bas, P. J. Shah, M. E. McConney, and M. R. Page, Optimization of acoustically-driven ferromagnetic resonance devices, *J. Appl. Phys.* **126**, 114501 (2019).

- [8] D. A. Bas, P. J. Shah, A. Matyushov, M. Popov, V. Schell, R. C. Budhani, G. Srinivasan, E. Quandt, N. Sun, and M. R. Page, Acoustically driven ferromagnetic resonance in diverse ferromagnetic thin films, *IEEE Trans. Mag.* **57**, 4300605 (2021).
- [9] P. J. Shah, D. A. Bas, A. Hamadeh, M. Wolf, A. Franson, M. Newburger, P. Pirro, M. Weiler, and M. R. Page, Symmetry and nonlinearity of spin wave resonance excited by focused surface acoustic waves, *Adv. Electron. Mater.* **9**, 2300524 (2023).
- [10] V. E. Zakharov, S. L. Musher, and A. M. Rubenchik, Hamiltonian approach to the description of non-linear plasma phenomena, *Phys. Rep.* **129**, 285 (1985).
- [11] V. E. Zakharov, V. S. L'vov, and G. Falkovich, *Kolmogorov spectra of turbulence I. Wave turbulence* (Springer-Verlag, Berlin, 1992).
- [12] See Supplemental Material at <http://link.aps.org/supplemental/10.1103/trxt-p1dt> for a detailed derivation of the coefficients in Eqs. (8)–(12), which was carried out based on models for magnons and phonons in an ultrathin film. The Supplemental Material also contains Refs. [20–26].
- [13] F. Strocchi, Complex coordinates and quantum mechanics, *Rev. Mod. Phys.* **38**, 36 (1966).
- [14] P. Krivosik and C. E. Patton, Hamiltonian formulation of nonlinear spin-wave dynamics: Theory and applications, *Phys. Rev. B* **82**, 184428 (2010).
- [15] V. L. Safonov, *Nonequilibrium Magnons* (Wiley-VCH, Weinheim, 2013).
- [16] C. Sulem and P.-L. Sulem, *The Nonlinear Schrödinger Equation: Self-Focusing and Wave Collapse* (Springer-Verlag, NY, 1999).
- [17] N. Gavish, G. Fibich, L. T. Vuong, and A. L. Gaeta, Predicting the filamentation of high-power beams and pulses without numerical integration: A nonlinear geometrical optics method, *Phys. Rev. A* **78**, 043807 (2008).
- [18] N. Vladimirova, S. Derevyanko, and G. Falkovich, Phase transitions in wave turbulence, *Phys. Rev. E* **85**, 010101(R) (2012).
- [19] G. Y. Guo, Orientation dependence of the magnetoelastic coupling constants in strained FCC Co and Ni: an ab initio study, *J. Magn. Magn. Mater.* **209**, 33 (2000).
- [20] L. D. Landau and E. M. Lifshitz, *Electrodynamics of Continuous Media, Vol. 8, Course of Theoretical Physics* (Pergamon, Oxford, 1984), ch. 5.
- [21] M. Mansuripur, *The Physical Principles of Magneto-Optical Recording* (Cambridge University Press., Cambridge, 1995), p. 488.
- [22] J. Villain, Quantum theory of one- and two-dimensional ferro- and antiferromagnets with easy magnetization plane. I. Ideal 1-D or 2-D lattices without in-plane anisotropy, *J. Phys.* **35**, 27 (1974).
- [23] V. G. Bar'yakhtar and D. A. Yablonskii, On a representation for spin operators and its use in the theory of magnetism, *Theor. Mat. Fiz.* **25**, 250 (1975). [*Theoretical and Mathematical Physics* **25**(2), 1109–1115 (1975)].
- [24] B. A. Kalinikos and A. N. Slavin, Theory of dipole-exchange spin wave spectrum for ferromagnetic films with mixed exchange boundary conditions, *J. Phys. C: Solid State Phys.* **19**, 7013 (1986).
- [25] Q. Shi, V. L. Safonov, M. Mino, and H. Yamazaki, Unitary transformations in weakly nonideal Bose gas, *Phys. Lett. A* **238**, 258 (1998).
- [26] L. D. Landau and E. M. Lifshitz, *Theory of Elasticity, Vol. 7, Course of Theoretical Physics* (Pergamon, Oxford, 1970), ch. 3.