

Unified admittance model for resonant tunneling diodes: Incorporating the space-charge dynamics of the quantum well and collector

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In this paper, we resolve a long-standing controversy: how and to what extent electron transport in the collector of a double-barrier resonant tunneling diode (RTD) affects its dynamics, particularly the rate of charge-relaxation processes and the RTD admittance. Here, we present a general analytical small-signal RTD admittance model that consistently incorporates space-charge effects in both the quantum well (QW) and the collector depletion region (spacer). If electron transport through the collector is sufficiently fast—this condition is met in contemporary subterahertz and terahertz RTDs under typical operating conditions—the general model reduces to a simple four-element *RLRC* equivalent circuit with a certain charge-relaxation time constant (τ), which, in general, largely deviates from the tunnel electron lifetime in the RTD QW. Collector space-charge effects influence both dynamic and static RTD characteristics. Specifically, in the negative differential conductance (NDC) region, RTDs exhibit increased τ and capacitance, resulting in slower operation. Additionally, the RTD I - V curve tilts to the right and the NDC increases. However, collector space-charge effects are typically negligible when electrons travel ballistically through the collector—this outcome of our analysis is important for the practical modeling and design of RTDs. Nevertheless, these effects become significant in RTDs with long collector spacers, high current density, and electron transport at saturation velocity, though such RTDs are unusual at subterahertz and terahertz frequencies.

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I. INTRODUCTION

Resonant tunneling diodes (RTDs) are the fastest active electronic devices that operate at room temperature [1,2]. Their negative differential conductance (NDC), high-frequency performance, and nonlinear current-voltage (I - V) characteristics make RTDs promising candidates for terahertz sources and detectors. RTD oscillators have been demonstrated to operate at frequencies approaching 2 THz [3,4]; when used in arrays, they have achieved output powers exceeding 10 mW at subterahertz frequencies [5]. Efforts to increase output power while reducing device footprint are actively underway [4,6–8]. Furthermore, RTDs are being explored for use in square-law, heterodyne, and injection-locking detection modes [9–11]. These advances underline the

potential of RTDs in diverse applications, such as terahertz telecommunication [10,12], imaging [13,14], and sensing [15].

To effectively design, optimize, and predict the performance limits of RTD oscillators, a simple, analytic, and intuitive model of double-barrier RTDs is essential. In particular, the frequency-dependent small-signal admittance model is crucial for determining the maximum operational frequency of these oscillators.

To develop a sufficiently general and accurate model, the analytic description of RTD admittance must incorporate two key dynamic effects: the charge dynamics within the RTD quantum well (QW) and the effects caused by the traveling space charge in the RTD collector depletion region (hereafter referred to simply as the spacer). Existing models vary in how they incorporate these effects. Some provide a precise description of the effects caused by the traveling space charge in the collector spacer but do not fully capture the complex charge dynamics within the QW [16,17]. Conversely, other models account for the QW charge dynamics but disregard the traveling space charge and its effect on the overall dynamic response [18–23].

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The models describing the effects of the traveling space charge in the collector region are largely based on those developed for barrier injection transit-time (BARITT) diodes [24], where the RTD is treated as an electron injector into the spacer layer. In this framework, the electrons constituting the space charge propagate through the spacer with a finite velocity and transit time [16,17]. These RTD models partially capture the dynamics of the QW charge by accounting for the delay in the wave function buildup following a voltage perturbation, an effect described in terms of a tunnel lifetime and associated with a quantum inductance [25]. However, these models neglect the Coulomb interaction for the QW electrons and the displacement currents induced by the QW charging.

Building on the sequential tunneling model [26], subsequent studies incorporated charge dynamics within the QW (displacement currents and Coulomb interaction), leading to a model that more accurately captures the QW charge behavior. This approach results in an *RLRC* equivalent circuit, which accounts for a tunnel charge-relaxation time constant (τ_{rel}) and a modified quantum inductance [18–21]. Importantly, this model predicts that, in the limit of high frequencies, the real part of the RTD admittance can remain negative and large, thereby enabling active RTD operation at frequencies far exceeding $1/\tau_{\text{rel}}$, a prediction that has been confirmed experimentally [21,27,28]. Nevertheless, the model does not include the electron transport time through the collector spacer.

The discrepancies between these models may have created some confusion in the field. In this paper, we seek to reconcile them by incorporating accurately both the effects of the traveling space charge in the spacer and the dynamics of the QW charge. To unify these mechanisms, we split the RTD into the regions where each effect occurs, assess their responses using appropriate parameters that enable their combination, and then analyze the overall response of the complete RTD. The analysis is performed for both ballistic transport in the spacer and transport limited by the saturation velocity of electrons.

The paper is organized as follows. In Sec. II, we split the RTD into two regions, the tunnel region (bare RTD) and the collector spacer, and describe how these regions are coupled. Section III derives the small-signal admittance of the bare RTD. In Sec. IV, we derive the impedance associated with the traveling space charge in the collector spacer. Section V combines these two components to obtain an admittance expression for the complete RTD and shows that, for short spacers, the device model reduces to an *RLRC* equivalent circuit. In Sec. VI, we examine the influence of the traveling space charge on the dc I - V characteristics and provide example admittance calculations for state-of-the-art RTDs with high and moderate current densities. Finally, Sec. VII concludes the paper.

II. SPLITTING THE RTD INTO TWO REGIONS

We begin by considering the conduction band diagram of a typical double-barrier RTD used in terahertz oscillators, as shown in Fig. 1. The diagram defines key geometrical parameters and potential distributions along the RTD.

Current transport via tunneling and through the spacer region occurs in separate spatial domains, allowing the device to be conceptualized as two distinct sections: the bare RTD (injector), comprising the double barriers and the emitter for $z < 0$, and the depleted collector spacer, for $z > 0$.

These two regions are linked by total current continuity: the RTD current (j_{RTD}) flowing through the external contacts equals the current inside the device at any point z , comprising electric and displacement currents. Consequently, the same total current flows through each region, permitting a model where the two regions are treated as two elements connected in “series.” The total voltage drop across the RTD can then be expressed as

$$V_{\text{RTD}} = U_{\text{RTD}} + U_s, \quad (1)$$

where U_{RTD} denotes the potential drop across the bare RTD region, i.e., between the emitter’s quasi-Fermi level and some arbitrarily chosen point on the other side. We choose the conduction band edge at the spacer’s start (immediately beyond the collector barrier) as one such point. Meanwhile, U_s is the potential drop across the spacer, between that conduction band edge and the collector’s quasi-Fermi level.

However, the coupling between these two regions is more complex. In conventional circuits, the total current

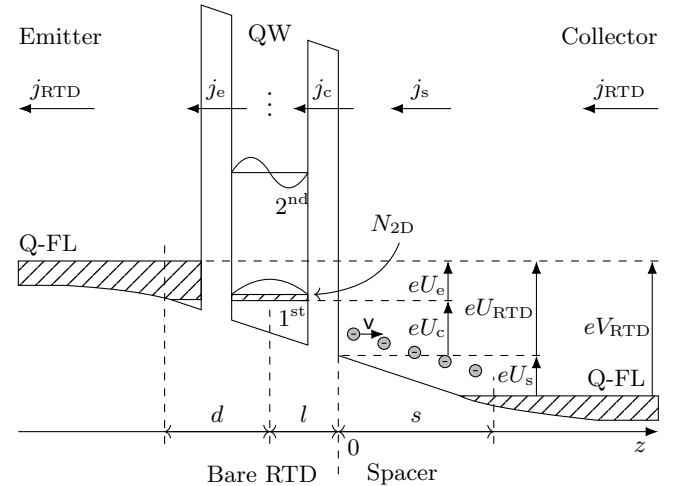


FIG. 1. Conduction band diagram of a resonant tunneling diode with a spacer in the collector. Q-FL, corresponding quasi-Fermi levels in emitter and collector contacts; QW, quantum well; RTD, resonant tunneling diode.

has to be continuous across the interface between two sequentially connected regions. In our case, the electric field is also continuous, as no charge accumulates at the interface. Therefore, not only is the total current continuous at the interface, but also its individual components, namely, the electric and displacement currents; the electric current corresponds to the tunneling current through the RTD collector barrier (j_c). Consequently, these two regions are not simply separate circuit elements connected in series; they are inherently interdependent, and this coupling must be properly accounted for in the modeling.

III. MODEL OF THE BARE RTD

Accurate dynamic models for RTDs that exclude traveling space charge effects in the collector have been developed and extensively described in the literature [18–20]. However, to integrate these models into our unified framework, we need to extract two specific parameters—the collector current density j_c and the electric field in the collector barrier F_c —and establish their interrelation. Therefore, we briefly review the general RTD model and adapt it to represent the bare RTD for our specific purposes.

We also emphasize an important aspect: charge-relaxation processes within the QW of an RTD can be characterized by a time constant, but this relaxation time is not universal. Instead, it depends on the nature of the relaxation and which parameters are held fixed. For example, if the electric fields in both the emitter and collector barriers remain constant during the relaxation of the QW charge (no QW shift with respect to emitter and collector), relaxation is governed solely by changes in the occupation of QW states, with the relevant time constant being the quasibound state lifetime in the QW, τ_{dwell} . In contrast, if the external bias on the RTD is held constant during the relaxation, the appropriate relaxation time is different; it is a certain τ_{rel} , as shown by Feiginov [18,19]. For our analysis of the bare RTD, we consider perturbations where the collector-barrier field F_c is held constant during the relaxation (F_c is the control variable). This defines a different relaxation time constant, internal to the RTD, which does not directly appear in measurable responses but is essential for accurate modeling of the RTD as an electron injector.

A. Small-signal model

For small variations in the electric field F_c within the collector barrier, we can linearize the set of equations derived from the sequential tunneling model [19,26]. In this framework, the bare RTD response is described in terms of the small-signal complex Fourier amplitudes, denoted with hats, as follows:

$$\hat{j}_{\text{RTD}} = \hat{j}_c + i\omega\epsilon\hat{F}_c, \quad (2)$$

$$\hat{j}_e = e(N_e - N_{2D})\hat{v}_e + e(\hat{N}_e - \hat{N}_{2D})v_e, \quad (3)$$

$$\hat{j}_c = eN_{2D}\hat{v}_c + e\hat{N}_{2D}v_c, \quad (4)$$

$$\hat{U}_{\text{RTD}} = \hat{U}_e + \hat{U}_c, \quad (5)$$

$$\hat{j}_e - \hat{j}_c = i\omega e\hat{N}_{2D}, \quad (6)$$

$$\hat{F}_e - \hat{F}_c = -\frac{e\hat{N}_{2D}}{\epsilon}, \quad (7)$$

where the variables without hats denote their steady-state values (time-averaged). Here, i is the imaginary unit; ω is the angular frequency; ϵ denotes the permittivity of the RTD semiconductor layers (which is assumed to be homogeneous); j_e is the tunneling current density through the emitter barrier; e is the modulus of an electron charge; N_e is the electron density available for tunneling from the emitter contact; N_{2D} is the electron density in the ground QW subband; v_e and v_c are the tunnel rates from the QW through the emitter and collector barriers, respectively; U_e is the potential difference between the emitter Fermi level and the ground QW subband bottom; U_c is the difference between the ground QW subband bottom and the conduction band edge at the spacer's start; and F_e is the electric field in the emitter barrier. Note that all currents are defined with the conventional sign.

In this model, backfilling current from the collector into the QW is neglected, assuming that the applied bias needed for the RTD to be biased in the NDC region is sufficiently high to ensure spacer depletion.

Next, we explicitly state the definitions of the Fourier components of the variables used in Eqs. (2)–(7) in terms of the electric fields $\hat{F}_e$ and $\hat{F}_c$, which are well-defined within the RTD barriers:

$$\hat{U}_e = \frac{\partial U_e}{\partial F_e}\hat{F}_e = d\hat{F}_e, \quad (8)$$

$$\hat{U}_c = \frac{\partial U_c}{\partial F_c}\hat{F}_c = l\hat{F}_c, \quad (9)$$

$$e\hat{N}_e = e\frac{\partial N_e}{\partial U_e}\hat{U}_e = e\frac{\partial N_e}{\partial U_e}d\hat{F}_e, \quad (10)$$

$$\hat{v}_e = \frac{\partial v_e}{\partial F_e}\hat{F}_e = v'_e\hat{F}_e, \quad (11)$$

$$\hat{v}_c = \frac{\partial v_c}{\partial F_c}\hat{F}_c = v'_c\hat{F}_c, \quad (12)$$

where the effective lengths d and l are defined by Eqs. (8) and (9) as follows: d is the sum of the emitter screening length, the emitter barrier thickness, and half of the QW thickness; l is the sum of the collector-barrier thickness and half of the QW thickness. In these definitions, the small-signal QW perturbation charge is assumed to be localized in the center of the QW. The derivatives v'_e and v'_c denote the sensitivities of the tunnel rates to the respective electric fields.

Our goal is to express the bare RTD's total current solely in terms of $\hat{F}_c$, which we treat as the control variable. This form is essential for describing the bare RTD as an electron injector for the collector spacer. We define the injector conductivity $\hat{\sigma}$ by

$$\hat{\sigma} = \frac{\hat{j}_c}{\hat{F}_c} \quad (13)$$

and rewrite the total RTD current from Eq. (2) as

$$\hat{j}_{\text{RTD}} = (\hat{\sigma} + i\omega\epsilon) \hat{F}_c. \quad (14)$$

To calculate $\hat{\sigma}$, we rewrite the continuity equation [Eq. (6)] in terms of $e\hat{N}_{2D}$ and use Eqs. (3), (4), and (7) to eliminate $\hat{j}_c$, $\hat{j}_e$, and $\hat{F}_e$, obtaining a relaxation-type equation controlled by $\hat{F}_c$:

$$\left(i\omega + \frac{1}{\tau_\sigma}\right) e\hat{N}_{2D} = \xi \hat{F}_c, \quad (15)$$

where the relaxation time τ_σ is given by

$$\frac{1}{\tau_\sigma} = \frac{1}{\tau_{\text{dwell}}} + e \frac{d}{\epsilon} \left(v_e \frac{\partial N_e}{\partial U_e} + (N_e - N_{2D}) \frac{v'_e}{d} \right); \quad (16)$$

$1/\tau_{\text{dwell}} = v_e + v_c$, ξ is given as

$$\xi = ev_e \frac{\partial N_e}{\partial U_e} d + e (N_e - N_{2D}) v'_e - e N_{2D} v'_c, \quad (17)$$

and $(1/e)\partial N_e/\partial U_e \approx \rho_{2D}$, when $eU_e \gtrsim kT$, where ρ_{2D} is the two-dimensional (2D) density of states in the ground QW subband, k is the Boltzmann constant, and T is the device temperature. The time constant τ_σ represents the QW charge-relaxation time for perturbations defined by $\hat{F}_c$, while ξ acts as an effective conductivity for the QW charging currents. The product $\xi \tau_\sigma$ further defines the quasistatic perturbation amplitude of the QW charge ($e\hat{N}_{2D}$) in response to changes in F_c , as given by Eq. (15).

Using Eqs. (4), (12), (13), and (15), the injector conductivity $\hat{\sigma}$ can be expressed as

$$\hat{\sigma} = eN_{2D}v'_c + \frac{\xi \tau_\sigma}{1 + i\omega\tau_\sigma} v_c, \quad (18)$$

where the first term represents the current contribution from charge stored in the QW and the second term—playing the dominant role—arises from the dynamic variation of eN_{2D} with F_c . The second term decreases with increasing frequency, governed by the relaxation time τ_σ , and therefore the injection becomes small as $\omega \rightarrow \infty$.

Further, to be able to connect the bare RTD to the spacer we need to determine the potential drop across the bare

RTD, $\hat{U}_{\text{RTD}}$. For that, we can express the potential drop using Eqs. (5), (7), (15), as

$$\hat{U}_{\text{RTD}} = (l + (1 - \hat{\alpha})d) \hat{F}_c, \quad (19)$$

where the scaling factor $\hat{\alpha}$ is defined as

$$\hat{\alpha} = \frac{\xi}{\epsilon} \frac{\tau_\sigma}{1 + i\omega\tau_\sigma}. \quad (20)$$

This scaling factor arises as a result of the electric field inhomogeneity across the bare RTD introduced by charge accumulation $e\hat{N}_{2D}$ in the QW; ignoring the discontinuity of the field due to the QW charge would set $\hat{\alpha} = 0$.

The injector conductivity $\hat{\sigma}$, its role in the total RTD current [see Eq. (14)], and the potential drop $\hat{U}_{\text{RTD}}$ are the key parameters necessary to connect the bare RTD to a collector spacer.

For completeness, we can write down the admittance of the bare RTD explicitly. Using Eqs. (14) and (19), the bare RTD admittance is given by

$$\hat{Y}_{\text{RTD,B}} = \frac{\hat{j}_{\text{RTD}}}{\hat{U}_{\text{RTD}}} = \frac{\hat{\sigma} + i\omega\epsilon}{l + (1 - \hat{\alpha})d}. \quad (21)$$

If traveling space charge in the collector spacer is neglected (as in older RTD models) [19,20], the collector spacer can be incorporated simply by replacing $l \rightarrow l + s$, where s is the length of the spacer plus the collector's screening length. Within this approximation, Eq. (21) then describes the admittance of the full RTD.

Equation (21) has an appealing simplicity: it resembles the admittance of a leaky parallel-plate capacitor with a homogeneous conductance $\hat{\sigma}$ and thickness $d + l$ (or an effective thickness modified by QW charge effects). This simplicity holds straightforwardly in the quasistatic limit. However, in general, the simplicity is deceptive, since $\hat{\sigma}$ [Eq. (18)] and the thickness-correcting factor $\hat{\alpha}$ [Eq. (20)] are complex frequency-dependent quantities.

This expression shares its structure with the results given in Brown *et al.* [16], but our form also explicitly incorporates Coulomb interaction for QW charge, displacement currents arising from QW charge dynamics, and variations in collector-barrier tunnel rate. The overall dynamic response can be effectively captured by an *RLRC* equivalent circuit [19,20], but the mathematical form of Eq. (21) resembles injector models for BARITT and impact ionization avalanche transit-time (IMPATT) diodes [24,29].

B. RLRC model of the bare RTD

As indicated in the foregoing, the terms in the equation for the RTD admittance, Eq. (21), can be rearranged in a different form—namely, as an *RLRC* circuit, described by

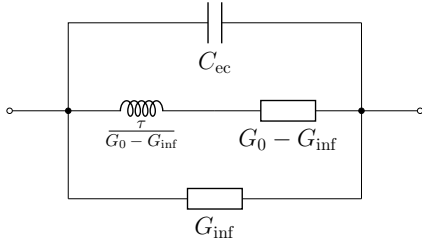


FIG. 2. Equivalent $RLRC$ small-signal model of a resonant tunneling diode. In a general case, it captures both the dynamics of the quantum well charge and the effects of the traveling space charge in the collector in the limit of $\omega \ll 1/\tau_s$.

Feiginov [19,20] and shown in Fig. 2:

$$Y_{\text{RTD},B} = i\omega C_{\text{ec},B} + G_{\text{inf},B} + \frac{G_{0,B} - G_{\text{inf},B}}{1 + i\omega\tau_{\text{rel},B}}, \quad (22)$$

with the dc conductance

$$G_{0,B} = v_c \frac{\varepsilon}{l} \left(1 - \frac{\tau_{\text{rel},B}}{\tau_{\text{dwell}}} \right) + eN_{2D} \frac{v'_c}{l} \left(\frac{l}{l+d} - v_c \tau_{\text{rel},B} \right), \quad (23)$$

the high-frequency conductance

$$G_{\text{inf},B} = \frac{d}{l+d} \frac{\varepsilon}{l} \left(\frac{1}{\tau_{\text{rel},B}} - \frac{1}{\tau_{\text{dwell}}} \right) + \frac{l-d}{l+d} eN_{2D} \frac{v'_c}{l}, \quad (24)$$

the geometrical capacitance of the bare RTD

$$C_{\text{ec},B} = \frac{\varepsilon}{l+d}, \quad (25)$$

and the relaxation time constant

$$\frac{1}{\tau_{\text{rel},B}} = \frac{1}{\tau_{\text{dwell}}} + \frac{e}{\varepsilon} \frac{ld}{l+d} \left(v_e \frac{\partial N_e}{\partial U_e} + (N_e - N_{2D}) \frac{v'_e}{d} + N_{2D} \frac{v'_c}{l} \right). \quad (26)$$

In these equations, we assume that

$$\frac{eN_{2D}}{\varepsilon} \frac{v'_c}{v_c} \ll 1. \quad (27)$$

This condition is usually well satisfied in typical terahertz RTDs with moderate emitter doping [3,4,30]. One can show, either using the quasiclassical approximation or via numerical estimation, that the ratio v'_c/v_c is approximately 10 nm/V for InGaAs RTDs with AlAs barriers, and that it depends only weakly on the barrier thickness. $\varepsilon = \varepsilon_r \varepsilon_0$, where ε_0 denotes the vacuum permittivity and ε_r denotes the relative dielectric constant, which is approximately 13 in InGaAs/InP RTDs. The parameter N_{2D} is more or less

universal across terahertz RTDs ($\pm 20\%$), typically having values at the current peak of around 10^{12} cm^{-2} . This yields a value of approximately 0.1 for the left-hand side of Eq. (27), confirming that this condition is indeed satisfied.

Here, we explicitly see that $\tau_{\text{rel},B}$ determines the frequency response of the RTD in Eq. (22), which appears measurable from the RTD's admittance (when neglecting charge dynamics in the collector spacer), although this dependence is spread over different terms in Eq. (21) and not immediately apparent. Equation (22) offers a more convenient and simple representation of RTD admittance for practical use. Again, this equation is applicable to both the bare and full RTD, depending on whether the spacer is included in the RTD collector or not (i.e., l vs. $l+s$). Notice that τ_σ defined by Eq. (16) is the limit of $\tau_{\text{rel},B}$ in Eq. (26) as $l \rightarrow \infty$.

IV. IMPEDANCE OF THE COLLECTOR SPACER

Next, to connect the collector spacer—incorporating the dynamics of the traveling space charge—to the bare RTD, we need to determine the potential drop across the spacer while ensuring total current continuity. This requires knowledge of the traveling charge distribution within the collector spacer. To keep the model simple, we assume that the electron drift velocity v in the spacer is a constant, and that the transit time is $\tau_s = s/v$. Under this assumption, the electric current density $\hat{j}_s(z)$ at position z in the spacer can be described as a delayed (i.e., phase-shifted in the Fourier domain) version of the tunneling current through the collector barrier of the bare RTD, $\hat{j}_c = \hat{\sigma} \hat{F}_c$, as

$$\hat{j}_s(z) = \hat{\sigma} \hat{F}_c \exp\left(-i\omega \frac{z}{v}\right). \quad (28)$$

Then the traveling space-charge density $\hat{n}_s(z)$ in the spacer is

$$e\hat{n}_s(z) = -\frac{\hat{j}_s(z)}{v}. \quad (29)$$

To calculate the potential drop across the spacer, $\hat{U}_s$, we include $\hat{n}_s$ in Poisson's equation. Integrating it over the spacer leads to the following expression:

$$\hat{U}_s = \left(\frac{s}{i\omega\varepsilon} \left(1 - \frac{1 - \exp(-i\omega\tau_s)}{i\omega\tau_s} \right) \hat{\sigma} + s \right) \hat{F}_c. \quad (30)$$

Calculating $\hat{U}_s$ in Eq. (30), we applied the boundary conditions that preserve the continuity of the electric field ($\hat{F}_c$) and current ($\hat{\sigma} \hat{F}_c$) at the left-hand side of the spacer ($z = 0$). That also guarantees continuity of the total current along the RTD.

Finally, defining the spacer impedance $\hat{Z}_s$ as the ratio of the potential drop across the spacer to the total current in the structure Eq. (14), we find that

$$\begin{aligned}\hat{Z}_s &= \frac{\hat{U}_s}{(\hat{\sigma} + i\omega\varepsilon)\hat{F}_c} \\ &= \frac{s}{i\omega\varepsilon} \left(1 - \frac{\hat{\sigma}}{\hat{\sigma} + i\omega\varepsilon} \frac{1 - \exp(-i\omega\tau_s)}{i\omega\tau_s} \right),\end{aligned}\quad (31)$$

which is identical to the impedance formula derived for BARITT diodes [24]. Note that the expression contains an oscillatory term, $1 - \exp(-i\omega\tau_s)$, that is strongly attenuated—roughly proportionally to ω^{-3} , if the frequency dependence of σ is neglected (which introduces an additional damping). Therefore, one should expect strongly suppressed ripples in the admittance of the full RTD at high frequencies (see Sec. VI).

V. SMALL-SIGNAL MODEL OF THE FULL RTD

A. Admittance of the full RTD

To express the admittance of the full RTD, we use the total current continuity condition and express the total current density from Eq. (14) together with the definition of the voltage across the RTD, V_{RTD} , given by Eq. (1):

$$\hat{Y}_{\text{RTD}} = \frac{\hat{j}_{\text{RTD}}}{\hat{V}_{\text{RTD}}} = \frac{(\hat{\sigma} + i\omega\varepsilon)\hat{F}_c}{\hat{U}_{\text{RTD}} + \hat{U}_s}. \quad (32)$$

Combining this expression with Eqs. (21) and (31), we can write the admittance of the full RTD as

$$\hat{Y}_{\text{RTD}} = \frac{\frac{\hat{\sigma} + i\omega\varepsilon}{l + s + (1 - \hat{\alpha})d}}{1 + \frac{\hat{\sigma}}{l + s + (1 - \hat{\alpha})d} \frac{s}{i\omega\varepsilon} \left(1 - \frac{1 - \exp(-i\omega\tau_s)}{i\omega\tau_s} \right)}. \quad (33)$$

We will refer to this model as the “full model” in the following text. Note that the admittance of the full RTD is effectively the admittance of the RTD without the space charge effects [Eq. (21)] when $l \rightarrow l + s$, scaled by a factor in the denominator of Eq. (33) that involves a part of this original admittance and a term with $\hat{\sigma}$ in Eq. (30).

Perhaps more intuitively, one can arrive at Eq. (33) from a different perspective; see further details in Ourednik *et al.* [31]. We start with an “old” full RTD model given by Eq. (21) with $l \rightarrow l + s$, where the traveling space charge in the collector is neglected. The RTD admittance is denoted $\hat{Y}_{\text{RTD}}^{\text{old}}$ in this case. Now we switch the traveling space charge in the collector on; the corresponding parameters are labeled with the superscript “new.” Further on, if we adjust the ac voltage on RTD in such a way that $\hat{F}_c$ remains

identical in both the “old” and “new” cases, the total RTD current remains the same:

$$\hat{j}_{\text{RTD}} = \hat{Y}_{\text{RTD}}^{\text{old}} \hat{V}_{\text{RTD}}^{\text{old}} = \hat{Y}_{\text{RTD}}^{\text{new}} \hat{V}_{\text{RTD}}^{\text{new}}, \quad (34)$$

since the voltage drop on the bare RTD and the potential profile there remain identical in both cases. Further, we note that $\hat{V}_{\text{RTD}}^{\text{new}}$ differs from $\hat{V}_{\text{RTD}}^{\text{old}}$ by an additional voltage drop ($\delta\hat{V}_s$) on the spacer, owing to the traveling space charge:

$$\hat{V}_{\text{RTD}}^{\text{new}} = \hat{V}_{\text{RTD}}^{\text{old}} + \delta\hat{V}_s. \quad (35)$$

That leads us to an equation for $\hat{Y}_{\text{RTD}}^{\text{new}}$:

$$\hat{Y}_{\text{RTD}}^{\text{new}} = \frac{\hat{Y}_{\text{RTD}}^{\text{old}}}{1 + \delta\hat{V}_s/\hat{V}_{\text{RTD}}^{\text{old}}}, \quad (36)$$

where the denominator is a scaling factor for $\hat{Y}_{\text{RTD}}^{\text{old}}$. The value of $\delta\hat{V}_s$ is given by the term with $\hat{\sigma}$ in Eq. (30). Substituting it into Eq. (36), and replacing $\hat{Y}_{\text{RTD}}^{\text{old}}$ with Eq. (21) and $\hat{V}_{\text{RTD}}^{\text{old}}$ with Eq. (19), where $l \rightarrow l + s$, we arrive at the same equation, Eq. (33).

The second term in Eq. (4) describes the contribution to $\hat{j}_c$ resulting from the variation of QW charge $\hat{N}_{2D}$; this contribution is usually dominant. If we retain it, but neglect the first term in Eq. (4) with $\hat{v}_c$ —on account of Eq. (12), this term leads to the terms with ν'_c in the RTD admittance and other parameters—then one can show that

$$\frac{\hat{\sigma}}{l + s + (1 - \hat{\alpha})d} \rightarrow \frac{G_0^{\text{old}}}{1 + i\omega\tau_{\text{rel}}}. \quad (37)$$

Substituting that into Eq. (33), we can rewrite it in a simplified form:

$$\hat{Y}_{\text{RTD}}^{\text{new}} = \frac{\hat{Y}_{\text{RTD}}^{\text{old}}}{1 + \frac{G_0^{\text{old}}}{1 + i\omega\tau_{\text{rel}}} \frac{s}{i\omega\varepsilon} \left(1 - \frac{1 - \exp(-i\omega\tau_s)}{i\omega\tau_s} \right)}. \quad (38)$$

The scaling factor of $\hat{Y}_{\text{RTD}}^{\text{old}}$ is now easily understood in terms of the quantities of the old *RLRC* model. Indeed, the injection is directly proportional to G_0^{old} at low frequencies—reflecting the injection current $\hat{j}_c$ —and decreases with frequency at the same rate as $\hat{N}_{2D}$ [see Eq. (4)], with τ_{rel} being the relevant time constant. The neglect of ν'_c in the foregoing is not always justified, particularly at very high frequencies ($\omega\tau_s \gg 1$) and close to the peak and valley bias points; however, the terms with ν'_c could be neglected in the typical operation regimes of subterahertz and terahertz RTDs, and this approximation is useful in qualitative analysis.

Although Eq. (33) is general, its equivalent circuit representation would be overly complex. Therefore, in the following subsection, we analyze a limiting case—which is nevertheless applicable to the state-of-the-art RTDs—that allows for a simple equivalent circuit representation.

B. Equivalent *RLRC* model of the full RTD for $\omega \ll 1/\tau_s$

We can identify two limiting cases for the assessment of the electron velocity in an RTD spacer. The first case is that of ballistic electron transport. As a rule of thumb, this case is realized when the energy of the electrons, injected through the collector barrier and traveling ballistically in the spacer, remains roughly below the L or X valley until the electrons reach the end of the spacer [32,33]. High-frequency (terahertz) RTDs are often designed according to this rule [1,2,27,34]. One can also state it as follows: the voltage drop on the spacer in the NDC region is roughly below 0.5 V (for RTDs with InGaAs or InP spacers), or the NDC region lies below a bias of roughly 1 V. Almost all terahertz RTDs satisfy these requirements [1,2,27,34]. In this case, the averaged ballistic electron velocity in the InGaAs or InP spacers can be estimated as approximately 10^8 cm/s [32,33,35]. For a typical thickness of the RTD collector depletion region or spacer of approximately 10 nm, that gives an estimate of $\tau_s \approx 10$ fs, or a frequency, $1/(2\pi\tau_s)$, of 16 THz, which greatly exceeds the typical operating frequencies of terahertz RTD oscillators. This electron velocity also roughly corresponds to the velocity of electrons injected from the QW in typical terahertz RTDs, see Sec. VI.

The second case is that of electron transport with a saturation velocity. This may be applicable to RTDs with thick spacers (of the order of 100 nm), where the voltage drops across the spacers are significantly more than 0.5 V or 0.25 V for RTDs with InGaAs or GaAs spacers, respectively. For spacer electric fields of more than 10 kV/cm, the saturation velocity in InGaAs is then approximately 1.5×10^7 cm/s, which is approximately an order of magnitude less than the ballistic velocity. Over these lengths, electrons have sufficient time during the transport to scatter into the L or X valleys. Nowadays, such RTDs are not commonly used at terahertz frequencies [16,36]. Nevertheless, we take this saturation velocity as the worst-case scenario in the subsequent calculations, which gives, for an approximately 10 nm spacer, $\tau_s \approx 70$ fs, or a frequency, $1/(2\pi\tau_s)$, of 2.4 THz. Even for this worst-case scenario, the condition $\omega \ll 1/\tau_s$ is satisfied for typical terahertz and subterahertz RTDs.

To cover the gap between these two extreme cases, one should perform a Monte Carlo analysis of the electron transport in the spacer to obtain an average velocity for use in the model [32,33,35]. However, this goes beyond the scope of this paper, and we confine ourselves to showing

the examples for the extreme cases, noting that state-of-the-art RTDs for terahertz applications use short spacers, and are covered by the first case, with the ballistic velocity.

We also note that (quasi)ballistic transport can persist over lengths of a few hundred nanometers [32,33,35]. This could be achieved in RTDs by appropriate modification of the doping profile or by grading. However, the condition to maintain the ballistic transport throughout the entire oscillation cycle might limit the amplitude of RTD oscillations [37]. A detailed discussion of such trade-offs is beyond the scope of this paper.

Therefore, we limit our analysis to frequencies $\omega \ll 1/\tau_s$, allowing us to approximate the oscillating term in the spacer impedance in Eqs. (31) and (33) as

$$\frac{1 - \exp(-i\omega\tau_s)}{i\omega\tau_s} \approx 1 - i\frac{\omega\tau_s}{2}. \quad (39)$$

Under this approximation, Eq. (33) reduces to

$$\hat{Y}_{\text{RTD}} \approx \frac{\frac{\hat{\sigma} + i\omega\epsilon}{l + s + (1 - \hat{\alpha})d}}{1 + \frac{\hat{\sigma}}{l + s + (1 - \hat{\alpha})d} \frac{s\tau_s}{2\epsilon}}. \quad (40)$$

This expression can be rewritten in the form of the *RLRC* model shown in Fig. 2, with modified parameters (compared with Eq. (22) with $l \rightarrow l + s$):

$$\hat{Y}_{\text{RTD}} = i\omega C_{\text{ec}} + G_{\text{inf}} + \frac{G_0 - G_{\text{inf}}}{1 + i\omega\tau}. \quad (41)$$

Under a few additional realistic conditions (see the following), the model parameters can be written in a simplified form. The geometrical capacitance of the full RTD is then

$$C_{\text{ec}} = \frac{\epsilon}{d + l + s}. \quad (42)$$

The relaxation time constant (which is a measurable parameter) of the full device, including the traveling space charge, is

$$\frac{1}{\tau} = \frac{1}{\tau_{\text{rel}}} \left(1 + v_c \tau_s \frac{s}{l + s} \frac{1}{2} \left(1 - \frac{\tau_{\text{rel}}}{\tau_{\text{dwell}}} \right) \right), \quad (43)$$

where τ_{rel} is the relaxation time constant of the device without traveling space charge, given by Eq. (26) with the substitution $l \rightarrow l + s$. The high-frequency conductance is

$$G_{\text{inf}} = \frac{d}{d + l + s} \frac{\epsilon}{l + s} \left(\frac{1}{\tau_{\text{rel}}} - \frac{1}{\tau_{\text{dwell}}} \right) \left(1 - \frac{sv_c\tau_s}{2d} \right) + \frac{l + s + s\frac{1}{2}v_c\tau_s - d}{d + l + s} eN_{2\text{D}} \frac{v'_c}{l + s}, \quad (44)$$

where G_{inf} refers to conductance for the case $\omega \gg 1/\tau$ but still $\omega \ll 1/\tau_s$. The dc conductance is

$$G_0 = \frac{\nu_c \frac{\varepsilon}{l+s} \left(1 - \frac{\tau_{\text{rel}}}{\tau_{\text{dwell}}}\right) + \left(\frac{l+s}{d+l+s} - \nu_c \tau_{\text{rel}}\right) eN_{2D} \frac{\nu'_c}{l+s}}{1 + \nu_c \tau_s \frac{s}{l+s} \frac{1}{2} \left(1 - \frac{\tau_{\text{rel}}}{\tau_{\text{dwell}}}\right)}. \quad (45)$$

Equations (42)–(45) are valid under the condition of Eq. (27) stated in Sec. III B and the following additional conditions:

$$eN_{2D} \frac{\nu'_c \tau_s}{2\varepsilon} \ll 1, \quad (46)$$

$$eN_{2D} \frac{\nu'_c \tau_s}{2\varepsilon} \ll \frac{1}{2} \frac{\tau_{\text{dwell}}}{\tau_{\text{rel}}}. \quad (47)$$

Equation (46) could be rewritten in the following form:

$$eN_{2D} \nu_c \frac{\nu'_c \tau_s}{2\varepsilon} \ll 1, \quad (48)$$

where the term $eN_{2D} \nu_c$ gives the dc RTD current (j_{RTD}), which can go up to 20–30 mA/ μm^2 in thin-barrier RTDs [1,2,4]. We assume ballistic transport in the spacer; then $\tau_s \approx 10$ fs. That leads to an assessment for the left-hand sides of Eqs. (46)–(48) of the order of 10^{-2} . Since the relaxation time constant τ_{rel} is typically 2–10 times greater than τ_{dwell} in the NDC region [18,19,21,27], Eq. (47) is more stringent than Eq. (46). Nevertheless, Eqs. (46) and (47) are typically well satisfied in contemporary terahertz RTDs. See also the evaluations in Sec. III B. In the following, we refer to the model given by Eqs. (41)–(45) as the “*RLRC* model.”

Additionally, we assume a reasonable distance from the singularity ($|G_0| \rightarrow \infty$), i.e., the denominator in Eq. (45) is not close to 0. As a rule of thumb, we assume

$$1 + \nu_c \tau_s \frac{s}{l+s} \frac{1}{2} \left(1 - \frac{\tau_{\text{rel}}}{\tau_{\text{dwell}}}\right) \gtrsim 0.1. \quad (49)$$

For the validity of this assumption, refer to the figures in the next section.

The terms involving ν'_c in the nominator of Eq. (45), in Eq. (23), and also in τ_{rel} given by Eq. (26) are always negligible (compared with the terms with ν_c), except near the current peak and valley, where $\tau_{\text{rel}} \approx \tau_{\text{dwell}}$ and the terms with ν_c disappear. In this case, $\tau_{\text{rel}} > \tau_{\text{dwell}}$ in the NDC region, as one can see from Eq. (23), i.e., RTD is slow in the NDC region. Inspecting Eq. (43), we can notice that including the space charge in the collector makes the RTD even slower in the NDC region: $\tau > \tau_{\text{rel}}$, since the second term with $\nu_c \tau_s$ is negative in Eq. (43). Note that a similar term also appears in the denominator of Eq. (45), making

it less than 1 and leading to an increase in the dc NDC, compared with the case when the collector space charge is neglected, see Eq. (23). See also the examples in Sec. VI.

To be complete, in the positive differential conductance (PDC) region, $\tau_{\text{rel}} < \tau_{\text{dwell}}$, which makes the RTD respond faster to a bias perturbation as, in that case $\tau < \tau_{\text{rel}}$, see Eq. (43). The dc and high-frequency PDCs are getting smaller.

C. Different operation regimes of an RTD

We can identify the following operation regimes of an RTD. First, an RTD with fast electron transport in the collector ($\omega\tau_s \ll 1$), while $\omega\tau$ can take arbitrary values. This regime covers the majority of cases relevant to terahertz and subterahertz RTDs, as indicated in Sec. V B. We have shown there that the RTD admittance can be described by an *RLRC* equivalent circuit with parameters rescaled by the contribution of the traveling collector space charge. This regime also covers the effect of the collector space charge on the dc and quasi-dc RTD I - V curve and conductance ($\omega\tau_s \ll 1$ and $\omega\tau \ll 1$), see Sec. VI A.

Second, a fast RTD ($\omega\tau_\sigma \ll 1$) with possibly slow transport in a long collector (arbitrary $\omega\tau_s$). This is the regime of a BARITT diode with an RTD as an injector. So far, RTDs have not been used in this regime.

Third, an intermediate regime (similar to IMPATT diodes), when $\omega\tau_\sigma \approx 1$ and $\omega\tau_s \approx 1$. It is a relatively slow RTD with a relatively slow transport in a long collector spacer. The general expression for RTD admittance [Eq. (33)] should be used in this case. However, when $\omega\tau_\sigma$ becomes $\gtrsim 1$, the collector electric ac current drops, since the QW charge cannot follow the rate of the ac variation of the RTD bias [18–20]. The collector ac current disappears almost completely when $\omega\tau_\sigma \gg 1$, see Eq. (18). As a result, the injection current necessary for operation in the BARITT regime is attenuated. In addition to $\hat{\sigma}$, the spacer impedance also goes to zero with increasing frequency [see Eq. (31)], so transport in the collector stops being important. The behavior of an RTD is reduced to the regime (no collector space charge) already considered by Feiginov [18–20]. In this case, the ac RTD current is localized between the RTD emitter and the QW; its contribution to the total ac RTD current is defined by a small (in this case) parameter $d/(d+l+s)$, and the NDC drops. Therefore, it is not an optimal operation regime for terahertz RTDs.

TABLE I. Parameters of the example resonant tunneling diode (RTD) wafers, with time constants taken at the point of maximum negative differential conductance.

RTD wafer	1.0-RTD	1.6-RTD
Nominal barrier thickness (nm)	1.0	1.6
Peak current density (mA/ μm^2)	23.6	6.2
Peak-to-valley current ratio (—)	3.7	6.8
Spacer length for τ_{rel} (nm)	7	16
τ_{rel} (fs)	87	378
τ_{dwell} (fs)	59	158
τ_{σ} (fs)	157	922

VI. QUALITATIVE DISCUSSION AND EXAMPLES OF RTD CHARACTERISTICS

For the qualitative discussion, we use two RTD wafers with differing barrier thicknesses that were used for state-of-the-art subterahertz and terahertz oscillators [4,8,30]. See their parameters in Table I. We refer to the thinner-barrier, high-current-density, wafer as 1.0-RTD, and to the thicker, lower-current-density, wafer as 1.6-RTD.

A. dc characteristics

It has been known for a long time that Coulomb interaction modifies (tilts) the RTD I - V curves [38,39]. Qualitatively, this effect can be explained as follows. We start with an RTD I - V curve, where the Coulomb interaction of the QW charge is ignored. In this case, the applied RTD bias drops linearly between the RTD emitter and collector. Then we switch the Coulomb interaction on. The QW charge acts repulsively on the electrons, i.e., the QW bottom shifts upward in the energy diagram in Fig. 1. To bring it back and to restore the RTD current (which is determined by the alignment of the QW states to the emitter), we need to apply more bias to the RTD. The additional bias (δV) is proportional to the QW charge—i.e., also to the dc RTD current $j_{\text{RTD}} = j_c = eN_{2D}v_c$ —therefore, each point of the initial I - V curve shifts to the right, with the shift proportional to the RTD current ($\delta V \propto j_{\text{RTD}}$). Note that the current peak shifts more than the valley, and that this results in a tilt of the I - V curve to the right: NDC is increasing [38,39]. In rare cases and at low temperatures, NDC can become infinitely large (singularity) and the I - V curve can even assume a Z-shape with a (dc) hysteresis region [40,41].

Now we add the collector space charge. The mechanism by which it affects the RTD I - V curve remains the same as described previously. The added collector space charge acts repulsively on electrons; the band bottom rises in the spacer region and (importantly) also in the QW. That requires an additional bias on the RTD to restore the alignment between the QW levels and the emitter. Since the collector space charge is caused by the RTD current, the previous relation $\delta V \propto j_{\text{RTD}}$ holds but the additional bias,

the amount of tilt of the I - V curve, and NDC are just getting larger. The infinitely large value of NDC (singularity) and hysteresis can appear earlier in the RTD I - V curve. Qualitatively, the longer the spacer, the more space charge is present there; the longer the length over which the corresponding field acts, the stronger is the effect ($\delta V \propto s^2$). Quantitatively, δV caused by the collector space charge is given as $j_{\text{RTD}}s^2/(2V\epsilon)$, which follows from Eq. (30).

An illustration of the collector space-charge effects for 1.0-RTD and 1.6-RTD is given in Fig. 3, where we show sets of example I - V curves for various spacer lengths and for infinite (space charge effects in the spacer are off), ballistic, and saturation electron velocities. Notice that, for long spacers and low electron velocities, the tilt induced by the traveling space charge can result in a Z-shaped I - V curve. In Fig. 3, we also show simulated I - V curves based on experimental data corresponding to the 1.0-RTD and 1.6-RTD [8,30].

A larger voltage difference between the peak and valley points is one of the large-signal figures of merit for RTDs, influencing the maximum voltage swing of RTD oscillations and thereby determining the maximum output power that the RTD oscillator can deliver [42,43]. It is often considered intuitive that increasing the spacer length results in a larger peak-to-valley voltage difference. This trend can also be observed in Fig. 3. However, owing to the space charge effects in the collector spacer, the increase might be strongly reduced or might not occur at all, particularly in the saturation-velocity regime and in high-current-density RTDs.

In real devices, the measured dc RTD I - V curves are tilted because of the nonzero contact resistance as well. Assuming a contact resistance of $2 \Omega \mu\text{m}^2$ [4], which is typical for state-of-the-art RTDs, the measured current peak would shift by 0.048 V and 0.012 V for the 1.0-RTD and 1.6-RTD, respectively. These shifts are comparable to those caused by the collector space-charge effect in the case of ballistic electron transport (see Fig. 3). However, the shifts caused by space charge become much larger if the electrons move with the saturation velocity.

B. Small-signal RTD dynamics

The RTD dynamics can be qualitatively discussed in terms similar to the dc RTD characteristics. We start with a quasistatic (low-frequency) limit for the RTD admittance. From Eq. (41), the RTD admittance in the low-frequency limit ($\omega\tau \ll 1$) can be approximated as

$$\hat{Y}_{\text{RTD}} \approx G_0 + i\omega C_0, \quad (50)$$

where C_0 is a low-frequency RTD capacitance, equal to

$$C_0 = C_{\text{ec}} - \tau (G_0 - G_{\text{inf}}). \quad (51)$$

In this limit, the RTD can be approximated by an RC circuit. G_0 is given by Eq. (45). However, G_0 is also a

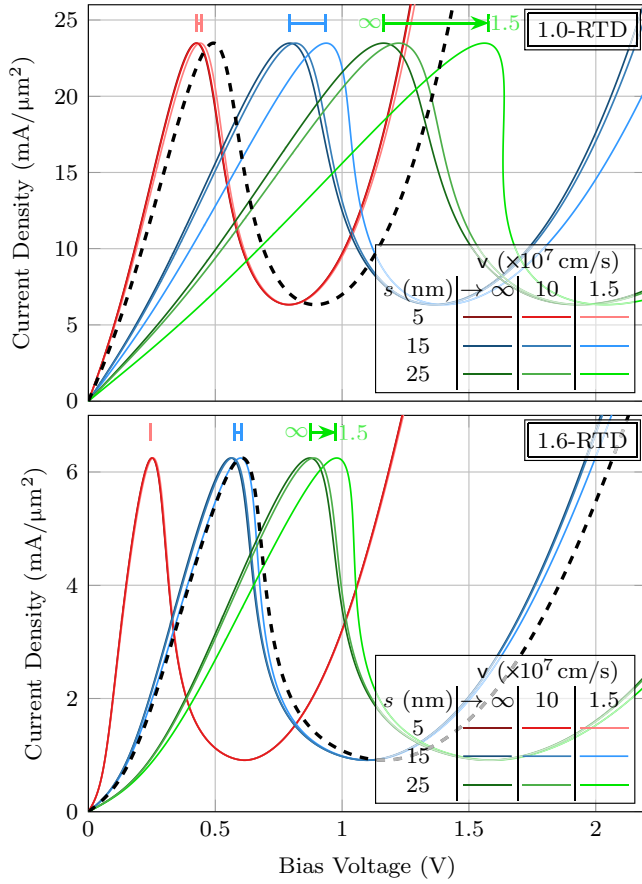


FIG. 3. I - V curves for various spacer lengths and electron velocities for 1.0-RTD and 1.6-RTD. Note that, for longer spacers and lower electron velocities, the traveling space charge introduces a tilt that makes the I - V curve Z-shaped. The dashed black lines represent simulated RTD I - V curves obtained from experimental data and thus provide a link to the measured devices [8,30]. The thicknesses of the depleted regions in the experimentally investigated structures were 7 nm and 16 nm for the 1.0-RTD and 1.6-RTD, respectively. The intervals above the curves indicate the voltage tilt differences between the infinite and saturation velocities.

derivative of the static I - V curve. As discussed previously, the RTD I - V curve is tilted, owing to the collector space charge. That leads to an increase of NDC in the corresponding region of the I - V curve. This is also reflected by the small denominator in Eq. (45) in the NDC region, see also Eq. (49). Conversely, C_0 is also governed by the RTD conductance [Eq. (51)] and becomes increased in the NDC region, compared with the geometrical capacitance C_{cc} . Qualitatively, the capacitance can be linked to the redistribution of charges between the QW and emitter: the steeper the I - V curve (the larger the tilt), the more charge is being shifted between the QW and emitter for a given bias variation, and the larger the capacitance. That leads to a considerable peak in C_0 in the NDC region [44,45], which gets even larger, on account of the collector space charge.

Equation (50) holds until $\omega\tau \ll 1$ and becomes strongly modified at higher frequencies, see Eq. (41).

Now we need to sort out different time constants appearing in the discussion on RTD dynamics in different circumstances. First, we switch the collector space charge off. Going through Eqs. (16) and (26), one can identify the following relations between several time constants in the NDC region: $\tau_{\text{dwell}} < \tau_{\text{rel}} < \tau_{\sigma}$. These interrelations could also be explained in terms of Coulomb-interaction effects. The time constants describe the QW charge relaxation. Without Coulomb interaction, the relaxation process for a charge perturbation in the QW is governed by τ_{dwell} . Now, we switch the Coulomb interaction on. When we add electrons to the QW (perturbation), the QW bottom shifts upward in the band diagram shown in Fig. 1. This shift is governed by the capacitive coupling between the QW and the emitter and collector: the smaller the QW-emitter and QW-collector capacitances are, the larger is the shift. In the NDC region, this shift upward leads to an increase in the current from the emitter into the QW; this increase will slow the relaxation process down (we added charge but its relaxation is slowed down by an increase in the incoming current). Therefore, $\tau_{\text{dwell}} < \tau_{\text{rel}}$ in the NDC region [1,18–20]. Notice that the opposite happens in the PDC region: the shift of the QW upward leads to a decrease of emitter-QW current and, as a result, to acceleration of the relaxation process ($\tau_{\text{dwell}} > \tau_{\text{rel}}$).

The time constant τ_{σ} corresponds to the case when the electric field in the collector is fixed. This situation can be mimicked by letting the collector thickness be large ($l \rightarrow \infty$). Then the QW-collector capacitance goes to zero and the QW bottom shift upward becomes the largest. As a consequence, $\tau_{\text{rel}} < \tau_{\sigma}$. These considerations are in agreement with Eqs. (26) and (16): $\tau_{\text{rel}} \rightarrow \tau_{\sigma}$ when $l \rightarrow \infty$, and $\tau_{\text{rel}} < \tau_{\sigma}$ for finite l in the NDC region.

Interrelation of τ to these time constants is more complicated (the collector space charge is switched on now). Addition of electrons to the QW will lead to more collector current and, therefore, to the appearance of an accompanying electron charge in the collector spacer. Effectively, a perturbation of the potential due to the QW charge becomes amplified by the collector space charge. As a result, the Coulomb shift of the QW bottom upward will also be enhanced and that leads to a further slowdown of the relaxation process: $\tau > \tau_{\text{rel}}$ in the NDC region, see also Eq. (43). In general, τ can be shorter as well as longer than τ_{σ} .

The significance and extent of collector-space-charge effects are illustrated in the following examples.

C. Examples of dynamic characteristics

We start with the calculation of the dc I - V curves for the example RTDs with various spacer lengths and for infinite, ballistic, and saturation electron velocities. For each I - V curve, we select the point with the highest NDC and

calculate the dynamic characteristics of the RTDs at these points. Beyond the point where the I - V curve becomes bistable (z-shaped), we use the same bias current density as at the highest-NDC point just before the onset of bistability. Equations (41)–(45) are used to evaluate the admittance of the full RTD. Since the $RLRC$ model given by Eqs. (41)–(45) was derived under the assumptions of Eqs. (46)–(48) and (27), for comparison, we also compute the full admittance without these assumptions by using Eq. (40).

We note that, for very short spacers, back-injection from the heavily doped collector may become non-negligible [46]; this regime is not included in our analysis, since short spacers can potentially reduce both the peak current and the peak-to-valley current ratio of the RTD. Hence, we restrict our analysis to spacer lengths no shorter than 5 nm.

Figures 4, 5, and 6 show the dependencies of G_0 , G_{inf} , low-frequency capacitance C_0 , geometrical capacitance C_{ec} (for the $RLRC$ model, independent of electron velocity and corresponding to the capacitance as $\omega \rightarrow \infty$), and time constant τ on spacer length. The dashed lines represent results calculated with Eq. (40) without the assumptions of Eqs. (46)–(48) and (27). The results show that, except near the singularity, the $RLRC$ model provides a good approximation for the RTD admittance.

In Figs. 4–6, the curves corresponding to ballistic electron velocity closely follow those obtained by neglecting the space-charge effects ($v \rightarrow \infty$), indicating that, for ballistic electron velocities, the space-charge effects are minor, and play a minimal role in the RTD admittance, across the considered range of spacer lengths. In contrast, for the saturation velocity, the impact of the collector space-charge effects on the RTD characteristics is clearly visible, particularly for 1.0-RTD, where the plots even exhibit a singularity ($|G_0| \rightarrow \infty$, $C_0 \rightarrow \infty$, $|\tau| \rightarrow \infty$), as the I - V curve becomes Z-shaped.

Space-charge effects are determined by the amount of charge in the collector, which is proportional to the product $v_c \tau_s$; notice that the space charge is proportional to the time electron spend in the spacer (τ_s) and that the space-charge density is proportional to the RTD current density, which in its turn is proportional to v_c . Indeed, inspecting Eqs. (43)–(45), we can notice a common factor $v_c \tau_s$ in all terms originating from the collector space charge. However, the space-charge terms in Eqs. (43)–(45) also contain the ratio of τ_{rel} and τ_{dwell} , which is a more subtle parameter related to the shape of the RTD I - V curves. Therefore, $v_c \tau_s$ and the RTD current density provide only a rough indication of the strength of the space-charge effects. Nevertheless, the parameter is quite informative. Indeed, the peak current density is 4 times greater in 1.0-RTD, compared with 1.6-RTD (see Table I); accordingly, Figs. 3–6 clearly show a much stronger impact of the space-charge effects in 1.0-RTD in the case of saturation velocity for the same spacer lengths.

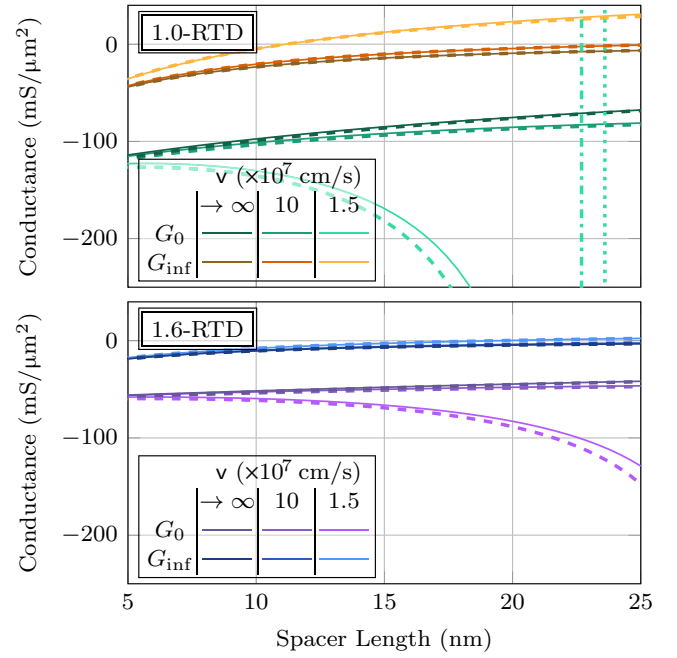


FIG. 4. Dependence of resonant tunneling diode (RTD) conductances G_0 and G_{inf} on spacer length. The solid lines depict the calculation results with the $RLRC$ model, valid for $\omega \ll 1/\tau_s$ and with the approximations of Eqs. (46)–(48) and (27), while the dashed lines represent the model given by Eq. (40) without those approximations. Dash-dotted and dotted lines indicate the positions of singularities in the respective models. The darkest lines correspond to the calculation results for G_0 and G_{inf} without the effects of the traveling space charge in the collector spacer ($v \rightarrow \infty$).

Figures 7 and 8 illustrate the frequency dependence of the RTD conductance [$\text{Re}(\hat{Y}_{\text{RTD}})$] and capacitance [$\text{Im}(\hat{Y}_{\text{RTD}})/\omega$], respectively, for various spacer lengths. The solid lines show the calculation results with the $RLRC$ model for the approximation $\omega \ll 1/\tau_s$, while the dashed lines correspond to the full model, as given by Eq. (33). Together with Fig. 6, these results show that increasing the spacer length slows down the device response: the transition from low- to high-frequency regime ($\omega \tau_s = 1$) shifts to lower frequencies.

Discrepancies between the $RLRC$ and full models in Figs. 7 and 8 are minor and have limited practical impact at typical operating frequencies of RTDs up to 2 THz. They become prominent only at high frequencies, when the condition $\omega \ll 1/\tau_s$ is no longer satisfied. In this case, ripples in the conductance curves calculated with the full model can be observed, see Fig. 7. These ripples are BARITT-like features (regime three from Sec. VC). The 1.0-RTD wafer exhibits stronger ripples, compared with the 1.6-RTD, owing to its higher current density and consequently larger $\hat{\sigma}$. At these frequencies, however, they are strongly attenuated [see Eq. (31)] with additional frequency-dependent reduction of $\hat{\sigma}$ (see values for τ_σ in Table I). An exception

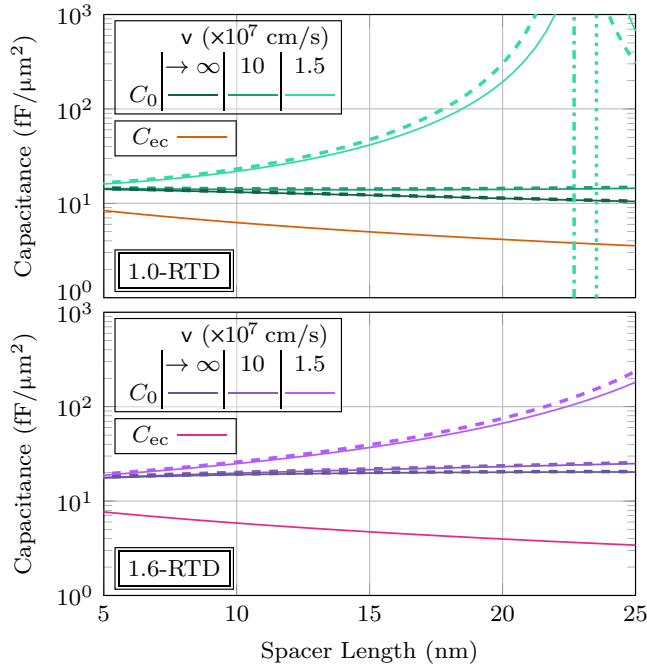


FIG. 5. Dependence of low-frequency resonant tunneling diode (RTD) capacitances on spacer length. Line styles follow the same convention as in Fig. 4. The small difference in C_{ec} between the cases with and without the approximations of Eqs. (46)–(48) and (27) lies within the linewidth and is, therefore, not shown.

occurs for moderate-to-long spacer lengths (15 nm) with electron saturation velocity, where the first ripple reaches frequencies below 2 THz. For the *RLRC* model, the ripples are missing, as the model neglects the oscillating term in the admittance of the spacer. In principle, the ripples

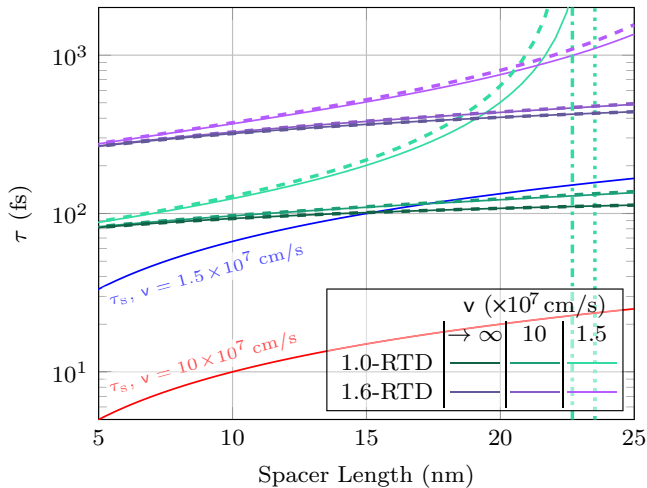


FIG. 6. Dependence of the resonant tunneling diode (RTD) time constant τ on spacer length. Line styles are consistent with those used in Fig. 4. The red and blue lines show τ_s for the corresponding electron velocities.

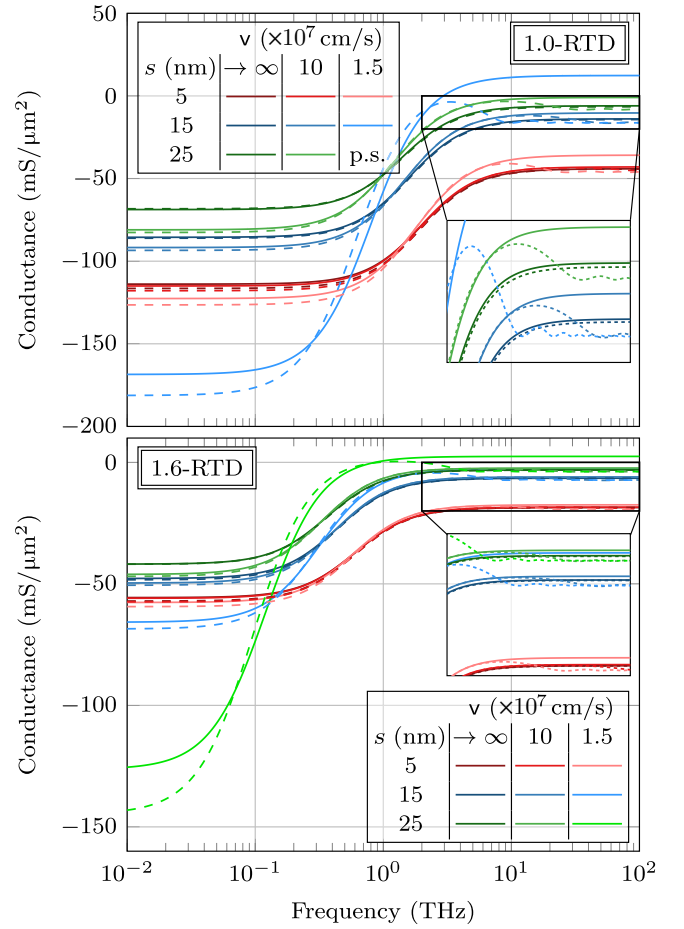


FIG. 7. Frequency dependence of resonant tunneling diode (RTD) conductance $[\text{Re}(\hat{Y}_{\text{RTD}})]$ for various spacer lengths. Solid lines correspond to the *RLRC* model, while dashed lines show results from the full model [Eq. (33)]. The insets show enlarged portions of the parts of the plots marked by black rectangles and illustrate high-frequency ripples in the regime $\omega\tau_s \gtrsim 1$. The case of the 1.0-RTD with the longest spacer and slow electrons is past the singular point (p.s.), where the RTD cannot be biased, and thus it is excluded from the frequency plot.

might lead to unconventional operation regimes of RTDs. However, the feasibility and practicality of such regimes require a separate investigation.

Apart from ripples, we can notice a particular asymptotic behavior of RTD conductance in Fig. 7 (see insets) at high frequencies and as $\omega \rightarrow \infty$: the conductances calculated with the full model for both saturation and ballistic velocities converge toward the same asymptotic values, which are slightly below G_{inf} calculated with the *RLRC* model with $v \rightarrow \infty$ (neglected collector space charge) for the same spacer length. The convergence to an asymptotic value indicates suppression of the collector-space-charge effects at high frequencies. On the one hand, that happens because of the reduction of the amplitude of the

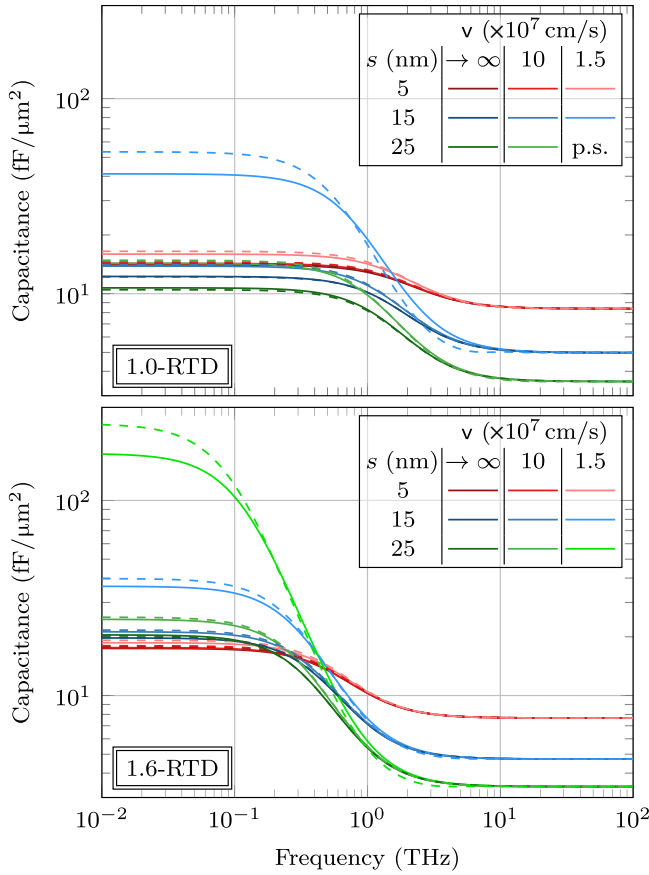


FIG. 8. Frequency dependence of the resonant tunneling diode (RTD) capacitance $[\text{Im}(\hat{Y}_{\text{RTD}})/\omega]$ for various spacer lengths. Line styles follow the same convention as in Fig. 7.

space-charge oscillations in the collector with increasing frequency, as mentioned previously. On the other hand, as predicted by the Shockley-Ramo theorem [47,48], these space-charge oscillations become averaged out when many of them are simultaneously present in the collector ($\omega \gtrsim 1/\tau_s$). As a consequence, not only do the space-charge effects become suppressed, but the contribution of the collector-barrier current $\hat{j}_c$ to the RTD admittance, as a whole, is also eliminated at high frequencies.

The latter effect is also responsible for the shift of the asymptotic conductance below G_{inf} . The injection current into the collector ($\hat{j}_c$) has a frequency-independent component due to the term with v'_c , see Eq. (18). This effect accounts for the continuous injection of electrons in response to bias variations, even when N_{2D} remains constant. If the space-charge effects in the collector are neglected, this current gives a positive contribution to G_{inf} , thereby reducing NDC at high frequencies [18–20]. When the space charge in the collector spacer is properly taken into account, the contribution of $\hat{j}_c$ is eliminated at high frequencies, as pointed out already. This explains why the dashed lines in Fig. 7 do not converge to the darkest solid

lines, which correspond to the case where space charge is neglected. As a result, the NDC of the RTD is expected to remain negative even in the high-frequency limit $\omega \rightarrow \infty$ and long spacers. The inclusion of the space charge effects therefore leads to a higher predicted NDC in this limit. However, this increase is mainly of academic interest, as it occurs at frequencies far beyond the practical operating range of RTDs and the effect is small.

It follows from the foregoing discussion that RTDs exhibit asymptotically NDC at high frequencies. Therefore, the intrinsic operating range of RTDs appears to be unbounded. In practical implementations of terahertz oscillators, however, it is the parasitic elements—external to the RTD layers—that impose limitations. The dominant contribution to high-frequency degradation of NDC comes from the contact resistance between the semiconductor layers and the metallic electrodes.

As stated previously, state-of-the-art RTDs can exhibit contact resistances as low as $2 \, \Omega\mu\text{m}^2$ [4]. Owing to the presence of a Schottky barrier, the contact capacitance can be estimated to be approximately $17 \, \text{fF}/\mu\text{m}^2$, appearing in parallel with the contact resistance. The maximum operating frequency can then be estimated as the frequency at which the real part of the total admittance—comprising the series combination of contact parasitics and RTD admittance—becomes positive.

The estimated maximum operating frequencies for the two example wafers are shown in Fig. 9. The figure indicates that, in general, shorter spacer lengths result in higher operating frequencies. In the ballistic transport regime, the spacer length has only a minor impact on the maximum frequency. However, in the case when electrons propagate at the saturation velocity, the spacer length becomes a significant factor, particularly in high-current-density RTDs; this can significantly reduce the operating frequencies of RTD oscillators [43].

We add here two comments. First, the impact of the space-charge effects is qualitatively different from that of the RTD parasitics. The space-charge effects modify the internal dynamics of an RTD, i.e., its charge-relaxation time constant and high-frequency conductance (see Figs. 6 and 7), whereas the parasitics affect only the external RTD admittance. Second, in practical RTD oscillators, additional parasitic effects, such as bottom-contact resistance, spreading resistance in the semiconductor layers, and resonator losses, further limit the maximum operating frequency. The values shown here, therefore, represent an idealized case and are higher than can be achieved in real devices, compared with those reported in Picco *et al.* [43].

As discussed in Sec. VIA, a longer spacer may lead to a larger peak-to-valley voltage difference, potentially increasing voltage swing or output power; see the discussions on this point given by Spudat *et al.* [42] and Picco *et al.* [43]. However, it also shifts the NDC region

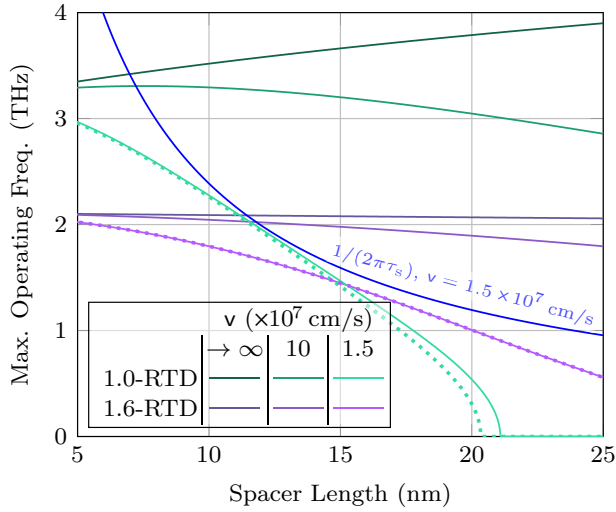


FIG. 9. Maximum operating frequency for the two example wafers. The solid blue line corresponds to the characteristic frequency determined by the electron transfer time in the saturation-velocity regime; the maximum frequencies for both wafers approach this limit, violating the condition $\omega \ll 1/\tau_s$. Therefore, the dashed lines indicate results obtained from the full model without applying the approximation [Eq. (39)]. The comparison shows that the deviation is minimal, confirming that the approximation remains valid.

toward higher voltages, increasing the power dissipated in the RTD (see Fig. 3). For high-current-density RTDs, this additional heating can approach the thermal limit and, in general, may even lead to device breakdown, as discussed by Tanaka *et al.* [49]. From a different perspective, a longer spacer reduces the maximum RTD operating frequency but also decreases the high-frequency RTD capacitance; the latter is favorable for RTD oscillators. Therefore, the spacer length is a critical design parameter that must be optimized as a trade-off between output power, frequency performance, and device reliability.

VII. CONCLUSIONS

In this paper, we present a general small-signal model of RTD admittance that incorporates both the charge dynamics within the QW and the dynamics of the traveling space charge in the collector spacer. The presented model extends an earlier (“old”) *RLRC* model, which accounts only for the QW dynamics.

If electron transport through the collector spacer is sufficiently fast ($\omega \ll 1/\tau_s$), the general model reduces to an equivalent *RLRC* circuit. This approximation holds for typical subterahertz and terahertz RTDs under typical operating conditions. This *RLRC* circuit retains the same configuration as the “old” *RLRC* model. However, the circuit parameters are rescaled when collector space-charge

effects are taken into account. In particular, QW charge relaxation slows down in the NDC region, resulting in a longer relaxation time constant (τ) and an increased low-frequency RTD capacitance.

As with the “old” model, τ can be significantly shorter in the PDC region or significantly longer in the NDC region, compared with τ_{dwell} . Similarly, as in the “old” model, the RTD admittance exhibits NDC at frequencies well beyond the relaxation time limit, where $\omega \gg 1/\tau$.

The traveling collector space-charge affects not only dynamic but also static RTD characteristics, particularly the RTD *I-V* curve. As a result, the RTD *I-V* curve tilts to the right, and the NDC increases. In principle, these effects can lead to an infinitely large NDC and a Z-shaped *I-V* curve with a hysteresis region. More importantly, collector space-charge effects can mitigate or reverse an increase in the peak-to-valley voltage difference with increasing spacer length. Note that the peak-to-valley voltage difference is a critical parameter in the large-signal analysis of conventional RTD oscillators.

The mechanism by which collector transport affects RTD characteristics is not merely due to the time delay caused by electron transit through the collector spacer. Instead, it is related to the charge buildup in the collector, which influences the band alignment between the QW and the RTD emitter, the tunnel current through the barriers, and, consequently, the dynamics of the QW charge. This mechanism is analogous to the effect of Coulomb interaction on QW charge dynamics in the “old” model and on the dc RTD *I-V* curves. The collector space-charge contribution can be viewed as an amplification mechanism for QW space-charge effects.

The strength of collector space-charge effects scales with the electron transit time through the collector spacer (τ_s) and the RTD current density. However, in typical subterahertz and terahertz InGaAs-AlAs RTDs, which are designed to maintain ballistic electron transport across the collector depletion region (spacer), the collector space-charge effects remain minor and can almost always be neglected.

Collector space-charge effects become significant only in RTDs with relatively high current density ($\gtrsim 10 \text{ mA}/\mu\text{m}^2$), a sufficiently long collector spacer ($\gtrsim 20 \text{ nm}$), and, importantly, slow electrons moving at saturation velocity; this is a rare situation in contemporary subterahertz and terahertz RTDs.

RTD admittance exhibits ripples associated with traveling space charge, a phenomenon well known in BARITT and IMPATT diodes at frequencies $\omega \gtrsim 1/\tau_s$. However, these ripples are strongly damped and are expected to be negligible in state-of-the-art RTDs. Additionally, the collector-barrier current contribution to RTD conductance is significantly suppressed by space-charge effects in the regime $\omega \gg 1/\tau_s$, increasing NDC in the asymptotic high-frequency regime.

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DATA AVAILABILITY

The data that support the findings of this article are not publicly available. The data are available from the authors on reasonable request.

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