

Protecting logical qubits with dynamical decoupling

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Demonstrating that logical qubits outperform their physical counterparts represents a milestone in achieving reliable quantum computation. Here, we propose to protect logical qubits with a dynamical decoupling scheme that implements iSWAP gates on nearest-neighbor physical qubits, and we experimentally demonstrate this scheme on superconducting transmon qubits. In our scheme, each logical qubit requires only two physical qubits. A universal set of quantum gates on the logical qubits can be achieved such that each logical gate comprises only one or two physical gates. Our experiments reveal that the coherence time of a logical qubit is extended by up to 357% compared to the better-performing physical qubit. Moreover, we demonstrate—to the best of our knowledge, for the first time—that multiple logical qubits can outperform their physical superconducting counterparts. We illustrate a set of universal gates through a logical Ramsey experiment and the creation of a logical Bell state. Given its scalable nature, our scheme holds promise as a component for future reliable quantum computation.

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I. INTRODUCTION

Physical qubits are fundamental building blocks of quantum computers, but they are inherently susceptible to errors due to unavoidable interactions with their environment [1,2]. To perform reliable quantum computation, one needs to employ logical qubits, which are defined using a set of physical qubits [3] to resist these errors. Demonstrating that logical qubits outperform their associated physical counterparts—known as the “break-even point”—is considered a milestone in quantum computing [4–7]. One available approach to achieving this goal is to use quantum error correction (QEC) [3,8–15]. Despite impressive achievements, connecting multiple logical qubits remains a significant challenge for these schemes. To accurately perform quantum algorithms before large-scale QEC is

realized, it is still essential to investigate scalable error-mitigation and error-suppression schemes.

Dynamical decoupling (DD) [16–29] can be used to manipulate interactions between physical qubits and their environment by applying tailored control pulses to the qubits. When a DD procedure selectively eliminates the noncollective interactions between qubits and their environment while preserving collective interactions, this enables those qubits to interact with the environment in a uniform manner [30–33], and this in turn introduces decoherence-free subspaces (DFSs) or noiseless subsystems. Logical qubits encoded in the DFSs are inherently immune to the remaining collective errors, thereby providing an effective error-suppression scheme for quantum computation.

Previous pioneering work has demonstrated the optimal generation of collective interactions using DD sequences composed of SWAP gates [33]; however, the SWAP-based DD approach retains all types of collective interactions among the qubits, which renders the DFS less conducive to efficient encoding and decoding procedures and complicates the implementation of logical operations. Moreover, SWAP gates cannot be directly implemented on various quantum computing platforms. Therefore, an ideal DD scheme for protecting logical qubits should use directly

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realizable quantum gates to construct DFSs, enabling simple encoding and decoding of logical qubits while supporting experimentally straightforward logical gates.

In this work, we propose to generate Z-type collective interactions for physical qubits via periodically performing iSWAP gates, removing the X - and Y -type interactions from the system evolution. This yields DFSs supporting logical qubits encoded in pairs of physical qubits. Our scheme exhibits remarkable scalability, as it requires only a linearly scaling number of iSWAP gates between nearest-neighbor qubits to protect multiple logical qubits. Moreover, universal gates for logical qubits can be realized using single-body and nearest-neighbor two-body Hamiltonians. We demonstrate our scheme on two superconducting processors with different energy-relaxation and coherence times. Compared with the existing DD schemes in superconducting qubits [34–38], our scheme can be regarded as many-body DD [39]. Our results show that the coherence time of a protected logical qubit is extended by up to 357% compared to that of the better-performing physical qubit. For two logical qubits, the error of an identity process with a duration of $\tau = 4.8 \mu\text{s}$ (39.49%) is reduced to approximately 57% of the error for physical qubits (68.81%). To the best of our knowledge, this is the first time that multiple logical qubits have been shown to outperform their physical counterparts in superconducting qubits. To showcase the universal control of logical qubits, we conduct a logical Ramsey experiment on one of the logical qubits and create a logical Bell state using logical operations. Additionally, we demonstrate a scalable protection scheme by enhancing the success probability in logical superdense coding.

The structure of the remainder of this paper is as follows. In Sec. II, we present our scheme for protecting logical qubits using iSWAP gates. In Sec. III, we provide experimental results that demonstrate the effectiveness of the proposed protection method. In Sec. IV, we showcase a set of universal gates for logical qubits, including a Ramsey experiment on a single logical qubit and the creation of a logical Bell state on two logical qubits. In Sec. V, we propose a scalable protecting procedure for multiple logical qubits and demonstrate its efficiency through logical superdense coding. In Sec. VI, we discuss the advantages of our scheme and its potential applications. Finally, in the concluding section (Sec. VII), we summarize the key findings of this work.

II. PROTECTING LOGICAL QUBITS WITH iSWAP GATES

When a set of physical qubits interact with their environment, the total Hamiltonian of the system can be described as

$$H_t = H_S + H_E + H_I, \quad (1)$$

where H_S represents the Hamiltonian of the qubits, H_E corresponds to the environment's Hamiltonian, and H_I describes the qubit-environment interaction [40]. In the general scenario where the qubits interact with the environment independently, H_I can be expressed as

$$H_I = \sum_{k,\alpha} \sigma_\alpha^k \otimes E_\alpha^k. \quad (2)$$

Here, σ_α^k represents the Pauli- α operator (where $\alpha = x, y, z$) acting on the k th qubit, and E_α^k is the corresponding environmental operator [18]. Furthermore, H_I may induce dephasing and dissipation in the qubits, which in turn distorts their evolution.

To elucidate our scheme, we begin by explaining how to encode a single logical qubit with a pair of physical qubits. The interaction Hamiltonian can be written as

$$H_I = \sum_{\alpha=x,y,z} \sigma_\alpha^1 \otimes E_\alpha^1 + \sigma_\alpha^2 \otimes E_\alpha^2. \quad (3)$$

The independent interaction H_I is transformed into a collective interaction through the DD procedure D_2 [shown in Fig. 1(a)], which consists of four iSWAP gates applied at equal time intervals. Assuming that τ is sufficiently small and the iSWAP gate duration is negligible compared to τ , we can calculate the effective Hamiltonian for D_2 as follows:

$$H_I^z = \frac{1}{4} [H_I + \text{i}S H_I \text{i}S^{-1} + \text{i}S^2 H_I (\text{i}S^2)^{-1} + \text{i}S^3 H_I (\text{i}S^3)^{-1}], \quad (4)$$

where the notation $\text{i}S$ represents the iSWAP gate, and we have applied the Baker-Campbell-Hausdorff formula. The resulting effective Hamiltonian takes the form

$$H_I^z = \frac{1}{2} (\sigma_z^1 + \sigma_z^2) \otimes (E_z^1 + E_z^2) = S_z \otimes \bar{E}_z, \quad (5)$$

which is a Z-type collective interaction. Note that H_I^z can also be generated using more iSWAP gates in the above DD procedure. For instance, we can perform eight iSWAPs within the time duration τ at equal time intervals ($\tau/8$). We denote this procedure as D_2^* [shown in Fig. 1(a)].

The effective Hamiltonian H_I^z supports a DFS, denoted as $\text{DFS}_2 = \text{Span}\{|01\rangle, |10\rangle\}$, which serves as the encoding space for a logical qubit defined by $|0\rangle_L = |01\rangle$ and $|1\rangle_L = |10\rangle$. Since these two logical basis states are invariant under the action of H_I^z , they act as dark states of H_I^z and are consequently unaffected by it. The encoding and decoding procedures for a logical qubit are explicitly illustrated in Fig. 1(b). Further details and circuits for the encoding and decoding of multiple logical qubits can be found in Appendix A. Notably, DFS_2 has also been used in other frameworks, such as the g -frame qubit [39],

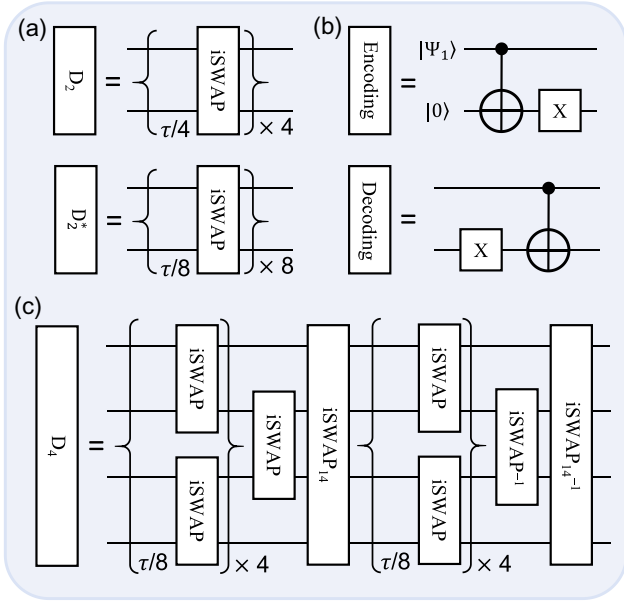


FIG. 1. Quantum circuits for various procedures. (a) Quantum circuits of D_2 and D_2^* . (b) Encoding and decoding circuits for a single logical qubit. (c) Quantum circuit for the DD procedure D_4 .

hardware-efficient architectures [41], and dual-rail qubits [42–46].

When considering two logical qubits encoded within the $\text{DFS}_4 = \text{Span}\{|00\rangle_L, |01\rangle_L, |10\rangle_L, |11\rangle_L\}$ using four physical qubits, an intuitive approach is to implement a four-qubit Z -type collective interaction. This can be effectively achieved via the D_4 procedure, as illustrated in Fig. 1(c). We denote the two logical qubits as L_1 and L_2 . By applying half of the decoupling procedure D_2^* (denoted as $D_2^*/2$) simultaneously to L_1 and L_2 , the effective Hamiltonian becomes

$$H'_{\text{eff}} = S_z^1 \otimes \bar{E}_z^1 + S_z^2 \otimes \bar{E}_z^2, \quad (6)$$

where $S_z^k = \frac{1}{2}(\sigma_z^{2k-1} + \sigma_z^{2k})$ and $\bar{E}_z^k = E_z^{2k-1} + E_z^{2k}$ ($k = 1, 2$). Since H'_{eff} is not a collective interaction—generally $\bar{E}_z^1$ and $\bar{E}_z^2$ are not identical—we apply the following DD procedure to transform H'_{eff} into a four-qubit collective interaction:

$$D_4 = [D_2^*/2, \text{i}S_{23}\text{i}S_{14}(D_2^*/2)\text{i}S_{23}^{-1}\text{i}S_{14}^{-1}], \quad (7)$$

where $\text{i}S_{mn}$ is the i SWAP gate acting on physical qubits Q_m and Q_n . Using the relation

$$\text{i}S_{mn}\sigma_z^m \otimes I^n \text{i}S_{mn}^{-1} = I^m \otimes \sigma_z^n, \quad (8)$$

where I^k ($k = m, n$) is the identity operator on the physical qubit Q_k , the resulting effective Hamiltonian is given by

$$H''_{\text{eff}} = S_z \otimes \bar{E}_z, \quad (9)$$

where $S_z = \frac{1}{2} \sum_{k=1}^4 \sigma_z^k$ and $\bar{E}_z = \frac{1}{2} \sum_{k=1}^4 E_z^k$. Consequently, H''_{eff} represents a Z -type collective interaction.

III. EXPERIMENTAL DEMONSTRATION OF PROTECTING LOGICAL QUBITS

In our experiments, we use two superconducting devices that share the same layout but have different energy-relaxation and coherence times. Figure 2(a) shows the configuration of the qubits: the four frequency-tunable

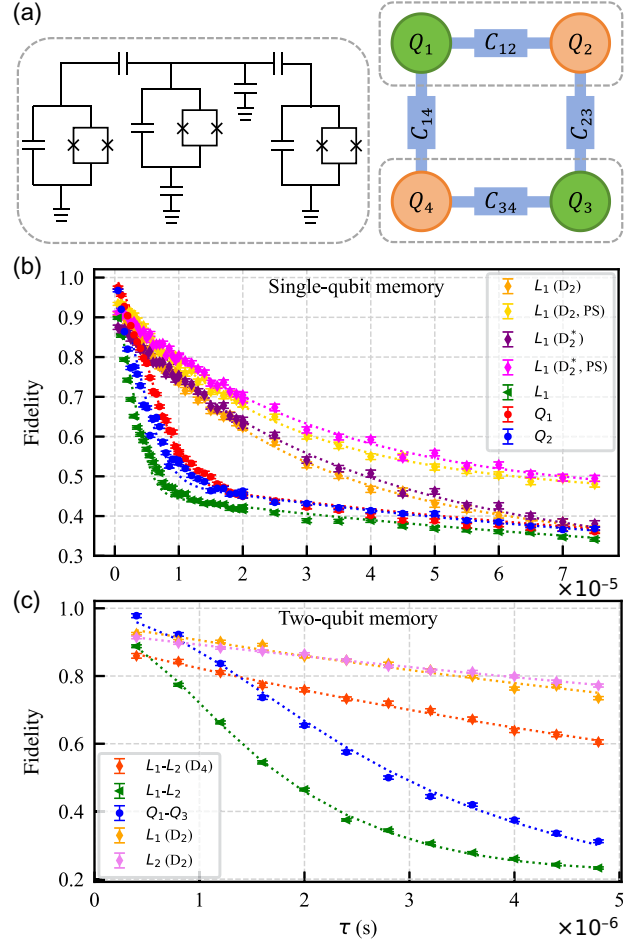


FIG. 2. Arrangement of logical qubits and process fidelities for various procedures. (a) Left: schematic circuit of a logical qubit consisting of two transmon qubits and one tunable coupler. Right: the physical qubits are denoted by green and orange circles. Couplers (C_{12} to C_{14}) between physical qubits are labeled with blue rectangles. Physical qubits Q_1 and Q_2 are used to define the logical qubit L_1 , while Q_3 and Q_4 constitute the logical qubit L_2 . (b) Process fidelities of single-qubit identity operation over increased durations. Here, PS denotes postselection. (c) Process fidelities of two-qubit identity operation over increased durations. The two-physical-qubit fidelity is measured with Q_1 and Q_3 . Details are shown in the inset. The dashed lines are fitting curves plotted using the fidelity functions (see Appendix E for details).

transmon qubits Q_1 to Q_4 are arranged in a circle with only nearest-neighbor coupling, and adjacent qubits are connected through tunable couplers [47], labeled as C_{12} to C_{14} . Each qubit is connected to a dedicated driving line for individual XY control. The coupling strength between neighboring qubits can be adjusted from negative to positive by tuning the couplers' resonance frequencies. This adjustability ensures high-contrast on-off switching of interqubit coupling, which is essential for implementing entangling gates such as iSWAP and controlled-Z (CZ) gates [48]. Additionally, we employ impedance-matched Josephson parametric amplifiers (JPAs) to achieve high-fidelity single-shot readout of the qubits [49]. We use the first processor to demonstrate the protection and operations of a single logical qubit, while the second processor is used to demonstrate the protection of two logical qubits and universal operations. More information about the experimental setup and parameters can be found in Appendices B and C.

We assess the memory times of both logical and physical qubits by employing quantum process tomography (QPT) [1] to quantify the fidelities of the identity operation across various durations (see Appendices D and E for details). We begin by evaluating the performance of the logical qubit L_1 and its physical counterparts, as shown in Fig. 2(b). Without any protection mechanisms, the process fidelities of the physical qubits Q_1 and Q_2 noticeably decrease with increasing duration τ . The fidelity for the physical qubit Q_i can be written as [50,51]

$$F_{Q_i} = \frac{1}{4} \left(2e^{-(\tau/T_{2,i}^*)^2} + e^{-\tau/T_{1,i}} + 1 \right), \quad (10)$$

where $T_{2,i}^*$ and $T_{1,i}$ represent the coherence and energy-relaxation times of the physical qubit Q_i , respectively.

Without protection, L_1 experiences faster fidelity degradation than a single physical qubit, as two physical qubits introduce more errors. The unprotected logical coherence time is given by $T_2^{\text{un}} = T_{2,1}^* T_{2,2}^* / (T_{2,1}^* + T_{2,2}^*)$, and the energy-relaxation time is dominated by $T_{1,1}$. The fidelity of L_1 is affected by T_1 error only from Q_1 due to the tailored decoding procedure (see Appendices A and E for details).

Remarkably, when L_1 is protected by D_2 , the process fidelity of L_1 decreases much more slowly than that of unprotected L_1 and any physical qubits. Compared with the Gaussian decay observed in the fidelity of unprotected qubits, the protected L_1 exhibits an exponential decay in process fidelity (see more details in Appendix E),

$$F_{L_1} = \frac{1}{4} \left(2e^{-\tau/T_2^p} + e^{-\tau/T_{1,1}} + 1 \right), \quad (11)$$

which we attribute to the modification of the interaction between L_1 and its environment by D_2 . A prolonged logical coherence time $T_2^p = 22.5$ (0.7) μs is found from F_{L_1} , far exceeding the coherence times of physical qubits $T_{2,1}^* = 8.6$ (0.4) μs and $T_{2,2}^* = 7.0$ (0.9) μs .

We also implement a protection procedure D_2^* that uses eight iSWAPs to protect L_1 . The application of D_2^* increases the logical coherence time to $T_2^p = 30.7$ (1.2) μs . Thus, our results convincingly demonstrate that the memory of the protected logical qubit surpasses that of the better-performing physical qubit. We anticipate that even longer logical coherence times can be achieved by employing more iSWAP gates in the protecting procedure, as these gates can provide better elimination of noncollective interactions.

Postselection is employed here to further enhance the fidelity of the logical qubit by mitigating T_1 errors ($|1\rangle \rightarrow |0\rangle$). Consider a logical state $|\psi_i\rangle = \alpha|0\rangle_L + \beta|1\rangle_L$, where $|\alpha|^2 + |\beta|^2 = 1$. When T_1 errors ($|1\rangle \rightarrow |0\rangle$) occur in the two physical qubits with probability $\gamma \in [0, 1]$ (assuming both physical qubits have the same probability γ for simplicity), the process is described by the Kraus operators $K_0 = |0\rangle\langle 0| + \sqrt{1-\gamma}|1\rangle\langle 1|$ and $K_1 = \sqrt{\gamma}|0\rangle\langle 1|$. The final state's density matrix after the amplitude-damping channel is $\rho_e = (1-\gamma)\rho_i + \gamma|00\rangle\langle 00|$. After decoding, ρ_e becomes

$$\rho_e^d = (1-\gamma)(\alpha|0\rangle + \beta|1\rangle)(\alpha^*\langle 0| + \beta^*\langle 1|) \otimes |0\rangle\langle 0| + \gamma|0\rangle\langle 0| \otimes |1\rangle\langle 1|. \quad (12)$$

In the absence of T_1 errors, the second physical qubit Q_2 will return to $|0\rangle$; however, with T_1 errors, Q_2 has a probability γ of being measured as $|1\rangle$. Thus, the state of Q_2 serves as a diagnostic signal for T_1 errors. We implement postselection by measuring both qubits after decoding: If Q_2 is in $|1\rangle$, we discard the result; if Q_2 is in $|0\rangle$, we accept the data. As shown in Fig. 2(b), postselection significantly improves the fidelity of L_1 . The number of accepted shots in the postselection procedure decreases with increasing τ . For the experiment with the D_2 procedure, when $\tau = 75$ μs , the average fraction of accepted shots decreases to 46.5%.

We demonstrate the protection of two logical qubits on the second processor. First, we replicate the single-logical-qubit experiments using procedure D_2 on both L_1 and L_2 , which also show a significant increase in the fidelity (coherence time) of the logical qubits. Second, we implement the protection for two logical qubits using the DD procedure D_4 . The protected two-logical-qubit process exhibits significant improvement over the two-physical-qubit process, as shown in Fig. 2(c). When $\tau = 4.8$ μs , the fidelity of the former remains at 60.51%, significantly surpassing that of the latter (31.19%).

IV. UNIVERSAL OPERATIONS FOR LOGICAL QUBITS

Our scheme enables universal control of logical qubits using single-body and nearest-neighbor two-body Hamiltonians. For logical qubit L_1 , the logical Pauli-Z operator corresponds to the physical σ_z^1 operator. Thus, a logical rotation about the z axis by an angle φ for L_1 is equivalent to the same rotation R_z^φ of the physical qubit Q_1 . The logical Pauli-X operator for L_1 ($\sigma_x^{L_1}$) is represented by the XY interaction $\sigma_x^1 \sigma_x^2 + \sigma_y^1 \sigma_y^2$. Thus, the $\sqrt{\text{ISWAP}}$ gate acting on Q_1 and Q_2 implements the logical rotation about the x axis by an angle $-\pi/2$ for L_1 . With the two rotations, we can perform arbitrary single-logical-qubit operations on a logical qubit, such as the logical Hadamard gate (H). Additionally, the $\sigma_z^{L_1} \sigma_z^{L_2}$ interaction corresponds to physical $\sigma_z^2 \sigma_z^3$, which is a nearest-neighbor interaction between Q_2 and Q_3 . Hence, a logical CZ gate can be achieved by implementing a CZ gate between Q_2 and Q_3 followed by an R_z^π gate on Q_3 . Therefore, we can establish a complete set of universal logical gates: $\{R_z^\varphi, R_x^{-\pi/2}, \text{CZ}\}$ [see Fig. 3(a)].

To demonstrate single-qubit logical operations, we conduct a logical Ramsey experiment on L_1 with both D_2 and postselection applied. The quantum circuit and experimental results are depicted in Fig. 3(b). The coherence times obtained from the logical Ramsey experiment and the QPT fidelity experiment are in good agreement. The universal logical control is demonstrated by preparing a logical Bell state $B_1 = (|00\rangle_L + |11\rangle_L)/\sqrt{2}$ on L_1 and L_2 , as shown in Fig. 3(c). We use quantum state tomography to evaluate the state's fidelity, which is determined to be 92.30%. Our analysis indicates that approximately

28% of the errors originate from the state tomography and readout process. The remaining errors predominantly arise from operations involving physical two-qubit gates ($\sqrt{\text{ISWAP}}$ and CZ). We stress that the main focus here is to demonstrate the capability of universal control in our scheme. Hence, we choose a rather tedious circuit comprising seven layers and 11 two-qubit gates to create B_1 . In practice, more concise methods are available for preparing the logical Bell state, which would be expected to result in a higher state fidelity.

V. EFFICIENT PROTECTION OF MULTIPLE LOGICAL QUBITS

To protect N logical qubits encoded in $2N$ physical qubits, a $2N$ -qubit Z -type collective interaction is a natural choice; however, it is important to note that the largest DFS supported by this collective interaction is $C(2N, N)$ -dimensional, significantly exceeding the required dimension of $\text{DFS}_2^{\otimes N}$. This implies a substantial dimension overhead, which can be optimized to reduce the time or number of gates needed for protecting multiple logical qubits. We propose a more efficient approach to protect multiple logical qubits. The encoding subspace $\text{DFS}_2^{\otimes N}$ for N logical qubits is also supported by the effective Hamiltonian $\sum_{k=1}^N H_{1,k}^z$, where

$$H_{1,k}^z = \frac{1}{2}(\sigma_z^{2k-1} + \sigma_z^{2k}) \otimes (E_z^{2k-1} + E_z^{2k}) = S_z^k \otimes \bar{E}_z^k, \quad (13)$$

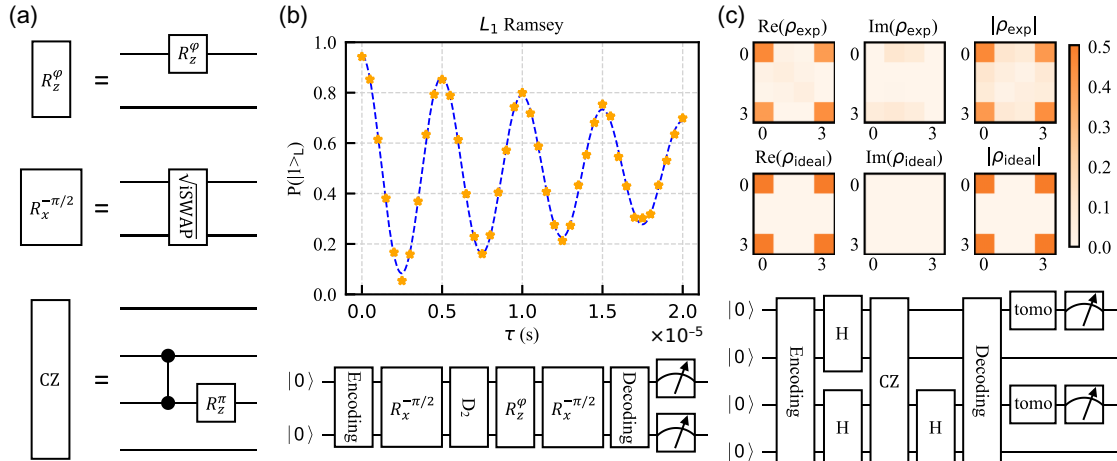


FIG. 3. Universal logical operations. (a) The set of universal logical gates. (b) A Ramsey experiment is conducted on logical qubit L_1 . The orange stars represent experimental data points, while the blue dashed line indicates the simulation results, which are calculated based on the energy-relaxation time $T_{1,1}$ and the coherence time T_2^p as input parameters. (c) Preparation of the logical Bell state $B_1 = (|00\rangle_L + |11\rangle_L)/\sqrt{2}$ from initial state $|0000\rangle$ using logical Hadamard and CZ gates. The obtained density matrix elements are shown at the top, where 0 to 3 represent the bases $|00\rangle$, $|01\rangle$, $|10\rangle$, and $|11\rangle$, respectively.

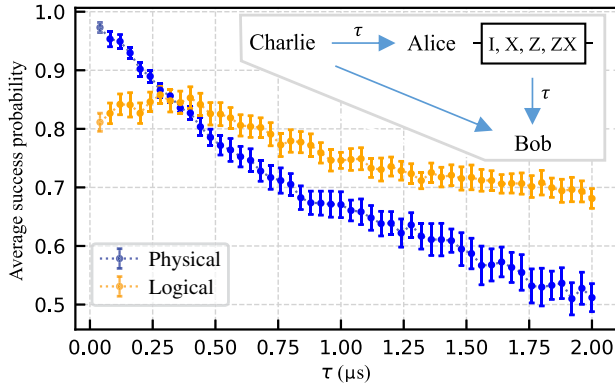


FIG. 4. Superdense coding, showing the average success probabilities for logical superdense coding (illustrated in orange) and physical superdense coding (illustrated in blue).

because the logical basis states $|ij \cdots m\rangle_L$ in $\text{DFS}_2^{\otimes N}$ are dark states of $\sum_{k=1}^N H_{1,k}^z$, i.e.,

$$\sum_{k=1}^N H_{1,k}^z |ij \cdots m\rangle_L = 0, \quad i, j, \dots, m \in \{0, 1\}. \quad (14)$$

This effective Hamiltonian can be realized by simultaneously applying the D_2 procedure to each logical qubit. Therefore, the control time required by this protection scheme does not increase with the number of logical qubits, indicating strong scalability.

As a demonstration, we use two logical qubits to enhance the performance of superdense coding [52]. The process is illustrated in the inset of Fig. 4. Initially, a trusted third party, Charlie, prepares the Bell state B_1 . Then, Charlie sends one qubit to Alice and the other to Bob, assuming that this transmission takes time τ . Upon receiving her qubit, Alice applies one of the four possible operations ($\{I, X, Z, ZX\}$) to it and then forwards it to Bob, a process that also takes time τ . Bob then performs a Bell measurement, yielding four possible outcomes according to Alice's operations. This allows Bob to extract the information transmitted by Alice after a total duration of 2τ . We vary the duration τ to observe its impact on Bob's average success probability.

The experiments are conducted with both physical and logical qubits. During each τ , we apply the D_2 procedure to the two logical qubits independently. For short values of τ , the physical qubits outperform the logical ones, because the physical qubits have not yet experienced substantial decoherence, but the logical ones have more control errors; however, for sufficiently long τ , the coherence protection shows its advantage, allowing the logical qubits to outperform the physical ones.

VI. DISCUSSION

A. Comparison between logical qubits and DD-protected physical qubits

We measure the coherence times of the physical qubits Q_1 and Q_2 , which are used in the single-logical-qubit experiment shown in Fig. 2(b). The Ramsey T_2 values of these qubits are 8.6 and 7.0 μs , respectively. When the qubits are protected by a single Carr-Purcell (CP) pulse, their coherence times increase to 14.3 and 14.1 μs . Additionally, we measure the coherence times for four CP pulses, eight CP pulses, and XY-4 decoupling pulses. The resulting coherence times are comparable, averaging 23.5 and 23.1 μs . We attribute this saturation to high-frequency and time-dependent noise. In contrast, when we use our scheme to protect the logical qubit, the coherence time under D_2 protection reaches 22.5 μs , which is close to the saturated values of the physical qubits. For the logical qubit protected by D_2^* , the coherence time is 30.7 μs , exceeding that of the protected physical qubits (shown in Fig. 5). We note that in a recent work [53], logical qubits protected by DD have also been shown to exhibit longer coherence times than DD-protected physical qubits, albeit using a different protection scheme. Furthermore, we believe that further elimination of noncollective interactions will lead to longer coherence times. Thus, we plan to employ continuous DD [43,45] to protect logical qubits in future investigations, which can significantly enhance the quality of DFSs and the coherence times of logical qubits.

B. Scalability analysis

Consider an $N \times M$ physical-qubit lattice in which each physical qubit interacts with its four nearest neighbors. As illustrated by the black rectangles in Fig. 6, nearest-neighbor qubits are paired to encode a logical

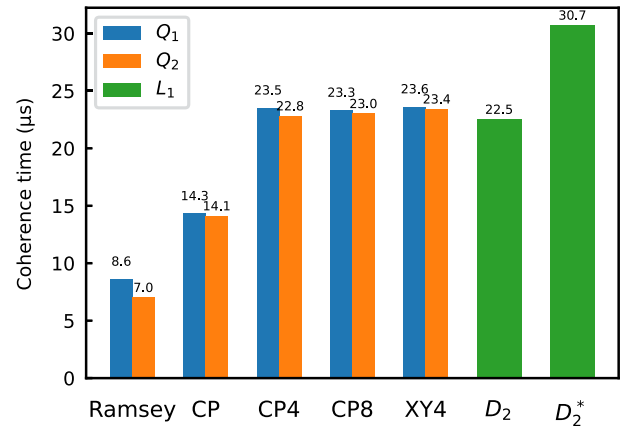


FIG. 5. Coherence times of physical qubits and the logical qubit under different procedures. With the protection of D_2^* , the T_2 value of L_1 surpasses that of both physical qubits protected by CP or XY pulses.

qubit. As discussed above, protecting multiple logical qubits requires applying the D_2 procedure individually to each logical qubit, resulting in a linear increase in the number of iSWAP gates while keeping the control time constant with respect to the number of logical qubits. The gray lines in the figure indicate interactions between logical qubits, where pairs connected by these lines are capable of implementing logical CZ gates. After encoding, the logical-qubit lattice becomes $N/2 \times M$, retaining the original topological structure of the physical-qubit lattice while introducing direct interactions between diagonal logical qubits, thereby enhancing connectivity. Therefore, the scheme is highly scalable.

C. Benefits and potential applications

A key feature of our scheme is the flexible transfer of quantum states between logical qubits and their physical counterparts. During the execution of a quantum circuit, one can decode a logical state to a single physical qubit and measure another physical qubit. By selecting an appropriate measurement period, we can detect whether a decay error has occurred during this duration. After the error-checking procedure, the information can be re-encoded back into the logical qubit. This error-checking procedure offers two primary advantages. On one hand, this process allows for the mitigation of decay errors through postselection, thereby enhancing the precision of the quantum circuit and improving computational accuracy in the noisy intermediate-scale quantum (NISQ) era. On the other hand, our scheme has the potential to realize erasure qubits [45],

which can increase the number of correctable errors and improve the threshold for QEC.

VII. SUMMARY

In this work, we propose and verify the protection of logical qubits using the DD technique. We explicitly demonstrate that the lifetimes of both single logical qubits and pairs of logical qubits are prolonged compared to those of their physical counterparts. Additionally, we conduct a logical Ramsey experiment and successfully create a logical Bell state to demonstrate universal control of logical qubits. We propose a scalable protection procedure and demonstrate its efficiency through a comparison between physical and logical superdense coding. Furthermore, we discuss the scalability and potential applications of our scheme.

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DATA AVAILABILITY

The data are not publicly available. The data are available from the authors upon reasonable request.

APPENDIX A: THE ENCODING AND DECODING OF THE LOGICAL QUBITS

In the main text, we use physical qubits Q_1 and Q_2 to define the first logical qubit L_1 , and physical qubits Q_3 and Q_4 for the second logical qubit L_2 . Consider an arbitrary single-qubit pure state $|\psi\rangle = a_0|0\rangle + a_1|1\rangle$, where $|a_0|^2 + |a_1|^2 = 1$. To encode $|\psi\rangle$ into DFS_2 , we initialize the two physical qubits in the product state $|\phi_0\rangle = |\psi\rangle \otimes |0\rangle$. Upon applying a CNOT gate to these qubits, we obtain the state $|\phi_1\rangle = a_0|00\rangle + a_1|11\rangle$. The CNOT gate is implemented by combining a CZ gate and two single-qubit Hadamard (H) gates. We then apply an X gate to the second qubit, transforming the state to $|\phi_2\rangle = a_0|01\rangle + a_1|10\rangle$ within DFS_2 . This encoding approach enables the parallel preparation of a multiqubit logical product state, as shown in Fig. 7(a). The decoding is simply the reverse of the encoding procedure. After decoding, the logical information resides only in the first physical qubit in each pair.

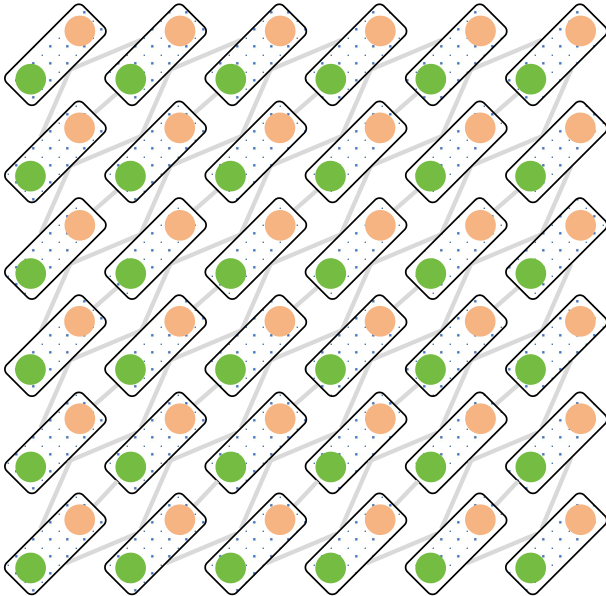


FIG. 6. Scalability diagram. Physical qubits are depicted as green and orange circles, while logical qubits are represented by black rectangles. Gray lines indicate the interactions between logical qubits.

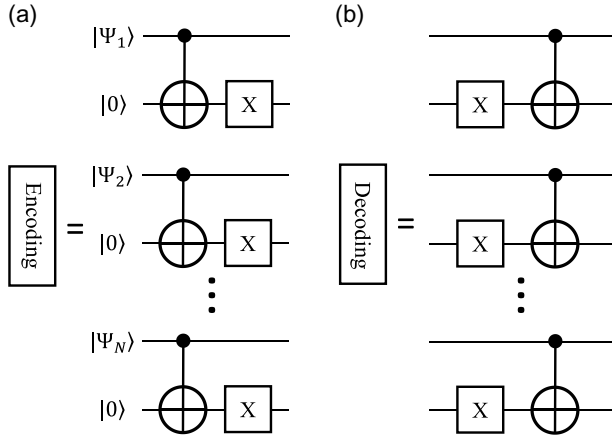


FIG. 7. Quantum circuits for the (a) encoding and (b) decoding procedures of multiple logical qubits.

APPENDIX B: EXPERIMENTAL SETUP

Our experimental setup is shown in Fig. 8. The sample, which comprises nine frequency-tunable qubits and 12 tunable couplers, is installed in the mixing chamber (MC) layer of the dilution refrigerator. To reduce wiring complexity, a single control line is used for both the XY and Z control signals of each qubit, with the XY and Z signals combined externally via a multiplexer. The signals, after 30-dB attenuation and infrared filtering using Eccosorb CR-124, are delivered to the sample. The CR-124 infrared filter attenuates the XY control signals by approximately 20–50 dB (depending on the filter length) while having negligible effects on the dc pulses (known as Z control signals). An additional 500-MHz low-pass filter is added to eliminate high-frequency noise, as the couplers do not require XY control signals. The probe signals, after 70-dB attenuation, low-pass filtering, and infrared filtering using Eccosorb CR-110, reach the sample. Compared to CR-124, CR-110 exhibits less frequency-selective attenuation. After passing through the sample, the probe signals are amplified by a JPA before entering a high-electron-mobility transistor (HEMT) at the 4-K stage. After exiting the refrigerator, the signals are further amplified by two microwave amplifiers, filtered, and digitized by a 4 GS/s analog-to-digital converter (ADC).

The Z control signals are generated using a digital-to-analog converter (DAC). The XY signals and the probe signals are generated via direct digital synthesis (DDS), and appropriate band-pass filters are used to filter out out-of-band frequencies, retaining only the desired signals. In comparison to using IQ mixers, this method avoids the need for repeated calibration, simplifying the classical control chain for quantum computing experiments, enabling more advanced pulse shaping, and reducing the number of components that need to be carefully calibrated.

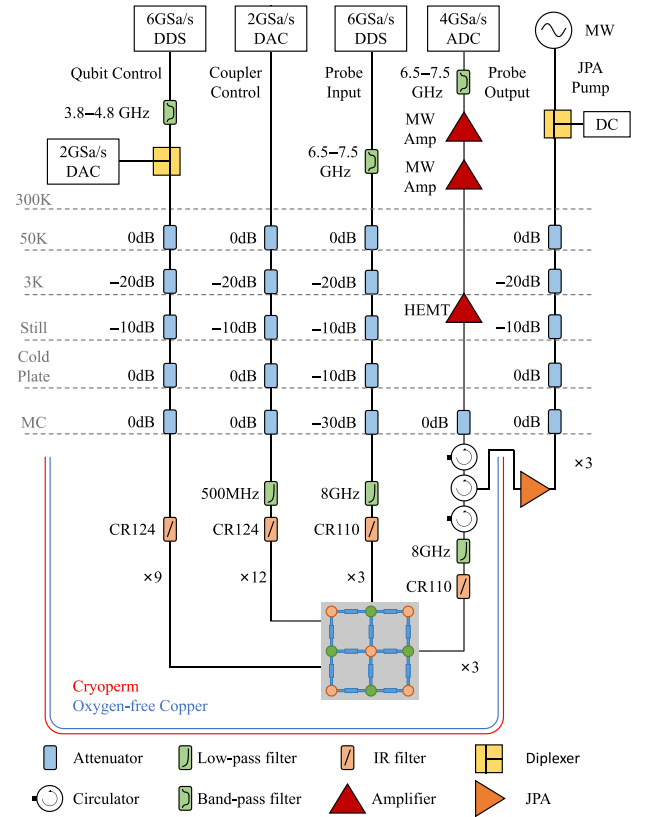


FIG. 8. Schematic diagram of the measurement setup.

APPENDIX C: EXPERIMENTAL PARAMETERS AND GATE FIDELITIES

The experimental procedures described in the main text are divided into two distinct segments. The first segment focuses exclusively on a single logical qubit. The experiment is conducted on the first processor, and this part corresponds to the results depicted in Fig. 2(b) in the main text. In the second segment, we use the other processor and investigate two logical qubits consisting of four physical qubits. Thus, our experimental framework encompasses two sets of parameters, which are detailed in Tables I and II. In both experiments, the durations for the quantum gates acting on the physical qubits are listed in Table III.

TABLE I. Parameters for the first part of the experiments.

Parameter	Q_1	Q_2
$\omega_{01}/2\pi$ (GHz)	4.207	4.309
T_1 (μ s)	100.42	95.52
T_2^* (μ s)	8.56	7.02
1Q RB e_1 (%)	0.04	0.04
Readout $e_{ 0\rangle}$ (%)	1.54	2.33
Readout $e_{ 1\rangle}$ (%)	1.26	4.19
CZ RB e_2 (%)		0.50
iSWAP XEB e_2 (%)		0.44

TABLE II. Parameters for the second part of the experiments.

Parameter	Q_1	Q_2	Q_3	Q_4
$\omega_{01}/2\pi$ (GHz)	4.513	4.431	4.192	4.349
T_1 (μ s)	44 ± 3	44 ± 3	41 ± 2	43 ± 2
T_2^* (μ s)	4.2 ± 0.4	4.2 ± 0.2	3.9 ± 0.2	4.1 ± 0.2
1Q RB e_1 (%)	0.08	0.09	0.10	0.12
Readout $e_{ 0\rangle}$ (%)	1.54	3.09	1.74	2.77
Readout $e_{ 1\rangle}$ (%)	4.59	5.77	6.45	5.18
CZ RB e_2 (%)	0.30	0.82	1.37	
iSWAP XEB e_2 (%)	0.73	0.86	0.97	1.08 (Q_1 - Q_4)

APPENDIX D: QUANTUM PROCESS TOMOGRAPHY FOR LOGICAL AND PHYSICAL QUBITS

In our study, we assess the memory performance of both logical and physical qubits using QPT. We measure the fidelity of identity operations at different time intervals. For physical qubits, process fidelity is determined using standard QPT protocols. In contrast, for logical qubits, we first encode the initial states into logical qubits. After a duration of τ or applying a DD procedure, we decode the information back to physical qubits for measurement. The quantum circuits used in the experiments shown in Fig. 2 of the main text are displayed in Fig. 9. We construct the χ matrix of the process using the measurement results and calculate the process fidelity as $\text{Tr}(\chi_{\text{ideal}}\chi_{\text{meas}})$, where χ_{ideal} and χ_{meas} denote the χ matrices for the ideal case and those obtained from the measurement, respectively [50,51].

APPENDIX E: FITTING THE PROCESS FIDELITIES

The process fidelity of the identity operation I on a single qubit is affected by the qubit's energy-relaxation and coherence times. Consider a single qubit in the state $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$. Under the action of an ideal identity, the state $|\psi\rangle$ is unchanged. The corresponding ideal process matrix

(χ_{ideal}) is given by

$$\chi_{\text{ideal}} = \begin{pmatrix} 1 & & & \\ & 0 & & \\ & & 0 & \\ & & & 0 \end{pmatrix}. \quad (\text{E1})$$

In the presence of amplitude damping and dephasing effects, the qubit's density matrix at time t can be written as

$$\rho(t) = \begin{pmatrix} 1 + (|\alpha|^2 - 1)e^{-\Gamma_1 t} & \alpha\beta^*e^{-\Gamma_2 t} \\ \alpha^*\beta e^{-\Gamma_2 t} & |\beta|^2 e^{-\Gamma_1 t} \end{pmatrix}, \quad (\text{E2})$$

where Γ_1 is determined by the amplitude damping rate and Γ_2 is determined by the dephasing rate and the amplitude damping rate. By comparing $\rho(0)$ and $\rho(t)$, the superoperator $\hat{L}$ of this process can be calculated as

$$\hat{L} = \begin{pmatrix} 1 & 0 & 0 & 1 - e^{-\Gamma_1 t} \\ 0 & e^{-\Gamma_2 t} & 0 & 0 \\ 0 & 0 & e^{-\Gamma_2 t} & 0 \\ 0 & 0 & 0 & e^{-\Gamma_1 t} \end{pmatrix}. \quad (\text{E3})$$

Based on $\hat{L}$, the corresponding χ matrix can be found by solving the equation $\hat{L} = \sum_{ij=I,x,y,z} \chi_{ij} \hat{A}_{ij}$, where $\hat{A}_{ij} = \sigma_i \otimes \sigma_j$ (with $\sigma_I = I$). Since the process fidelity $F = \text{Tr}(\chi_{\text{ideal}}\chi_{\text{meas}})$, the primary interest lies in χ_{II} , based on which the process fidelity F is calculated as $F = \frac{1}{4}(2e^{-\Gamma_2 t} + e^{-\Gamma_1 t} + 1)$.

The dephasing characteristics of superconducting qubits are influenced by the nature of the noise. For example, Gaussian flux noise typically induces dephasing that follows a Gaussian decay pattern, represented as $e^{-(\Gamma_2 t)^2}$. In contrast, decoherence associated with relaxation processes (T_1) exhibits an exponential decay. Consequently, the fitting of process fidelity in such systems may alternate between Gaussian and exponential dephasing forms, as shown in Eqs. (10) and (11) in the main text. We fit all the experimental data in Fig. 2(b) in the main text using both Gaussian and exponential functions, and we use the coefficient of determination (R^2) as a metric to evaluate

TABLE III. Durations for the physical quantum gates.

Gates	Gate time
$R_x^{\pi/2}$	20 ns
$R_y^{\pi/2}$	20 ns
$R_x^{-\pi/2}$	20 ns
H	40 ns
σ_x	40 ns
$\sqrt{\text{iSWAP}}$	20 ns
iSWAP	40 ns
CZ	70 ns

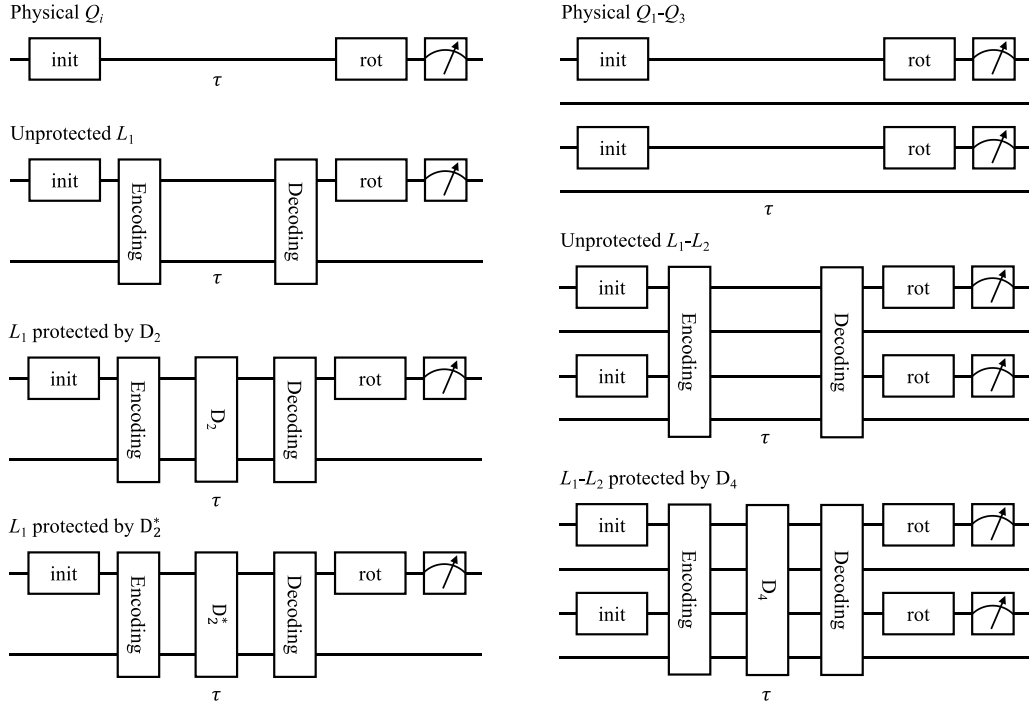


FIG. 9. Quantum circuits for the experiments shown in Fig. 2 in the main text.

the fit quality. The R^2 values for all fittings are shown in Table IV.

To account for state preparation and measurement (SPAM) errors in our experiments, we use the following fitting function for a single physical qubit:

$$f(x) = \frac{1}{4} \left[A \left(2e^{-(x/T_2^*)^2} + e^{-x/T_1} \right) + B \right], \quad (\text{E4})$$

where A and B account for SPAM errors, each typically taking values close to 1. This fitting yields the qubit's T_1 and T_2^* values, consistent with its measured energy-relaxation and coherence times, respectively.

For an unprotected logical qubit, the fidelity of an identity operation is described by

$$F = \frac{1}{4} \left(2e^{-(\tau/T_2^{\text{un}})^2} + e^{-\tau/T_{1,1}} + 1 \right), \quad (\text{E5})$$

where $T_{1,1}$ is the energy-relaxation time of the first physical qubit. The term T_2^{un} , defined as $T_{2,1}^* T_{2,2}^* / (T_{2,1}^* + T_{2,2}^*)$, denotes the unprotected effective coherence time of the logical qubit. This effective coherence time is determined

by the coherence times of the two physical qubits constituting a logical qubit. The process fidelity is only influenced by the relaxation time of the first physical qubit. This is due to the specialized decoding process employed, which effectively renders the relaxation of the second physical qubit inconsequential to the decoded result.

The inclusion of D_2 and D_2^* protection mechanisms significantly alters the dephasing characteristics of our logical qubits, leading to an exponential behavior. Consequently, the process fidelity of a logical qubit under such protection is described by Eq. (11) in the main text.

Similarly, we can also obtain the process fidelity of a two-qubit system, which is determined by the energy-relaxation and coherence times of each qubit. For two physical qubits, the fidelity can be represented by

$$F = \frac{1}{16} \left(e^{-\tau/T_{1,1}} + e^{-\tau/T_{1,2}} + e^{-\tau/T_{1,1} + T_{1,2}/T_{1,1}} \right. \\ + 2e^{-(\tau/T_{2,1}^*)^2} + 2e^{-(\tau/T_{2,2}^*)^2} \\ + 2e^{-\tau/T_{1,1} - (\tau/T_{2,2}^*)^2} \\ + 2e^{-\tau/T_{1,2} - (\tau/T_{2,1}^*)^2} \\ \left. + 4e^{-\tau^2 / \left(\frac{T_{2,1}^* T_{2,2}^*}{T_{2,1}^* + T_{2,2}^*} \right)^2} + 1 \right). \quad (\text{E6})$$

TABLE IV. The R^2 values for all the fittings in the Fig. 2(b).

Function type	Q_1	Q_2	L_1	$L_1(D_2)$	$L_1(D_2^*)$
Exponential	0.966	0.958	0.912	0.994	0.997
Gaussian	0.993	0.997	0.993	0.915	0.895

For two logical qubits without any protection, the fidelity is described by

$$F = \frac{1}{16} \left(e^{-\tau/T_{1,1}} + e^{-\tau/T_{1,3}} + e^{-\tau/T_{1,1} + T_{1,3}} \right. \\ + 2e^{-(\tau/T_{2,e_1})^2} + 2e^{-(\tau/T_{2,e_2})^2} \\ + 2e^{-\tau/T_{1,1} - (\tau/T_{2,e_2})^2} \\ + 2e^{-\tau/T_{1,3} - (\tau/T_{2,e_1})^2} \\ \left. + 4e^{-\tau^2 / \left(\frac{T_{2,e_1} T_{2,e_2}}{T_{2,e_1} + T_{2,e_2}} \right)^2} + 1 \right). \quad (\text{E7})$$

Here, $T_{2,e_1} = T_{2,1}^* T_{2,2}^* / (T_{2,1}^* + T_{2,2}^*)$ and $T_{2,e_2} = T_{2,3}^* T_{2,4}^* / (T_{2,3}^* + T_{2,4}^*)$ represent the effective coherence times of the first and second logical qubits, respectively. When these qubits are protected by D_4 , the Gaussian terms in Eq. (E7) are substituted with exponential terms.

All these relations are integral to fitting the experimental fidelity curves in Fig. 2 of the main text. The additional parameters A and B are also incorporated, analogous to those in Eq. (E4), to account for SPAM errors.

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