

Doppler pulse amplification

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The ability to amplify ultrashort pulses has revolutionized modern laser science, driving advances in various fields such as ultrafast optics and spectroscopy. A pivotal development in this field is chirped pulse amplification (CPA), which stretches, amplifies, and recompresses ultrashort optical pulses using dispersive elements to overcome amplification limits. However, CPA faces limitations due to gain narrowing, restricting the final pulse duration. Here we propose Doppler pulse amplification (DoPA), an approach for amplifying ultrashort pulses. While DoPA shares similarities with CPA in that it also stretches, amplifies, and recompresses pulses, it differs in how it achieves this temporal compansion (expansion and compression). Unlike CPA, DoPA exploits Doppler shifts induced by space-time-modulated interfaces through a space-time wedge implementation without chirping. We show that DoPA dynamically shifts the pulse spectrum, effectively mitigating the gain narrowing issue of CPA. Additionally, we show that DoPA enables more compact amplification systems via a space-time Fresnel implementation. This approach may pave the way for more efficient, high-intensity laser systems and expand the potential for applications in both laboratory research and practical environments.

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I. INTRODUCTION

Pulse amplification techniques have continuously evolved to meet the growing demands of various fields, from telecommunications and medical applications to scientific research. Some applications, such as communication [1,2] and radar [3,4] systems, require only moderate gain amplification. However, others—including laser light-matter interactions [5,6], particle acceleration [7,8], ultrafast optics [6,9], and precision eye surgery [10,11]—require advanced techniques to amplify ultrashort pulses to very high peak powers. In these cases, direct amplification of ultrashort pulses exceeds the saturation level of the amplifier, potentially damaging optical components and introducing nonlinear effects such as self-phase modulation [12,13]. To address this, two main approaches have been developed. The first, chirped pulse amplification (CPA) [14–18], uses dispersive elements, such as diffraction gratings, to temporally stretch the pulse, reducing its peak power and spreading its energy over time. This enables greater amplification before the pulse is recompressed to its original shape and duration, now with significantly higher peak power. Developed by Strickland

and Mourou [14], CPA revolutionized laser science and earned them the 2018 Nobel Prize in Physics [15]. The second approach, divided-pulse amplification [19–25], splits the input pulse into multiple identical lower-power copies using a sequence of birefringent crystals. These copies are individually amplified and then recombined, resulting in a pulse with significantly higher peak power.

Despite their success, both approaches come with intrinsic limitations that restrict their amplification performance. On one hand, CPA suffers from gain narrowing, a spectral bottleneck that arises when the spectral width of the pulse exceeds the bandwidth of the gain amplifier, leading to temporal broadening and pulse distortion in the output [18,26,27]. Additionally, CPA is constrained by its inability to accommodate the large diffraction grating sizes required for relatively long pulses [28]. On the other hand, divided-pulse amplification is limited by the number of pulse divisions, restricting the reduction in peak power of the individual copies. Consequently, amplification is constrained, as the copies quickly reach the saturation level of the amplifier [29–31].

We propose a method for amplifying ultrashort pulses, called “Doppler pulse amplification” (DoPA). DoPA shares the same top-level structure with CPA—stretching, amplifying, and compressing—but instead of using dispersive elements, it relies on Doppler frequency shifts [32] induced by moving modulated interfaces [33–36] to control the pulse compansion (expansion and compression), without any chirping. It mitigates the gain narrowing problem by dynamically alternating the pulse spectrum, making

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it suitable for the amplification of ultrashort pulses, and may lead to compact systems.

This paper is structured as follows. DoPA is introduced along with CPA in Sec. II. In Sec. III the DoPA system is analyzed with the use of a space-time wedge implementation. This analysis is demonstrated in Sec. IV. In Sec. V we discuss how the device size can be reduced by replacing the space-time wedge with a space-time Fresnel profile. In Sec. VI potential experimental realizations of the DoPA system are outlined. Section VII concludes the paper.

II. CONCEPT

Although based on a fundamentally different mechanism, DoPA shares the principle of temporal compansion with CPA. Therefore, it makes sense to introduce DoPA alongside CPA. Figure 1 depicts the two approaches, with Figs. 1(a) and 1(b) corresponding to CPA and DoPA, respectively.

A CPA system, shown in Fig. 1(a), consists of three main stages [14]. First, the input pulse—with peak power P_{in} —passes through a chirp filter characterized by a phase modulation $\varphi[\omega]$, whose group velocity dispersion temporally expands and chirps the pulse [9]. This causes different frequency components to experience different group delays, resulting in temporal stretching of the pulse. This chirping process reduces the peak power from P_{in} to P_{in}/a , where $a > 1$ is the compansion factor while spreading its energy over a longer duration. Second, the stretched pulse is amplified by a gain amplifier, e.g., Ti:sapphire or Nd:glass, up to its saturation threshold P_{th} to mitigate nonlinear effects and prevent damage to the optical components, resulting in an intermediate peak power $P'_{\text{out}} =$

GP'_{in} , where G is the amplifier gain. Finally, a second chirp filter, with opposite dispersion characteristics, compresses the pulse back to its original duration, eliminating the chirp introduced in the first step and significantly increasing the output peak power to $P_{\text{out}} = aP'_{\text{out}} = GP_{\text{in}}$.

Since the input pulse is ultrashort, it has a broad spectral width that remains unchanged after it has passed through the chirp filter [9]. This imposes strict constraints on the amplifier, whose limited bandwidth may amplify only a portion of the spectrum, leading to distortions that undermine the original goal of the system. This phenomenon, known as gain narrowing, is one of the principal limitations of CPA. In Fig. 1(a), it is avoided by the use of a very broadband amplifier, but it would arise if the narrower-band amplifier corresponding to the dotted curve in Fig. 1(b) were used [18,26,27].

While DoPA also involves sequential pulse expansion, amplification, and recompression, as illustrated in Fig. 1(b), it achieves these transformations through Doppler frequency shifts induced by moving interfaces, instead of chirp filters. DoPA uses two paired space-time wedges, recently introduced in Ref. [36]. The wedge edges are assumed to act as perfect electric conductor moving interfaces, ensuring that the pulse remains confined within the structure. This configuration allows the pulse to bounce back and forth between two moving interfaces, and depending on the wedge type—either opening or closing—the temporal envelope of the pulse either expands or compresses upon each interaction with the interfaces. In the case of an opening wedge, the pulse interacts only with comoving interfaces, leading to Doppler downshifting, which stretches the pulse duration by a factor a and reduces its peak power to P_{in}/a , facilitating safe amplification.

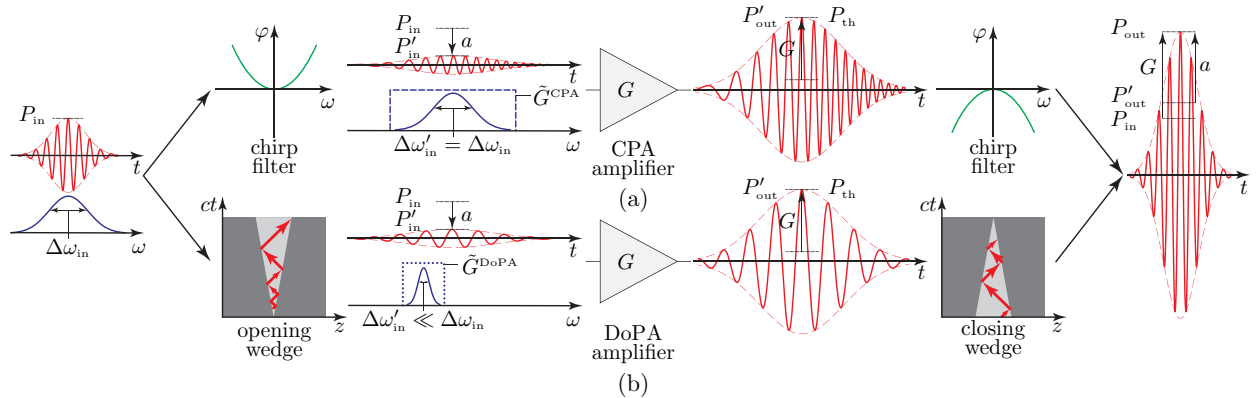


FIG. 1. General concept of (a) CPA and (b) proposed DoPA. In both CPA and DoPA, the input pulse is first stretched out in time to reduce its peak power from P_{in} to $P'_{\text{in}} = P_{\text{in}}/a$, where $a > 1$ is the compansion factor. This temporal stretching—achieved via a chirp filter in CPA and an opening space-time wedge in DoPA—lowers the peak power enough to allow amplification up to the saturation threshold P_{th} of the amplifier. With gain G , the amplified stretched pulse reaches $P'_{\text{out}} = GP'_{\text{in}} = P_{\text{th}}$. The pulse is then recompressed—with the use of a matched chirp filter (CPA) and a matched closing space-time wedge (DoPA)—restoring its duration and boosting the peak power to $P_{\text{out}} = aP'_{\text{out}} = GP_{\text{in}}$. Unlike CPA, where the spectral width remains constant through the chirp filter ($\Delta\omega'_{\text{in}} = \Delta\omega_{\text{in}}$) [9] and, therefore, requires a broad amplifier bandwidth, DoPA dynamically compresses the spectrum during the stretching stage ($\Delta\omega'_{\text{in}} \ll \Delta\omega_{\text{in}}$) [36], enabling amplification with a much-narrower-band amplifier.

Conversely, for a closing wedge, the wave encounters only contramoving interfaces, resulting in Doppler upshifting and compressing of the pulse back to its original duration and shape. Notably, no chirping occurs here, given the uniformity of the space-time structure [37,38].

This Doppler-driven compansion mechanism causes progressive spectral narrowing as the pulse propagates through the opening wedge (Sec. III), in contrast to CPA. This dynamic narrowing enables the use of much-narrower-band amplifiers without incurring gain narrowing and preserving spectral integrity after amplification. Additionally, DoPA offers the potential for a compact implementation (Sec. V), which could be particularly advantageous in applications requiring high-power pulses where system size and complexity are critical constraints.

III. SYSTEM ANALYSIS

Figure 2 shows a practical implementation of the double space-time wedge structure of the DoPA system [Fig. 1(b)] with corresponding spectral transition diagrams. Unlike idealized space-time wedges [36], this implementation includes space-time apertures that allow the pulse to enter and exit the structure without spurious reflections. In the opening wedge configuration [Fig. 2(a)], the input aperture is positioned beyond the wedge apex, while the output aperture is before the wedge basis. In the closing wedge configuration [Fig. 2(b)], the input and output apertures correspond to the time-reversed versions of their opening-wedge counterparts. Once inside the structure, the incident pulse—initially at $X_0 = (z_0, ct_0)$ —undergoes multiple space-time reflections and related spectral transitions at distinct space-time scattering points $X_1, X_2, \dots$, until its final reflection at X_p , after which it exits the structure. The opening and closing wedges both move with a constant normalized modulation velocity $\beta = v_m/c$, where $0 < \beta < 1$, which governs the Doppler shifts and temporal dynamics of the pulse as it propagates through the structure.

To ensure full recovery of the original pulse shape and duration, the opening and closing wedges must be paired with precisely opposite space-time trajectories, i.e., the same velocity magnitude, and an identical number of reflections. This design is analogous to chirp filters in CPA, where the phase modulation in the second filter compensates for the chirp introduced by the first filter [Fig. 1(a)]. Likewise, in DoPA, the opening wedge modifies the pulse in a controlled manner, while the closing wedge precisely reverses these modifications.

When the incident pulse impinges on the moving interface, at X_1 , it undergoes the Doppler frequency shift of a moving interface with constant velocity β , viz., $\omega_1 = R\omega_0$, where $R = (1 \mp \beta) / (1 \pm \beta)$ and ω_0 is the center frequency of the incident wave [34]; see Fig. 2. The top sign corresponds to an interface that is comoving with

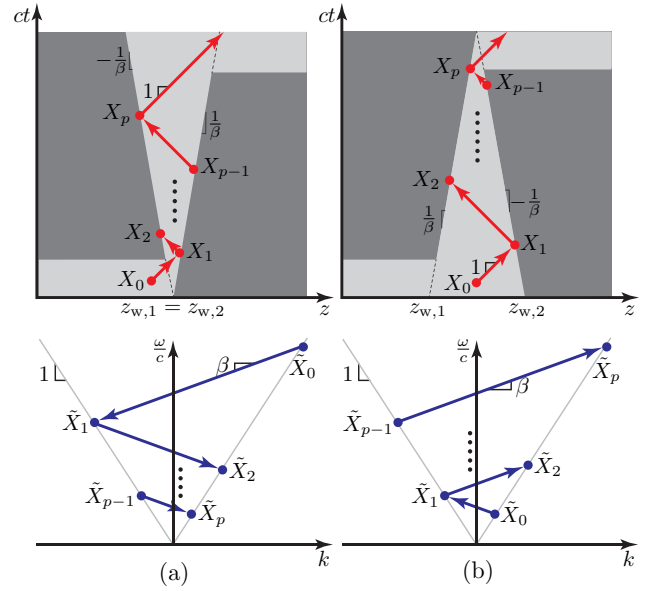


FIG. 2. Implementation of the space-time wedges in the DoPA system shown in Fig. 1(b) with space-time apertures (top) and corresponding spectral transition diagrams (bottom). The wedges are the light-gray triangular areas, representing propagation media of unit refractive index ($n = 1$), that compand pulses via Doppler shifting [Eq. (1)] by compansion factor a [Eq. (2)]. The two dark-gray areas sandwiching the wedges act as perfect electric conductors. The modulation interfaces travel with normalized velocity $\beta = v_m/c$. The direct space-time points are denoted by $X_p = (z_p, ct_p)$, while the inverse space-time points are denoted by $\tilde{X}_p = (k_p, \omega_p/c)$. (a) Opening wedge: pulses broaden temporally and compress spectrally. (b) Closing wedge: pulses compress temporally and broaden spectrally.

respect to the incident wave, resulting in $R < 1$ and thus a decrease in frequency. The bottom sign corresponds to an interface that is contramoving with respect to the incident wave, resulting in $R > 1$ and thus an increase in frequency. Because this scaling applies uniformly to the entire spectrum, both the center frequency ω_1 and the spectral width $\Delta\omega_1$ are multiplied by the same factor, R , resulting in spectral compression for comoving interfaces ($R < 1$) and spectral broadening for contramoving interfaces ($R > 1$).

Within the wedge structure (Fig. 2), the pulse experiences multiple space-time reflections, each introducing its own Doppler shift. However, because of the uniformity of the structure, the pulse undergoes the same Doppler shift at each scattering event. Therefore, the Doppler-shifted frequency ω_p after p space-time reflections is simply (see Appendix A 2)

$$\omega_p = R^p \omega_0. \quad (1)$$

For an opening wedge [Fig. 2(a)], the wave encounters only comoving interfaces ($R < 1$), and thus both the

frequency and the spectral width decrease with each reflection. For a closing wedge [Fig. 2(b)], the pulse encounters only contramoving interfaces ($R > 1$), and thus both the frequency and the spectral width increase as the wave propagates through the structure.

In addition to the Doppler shift [Eq. (1)], the temporal envelope of the pulse is affected, exhibiting an inverse relationship with frequency. We define the temporal compansion factor a as the ratio of the width τ_p of the reflected wave after p bounces and the temporal width τ_0 of the incident wave, viz. (see Appendix A 3),

$$a = \frac{\tau_p}{\tau_0} = R^{-p}. \quad (2)$$

For an opening wedge, $a > 1$, meaning that the pulse stretches in time, making a a stretching factor. For a closing wedge, $a < 1$, meaning that the pulse compresses, making a a compression factor. Notably, Eqs. (1) and (2) exhibit an inverse relationship: compression (expansion) in the spectral domain [Eq. (1)] corresponds to expansion (compression) in the time domain [Eq. (2)], and vice versa.

The physical size of the DoPA system is a key consideration for its practical implementation. The total length l_w of the wedge implementation depends on the initial pulse duration τ_0 , the compansion factor a , and the interface velocity β , following the relation (see Appendix B 1)

$$l_w = a \left(\frac{2}{1 + \beta} + \beta \right) c \tau_0. \quad (3)$$

For instance, for a femtosecond pulse with $\tau_0 = 10$ fs, a compansion factor a of 10^6 , and an interface velocity β of 0.01, the required device length l_w is approximately 6 m, which is comparable to the size of a CPA system, and requires a complex meandrous structure [39]. Importantly, there is an inherent trade-off between the interface velocity and device size. At low velocities, the compansion factor per bounce is small, requiring a large number of reflections to achieve the desired compansion. As the interface velocity increases, the compansion factor per bounce increases, reducing the number of reflections required. Notably, this increase in compansion factor per bounce outpaces the reduction in the number of reflections, leading to an overall decrease in device size. This trend continues until a critical velocity $\beta_c = \sqrt{2} - 1$, at which the device size is minimized, independent of the compansion factor and initial pulse duration. Beyond this point, while the compansion factor per bounce continues to increase—further reducing the number of reflections required—the pulse takes longer to catch up with the moving interface, increasing the required size.

IV. SYSTEM DEMONSTRATION

Figure 3 demonstrates the space-time evolution, time-domain waveforms, and frequency spectra of a

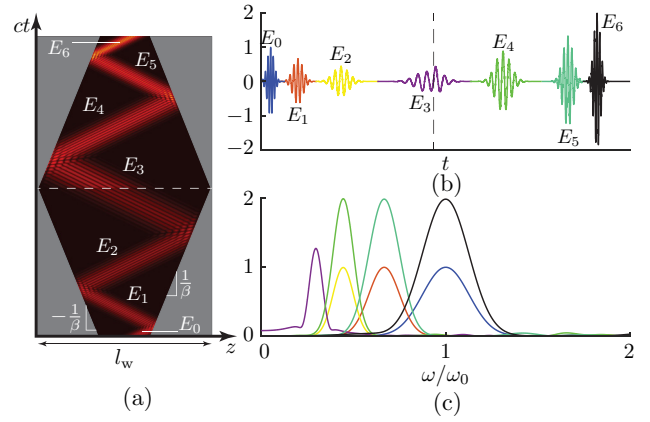


FIG. 3. Demonstration of the DoPA system in Fig. 1(b) with the space-time wedge implementation in Fig. 2. (a) Space-time diagram of the cascaded open-closed wedge structure with a central amplifier. (b) Time-domain waveforms, with the dashed line indicating the amplifying time. (c) Corresponding spectrum.

space-time wedge implementation (Fig. 2) of the DoPA system [Fig. 1(b)], governed by the Doppler [Eq. (1)] and temporal width [Eq. (2)] scaling relations. Initially, the incident pulse undergoes multiple comoving space-time reflections within the opening wedge, resulting in successive temporal stretching [Fig. 3(b)] and Doppler downshifting [Fig. 3(c)]. Then the expanded pulse is amplified, at the time indicated by the dashed lines in Figs. 3(a) and 3(b). Finally, the interfaces begin moving toward each other (closing wedge), causing the pulse to undergo multiple contramoving space-time reflections, leading to successive temporal compression and Doppler upshifting. Ultimately, the pulse exits the system with its original duration and shape, but with significantly higher peak power. Notably, the amplifier gain G in Fig. 3 is only 2, due to space constraints, although in practice it can reach 10^3 – 10^4 .

V. DEVICE SIZE REDUCTION

In the space-time wedge implementation in Fig. 3, an issue arises due to the continuous motion of the interfaces. As the wedge width increases with the required compansion factor [Eq. (2)], its device length [Eq. (3)] may become excessively large. To address this drawback, we adopt the concept of the space-time Fresnel prism [40]. In this approach, the interface positions reset after each reflection, producing an oscillatory, sawtoothlike trajectory.

Figure 4 depicts the space-time evolution, time-domain waveforms, and frequency spectra of the space-time Fresnel implementation of the DoPA system [Fig. 1(b)]. The pulse experiences the same sequence of Doppler-induced shifts and compansion as the wedge implementation (Fig. 3), as evidenced by the similar waveform and

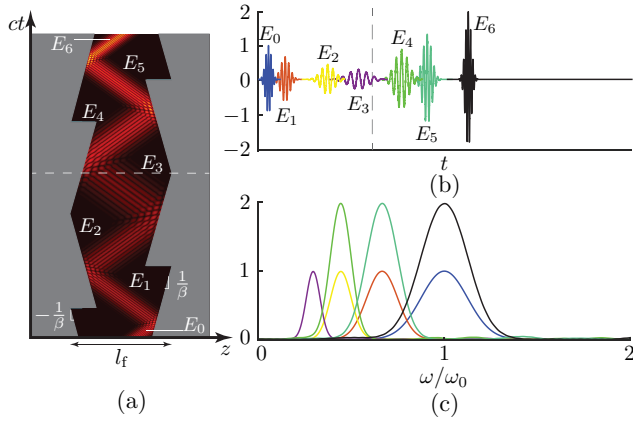


FIG. 4. Realization of DoPA [Fig. 1(b)] using a space-time Fresnel structure with the same number of reflections as in Fig. 3. (a) Space-time diagram. (b) Time-domain waveforms, with the dashed line indicating the amplifying time. (c) Corresponding spectrum.

spectral evolution. The key difference lies in the oscillatory motion of the interfaces and, hence, the shorter spacings between the temporal pulses, which allows the device length to be significantly reduced. Note that precise synchronization between the two moving boundaries is critical to avoid spurious reflections from the sharp discontinuities. If not carefully managed, these interactions may degrade pulse quality and reduce overall efficiency.

The device size l_F of the space-time Fresnel implementation is given by (see Appendix B 2)

$$l_F = 2(1 + a\beta) c\tau_0. \quad (4)$$

For the same values as those below Eq. (3), Eq. (4) yields $l_F \approx 6$ cm, which is about 100 times smaller than for the wedge implementation. Unlike Eq. (3), where the device length scales nonlinearly with the interface velocity, here it grows linearly. This is because the wavelength of the (periodic) oscillatory motion is directly proportional to the velocity.

The contrast in device size between the wedge and Fresnel configurations is evident from comparison of Figs. 3 and 4. To further illustrate this, Fig. 5 shows the case when $l_F = l_w$. In this scenario, the Fresnel profile allows for an additional expansion-compression pair, further reducing the peak power of the pulse. This enables a larger amplification gain—4 in this case—before the saturation threshold of the amplifier is reached.

VI. EXPERIMENTAL OUTLOOK

The effectiveness of DoPA system hinges on the ability to generate interfaces moving at relatively high speeds, posing significant experimental challenges. Although no direct method currently exists for creating such interfaces,

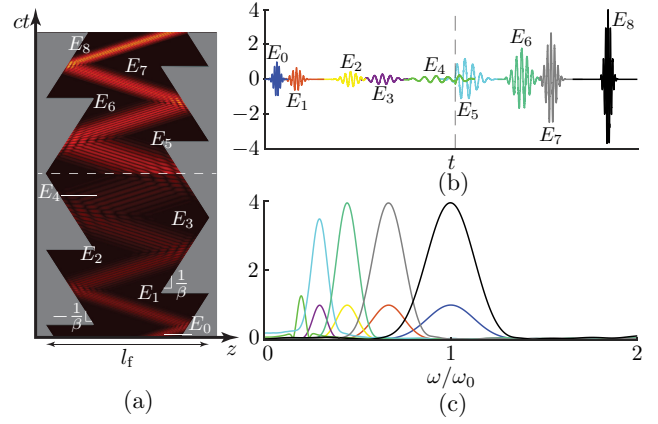


FIG. 5. DoPA using a space-time Fresnel structure with the same device size as in Fig. 3 ($l_F = l_w$). (a) Space-time diagram. (b) Time-domain waveforms, with the dashed line indicating the amplifying time. (c) Corresponding spectrum. Note that the dynamic ranges in (b),(c) are twice those in Figs. 3 and 4.

recent advances in time-varying media [41–46] provide a foundation that could be extended to space-time-varying systems. Here we outline how these existing platforms might be adapted to demonstrate a proof-of-concept realization of DoPA.

In the microwave regime, pure time interfaces have been realized by temporal modulation of the effective impedance of artificial transmission lines, controlled via high-speed electronic components, such as electronic switches [41,42] or high-speed photodiodes [43]. Extending this concept to space-time systems requires sequential, spatially coordinated switching to the ground that emulates a moving perfect electric conductor interface, demanding precise spatial control of the switching signals. One strategy is to drive the switches with the use of a field-programmable gate array, allowing precise tunable delays and sequencing. By synchronization of the activation of switches in a controlled pattern, it is possible to form two contramoving interfaces along the line, effectively realizing space-time wedges. Microwave implementations using corporate feed networks with tailored optical delays offer another route. The key DoPA parameter is the normalized modulation velocity $\beta = \Delta x/c\Delta t$, where Δx is the distance between adjacent switching elements and Δt is the time delay between their activation. To emulate sharp moving interfaces corresponding to the wedges, two conditions must be met. First, the local rise time at each site must remain shorter than Δt , ensuring that the wave perceives a distinct discontinuity. Second, Δx must be small compared with the wavelength of the incoming pulse to ensure a smooth modulation profile. For instance, with the use of parameters from Ref. [42], $\Delta x = 0.208$ m and $\Delta t = 4$ ns yield $\beta = 0.17$. With $\tau_0 = 0.5$ ns and $a = 10^2$, the Fresnel device length l_F [Eq. (4)] is approximately 5.5 m, which is

smaller than the 6.24 m transmission line used in the work reported in Refs. [41,42].

Experiments in the optical regime have also demonstrated temporal discontinuities using pump-probe setups [44–46], which induce refractive index changes via intense pump pulses in nonlinear media. The main challenge lies in achieving index variations that are both sufficiently large and sufficiently fast. Transparent conducting oxides, especially in the ϵ -near-zero regime, have emerged as promising candidates, capable of refractive index modulations approaching unity on femtosecond timescales. Space-time-modulated configurations can be achieved by tilting of the pump beam with respect to the probe beam [34]. Because of the oblique geometry, the pump-induced refractive index change sweeps across the medium, effectively forming a moving interface. The use of two contratilted pump beams allows the creation of a space-time wedge structure. For instance, if the angle between the probe pulse and the pump pulse is 60° and the pump is traveling at the speed of light, the normalized modulation velocity is given by $\beta = 1/\tan 60^\circ = 1/\sqrt{3}$. Next, if we take $a = 10^3$ and $\tau_0 = 10$ fs, a Fresnel length l_F of approximately 3.5 mm is obtained.

VII. CONCLUSIONS

We introduced DoPA, an approach that harnesses space-time modulation interfaces to amplify ultrashort pulses via Doppler frequency shifts, complementing established techniques such as CPA and divided-pulse amplification. We analyzed the system with a space-time wedge implementation, demonstrating how multiple space-time scattering events and associated Doppler frequency shifts can be utilized to achieve compansion, while also mitigating the gain-narrowing problem inherent in CPA. Furthermore, we explored a space-time Fresnel architecture approach, which reduces the system size without sacrificing amplification efficiency, making DoPA suitable for amplification of ultrashort pulses. This compact design makes DoPA particularly suitable for applications requiring high-power ultrashort pulses, especially where system size and complexity are critical factors. Finally, we outlined potential experimental realizations of the DoPA system in both the microwave regime and the optical regime.

K.D.K. performed the bulk of the work. A.B. assisted with the calculations. C.C. supervised the work. All authors accepted responsibility for the entire content of the manuscript and approved its submission.

The authors declare that they have no competing interests.

DATA AVAILABILITY

The data supporting this study's findings are available within the article.

APPENDIX A: SPACE-TIME WEDGE SCATTERING SOLUTION

1. Scattering solution

The scattering solution E_w for an electromagnetic wave inside a space-time perfect electric conductor wedge with both interfaces moving at a constant normalized velocity β is a particular case of the expressions in Eq. (5) in Ref. [36], with $v_1 = \mp\beta c$, $v_2 = \pm\beta c$, and $u = c$, where the top sign refers to an opening space-time wedge and the bottom sign refers to a closing wedge. The initial positions of the interfaces are denoted by $z_{w,1}$ and $z_{w,2}$. The solution is a superposition of forward and backward waves, and is given by

$$E_w = \sum_{p=2k} R^p E_i \left[\underbrace{\Delta\phi_{k-1}^- + R^p \left(\frac{z}{c} - t \right)}_{\Phi_p} - \sum_{p=2k+1} R^p E_i \left[\Delta\phi_k^+ - R^p \left(\frac{z}{c} + t \right) \right] \right], \quad (\text{A1})$$

where E_i is the incident wave,

$$R = \frac{1 \mp \beta}{1 \pm \beta}, \quad (\text{A2})$$

and p denotes the number of scattering events. If p is even, the solution represents a forward wave, while if p is odd, it represents a backward wave. The phase shifts in Eq. (A1) are defined as

$$\begin{aligned} \Delta\phi_k^+ &= A - R \frac{1 - R^{2k}}{1 - R^2} B, \\ \Delta\phi_k^- &= \Delta\phi_k^+ - R^{2k+1} C, \end{aligned} \quad (\text{A3})$$

where

$$\begin{aligned} A &= (1 + R) \frac{z_{w,2}}{c} - R\phi_0^- - \phi_0^+, \\ B &= (1 + R) \frac{z_{w,1} - Rz_{w,2}}{c} - (1 - R^2) \phi_0^-, \\ C &= (1 + R) \frac{z_{w,1}}{c} - R\phi_0^+ - \phi_0^-. \end{aligned} \quad (\text{A4})$$

The initial phase shifts are defined as

$$\phi_0^\pm = \frac{z_0}{c} \mp t_0. \quad (\text{A5})$$

2. Doppler shift

The shifted frequency ω_p after p bounces may be calculated by taking a time derivative of the phase argument in Eq. (A1):

$$\omega_p = \frac{\partial \Phi_p}{\partial t} \omega_0 = R^p \omega_0. \quad (\text{A6})$$

3. Compansion factor

In addition to the frequency Doppler shift [Eq. (A6)], the temporal envelope of the pulse also undergoes modification. Because of the Fourier limit, the width of the optical pulse changes by a factor R^{-p} . Thus, the compansion factor $a = \tau_p/\tau_0$, defined as the ratio of the width τ_p of the reflected wave and the width τ_0 of the incident wave is given by

$$a = \frac{\tau_p}{\tau_0} = R^{-p}. \quad (\text{A7})$$

APPENDIX B: DEVICE SIZE FOR A GIVEN NUMBER OF SPACE-TIME REFLECTIONS

1. Wedge profile

In this section, we determine the minimal size of the wedge profile (Fig. 2) required for the compansion factor [Eq. (2)] to equal a after p bounces. We perform the analysis for the pulse expansion (first step in DoPA, Fig. 1), i.e., an opening space-time wedge [Fig. 2(a)]. The two edges of the space-time wedge may be parametrized as

$$\begin{aligned} z_L[t] &= -\beta ct - z_{w,0}, \\ z_R[t] &= \beta ct + z_{w,0}, \end{aligned} \quad (\text{B1})$$

where we assume a symmetric setup around the ct axis and $z_L[t]$ represents the left edge and $z_R[t]$ represents the right edge of the wedge. To ensure that the initial pulse can pass through the space-time aperture of the wedge, we require that the initial width of the wedge, given by $z_R[0] - z_L[0] = 2z_{w,0}$, must be at least as large as twice the initial width of the incident pulse. This leads to the condition

$$z_{w,0} \geq c\tau_0, \quad (\text{B2})$$

where we assume equality for minimal device size. The trajectory of the reflected electromagnetic pulse after i bounces is parametrized as

$$z_i[t] = (-1)^i c(t - t_i) + z_i, \quad (\text{B3})$$

where t_i and z_i are the scattering time and position respectively (Fig. 2), with $t_0 = 0 = z_0$. When i is even, Eq. (B3) represents a forward wave, and when i is odd, it represents a backward wave [Eq. (A1)].

We are seeking the location of the p th bounce, i.e., $z_p = z_{p-1}[t_p]$, and we proceed iteratively. The position z_1 of the first reflection is the intersection of the world line of the incident wave [Eq. (B3)] and the world line of the right edge of the wedge [Eq. (B1)], i.e., $z_0[t_1] = z_R[t_1]$, which

can be solved for t_1 as

$$t_1 = \frac{1}{1 - \beta} \tau_0. \quad (\text{B4})$$

The position of the first space-time reflection is then given by

$$z_1 = z_R[t_1] = \frac{1}{1 - \beta} c\tau_0. \quad (\text{B5})$$

The position z_2 of the second reflection is the intersection of the world line of the first reflected wave [Eq. (B3)], with the first scattering time t_1 [Eq. (B4)] and position z_1 [Eq. (B5)] and the left leg of the wedge interface [Eq. (B1)], i.e., $z_1[t_2] = z_L[t_2]$, which can be solved for t_2 , yielding

$$t_2 = \frac{3 - \beta}{(1 - \beta)^2} \tau_0. \quad (\text{B6})$$

Hence, the second scattering event position is given by

$$z_2 = z_L[t_2] = -\frac{1 + \beta}{(1 - \beta)^2} c\tau_0. \quad (\text{B7})$$

This process repeats. By induction, we find that the time and position of the p th bounce are given by

$$\begin{aligned} t_p &= \frac{1}{\beta} \left(\frac{a}{1 + \beta} - 1 \right) \tau_0, \\ z_p &= (-1)^p \frac{a}{1 + \beta} c\tau_0, \end{aligned} \quad (\text{B8})$$

where the compansion factor is given by Eq. (2). Equation (B8) refers to the scattering time and position of the *center* of the pulse. At this point, the wave has already expanded to a temporal width $a\tau_0$, and thus the final part of the expanded wave extends $a\tau_0/2$ above the center. Consequently, the last part of the wave reaches the wedge at time $t_p + a\tau_0/2$ and needs to be taken into account. Therefore, we compute $z_{L,R}[t_p + a\tau_0/2]$ using Eq. (B1), viz.,

$$z_{L,R}\left[t_p + \frac{a}{2}\tau_0\right] = \frac{(-1)^p}{2} \left(\frac{2}{1 + \beta} + \beta \right) ac\tau_0. \quad (\text{B9})$$

The device size l_w is twice the value in Eq. (B9), i.e.,

$$l_w = a \left(\frac{2}{1 + \beta} + \beta \right) c\tau_0. \quad (\text{B10})$$

2. Fresnel profile

The space-time Fresnel trajectory can be parametrized as a sawtooth profile with period T and constant velocity

β . The characteristic wavelength λ_F of this profile is given by

$$\lambda_F = \beta cT. \quad (\text{B11})$$

The total device length l_F consists of two contributions: (1) twice the distance traveled by the interface in a single oscillation [Eq. (B11)] and (2) the fixed separation between the two edges. Here we assume this separation to be the same as in the space-time wedge profile [Eq. (B2)]. This leads to

$$l_F = 2\beta cT + 2c\tau_0. \quad (\text{B12})$$

To ensure that the expanded wave remains confined within a single oscillation and does not interact with the sharp edges, we impose the condition

$$T \geq a\tau_0. \quad (\text{B13})$$

Substituting this constraint into Eq. (B12), we obtain the final expression for the device length of the Fresnel trajectory profile:

$$l_F = 2(1 + a\beta)c\tau_0. \quad (\text{B14})$$

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