

# Electrically small Rydberg sensor for three-dimensional determination of radio-frequency $k$ -vectors

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We present an electrically small Rydberg atom electric field sensor with the ability to extract the three-dimensional  $k$ -vector of an elliptically polarized radio-frequency field. In most mediums, the  $k$ -vector (or wave vector) provides the direction of propagation of an electromagnetic wave. Our method uses a field vector measurement at a single point in space and is thus compatible with a sensor volume that is arbitrarily small compared with the carrier wavelength. We measure the  $k$ -vector of a circularly polarized signal field with average absolute errors in the polar and azimuthal angles of 33 and 43 mrad, respectively, and statistical noise of 1.3 and 1.5 mrad/ $\sqrt{\text{Hz}}$ , respectively. Additionally, we characterize the performance of the sensor as a function of the ellipticity of the input field and the size of the sensing region. We find that the sensor works over a broad range of ellipticities, and validate that an electrically small sensing region is optimal.

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## I. INTRODUCTION

Electric field sensors are powerful tools for field characterization and communication. However, measuring the electric field vector at a single point in space is insufficient to fully characterize an electromagnetic wave. To determine the  $k$ -vector of an incoming linearly polarized plane wave, which is defined to be perpendicular to the wave fronts of the wave, one must make a spatially extended measurement, make simultaneous measurements of the electric and magnetic field vectors [1], or measure the momentum vector of the wave, for example, through particle recoil [2]. With electric field measurements alone, only spatially extended measurements that cover an appreciable fraction of the wavelength will provide the  $k$ -vector, such as with an interferometer [3], focusing element [4], or other highly directional antenna, and the signal decreases to zero when spatial extent of the measurement is decreased well below the wavelength [5]. These extended methods are the most common, and work by measuring the phase difference between points in space, either directly or through interference, to deduce the  $k$ -vector. This restriction is relaxed for elliptically polarized plane waves, where the electric field vector traces out an ellipse and the  $k$ -vector is constrained to be normal to this ellipse. Thus, for elliptically polarized

plane waves, the  $k$ -vector can be determined up to a sign by one measuring the electric field vector at a single point in space.

Rydberg electric field sensors offer compelling capabilities beyond the capabilities of classical antennas, including their large tunability [6–8], nonabsorptive sensing [9], traceability to the International System of Units [9,10], and small size [11]. However, previous  $k$ -vector measurement methods with Rydberg electric field sensors have relied on at least two spatially separated measurements to determine the angle of a source [12–15]. This method becomes less effective as the sensor becomes significantly electrically small.

In this work, we demonstrate an electrically small quantum sensor that measures the  $k$ -vector for elliptically polarized radio-frequency (rf) fields. Building on our previous development of a Rydberg atom polarimeter [16], we extract the three-dimensional  $k$ -vector by measuring the polarization ellipse with a single optical probe measurement. This method allows the sensor to be arbitrarily small relative to the carrier wavelength, and thus operates down to quasi-dc fields. We quantitatively assess the  $k$ -vector measurement, the impact of interaction region size relative to the rf wavelength, and the dependence of the accuracy and precision of the measurement on the ellipticity of the input field. The three-dimensional and inherently electrically small nature of our method opens up new possibilities in subwavelength field characterization and alternatives to extended apertures or arrays in radar systems or radio-frequency telescopes as in most cases the  $k$ -vector corresponds to the propagation direction of the wave.

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## II. METHODS

The basic principle of our sensor is discussed in Ref. [16], and the apparatus is detailed in Fig. 1.  $^{85}\text{Rb}$  atoms in a vapor cell are excited to a Rydberg state through a two-photon process with probe and coupling lasers where they become sensitive to a signal rf field ( $S$ ) resonant with an atomic transition between Rydberg states. We measure the amplitude and phase of the incoming signal field projected along each cardinal axis through rf heterodyne measurements [8,17,18] with three local oscillators ( $\text{LO}_X$ ,  $\text{LO}_Y$ ,  $\text{LO}_Z$ ) polarized along each axis. The rf heterodyne beats are present in the Autler-Townes splitting of the Rydberg states, and are measured by the absorption of the probe beam through electromagnetically induced transparency spectroscopy. We record the transmission of the probe beam for 100 ms per measurement unless otherwise stated. By applying different detunings to each LO, we separately extract from the Fourier transform of the probe beam transmission the beat amplitude for each axis  $|b_x|$ ,  $|b_y|$ ,  $|b_z|$  and the relative phases  $\phi_{xy}$ ,  $\phi_{xz}$  of the field projection along each axis, where  $\phi_{ij}$  is the phase difference between the projection along  $\hat{i}$  and the projection along  $\hat{j}$  ( $\phi_i - \phi_j$ ). These quantities provide a measurement of the electric field vector in three dimensions.

We have modified the previous apparatus [16] to implement  $k$ -vector measurements. Using a gimble, we vary the azimuthal  $\Phi$  and polar  $\theta$  angles of the rf horn antenna that transmits the signal field as shown in Fig. 1. Previously, this horn was restricted to the  $x$ - $y$  plane. The azimuthal angle is set as  $\Phi \in [\pi/8, 3\pi/8]$  and the polar angle is set as  $\theta \in [\pi/4, 3\pi/4]$ . The signal field is transmitted by a dual-polarization horn, allowing us to adjust the ellipticity of the

field by changing the relative phases of the two separate inputs (labeled  $S_v$  and  $S_h$ ). We maintain an rms amplitude of  $0.06(1) \text{ V m}^{-1}$  for the signal field and  $0.14(2) \text{ V m}^{-1}$  for the LOs throughout the experiment. Unless otherwise specified, we use a 2.5-cm-long vapor cell and a signal field on resonance with the Rydberg transition  $70D_{5/2} \rightarrow 69F_{7/2}$  at 6.64 GHz (4.51 cm). This configuration differs from the previous apparatus in that it keeps the interaction region smaller than the rf wavelength, and suppresses the signal averaging effect discussed later. We also keep LO detunings small in comparison with the previous iteration ( $\delta_X = 27 \text{ kHz}$ ,  $\delta_Y = -20 \text{ kHz}$ ,  $\delta_Z = 30 \text{ kHz}$ ) to avoid any negative effects from the presence of Earth's magnetic field. The coordinate axes are defined such that  $\hat{z}$  is along the optical axis, and the spherical coordinates are conventionally defined relative to our gimbal mechanism (this differs from the coordinate system in Ref. [16]). These improvements allow us to fully investigate the sensor's ability to measure  $k$ -vectors over a significant solid angle.

## III. $k$ -VECTOR MEASUREMENT

The three beat amplitudes ( $|b_x|$ ,  $|b_y|$ ,  $|b_z|$ ) and two relative phases ( $\phi_{xy}$ ,  $\phi_{xz}$ ) measured by the system are sufficient to extract the  $k$ -vector up to a sign in the following way. The field vector in time is reconstructed (up to an absolute phase) as

$$\begin{aligned} \vec{E}(t) \propto & |b_x| \cos(\omega_0 t) \hat{x} \\ & + |b_y| \cos(\omega_0 t - \phi_{xy}) \hat{y} \\ & + |b_z| \cos(\omega_0 t - \phi_{xz}) \hat{z}, \end{aligned} \quad (1)$$

where  $\omega_0$  is the signal frequency. Provided that the signal field is not purely linearly polarized, the field traces out an ellipse in three dimensions, and we can extract two orthogonal vectors along this ellipse when  $t = 0$  and  $t = \pi/2\omega_0$ :

$$\vec{v}_1 = |b_x| \hat{x} + |b_y| \cos(-\phi_{xy}) \hat{y} + |b_z| \cos(-\phi_{xz}) \hat{z}, \quad (2)$$

$$\vec{v}_2 = |b_y| \sin(-\phi_{xy}) \hat{y} + |b_z| \sin(-\phi_{xz}) \hat{z}. \quad (3)$$

For a plane wave, the  $k$ -vector must be orthogonal to the field vector at all times, and we can extract it up to a sign from the cross product of  $\vec{v}_1$  and  $\vec{v}_2$ :

$$\hat{k} = \pm \frac{\vec{v}_1 \times \vec{v}_2}{|\vec{v}_1| |\vec{v}_2|}. \quad (4)$$

In this work, we are considering only the unit  $k$ -vector, as the frequency of the field is presumed to be known or easily determined from the heterodyne measurements. The polar

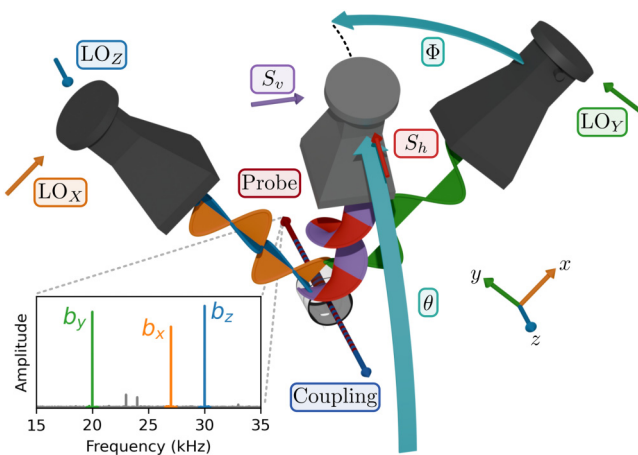


FIG. 1. The sensor. This diagram shows the orientation of the LO and signal field inputs and rf polarizations relative to the optical beams and the vapor cell. The Cartesian coordinate system for the field vectors and the spherical coordinate system for the  $k$ -vector are defined here. An example spectrum extracted from the probe transmission is shown in the inset.

and azimuthal angles can then be found as

$$\theta = \arccos(\hat{k} \cdot \hat{z}), \quad \Phi = \arccos\left(\frac{\hat{k} \cdot \hat{x}}{\sqrt{(\hat{k} \cdot \hat{x})^2 + (\hat{k} \cdot \hat{y})^2}}\right). \quad (5)$$

To assess this approach to measuring the signal  $k$ -vector, we rotate the signal horn around the cell, sweeping over a grid of polar angles from  $\pi/4$  to  $3\pi/4$  and azimuthal angles from  $\pi/8$  to  $3\pi/8$  with a step size of  $\pi/36$  ( $5^\circ$ ), representing a solid angle of  $\pi/2\sqrt{2}$ . We also sweep the phase offset  $\phi_{vh}$  over  $2\pi$  to apply fields with different ellipticities. We then use the method described above to extract our measured polar and azimuthal angles.

At each  $k$ -vector, we model variations from the expected behavior due to rf reflections present in the interaction region. This modeling, which also corrects for the LO amplitudes and phases, is described in Ref. [16]. By fitting data over a variety of input polarizations, the model provides a matrix ( $A$ ) that represents the relative amplitudes and phases of the reflections from one axis into another. For instance,  $A_{ij}$  represents the combined amplitude and phase from all reflections of  $\text{LO}_i$  that project along axis  $\hat{j}$ . This model assumes that the reflections are constant across the interaction region and do not depend on the amplitude of the input fields. As the signal horn moves to different  $k$ -vectors, the character of the rf reflections changes. We first ignore this variation and apply the average correction equivalently across all  $k$ -vectors, which works mostly to calibrate the LO amplitudes and phases. The average correction parameter matrix  $A$  is as follows:

$$A = \begin{bmatrix} 1.003 & 0.028e^{1.75} & 0.034e^{0.20} \\ 0.050e^{0.24} & 0.948e^{-2.10} & 0.020e^{0.98} \\ 0.003e^{0.88} & 0.017e^{-3.13} & 1.000e^{0.47} \end{bmatrix}, \quad (6)$$

where the diagonal elements primarily represent corrections to the LO amplitudes and phases, and the off-diagonal elements represent reflection amplitudes and phases. The resulting polar and azimuthal angles from this averaged correction, restricted to purely left or right circularly polarized input fields, are presented in Figs. 2(a), 2(c), 2(e), and 2(g). The accuracy of this method, defined as the average difference between the applied and measured angles, is 60 mrad in  $\theta$  and 63 mrad in  $\Phi$ . In these data we see a systematic coupling between  $\theta$  and  $\Phi$ . Most clearly, for left circularly polarized light,  $\Phi$  is underestimated at low  $\theta$ . This effect is likely due to the changing character of the rf reflections as a function of the  $k$ -vector. Specifically, at high and low polar angles, the projection of the input field along  $\hat{z}$  is small, and thus systematic effects of reflections on the measurement have greater impact.

Over the measured  $k$ -vectors, the elements of the parameter matrix  $A$  vary with standard deviations in amplitude

( $\sigma_{|A|}$ ) and phase ( $\sigma_\phi$ ), respectively, of

$$\sigma_{|A|} = \begin{bmatrix} 0.144 & 0.036 & 0.036 \\ 0.027 & 0.097 & 0.033 \\ 0.033 & 0.038 & 0.119 \end{bmatrix}, \quad \sigma_\phi = \begin{bmatrix} 0 & 1.20 & 0.99 \\ 0.48 & 0.09 & 1.12 \\ 1.85 & 2.23 & 0.13 \end{bmatrix}. \quad (7)$$

We account for these changing reflections and address the systematic coupling between  $\theta$  and  $\Phi$  by applying an iterative correction approach. First, we create a map of reflection parameters as a function of the true input  $k$ -vector. Then, for a given data point, we take the measured  $k$ -vector using the reflection averaging method described above as a best guess, and apply the reflection parameters from the closest  $k$ -vector in the map. This generates a new and more accurate  $k$ -vector, and the process converges rapidly when iterated. The measured angles found with this method with three iterations are shown in Figs. 2(b), 2(d), 2(f), and 2(h). In these data, the systematic coupling between  $\theta$  and  $\Phi$  is largely removed, and the accuracy is improved to 33 mrad in  $\theta$  and 43 mrad in  $\Phi$ . The remaining deviations are likely from instabilities in the iterative correction where the initially measured  $k$ -vector is too far from the true value, or the structure in the rf reflections is too fine to be captured by the map.

#### IV. INTERACTION REGION AVERAGING

Our measurement averages over the spatial extent of the interaction region. When the  $k$ -vector of the signal field is not aligned with the  $k$ -vector of a particular LO, and the interaction region is an appreciable fraction of the carrier wavelength, the phase of the respective heterodyne beat will vary in space. Our measured signal is the result of averaging over this phase gradient. In our system, the optical beams are negligibly small relative to the carrier wavelength with a ratio of approximately 0.02, so only the averaging along the optical axis ( $\hat{z}$ ) is significant. Along this axis, the phase gradient of the beat has a wavelength  $\lambda_p$  given by

$$\lambda_p = \lambda_0 / |\cos(\theta)|, \quad (8)$$

where  $\lambda_0$  is the wavelength of the rf carrier. When we perform averaging over the interaction region of length  $d$ , we get a normalized amplitude reduction of

$$|b_i|/|S_i| \propto |\text{sinc}(\pi d/\lambda_p)|, \quad (9)$$

where  $S_i$  is the predicted amplitude of the signal field projected onto axis  $\hat{i}$ . Since in our system all LO's are perpendicular to the optical axis, all the beat amplitudes

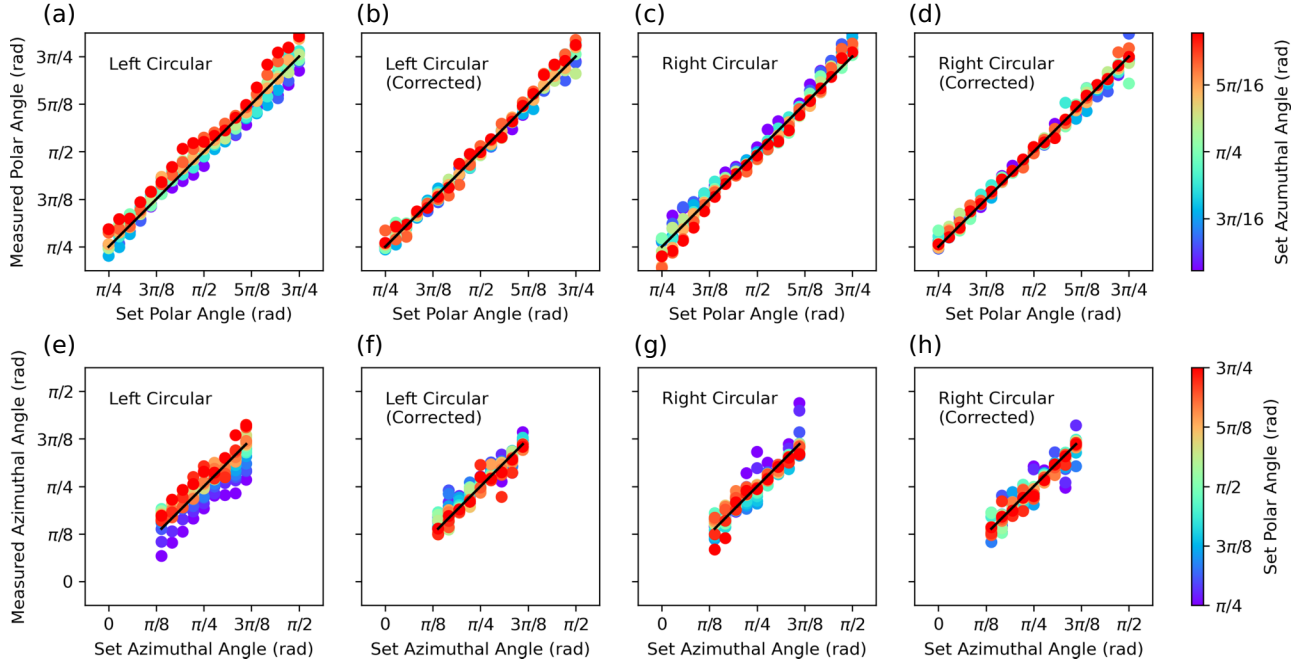


FIG. 2. Comparison of the set and measured polar [(a)–(d)] and azimuthal [(e)–(h)] angles of the signal field  $k$ -vector. The results for left circularly polarized light are shown on the left [(a),(b),(e),(f)] and the results for right circularly polarized light are shown on the right [(c),(d),(g),(h)]. The data in (a),(c),(e),(f) have an average calibration applied, as described in the main text, which largely accounts for LO amplitudes and phases. These data have an average accuracy of 60 mrad in  $\theta$  and 63 mrad in  $\Phi$ . The data in (b),(d),(f),(h) are corrected with the use of the iterative approach described in the main text. These data have an average accuracy of 33 mrad in  $\theta$  and 43 mrad in  $\Phi$ . The black lines with a slope of 1 are included to guide the eye.

should average equally. This should affect only the measured total amplitude of the field and the noise in the  $k$ -vector measurement, but not the accuracy.

We evaluate this averaging effect by testing the system with three different vapor cell lengths (1, 2.5, and 7.6 cm) and two different carrier wavelengths (4.5 and 2.4 cm). The carrier wavelengths correspond to the  $70D_{5/2} \rightarrow 69F_{7/2}$  resonance at 6.64 GHz and the  $57D_{5/2} \rightarrow 56F_{7/2}$  resonance at 12.47 GHz. These parameters yield values for  $d/\lambda_p$  ranging from 0.2 to 3.2, where the interaction region length  $d$  is the full length of the vapor cell. The result of this averaging effect on  $|b_z|/|S_z|$ , normalized to the value at  $\theta = \pi/2$ , can be seen in Fig. 3. The data exhibit the expected behavior, with deviations likely due to changes in the rf absorption and reflection through the cell at different angles. The minima of the sinc function correspond to locations where the signal is averaged over exactly a single period of the phase gradient, and are resolvable in the data for  $d/\lambda_0 = 1.7$ , where this constraint is satisfied at  $\theta = \pi/2 \pm 0.63$ . Although not shown here, the averaging effect looks similar in  $|b_x|/|S_x|$  and  $|b_y|/|S_y|$  for the longer wavelength  $\lambda_0 = 4.5$  cm. However, when  $\lambda_0 = 2.4$  cm, close to the cell diameter of 2.54 cm, and the signal is normal to the cylindrical walls of the cell,  $|b_x|/|S_x|$  and  $|b_y|/|S_y|$  differ significantly from theory, and from cell to cell. This could be due to variation in the cell geometry or the distribution

of rubidium condensed on the walls. This effect is present to a lesser extent in  $|b_z|/|S_z|$ , where the largest deviations are for  $d/\lambda_0 = 1.0$  and  $d/\lambda_0 = 3.2$ , which also correspond

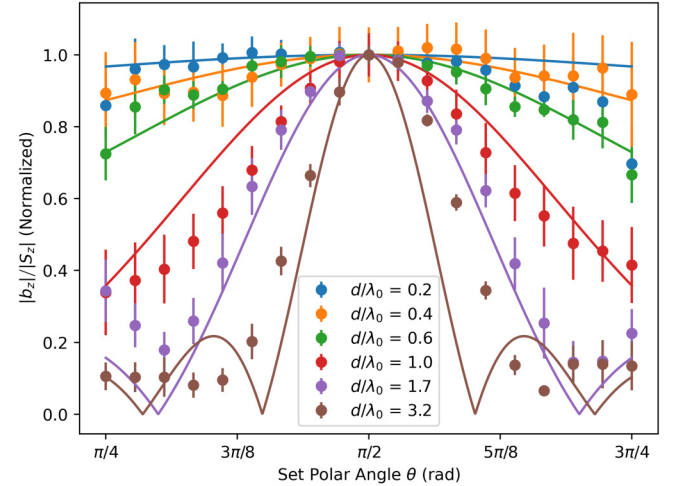


FIG. 3. Effect of averaging over the interaction region on the ratio of the measured beat amplitude  $|b_z|$  to the expected  $z$  signal projection  $S_z$  as a function of  $\theta$  for a variety of cell lengths and carrier wavelengths. The solid lines correspond to the effect predicted from Eq. (9).

to  $\lambda_0 = 2.4$  cm. We attribute this wavelength- and angle-dependent effect to interference from reflections off the cell walls, providing evidence that most of the reflection effects we see in the system are due to the vapor cell rather than the external environment.

These data confirm that the solid angle over which the  $k$ -vector can be measured increases as the interaction region becomes small relative to the carrier wavelength. In general, there will be a trade-off between absolute sensitivity, which typically improves with interaction volume through increased atom number, and the sensor's field of view. However, in principle, the sensor can be made arbitrarily electrically small. For instance, we are able to measure the  $k$ -vector in three dimensions even when the sensor is significantly electrically small along  $\hat{x}$  and  $\hat{y}$ .

### V. ELLIPTICITY DEPENDENCE

For circular or elliptical signal polarization one can define a single  $k$ -vector perpendicular to the electric field at all times. For pure linear polarization, the  $k$ -vector can be restricted only to a plane. Thus, our ability to extract the  $k$ -vector depends on the ellipticity of the input field. To investigate this, we scan the phase offset between the  $S_v$  input and the  $S_h$  input, which changes the signal between linear and circular polarization. Figure 4(a) shows the average magnitude of the error in the measured  $\theta$  and  $\Phi$  as a function of the signal phase offset  $\phi_{vh}$  averaged over all measured  $k$ -vectors. These data are corrected with the use of the iterative correction method described above. The error exhibits the expected sharp peaks near linear polarization, but is flat near circular polarization. The values at

pure linear polarization are likely due to a combination of residual ellipticity and the error associated with restricting the  $k$ -vector to a plane. There is some asymmetry that most notably causes the minimal error in  $\theta$  to occur at an ellipticity that is not perfectly circular, and which we attribute to residual effects from reflections.

We also investigate the statistical error per  $k$ -vector by splitting the data trace of each measurement into ten time bins, each 10 ms long, extracting the standard deviation in the  $\theta$  and  $\Phi$  measurements, and normalizing the noise to the integration time. The average noise in the angle measurements as a function of signal phase offset is shown in Fig. 4(b). Compared with the accuracy, the noise has a slightly flatter dependence on signal ellipticity, but the asymmetries remain. The minimal noise is  $1.3 \text{ mrad}/\sqrt{\text{Hz}}$  for  $\theta$  and  $1.5 \text{ mrad}/\sqrt{\text{Hz}}$  for  $\Phi$ . The difference in the noise for  $\theta$  and  $\Phi$  is expected as  $\Phi$  involves more measured quantities in its calculation. The statistical noise here corresponds to a field sensitivity of  $58 \mu\text{V}/\text{m}\sqrt{\text{Hz}}$ , which is similar to the sensitivity of the previous experimental iteration [16]. In all, these results show that the  $k$ -vector can still be extracted over a wide range of ellipticities.

### VI. CONCLUSIONS

We have demonstrated  $k$ -vector measurements with an electrically small Rydberg electric field sensor for elliptically polarized input fields. The accuracy of the current sensor is likely limited by rf reflections inside the vapor cell. Vapor cell design, or more precise modeling of reflections, could lessen this effect. Additional development could also address the trade-off between a small interaction region and signal size. For instance, our method should be compatible with a noncollinear three-photon Doppler-free excitation scheme [19], which could provide higher signal sizes for small interaction regions. The inability of the sensor to measure the  $k$ -vector of linearly polarized fields could also be addressed by the incorporation of rf phase plates to circularize the input field [20–22]. However, in some cases the birefringence of the propagation medium, such as the ionosphere, may provide sufficiently elliptical signals [23,24].

Our sensor uses the power of rf heterodyne measurements in Rydberg atom electric field sensors to extract the  $k$ -vector in three dimensions with a single measurement in a platform that can be arbitrarily electrically small. This expands the potential applications of such sensors and demonstrates a more comprehensive characterization of the measured field than was previously possible with this platform.

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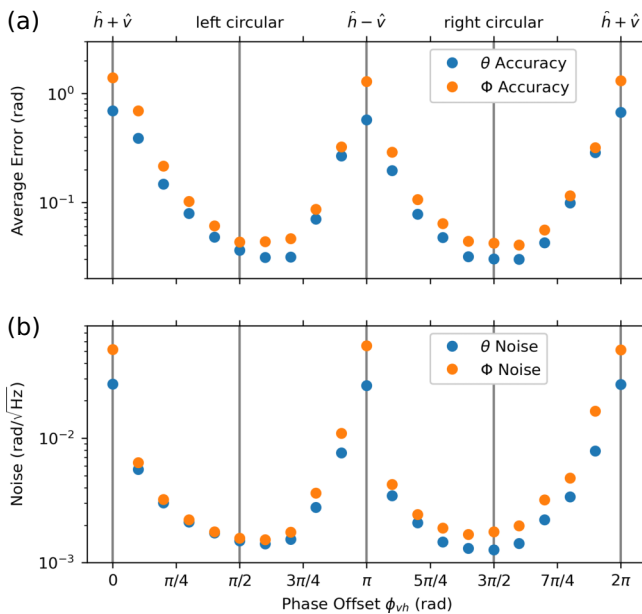


FIG. 4. (a) Accuracy and (b) noise in the measured polar and azimuthal angles as a function of the polarization ellipticity.

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### DATA AVAILABILITY

The data that support the findings of this article are not publicly available upon publication because it is not technically feasible and/or the cost of preparing, depositing, and hosting the data would be prohibitive within the terms of this research project. The data are available from the authors upon reasonable request.

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