

Current-control of chaos and effects of thermal fluctuations in magnetic tunnel junctions

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We theoretically investigate the chaotic behavior of spin-torque ferromagnetic resonance in magnetic tunnel junctions (MTJs) with perpendicular magnetic anisotropy under thermal fluctuations. By calculating the Lyapunov exponent based on the Landau–Lifshitz–Gilbert equation, we demonstrate that an MTJ characterized by a double-well potential, composed of uniaxial magnetic anisotropy and an external dc magnetic field, exhibits chaotic magnetization dynamics that can be controlled by means of the dc bias via spin-transfer torque. Furthermore, we find that thermal fluctuations help to induce these chaotic magnetization dynamics, which can be regarded as noise-induced chaos. This research provides a basis for brain-inspired computing using spintronic devices and advances the understanding of the interplay between thermal fluctuations and chaos in magnetization dynamics.

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Introduction. In recent years, there has been growing interest in the nonlinear and complex dynamics of physical systems for brain-inspired computing, including chaotic neural networks [1–3], reservoir computing [4–6], and probabilistic computing [7–9]. As demonstrated in previous studies, it is often reported that reservoir computing achieves high computational performance when its elements exhibit “edge of chaos,” which is a transient state between the periodic and chaotic states [1,10–13]. Owing to the rich nonlinear effects and fast response of magnetization, spintronic devices have been actively implemented in experiments for brain-inspired computing [6–8,14]. At the same time, theoretical studies of nonlinear and chaotic dynamics in spintronic devices have been actively conducted across various platforms, including ferromagnetic resonance (FMR) [15–17], spin waves [18,19], magnetic-vortex-based devices [20,21], and spin-torque oscillators [22–24].

Among these studies, the magnetic Duffing oscillator is proposed [16,17], which exhibits chaotic magnetization

dynamics controlled by dc and ac magnetic fields. The magnetic Duffing oscillator is characterized by the homoclinic orbit [25], which is the origin of chaos in the phase space of magnetization dynamics. The homoclinic orbit resides in a double-well potential composed of uniaxial magnetic anisotropy and an external dc magnetic field. By tuning the dc magnetic field applied perpendicular to the magnetic anisotropy axis, one can shift the position of the homoclinic orbit, adjusting the onset of chaos. Although the magnetic Duffing oscillator has substantial practical value in spintronic devices, it requires the adjustment of both ac and dc magnetic fields for the generation and control of chaos, which makes it less suitable for integration into nanodevices.

Magnetic tunnel junctions (MTJs) [26,27] enable electrical input and readout of the magnetization state without relying on additional magnetic sensors, making them highly suitable for miniaturization and integration. These characteristics have attracted considerable attention to their magnetization dynamics [28–34]. Since MTJs are primarily designed for room-temperature operation, it is essential to consider the influence of thermal fluctuations, that is, stochastic forces, when investigating their nonlinear dynamics. Up to now, nonlinear dynamics under stochastic forces have been extensively studied in various physical systems [35–41]. These studies report a wide range of phenomena, including stochastic resonance [35–37], noise-induced chaos [38–40], and noise-induced

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order [41]. When a stochastic force is introduced into chaotic systems, the stability of chaos becomes a highly nontrivial issue. Nevertheless, from the perspective of utilizing strong nonlinear or chaotic dynamics in real devices, it is crucial that these behaviors remain robust against noise. However, the influence of thermal fluctuations on chaotic magnetization dynamics in MTJs has remained largely unexplored.

In this Letter, we theoretically investigate nonlinear and chaotic magnetization dynamics in spin-torque FMR [28,29] based on an MTJ with perpendicular magnetic anisotropy [30–33]. By applying an external dc magnetic field along the in-plane direction of the free layer, we create a potential landscape analogous to that of the magnetic Duffing oscillator. When an ac is applied to drive the magnetization, the emergence of chaos is confirmed through the Lyapunov exponent [42,43] and the magnetization trajectory in the phase space. We then examine how the magnitudes of the dc and ac affect the onset of chaos via the spin-transfer torque. In particular, we find that the dc acts as an effective control parameter that can suppress chaotic behavior. Furthermore, we take into account thermal fluctuations as stochastic magnetic fields, demonstrating that they help to excite the magnetization and enhance chaotic dynamics.

Model and mechanism of chaos. We introduce the system: an MTJ consisting of a magnetized free layer with an easy uniaxial z axis and a magnetized reference layer in the plane, as shown in Fig. 1. The unit vectors, denoted by $\mathbf{m}$ and $\mathbf{p}$, represent the magnetization in the free and reference layers, respectively. The magnetization in the reference layer is pinned along the y axis and induces a spin-transfer torque when an electric current is applied along the z direction. Magnetization dynamics in the free layer is described by the Landau–Lifshitz–Gilbert equation:

$$\frac{d\mathbf{m}}{dt} = -\gamma\mathbf{m} \times \mathbf{B}_{\text{eff}} - \gamma\tau(\mathbf{m}, j)\mathbf{m} \times (\mathbf{p} \times \mathbf{m}) + \alpha\mathbf{m} \times \frac{d\mathbf{m}}{dt}, \quad (1)$$

where γ is the gyromagnetic ratio and α is the Gilbert damping constant. Then, the Hamiltonian is described as:

$$\mathcal{H} = -\gamma M_s \left(B_x m_x + \frac{1}{2} B_{\text{ani}} m_z^2 \right), \quad (2)$$

and the effective magnetic field is

$$\mathbf{B}_{\text{eff}} = -\frac{1}{M_s} \frac{\delta \mathcal{H}}{\delta \mathbf{m}} = B_x \mathbf{e}_x + B_{\text{ani}} m_z \mathbf{e}_z, \quad (3)$$

where $\{\mathbf{e}_x, \mathbf{e}_y, \mathbf{e}_z\}$ are the unit vectors along the respective Cartesian axes. B_x is the dc magnetic field applied along

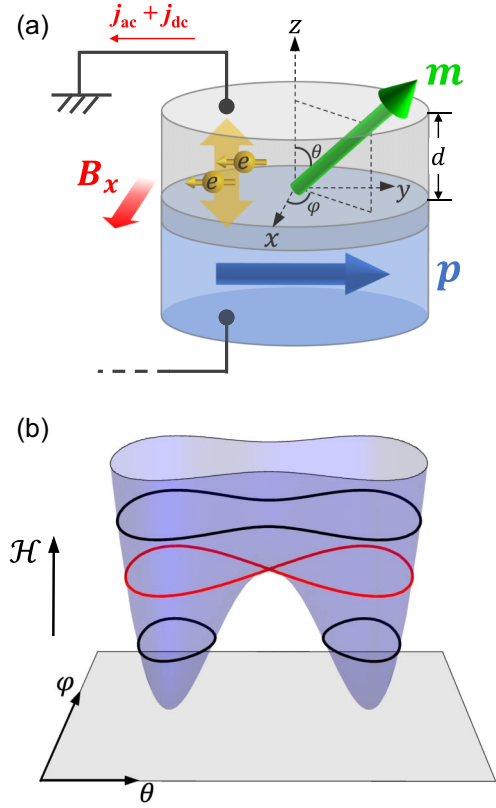


FIG. 1. (a) Schematic illustration of the system. The MTJ consists of a free layer and a reference layer underneath. An external dc magnetic field B_x is applied along the x axis, and the uniaxial magnetic anisotropy field B_K is oriented along the z axis. The polar and azimuthal angles of the magnetization vector in the free layer, $\mathbf{m}$, are denoted by θ and ϕ . The magnetization vector pointing along the y axis in the reference layer is represented by $\mathbf{p}$. The applied ac and dc along the z direction generate spin-transfer torques. (b) Hamiltonian ($\mathcal{H}$) landscape of the MTJ for $B_x = 32$ mT and $B_{\text{ani}} = 41.9$ mT. The red energy contour line is the homoclinic orbit, which starts from and ends at the same saddle point.

the x axis, $B_{\text{ani}} = B_K - \mu_0 M_s$ is the total magnetic anisotropy field consisting of B_K , and $\mu_0 M_s$ (μ_0 is the permeability of free space) are the interfacial [30–33] and shape anisotropy fields, respectively. The second term of the right-hand side of Eq. (1) describes the spin-transfer torque, which has the factor [23]:

$$\tau(\mathbf{m}, j) = \frac{\hbar j}{2e M_s d} \frac{\eta}{1 + \eta^2 \mathbf{m} \cdot \mathbf{p}}, \quad (4)$$

where $\hbar$, e , d , j , and η are the Dirac constant, elementary charge, the thickness of the free layer, the interfacial current density through z axis, and the spin polarization of the current, respectively. The interfacial current density j is composed of two parts:

$$j = j_{\text{dc}} + j_{\text{ac}} \cos(2\pi f t), \quad (5)$$

where j_{dc} is the interfacial dc density and j_{ac} and f denote the amplitude and frequency of the interfacial ac density with sinusoidal time dependence, respectively. Because of the factor τ , the auto-oscillation of the magnetization and the corresponding limit cycle in the phase space can appear in the presence of the dc. Throughout this paper, we use the following parameters: $\gamma = 1.76 \times 10^{11} \text{ T}^{-1} \text{ s}^{-1}$, $d = 2 \text{ nm}$, $\eta = 0.537$, $\mu_0 M_s = 1.821 \text{ T}$, $\alpha = 0.05$, $B_x = 32 \text{ mT}$, and $B_K = 1.862 \text{ T}$. These values represent typical parameters for an MTJ with a perpendicularly magnetized free layer consisting of a CoFeB/MgO structure, with a few nanometers of thickness [23,30]. The fundamental principle of the giant value of B_K is broadly considered to be the hybridization of Fe-3d and O-2p orbitals at the metal-insulator interface.

In order to calculate the magnetization dynamics and the Lyapunov exponent, we express Eq. (1) in polar coordinates as:

$$\begin{aligned} \frac{d\theta}{dt} &= \gamma B_\varphi(\theta, \varphi, t) + \alpha \gamma B_\theta(\theta, \varphi, t) + \gamma \tau_\theta(\theta, \varphi, z), \\ \frac{d\varphi}{dt} &= \frac{1}{\sin \theta} [-\gamma B_\theta(\theta, \varphi, t) + \alpha \gamma B_\varphi(\theta, \varphi, t) + \gamma \tau_\varphi(\theta, \varphi, z)], \\ \frac{dz}{dt} &= 2\pi f, \end{aligned} \quad (6)$$

where B_θ and B_φ denote the θ and φ components of the effective magnetic field $\mathbf{B}_{\text{eff}}$, respectively, and τ_θ and τ_φ represent the corresponding components of the spin-transfer torque term. Note that the variable z represents the phase of the ac and is introduced to convert a non-autonomous system into an autonomous system. Each component of the effective magnetic field and spin-transfer torque is described as:

$$\begin{aligned} B_\theta(\theta, \varphi, t) &= (B_x + B_x^{\text{th}}(t)) \cos \theta \cos \varphi \\ &\quad + (B_{\text{ani}} \cos \theta + B_y^{\text{th}}(t)) \sin \theta - B_z^{\text{th}}(t) \sin \theta, \\ B_\varphi(\theta, \varphi, t) &= -(B_x + B_x^{\text{th}}(t)) \sin \varphi + B_y^{\text{th}}(t) \cos \varphi, \\ \tau_\theta(\theta, \varphi, z) &= \tau(\theta, \varphi, z) \cos \theta \sin \varphi, \\ \tau_\varphi(\theta, \varphi, z) &= \tau(\theta, \varphi, z) \cos \varphi, \end{aligned} \quad (7)$$

where $B_i^{\text{th}}(t)$ denotes the i -th component of the Gaussian random magnetic field, which represents the thermal fluctuation. $B_i^{\text{th}}(t)$ satisfies $\langle B_i^{\text{th}}(t) \rangle = 0$ and $\langle B_i^{\text{th}}(t) B_j^{\text{th}}(s) \rangle = \sigma^2 \delta_{ij} \delta(t - s)$ [44,45], where

$$\sigma = \sqrt{\frac{2\alpha k_B T}{\gamma M_s \mathcal{V}}}, \quad (8)$$

where k_B , T , δ_{ij} , and $\delta(t)$ are the Boltzmann constant, the system temperature, Kronecker's delta, and Dirac's delta, respectively. The volume of the free layer $\mathcal{V}$ is assumed to be $\pi \times 120 \times 120 \times 2 \text{ nm}^3$. We simulated the

magnetization dynamics by using the Runge–Kutta method with the DifferentialEquation package in Julia 1.10.

The mechanism of chaos, as in the magnetic Duffing oscillator, can be understood from the homoclinic orbit created by the anharmonic magnetic potential [16,17]. Owing to the uniaxial magnetic anisotropy field and the dc magnetic field, a double-well potential appears in the x - z plane (i.e., along the θ axis), whereas a single-well potential emerges along the y axis [see Fig. 1(b)]. The saddle exists at $(\theta, \varphi) = (\pi/2, 0)$. Then, the orbits starting from and ending at this saddle point arise; these orbits correspond to the homoclinic orbits. Both the Duffing oscillator and the magnetic Duffing oscillator exhibit chaotic behavior when the state is excited to the strange attractor created by the perturbed homoclinic orbit due to a driving force and damping term. Thus, our strategy for generating or controlling chaos in the MTJ is as follows: by applying an ac, we excite the magnetization up to the homoclinic orbit and create a strange attractor around this orbit. By adjusting the dc, we shift the magnetization state to modify its distance from this orbit and collapse the strange attractor.

Current-control of chaos and effects of thermal fluctuations. To confirm the presence of chaos and clarify the effect of the dc on the magnetization dynamics, we numerically calculate the Lyapunov exponent. Figure 2(a) shows the Lyapunov exponent as a function of the ac density j_{ac} with $f = 0.57 \text{ GHz}$ and the dc density j_{dc} without thermal fluctuations. The frequency f is chosen as the one that most effectively promotes chaos (Supplemental Material [46]). In this figure, regions with $\lambda > 0$ (bright areas) indicate chaotic magnetization dynamics and $\lambda = 0$ corresponds to the limit cycle. As expected, when an ac is applied, the magnetization exhibits chaotic dynamics. In fact, as shown in Fig. 3(a), the ac excites the magnetization state from the bottom of the potential well. On the other hand, the dc increases the threshold amplitude of the ac required for the onset of chaotic dynamics in Fig. 2(a). Note that some dark regions with a Lyapunov exponent $\lambda = 0$ appear between chaotic domains, for example, around $(j_{ac}, j_{dc}) = (15, 0)$. This phenomenon is called “periodic window” [47] and is observed in many dynamical systems that exhibit chaos. A clear boundary between chaos and limit-cycle regimes is observed in Fig. 2(a). Note that although high ac and dc densities appear to be required to excite chaos, these thresholds can be reduced in two ways. The first approach is to reduce B_x and B_{ani} , thereby lowering the overall energy scale in MTJs. The second approach is to lower the energy levels of the saddle point and the homoclinic orbit by setting the strength of B_x close to that of B_{ani} [16].

Figure 2(b) shows the heatmap of the Lyapunov exponent with random magnetic fields at room temperature, i.e., $T = 300 \text{ K}$. Comparing Figs. 2(a) and 2(b), the

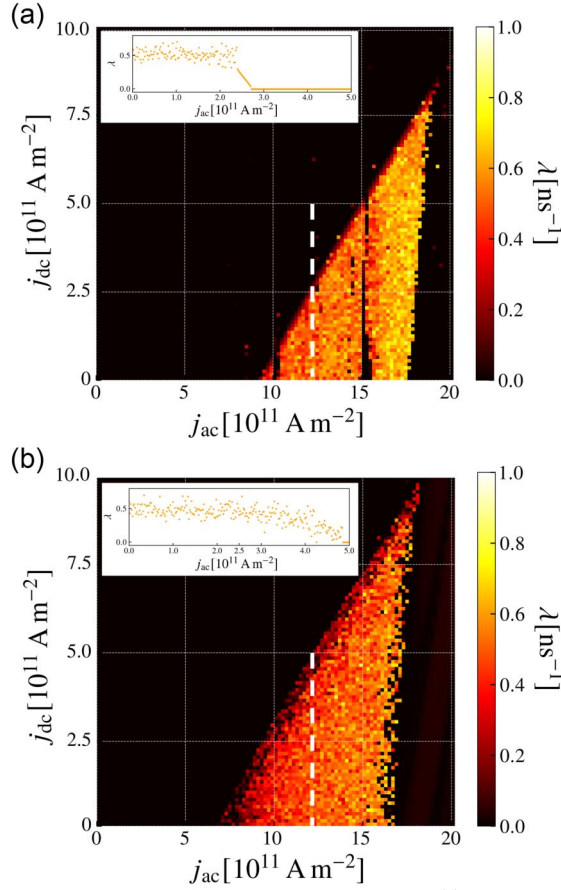


FIG. 2. Heatmaps of the Lyapunov exponent λ as functions of the dc density j_{dc} and the amplitude of ac density j_{ac} for (a) $\sigma = 0 \text{ mT s}^{1/2}$ and (b) $\sigma = 0.366 \text{ mT s}^{1/2}$ (corresponding to room temperature). The insets show cross-sectional views of the Lyapunov exponent λ taken along the white dashed lines.

boundary becomes less distinct and shifts to the left due to the thermal fluctuations. The shift in the parameter range of the chaotic region indicates not only that the strange attractors are robust against stochastic forces but also that stochastic forces assist in exciting the magnetization, leading to chaotic dynamics. Since the boundary of the bright region is blurred, the edge of chaos may also become indistinct. As shown in the insets of Fig. 2(a), in the absence of a stochastic force, the Lyapunov exponent decreases linearly at the edge of chaos. However, in the presence of a stochastic force, the decrease in the Lyapunov exponent becomes scattered in the insets of Fig. 2(b). Note that several studies on Duffing oscillators with stochastic forces have reported that stochastic forces can facilitate the onset of chaotic dynamics [39,40]. Thus, the MTJ exhibits behavior similar to that of the Duffing oscillator when subjected to stochastic forces.

Next, we examine the roles of the dc and stochastic forces using the magnetization trajectory. Figure 3 shows the magnetization dynamics in the phase space and the Fourier spectrum of the y-component of magnetization m_y ,

which can be detected by the tunnel magnetoresistance effect via the spin-torque diode effect [26–28]. Figure 3(a) presents the chaotic dynamics over the double-well potential driven by an ac density $j_{ac} = 1.2 \times 10^{12} \text{ A m}^{-2}$ without a dc. This trajectory covers a wide area of the phase space, and the Fourier spectrum is broad. When a dc is applied, a limit cycle appears, accompanied by several Fourier peaks, and the chaotic dynamics disappear in the phase space, as shown in Fig. 3(b). This is because the dc generates a spin-transfer torque along the y axis, which causes the magnetization to remain on one side of the double-well potential. Hence, due to the dc, it is difficult to reach the homoclinic orbits, and chaotic dynamics are suppressed. In terms of dynamical systems theory, the dc acts as a bifurcation parameter that alters the qualitative structure of attractors in the phase space, such as limit cycles or strange attractors. Figure 3(c) shows the magnetization dynamics with stochastic forces at room temperature. As seen in Figs. 2 and 3, the chaotic dynamics are robust against the stochastic forces, indicating the feasibility of observing chaos experimentally. Moreover, when stochastic forces are applied to a state in which chaos is not originally present, a strange attractorlike structure emerges, as shown in Fig. 3(d), leading to the onset of chaotic dynamics. This phenomenon is known as noise-induced chaos and has been reported in studies of the Duffing oscillator [39,40]. Physically, stochastic forces can be regarded as an additional energy source that pushes the magnetization state toward the homoclinic orbit in the phase space, leading to the emergence of chaotic dynamics. That is why the threshold of chaos is reduced in the presence of thermal fluctuations.

Also, we demonstrate noise-induced order, a phenomenon in which stochastic forces suppress the characteristic features of chaos, such as strange attractors and high sensitivity to initial conditions. Figure 3(e) displays the chaotic dynamics without thermal fluctuations for $j_{ac} = 1.75 \times 10^{12} \text{ A m}^{-2}$ and $j_{dc} = 2.9 \times 10^{11} \text{ A m}^{-2}$. When thermal fluctuations are introduced, the trajectories no longer wander erratically through the phase space; instead, blurred periodic trajectories emerge in Fig. 3(f). The Fourier spectrum contains two main peaks and sidebands around them. In this model, the energy contours are directly related to the homoclinic orbits. Hence, this phenomenon can be interpreted by considering thermal fluctuations as an effective energy source. The stochastic forces push the magnetization state above the homoclinic orbits and prevent the emergence of a strange attractor around these orbits. Thus, analyzing the occurrence of chaos through the homoclinic orbit clarifies the physical roles of terms such as spin torque or random magnetic field in generating chaotic behavior. In this way, the MTJ model can experimentally demonstrate not only simple chaotic magnetization dynamics but also various noise-induced phenomena known in the field of dynamical systems.

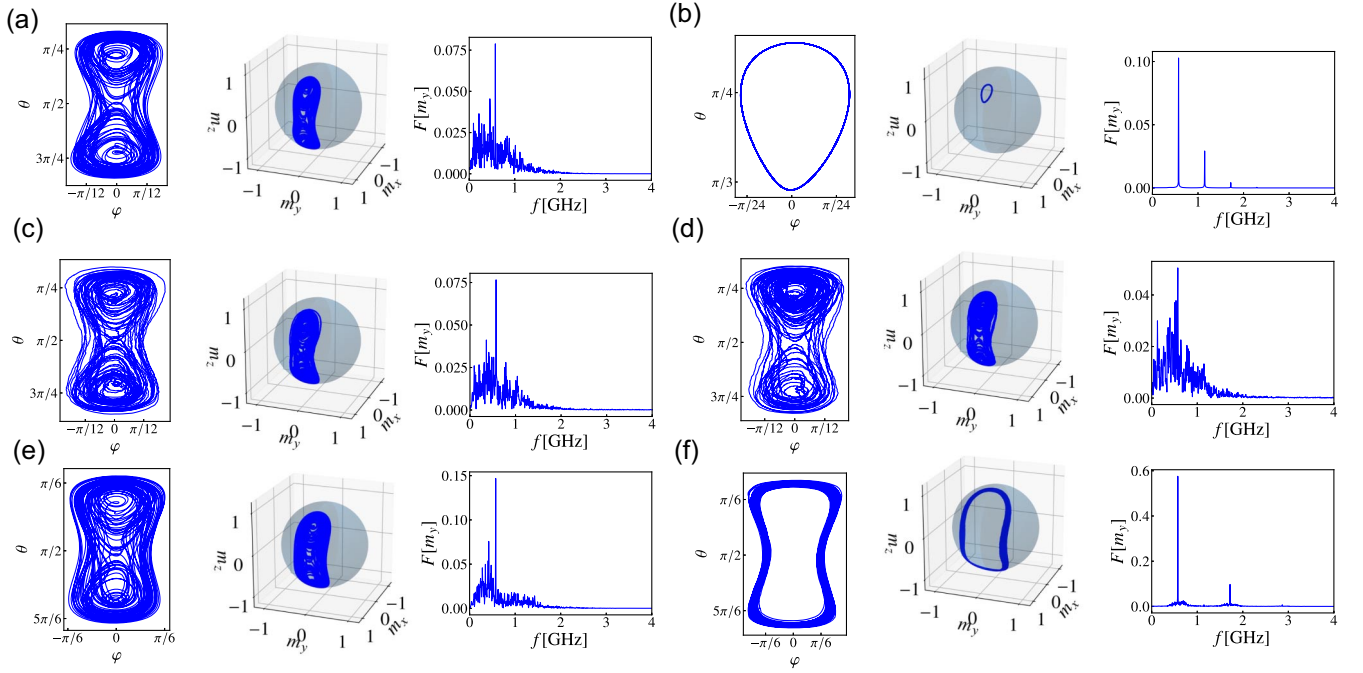


FIG. 3. Magnetization dynamics and corresponding Fourier spectrum. The magnetization trajectory in the phase space of θ and φ is shown in the left panel. The central panel displays the trajectory in three-dimensional real space represented by (m_x, m_y, m_z) . The right panel shows the Fourier spectrum of the y -component of the magnetization m_y . The parameters are as follows: (a),(c) $j_{ac} = 1.2 \times 10^{12} \text{ A m}^{-2}$, $j_{dc} = 0$; (b),(d) $j_{ac} = 1.2 \times 10^{12} \text{ A m}^{-2}$, $j_{dc} = 2.9 \times 10^{11} \text{ A m}^{-2}$; and (e),(f) $j_{ac} = 1.75 \times 10^{12} \text{ A m}^{-2}$, $j_{dc} = 2.9 \times 10^{11} \text{ A m}^{-2}$, respectively. Panels (a), (b), and (e) show the results without thermal fluctuations ($\sigma = 0 \text{ mT s}^{1/2}$), whereas panels (c), (d), and (f) include thermal fluctuations at room temperature ($\sigma = 0.366 \text{ mT s}^{1/2}$).

At the end of this section, it is noted that we investigate the dynamics under a more realistic situation in which the magnetization of the reference layer, $\mathbf{p}$, is tilted from the y direction and unexpected in-plane magnetic anisotropy is present due to fabrication tolerances (see Supplemental Materials [46]). As found in Ref. [46], chaos still occurs under such realistic conditions.

Stochastically driven chaos. Here, we demonstrate chaotic magnetization dynamics driven solely by thermal fluctuations. The method for evaluating the Lyapunov exponent differs from Fig. 2 because the system is a two-dimensional autonomous system with a stochastic force. We use the norm defined in the phase space $(\theta, \varphi) \in \mathbb{R}^2$. It is noted that the stochastic force can be considered as the third dimension in the phase space, which allows for the emergence of chaos. Figure 4(a) illustrates the Lyapunov exponent as a function of the standard deviation of the stochastic force σ . As σ increases, the Lyapunov exponent also increases and becomes positive around $\sigma = 0.6$. This indicates that under small stochastic forces, the trajectory is attracted to the minimum of the potential, and the system remains stable. According to Fig. 4(b), the magnetization is trapped on one side of the potential well. In contrast, when $\sigma = 0.8$, the magnetization reverses between the double-well potential repeatedly, as shown in Fig. 4(c), and the Lyapunov exponent

becomes positive. This shows that the homoclinic orbit imparts the feature of chaos and sensitivity to initial states to the magnetization dynamics driven solely by the stochastic force. This phenomenon indicates that the magnetization dynamics in stochastic MTJ driven by thermal fluctuations [7–9] exhibit certain characteristics of chaotic behavior.

Furthermore, in Supplemental Material [46], we calculated the distribution of the residence time, defined as the interval between consecutive sign reversals of m_z . We found a power-law distribution rather than an exponential distribution. While the latter is a typical distribution observed in stochastic magnetization switching [48–50], the former (power-law distribution) exhibits scale-free behavior and is regarded as a signature of noise-induced chaotic phenomena. Therefore, the magnetization dynamics exhibits not only simple diffusive behavior but also complex phenomena such as chaotic itinerancy, on-off intermittency [51,52], and related phenomena [53,54]. Thermal fluctuations are introduced as Gaussian white noise in the effective magnetic field, which primarily induces diffusive motion in phase space. However, owing to the nonlinear structure of the underlying deterministic dynamics—particularly the presence of the double-well potential and the homoclinic orbit—the noise does not act as simple diffusion alone, even under the combined influence of stochastic and periodic driving forces.

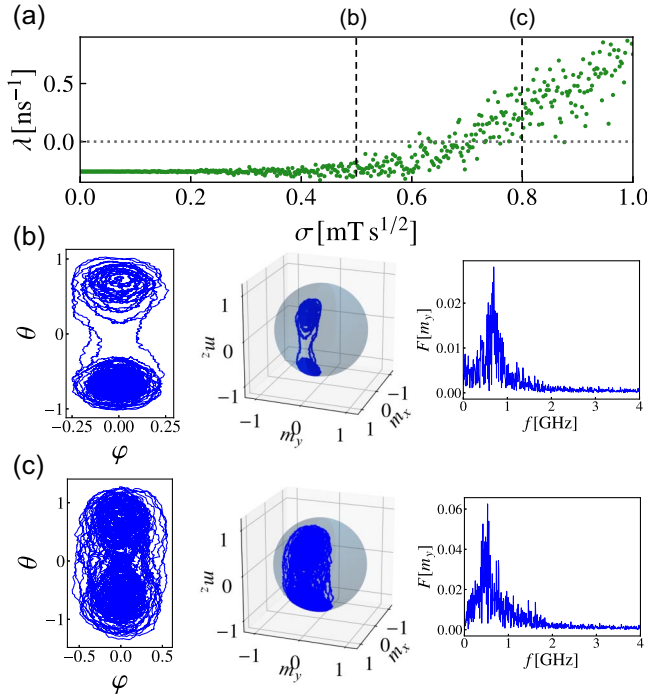


FIG. 4. Magnetization dynamics driven solely by stochastic forces. (a) Lyapunov exponent λ as a function of the noise strength σ . The dynamics exhibits chaotic behaviors for $\sigma \gtrsim 0.6$ mT s^{1/2}. (b),(c) Magnetization trajectory and the corresponding Fourier spectrum at (b) $\sigma = 0.5$ mT s^{1/2} for the radius of the free layer being 88 nm and (c) $\sigma = 0.8$ mT s^{1/2} for the radius with 55 nm, at room temperature.

Conclusion. In conclusion, we theoretically studied non-linear and chaotic magnetization dynamics controlled by ac and dc via spin-transfer torque in MTJs. The perpendicular magnetic anisotropy and the external dc magnetic field in the free layer generate a homoclinic orbit in the phase space, which leads to chaotic behavior of magnetization. The ac excites the magnetization and induces chaotic dynamics, whereas the dc shifts the magnetization away from the homoclinic orbit, resulting in the suppression of chaos. In order to evaluate the robustness of chaos against thermal fluctuations, we examined how a stochastic force influences the emergence of chaotic magnetization dynamics. We found that chaos not only persists in the presence of the stochastic force, but that the stochastic force also assists to excite the magnetization and promotes chaotic behavior, indicating that chaotic magnetization dynamics could be detected experimentally. This result is consistent with the noise-induced chaos and order reported in the original Duffing oscillator [47,55], thereby reinforcing the analogy between the two systems. Furthermore, we evaluate the characteristics of chaos in magnetization dynamics driven solely by stochastic forces. According to the Lyapunov exponent, as the variance of the stochastic force increases, the system becomes more sensitive to initial conditions, which is a feature typical of chaotic

dynamics. In addition, we identified an approximately power-law distribution of the residence time in the chaotic regime, which plays a significant role in the theory of stochastic dynamical systems. This behavior is fundamentally different from the conventional stochastic switching statistics reported for MTJs. To the best of our knowledge, this represents the first demonstration of noise-induced chaos associated with thermal fluctuations in MTJs. This work not only advances the practical implementation of spintronic devices that utilize chaos but also contributes to a deeper understanding of the interplay between thermal fluctuations and chaos in magnetization dynamics.

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Data availability. The data that support the findings of this letter are not publicly available upon publication because it is not technically feasible and/or the cost of preparing, depositing, and hosting the data would be prohibitive within the terms of this research project. The data are available from the authors upon reasonable request.

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