

Universal scaling of adiabatic tunneling out of a shallow confinement potential

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The ability to tune quantum tunneling is key for achieving selectivity in manipulation of individual particles in quantum technology applications. In this work we count electron escape events out of a time-dependent confinement potential, realized as a dynamic quantum dot in a GaAs/AlGaAs heterostructure. A universal scaling relation of the escape probability as a function of potential barrier rise time and depth is established and developed as a method to probe tunneling rates over many orders of magnitude reaching the limit of shallow anharmonic confinement. Crossover to thermally activated transport is used to estimate the single time-energy scale of the universal model. In application to metrological single-electron sources, *in situ* calibrated control signals greatly extend the accessible dynamical range for probing the quantization mechanism. Validation of the cubic potential approximation sets a foundation for microscopic modeling of quantum tunneling devices in the shallow confinement regime.

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I. INTRODUCTION

Tunneling through a potential barrier is a hallmark quantum phenomenon and a fundamental element for evolving quantum technologies, enabling future and limiting existing applications [1–3]. The exponential sensitivity of tunneling rates to the barrier shape provides discriminative power for quantum state initialization [4,5], read-out [6], or quantum logic operations [7]. The full range of tunneling rates that can be exploited by a particular technology is limited by time-energy uncertainty as the minimal barrier height of a confining potential sets the corresponding maximal tunneling rate. This shallow confinement limit constrains the capabilities of tunneling devices for quantum metrology and sensing applications such as the frequency-to-current conversion [4,8] or generation of ultrashort electronic pulses [9–11]. A key to engineering quantum tunneling is quantitative control of the anharmonic confinement potential accessible *in situ* by experimental realizations. While such an achievement proved to be an essential stepping stone [12] for exploiting quantum tunneling at the macroscopic level [13], the

energy and time scales for fast single-electron control are challenging to probe directly.

Here we map out rates for the last electron escape [14] from an emerging quantum dot (QD) over many orders of magnitude, by deriving and experimentally verifying a scaling relation for the integrated escape rate. This makes it possible to stitch together accurate counting data of tunneling events from a variation of the rise time of the confining potential barrier. Utilizing this scaling to calibrate *in situ* the speed of control voltages yields a method to access the regime of shallow confinement. Crossover to activated transport is used to estimate the single energy scale, which limits the maximal attainable tunneling rate. A minimal microscopic model of ground-state tunneling from a cubic potential predicts a generalized scaling curve linking tunneling rate to potential depth. This scaling curve applies universally to different realizations and we find excellent agreement across experiments, which validates a fundamental trade-off between speed and accuracy of on-demand single-electron manipulation. This work demonstrates practical methodology to facilitate comparability between tunneling devices and provides a benchmark to engineer the optimal speed of device operation while confirming its robustness over the full relevant parameter range.

II. CAPTURE OF AN ELECTRON IN SHALLOW CONFINEMENT

In the experiment, the QD potential is defined in a GaAs/AlGaAs heterostructure by a shallow etched mesa

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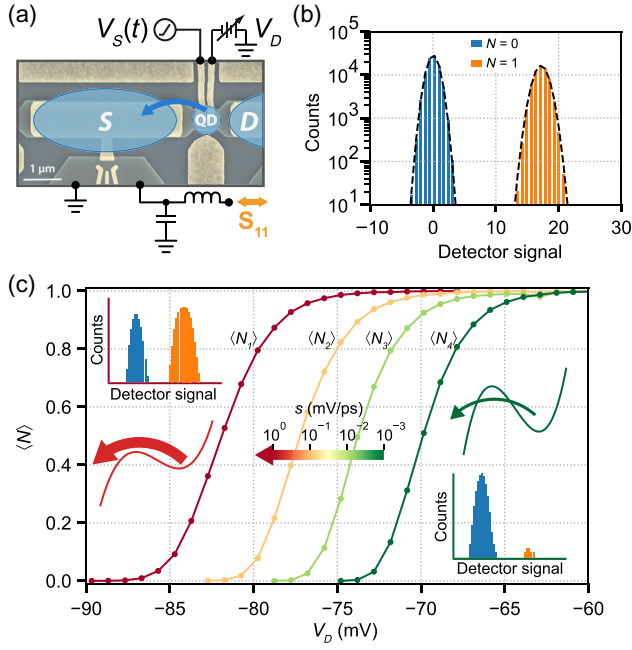


FIG. 1. (a) Schematic circuit diagram of the experimental setup over a sample micrograph illustrating the charge-state measurement via reflected rf signal S_{11} . (b) Example counting histogram of detector signal in units of average detector noise (standard deviation). (c) Shift of the capture probability $\langle N \rangle$ as a function of V_D due to variation of the ramp rate s . Two counting histograms (small insets) illustrate how for one V_D setting the counts of captured electrons increase with an increase of the ramp rate.

channel for confinement in the transverse direction, and two metallic top gates inducing two tunneling barriers for longitudinal confinement [15]. See Fig. 1(a) for a schematic of the experiment. The corresponding gate voltages, V_S and V_D , are tuned such that a very shallow QD can emerge from the source lead depending on the source gate setting V_S , while being isolated from the drain lead at all times. A linear voltage ramp $V_S(t) = -st$, approximated by the staircase of a digital waveform with an optional low-pass filter, is added to the source gate, while V_D remains constant. The voltage ramp deforms the potential to raise the source tunneling barrier, which then gradually grows and eventually isolates the created QD from the source lead with a decoupling speed proportional to the ramp rate s . A fundamental technical challenge in addressing fast tunneling rates is the systematic uncertainty of the effective ramp rate s' of the gate potential in response to the nominal rate s of the generator output $V_S(t)$. A constant operating point was chosen such as to robustly remain in a charge-capture regime limited by the escape rate [16] independent of the choice of s . The scaling technique detailed below then makes it possible to infer the mapping $s \rightarrow s'$ from the data. Establishing this link to external control voltages and validating the function of key drive parameters despite

signal distortions are essential for employing modulated tunneling barriers in high-speed nanoscale devices [16,17].

To measure the capture probability $\langle N \rangle$ (only $N = 0$ and $N = 1$ are considered for the number of captured electrons N) accurately, a high-fidelity counting scheme is employed [18], where the charge state of a large island serving as the source lead is read out by a capacitively coupled detector dot. Statistics for N [see Fig. 1(b)] are obtained by repeatedly executing a sequence of two charge-state measurements before and after a single decoupling cycle, followed by a reset operation in which the island is briefly connected to the ground potential. The overall repetition rate of this sequence is 335 Hz, which ensures a negligible readout error due to the long integration time independent of the value of s . Experiments were performed at a base temperature of 20 mK and without a magnetic field applied. Figure 1(c) shows the measured $\langle N_i \rangle$ for various values of s_i spanning several orders of magnitude ($i = 1, 2, \dots$ enumerates datasets with fixed $s = s_i$) as a function of V_D . A transition from 0 to 1 is evident. With increasing decoupling speed, this transition is shifted towards more negative V_D and therefore towards a shallower potential and more strongly coupled QD, consistent with the logarithmic rise-time dependence [4] observed in a Si-based single-electron ratchet [19].

Our baseline approximation to model $\langle N \rangle$ relies on time-scale separation [20,21] between the instantaneous reaction of the QD charging $\Gamma_{\text{in}}(t)$ and discharging $\Gamma_{\text{out}}(t)$ rates to the driving parameter $V_S(t)$, and the rates themselves, which go to zero as the QD closes, $t \rightarrow \infty$. The nonequilibrium occupation probability $P(t)$ of the QD is governed by [21–23] $dP/dt = \Gamma_{\text{in}}[1 - P] - \Gamma_{\text{out}}P$, which yields a model for $\langle N \rangle = P(t \rightarrow \infty)$. We further consider the limit in which the dot switches from $\Gamma_{\text{in}} \gg \Gamma_{\text{out}}$ to $\Gamma_{\text{in}} \ll \Gamma_{\text{out}}$ at a sufficiently well-defined moment t_0 compared with the characteristic time scale $\tau = -(d \ln \Gamma_{\text{out}}/dt)^{-1} \propto 1/s'$ for the subsequent reduction of the escape rate at $t > t_0$. In terms of the QD's electrochemical potential $\mu(t)$ given by the condition of detailed balance $\exp[\mu(t)/kT_L] = \Gamma_{\text{out}}(t)/\Gamma_{\text{in}}(t)$ with the source lead at temperature T_L , this requires lifting Pauli blockade much faster than closing the dot, $kT_L \ll \dot{\mu}\tau$ [21]. Assuming the dot is initially occupied, we get

$$\langle N \rangle = e^{-\int_{t_0}^{\infty} \Gamma_{\text{out}} dt}. \quad (1)$$

III. SCALING RELATIONS

The key idea to probe the scaling of tunneling rates up to the regime of shallow confinement is to overlap measurements of $\langle N_i \rangle$ with Γ_{out} going through the same range of values but with different rates s'_i [see sketch in Fig. 2(a)]. At any given parameter setting of s finite measurement time limits the accurate estimation of $\langle N \rangle$ due to the rarity of events as $\langle N \rangle$ approaches 0 or 1 with V_D . If the ramp rate

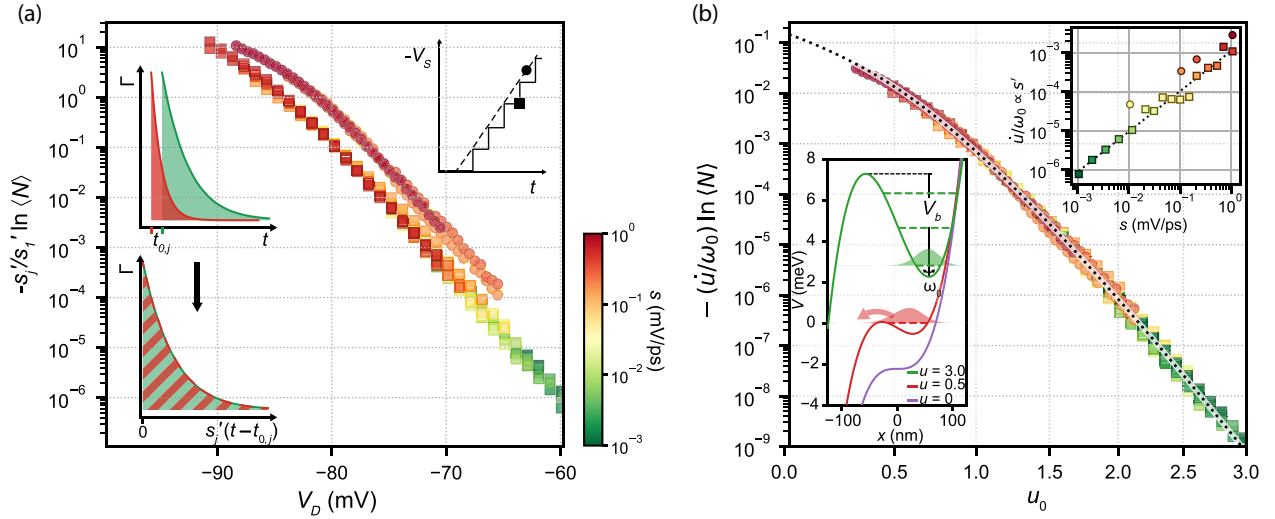


FIG. 2. (a) Data collapse of scaled capture probability $-s'_j/s'_1 \ln \langle N_i \rangle$ (symbols) as a function of V_D for filtered (circles) and unfiltered (squares) digital voltage ramps color-mapped by s . Left insets: schematics of the escape rate integration for two ramp rates collapsing on a $1/s_i$ -scaled t axis. Right inset: schematic of the (filtered) digital staircase of the voltage ramp. (b) Datasets $\langle N_i \rangle$ of (a) aligned to the universal scaling curve $M(u_0) = -(\dot{u}/\omega_0) \ln \langle N \rangle$ (dotted line) as a function of initial depth u_0 . Left inset: Conceptual sketch of the cubic potential model $V(x, t)$ for three values of dimensionless depth $u(t)$ with resonance energies and ground-state probability densities of the corresponding anharmonic oscillator. The zero level is fixed to the Fermi energy of the source by setting $V_0(t) = \mu(t) - E_0(t) + V_b(t)/2$. Right inset: inferred absolute values of the decoupling speeds $\dot{u}/\omega_0$ as a function of the nominal ramp rate s for the two datasets.

is switched from s_i to s_j with all other time-independent parameters kept constant, then the integral of the escape rate in Eq. (1) is multiplied by s'_i/s'_j . This implies a relation between any pair of datasets taken as a function of V_D :

$$[\langle N_i \rangle (V_D)]^{s'_i} = [\langle N_j \rangle (V_D)]^{s'_j}. \quad (2)$$

This must hold regardless of the functional dependence of Γ_{out} on $V_S(t)$ and V_D as long the systems stays in the escape-dominated regime governed by Eq. (1).

In order to robustly test Eq. (2) and infer $s'(s)$, we first estimate the exponent matrix $m_{ij} = s'_j/s'_i$ for the predicted power law $\langle N_i \rangle = \langle N_j \rangle^{m_{ij}}$ averaging over data points with different V_D but common (s_i, s_j) . Diagonalizing m_{ij} yields a single dominating eigenvalue with the corresponding eigenvector proportional to the set of s'_i . The scaling relation Eq. (2) is validated by the data collapse of $s'_i/s'_1 \ln \langle N_i \rangle$ shown by square symbols in Fig. 2(a). We observe a single curve for the full range of the input ramp rates s . To further extend this curve towards the shallow limit as much as is accessible, a second set of $\langle N_i \rangle$ is shown as circles, where the measurement time was concentrated on the fastest ramp rates and a flat group delay filter was added to linearize the staircase ramp. In the coordinates of the physical parameters V_D and s'_i , the two experiments both collapse to the respective reference speed s'_1 but appear offset due to differing operating points. The corresponding inferred decoupling speeds s'_i are plotted in the upper-right inset of Fig. 2(b) (with an absolute normalization defined

by a model below) and illustrate robustness and sensitivity of the method by revealing the discretization steps of the unfiltered waveform.

Predicting $\langle N \rangle$ from the backtunneling rate as in Eq. (1) is essential to model the fidelity of charge capture [4,8,19,24–27], yet these previous studies so far relied on the linearization of $\ln \Gamma_{\text{out}}$ over the relevant range of voltages. The experimental evidence in form of the empirical scaling curves in Fig. 2(a) shows, however, this linearity to be violated at high speeds, i.e. $\ln(-\ln \langle N \rangle)$ is not linear in V_D . This capping of the exponential growth of the integrated tunneling rate at increasingly negative V_D indicates a disappearing source barrier of an increasingly shallow dot. This connection between barrier height and tunneling rate will be probed over a wide parameter range by combining evidence of multiple experiments in terms of a generic model of tunneling out of shallow confinement [28], which we summarize below.

IV. CUBIC POTENTIAL MODEL

Confinement emerges near the stationary inflection point of the potential energy for which a cubic approximation is generic [12,29,30]. Hence we model the microscopic potential of the QD as $V(x, t) = bx^3/3 - F(t)x + V_0(t)$ [see lower-left inset in Fig. 2(b)]. Here x is measured from the uniformly moving inflection point in the longitudinal direction and $F(t) = (t - t_i)\dot{F}$ is the lowest-order

term to capture the transition at time t_i marking the formation of a barrier and a well. The closing speed $\dot{F} \propto s'$ is constant as $F(t)$ is assumed to be linearly driven by $V_S(t)$. The starting shape $V(x, t_0)$ at the onset of backtunneling is set by $F(t_0) = (t_0 - t_i)\dot{F}$, which is similarly set to be linear in the tuning gate voltage V_D .

Motion in the transverse direction is assumed to be confined to the lowest-energy mode and decoupled from x . Expansion near the inflection point implies nontrivial power laws for the barrier height $V_b = 4F^{3/2}b^{-1/2}/3 \propto (t - t_i)^{3/2}$ and the instantaneous linear oscillation frequency $\omega_0 = (2/m)^{1/2}(Fb)^{1/4} \propto (t - t_i)^{1/4}$ instead of $V_b \propto t$ and $\omega_0 = \text{const.}$ often assumed [19,26,31] for deeper dots (here m is the effective mass). We present the results in terms of a dimensionless depth, $u = V_b/(\hbar\omega_0)$, which counts the number of well-localized quasibound states, and the time-independent, device-specific $\Omega_b = \omega_0 u^{-1/5}$, which sets the absolute frequency and energy scales. A dimensionless time-independent speed parameter, $\dot{u}/\omega_0 = 5\dot{F}m/(6\hbar b) \propto s'$, sets an absolute scale for the decoupling speed s' for which the relative scale was determined earlier.

In the low-temperature, quantum adiabatic modulation limit we equate Γ_{out} in Eq. (1) to the decay rate Γ_0 of the resonance with the lowest real part $E_n = E_0$ of the corresponding complex energy eigenvalues $E_n - i\hbar\Gamma_n/2$ of the cubic potential [32]. Γ_0/ω_0 is a universal function of u (computed numerically using the complex scaling method [28,33]). This results in

$$\langle N \rangle = \exp \left[-\frac{\omega_0}{\dot{u}} \int_{u_0} [\Gamma_0(u)/\omega_0(u)] du \right], \quad (3)$$

where $u_0 = u(t_0)$ is the initial depth. The linear relation between $F(t_0)$ and V_D gives the power law $u_0 = [\tilde{\alpha}(V_D - V_D^c)]^{5/4}$. Here V_D^c , $\tilde{\alpha}$, and $\dot{u}/\omega_0$ constitute experiment-specific fitting parameters to align an empirical scaling curve $-s' \ln \langle N \rangle$ to the universal one, $M(u_0) \equiv \int_{u_0} [\Gamma_0(u)/\omega_0(u)] du$. This is demonstrated in Fig. 2(b) for the two empirical curves from Fig. 2(a). We find excellent agreement with the experiment over the full range of probed ramp rates, including the shallow confinement regime of $u_0 < 1$, evidence for the anharmonicity of the driven oscillator model. The shape of $M(u_0)$ contains no device-specific parameters. Examples of experiments conforming to this universality [Fig. 2 and more in Fig. 4(b)] provide evidence of tunneling as the fundamental microscopic mechanism of electron escape in contrast to the purely phenomenological decay cascade model [8,25]. The energy gap protecting this universality is set by the device-specific scale Ω_b , which we estimate in the following.

V. ENERGY SCALE ESTIMATION

For sufficiently fast decoupling speeds, nonadiabatic effects, such as intradot excitation [31,34,35] and non-Markovian effective temperature [36–38], are predicted to modify escape dynamics beyond tunneling out of the ground state. In our experiment, the quantum adiabaticity is maintained as the increased decoupling speed shifts the transition $\langle N \rangle = 0 \leftrightarrow 1$ into the shallow limit ($u_0 < 1$); hence we use thermal activation in the regime with several quasibound states ($u_0 > 1$) to probe the excitation spectrum and thus estimate the energy scale $\hbar\Omega_b$.

Thermal broadening of the Coulomb resonances used to read out the charge state and infer the capture probability limits the temperature range accessible to counting to $T < 2$ K. Up to this temperature, however, no discernible change of the capture probability can be observed. At higher temperatures $\langle N \rangle$ is therefore measured using a precision current amplifier [39], detecting the continuous current at a fixed ramp rate of 0.1 mV/ps as the captured electrons are emitted towards the drain by further raising the potential [4]. Figure 3(a) shows $-(\dot{u}/\omega_0) \ln \langle N \rangle$ for temperatures up to 6 K with $\dot{u}/\omega_0$ inferred from Fig. 2(b). In comparison with the baseline of the counting measurement (black dashed line), the good agreement with the lowest temperatures validates the consistency between the different measurement techniques. Furthermore, at $T \gtrsim 2.5$ K the capture probability decreases with temperature in the plateau region ($\langle N \rangle$ close to 1), a distinct qualitative difference from behavior observed in the tail ($V_D < -75$ mV in the inset). We attribute the onset of this temperature dependence, seen as a change of slope in Fig. 3(a), to crossover from tunneling to hopping [40]. Above this crossover the data points deviate from the universal scaling curve, which conversely corroborates the ground-state interpretation of the base-temperature data (Figs. 2 and 4) and contradicts speed-dependent heating.

In order to describe the temperature dependence of the metastability, thermally activated escape via states near the top of the barrier has to be included [29,30,41].

In a deep confinement regime, $V_b \gg \hbar\omega_0$, crossover from tunneling to hopping is expected at a temperature $k_B T_0 = \hbar\omega_0/(2\pi)$ [30]. However, there are two distinctive features of our problem that need to be accounted for: the potential may be shallow ($V_b \sim \hbar\omega_0$) and ω_0 is time dependent. To account for shallowness, we model a thermally averaged $\langle \Gamma \rangle$ of a fixed-depth cubic potential adapting the quantum transition state theory [30]: a Boltzmann distribution defined by a heat bath of temperature T controls the average over discrete resonances with decay rates Γ_n at energies $E_n \lesssim V_b$ and a continuum above the barrier [41]. Both contributions can be continuously connected within the cubic potential approximation, as derived in Ref. [28], to provide an accurate account for the transition from tunneling to hopping [30,41] with the crossover

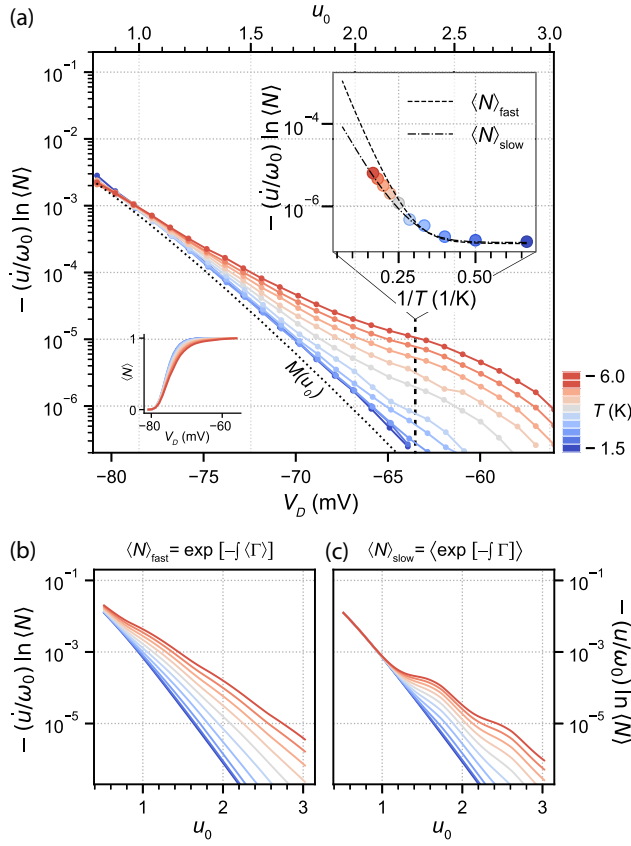


FIG. 3. (a) Capture probability as a function of V_D , inferred from current measurements, showing thermally activated escape as T is increased up to 6 K. The vertical axis is scaled using the speed parameter as in Fig. 2(b) matching the universal scaling curve (dotted line). Lower-left inset: unscaled data; top-right inset: Arrhenius plot for $u_0 \approx 2.3$ compared with activated-transport models $\langle N \rangle_{\text{fast}}$ and $\langle N \rangle_{\text{slow}}$ with $\hbar\Omega_b = 1.6$ meV. The model predictions as functions of u_0 are shown for (b) fast and (c) slow thermalization limits.

becoming broad at $u \sim 1$. To account for time dependence, we compare two opposite limits of the competition between thermalization and electron escape time scales. In the fast thermalization limit, the potential closes quasistatically and Eq. (1) holds with $\Gamma_{\text{out}}(t) = \langle \Gamma \rangle$ given by instantaneous depth $u(t)$ at a constant T . This gives $\langle N \rangle_{\text{fast}}$ as a function of u_0 and $\hbar\Omega_b/k_B T$. For the opposite limit of slow thermalization, we use the initial Boltzmann distribution at $t = t_0$ over a finite number (approximately u_0) of discrete metastable states to average the corresponding escape probabilities (3) computed from each state separately (with Γ_0 replaced by the appropriate Γ_n) [28]. The corresponding $\langle N \rangle_{\text{slow}}$ is thus an average of escape probabilities in contrast to the escape probability $\langle N \rangle_{\text{fast}}$ with an evolving averaged rate. The two approximations are contrasted in Figs. 3(b) and 3(c) for the same set of temperatures as in Fig. 3(a) and a fixed Ω_b . Both limits capture the threshold behavior indicative of the thermally activated

escape, as seen experimentally in Fig. 3(a), but the differences show that competition with an additional time scale for thermalization is important if excited states are engaged.

For a quantitative estimate, we choose a fixed value of $u_0 = 2.3$ and plot both model predictions and the experimental data as a function of $1/T$ (Arrhenius plot) in the inset to Fig. 3(a). We find $\langle N \rangle_{\text{slow}}$ with $\hbar\Omega_b = 1.6$ meV provides the best fit to our experiment, thus yielding an absolute time and energy scale of the microscopic potential inferred from the escape rates enhanced by thermal activation.

VI. CONCLUSIONS AND OUTLOOK: A SPEED-DEPTH BENCHMARK FOR TUNNELING DEVICES

In conclusion, matching of the counting data to the universal scaling relation over a broad range of decoupling speeds and the estimation of Ω_b from the temperature dependence provide an experimental technique for inferring the ground-state escape rate Γ_0 and the barrier height V_b in physical units down to the shallow limit where $V_b/\hbar \sim \Gamma_0 \sim \Omega_b$ and the confinement is eventually lost [see Fig. 4(a)]. Using the dimensionless coordinates u_0 and $\dot{u}/\omega_0$, different measurements can be put on a single universal speed-depth chart, Fig. 4(b), which follows the level lines of constant $\langle N \rangle$. We find that the devices

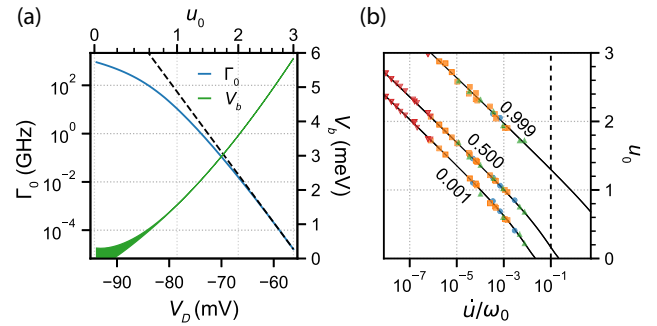


FIG. 4. (a) Inferred ground-state tunneling rate Γ_0 and the classical barrier height V_b as a function of gate voltage; the width of the V_b line shows the quantum uncertainty bracket $\hbar\Gamma_0$. As an indicator of anharmonicity, exponential extrapolation from the harmonic confinement limit is shown for comparison by the dashed line. (b) Trade-off between initial depth and speed to reach the same capture probability in the dimensionless coordinates of the universal model (3). Lines mark three representative levels of $\langle N \rangle$, symbols are various experimental realizations: a different QD implementation with sinusoidal driving waveform (green triangles), linear ramp waveform as in Figs. 2(a) and 2(b) (blue circles and orange squares), and a set of linear ramp waveforms focused on slow decoupling speeds (red down triangles). The vertical line in (b) marks the theoretical limit on adiabaticity ($\dot{u}/\omega_0 < 0.1$; see Ref. [28]).

studied here robustly remain in the regime of ground-state tunneling, constrained to operating frequencies of a few gigahertz by the shallow limit before nonadiabatic excitations become relevant [28]. The speed-depth scaling provides a benchmark to develop electronics manipulating individual particles towards their quantum bandwidth limit.

In particular, the generation of a quantized current in quantum metrology applications requires precision in the control over the stochastic tunneling out of the confinement potential through which electrons are shuttled. The accuracy of the shuttle operation is tied to the complete escape of excess electrons while the escape of the last remaining electron is suppressed. For best accuracy deep initial confinement is preferred as the high tunnel barrier makes $\langle N \rangle$ approach the quantized value of one (as long as the adiabatic condition is not violated). Yet maximizing the speed of operation introduces a cap on the initial depth as the excess electrons need to be purged with sufficient accuracy. These two sides of the trade-off correspond to the two limits of the universal ground-state tunneling model that can be traced in Fig. 4(b) following lines of $\langle N \rangle \approx 1$ (nonadiabaticity) and $\langle N \rangle \approx 0$ (loss of confinement), respectively. With the estimation of the scaling factors, a particular realization can be placed on the universal diagram, thus helping to engineer an optimal compromise between operating frequency and accuracy of selective confinement. Thus this work introduces a capability gauge for technology platforms [3,42] and a path to certify that the residual error is limited by a universal quantum effect.

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DATA AVAILABILITY

The data that support the findings of this article are openly available [43].

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