

Elasticity-induced quasi-bound states in the continuum in compact acoustic resonators

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Quasi-bound states in the continuum (QBIC) have attracted considerable attention due to their exceptionally high Q factors. In this work, we reveal the critical role of elastic (shear) waves in the formation of acoustoelastic Friedrich-Wintgen QBIC in a simple and compact elastodynamic resonator consisting of a silicon frame enclosing an airborne acoustic cavity. Full-wave numerical simulations support the theoretical predictions based on temporal coupled mode theory, allowing for precise prediction of the near-field coupling. Of note, we demonstrate the emergence of a shear hybrid mode in conjunction with the classical coupled modes, which enhances the coupling strength between the cavity's eigenmodes. We further analyze how material properties and frame thickness influence the characteristics of the elastic mode. Owing to its structural simplicity and ease of fabrication, the proposed resonator offers promising potential for applications in acoustic sensing, filtering, and beyond.

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I. INTRODUCTION

Bound states in the continuum (BIC) are nonradiative states trapped within the continuum of the radiation spectrum [1–3]. Originally, these intriguing quantum states were predicted by Wigner and von Neumann via tailored complex potentials [4,5], which was difficult to realize in practical systems [6]. The growing interest in BIC physics stems from its inherent diverging quality factors (or Q factors) that result in low-loss devices, energy confinement, and enhancement of wave-matter interactions [7]. Based on the underlying physical principles, BIC can be classified into symmetry-protected (SP) BIC [1,8–10] and accidental BIC [11–13]. The latter can be further divided into Fabry-Perot (FP) BIC [1,14–16] and Friedrich-Wintgen (FW) BIC [8,9,17]. Although achieving the infinite Q factor, the hallmark of true BIC, is unattainable, a significantly high Q factor can still be obtained by leveraging the concept of quasi-BIC (QBIC) via small perturbation of the symmetric system or by considering losses. This achievement has thus

led to key applications such as low-threshold lasing [18–21], ultrasensitive sensing [22,23], imaging [24], vortex beam generation [25], and filtering [26,27].

Over the past few years, BIC have continued to arouse considerable interest across diverse fields such as optical [16,28–32], acoustic [8,13,14,33,34], and elastic waves [35–39]. Notably, several pioneering studies explored BIC-like properties in acoustic resonators long before the terminology of BIC was widely adopted [9,40,41]. Building on these foundational works, numerous studies have employed various acoustic resonator systems to study BIC [42], making significant progresses in both theories and experiments. Recent research also considered the intrinsic interplay between elastic and acoustic waves in elastodynamic resonators, which has further advanced the understanding of BIC in such hybrid systems [39,43–45]. For example, in 2021, researchers reported the first experimental demonstration of elastic-acoustic BIC in simple solid-liquid layered structures [39]. Moreover, Deriy *et al.* proposed an elastic BIC within a compact solid spherical resonator surrounded by a fluid, by exploiting the decoupling between the torsion mode within the solid material and the longitudinal acoustic wave in the fluid [43]. More recently, Farhat *et al.* put forward an elastodynamic structure that supports FP-BIC, comprising a periodically slitted silicon metasurface embedded in water [44]. This study demonstrated, both theoretically and experimentally, the existence of a high- Q -factor in the ultrasound regime.

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Although extensive research has been devoted to the study of BIC within elastodynamic resonators, there remains a notable gap regarding the impact of elastic waves on the generation and dynamics of acoustic BIC in elastodynamic resonators, in particular compact ones. In fact, most of the previous studies considered a single type of wave (acoustic or elastic). For example, in these works [8,11,14] the solid media was considered as a hard wall (i.e., perfectly rigid, with no elastic waves). Yet, this idealization may not always be accurate due to the finite rigidity of the materials and the effect played by shear waves, which impact the BIC. To address this limitation, we introduce a simple and compact design that leverages the interplay between elastic waves and airborne acoustic cavity modes to create a hybrid QBIC endowed with extremely high- Q -factor even though the structure consists of a single compact (approximately wavelength) resonator made of silicon. We quantitatively characterize the effect of the shear mode in the silicon frame on the destructive interference with acoustic modes of the same parity that ultimately leads to the acoustoelastic accidental FW-QBIC. We believe that this understanding of multiphysics-induced QBIC may advance the field of acoustic BIC in elastodynamic resonators and foster their potential applications in amplifying, detecting, and/or filtering acoustic waves, presenting an exciting frontier on acoustic metamaterials and metasurfaces [46].

The remainder of this paper is organized as follows: We start by presenting the main results and discussion in several subsections. In the first one, we present the mode analysis and the mathematical framework of the model. Then, we explore the physics behind the interplay between elastic waves and the acoustic square-shaped resonator. Next, we provide a brief overview on the general mechanism of FW-QBIC and quantitatively analyze the near-field coupling between the two hybrid modes utilizing the temporal coupled-mode theory (TCMT). In the following subsection, we further unveil the pivotal role played by shear waves in constructing FW-QBIC as well as explore the essential conditions for achieving a high- Q -factor resonance. Finally, we summarize the main highlights of this contribution in the conclusion section.

II. RESULTS AND DISCUSSION

A. System and mathematical setup

Figure 1(a) illustrates the compact elastic resonator under investigation. The structure consists of a square air cavity with side length L surrounded by a square-shaped silicon frame of thickness d . The densities of silicon and air are taken as $\rho_s = 2329 \text{ kg/m}^3$, $\rho_a = 1.23 \text{ kg/m}^3$, respectively. The speed of sound in air is $c_a = 343 \text{ m/s}$, whereas in the silicon, the longitudinal and transverse velocities are $c_1 = 9660 \text{ m/s}$ and $c_2 = 5339 \text{ m/s}$, respectively.

To understand the wave-propagation behavior in such a system, we begin with decomposing the structure into two parts: an acoustic square cavity surrounded by hard walls and a square hollow elastic frame with free boundaries as schematized in the right panel of Fig. 1(a). In fact, the pressure field inside the acoustic cavity obeys the well-known Helmholtz equation, i.e., $(1/\rho_a)\Delta p + (\omega^2/K)p = 0$, with p , ρ_a , K denoting the pressure field, density, and bulk modulus of air, respectively. The boundary condition at the edge of the cavity, i.e., the hard wall, is expressed by $v_n = (1/\rho_a)(\partial p/\partial n) = 0$, where v_n denotes the normal component of the velocity field. It ensures a perfect cavity, i.e., without leaking to the outside. Inside the elastic frame with thickness d , the governing equation is $(\lambda + 2\mu)\nabla(\nabla \cdot \mathbf{u}) - \mu\nabla \times \nabla \times \mathbf{u} + \rho_s\omega^2\mathbf{u} = \mathbf{0}$, where $\mathbf{u}$ is the displacement field and $\mu = E/(2(1 + \nu))$ and $\lambda = Ev/((1 + \nu)(1 - 2\nu))$ are the Lamé coefficients, with E and ν being Young's modulus and Poisson's ratio of the elastic medium [47–49], taken here as silicon. We analyze the eigenmodes of the air cavity and the elastic frame and plot the representative ones in Fig. S1 within the Supplemental Material [50]. The cavity modes are labeled as M_{lm} where $l - 1$ and $m - 1$ represent the number of nodes (zeros) of the pressure-field distribution along the x and y directions, respectively. It is interesting to find that the eigenfrequencies of degenerate acoustic cavity modes (M_{13} and M_{31}), 6860 Hz, and the elastic mode (M_e), 6850 Hz, are very close. Therefore, we focus our study on the nearby frequencies to investigate the interplay among these modes when both acoustic and elastic components are considered together.

When the air cavity and the elastic frame are considered in a single system, the hard wall should be replaced by the real physical wall, and the acoustic cavity modes interact with the elastic frame mode via the interface between the silicon frame and the air cavity. The boundary conditions become (i) the pressure field must be equal to the normal component of the stress tensor [48] from the elastic frame, i.e., $p = -\sigma_{nn}$; (ii) the normal displacement across the interface is continuous, i.e., $u^p = u^s$, where u^p and u^s represent the displacement inside the fluid and solid, respectively; and (iii) vanishing of the tangential components of the shear stress at the interface of the silicon, i.e., $\sigma_{nt} = \sigma_{nz} = 0$, where n and t represent the normal and tangential directions, respectively, and z is the off-plane direction [49]. In the real structure, because of the existence of the elastic frame, the eigenmode of the ideal acoustic cavity with hard-wall boundaries (or the so-called normal mode: NM) becomes a quasinormal mode (QNM), in the partially open resonator system [left panel of Fig. 1(a)], characterized by a finite yet ultrahigh- Q -factor [51]. The original purely real eigenfrequency ω_0 turns into a complex eigenfrequency $\omega = \omega_0 + i\gamma_0$ where γ_0 denotes its decay rate.

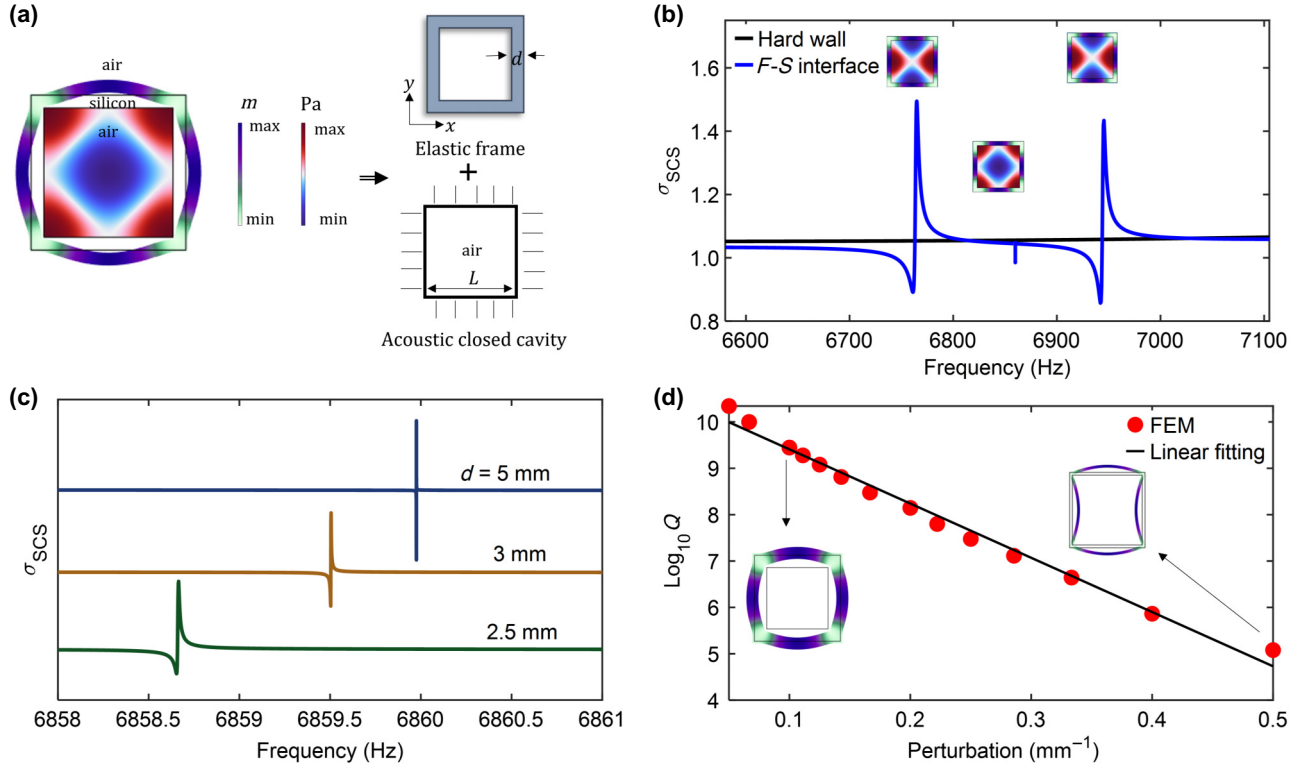


FIG. 1. Elastodynamic resonator. (a) Left panel: schematic of an elastodynamic resonator composed of a silicon frame embedded in air. The displacement field within the silicon frame is shown in purple and teal, while the pressure field inside the cavity is depicted in red and blue. Right panel: decomposition of the system into an elastic frame with thickness d and a closed acoustic cavity with side length L . (b) Normalized scattering cross section σ_{SCS} for a frame thickness of $d = 5.3$ mm, showing the emergence of three hybrid resonances near the degenerate acoustic modes M_{13} and M_{31} . The pressure-field distributions corresponding to each resonance are displayed above the peaks, highlighting the hybridization characteristics. (c) σ_{SCS} around the QBIC region in panel (b) illustrating its evolution as the frame thickness varies. (d) Evolution of the Q factor of the QBIC VS increasing perturbation ($1/d$) (red circles: in logarithmic scale). Linear fit ($\text{Log}_{10} Q = -11.7(1/d) + 10.6$) (solid black line).

B. Mechanism of the shear-induced FW-QBIC

The structure is excited by a plane pressure wave incident along the x axis, covering the frequency band from 6600 to 7100 Hz. The total acoustic scattering cross section of the structure is evaluated by $\sigma_{\text{SCS}}^0 = \int_0^{2\pi} |f(\theta)|^2 d\theta$, where $f(\theta)$ is the acoustic scattering amplitude [52] obtained using COMSOL Multiphysics [53]. In Fig. 1(b), we plot the normalized scattering cross section, defined as $\sigma_{\text{SCS}} = \sigma_{\text{SCS}}^0 / (L + 2d)$, in which we found three resonances occurring at frequency $6945.2 + i1.39$ Hz, $6860 + i2.5 \times 10^{-5}$ Hz, and $6765 + i1.57$ Hz respectively. Their corresponding field distributions are also shown in Fig. 1(b), which exhibits completely different behaviors from the closed-acoustic-cavity modes and the elastic mode in the frame shown in Fig. S1 within the Supplemental Material [50]. Two resonance modes at $6945.2 + i1.39$ Hz and $6765 + i1.57$ Hz show similar field-distribution patterns. The middle one at $6860 + i2.5 \times 10^{-5}$ Hz with a high-quality-factor is a QBIC. The evolution of this QBIC as a function of thickness of the elastic frame is plotted

in Figs. 1(c)–1(d), which show that the thinner the elastic frame, the broader the resonance, and the smaller the Q factor. This is not surprising because when the elastic frame gets thinner, the acoustic cavity modes are easier to leak from the cavity, resulting in stronger interaction between the interior and exterior of the structure via the elastic frame, despite the significant impedance mismatch between the two media.

To better understand these three modes, we fix the thickness of the square silicon frame $d = 5$ mm and the side length of the air cavity $L = 5$ cm. By solving the eigenproblem of this quasi-open system, we notice that some of the eigenfrequencies exhibit a negligible imaginary component, i.e., radiative decay rate $\gamma_0 \ll \omega_0$, which is a signature of an ultrahigh- Q -factor. The corresponding eigenstates, characterized thus by minimal loss, are reminiscent of the cavity modes: despite slightly radiating energy into the surrounding as $\gamma_0 \neq 0$, the fields of these eigenstates remain localized within the resonator and exhibit a similar pressure-field distribution to that of the

corresponding cavity modes, albeit with a slightly shifted frequency.

For the two degenerate cavity modes M_{13} and M_{31} , the degeneracy is induced by the square symmetry, i.e., the side lengths along the x and y directions are identical ($L_x = L_y$). We define the aspect ratio $R = L_x/L_y$ to characterize this symmetry. The eigenfrequencies of M_{13} and M_{31} vary as R changes, and only when $R = 1$ they overlap. Upon introducing the silicon frame with finite thickness, these two degenerate cavity modes interact with the elastic eigenmode of the frame, resulting in the emergence of three distinct hybrid modes that we denote by M_1 , M_2 , and M_3 . By assuming the elastic mode as a perturbation, we use the original cavity modes M_{13} and M_{31} as a basis to expand the hybrid modes as follows:

$$\psi^i(x, y) = A^i \psi_{13}(x, y) + B^i \psi_{31}(x, y), \quad (1)$$

where ψ_{13} and ψ_{31} represent the field distributions of M_{13} and M_{31} , respectively. A^i and B^i are the expansion

coefficients, with $i = 1, 2$ representing the mode index (see Note 2 within the Supplemental Material [50]). Notably, among these hybrid modes, M_2 exhibits a negligible imaginary component in its eigenfrequency at $6860 + i2.9 \times 10^{-5}$ Hz. In contrast, M_1 and M_3 have frequencies of $6886 + i0.23$ Hz and $6520.4 + i2.95$ Hz, respectively. In Fig. 2(a), we plot the real part of the eigenfrequencies of the three hybrid modes in red circles (M_1), blue diamonds (M_2), and green squares (M_3) versus the aspect ratio. The eigenfrequencies of two original cavity modes, as well as the elastic frame mode, are also plotted in Fig. 2(a) in black-dashed (M_{13} and M_{31}) and dotted lines (M_e). It is seen that the lines almost agree with the symbols, indicating the hybrid modes M_1 and M_2 are mostly contributed by the cavity modes, while M_3 is mainly from the elastic frame mode M_e .

To further clarify the development of the hybrid modes, we plot in Fig. 2(b) the pressure-field distribution corresponding to the points shown in Fig. 2(a), labeled from 1 to 12. From Fig. 2(b), we can find that away from

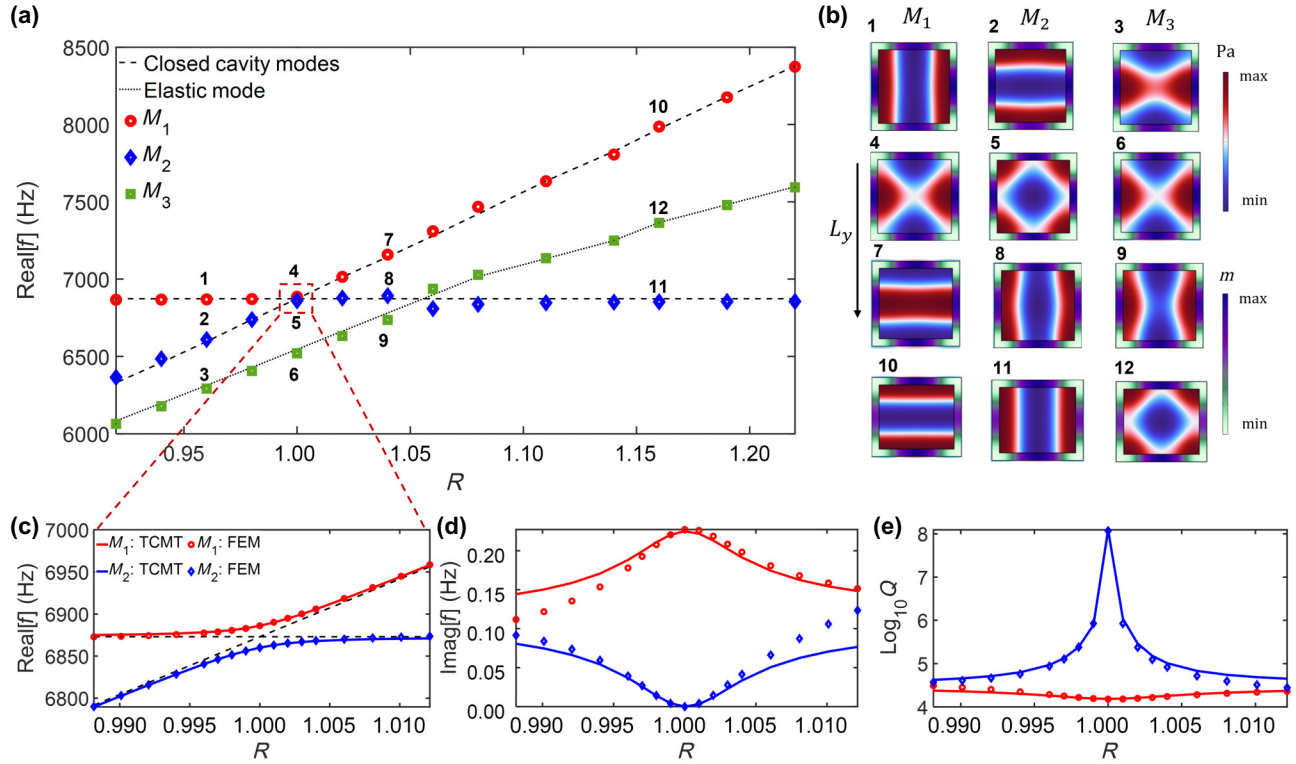


FIG. 2. TCMT for acoustoelastic FW-QBIC. (a) Real part of the eigenfrequencies of the three hybrid eigenmodes resulted from the interplay between the silicon frame and the acoustic cavity vs the aspect ratio $R = L_x/L_y$ of the resonator. The red circles, blue diamonds, and green squares represent elastodynamic eigenmodes M_1 , M_2 , and M_3 , respectively. The black-dashed and dotted lines depict the closed cavity modes (or NN) M_{13} and M_{31} and the pure elastic mode M_e , respectively. (b) Real part of the total pressure-field distribution inside the cavity as well as the corresponding displacement field in the silicon frame at each point denoted by numbers from 1 to 12, as marked in (a). The direction of the vertical arrow indicates increasing R (i.e., decreasing L_y). (c) Zoom in around the avoided-crossing region of the real part of the two eigenfrequencies modes. The red and blue continuous lines represent the analytically computed eigenfrequencies [via Eq. (3)]. The red circles and blue diamonds represent the numerical simulation data. The black dashed lines depict the closed cavity modes M_{13} and M_{31} . (d) Imaginary part of the two main modes vs aspect ratio R . (e) Q factor in logarithm scale of the two main modes vs the aspect ratio R .

$R = 1$, M_1 and M_2 modes are primarily cavity modes, with M_1 (M_2) transitioning from M_{31} (M_{13}) to M_{13} (M_{31}). Interestingly, the pressure distribution of the mode M_3 is influenced by its nearest eigenmode (i.e., closest eigenfrequency). At $R = 1$, the field patterns differ significantly from the other points. The node lines are no longer parallel and the fields are highly hybrid.

C. Eigenvalue formulation of the FW-QBIC

The evolution of M_1 and M_2 with varying R closely mirrors the classical process observed in the formation of FW-QBIC [8,33], which emerges as a result of destructive interference between two degenerate states of peculiar symmetries in a given wave system. In order to gain a quantitative understanding on the dynamics of eigenmode coupling and hybridization that leads to the acoustoelastic FW-QBIC, we consider a general two-level system interacting with its environment (cavity modes M_{13} and M_{31}) and employ the temporal coupled-mode theory (TCMT), which allows to monitor the temporal evolution of coupled modes via various mechanisms when the resonance frequency is much larger than its bandwidth [54–58]. We consider that the two-level system is described by its state vector $\mathbf{A} = (\psi_{13}, \psi_{31})^T$. The Hamiltonian is the operator that captures the temporal dynamics of the mode amplitudes in a way similar to the Schrödinger equation, i.e., $\mathcal{H}\mathbf{A} = -i\partial\mathbf{A}/\partial t$ [59]. In our case, $\mathcal{H}$ can be expressed by [1,60]

$$\mathcal{H} = \begin{bmatrix} \omega_{13} - i\gamma_{13} & \kappa - i\tilde{\gamma} \\ \kappa - i\tilde{\gamma} & \omega_{31} - i\gamma_{31} \end{bmatrix}, \quad (2)$$

where ω_{13} and ω_{31} are the angular frequencies of acoustic modes M_{13} and M_{31} where γ_{13} and γ_{31} are their respective decay rates. The parameters κ and $\tilde{\gamma}$ represent the near-field and far-field couplings between the two modes, respectively. The eigenvalues of this Hamiltonian are

$$\omega_{1,2} = \frac{1}{2} [(\omega_{13} + \omega_{31}) + i(\gamma_{13} + \gamma_{31})] \pm \frac{1}{2} [(\omega_{13} - \omega_{31})^2 - (\gamma_{13} + \gamma_{31})^2 + 4\kappa^2 - i2[(\omega_{13} - \omega_{31})(\gamma_{13} - \gamma_{31}) + 4\kappa\tilde{\gamma}]]^{1/2}. \quad (3)$$

In the absence of coupling (i.e., the closed cavity modes M_{13} and M_{31}) γ_{13} , γ_{31} , κ , and $\tilde{\gamma}$ are zero. Their angular frequencies ω_{13} and ω_{31} linearly depend on the aspect ratio R of the cavity [i.e., $\omega_{13} = (2\pi c/L_x) \times R$ and $\omega_{31} = 2\pi c/L_x$. Here L_x is constant and L_y is varying]. These two modes become degenerate, i.e., $\omega_{13} = \omega_{31}$, when $R = 1$. In the case of finite near-field coupling ($\kappa \neq 0$), the two newly formed eigenstates are M_1 and M_2 with amplitudes ψ^1 and ψ^2 and angular frequencies ω_1 and ω_2 , respectively. These hybrid modes experience an avoided-crossing at R , typical

of these systems in the presence of coupling [59]. Next, we focus on the behavior of the modes at the degenerate point (i.e., $\omega_{13} = \omega_{31} = \omega$) and suppose that the decay rates are similar (i.e., $\gamma_{13} \approx \gamma_{31} \approx \gamma$). In the case of strong coupling, i.e., the regime where $\kappa \gg \gamma_{13} + \gamma_{31}$, Eq. (3) can be approximated as (see Note 3 within Supplemental Material [50] for the detailed derivation):

$$\omega_{1,2} = \begin{cases} \omega_1 = (\omega + \kappa) + i\left(2\gamma - \frac{\gamma^3}{2\kappa^2}\right) + o\left(\frac{\gamma^3}{\kappa^2}\right), \\ \omega_2 = (\omega - \kappa) + i\left(\frac{\gamma^3}{2\kappa^2}\right) + o\left(\frac{\gamma^3}{\kappa^2}\right). \end{cases} \quad (4)$$

As expected, the real parts of the alternative eigenfrequencies [i.e., $\text{Re}(\omega_{1,2})$] linearly depend on the near-field coupling κ , resulting in an angular frequency separation of $\Delta\omega = 2\kappa$ between them. Furthermore, one of these eigenfrequencies (ω_2) exhibits a negligible imaginary part while the other one (ω_1) has an increased damping [i.e., $\text{Im}(\omega_1) \approx 2\gamma$]. Although the correction $\gamma^3/2\kappa^2$ is tiny, it cannot be completely ignored, since it indicates that the second eigenstate M_2 represents a QBIC rather than a true BIC, which is a key finding in our work.

To further confirm the existence of the multiphysics FW-QBIC, our investigation primarily focuses on the phenomenon of avoided-crossing between the two modes M_1 and M_2 . A closer examination around points 4 and 5 in Fig. 2(a) reveals the behavior of modes M_1 and M_2 near the degenerate point for different aspect ratios as shown in Fig. 2(c): the red and blue curves represent the eigenfrequencies $\omega_{1,2}$ derived from Eq. (3), while the red circles and blue diamonds correspond to ω_1 and ω_2 obtained from the full-wave simulations using COMSOL Multiphysics [53]. The excellent agreement between the simulated results and the TCMT ones verifies the validity of the model. It is useful to highlight that the modes M_1 and M_2 display avoided-crossing behavior, which is a result of the near-field coupling with a magnitude of $\kappa = 81.6 \text{ rad s}^{-1}$ in this case. The coupling at $R = 1$ plays a crucial role in facilitating the exchange of acoustic energy between the two eigenmodes, resulting in destructive interference between them (see Note 4 within Supplemental Material [50]). This can be observed in Figs. 2(d) and 2(e): Notably, at $R = 1$, the imaginary part of the first mode M_1 experiences a pronounced increase (red line and circles), resulting in a reduced Q factor to 1.5×10^4 . Conversely, the imaginary part of the second mode almost approaches zero (blue line and diamonds), leading to a maximized Q factor with a higher value of 1.17×10^8 , which indicates the emergence of FW-QBIC. It is crucial to note here that in the calculation of TCMT, we must account for the interaction among three original modes: M_{13} , M_{31} , and M_e . This system can be represented by a 3×3 Hamiltonian matrix

as well as three near-field coupling parameters. Yet, due to the strong acoustic-elastic impedance mismatch, the elastic mode couples only weakly to the acoustic eigenmodes away from the critical thickness. We therefore focus on a frame thickness of 5 mm (near the critical thickness), where the interaction between the two acoustic modes remains strong, while the elastic mode can be treated as a perturbation in the calculation of TCMT and near-field coupling. In this case we consider a 2×2 Hamiltonian matrix and as we can see from Figs. 2(c)–2(e) this approximation fits well with the simulation results, confirming our analysis.

D. Specific role of shear waves in FW-QBIC formation

The elastic frame enclosing an acoustic cavity does not guarantee the emergence of the FW-QBIC. Indeed, for a FW-QBIC to occur, the thickness of the frame must fall within a range of critical thicknesses, for which the resonant frequency of the elastic frame lies within the range of frequencies of the enclosed acoustic cavity. For example, when using silicon as the elastic material around a cavity with a side length of 5 cm, the critical thickness for the emergence of FW-QBIC lies near 5.3 mm. At this specific thickness, the elastic eigenmode M_e approaches the degeneracy point ($R = 1$), i.e., the crossing point in the closed cavity, thus intensifying the destructive interference between the cavity modes and thereby increasing the near-field coupling. Away from this critical thickness, the eigenfrequency of mode M_e shifts away from that of the cavity modes, reducing the destructive interference. For instance, Figs. 3(a) and 3(b) illustrate the relationship between the thickness of the silicon frame and the frequency of M_e , and the ratio of $Q_{\text{QBIC}}/Q_{\text{loss}}$ (corresponding in fact to Q_2/Q_1) between modes M_1 (lossy mode) and M_2 (QBIC). In Fig. 3(a), the black dashed and dotted lines represent the real part of the frequency of the closed-square cavity eigenmodes $f_{13} = f_{31}$ and the elastic eigenmode f_e with free boundaries, respectively, while the green squares represent the real part of the frequency of the shear eigenmode f_3 VS the thickness of the silicon frame d in mm. Notably, the two curves (cavity NM and elastic mode) intersect at approximately 5.3 mm, i.e., at the critical thickness, responsible for the acoustoelastic QBIC that we observe in our study. In Fig. 3(b), the ratio $Q_{\text{QBIC}}/Q_{\text{loss}}$ attains its maximum value, 4.4×10^4 , precisely at 5.3 mm, signifying the strong destructive interference occurring at that specific thickness. Interestingly, the ratio $Q_{\text{QBIC}}/Q_{\text{loss}}$ is proportional to $1/|\Delta f|$ where Δf represents the difference between the real parts of frequencies of the shear eigenmode f_e and the closed cavity mode $f_{13} = f_{31}$, which is a proof that the multiphysics QBIC observed within our resonator takes roots in the peculiar interaction between elastic and acoustic waves, a behavior that was shown in

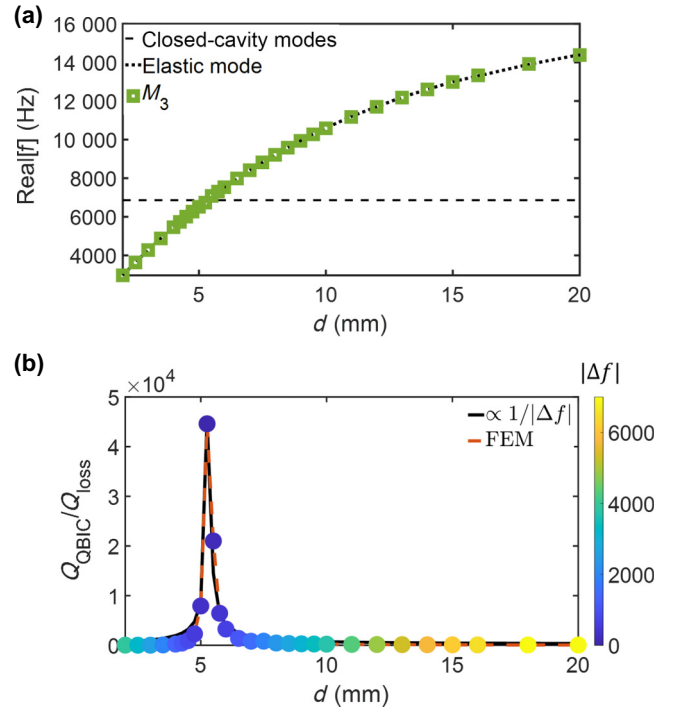


FIG. 3. Role of the shear mode. (a) Real part of the shear eigenfrequency (green squares) and the degenerate mode in the closed cavity $f_{13} = f_{31}$ (black dashed line). The two lines intersect at the thickness $d = 5.3$ mm. (b) Ratio between the Q factors of modes M_2 (QBIC) and M_1 (loss) as a function of the frame thickness d . The destructive interference maximizes at the critical thickness $d = 5.3$ mm. The color bar indicates the differences between the real parts of the shear eigenfrequency and the degenerate closed cavity modes $|\Delta f| = |f_e - f_{13/31}|$.

a related study [44], albeit not explained quantitatively as we do here.

It is also worthwhile to emphasize that the critical thicknesses vary if the material of the elastic frame changes. For a steel frame enclosing the same acoustic cavity, the elastic eigenmode approaches the degeneracy point at a critical thickness of 11.6 mm, at which $Q_{\text{QBIC}}/Q_{\text{loss}}$ reaches its maximum value 3×10^5 . Beyond the critical point, it gradually diminishes until it eventually reaches a minimum level on the order of 10 as shown in Fig. S4 in Note 5 of the Supplemental Material [50]. Table I provides an overview of the critical thickness for different materials, along with the corresponding ratios of the two Q factors.

TABLE I. Critical thickness (CT) for different materials.

Material	CT (mm)	Q_{QBIC}	Q_{loss}	$Q_{\text{QBIC}}/Q_{\text{loss}}$
Silicon	5.3	1.5×10^8	3.4×10^3	4.4×10^4
Aluminum	11.3	6×10^8	4.8×10^3	1.25×10^5
Steel	11.6	5.2×10^9	1.8×10^4	3×10^5
Iron	11.8	5.4×10^9	1.8×10^4	9.1×10^4

To delve deeper into the impact of shear waves within the elastic material on the formation of this peculiar FW-QBIC, we conducted two separate investigations: In the first one, the solid medium in the frame is replaced with a fictitious fluidlike medium that shares the same density as silicon, i.e., $\rho = 2329 \text{ kg/m}^3$ and the same longitudinal velocity $c_1 = 9659.9 \text{ m/s}$, referred to as “silicon fluid.” Only two modes exist in this case, as further detailed in Fig. S5 within Supplemental Material [50]. The destructive interference quantified by $Q_{\text{QBIC}}/Q_{\text{loss}}$, associated with these modes is only 1.2. The absence of the shear modes results in diminished destructive interference, leading to the disappearance of FW-QBIC within the system. In the second investigation, we artificially alter the shear-wave velocity of the elastic frame around the known value of silicon ($c_2 = 5339 \text{ m/s}$), while maintaining a constant longitudinal velocity of sound as well as density equal to those of silicon. We constructed four materials with different shear-wave velocities and calculated the corresponding eigenfrequencies of the elastic mode: $c_2 = 2669 \text{ m/s}$, $f_e = 3763.6 \text{ Hz}$; $c_2 = 6000 \text{ m/s}$, $f_e = 6925 \text{ Hz}$; $c_2 = 6830 \text{ m/s}$, $f_e = 7124 \text{ Hz}$. A clear redshift of the eigenfrequency can be observed as the shear velocity decreases, as shown in Fig. 4. As the elastic mode M_e approaches the degenerate point at 6860 Hz, depicted in Fig. 4(c), the destructive

interference between modes M_1 and M_2 becomes increasingly pronounced, leading to stronger near-field coupling ($\kappa = 207.3 \text{ rad s}^{-1}$). Consequently, the avoided-crossing between M_1 and M_2 is more prominent. In contrast, when the elastic eigenmode lies far from the degenerate point, as shown in Fig. 4(a), the destructive interference weakens. This reduction results in a much smaller near-field coupling ($\kappa = 12.6 \text{ rad s}^{-1}$) and a lower quality factor ratio $Q_{\text{QBIC}}/Q_{\text{loss}}$, as further demonstrated in Fig. S5 within the Supplemental Material [50].

In our study, we treat a two-dimensional air-filled square frame, which is indeed a simplified version. A practical setup consists of fabricating a long square waveguide (inner side $a = 5 \text{ cm}$, length $L = 50 \text{ cm} \gg a$) from, e.g., silicon, steel, or aluminum; and mount it vertically in an anechoic (absorbing) environment to suppress any reflections. The excitation can be provided by a loudspeaker or a small phased array to approximate a plane wave at one end. At the other end, use a calibrated microphone array to record the transmitted pressure over the frequency band of interest. From the optical theorem, the total scattering cross section can be calculated from the forward channel, so only the forward scattered field needs to be measured rather than its full angular distribution [61].

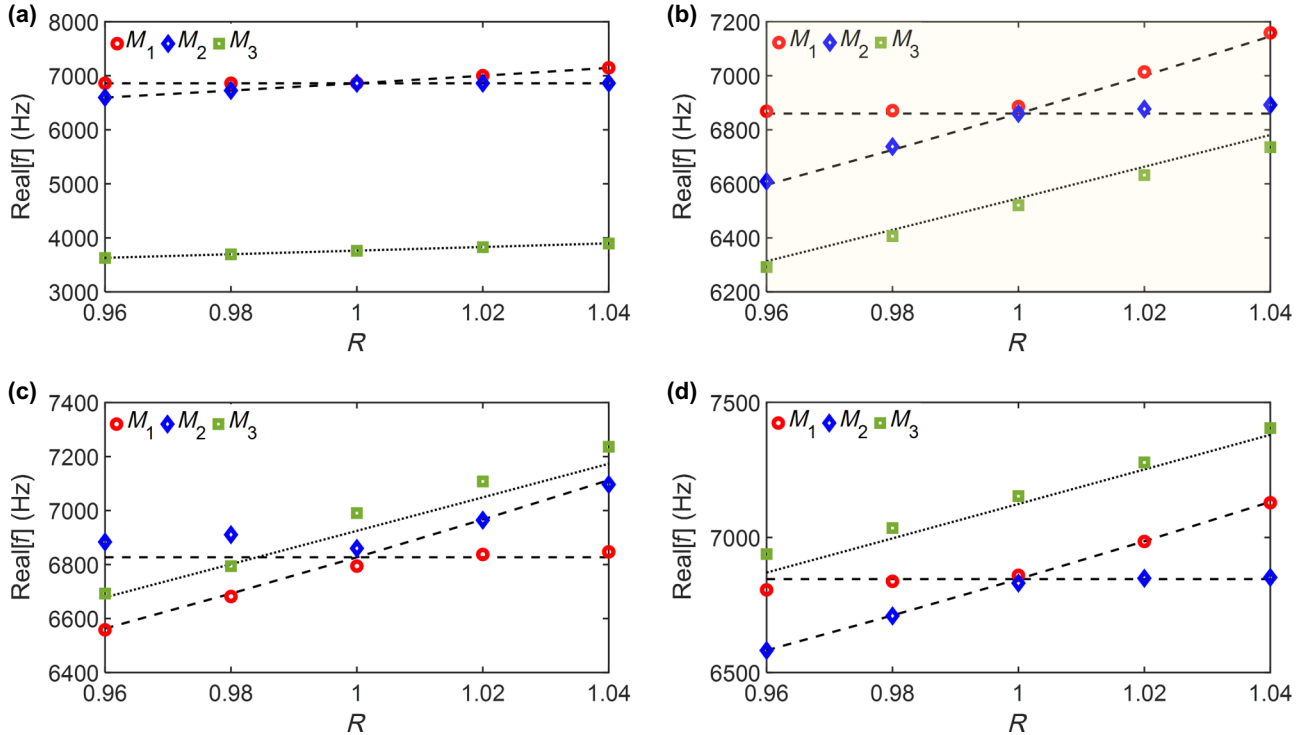


FIG. 4. Role of the shear-wave velocity. Real frequencies of hybrid modes M_1 , M_2 , and M_3 . Dashed lines indicate the acoustic cavity modes, and dotted lines depict the elastic mode M_e varying with shear velocity in the frame. (a) $c_2 = 2669 \text{ m/s}$, (b) $c_2 = 5339 \text{ m/s}$ (silicon), (c) $c_2 = 6000 \text{ m/s}$, and (d) $c_2 = 6830 \text{ m/s}$. Near-field coupling constants: $\kappa = 12.6, 81.7, 207.3$, and 88 rad s^{-1} , respectively.

III. CONCLUSION

In this work, we find an accidental FW-QBIC induced by the interplay of acoustic and elastic waves in silicon compact resonators. We conduct full-wave numerical simulations as well as theoretical analysis based on the temporal coupled-mode theory to determine the eigenmodes of the resonator and evaluate the near-field coupling. The agreement between the two methods is excellent. Significantly, we show that the presence of shear waves leads to the emergence of an additional eigenmode associated with M_{13} and M_{31} within the closed cavity. This extra shear mode, located near the degenerate point, induces destructive interference between the two primary modes, resulting in the formation of FW-QBIC with extremely high- Q -factors, even though the system is compact (on the order of background sound wavelength). The frequency of the elastic eigenmode (or shear mode) is demonstrated to be dependent on the thickness of the material's frame. Each material showcases a critical thickness at which the acoustic cavity modes intersect with the elastic mode, maximizing the destructive interference between the two acoustic cavity modes, thereby enabling the emergence of the accidental acoustoelastic FW-QBIC.

This model can be used as a practical acoustic sensor for long square waveguides. The QBIC is sensitive to the changes of air velocity c_a inside the waveguide, with velocity sensitivity $S = \Delta f / \Delta c$ and a figure of merit given by $FoM = S \times Q / f_0$, where f_0 is the resonance frequency and Δf represents the frequency shift. In our study, FW-QBIC shows a velocity sensitivity of roughly 20 Hz/m s^{-1} together with an extremely high $FoM \simeq 3.4 \times 10^5 \text{ m}^{-1} \text{ s}$, which makes it ideal for precise fluid property monitoring. In future work, we will extend this elastodynamic-BIC framework to enhance acoustic sensing by exploiting coupled acoustoelastic wave interactions in related design.

In this context, our contribution may offer several advantages on the design of innovative multiphysics BIC devices. Our structure is simple to fabricate, as it consists of a single open resonator setup, made of silicon and empowered with an extremely high- Q -factor. This design also permits to avoid the experimental imperfections from periodic materials such as metamaterials or metasurfaces, especially that it is well known that BIC are very sensitive to such tiny defects. Hence, we believe that our design can be easily fabricated and characterized experimentally, which may result in a precise toolbox for future high- Q devices. Yet, it would be interesting to explore different mechanisms of coupling of waves to the cavity or between multiresonator systems.

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DATA AVAILABILITY

The data that support the findings of this article are not publicly available. The data are available from the authors upon reasonable request.

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