Optical interferometric readout of a magnetically levitated superconducting microsphere

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We probe the motion of a 6-μg magnetically levitated superconducting microsphere using optical interferometry at 3 K, achieving a resolution of the order of $1\,\mathrm{nm}/\sqrt{\mathrm{Hz}}$, and use the measured signal to feedback-cool its motion. The resolution exceeds the shot-noise limit of $11\,\mathrm{pm}/\sqrt{\mathrm{Hz}}$ primarily due to technical noise arising from the roughness of the particle, in combination with its free rotation. These measurements demonstrate that the combination of interferometric optical readout and superconducting magnetic levitation, together with established techniques of cavity optomechanics, provides all the ingredients required for quantum physics experiments with microgram masses.

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I. INTRODUCTION

Mechanical quantum systems are of interest for the exploration of quantum phenomena in progressively larger and more massive systems, thereby pushing the boundaries in testing quantum mechanics [1–8]. Compared to clamped mechanical oscillators, levitated systems offer unique advantages, such as highly controllable trapping potentials that can, for example, facilitate sophisticated protocols in nonharmonic potentials [9,10]. In particular, significant strides have been made by optically levitated nanoparticles with quantum control over the center-of-mass motion [11–16], levitated permanent magnets [17, 18], and electrical levitation with mechanical quality factors exceeding 10^{10} [19]. Here we use all-superconducting magnetic levitation, allowing the virtually dissipation-free trapping of massive particles [20–23]. Recent experiments have demonstrated the levitation of microgram masses with mechanical quality factors reaching 2.6×10^7 [24].

Precise readout of the levitator motion is a critical element on the path toward quantum experiments. Superconducting quantum interference devices have previously been used to track the particle motion indirectly through its effect on the magnetic field, although quantum-limited detection remains challenging [24–27]. Optical interferometers are the most precise measurement tools for relative position changes [28], promising the resolution required to enable quantum control of magnetically levitated particles. However, they are generally incompatible with superconductors because the single-photon energy is above typical Cooper-pair binding energies, causing overexposure to break the superconductivity. Recently, progress has been made to combine optical photons with superconducting circuits [29–31]. In this paper we successfully demonstrate the direct optical readout of a levitated superconductor, establishing a realm of compatibility for these experimental methods, and providing a pathway to quantum control of a microgram mass.

II. EXPERIMENTAL SETUP

Our experimental setup is shown schematically in Fig. 1. We levitate a 6-μg lead-tin (PbSn) superconducting sphere of radius 50 μm in an anti-Helmholtz magnetic trap made from coils of niobium-titanium wire within a closed-cycle cryostat at 3 K. The particle oscillates about the trap center with frequencies $f_i = \sqrt{3/(8\pi^2\mu_0\rho)}b_i$ [22], where μ_0 is the permeability of free space, ρ is the particle density, and

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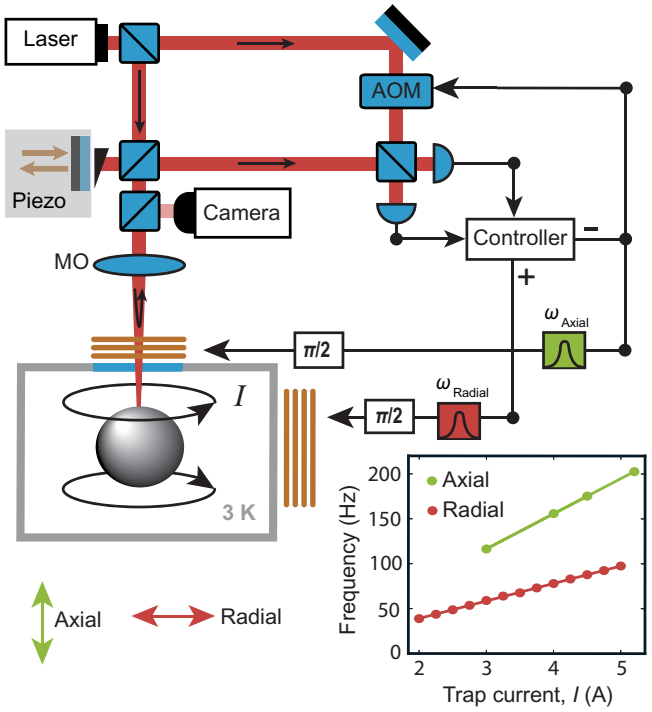


FIG. 1. Schematic of the setup. We levitate a superconducting microsphere in the magnetic field minimum between anti-Helmholtz coils. The axial position of the particle is measured by interferometry. We lock the phase with an acousto-optic modulator (AOM) to increase the linear dynamic range of the interferometric readout. One arm of the interferometer is focused onto the particle by a microscope objective (MO). The sum of the detector's output contains intensity information which is used to feed back the radial motion, while the difference contains phase information corresponding mainly to the axial motion. Feedback is applied by bandpass filtering around the corresponding frequency, adding a phase delay and amplification. The resulting feedback current is applied to additional wires outside the cryostat (brown). While not depicted, independent feedback lines are used for x and y. The interferometer signal can be calibrated using a mirror mounted on a piezoelectric actuator (gray). We also record the particle position using a camera. (Inset) The motional frequencies are linearly dependent on the trap current.

b_i is the gradient in the i-direction that is linearly dependent on the coil current. In the azimuthally symmetric anti-Helmholtz trap, the axial field gradient b_z and the axial frequency f_z are double the radial-plane field gradients b_x, b_y and degenerate frequencies f_x,f_y [32]. Optical interferometry in the axial z-direction is performed with the particle as one of the two mirrors in a Mach-Zehnder-like interferometer, using a $\lambda = 637\,\text{nm}$ laser and balanced homodyne detection. Because radial motion of the particle affects the amount of light collected at the detectors, we precool the particle with camera-based and laser-intensity-based methods that take advantage of the axial optical access. Further details on the experimental setup are available in Appendix A.

An experimental run begins with the superconducting microsphere resting on a silicon carbide (SiC) substrate at the cryostat base temperature of 3 K. We start by applying a large current through the magnetic trap coils to overcome any adhesion forces and lift the particle off the substrate. To reduce residual axial motion, we use friction. Specifically, we briefly lower the current after liftoff, such that the microsphere returns to the SiC surface, where it is frictionally damped through surface contact. The trap current is then slowly ramped up again until the particle is placed into the center of the trap. Initially, the particle moves with a large radial amplitude of around 50 μm, and is feedback-cooled using external magnetic fields generated with dedicated wires outside the cryostat. The first cooling step uses a camera-based readout. We illuminate the particle with an LED flash and image the reflected light through the microscope objective. Distances are calibrated using the known microsphere diameter. Using two camera snapshots separated by 20% of the radial period, we discern the particle motion and apply a counteracting magnetic pulse through the external feedback wires. Repeating this process, we reduce the radial amplitude to around 200 nm.

Next, laser-intensity-based measurements are used to continuously stabilize the particle position. A piezoelectric stage holding the microscope objective is used to position the laser beam onto the particle. Because the microsphere is curved, the angle of the reflected light, and hence the collection efficiency, depends on the radial displacement. We position the laser beam slightly off-center, such that radial motion of the particle induces a power modulation of the detected light. The summed signal of the photodetectors,

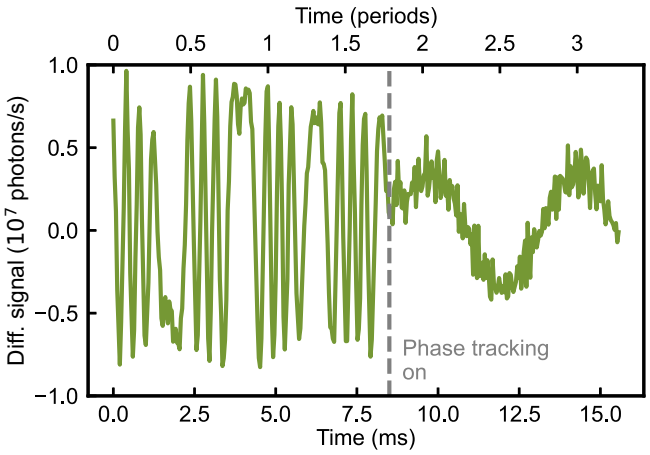


FIG. 2. Linearization of the interferometric signal with balanced homodyne detection. Throughout this time trace the particle motional amplitude is approximately λ, resulting in multiple oscillations in the time trace. Linearization by phase tracking allows the extraction of a sinusoidal signal proportional to the particle displacement.

corresponding to intensity, is then bandpass-filtered and phase-shifted to supply a current to the radial feedback wires. The current creates an oscillating magnetic field that opposes the radial motion of the particle. Further information on the precooling protocol can be found in Appendix B. Finally, the difference signal of the photodetectors is used for interferometric readout. A conventional interferometer responds linearly only to phase shifts far below $\pi/2$ [33], corresponding to a displacement of $\lambda/8$ in our setup. To increase the dynamic range of the interferometer to displacements of several λ, we employ a high-gain closed loop [34] with frequency feedback. This method uses an acousto-optic modulator (AOM) to track the frequency difference between the signal and reference arms of the interferometer; a mathematical description is provided in Appendix D. Figure 2 compares the interferometer signals with and without phase tracking. When the lock is active, the signal is linearly proportional to the particle axial displacement, with the constant selected to achieve the desired dynamic range. The axial motion is cooled in the same way as above.

At infinite gain and instantaneous measurement, the feedback lock would provide a self-calibrated measure of displacement. However, our experiment does not reach this regime. A more accurate calibration of the signal is performed by applying an oscillating driving force through the axial feedback coil and measuring the response. An estimate of the applied magnetic field at the sphere's location is calculated using numerical integration with the coil geometry. Measurements of the response are then carried out on specific sphere levitators at fixed lock gains to emulate experimental conditions as closely as possible. Figure 3 depicts three such calibration measurements at different trapping frequencies about a drive frequency that corresponds to the experimental-run operating trap

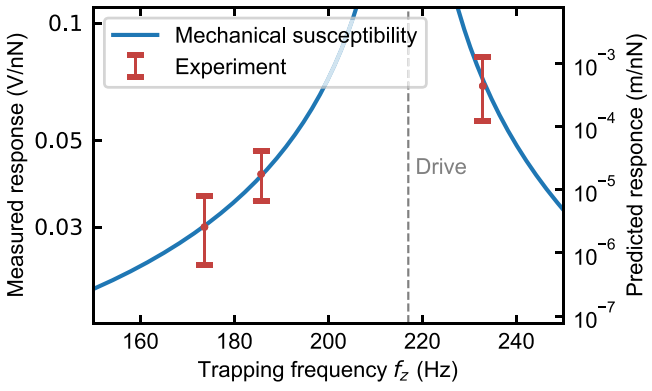


FIG. 3. Calibration of the interferometer signal. An oscillating magnetic force is applied at 217 Hz with an external coil. The response of the particle is measured for three trapping frequencies, {174, 186, 233} Hz . A careful characterization of the applied force allows calibration of displacement units (right axis) from the measured response (left axis).

frequency. More details on the calibration methods are given in Appendix E.

III. RESULTS

We measure a calibrated axial-displacement-amplitude one-sided spectral density noise floor of $\sqrt{2S_{zz}} = (1.52 \pm 0.35)$ nm/$\sqrt{\text{Hz}}$ near the mechanical frequency. Ideally, our precision would be limited by the photon shot noise $S_{zz}^{\text{shot}} = \lambda^2/(64\pi^2 n_{\text{det}})$, where λ is the laser wavelength, and n_{det} is the average collection rate of photons scattered off the particle [35]. With $\lambda = 637$ nm and collection rate $n_{\text{det}} = 10^7$ photons/s, this theoretical limit is $\sqrt{2S_{zz}^{\text{shot}}} = 11\,\text{pm}/\sqrt{\text{Hz}}$. The greater noise floor observed in our experiment is dominated by the microsphere roughness, together with rotations and drifts within the trap. Using white light interferometry, the sphere roughness was measured to be 104 ± 45 nm, a significant fraction of the red laser wavelength. This results in a wildly fluctuating reflectivity, and hence noise in our measured signal.

Despite direct laser incidence, the sphere can remain superconducting for hundreds of seconds. At low power levels, the levitation lifetime follows an inverse power relation from a buildup of heat that cannot be sufficiently dissipated. Due to the optical access windows, a small amount of background radiation is always present in the cryostat, and the same qualitative behavior is observed when changes are made to the aperture. At high laser power there is a sudden collapse in levitation which we attribute to Cooper-pair breaking exceeding the recombination rate. To avoid any quenching, we limit experimental runtimes to 100 s and laser-photon fluxes to 10^7 photons/s, well sufficient to implement our cooling protocol. As such, the use of optical measurements is compatible with the superconducting levitator over timescales needed to carry out experiments. Between runs, we rest the particle on the 3 K substrate, allowing it to rethermalize for 100 s. Further discussion of the trapping lifetime is provided in Appendix C.

Cooling dynamics for interferometric feedback on the axial motion are shown in Fig. 4. With moderate gain, the motion of the particle is cooled to an amplitude below λ to be in the linear regime of the detection. By increasing the feedback gain, the amplitude is further reduced. Optimal feedback cooling, corresponding to a flat spectral density, is found around $\Gamma_{\text{fb}}^{\text{opt}} = 21$ Hz, and gives a minimum cooled displacement oscillation of $\sqrt{\langle z^2 \rangle_{\text{min}}} \approx 8$ nm corresponding to mechanical phonon occupation of around $\bar{n} \approx 5 \times 10^{12}$ and a temperature equilibrium of 52×10^3 K. For high gains, the particle dynamics are dominated by the fed-back measurement noise, and classical noise squashing [12,36] below the measurement noise floor can be observed. The initial effective environmental temperature of the levitator motion is significantly above the thermal temperature due to vibrational noise.

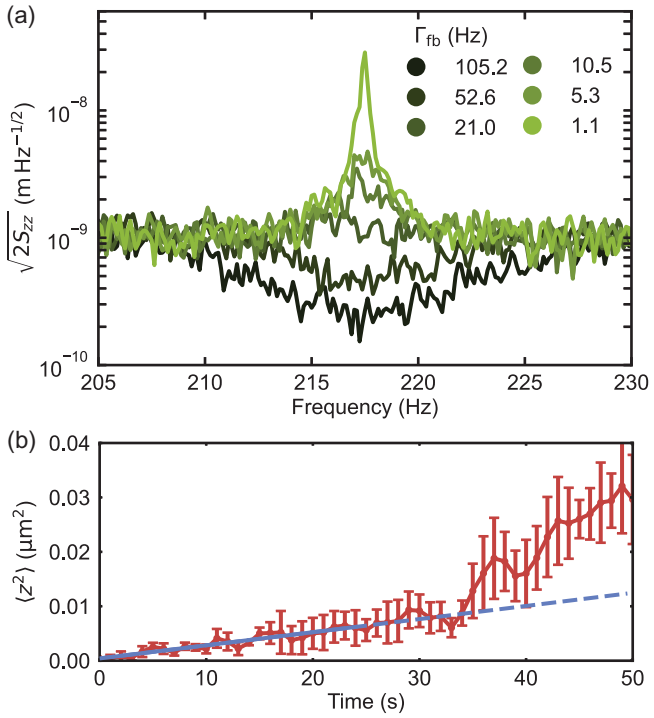


FIG. 4. (a) Measured one-sided power spectral density, in units of displacement noise, taken with different feedback rates. Each trace is produced from the Fourier transform of time series data. Increasing feedback strength (darker curves) reduces the displacement amplitude, pushing the measured response below the noise floor when the optimal feedback gain is exceeded. (b) Ring-up of the motional energy of the mechanical oscillator from its cooled state is used to fit the thermal decoherence rate ($\Gamma_{th} = 6.4 \times 10^{12}$ Hz at a trap frequency of 160 Hz) at low amplitudes. At higher amplitudes (after about 35 s), faster heating is observed, likely caused by coupling between modes due to trap anharmonicity.

After the motional amplitude of the levitator is cooled, its energy is below that of the environmental bath dominated by mechanical vibrations. When cooling is stopped, the levitator motion at the resonant frequency ω_z is heated by external excitations mediated by the mechanical damping rate $\gamma = \omega_z/Q$ [Fig. 4(b)]. The linear increase of the particle energy can be expressed by $n_0 + n_{env}\gamma t = n_0 + \Gamma_{env} t$ where n_0 is the initial cooled phonon occupation of the oscillator, n_{env} is the effective environmental bath occupation, and $\Gamma_{env} = n_{env}\gamma$ is the decoherence rate [12,19]. We observe a change in the rate after a certain amplitude, consistent with behavior observed in other experiments and related to an amplitude-dependent coupling to other modes [26]. With the measured environmental decoherence rate and the induced back-action rate Γ_{ba} from 10^7 photons per second, the quantum cooperativity, $\mathcal{C}_q = \Gamma_{ba}/\Gamma_{env} \approx 1.4 \times 10^{-21}$, of the system is far from the required value to enable feedback-based quantum control of the oscillator.

IV. OUTLOOK

We now outline a path to cooling a magnetically levitated, superconducting microgram object to its mechanical quantum ground state using optical measurement-based feedback. This method requires high resolution in the particle position, which implies a high collection and detection efficiency η [12,13]. Even with $\eta = 1$, and a quality factor of 2.6×10^7 at 3 K, our system would require at least 7.5×10^{17} photons/s or 235 mW to sufficiently suppress the shot noise for ground-state cooling. This is practically infeasible due to associated heating, but can be avoided by implementing an optical cavity [28,37–39] where the phase information acquired by each photon is enhanced so that fewer photons are required to probe the mechanics with the same precision.

An implementation of this concept is shown in the inset of Fig. 5, where we consider the levitator as one of the mirrors in an optical cavity. Describing the optical cavity with resonant angular frequency ω_{cav}, power loss rate κ, and average occupation n_{cav}, the optomechanical system has $\mathcal{C}_q = \mathcal{C}_{om}/n_{th}$, where $\mathcal{C}_{om} = 4g^2 n_{cav}/(\kappa\gamma)$ is the optomechanical cooperativity and $g = z_{zpf}\delta\omega_{cav}/\delta z$ is the (bare) optomechanical coupling with zero point fluctuation $z_{zpf} = \sqrt{(\hbar/(2m\omega_z)}$ and optical cavity frequency shift per displacement $\delta\omega_{cav}/\delta z$. The minimum requirement to cool to a mechanical occupation below 1 phonon, ignoring other noise sources, is given by

$$n < 1 \quad \Rightarrow \quad \frac{4g^2 n_{cav}}{\kappa \Gamma_{th}} > \frac{1}{9\eta - 1} > 0,$$

where $\eta = n_{det}/n_{in}$ is the total efficiency. Cooling to the ground state thus requires a minimum number of probing

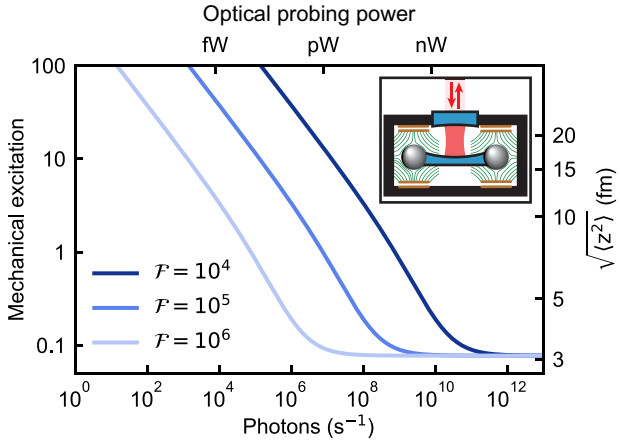


FIG. 5. Simulated number of incoming optical photons needed to cool a mirror levitator assuming optimal feedback. System parameters given in the main text with varying optical cavity finesse, $\mathcal{F}$. The number of incoming photons required to achieve the desired mechanical excitation depends inversely on the square of the finesse. (Inset) Superconductors are used to levitate a mirror which forms part of an optical cavity.

photons.If we consider an optical cavity with a micromirror [40–44] operating at $\lambda = 1.55\,\mu\text{m}$ with finesse $\mathcal{F} = 10^5$ in the one-sided overcoupled regime ($n_{\text{cav}} = 4n_{\text{in}}/\kappa$) and efficiency $\eta = 0.75$, together with a magnetically levitated superconductor [24] of mass $m = 6\,\mu\text{g}$ at resonant frequency $f_z = 200\,\text{Hz}$ and quality factor $Q = 2.6 \times 10^7$, in a cryogenic environment $T = 15\,\text{mK}$, then ground-state cooling needs an input of only $n_{\text{in}} = 7 \times 10^6$ photons/s or 0.9 pW. In Fig. 5 we plot this requirement, showing the mechanical excitation as a function of input photons, assuming optimal feedback cooling, for varying optical cavity finesse. The measurement noise beyond shot noise $S_{zz}^{\text{imp+}}$ needs to be below $\sqrt{2S_{zz}^{\text{imp+}}} < 0.8\,\text{fm}/\sqrt{\text{Hz}}$ for the given parameters to realize measurement-based feedback cooling. An overview of optomechanical feedback cooling is provided in Appendix F. Other improvements, such as higher mechanical frequencies by using ring- or disk-shaped superconductor levitators [45–48], are also possible.

V. CONCLUSION

In summary, we precisely read out the motion of a levitated superconducting microsphere using optical interferometry, with a sensitivity around $1\,\text{nm}/\sqrt{\text{Hz}}$, and validate the feasibility of this combined experimental technique. Modifications to the system, in particular the integration of high-finesse optical cavity, will allow cooling of the particle center-of-mass motion to the quantum mechanical ground state. The high resolution and isolation from the surroundings make this platform a promising path towards quantum experiments with masses in the microgram regime, eventually enabling the exploration of gravitational quantum phenomena [49–52].

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DATA AVAILABILITY

The data that support the findings of this article are openly available [53]. Embargo periods may apply.

APPENDIX A: EXPERIMENTAL SETUP

The superconducting sphere is levitated inside a Montana Cryostation with a high numerical aperture kit, which reaches a temperature of 3 K at the experimental platform. The particle is kept in an enclosed housing, where the bottom is made of silicon carbide for its high thermal conductivity [54], and the top is covered by a 30-μm- thick glass lid to prevent escape. To minimize room temperature radiation from heating the particle, the setup has a fused silica window at the 30 K stage and an optical bandpass filter at the 3 K stage. The trap coils are made from copper-matrix niobium titanium wire, wound around an aluminum holder in an anti-Helmholtz configuration. A voltage-controlled current supply (Delta Elektronika), connected via a high-current feedthrough, supplies the trap with up to 6 A. Conceptually, the trap is comparable to the one presented in [24], but without vibration isolation.

The particle is a commercial Sn10Pb90 (10% tin, 90% lead, by weight) solder ball from Easyspheres, with a specified diameter of $100 \pm 6\,\mu\text{m}$. The displacement resolution reached in this experiment exceeds the shot-noise limit by two orders of magnitude. We ascribe this discrepancy to surface roughness and shape imperfections of the levitated PbSn particle, which lead to intensity noise and phase fluctuations as the particle rotates in the trap. A microscope image of a sample of solder balls is shown in Fig. 6. Shape imperfections are clearly discernible. A white light interferometer scan of the surface of four different solder balls was performed, one of which is shown in Fig. 7, giving a surface roughness of 104 ± 45 nm over a region of $707\,\mu\text{m}^2$.

At rest on the SiC surface, the sphere thermalizes with the cryostat and is assumed to reach base temperature before liftoff. Levitation was achieved with starting temperatures ranging from 3 K to 6.4 K. Once levitated, the particle can dissipate internal energy only very slowly

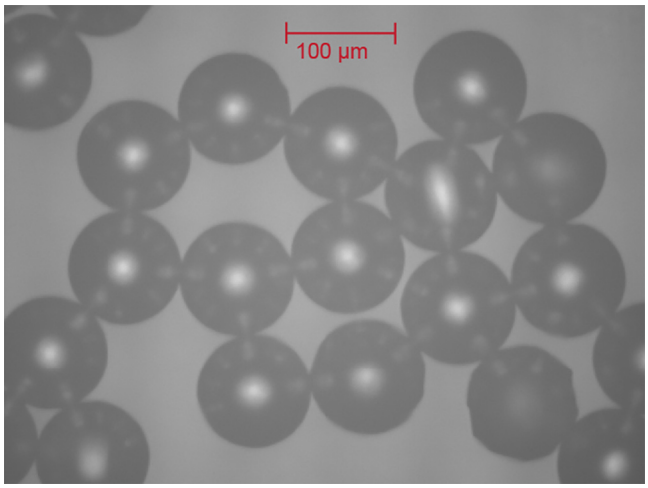


FIG. 6. Microscope image of Sn10Pb90 solder balls.

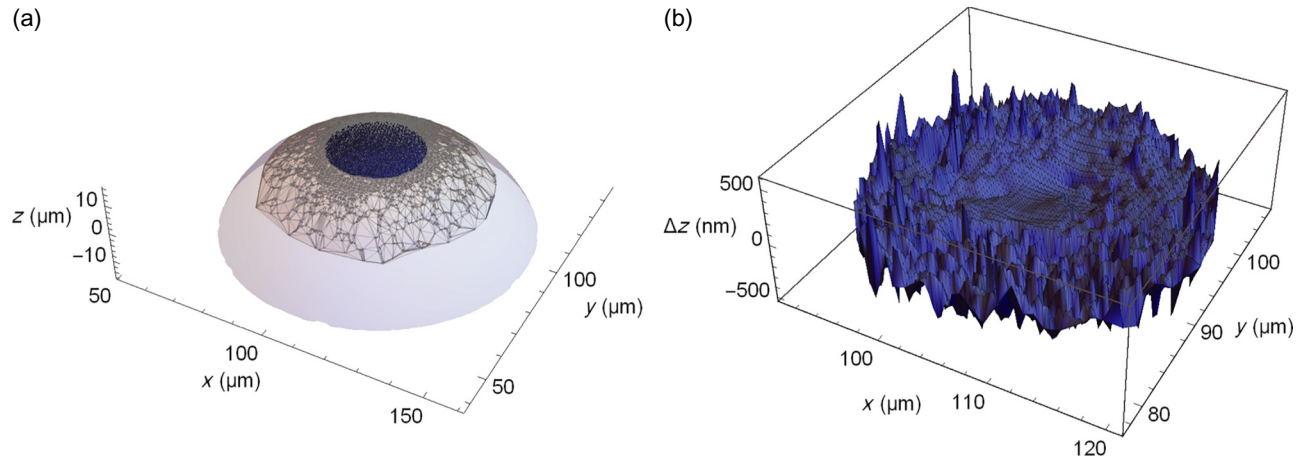


FIG. 7. The sphere is scanned with a white light interferometer to get a measure of the surface roughness. As shown in (a) the whole scan is used to fit the surface to a sphere. The curvature is subtracted from the data in the center of the scan with a radius of 15 μm (b) where the surface roughness is extracted.

through blackbody radiation. Heating due to illumination and stray light therefore eventually quenches the superconductor, leading to finite levitation times. An investigation of the relationship between laser power and levitation lifetime is described in Appendix C.

Figure 8 provides an overview of the experimental setup. Global experimental sequence control is provided by a LabVIEW program, which also controls the feedback loops. To image the position of the particle, a camera (The Imaging Source Europe GmbH, DMK 37BUX287 CMOS with a resolution of 560 × 720 pixels) and an LED flashlight with an analog trigger are installed above the chamber. Laser light is provided by a Toptica DLC Pro with a wavelength of 637 nm (red), which is then split into two paths by a 50 : 50 beam splitter. Both paths are equipped with an acousto-optic modulator (AOM) to change the differential frequency between the two paths. One path represents the reference arm of the interferometer and is directly coupled to the fiber beam splitter, while the other path is mode-cleaned with a single-mode fiber and directed onto the particle. The alignment of the laser onto the particle is performed with a microscope objective mounted on a three-dimensional piezo stage. An intensity scan of the position, as shown in Fig. 11, is used to choose the location of the laser probe for an experimental run.

The reflection of light from the particle is mixed with the local oscillator arm in a 50 : 50 fiber beam splitter and detected by two photon-counting modules (Excelitas SpCM-AQRH-10-FC) in a balanced homodyne configuration. Logic voltage pulses corresponding to detected photons are converted within a time-bin to a number-proportional voltage by a field-programmable gate array (FPGA, Liquid instruments Moku:Go) with a rate of 200 kHz. Another FPGA (Moku:Go) combines these two detector outputs into a sum and difference signal. For interferometric-based axial-motion cooling, the difference signal is sent to an FPGA-based bandpass filter (Red Pitaya using PYRPL [55]) to convert it to the direct axial feedback signal applied to a custom-built voltage-controlled current amplifier, and to an FPGA-based voltage-controlled oscillator (Moku:Go) to shift the frequency of the AOM locking the interferometer. The sum of the signal corresponds to the intensity and is used to cool the radial motion. The signal is processed by the LabVIEW program which controls a custom-built current supply to the radial feedback coils.

APPENDIX B: EXPERIMENTAL SEQUENCE

As illustrated in Fig. 9, cooling of the particle is performed in four steps.

1. Frictional cooling

In order to get the particle into the trap with minimal excess axial motion, we start by launching the particle off the ground with a rapid ramp to 2.6 A of trap current in 0.07 s, such that any adhesion forces are overcome. Then the current is ramped down to 0.7 A such that the sphere returns to the SiC surface but can roll across it, eventually approaching the local minimum of the magnetic field. The trap current is then increased following a quadratic ramp to 3 A in 4.4 s, so as to adiabatically increase the axial trapping frequency to a value of 120 Hz.

2. Camera-based cooling

Next, it is necessary to reduce the radial motion of the particle such that its apex remains within the laser spot of 2 μm diameter. Using the known frequency of the particle's trajectory in the trap, we apply camera-based feedback to reduce the radial amplitude. First, the center of the radial motion is found by recording a video with 30 frames.

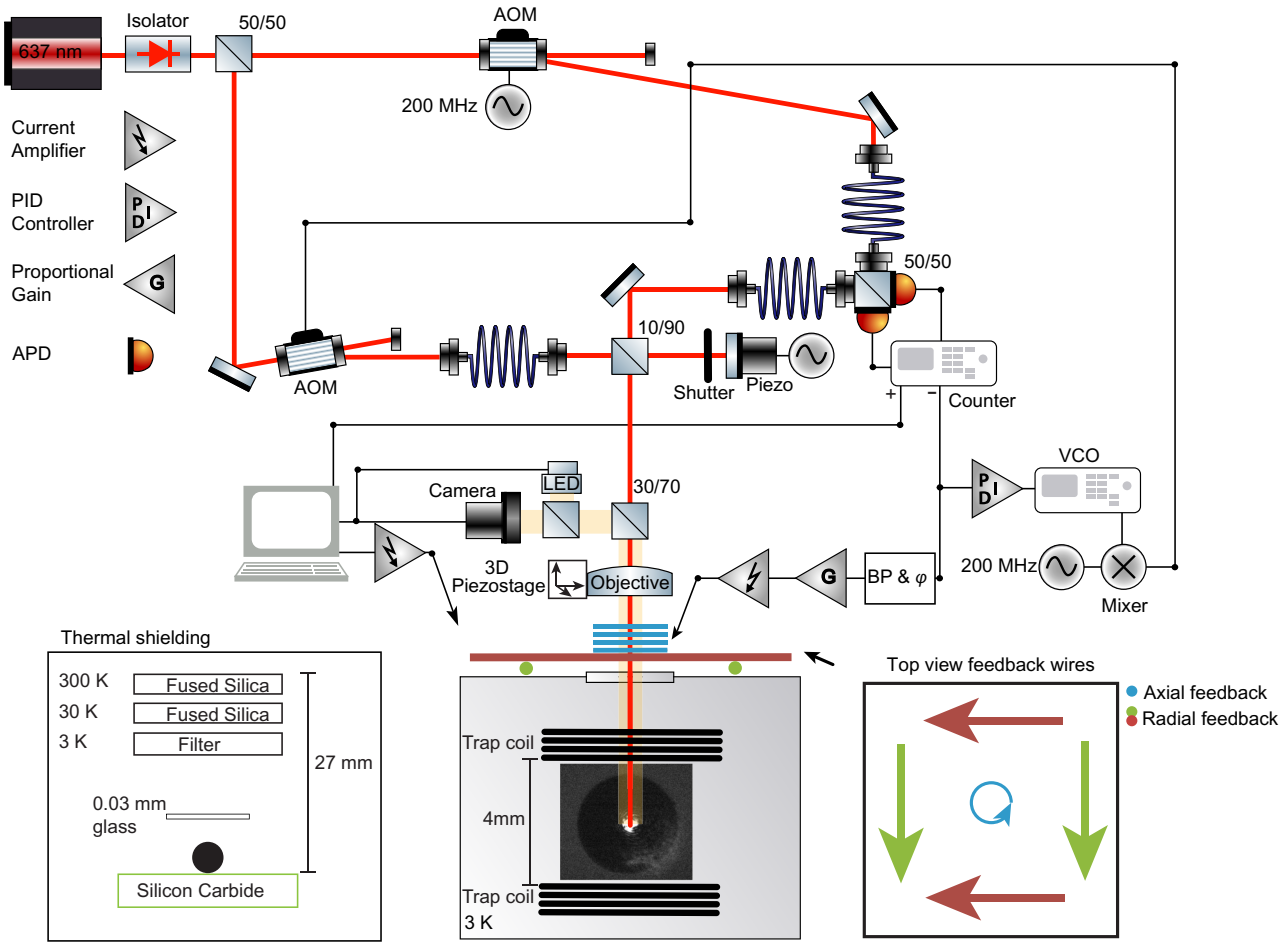


FIG. 8. Schematic of the experimental setup. A diode laser supplies coherent light with a wavelength of 637 nm to a 50 : 50 beam splitter, which separates the light into two paths, creating a Mach-Zehnder-like interferometer. For the linearization lock described in the text, the laser is modulated in both the experiment path and the local oscillator path by two independent acousto-optic modulators (AOM), which shift the frequency difference between the two arms. The laser beams are recombined and directed onto the top of the levitated particle using a wavelength-correcting microscope objective (MO) mounted on a three-dimensional piezo stage with a travel range of 20 µm on all axes. Levitation of the superconducting sphere is carried out with a superconducting anti-Helmholtz coil-pair magnetic trap with up to 6 A of current inside a closed-cycle cryostat at 3 K. A camera is placed above the objective to image the radial position of the particle. The objective collimates light reflected by the particle. Here, we lose 30% at the camera beam splitter and 10% at the beam splitter redirecting the light into the Mach-Zehnder interferometer. The remaining 63% recombines with the local oscillator at the 50 : 50 beam splitter. The outputs of the beam splitter are connected to two single-photon counting modules (APD), creating a balanced homodyne detection scheme. The counts are converted to a proportional voltage signal by an counter based on a field-programmable gate array (FPGA). The difference in the photon counts is used to feed back on the motion of the particle in the axial direction, while the sum of the photon counts carries information about the radial motion. The sum is passed on to a computer running a LabVIEW program, calculating a feedback signal at the radial frequency and passing it onto a current amplifier. The differential signal is passed on to a PID controller, which creates an error signal controlling a voltage-controlled oscillator (VCO) whose output is mixed onto the AOM drive in the signal arm. When the signal in the differential output is linearized, it is passed onto an FPGA-based bandpass filter and passed with a time delay and gain onto a current amplifier driving the axial feedback coil (blue). The thermal shielding consists of a fused silica window at 30 K, an optical bandpass filter thermalized to the 3 K stage, and a narrow opening to the particle holder. Additionally, a 30-µm glass plate is placed on the holder, preventing the particle from escaping the housing. On top of the vacuum chamber, copper wires are mounted in two counterpropagating coils to exert a horizontal force on the particle (green, x axis; red, y axis). For the axial control, a radial coil is placed directly above the particle (blue).

Then damping sequences with timed, directional magnetic pulses are applied. This sequence begins with two images taken 3 ms apart, followed by a waiting time of two oscillation periods to allow the computer to calculate the required feedback direction, and then the appropriate pulse is applied through the radial feedback coils. After 25 iterations, this pulsed cooling method reduces the radial amplitude of the particle to below 1 µm rms as shown in Fig. 10. This is sufficient to keep the particle within the laser beam.

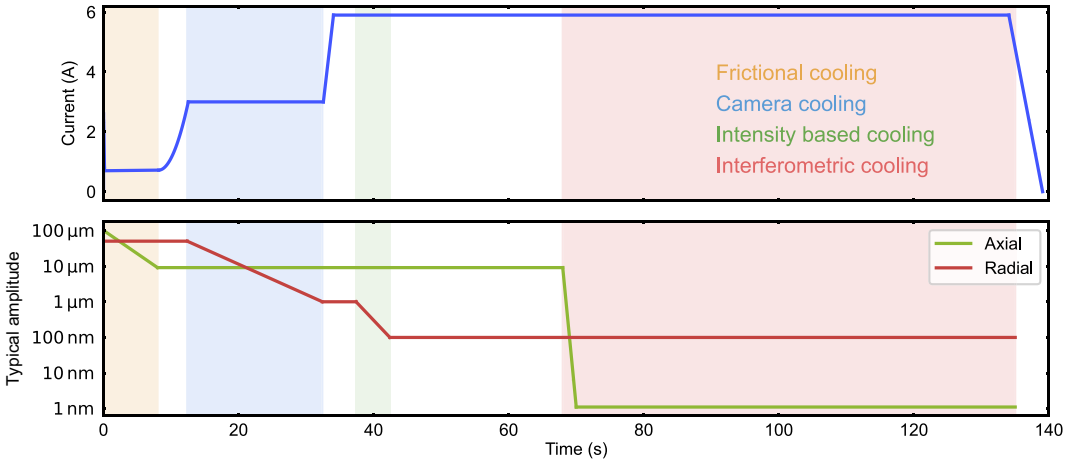


FIG. 9. A typical experimental sequence. The particle is first cooled on the bottom of the trap using friction, by choosing the current to be large enough to roll the particle around on the ground. After the liftoff of the particle, the radial motion is damped using pulsed camera detection-based feedback. Then, focusing the laser onto the sphere with a known offset in the radial plane, the intensity of the reflected laser is used to suppress the radial motion of the sphere further. Finally, it is possible to recollect enough reflected light to lock the interferometer and feed back on the axial motion to the noise floor of the interferometric detector.

3. Intensity feedback cooling

With the particle radial displacements small enough, we now position the laser beam and use the reflected intensity (summed signal) to dampen the radial motion further. To map out the sphere's location relative to the laser, the piezoelectrical controllable stage holding the focusing lens is scanned in the microscope plane, measuring the intensity at each point for multiple periods of the radial motion to obtain an average intensity. As shown in Fig. 11, the intensity profile has a Gaussian shape with a full width at half-maximum of roughly 4 μm. The lens is positioned slightly off-center on the slope of the Gaussian, such that the instantaneous variation in reflected intensity is monotonic with respect to the particle's radial displacement. This sum signal, filtered around the radial frequency and with appropriate phase delay, is then applied to the radial feedback coils, reducing the transverse amplitude to approximately 100 nm.

4. Interferometric phase tracking and cooling

We now cool the axial motion with the interferometric (difference) signal.

The axial amplitude of the particle exceeds $\lambda/8$, which corresponds to a phase shift of $\pi/2$ in the interferometer; the signal is wrapped as shown in the main text. To increase the dynamic range of the interferometer and also account for drifts and the surface roughness of the particle, we apply a strong lock on the interferometric signal. The lock consists of a proportional controller using the voltage signal of the difference between the two detectors. The lock controls a voltage-controlled oscillator, the output of which

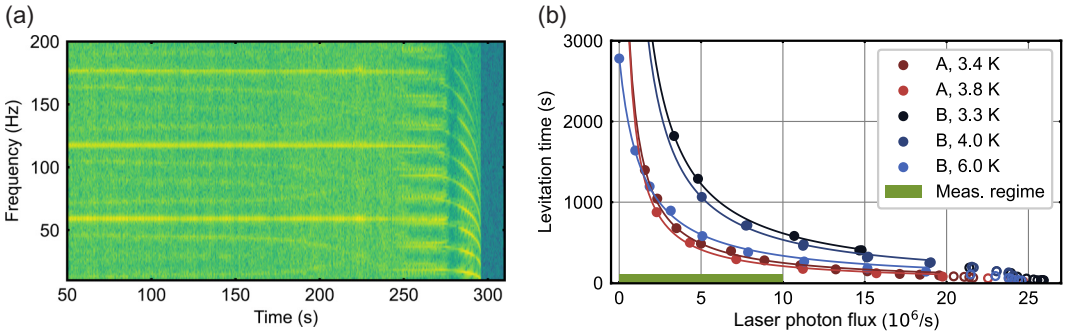


FIG. 12. (a) Sample spectrogram of the intensity readout of a levitated particle with a photon flux of around $1.7\times10^7\ \mathrm{s}^{-1}$ until the particle quenches and drops. The in-plane radial motion dominates with a frequency of 60 Hz. Due to the nonlinear detection caused by the Gaussian reflection profile, higher-order peaks at 120 Hz and 180 Hz are also visible. (b) Plot of levitation time for two different superconducting particles, A and B, for different laser powers and starting temperatures. The laser power applied is calibrated with the calibration mirror at the start of each measurement and referenced to what reaches the detectors. The curves are fitted according to $\tau_{\mathrm{lev}} = a/(b + n_{\mathrm{laser}})$. The parameter region in which all other measurements in the paper were taken is indicated by the green area.

is mixed into the drive of the AOM in the signal arm. This frequency shift compensates the Doppler shift induced in the reflected light by the velocity of the particle. Using this method, we increase the linear regime of the signal to several micrometers. As it also suppresses the actual signal from the oscillator, we calibrate the conversion to meters as described in Appendix E below. Once the signal is linearly dependent on the axial displacement of the particle, it can be used for direct feedback, for which we employ a Red Pitaya which filters the homodyne signal around the axial frequency and provides a gain and phase delay to the signal before applying it to a custom-built, bidirectional current amplifier which drives the axial feedback coil.

APPENDIX C: LEVITATION TIME

As soon as the particle begins levitating, it loses its direct thermal connection to the 3 K bath and begins to heat up. By installing a 3-mm pinhole in the 30 K stage, ambient thermal radiation was reduced to a minimum, allowing for levitation on the order of hours without applied laser power. When laser light is applied at low powers, the particle's levitation lifetime behaves as

$$\tau_{\mathrm{lev}} = \frac{\Delta E}{P}, \tag{C1}$$

where ΔE is an energy budget for the particle that depends on its starting temperature (and critical temperature), and P is heat absorbed by the particle. The power absorbed by the particle will be some fraction of the applied laser power and will depend on its reflectivity (for lead, this is 63% [56]). In Fig. 12 we show the lifetime dependence on laser power for two different particles, A and B, for different starting temperatures.

At laser fluxes above 20×10^6 photon/s, we observe a sudden drop in lifetime, the precise details of which are not fully understood. Furthermore, as seen in Fig. 12, during the final 10–20% of flight time, an additional lower-frequency motional mode appears. These modes are not reproducible and are probably related to the rotation or libration of the imperfect particle. During the quench, frequencies then collapse to zero as the superconducting portion of the sphere decreases, reducing the restoring forces in the trap. Our readout and cooling protocols are carried out over much shorter time periods and with lower laser power, as indicated in Fig. 12, such that these levitation lifetime effects do not affect the measurements.

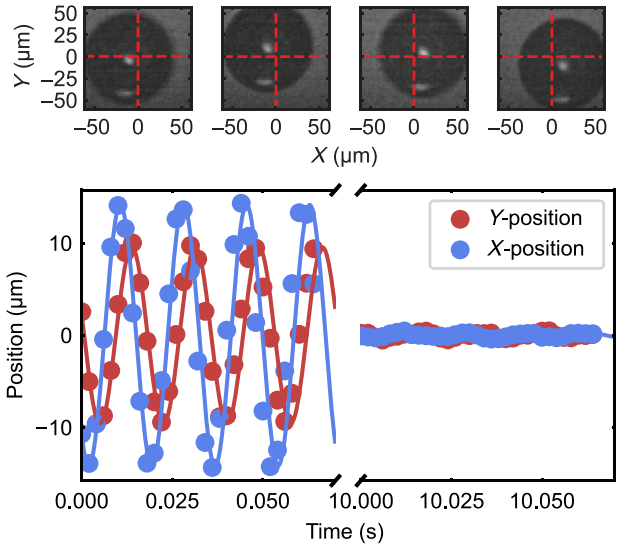


FIG. 10. (Upper panels) Using the camera pictures, the program determines the position of the particle in both radial coordinates. To feed back on the motion of the particle, the program finds the quadrant in which the particle is located and triggers a corresponding magnetic kick toward the center of rotation. The four pictures show four different images of the particle, where it is respectively in one of each of the four quadrants. (Lower panel) The radial motion of the sphere is recorded with a camera mounted above the microscope objective. Camera-based feedback reduces the motion of the sphere from tens of micrometers to less than 1 μm rms over a period of 10 s. The blue and red data points correspond to the particle's displacement from the trap's center on two orthogonal radial axes before and after performing the camera cooling protocol.

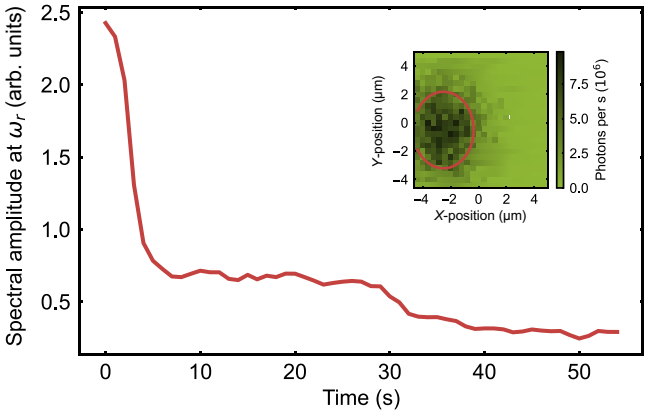


FIG. 11. The graph shows the amplitude of the radial frequency component over time, while intensity-based radial feedback cooling is applied. Continuous feedback is applied to the radial frequency in the direction chosen using the intensity profile. This feedback scheme can cool the radial motion, even when the motion becomes larger than the linear regime of the intensity signal. After 30 s, the axial motion begins cooling independently, decreasing the intensity noise such that the radial feedback limit improves. The vertical axis is calibrated to micrometers. However, this calibration only holds for small displacements along the gradient of the intensity profile around the rest position. (Inset) By scanning the microscope objective in the radial plane above the levitated particle, an intensity profile can be measured depending on the particle's position. This profile is used to choose a position at an intensity slope along the axis of the radial feedback coil.

We estimate the thermal energy budget of the particle as follows, approximating it as pure lead. At low temperature, in the superconducting state, lead has a (combined phononic and electronic) heat capacity of $c = 0.0115(T/\mathrm{K})^3$ J kg^{-1} K^{-1} [57]. Furthermore, lead has a superconducting critical temperature of $T_c = 7.2$ K, and a critical field of $H_0 = 6.4 \times 10^4$ A/m [58]. In the center of the trap with field gradients of 100 T/m and displacements up to 50 μm, the particle experiences magnetic fields approaching 5000 A/m. Using a simple empirical relationship, $H_c = H_0(1 - T^2/T_c{}^2)$ [59], the expected quenching temperature occurs around $T = 6.9$ K. For a 6-μg particle, integrating $\mathrm{d}E = cm\,\mathrm{d}T$ between $T_{\mathrm{start}} = 3.5$ K and $T_{\mathrm{end}} = 6.9$ K gives $\Delta E = 3.6 \times 10^{-8}$ J. The heat conductivity of lead, even while cold and superconducting, is great enough such that, over a timescale of seconds, the temperature of the sphere is effectively uniform. If an incident photon flux of 10^7/s ~ 3 pW was completely absorbed by the levitating particle as heat (in reality, it must be far less due to reflection), the corresponding levitation lifetime would be $\tau_{\mathrm{lev}} \sim 12\,000$ s. This is over an order of magnitude greater than what is observed, suggesting that the quench dynamics is governed by more complex processes.

A photon with a wavelength of 637 nm has an energy of 1.946 eV, far greater than the superconducting band gap of lead (2.73 meV); thus, direct impact will break Cooper pairs and excite quasiparticles. However, the recombination time is of the order of nanoseconds [60,61], such that, at low laser photon fluxes, the quasiparticle density equilibrates. At higher photon fluxes, a splitting rate exceeding the recombination rate may explain the abrupt drop in levitation lifetime. If the coupling between electron temperature and phonon temperature is sufficiently poor (over thousands of seconds), this could explain the lower energy budget. Using only the electron heat capacity of lead, $c = 0.0010\,(T/\mathrm{K})^3$ J kg^{-1} K^{-1} [57], the expected lifetime from 3 pW of heating is only around 1000 s.

APPENDIX D: INTERFEROMETER FREQUENCY LOCK

The interferometric lock is used to track the particle faster than it oscillates and allows for measurements of displacement larger than $\lambda/4$ that would normally exceed the "rails" of the sinusoidally encapsulated signal. In order to accommodate constant drifts over long timescales, a frequency-based feedback is needed, which results in measuring the Doppler shift and hence instantaneous velocity of the particle.

Our nominal homodyne difference signal from reflecting off the levitator, including contribution from the lock and some drift, is

$$V = V_0 \sin[2kz - \phi_{\mathrm{lock}} - \phi_{\mathrm{drift}}],$$

where $k = 2\pi/\lambda$. For a frequency-based feedback lock, we have $\phi_{\mathrm{lock}} = \omega_{\mathrm{lock}} t = GVt$ for gain G in radial frequency per volt. When the lock is operating correctly, the argument in the sine is kept small, and we have

$$\frac{V(t)}{V_0} \approx 2kz(t) - GV(t)t - \phi_{\mathrm{drift}}(t) \ll 1.$$

If our particle is moving slowly (with time t) we can Taylor expand over short times t' in which the lock operates,

$$z(t + t') \overset{t' \approx 0}{\approx} z(t) + z'(t)t' + \mathcal{O}[t']^2,$$

giving

$$\frac{V(t + t')}{V_0} \approx \frac{2kz(t) + 2kz'(t)t' - \phi_{\mathrm{drift}}(t) - \phi'_{\mathrm{drift}}(t)t'}{1 + GV_0 t'}.$$

For a sufficiently high gain $GV_0 t' \gg 1$, while $t' \ll 1$ remains small,

$$V(t) \approx \frac{2kz'(t) - \phi'_{\mathrm{drift}}(t)}{G} : \quad \begin{cases} G \gg \frac{\omega_z}{2\pi V_0} \\ |V| \ll |V_0|, \end{cases}$$

which for a harmonic signal $z(t) = z_0 \sin[\omega_z t]$ produces

$$V(t) \approx \underbrace{\frac{2k\omega_z}{G}}_{C} z_0 \cos[\omega_z t] - \frac{\phi'_{\mathrm{drift}}(t)}{G}.$$

Under ideal conditions this provides a self-calibrated measurement of displacement; however, in practice the combined situation of $Gt' \gg 1$ and $t' \approx 0$ is difficult to realize (too much gain will suppress the signal into the noise floor), and neglected terms will contribute to the signal. Nevertheless, the lock can provide a proportional signal for harmonic oscillation of our levitator.

To test the frequency feedback lock, a piezoelectric mounted mirror was used as the reflecting surface. In Fig. 14, the inferred measurement of displacement from $C = 2k\omega_z/G$ is seen to be still trending toward the true value that exists at gains beyond what can be realized in our experiment. Comparing to the probe-tone calibration, a gain of $G = 2\pi\ 8000$ Hz/V at $\omega_z = 2\pi\ 217$ Hz gives theoretical ideal $C = (5.35) \times 10^5$ V/m.

APPENDIX E: PROBE-TONE CALIBRATION OF THE FEEDBACK

When interferometers are used to probe a relative distance, the conversion from the signal (in most cases, voltage) to meters is determined by the wavelength of the light. In some cases, where effects other than arm's-length difference change the phase of the interferometer, it is necessary to calibrate the conversion by other means [62]. In

our case, the roughness of the particle in combination with free rotation requires a strong lock to stabilize the phase of the interferometer and avoid leaving the linear regime. Here, we calibrate displacements by applying a known driving force and measuring the response. For a particular experimental setup (sphere, laser power, lock gain) we can formulate our signal as $V = Cz$.

The equilibrium displacement of the particle in the magnetic quadrupole field, with the magnetic field gradient $\mathrm{d}B_{\mathrm{trap}}/\mathrm{d}z = b_z$, due to a constant external magnetic field offset, $B_{\mathrm{ext}}\hat{\hat{z}}$, can be approximated by

$$\Delta z = -\frac{B_{\mathrm{ext}}}{\mathrm{d}B_{\mathrm{trap}}/\mathrm{d}z}.$$

The gradient $dB_{\mathrm{trap}}/\mathrm{d}z$ can be calculated from the frequency of the sphere with the density of the particle given by $\rho = 1.1 \times 10^4\,\mathrm{kg/m^3}$ [24] as

$$|b_z| = \omega_z\sqrt{\frac{2\mu\rho}{3}}.$$

From the displacement Δz, the force acting on the mechanics can be calculated as

$$F = m\omega_z^2\,\Delta z,$$

which we drive as $F = F_0 \sin[\omega_{\mathrm{dr}}t]$ with current $I = I_0 \sin[\omega_{\mathrm{dr}}t]$.

Proceeding in terms of two-sided spectral densities, we have

$$S_{F_{\mathrm{dr}}F_{\mathrm{dr}}}(\omega) = {F_0}^2\frac{\pi}{2}\left(\delta[\omega-\omega_{\mathrm{dr}}]+\delta[\omega+\omega_{\mathrm{dr}}]\right),$$

$$S_{zz}(\omega) = \frac{S_{F_{\mathrm{dr}}F_{\mathrm{dr}}}(\omega)+S_{F_{\mathrm{N}}F_{\mathrm{N}}}(\omega)}{m^2\left(\left(\omega^2-{\omega_z}^2\right)^2+\gamma^2\omega^2\right)},$$

$$S_{VV}(\omega) = |C|^2\,S_{zz}(\omega).$$

Putting these together, the signal power collected at ω_{dr} is

$$\begin{aligned}P_{V(\omega_{\mathrm{dr}})} &= \frac{1}{2\pi}\int_{-\infty}^{\infty} S_{VV}(\omega)\,\mathrm{d}\omega \\ &= \frac{|C|^2\,{F_0}^2}{2m^2\left(\left({\omega_{\mathrm{dr}}}^2-{\omega_z}^2\right)^2+\gamma^2{\omega_{\mathrm{dr}}}^2\right)} \\ &= \left(\frac{B_{\mathrm{ext}}}{I_{\mathrm{dr}}}\right)^2\frac{3{\omega_z}^2}{4\mu\rho}\,\frac{|C|^2\,{I_0}^2}{\left({\omega_{\mathrm{dr}}}^2-{\omega_z}^2\right)^2+\gamma^2{\omega_{\mathrm{dr}}}^2}.\end{aligned}$$

The feedback coil used to apply the external magnetic field is flat with 30 coils in a squircle shape. An $n = 4$ superellipse is used to describe the geometry with parametric equations $\phi \mapsto \{r\sqrt{\cos[\phi]}, r\sqrt{\sin[\phi]}\} : 0 < \phi < \pi/2$. Using the Biot-Savart law and numerical integration, the magnetic field along z at the location of the sphere is estimated to be $B_{\mathrm{ext}}/I_{\mathrm{dr}} = (2.1 \pm 0.4) \times 10^{-4}$ T/m. Uncertainty is dominated by the relative positioning of the coil, and is the major contribution to the calibration uncertainty.

To rely on less system information, the drive is applied sufficiently far from the mechanical frequency such that $\gamma^2\omega_{\mathrm{dr}}^2 \ll \left(\omega_{\mathrm{dr}}^2-\omega_z^2\right)^2$, where γ is the damping rate of the mechanical oscillator [62]. At a drive frequency of 217 Hz, corresponding to the future operational resonance frequency, we measured the response of the oscillator to an external probe tone at three different resonance frequencies. An example of such a measurement is given in Fig. 13(a), with the linear dependence on current shown in Fig. 13(b). Each measurement provides a determination of the calibration factor C, and they are combined together to produce a final value. For the data presented, at gain = 8000 Hz/V, we find $C = (3.73 \pm 0.66) \times 10^5$ V/m.

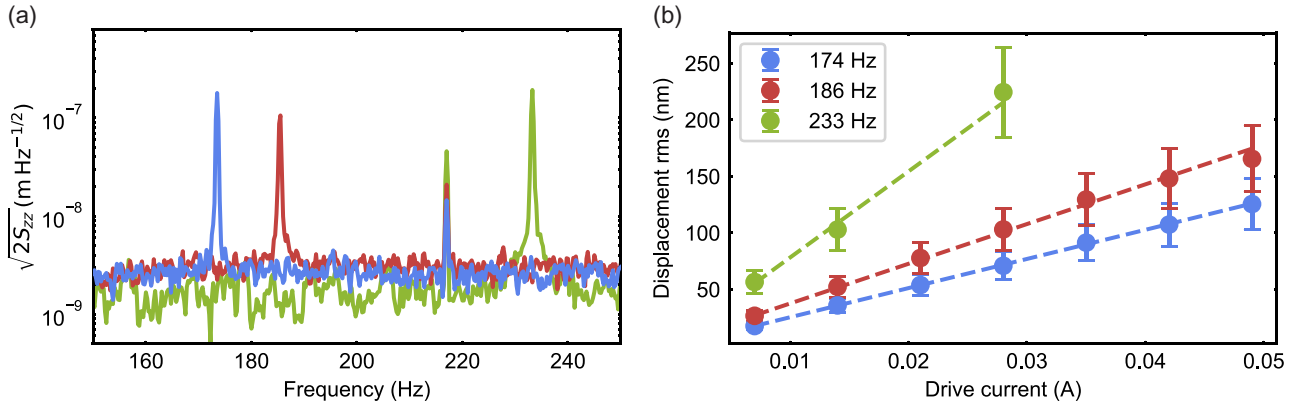


FIG. 13. (a) The power spectral densities for the same drive amplitude applied at 217 Hz as the trap frequency is varied between 175, 185, and 233 Hz. (b) The measured amplitudes at drive frequency 217 Hz while the trap is detuned to the indicated frequency. The amplitude is calibrated by the known dimensions of the driving solenoid and its distance to the levitated particle.

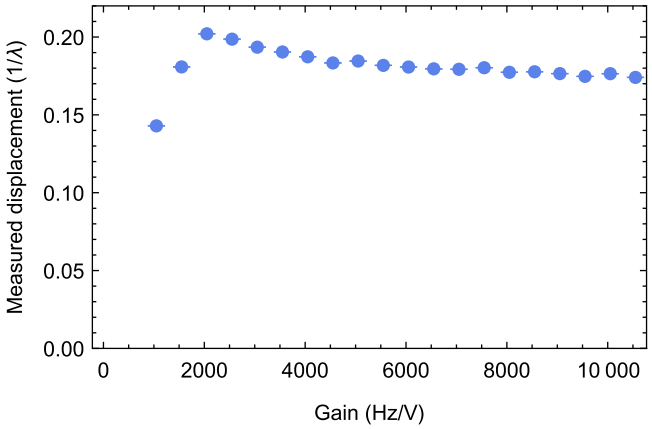


FIG. 14. The amplitude of a piezoelectric mounted mirror is set to a fixed drive (and hence amplitude) at 217 Hz. The inferred amplitude with the theoretical-ideal lock conversion is converging at gains much higher than accessible.

APPENDIX F: FEEDBACK COOLING

A classical harmonic oscillator is governed by the equation of motion,

$$m\ddot{x} + m\gamma\dot{x} + m\omega_0^2 x = F_N + F_{\mathrm{fb}},$$

where F_N is noise, and F_{fb} a feedback drive. If $x(t)$ can be accurately and continually measured, then a drive $F_{\mathrm{fb}}(t) = -m\gamma_{\mathrm{fb}}\dot{x}(t)$ can be applied to dampen the resonator and cool it below its normal (thermal or otherwise) occupancy from F_N. Classically, the (two-sided) thermal noise is $S_{F_{\mathrm{th}}F_{\mathrm{th}}} = 2\gamma m k_B T$, due to the fluctuation-dissipation theorem [63].

1. Sensitivity requirements

The achievable damping depends on the noise in the system. Including some error in the applied drive, its Fourier transform is $F(\omega) = \mathrm{i}\omega m\gamma_{\mathrm{fb}}\,(x(\omega) + \varepsilon)$ where ε is some error in the applied feedback. The full Fourier-transformed equation of motion is then

$$x(\omega) = \frac{F_N + \mathrm{i}\omega m\gamma_{\mathrm{fb}}\,\varepsilon}{m\left(\omega_0^2 - \omega^2 - \mathrm{i}\omega(\gamma + \gamma_{\mathrm{fb}})\right)},$$

and taking the (two-sided) spectral density,

$$S_{xx}(\omega) = \frac{S_{F_NF_N} + \omega^2 m^2 \gamma_{\mathrm{fb}}^2 S_{\varepsilon\varepsilon}(\omega)}{m^2\left((\omega_0^2 - \omega^2)^2 + \omega^2(\gamma + \gamma_{\mathrm{fb}})^2\right)},$$

which we can integrate to get $\langle x^2\rangle = (1/2\pi)\int_{-\infty}^{\infty} S_{xx}(\omega)\,\mathrm{d}\omega$.

For a high-Q resonator, we can approximate

$$\frac{1}{(\omega_0^2 - \omega^2)^2 + \omega^2\gamma^2} \approx \frac{1}{4\omega_0^2}\left(\frac{1}{(\omega - \omega_0)^2 + \frac{\gamma^2}{4}} + \frac{1}{(\omega + \omega_0)^2 + \frac{\gamma^2}{4}}\right)$$

and

$$\frac{f(\omega)}{(\omega - \omega_0)^2 + \frac{\gamma^2}{4}} \approx \frac{f(\omega_0)}{(\omega - \omega_0)^2 + \frac{\gamma^2}{4}},$$

when $f(\omega)$ is sufficiently constant over the resonator bandwidth. This will allow us to use the integral

$$\int_{-\infty}^{\infty} \frac{1}{(\omega - \omega_0)^2 + \frac{\gamma^2}{4}}\,\mathrm{d}\omega = \frac{2\pi}{\gamma}.$$

These relationships can be summarized to yield the expectation value,

$$\langle x^2\rangle = \frac{S_{F_NF_N}}{2m^2\omega_0^2(\gamma + \gamma_{\mathrm{fb}})} + \frac{\gamma_{\mathrm{fb}}{}^2 S_{\varepsilon\varepsilon}}{2(\gamma + \gamma_{\mathrm{fb}})}.$$

An effective temperature can be stated by equating to the harmonic oscillator energy, $k_B T_{\mathrm{eff}} = m\omega_0^2\langle x^2\rangle$.

The minimal displacement variance with optimal feedback parameter can be found by $\mathrm{d}\langle x^2\rangle/\mathrm{d}\gamma_{\mathrm{fb}} = 0$, giving

$$\gamma_{\mathrm{fb}}^{\mathrm{opt}} = \sqrt{\frac{S_{F_NF_N}}{m^2\omega_0^2 S_{\varepsilon\varepsilon}} + \gamma^2} - \gamma \approx \sqrt{\frac{S_{F_NF_N}}{m^2\omega_0^2 S_{\varepsilon\varepsilon}}},$$

and minimum variance

$$\begin{aligned}\langle x^2\rangle_{\mathrm{min}} &= S_{\varepsilon\varepsilon}\sqrt{\frac{S_{F_NF_N}}{m^2\omega_0^2 S_{\varepsilon\varepsilon}} + \gamma^2} - S_{\varepsilon\varepsilon}\gamma \approx \sqrt{\frac{S_{F_NF_N}S_{\varepsilon\varepsilon}}{m^2\omega_0^2}} \\ &= (\gamma + \gamma_{\mathrm{fb}}^{\mathrm{opt}})S_{\varepsilon\varepsilon} \approx \gamma_{\mathrm{fb}}^{\mathrm{opt}} S_{\varepsilon\varepsilon},\end{aligned}$$

where the approximations correspond to high-noise and large-feedback damping $\gamma_{\mathrm{fb}} \gg \gamma$. Under this approximation, cooling is limited to

$$S_{\varepsilon\varepsilon}S_{F_NF_N} \approx \hbar^2\left(\bar{n}_{\mathrm{min}} + \frac{1}{2}\right)^2, \tag{F1}$$

where we have $\langle x^2\rangle = \hbar/m\omega_0\left(\bar{n} + \frac{1}{2}\right)$.

In a continuous feedback system, this error will depend on the measurement noise for x. Most generally, our measurement will be $X = x + \sigma$, where σ is added measurement noise. With no additional noise in the feedback loop, $\varepsilon = \sigma$. Taking care of cross-terms appropriately, the measured spectral density is [36]

$$\begin{aligned}&S_{(x+\sigma)(x+\sigma)}(\omega) \\ &\quad = \frac{S_{F_NF_N} + m^2\left((\omega_0^2 - \omega^2)^2 + \omega^2\gamma^2\right)S_{\sigma\sigma}(\omega)}{m^2\left((\omega_0^2 - \omega^2)^2 + \omega^2(\gamma + \gamma_{\mathrm{fb}})^2\right)},\end{aligned} \tag{F2}$$

which for constant $S_{\sigma\sigma}$ is not integrable. For sufficiently large γ_{fb} this produces a dip below the noise background in what is known as squashing. Away from the resonance, $S_{(x+\sigma)(x+\sigma)}(\infty) = S_{\sigma\sigma}$. When optimal feedback is applied, $S_{(x+\sigma)(x+\sigma)}(\omega_0) = S_{\sigma\sigma}$.

2. Measurement and back-action noise

Generally, our measurement imprecision noise will be related to back-action force noise on the oscillator. The standard quantum limit for this is [35,37]

$$S_{xx}^{\mathrm{imp}} S_{F_{\mathrm{ba}}F_{\mathrm{ba}}} = \frac{\hbar^2}{4}.$$

Including some detection efficiency, η_{det}, and thermal noise,

$$S_{\sigma\sigma} = \frac{S_{xx}^{\mathrm{imp}}}{\eta_{\mathrm{det}}}, \quad S_{F_N F_N} = S_{F_{\mathrm{ba}}F_{\mathrm{ba}}} + S_{F_{\mathrm{th}}F_{\mathrm{th}}}.$$

Now, defining the back-action decoherence rate and thermal decoherence rate

$$\Gamma_{\mathrm{ba}} = \frac{S_{F_{\mathrm{ba}}F_{\mathrm{ba}}}}{4{p_{\mathrm{zpf}}}^2} = \frac{S_{F_{\mathrm{ba}}F_{\mathrm{ba}}}}{2\hbar m\omega_0}, \quad \Gamma_{\mathrm{th}} = \bar{n}_{\mathrm{th}}(\omega_0)\gamma,$$

where $\bar{n}_{\mathrm{th}}(\omega_0) \approx k_B T/\hbar\omega_0$, the quantum cooperativity is [12]

$$\mathcal{C}_{\mathrm{q}} = \frac{\Gamma_{\mathrm{ba}}}{\Gamma_{\mathrm{th}}} = \frac{S_{F_{\mathrm{ba}}F_{\mathrm{ba}}}}{S_{F_{\mathrm{th}}F_{\mathrm{th}}}},$$

and ground-state cooling requires, from (F1),

$$\bar{n} < 1 \quad \Rightarrow \quad \mathcal{C}_{\mathrm{q}} > \frac{1}{9\eta_{\mathrm{det}} - 1}.$$

For photons, with wavenumber $k = \omega/c$, measuring the position of some object, the associated noise spectral densities are [35]

$$S_{xx}^{\mathrm{imp}} = \frac{1}{16k^2 n_{\mathrm{in}}}, \quad S_{F_{\mathrm{ba}}F_{\mathrm{ba}}} = 4\hbar^2 k^2 n_{\mathrm{in}},$$

where n_{in} is the number of photons per second.

3. Optomechanics

An optomechanical system [37,64] has imprecision noise

$$\bar{S}_{\hat{x}\hat{x}}^{\mathrm{imp}}(\omega_0) = \frac{\kappa^2}{16G^2\bar{n}_{\mathrm{cav}}\kappa_{\mathrm{ext}}}\left(1 + \frac{4\omega_0^2}{\kappa^2}\right),$$

and back-action noise

$$S_{F_{\mathrm{ba}}F_{\mathrm{ba}}}(\omega_0) = \frac{4\hbar^2 G^2 \bar{n}_{\mathrm{cav}}}{\kappa}\left(1 + \frac{4\omega_0^2}{\kappa^2}\right)^{-1}.$$

Here G is the optical angular resonance frequency shift per displacement in the cavity. A one-sided, infinitely overcoupled optical cavity has $\kappa = \kappa_{\mathrm{ext}}$ and $\bar{n}_{\mathrm{cav}} = (4\kappa_{\mathrm{ext}}/\kappa^2)\bar{n}_{\mathrm{in}} = 4\bar{n}_{\mathrm{in}}/\kappa$.

Force white noise (such as a thermal or back-action) can always be cooled by increasing the feedback gain. Considering only back-action, (effective) thermal, and imprecision noise, with detector efficiency η_{det}, we have $S_{\varepsilon\varepsilon} = S_{\sigma\sigma} = (1/\eta_{\mathrm{det}})\bar{S}_{\hat{x}\hat{x}}^{\mathrm{imp}}$ and $S_{F_N F_N} = S_{F_{\mathrm{ba}}F_{\mathrm{ba}}} + S_{F_{\mathrm{th}}F_{\mathrm{th}}}$. Combining this detector efficiency with the external coupling ratio, defining $\eta = \eta_{\mathrm{det}}\kappa_{\mathrm{ext}}/\kappa$, then substituting everything into (F1),

$$n < 1 \quad \Rightarrow \quad \bar{n}_{\mathrm{cav}} > \frac{\kappa S_{F_{\mathrm{th}}F_{\mathrm{th}}}}{4\hbar^2 G^2 (9\eta - 1)}$$

$$\Rightarrow \quad \frac{\mathcal{C}_{\mathrm{om}}}{\bar{n}_{\mathrm{th}}(\omega_0)} > \frac{1}{9\eta - 1} \quad : \quad \mathcal{C}_{\mathrm{om}} = \frac{4g^2 n_{\mathrm{cav}}}{\kappa\gamma_{\mathrm{m}}},$$

where $\mathcal{C}_{\mathrm{om}}$ is the optomechanical cooperativity, $g = x_{\mathrm{zpf}}G = \sqrt{\hbar/2m\omega_0}G$ is the optomechanical coupling, and $\bar{n}_{\mathrm{th}}(\omega_0)$ is the thermal mechanical population. The quantum cooperativity is $\mathcal{C}_{\mathrm{q}} = \mathcal{C}_{\mathrm{om}}/\bar{n}_{\mathrm{th}}(\omega_0)$.

In addition to the quantum-limited noise, additional measurement noise $S_{\sigma^+\sigma^+}$ beyond the shot-noise imprecision needs to be considered, as well as force noise $S_{F_N^+F_N^+}$ excluding back-action, which can be described by a total effective temperature. With only $\bar{S}_{\hat{x}\hat{x}}^{\mathrm{imp}}$ and $S_{F_{\mathrm{ba}}F_{\mathrm{ba}}}$ dependent on cavity photon number, there exists an optimal value

$$n_{\substack{\mathrm{cav} \\ \mathrm{opt}}} = \frac{\kappa}{8G^2\hbar\sqrt{\eta}}\sqrt{\frac{S_{F_N^+F_N^+}}{S_{\sigma^+\sigma^+}}}\left(1 + \frac{4\omega_0^2}{\kappa^2}\right).$$

Putting this into (F1), we find that ground-state cooling requires

$$n < 1 \quad \Rightarrow \quad \sqrt{S_{\sigma^+\sigma^+}S_{F_N^+F_N^+}} < \frac{\hbar}{2}\left(3 - \frac{1}{\sqrt{\eta}}\right),$$

and if our force noise can be described by an (effective) temperature, $S_{F_N^+F_N^+} = 2m\gamma_{\mathrm{m}}k_B T$, we need

$$n < 1 \quad \Rightarrow \quad S_{\sigma^+\sigma^+} < \frac{1}{\eta}\frac{\hbar^2}{8m\gamma_{\mathrm{m}}k_B T}\left(3\sqrt{\eta} - 1\right)^2,$$

with $\eta > 1/9$.

A Fabry-Pérot optical cavity with one moving mirror has optomechanical coupling factor $G \approx -\omega_{\mathrm{cav}}/L$ and finesse $F = c\pi/(\kappa L)$, where ω_{cav} is the angular resonance frequency of interest and L is the cavity length.

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