

Engineering propagating cat states with driven four-level systems inside a cavity

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We propose a method for generating optical cat states in propagating pulses based on cavity quantum electrodynamics (QED). This scheme uses multiple four-level systems (FLSs) inside an optical cavity as a light source. Time-modulating driving stimulates it to produce a superposition of coherent states entangled with the FLSs. The postselection of an appropriate state of the FLSs leads to a multicomponent cat state in a propagating pulse. Taking atomic decay and cavity loss into account, we optimize the cavity external coupling rate to maximize the fidelity. We find that its optimum value is formulated similarly to those of other generation methods for propagating states, suggesting a universal property of cavity-QED systems interacting with fields outside the cavity.

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I. INTRODUCTION

Propagating nonclassical light is one of the promising systems for quantum information processing. The propagating optical state has several advantages: high scalability using time- or frequency-domain multiplexing methods [1–3] and high feasibility for long-distance quantum communication [4]. To generate nonclassical light with “Wigner negativity” [5], two exemplary methods are known. The first is the use of quantum entanglement and measurements of light [6,7]. The second is based on the nonlinear interaction between matter and light.

Whereas the former method will need photon number counting to generate complex optical states, the latter does not require such measurements. Notably, a nonlinear system inside an optical cavity, which is treated in cavity quantum electrodynamics (QED), is a promising system for generating nonclassical light because it can harness the strong interaction between matter and light. Generating nonclassical light with cavity-QED systems has been studied on both the theoretical and experimental sides. For example, single-photon states have been generated by various systems, such as neutral atoms [8,9], ions [10,11], and defects in solids [12,13], and

also a method for generating N -photon states has been proposed [14]. These discrete-variable (DV) states are generated by driving a nonlinear system inside the cavity. In contrast, continuous-variable (CV) states, such as Schrödinger cat states (superpositions of coherent states) and Gottesman-Kitaev-Preskill states [15], can be generated by reflecting an optical state off the cavity-QED systems [16–18]. However, in these schemes, (nonclassical) seed light must repeatedly pass through circulators and reflect off the cavity, which introduces additional losses, pulse distortions, and complications of the setup, and thus it will be difficult to generate the desired states with high fidelity.

In this work, we propose a protocol for generating cat states by driving four-level systems (FLSs) inside a cavity. Our protocol generates entanglement between the FLSs and a superposition of coherent states in a propagating pulse output from the cavity. If we use a single FLS and measure it after driving, we can generate a Schrödinger cat state, namely a superposition of two coherent states $|\alpha\rangle \pm |-\alpha\rangle$. Moreover, by driving and measuring two FLSs and postselecting the measurement result, we can generate a four-component cat state $|\beta\rangle + |-\beta\rangle + |i\beta\rangle + |-i\beta\rangle$. Thus, our protocol generates the representative CV states, which are practical resources for optical quantum information processing [19–23], without circulators or repeating reflection, thereby preventing additional optical loss. This further enables controlling the wave packet of emitted quantum light, similar to the conventional driving methods for DV states [14,24,25], which plays a crucial role in high-fidelity interference [26] and temporal mode encoding [2,3,27].

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To maximize the fidelity, we optimize the cavity external coupling rate and find that its optimum value and maximized fidelity are formulated similarly to those of the other generation methods for propagating states [18,28]. These results not only elucidate the potential of our protocol but also suggest a universal property of cavity-QED systems interacting with fields outside the cavity.

II. SCHRÖDINGER CAT ENTANGLED STATES WITH A SINGLE FOUR-LEVEL SYSTEM

We consider an FLS with two ground and excited states inside a one-sided optical cavity, where two optical transitions couple with a single cavity mode $\hat{c}$ and the two other diagonal transitions are driven by σ_+ - and σ_- -polarized laser fields (see Fig. 1). The Hamiltonian of the FLS in the rotating-wave approximation and a proper rotating frame is given by [29]($\hbar = 1$)

$$\begin{aligned} \hat{H}_{\text{FLS}} = & \left(\Delta - \frac{\delta_e}{2} \right) |e_1\rangle\langle e_1| + \left(\Delta + \frac{\delta_e}{2} - \delta_g \right) |e_2\rangle\langle e_2| \\ & + [\Omega(t)(|e_2\rangle\langle \downarrow| + |e_1\rangle\langle \uparrow|) \\ & + g(|e_1\rangle\langle \downarrow| + |e_2\rangle\langle \uparrow|)\hat{c} + \text{h.c.}]. \end{aligned} \quad (1)$$

Here, δ_g (δ_e) is the frequency difference between the ground spin (excited) states, $\Delta = \omega_e - \omega_c$ is the detuning between the optical transition frequency and the cavity frequency, and g is the cavity coupling strength. The frequency of the laser field that couples $|\downarrow\rangle - |e_2\rangle$ ($|\uparrow\rangle - |e_1\rangle$) is $\omega_c + \delta_g$ ($\omega_c - \delta_g$). For a sufficiently large $|\Delta|$, the excited states can be adiabatically eliminated (see below for the validity of this), leading to the simplified effective Hamiltonian (see Appendix A 1, as well as Refs. [29–31])

$$\hat{H}_{\text{FLS}}^{\text{eff}} = \omega_0 \hat{c}^\dagger \hat{c} + \hat{\sigma}_x [\lambda^*(t) \hat{c} + \lambda(t) \hat{c}^\dagger], \quad (2)$$

where we have dropped δ_g and δ_e by assuming $|\delta_g|, |\delta_e| \ll |\Delta|$ and have introduced $\hat{\sigma}_x = |\uparrow\rangle\langle \downarrow| + |\downarrow\rangle\langle \uparrow|$,

$$\omega_0 = -\frac{g^2}{\Delta}, \quad \lambda(t) = -\frac{g\Omega(t)}{\Delta}. \quad (3)$$

Note that such a simplified model was originally proposed for mimicking the Dicke model [30] and demonstrated by using ^{87}Rb atoms within an optical cavity [32].

To present the essence of our protocol, we first neglect the atomic decay. The cavity mode is coupled to a desired output mode at rate $2\kappa_{\text{ex}}$ and an unwanted mode at rate $2\kappa_{\text{in}}$, which corresponds to internal loss and scattering at the mirrors. The FLS is initially in the spin manifold $\{|\downarrow\rangle, |\uparrow\rangle\}$, and both of the cavity and output modes are in vacuum states. We use the input-output theory [33] to formulate the dynamics. From the commutation relation $[\hat{\sigma}_x, \hat{H}_{\text{FLS}}^{\text{eff}}] = 0$, $\hat{\sigma}_x(t)$ is time-independent and can be

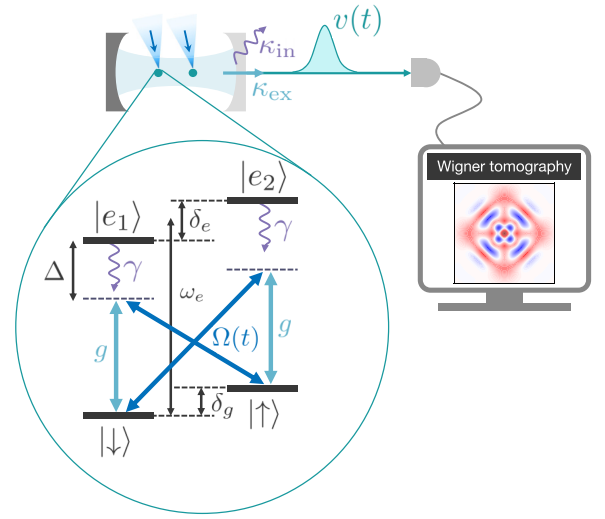


FIG. 1. Schematic of the proposed protocol.

denoted simply by $\hat{\sigma}_x(0)$. For $\hat{c}(t)$, we obtain

$$\dot{\hat{c}} = -(i\omega_0 + \kappa)\hat{c} - i\lambda(t)\hat{\sigma}_x(0), \quad (4)$$

where the dot denotes the time derivative, and $\kappa = \kappa_{\text{ex}} + \kappa_{\text{in}}$. Now, we choose the Rabi frequency $\Omega(t) = -\lambda(t)\Delta/g$ with

$$\lambda(t) = \frac{i\alpha}{\sqrt{2\kappa_{\text{ex}}}} [\dot{v}(t) + (i\omega_0 + \kappa)v(t)], \quad (5)$$

where α is a complex number and $v(t)$ is an envelope function satisfying $\int_0^\infty dt |v(t)|^2 = 1$ and $v(0) = \lim_{t \rightarrow \infty} v(t) = 0$. This choice gives the cavity mode as

$$\hat{c}(t) = \frac{v(t)}{\sqrt{2\kappa_{\text{ex}}}} \alpha \hat{\sigma}_x(0), \quad (6)$$

and we find that the desired output mode is given as $\hat{a}_{\text{out}}(t) = v(t)\alpha\hat{\sigma}_x(0)$, by using the input-output relation $\hat{a}_{\text{out}}(t) = \sqrt{2\kappa_{\text{ex}}}\hat{c}(t)$. Thus, after driving the FLS, the system emits a coherent state with the envelope function $v(t)$ whose sign of the amplitude depends on the initial spin state of the FLS. In the ideal case where $\kappa_{\text{in}} = 0$, this means that we can generate the entangled state $(|+\rangle|\alpha; v\rangle_{\text{ex}} + |-\rangle|-\alpha; v\rangle_{\text{ex}})/\sqrt{2}$, which is referred to as the Schrödinger cat entangled state, by preparing the initial spin state in $|\downarrow\rangle$. Here, $|\pm\rangle = (|\downarrow\rangle \pm |\uparrow\rangle)/\sqrt{2}$ are the eigenstates of $\hat{\sigma}_x$, and $|\alpha; v\rangle_{\text{ex}}$ is a coherent state with an envelope function $v(t)$ in the desired output mode. Measuring the spin state in $|\downarrow\rangle$ ($|\uparrow\rangle$) projects the light onto the even (odd) cat states $(|\alpha; v\rangle_{\text{ex}} + |-\alpha; v\rangle_{\text{ex}})/\mathcal{N}_+$, $[(|\alpha; v\rangle_{\text{ex}} - |-\alpha; v\rangle_{\text{ex}})/\mathcal{N}_-]$, where $\mathcal{N}_\pm = \sqrt{2(1 \pm e^{-2|\alpha|^2})}$ are normalization factors.

Given the desired amplitude α and envelope $v(t)$, applying classical driving given by Eq. (5) generates the desired

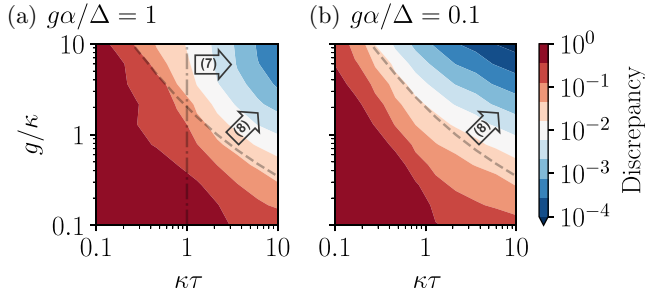


FIG. 2. Discrepancy between the full model $\hat{\rho}^{\text{full}}$ and the effective model $\hat{\rho}^{\text{eff}}$. The color plots show the average infidelity $1 - \int_0^T dt F(\hat{\rho}^{\text{full}}(t), \hat{\rho}^{\text{eff}}(t))/T$ as a function of g/κ and $\kappa\tau$ (see Appendix A 2 for the details). The initial spin state is $|+\rangle$, and the parameters are $\kappa = 100\gamma$, $T = 10\tau$, and $\alpha = 1$, with $\Delta = g\alpha$ in (a) and $\Delta = 10g\alpha$ in (b). The dash-dotted (dashed) lines show where the left-hand side is equal to the right-hand side of Eq. (7) [Eq. (8)], where the dash-dotted line in (b) lies outside the plotted parameter range.

entangled state if the system is described by the effective model (2), which is derived from the full model (1) via the adiabatic elimination. However, increasing the amplitude $|\alpha|$ or shortening the pulse $v(t)$ can invalidate the adiabatic elimination as follows. For simplicity, we set the pulse $v(t)$ as the Gaussian function with standard deviation τ . The adiabatic elimination requires $|\Delta| \gg g\sqrt{\langle \hat{c}^\dagger(t)\hat{c}(t) \rangle}$, $\Omega(t)$, yielding the following conditions (see Appendix A 2 for the details):

$$\kappa\tau \gg \left| \frac{g\alpha}{\Delta} \right|^2, \quad (7)$$

$$\frac{g^2}{\kappa}\tau \left(1 + \frac{1}{\kappa\tau} \right)^{-2} \gg |\alpha|^2. \quad (8)$$

Figure 2 shows the discrepancy between the full model and the effective model, where we have taken a small atomic decay rate γ into account, supporting the validity of the above conditions. In the following, to safely employ the effective model, we assume

$$|\Delta| \gg g|\alpha|, \quad \frac{g^2}{\kappa}\tau \gg |\alpha|^2, \quad \kappa\tau \gg 1. \quad (9)$$

The fidelity of the protocol is limited by two quantum jumps: the atomic spontaneous decay and the cavity decay to the unwanted mode. The event with no quantum jumps, which is described by a non-Hermitian Schrödinger equation [34], yields the unnormalized pure state $|\Phi\rangle$, where the decrease of the norm corresponds to the total jump probability, $P_j = 1 - \langle \Phi | \Phi \rangle$. The final density operator $\hat{\rho}$ can be decomposed as $\hat{\rho} = |\Phi\rangle\langle\Phi| + P_j\hat{\rho}_{\text{jumps}}$, where $\hat{\rho}_{\text{jumps}}$ is the state yielded by the event with one or more

jumps. Thus, the state $|\Phi\rangle$ gives the lower bound of the fidelity to the ideal final state $|\psi_{\text{id}}\rangle$ as follows:

$$F = \langle \psi_{\text{id}} | \hat{\rho} | \psi_{\text{id}} \rangle \geq |\langle \psi_{\text{id}} | \Phi \rangle|^2 =: F_{\text{min}}. \quad (10)$$

We further find that $F - F_{\text{min}} = P_j \langle \psi_{\text{id}} | \hat{\rho}_{\text{jumps}} | \psi_{\text{id}} \rangle \leq P_j$, meaning that F can be well approximated by F_{min} for small P_j , which is required for high fidelity.

To elucidate how the fidelity depends on the system parameters, we analytically derive $|\Phi\rangle$. To this end, we assume that classical driving is the dominant source of the atomic excitation, which should be valid for $|\Delta| \gg g^2/\kappa$ ensuring $\Omega(t) \gg g\sqrt{\langle \hat{c}^\dagger(t)\hat{c}(t) \rangle}$ (see Appendix B 1). This leads to $|\Phi\rangle \simeq \sqrt{1 - P_j} |\psi_{\text{id}}\rangle$ and $P_j \simeq 1 - (1 - P_\gamma)(1 - P_{\kappa_{\text{in}}})$, where P_γ and $P_{\kappa_{\text{in}}}$ are the atomic and the intracavity decay probability respectively, resulting in $F_{\text{min}} \simeq (1 - P_\gamma)(1 - P_{\kappa_{\text{in}}})$. Since the population of excited states and the cavity photon are approximately given by $|\Omega(t)/\Delta|^2$ and $|\alpha v(t)/\sqrt{2\kappa_{\text{ex}}}|^2$ respectively, we find

$$\begin{aligned} P_\gamma &\simeq 1 - \exp \left[-2\gamma \int_0^\infty dt \left| \frac{\Omega(t)}{\Delta} \right|^2 \right] \\ &\simeq 1 - \exp \left(-\frac{\kappa}{\kappa_{\text{ex}}} \frac{|\alpha|^2}{2C} \right), \end{aligned} \quad (11)$$

where $C = g^2/(2\gamma\kappa)$ is the cooperativity, and

$$\begin{aligned} P_{\kappa_{\text{in}}} &\simeq 1 - \exp \left[-2\kappa_{\text{in}} \int_0^\infty dt \left| \frac{\alpha v(t)}{\sqrt{2\kappa_{\text{ex}}}} \right|^2 \right] \\ &= 1 - e^{-(\kappa_{\text{in}}/\kappa_{\text{ex}})|\alpha|^2}. \end{aligned} \quad (12)$$

Thus, we obtain

$$F_{\text{min}} \simeq e^{-(1/M-1)|\alpha|^2}, \quad (13)$$

where

$$M = \frac{\kappa_{\text{ex}}}{\kappa} \frac{2C}{1 + 2C} (\leq 1), \quad (14)$$

characterizing the system performance.

Given $(g, \gamma, \kappa_{\text{in}})$, we can optimize the external coupling rate κ_{ex} to maximize the fidelity, by tuning the transmittance of the output coupler. From the analytical result (13), the optimal κ_{ex} to maximize F_{min} is given by

$$\kappa_{\text{ex}} = \kappa_{\text{in}} \sqrt{1 + 2C_{\text{in}}}, \quad (15)$$

where $C_{\text{in}} = g^2/(2\gamma\kappa_{\text{in}})$ is known as the internal cooperativity [28], leading to

$$M = \frac{\sqrt{1 + 2C_{\text{in}}} - 1}{\sqrt{1 + 2C_{\text{in}}} + 1}, \quad (16)$$

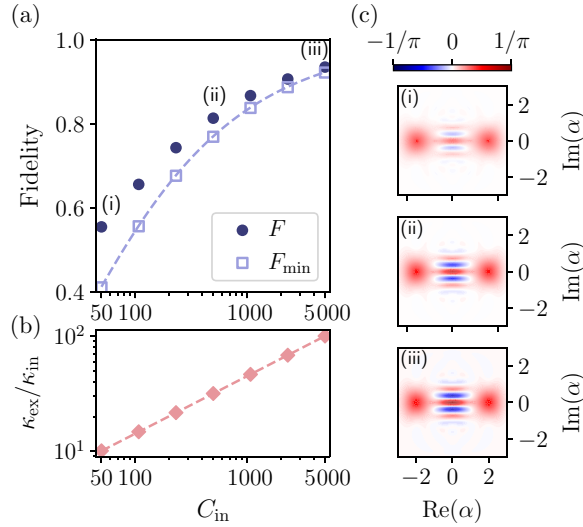


FIG. 3. Generation of the Schrödinger cat entangled state with $\alpha = 2$. The system parameters are $(\Delta, g) = (200, 10)\gamma$, and the branching ratio of the atomic decay into the spin states is 1 : 1. (a) Fidelity F and F_{min} as functions of C_{in} , where κ_{ex} is numerically optimized to maximize F_{min} , by simulating the effective model for efficient optimization (see Appendix C). The dashed line plots Eq. (17). (b) Numerically optimized κ_{ex} . The dashed line plots $\kappa_{ex}/\kappa_{in} = \sqrt{1 + 2C_{in}}$ [Eq. (15)]. (c) Wigner functions of the output field, postselected on $|\downarrow\rangle$, for $C_{in} = 50, 500$, and 5000 in (i), (ii), and (iii), respectively.

$$F_{min} = \exp\left(-\frac{2|\alpha|^2}{\sqrt{1 + 2C_{in}} - 1}\right). \quad (17)$$

To confirm the above analytical results, we numerically optimize κ_{ex} to maximize $F_{min} = |\langle\psi_{id}|\Phi\rangle|^2$. The numerical results for κ_{ex} and F_{min} are in good agreement with the analytical expressions (15) and (17), respectively [see Figs. 3(a) and 3(b)]. We also confirm that the lower bound F_{min} approaches the fidelity F as F increases. We note that κ_{ex} can be optimized to maximize the total fidelity F , which depends on the branching ratio of the atomic decay, once a specific physical system for realizing the FLS is identified. In such optimization, the analytical expression (15) can be used as a good initial guess. Figure 3(c) shows the Wigner functions of the propagating state after postselecting the FLS in $|\downarrow\rangle$, which is expected to be the even cat state. We find two circular spots centered around $\alpha = \pm 2$ and interference fringes between them, distinguishing the state from the classical mixture of $|\alpha = \pm 2\rangle$, even when the fidelity F is below 0.6 [Fig. 3(c)(i)].

The analytical formula (17) indicates that C_{in} should determine the protocol performance, given the desired α . Intriguingly, we can see C_{in} as the performance measure in various protocols based on cavity QED, including the generation of single photons in a driving manner and cat states by a reflection manner [17, 18, 28, 35], as discussed below.

We have shown that the system performance in our protocol is characterized by M (14), which is consistent with the reflection-based cat generation, where complicated setups will incur additional optical loss [17]. The measure M further determines the performance limit of the single-photon generation [28]. Thus, M serves as a performance measure of cavity-QED systems at least in the three cases. We have further discussed that M can be maximized by tuning κ_{ex} as Eq. (15), resulting in the value (16) being solely determined by C_{in} . We note that the performance limit of an adiabatic state transfer between two atoms inside a cavity, with $\kappa_{ex} = 0$, also depends only on C_{in} [36]. These results suggest that C_{in} serves as a universal figure of merit for cavity-QED systems, offering a clear guideline for achieving high-fidelity operations—that is, attaining high internal cooperativity.

III. FOUR-COMPONENT CAT STATES WITH TWO FOUR-LEVEL SYSTEMS

Adding another FLS inside the cavity allows for generating more complex entangled states. We individually apply laser fields to the FLS j ($j = 1, 2$) with the time-dependent Rabi frequency $\Omega_j(t) = -\lambda_j(t)\Delta/g$, where

$$\lambda_j(t) = \frac{i\alpha_j}{\sqrt{2\kappa_{ex}}} [\dot{v}(t) + (2i\omega_0 + \kappa)v(t)]. \quad (18)$$

By driving two FLSs initially prepared in $|\downarrow\rangle_1 |\downarrow\rangle_2$ with $\alpha_1 = \beta e^{i\pi/4}/\sqrt{2}$ and $\alpha_2 = \beta e^{-i\pi/4}/\sqrt{2}$, we can generate the entangled state

$$\frac{1}{2} (|+\rangle_1 |+\rangle_2 |\beta; v\rangle_{ex} + |+\rangle_1 |-\rangle_2 |i\beta; v\rangle_{ex} + |-\rangle_1 |+\rangle_2 |-i\beta; v\rangle_{ex} + |-\rangle_1 |-\rangle_2 |-\beta; v\rangle_{ex}). \quad (19)$$

Postselected on $|\downarrow\rangle_1 |\downarrow\rangle_2$, the propagating light becomes the four-component cat state $(|\beta; v\rangle_{ex} + |-\beta; v\rangle_{ex} + |i\beta; v\rangle_{ex} + |-i\beta; v\rangle_{ex})/N_\beta$ where N_β is a normalization factor. This state has various applications in optical quantum computation and quantum communication [20–23]; however, no study has yet succeeded in generating such a state in the optical domain, possibly due to the demanding setups of proposed generation methods [37–39].

Similarly to the inequalities (9) in Sec. II, we assume

$$|\Delta| \gg g \sum_j |\alpha_j|, \quad \frac{g^2}{\kappa} \tau \gg \sum_j |\alpha_j|^2, \quad \kappa \tau \gg 1, \quad (20)$$

for the adiabatic elimination (see Appendix A 2). Moreover, we again assume that classical driving is the primary

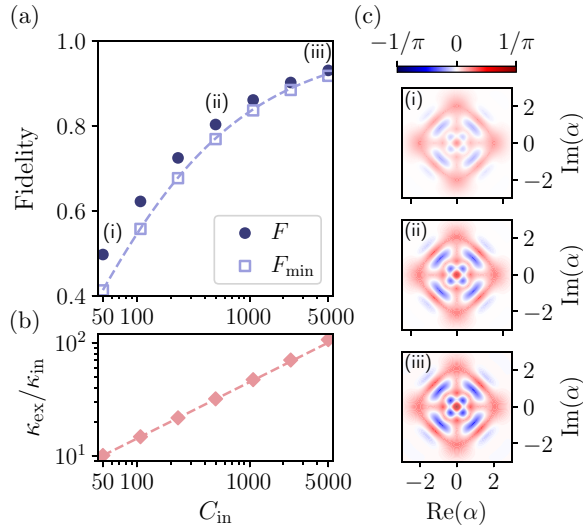


FIG. 4. Generation of the four-component cat entangled state with $\beta = 2$. The system parameters are $(\Delta, g) = (400, 10)\gamma$, and the branching ratio of the atomic decay into the spin states is 1 : 1. (a) Fidelity F and F_{min} as functions of C_{in} , where κ_{ex} is numerically optimized to maximize F_{min} . The dashed line plots Eq. (17), with α being replaced by β . (b) Numerically optimized κ_{ex} . The dashed line plots $\kappa_{ex}/\kappa_{in} = \sqrt{1 + 2C_{in}}$ [Eq. (15)]. (c) Wigner functions of the output field, postselected on $|\downarrow\rangle_1 |\downarrow\rangle_2$, for $C_{in} = 50, 500$, and 5000 in (i), (ii), and (iii), respectively.

source of atomic excitation and find

$$F_{min}^{(2)} \simeq (1 - P_{\gamma}^{(2)})(1 - P_{\kappa_{in}}^{(2)}) \simeq e^{-(1/M-1)|\beta|^2}, \quad (21)$$

taking the same form as that in the two-component cat case (13) (see Appendix B2 for the derivation). Thus, the optimal κ_{ex} to maximize $F_{min}^{(2)}$ is given by Eq. (15). As in Sec. II, we find that the above analytical results are still in good agreement with the numerical results, as shown in Figs. 4(a) and 4(b). We observe the characteristic structure of the four-component cat in the phase space, which is postselected on $|\downarrow\rangle_1 |\downarrow\rangle_2$, as shown in Fig. 4(c). The four-component cat state, known as a compass state in the context of quantum sensing [40,41], is expected to outperform coherent states in detecting phase-space displacements, even when its fidelity is below 0.5, as indicated by evaluations of the quantum Fisher information [37]. Thus, our protocol has the potential to support such applications, without demanding internal cooperativity.

IV. CONCLUSION

We have proposed a protocol to generate propagating cat states by driving FLSs inside a cavity. Using a single FLS, we can deterministically generate a Schrödinger

cat entangled state. This has applications in quantum communication and even the potential to implement resource-efficient and high-threshold fault-tolerant quantum computation harnessing DV-CV hybrid qubits [42–44]. Moreover, introducing the postselection of the spin state enables the generation of an arbitrary superposition of coherent states, $\mu |\alpha\rangle + \nu |-\alpha\rangle$, which is useful for coherent state quantum computation (CSQC) and quantum error correction with cat codes [19,22]. For CSQC whose optimal amplitude is $\alpha \sim 1.5$ [19], $C_{in} \sim 10^3$ is required to generate resource states with fidelity around 0.9. Although such high internal cooperativity may be challenging, cavity designs that achieve internal cooperativity exceeding 10^3 have been proposed and even realized experimentally [45–47], making the generation of optimal coherent states feasible in the near future.

Using two FLSs further enables the generation of a four-component cat state, which is a practical resource for quantum metrology [40,41] and quantum information processing [20–23]. The Schrödinger cat entangled state has already been generated based on cavity QED, using the reflection scheme proposed by Wang and Duan [16,17]. In principle, a four-component cat state can be generated by repeated use of the reflection scheme and postselection twice. In this case, however, the propagating Schrödinger cat state, which is fragile to optical loss, must pass through a circulator and reflect off a cavity with nonzero loss, which was 19% in a previous experiment [17]. Thus, the high-fidelity generation of a four-component cat state based on the reflection method may not be feasible. Our protocol can generate it without circulators and multiple reflections, and hence it will be a more promising method to generate complex CV states.

Finally, we note that our protocol can generate entanglement between N FLSs and 2^N coherent states (see Appendix B2). Thus, combining linear optics or squeezers with our protocol would provide a method to generate more complex and useful CV states with high fidelity.

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DATA AVAILABILITY

The data that support the findings of this article are not publicly available upon publication because it is not technically feasible and/or the cost of preparing, depositing, and hosting the data would be prohibitive within the terms of this research project. The data are available from the authors upon reasonable request.

APPENDIX A: SYSTEM MODELS AND EFFECTIVE OPERATOR FORMALISM

The Hamiltonian of N FLSs under the rotating-wave approximation is given by [29,30]

$$\begin{aligned}\hat{H}_{\text{FLSs}}^{(\text{rw})} &= \omega_c \hat{c}^\dagger \hat{c} + \sum_{j=1}^N \left(\hat{H}_{\text{atom},j} + \hat{H}_{\text{drive},j} \right), \\ \hat{H}_{\text{atom},j} &= \delta_g |\uparrow\rangle_j \langle \uparrow| \\ &\quad + \left(\omega_e - \frac{\delta_e}{2} \right) |e_1\rangle_j \langle e_1| + \left(\omega_e + \frac{\delta_e}{2} \right) |e_2\rangle_j \langle e_2|, \\ \hat{H}_{\text{drive},j} &= \Omega_j(t) (e^{-i\omega_+ t} |e_2\rangle_j \langle \downarrow| + e^{-i\omega_- t} |e_1\rangle_j \langle \uparrow|) + \text{h.c.} \\ &\quad + g(|e_1\rangle_j \langle \downarrow| + |e_2\rangle_j \langle \uparrow|) \hat{c} + \text{h.c.},\end{aligned}\quad (\text{A1})$$

where ω_c and $\omega_\pm$ are respectively the frequency of the cavity field and the $\sigma_\pm$ -polarized laser. In the rotating frame of the unitary operator $\hat{U}_f = e^{-i\hat{H}_f t}$ with

$$\begin{aligned}\hat{H}_f &= \omega_c \hat{c}^\dagger \hat{c} \\ &\quad + \sum_{j=1}^N \left[\delta_g |\uparrow\rangle_j \langle \uparrow| + \omega_c |e_1\rangle_j \langle e_1| + (\omega_c + \delta_g) |e_2\rangle_j \langle e_2| \right],\end{aligned}\quad (\text{A2})$$

we find the transformed Hamiltonian

$$\begin{aligned}\hat{H}_{\text{FLSs}} &= \hat{U}_f^\dagger \hat{H}_{\text{FLSs}}^{(\text{rw})} \hat{U}_f + i \left(\frac{d\hat{U}_f^\dagger}{dt} \right) \hat{U}_f = \sum_{j=1}^N \hat{H}_{\text{FLS},j}, \\ \hat{H}_{\text{FLS},j} &= \hat{H}_{e,j} + \hat{V}_j(t) + \hat{V}_j^\dagger(t), \\ \hat{H}_{e,j} &= \left(\Delta - \frac{\delta_e}{2} \right) |e_1\rangle_j \langle e_1| + \left(\Delta + \frac{\delta_e}{2} - \delta_g \right) |e_2\rangle_j \langle e_2|, \\ \hat{V}_j(t) &= \Omega(t) (|e_2\rangle_j \langle \downarrow| + |e_1\rangle_j \langle \uparrow|) \\ &\quad + g(|e_1\rangle_j \langle \downarrow| + |e_2\rangle_j \langle \uparrow|) \hat{c},\end{aligned}\quad (\text{A3})$$

where we have tuned the laser frequencies as $\omega_\pm = \omega_c \pm \delta_g$, and $\Delta = \omega_e - \omega_c$. For $N = 1$, we just denote $\hat{H}_{\text{FLSs}}$ as $\hat{H}_{\text{FLS}}$, which is described in Eq. (1).

1. Derivation of the effective operators

We first consider the case with $N = 1$ for brevity. The atomic decays are described by the Lindblad operators as follows:

$$\begin{aligned}\hat{L}_2 &= \sqrt{2r_1\gamma} |\downarrow\rangle \langle e_1|, \quad \hat{L}_3 = \sqrt{2(1-r_1)\gamma} |\uparrow\rangle \langle e_1|, \\ \hat{L}_4 &= \sqrt{2r_2\gamma} |\uparrow\rangle \langle e_2|, \quad \hat{L}_5 = \sqrt{2(1-r_2)\gamma} |\downarrow\rangle \langle e_2|,\end{aligned}\quad (\text{A4})$$

where r_1 and r_2 are branching ratios, which are set as $r_1 = r_2 = 0.5$ when getting numerical results. For the

case where excited states $\{|e_1\rangle, |e_2\rangle\}$ quickly reach steady states, we can adiabatically eliminate excited states. Using a formalism originally proposed by Reiter and Sørensen [31], where we apply the adiabatic elimination and a perturbative treatment of $\hat{V}$, gives the effective model with the time-dependent coupling, as explained in the following [48]. First, we define a non-Hermitian Hamiltonian

$$\hat{H}_{\text{NH}} = \hat{H}_e - \frac{i}{2} \sum_{m=2}^5 \hat{L}_m^\dagger \hat{L}_m. \quad (\text{A5})$$

The effective operators are given by

$$\begin{aligned}\hat{H}_{\text{FLS}}^{\text{eff}}(t) &= -\frac{1}{2} \hat{V}^\dagger(t) [\hat{H}_{\text{NH}}^{-1} + (\hat{H}_{\text{NH}}^{-1})^\dagger] \hat{V}(t), \\ \hat{L}_m^{\text{eff}}(t) &= \hat{L}_m \hat{H}_{\text{NH}}^{-1} \hat{V}(t) \quad (m = 2, 3, 4, 5).\end{aligned}\quad (\text{A6})$$

We now assume $|\Delta| \gg |\delta_g|, |\delta_e|, \gamma$, leading to $\hat{H}_{\text{NH}}^{-1} \simeq (1/\Delta)(|e_1\rangle \langle e_1| + |e_2\rangle \langle e_2|)$, and then the effective operators can be approximated as

$$\begin{aligned}\hat{H}_{\text{FLS}}^{\text{eff}} &= \omega_0 \hat{c}^\dagger \hat{c} + \hat{\sigma}_x [\lambda^*(t) \hat{c} + \lambda(t) \hat{c}^\dagger], \\ \hat{L}_2^{\text{eff}} &= -\frac{\sqrt{2r_1\gamma}}{g} [\lambda(t) \hat{\sigma}_- + \omega_0 |\downarrow\rangle \langle \downarrow| \hat{c}], \\ \hat{L}_3^{\text{eff}} &= -\frac{\sqrt{2(1-r_1)\gamma}}{g} [\lambda(t) |\uparrow\rangle \langle \uparrow| + \omega_0 \hat{\sigma}_+ \hat{c}], \\ \hat{L}_4^{\text{eff}} &= -\frac{\sqrt{2r_2\gamma}}{g} [\lambda(t) \hat{\sigma}_+ + \omega_0 |\uparrow\rangle \langle \uparrow| \hat{c}], \\ \hat{L}_5^{\text{eff}} &= -\frac{\sqrt{2(1-r_2)\gamma}}{g} [\lambda(t) |\downarrow\rangle \langle \downarrow| + \omega_0 \hat{\sigma}_- \hat{c}],\end{aligned}\quad (\text{A7})$$

where we have neglected the term of the identity operator in the Hamiltonian and defined

$$\omega_0 = -\frac{g^2}{\Delta}, \quad \lambda(t) = -\frac{g\Omega(t)}{\Delta}. \quad (\text{A8})$$

Similarly, the effective Hamiltonian of N FLSs is approximately given by

$$\begin{aligned}\hat{H}_{\text{FLSs}}^{\text{eff}} &= \sum_{j=1}^N \hat{H}_{\text{FLS},j}^{\text{eff}}, \\ \hat{H}_{\text{FLS},j}^{\text{eff}} &= \omega_0 \hat{c}^\dagger \hat{c} + \hat{\sigma}_{x,j} [\lambda_j^*(t) \hat{c} + \lambda_j(t) \hat{c}^\dagger],\end{aligned}\quad (\text{A9})$$

where we have defined $\hat{\sigma}_{x,j} = |\uparrow\rangle_j \langle \downarrow| + |\downarrow\rangle_j \langle \uparrow|$ and $\lambda_j(t) = -g\Omega_j(t)/\Delta$. The effective Lindblad operators $\hat{L}_m^{j,\text{eff}}$ are given by replacing $(\hat{\sigma}_\pm, |\uparrow\rangle \langle \uparrow|, |\downarrow\rangle \langle \downarrow|)$ with $(\hat{\sigma}_{\pm,j}, |\uparrow\rangle_j \langle \downarrow|, |\downarrow\rangle_j \langle \downarrow|)$ in Eqs. (A7).

2. Validity of the effective models

We first analytically address the adiabatic elimination condition. For simplicity, we consider $\kappa_{\text{in}} = 0$ and the pulse $v(t)$ to be the Gaussian function with the standard deviation τ :

$$v(t) = \frac{1}{(\pi\tau^2)^{1/4}} e^{-t^2/2\tau^2}. \quad (\text{A10})$$

For $N = 1$, to employ the adiabatic elimination of excited states, the detuning $|\Delta|$ should be sufficiently larger than the time scale of the other dynamics:

$$|\Delta| \gg g\sqrt{\langle \hat{c}^\dagger(t)\hat{c}(t) \rangle}, |\Omega(t)|, \quad (\text{A11})$$

where the first and second terms on the right-hand side depend on $[\alpha, v(t)]$. Considering $\langle \hat{c}^\dagger(t)\hat{c}(t) \rangle \simeq |\alpha v(t)|^2/2\kappa$ because of Eq. (6), we find

$$\left| \frac{g\sqrt{\langle \hat{c}^\dagger(t)\hat{c}(t) \rangle}}{\Delta} \right|^2 \simeq \frac{1}{2\sqrt{\pi}} \left| \frac{g\alpha}{\Delta} \right|^2 \frac{1}{\kappa\tau} e^{-(t/\tau)^2} \ll 1, \quad (\text{A12})$$

leading to $\kappa\tau \gg |g\alpha/\Delta|^2$ [Eq. (7)]. For the second condition, $|\Delta| \gg |\Omega(t)|$, we find

$$\begin{aligned} \left| \frac{\Omega(t)}{\Delta} \right|^2 &= \left| \frac{\lambda(t)}{g} \right|^2 \\ &= \frac{1}{2\kappa} \left| \frac{\alpha[\dot{v}(t) + \kappa v(t)]}{g} \right|^2 + \frac{1}{2\kappa} \left| \frac{g\alpha v(t)}{\Delta} \right|^2 \ll 1, \end{aligned} \quad (\text{A13})$$

where we have used Eqs. (A8) and (5). We have found that $\kappa\tau \gg |g\alpha/\Delta|^2$ ensures the second term is sufficiently small [see Eq. (A12)]. We further evaluate the first term as follows:

$$\begin{aligned} \frac{1}{2\kappa} \left| \frac{\alpha[\dot{v}(t) + \kappa v(t)]}{g} \right|^2 &\leq \frac{1}{2\sqrt{\pi}} \frac{|\alpha|^2}{g^2\kappa\tau^3} \left[(\kappa\tau)^2 + \frac{2\kappa\tau}{\sqrt{2e}} + \frac{1}{e} \right] \\ &\leq \frac{1}{2\sqrt{\pi}} \frac{|\alpha|^2\kappa}{g^2\tau} \left(1 + \frac{1}{\kappa\tau} \right)^2. \end{aligned} \quad (\text{A14})$$

Thus, to summarize, the conditions [Eqs. (7) and (8)],

$$\kappa\tau \gg \left| \frac{g\alpha}{\Delta} \right|^2, \quad (\text{A15})$$

$$\frac{g^2}{\kappa}\tau \left(1 + \frac{1}{\kappa\tau} \right)^{-2} \gg |\alpha|^2, \quad (\text{A16})$$

guarantee the validity of the adiabatic elimination.

To confirm those conditions, we compare the effective model with the full model by numerical calculation, where we treat the output modes as the environment. When getting numerical results, we use the following Gaussian envelope:

$$v(t) = \frac{1}{(\pi\tau^2)^{1/4}} \exp \left[-\frac{1}{2} \left(\frac{t-t_0}{\tau} \right)^2 \right], \quad (\text{A17})$$

where we set t_0 to be sufficiently large to ensure $v(0) \simeq 0$. The full master equation for N FLSs and the cavity is as follows:

$$\frac{d\hat{\rho}^{\text{full}}}{dt} = -i[\hat{H}_{\text{FLSs}}, \hat{\rho}^{\text{full}}] + \mathcal{D}[\hat{L}_c]\hat{\rho}^{\text{full}} + \sum_{j=1}^N \sum_{m=2}^5 \mathcal{D}[\hat{L}_m^j]\hat{\rho}^{\text{full}}, \quad (\text{A18})$$

where $\mathcal{D}[\hat{L}]\hat{\rho} := \hat{L}\hat{\rho}\hat{L}^\dagger - \{\hat{L}^\dagger\hat{L}, \hat{\rho}\}/2$, $\hat{L}_c = \sqrt{2\kappa}\hat{c}$, and the atomic decay terms are given by

$$\begin{aligned} \hat{L}_2^j &= \sqrt{2r_1\gamma}|\downarrow\rangle_j\langle e_1|, & \hat{L}_3^j &= \sqrt{2(1-r_1)\gamma}|\uparrow\rangle_j\langle e_1|, \\ \hat{L}_4^j &= \sqrt{2r_2\gamma}|\uparrow\rangle_j\langle e_2|, & \hat{L}_5^j &= \sqrt{2(1-r_2)\gamma}|\downarrow\rangle_j\langle e_2|, \end{aligned} \quad (\text{A19})$$

In contrast, the effective master equation is given by

$$\frac{d\hat{\rho}^{\text{eff}}}{dt} = -i[\hat{H}_{\text{FLSs}}^{\text{eff}}, \hat{\rho}^{\text{eff}}] + \mathcal{D}[\hat{L}_c]\hat{\rho}^{\text{eff}} + \sum_{j=1}^N \sum_{m=2}^5 \mathcal{D}[\hat{L}_m^{j,\text{eff}}]\hat{\rho}^{\text{eff}}. \quad (\text{A20})$$

We evaluate the fidelity between the states at time t and obtain the average infidelity as

$$1 - \frac{1}{T} \int_0^T dt F(\hat{\rho}^{\text{full}}(t), \hat{\rho}^{\text{eff}}(t)), \quad (\text{A21})$$

where $F(\hat{\rho}, \hat{\sigma}) := \left(\text{Tr} \sqrt{\sqrt{\hat{\rho}}\hat{\sigma}\sqrt{\hat{\rho}}} \right)^2$. We have shown the numerical results for the case $N = 1$ in Fig. 2, supporting the validity of the conditions (A15) and (A16). In the following, to safely employ the effective model, we assume

$$|\Delta| \gg |g\alpha|, \quad \frac{g^2}{\kappa}\tau \gg |\alpha|^2, \quad \kappa\tau \gg 1. \quad (\text{A22})$$

For $N = 2$, we also numerically evaluate the discrepancy as shown in Fig. 5, indicating that the conditions

$$|\Delta| \gg g \sum_j |\alpha_j|, \quad \frac{g^2}{\kappa}\tau \gg \sum_j |\alpha_j|^2, \quad \kappa\tau \gg 1 \quad (\text{A23})$$

ensure the adiabatic elimination. For numerical calculations, we use QuTiP [49] and QuantumOptics.jl [50].

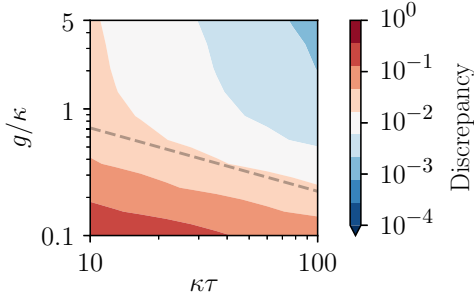


FIG. 5. Discrepancy between the full model $\hat{\rho}^{\text{full}}$ and the effective model $\hat{\rho}^{\text{eff}}$. The color plot shows the average infidelity (A21) for $N = 2$. The initial spin state is $|+\rangle_1 |+\rangle_2$, and the parameters are $\kappa = 100\gamma$, $T = 10\tau$, $\alpha_1 = e^{i\pi/4}/\sqrt{2}$, $\alpha_2 = e^{-i\pi/4}/\sqrt{2}$, and $\Delta/g = 50$. The dotted line represents $(g^2/\kappa)\tau = 25$.

APPENDIX B: PERFORMANCE OF THE PROTOCOL WITH N FOUR-LEVEL SYSTEMS

1. The $N = 1$ case

a. Unitary dynamics of the effective Hamiltonian

To evaluate the fidelity of our protocol with a single FLS, we first solve the unitary dynamics with the effective Hamiltonian. The total Hamiltonian consisting of the local system, the desired output mode, and the unwanted mode is given by

$$\begin{aligned}\hat{H}(t) &= \hat{H}_{\text{FLS}}^{\text{eff}}(t) + \hat{H}_B + \hat{H}_{\text{int}}, \\ \hat{H}_{\text{FLS}}^{\text{eff}}(t) &= \omega_0 \hat{c}^\dagger \hat{c} + \hat{\sigma}_x [\lambda^*(t) \hat{c} + \lambda(t) \hat{c}^\dagger], \\ \hat{H}_B &= \sum_{l=0,1} \int dk k \hat{a}_l^\dagger(k) \hat{a}_l(k), \\ \hat{H}_{\text{int}} &= \sum_{l=0,1} i \sqrt{\frac{\kappa_l}{\pi}} \int dk [\hat{a}_l^\dagger(k) \hat{c} - \hat{a}_l(k) \hat{c}^\dagger],\end{aligned}\quad (\text{B1})$$

where $\hat{a}_0$ ($\hat{a}_1$) is an annihilation operator of the desired (unwanted) output mode, and we have defined $(\kappa_0, \kappa_1) = (\kappa_{\text{ex}}, \kappa_{\text{in}})$ and set the speed of light as $c = 1$. This Hamiltonian dynamics has been analyzed in the main text, with the Heisenberg picture. With the Schrödinger picture, however, the time-dependent driving term in $\hat{H}_{\text{FLS}}^{\text{eff}}(t)$ may hinder a simple analysis. Nevertheless, applying an appropriate time-dependent unitary transformation yields a time-independent transformed Hamiltonian, as explained below. In a similar spirit to that of Ref. [51], which has developed an efficient numerical approach for simulating a dispersive readout in circuit QED, we use the time-dependent unitary operator

$$\begin{aligned}\hat{U}_d(t) &= e^{\hat{\sigma}_x [\beta(t) \hat{c}^\dagger - \beta^*(t) \hat{c}]} \\ &\times \prod_{l=0,1} \exp \left\{ \hat{\sigma}_x \int dk [\zeta_l(k, t) \hat{a}_l^\dagger(k) - \text{h.c.}] \right\},\end{aligned}\quad (\text{B2})$$

with free parameters $[\beta(t), \zeta_0(k, t), \zeta_1(k, t)]$ fulfilling $\beta(0) = \zeta_0(k, 0) = \zeta_1(k, 0) = 0$, to ensure $\hat{U}_d(0) = \hat{I}$. The transformed dynamics is given by

$$i \frac{d}{dt} |\Psi(t)\rangle_{U_d} = \hat{H}_{U_d}(t) |\Psi(t)\rangle_{U_d}, \quad (\text{B3})$$

where

$$\begin{aligned}|\Psi(t)\rangle_{U_d} &= \hat{U}_d^{-1}(t) |\Psi(t)\rangle, \\ \hat{H}_{U_d}(t) &= \hat{U}_d^{-1}(t) \hat{H}(t) \hat{U}_d(t) + i \left[\frac{d\hat{U}_d^{-1}(t)}{dt} \right] \hat{U}_d(t).\end{aligned}\quad (\text{B4})$$

Setting free parameters to fulfill the condition,

$$-i\dot{\zeta}_l(k, t) + k\zeta_l(k, t) + i\sqrt{\frac{\kappa_l}{\pi}}\beta(t) = 0 \quad (l = 0, 1), \quad (\text{B5})$$

leading to

$$\zeta_l(k, t) = \sqrt{\frac{\kappa_l}{\pi}} \int_0^t dt' \beta(t') e^{-ik(t-t')}, \quad (\text{B6})$$

$$\sqrt{\frac{\kappa_l}{\pi}} \int dk \zeta_l(k, t) = \kappa_l \beta(t), \quad (\text{B7})$$

the transformed Hamiltonian is given by

$$\hat{H}_{U_d}(t) = \hat{H}(t) - \hat{\sigma}_x \{ i[\dot{\beta}(t) + (i\omega_0 + \kappa)\beta(t)] \hat{c}^\dagger + \text{h.c.} \}. \quad (\text{B8})$$

Setting $\beta(t)$ such that $i[\dot{\beta}(t) + (i\omega_0 + \kappa)\beta(t)] = \lambda(t)$ cancels out the driving term and yields the time-independent Hamiltonian $\hat{H}_{U_d} = \omega_0 \hat{c}^\dagger \hat{c} + \hat{H}_B + \hat{H}_{\text{int}}$. Using Eq. (B6) gives

$$\int dk \zeta_l(k, t) \hat{a}_l^\dagger(k) = \sqrt{2\kappa_l} \int_0^t dt' \beta(t') \hat{a}_l^\dagger(z = t - t'), \quad (\text{B9})$$

where we have defined the annihilation operator at position z ,

$$\hat{a}_l(z) = \frac{1}{\sqrt{2\pi}} \int dk e^{ikz} \hat{a}_l(k). \quad (\text{B10})$$

To summarize, the time-dependent unitary operator is given by

$$\begin{aligned}\hat{U}_d(t) &= e^{\hat{\sigma}_x [\beta(t) \hat{c}^\dagger - \beta^*(t) \hat{c}]} \\ &\times \prod_l e^{\hat{\sigma}_x \sqrt{2\kappa_l} \int_0^t dt' [\beta(t') \hat{a}_l^\dagger(z = t - t') - \text{h.c.}]},\end{aligned}\quad (\text{B11})$$

with $\beta(t)$ fulfilling $i[\dot{\beta}(t) + (i\omega_0 + \kappa)\beta(t)] = \lambda(t)$.

For the initial state $|\Psi(0)\rangle_{U_d} = |\Psi(0)\rangle = \sum_{\mu=\pm} b_{\mu} |0\rangle_c |\mu\rangle |\text{vac}\rangle$, where $|n\rangle_c$ denotes the n -photon state of the cavity mode and $|\text{vac}\rangle$ denotes a vacuum state of all output modes, we find $|\Psi(t)\rangle_{U_d} = e^{-i\hat{H}_{U_d}t} |\Psi(0)\rangle_{U_d} = |\Psi(0)\rangle$, resulting in $|\Psi(t)\rangle = \hat{U}_d(t) |\Psi(t)\rangle_{U_d} = \hat{U}_d(t) |\Psi(0)\rangle$. Setting $\lambda(t)$ as [Eq. (5)]

$$\lambda(t) = \frac{i\alpha}{\sqrt{2\kappa_{\text{ex}}}} [\dot{v}(t) + (i\omega_0 + \kappa)v(t)], \quad (\text{B12})$$

which leads to $\beta(t) = \alpha v(t)/\sqrt{2\kappa_{\text{ex}}}$, we obtain the final state $|\Psi_f\rangle = |\Psi(t \rightarrow \infty)\rangle$ after finishing the driving as follows:

$$|\Psi_f\rangle = \sum_{\mu=\pm} b_{\mu} |0\rangle_c |\mu\rangle |\alpha_{\mu}; v\rangle_{\text{ex}} \left| \sqrt{\frac{\kappa_{\text{in}}}{\kappa_{\text{ex}}}} \alpha_{\mu}; v \right\rangle_{\text{in}}, \quad (\text{B13})$$

where $\alpha_{\pm} = \pm\alpha$, resulting in the ideal entangle state if $\kappa_{\text{in}} = 0$. The photon-loss probability due to the internal loss is given by

$$\begin{aligned} P_{\kappa_{\text{in}}} &= 1 - \text{Tr}[|\text{vac}\rangle_{\text{in}} \langle \text{vac}| |\Psi_f\rangle \langle \Psi_f|] \\ &= 1 - \exp\left(-\frac{\kappa_{\text{in}}}{\kappa_{\text{ex}}} |\alpha|^2\right). \end{aligned} \quad (\text{B14})$$

b. Atomic decay effect

We now incorporate the atomic decay effect, which is described by the effective Lindblad operators in Eq. (A7). In those operators, the first term describes atomic decay induced by classical-laser excitation, while the second term corresponds to decay upon absorbing the cavity photon. From the relation

$$\left| \frac{\Omega(t)}{g\sqrt{\hat{c}^\dagger(t)\hat{c}(t)}} \right|^2 \simeq 1 + \left| \frac{\Delta}{g^2} \frac{\dot{v}(t) + \kappa v(t)}{v(t)} \right|^2 \geq 1 + \left| \frac{\kappa\Delta}{g^2} \right|^2, \quad (\text{B15})$$

where we have used Eq. (A10) and the left-hand sides of Eqs. (A12) and (A13), we find that classical driving is the dominant source of the atomic decay for $|\Delta| \gg g^2/\kappa$, leading to the Lindblad operators being approximated as

$$\begin{aligned} \hat{L}_2^{\text{eff}} &\simeq -\frac{\sqrt{2r_1\gamma}}{g} \lambda(t) \hat{\sigma}_-, \\ \hat{L}_3^{\text{eff}} &\simeq -\frac{\sqrt{2(1-r_1)\gamma}}{g} \lambda(t) |\uparrow\rangle \langle \uparrow|, \\ \hat{L}_4^{\text{eff}} &\simeq -\frac{\sqrt{2r_2\gamma}}{g} \lambda(t) \hat{\sigma}_+, \\ \hat{L}_5^{\text{eff}} &\simeq -\frac{\sqrt{2(1-r_2)\gamma}}{g} \lambda(t) |\downarrow\rangle \langle \downarrow|. \end{aligned} \quad (\text{B16})$$

The dynamics under the condition of no atomic decays is described by the non-Hermitian Schrödinger equation

$i(d/dt)|\bar{\Psi}\rangle = \hat{\mathcal{H}}|\bar{\Psi}\rangle$ [34], where

$$\begin{aligned} \hat{\mathcal{H}} &= \hat{\mathcal{H}}_{\text{FLS}}^{\text{eff}} + \hat{H}_B + \hat{H}_{\text{int}}, \\ \hat{\mathcal{H}}_{\text{FLS}}^{\text{eff}} &= \hat{H}_{\text{FLS}}^{\text{eff}}(t) - \frac{i}{2} \sum_{m=2}^5 (\hat{L}_m^{\text{eff}})^\dagger \hat{L}_m^{\text{eff}} \\ &= \omega_0 \hat{c}^\dagger \hat{c} + \hat{\sigma}_x [\lambda^*(t) \hat{c} + \lambda(t) \hat{c}^\dagger] - i\gamma \left| \frac{\lambda(t)}{g} \right|^2 \hat{I}. \end{aligned} \quad (\text{B17})$$

Using the transformation (B3), which can be straightforwardly expanded for a nonunitary operator $\hat{O}(t)$, we recover the Hermitian Hamiltonian $\hat{H}_{\text{FLS}}^{\text{eff}}(t)$ with

$$\hat{O}(t) = \exp \left[-\gamma \int_0^t dt' \left| \frac{\lambda(t')}{g} \right|^2 \right] \hat{I}. \quad (\text{B18})$$

Thus, the final unnormalized state $|\bar{\Psi}_f\rangle = \hat{O}(t \rightarrow \infty) |\Psi_f\rangle$ is given by

$$|\bar{\Psi}_f\rangle = \exp \left[-\gamma \int_0^\infty dt \left| \frac{\lambda(t)}{g} \right|^2 \right] |\Psi_f\rangle, \quad (\text{B19})$$

where the decrease of $\langle \bar{\Psi}_f | \bar{\Psi}_f \rangle$ from unity corresponds to the total probability of atomic decays

$$P_\gamma = 1 - \langle \bar{\Psi}_f | \bar{\Psi}_f \rangle = 1 - \exp \left[-2\gamma \int_0^\infty dt \left| \frac{\Omega(t)}{\Delta} \right|^2 \right], \quad (\text{B20})$$

reflecting that the excited-state population is approximately $|\Omega(t)/\Delta|^2$. From Eq. (B12), we find

$$\begin{aligned} \int dt |\lambda(t)|^2 &= \frac{\kappa^2 |\alpha|^2}{2\kappa_{\text{ex}}} \left[1 + \left(\frac{\omega_0}{\kappa} \right)^2 + \frac{\int dt |\dot{v}(t)|^2}{\kappa^2} \right. \\ &\quad \left. - \frac{2\omega_0}{\kappa^2} \int dt \text{Im}[v(t)\dot{v}^*(t)] \right], \end{aligned} \quad (\text{B21})$$

and thus, for $v(t)$ as the Gaussian function with standard deviation τ , we obtain

$$\begin{aligned} P_\gamma &= 1 - \exp \left\{ -\frac{\kappa}{\kappa_{\text{ex}}} \frac{|\alpha|^2}{2C} \left[1 + \left(\frac{g^2}{\kappa\Delta} \right)^2 + \frac{1}{2(\kappa\tau)^2} \right] \right\}, \\ &\simeq 1 - \exp \left(-\frac{\kappa}{\kappa_{\text{ex}}} \frac{|\alpha|^2}{2C} \right). \end{aligned} \quad (\text{B22})$$

Including the dynamics with one or more atomic decays, we obtain the normalized final state $\hat{\rho} = (1 -$

$P_\gamma)|\Psi_f\rangle\langle\Psi_f| + P_\gamma\hat{\rho}_\gamma$, where $\hat{\rho}_\gamma$ represents the state yielded by the event with one or more atomic decays. Tracing out the unwanted mode gives the system state

$$\begin{aligned}\hat{\rho} &= \text{Tr}_{\text{in}}[|\text{vac}\rangle_{\text{in}}\langle\text{vac}|\hat{\rho}] + \text{Tr}_{\text{in}}[(\hat{I}_{\text{in}} - |\text{vac}\rangle_{\text{in}}\langle\text{vac}|)\hat{\rho}] \\ &= (1 - P_\gamma)(1 - P_{\kappa_{\text{in}}})|\psi_{\text{id}}\rangle\langle\psi_{\text{id}}| + P_\gamma \text{Tr}_{\text{in}}[|\text{vac}\rangle_{\text{in}}\langle\hat{\rho}_\gamma|\text{vac}\rangle_{\text{in}} \\ &\quad + \text{Tr}_{\text{in}}[(\hat{I}_{\text{in}} - |\text{vac}\rangle_{\text{in}}\langle\text{vac}|)\hat{\rho}],\end{aligned}\quad (\text{B23})$$

where we have extracted the ideal final state $|\psi_{\text{id}}\rangle = \sum_\mu b_\mu |0\rangle_c |\mu\rangle |\alpha_\mu; v\rangle_{\text{ex}}$. Thus, the generated state can be denoted by $\hat{\rho} = (1 - P_j)|\psi_{\text{id}}\rangle\langle\psi_{\text{id}}| + P_j\hat{\rho}_{\text{jumps}}$, where $\hat{\rho}_{\text{jumps}}$ represents the state yielded by the event with one or more jumps, and the total jump probability is given by

$$P_j = 1 - (1 - P_\gamma)(1 - P_{\kappa_{\text{in}}}). \quad (\text{B24})$$

2. The $N \geq 2$ case

a. Unitary dynamics of the effective Hamiltonian

We consider the case for N FLSs within a cavity, where the input-output theory still provides a simple solution for the unitary dynamics. From the relation $[\hat{\sigma}_{x,j}, \hat{H}_{\text{FLSs}}^{\text{eff}}] = 0$, $\hat{\sigma}_{x,j}(t)$ is time-independent and can be denoted simply by $\hat{\sigma}_{x,j}(0)$. For $\hat{c}(t)$, we obtain

$$\dot{\hat{c}} = -(iN\omega_0 + \kappa)\hat{c} - i \sum_{j=1}^N \lambda_j(t) \hat{\sigma}_{x,j}(0). \quad (\text{B25})$$

Choosing $\lambda_j(t)$ as

$$\lambda_j(t) = \frac{i\alpha_j}{\sqrt{2\kappa_{\text{ex}}}} [\dot{v}(t) + (iN\omega_0 + \kappa)v(t)] \quad (\text{B26})$$

gives

$$\hat{c}(t) = \frac{v(t)}{\sqrt{2\kappa_{\text{ex}}}} \sum_j \alpha_j \hat{\sigma}_{x,j}(0), \quad (\text{B27})$$

and we find that the desired and unwanted modes are given by

$$\begin{aligned}\hat{a}_{\text{out},0}(t) &= v(t) \sum_j \alpha_j \hat{\sigma}_{x,j}(0), \\ \hat{a}_{\text{out},1}(t) &= v(t) \sqrt{\frac{\kappa_{\text{in}}}{\kappa_{\text{ex}}}} \sum_j \alpha_j \hat{\sigma}_{x,j}(0)\end{aligned}\quad (\text{B28})$$

by using the input-output relation $\hat{a}_{\text{out},l} = \sqrt{2\kappa_l}\hat{c}(t)$ ($l = 0, 1$). Thus, the final state $|\Psi_f^{(N)}\rangle$ after finishing the driving

is given by

$$|\Psi_f^{(N)}\rangle = \sum_\mu b_\mu |0\rangle_c |\mu\rangle |\alpha_\mu; v\rangle_{\text{ex}} \left| \sqrt{\frac{\kappa_{\text{in}}}{\kappa_{\text{ex}}}} \alpha_\mu; v \right\rangle_{\text{in}}, \quad (\text{B29})$$

where $\mu_j = \pm$, $\mu = \{\mu_1, \mu_2, \dots, \mu_N\}$, $|\mu\rangle = |\mu_1\rangle_1 |\mu_2\rangle_2 \dots |\mu_N\rangle_N$, and $\alpha_\mu = \sum_j (\alpha_j)_{\mu_j}$. The photon-loss probability depends on the setting $\{b_\mu, \alpha_j\}$.

In the main text, we have focused on the case $N = 2$ with $\alpha_1 = \beta e^{i\pi/4}/\sqrt{2}$, $\alpha_2 = \beta e^{-i\pi/4}/\sqrt{2}$, leading to

$$|\Psi_f^{(2)}\rangle = \sum_\mu b_\mu |0\rangle_c |\mu\rangle |\beta_\mu; v\rangle_{\text{ex}} \left| \sqrt{\frac{\kappa_{\text{in}}}{\kappa_{\text{ex}}}} \beta_\mu; v \right\rangle_{\text{in}}, \quad (\text{B30})$$

where $\beta_{\{+,+\}} = \beta$, $\beta_{\{+,-\}} = i\beta$, $\beta_{\{-,+\}} = -i\beta$, and $\beta_{\{-,-\}} = -\beta$. In this case, the photon-loss probability is given by

$$P_{\kappa_{\text{in}}}^{(2)} = 1 - \exp\left(-\frac{\kappa_{\text{in}}}{\kappa_{\text{ex}}} |\beta|^2\right). \quad (\text{B31})$$

b. Atomic decay effect

As in the case $N = 1$, we again assume that classical driving is the primary source of atomic excitation, which should be valid for $|Ng^2/\kappa\Delta| \ll 1$. In this case, we straightforwardly expand the discussion for $N = 1$ and obtain the final unnormalized state

$$|\bar{\Psi}_f^{(N)}\rangle = \exp\left[-\gamma \int_0^\infty dt \sum_{j=1}^N \left|\frac{\lambda_j(t)}{g}\right|^2\right] |\Psi_f^{(N)}\rangle, \quad (\text{B32})$$

leading to the total atomic decay probability

$$\begin{aligned}P_\gamma^{(N)} &= 1 - \exp\left[-2\gamma \int_0^\infty dt \sum_{j=1}^N \left|\frac{\lambda_j(t)}{g}\right|^2\right] \\ &\simeq 1 - \exp\left(-\frac{\kappa}{\kappa_{\text{ex}}} \frac{\sum_j |\alpha_j|^2}{2C}\right).\end{aligned}\quad (\text{B33})$$

When generating the four-component cat state in the Gaussian envelope, as discussed in the main text, this probability reduces to

$$P_\gamma^{(2)} = 1 - \exp\left(-\frac{\kappa}{\kappa_{\text{ex}}} \frac{|\beta|^2}{2C}\right). \quad (\text{B34})$$

APPENDIX C: VIRTUAL CAVITY METHOD

To treat the output desired mode as a system for evaluating the protocol fidelity, we use the virtual-cavity method proposed in Refs. [52,53]. The density operator of the total system, which includes N FLSs, the cavity mode, and the

propagating light with an envelope function $v(t)$, evolves according to the master equation as follows:

$$\frac{d\hat{\rho}}{dt} = -i[\hat{H}, \hat{\rho}] + \sum_{m=1,2} \mathcal{D}[\hat{L}_m]\hat{\rho} + \sum_{j=1}^N \sum_{m=2}^5 \mathcal{D}[\hat{L}_m^j]\hat{\rho}, \quad (C1)$$

where the Hamiltonian and the Lindblad operators for the cavities are given by

$$\begin{aligned} \hat{H}(t) &= \hat{H}_{\text{FLSS}}(t) + \frac{i}{2}\sqrt{2\kappa_{\text{ex}}}[g_v^*(t)\hat{c}^\dagger\hat{a}_v - g_v(t)\hat{c}\hat{a}_v^\dagger], \\ \hat{L}_0(t) &= \sqrt{2\kappa_{\text{ex}}}\hat{c} + g_v^*(t)\hat{a}_v, \quad \hat{L}_1 = \sqrt{2\kappa_{\text{in}}}\hat{c}. \end{aligned} \quad (C2)$$

The master equation (C1) describes a Gedankenexperiment where we put another one-sided cavity with the complex input coupling

$$g_v(t) = -\frac{v^*(t)}{\sqrt{\int_0^t dt' |v(t')|^2}} \quad (C3)$$

that completely absorbs a propagating state with the envelope function $v(t)$. The operator $\hat{a}_v$ is an annihilation operator of the virtual cavity mode.

We now focus on the regime where the effective model is valid. In this case, we can reduce the master equation (C1) to the effective one as follows:

$$\frac{d\hat{\rho}}{dt} = -i[\hat{H}^{\text{eff}}, \hat{\rho}] + \sum_{m=1,2} \mathcal{D}[\hat{L}_m]\hat{\rho} + \sum_{j=1}^N \sum_{m=2}^5 \mathcal{D}[\hat{L}_m^{j,\text{eff}}]\hat{\rho}, \quad (C4)$$

where

$$\hat{H}^{\text{eff}}(t) = \hat{H}_{\text{FLSS}}^{\text{eff}}(t) + \frac{i}{2}\sqrt{2\kappa_{\text{ex}}}[g_v^*(t)\hat{c}^\dagger\hat{a}_v - g_v(t)\hat{c}\hat{a}_v^\dagger]. \quad (C5)$$

Finally, to numerically solve the dynamics with no quantum jumps, we solve the following effective non-Hermitian Schrödinger equation:

$$\begin{aligned} i\frac{d|\Phi\rangle}{dt} &= \hat{\mathcal{H}}^{\text{eff}}|\Phi\rangle, \\ \hat{\mathcal{H}}^{\text{eff}}(t) &= \hat{H}^{\text{eff}}(t) \\ &\quad - \frac{i}{2} \left[\sum_{m=0,1} \hat{L}_m^\dagger \hat{L}_m + \sum_{j=1}^N \sum_{m=2}^5 (\hat{L}_m^{j,\text{eff}})^\dagger \hat{L}_m^{j,\text{eff}} \right]. \end{aligned} \quad (C6)$$

From this result, we numerically evaluate $F_{\text{min}} = |\langle \psi_{\text{id}} | \Phi \rangle|^2$. For the numerical calculation shown in Figs. 3, 4, we use QuantumOptics.jl [50].

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