

Circulators based on coupled quantum anomalous Hall insulators and resonators

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Integrated plasmonics is advancing rapidly, enabling a wide range of functionalities to be incorporated onto a single chip. Applications span information processing, computation, quantum sensing, and dark matter detection. This progress has driven the development of integrated nonreciprocal devices, which are essential for preventing unwanted feedback that can degrade system performance. While nonreciprocal devices have been realized in edge magnetoplasmon (EMP) materials via classical interference effects, their operation is often limited by the input power range. Here, we demonstrate that topological circulators utilizing asymmetric coupling offer improved input power range, isolation, and insertion loss. In this configuration, we demonstrate that the coupling between a chiral edge magnetoplasmonic resonator and a pair of LC resonators is well described by an effective non-Hermitian two-site Hatano–Nelson model with asymmetric directional couplings, resulting in nonreciprocal behavior. The coherent photon-plasmon interaction enables a circulator with up to 50 dB of isolation across a broad range of excitation power. These results suggest that magnetic topological insulators provide a promising platform for realizing asymmetric non-Hermitian couplings at radio frequencies and for exploring regimes of strong directional suppression and possible exceptional-point physics. More broadly, they highlight the potential of topological-material-based microwave devices for future integration with superconducting quantum information platforms.

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I. INTRODUCTION

The chiral nature of the edge magnetoplasmon (EMP) makes it a promising candidate for advancing chip-scale nonreciprocal devices at quantum-classical interfaces [1–3]. EMP propagation has been extensively studied in quantum Hall (QH) systems [4–6]. Their coupling to external devices in the non-Hermitian regime has been investigated, while the possibility of accessing strongly asymmetric transport and exceptional-point-related phenomena remains largely unexplored. Recent studies have demonstrated that non-Hermitian systems, particularly those of the Hatano–Nelson type, offer potential for realizing isolators and switches because their intrinsically

asymmetric coupling strengths can induce exceptional points (EPs), leading to highly nonreciprocal transport phenomena [7]. In this context, EMPs can serve as nonreciprocal links that asymmetrically couple radio-frequency (rf) resonators, enabling circulation or isolation behavior.

Compared with QH systems, QAH materials offer distinct advantages, such as maintaining a stable number of edge channels across a broad range of applied magnetic fields, including zero magnetic field [8,9]. Moreover, QAH states can be engineered in multilayer magnetic topological insulators or heterostructures, providing design flexibility that can be tailored to meet specific circuit requirements [10,11]. This makes them well-suited for integration with superconducting circuits, where magnetic fields can be detrimental to performance, and semiconductor devices, which can tolerate or even require moderate magnetic fields for operation. The QAH insulator, Cr-doped $(\text{Bi}_x\text{Sb}_{1-x})_2\text{Te}_3$, provides a promising platform for realizing strong plasmon-photon interactions. This is due to its high-impedance plasmon mode, with an impedance of $Z = h/Ce^2 \sim 25.8 \text{ k}\Omega$ and a Chern number

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$C = 1$. Because the zero-point voltage fluctuation scales as $\sqrt{Z}$, this large Z markedly amplifies the electric-dipole coupling between the plasmon and nearby quantum emitters, making high-impedance edge magnetoplasmonic resonators ideal for achieving strong asymmetric plasmon-photon coupling and opening up exciting opportunities in nonreciprocal electronics.

In this work, we investigate a coupled plasmon-photon system, in which an on-chip QAH plasmon resonator is asymmetrically coupled to a pair of LC resonators. While Mahoney *et al.* studied a similar architecture in a QH system, they did not explore the connection to Hatano–Nelson-type couplings [1]. In our hybrid system, we observe a pronounced nonreciprocal transmission dip accompanied by a positive π phase shift. By leveraging the chiral nature of the EMP modes, we interpret the observed behavior as being consistent with an effective two-site Hatano–Nelson-type plasmonic system [7,12,13], in which asymmetric coupling may drive the system toward a strongly nonreciprocal non-Hermitian transport regime. Furthermore, the system achieves an *average isolation* of 20 dB over an approximately 50 MHz bandwidth and a peak isolation exceeding 50 dB at 543.8 MHz for an input microwave power of -119 dBm. The circulator remains operational down to an input power of -149 dBm. These results suggest a new mechanism for harnessing the intrinsic unidirectionality of QAH edge states to engineer nonreciprocal devices, with promising implications for scaling superconducting quantum processors and improving axion haloscope sensitivity.

II. RESULTS

A. Circulator configuration and effective Hatano–Nelson-type coupling

The configuration of the three-port circulator device is shown in Fig. 1(a), which consists of an EMP resonator with a radius of $100\ \mu\text{m}$ and three lumped-element inductors, each with an inductance $L = 180\ \text{nH}$. The inductor and the stray capacitor C_{stray} [Fig. 1(b)] form an LC resonator, whose resonance frequency $\omega_{LC} = 1/\sqrt{LC_{\text{stray}}}$ is set to closely match the EMP resonance frequency $\omega_m = [\frac{\sigma_{xy}}{2\pi\epsilon_0\epsilon_{\text{eff}}}(\ln\frac{2}{|q|w} + 1) + v_D]q$, where q is the wave vector, $\sigma_{xy} = Ce^2/h$ is the Hall conductivity, ϵ_{eff} is the effective dielectric constant, v_D is the drift velocity of the 1D edge state, and w is the physical width of the EMP mode at the edge. The value of ω_m can be directly extracted from experimental characterization of the circulator without requiring an impedance-matching circuit. At each port of the circulator, the EMP resonator is capacitively coupled to an LC resonator through an edge capacitor C_{edge} , which is formed between the Au pad and the EMP resonator [Fig. 1(b)]. The microwave transmission coefficients S_{31} and S_{32} are characterized using a measurement

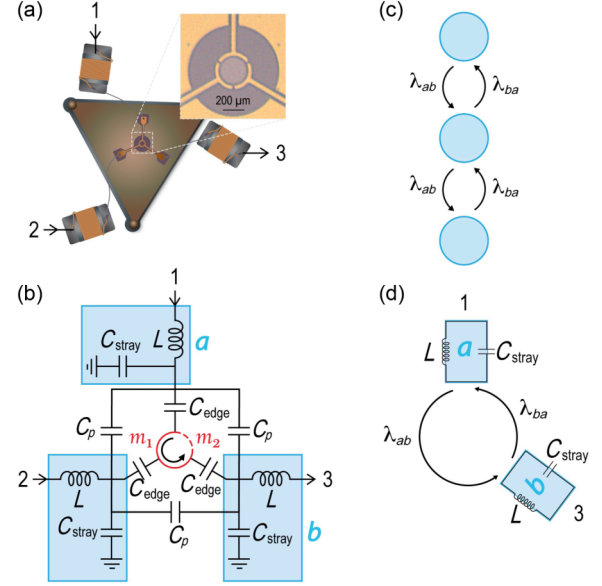


FIG. 1. Device and Hatano–Nelson model. (a) Schematic of the three-port circulator device, consisting of a quantum anomalous Hall (QAH) mesa with an edge conductance of e^2/h , and three $180\ \text{nH}$ lumped-element wire-loop inductors are wire-bonded between the mesa and the transmission line of the printed circuit board. Edge magnetoplasmon resonant modes are excited and propagate along the edge of the circular mesa. The inset shows the optical image of the circulator, with a $100\text{-}\mu\text{m}$ -radius QAH mesa separated from the gold pad that defines the three signal ports by a $15\ \mu\text{m}$ gap. (b) Equivalent circuit diagrams illustrating parasitic capacitance C_p , stray capacitance C_{stray} , and edge capacitance C_{edge} . Components a and b (blue pads) represent the LC resonators at ports 1 and 3, respectively. m_1 and m_2 represent the anticlockwise transport of the EMPs from resonator a to b , and from b to a , respectively. (c) Schematic of a one-dimensional (1D) Hatano–Nelson chain, where asymmetric couplings λ_{ab} and λ_{ba} between cells give rise to nonreciprocal transport in a non-Hermitian system. (d) Sketch of the circulator configuration composed of two LC resonators, a and b , coupled nonreciprocally through the interference of EMP and the parasitic capacitive pathways, leading to asymmetric coupling λ_{ab} and λ_{ba} .

scheme described in Appendices A and B. All cryogenic measurements were performed in a dilution refrigerator at a temperature of approximately $10\ \text{mK}$.

The circulator architecture, consisting of two LC resonators (a and b) and the two directional EMP channels, is described by a minimal non-Hermitian coupled-mode matrix that assumes idealized nonreciprocity propagation [7,14–16]:

$$H_{\text{eff}}/\hbar = \begin{pmatrix} \omega_a - i\kappa_a & g & 0 & 0 \\ 0 & \omega_{m_1} - i\kappa_{m_1} & g & 0 \\ 0 & 0 & \omega_b - i\kappa_b & g \\ g & 0 & 0 & \omega_{m_2} - i\kappa_{m_2} \end{pmatrix}, \quad (1)$$

where κ_a and κ_b are the decay rates of the LC resonators a and b , respectively. κ_{m_1} and κ_{m_2} are the decay rates of the EMP resonators along two propagation paths, m_1 and m_2 , as illustrated in Fig. 1(b). ω_{a,b,m_1,m_2} are the bare resonance frequencies of the LC and EMP modes, respectively, and g denotes the coupling strength between the LC resonators and the EMP modes. In the frequency-domain coupled-mode treatment (see Appendix E), the response coefficients are written as $\alpha_j(\omega) = -i\omega + i\omega_j + \kappa_j = \kappa_j - i(\omega - \omega_j)$.

By eliminating the intermediate EMP modes, Eq. (1) can be reduced to an effective two-site Hatano–Nelson-type model [Fig. 1(c)] with unequal direction-dependent couplings between neighboring unit cells (see Appendix E),

$$\lambda_{ab}(\omega) = \frac{g^2}{\alpha_{m_1}}, \quad \lambda_{ba}(\omega) = \frac{g^2}{\alpha_{m_2}}, \quad (2)$$

where $\alpha_{m_1}(\omega)$ and $\alpha_{m_2}(\omega)$ describe the frequency-domain complex response of the two EMP propagation paths. Because these paths have different lengths and dissipation, $\alpha_{m_1} \neq \alpha_{m_2}$ in general, leading to asymmetric effective coupling. In our effective model, the nonreciprocal inter-resonator couplings, λ_{ab} and λ_{ba} , arise from destructive and constructive interference between the EMP and parasitic capacitive pathways [Figs. 1(b) and 1(d)]. As a result, they are determined by the EMP propagation lengths and depend sensitively on both the input microwave power and the device operating temperature. Within this effective model, sufficiently strong suppression of the one-directional coupling could in principle drive the reduced two-site system toward an EP, marked by eigenvalue and eigenvector coalescence [17].

B. Circulator characterization

We begin by examining the transmission in an EMP resonator, both without and with LC resonators, across a wide spectral range. Figures 2(a)–2(c) show the transmission through the EMP resonator, without coupling to an LC resonator. Owing to the constructive and destructive interference between the EMP mode (red traces), coupled via C_{edge} , and the port-to-port rf signal transmitted through the parasitic capacitance C_p [Fig. 1(b)], S_{31} and S_{32} exhibit a nonreciprocal response with a bandwidth of approximately 100 MHz [Figs. 2(d)–2(f)]. The isolation magnitude, $|S_{32}| - |S_{31}|$, reaches a maximum of 20 dB over an input power range spanning approximately 5 dBm.

Figures 2(g)–2(i) show the transmission spectra as a function of frequency and input power when the EMP and LC resonators are coupled. Notably, the magnitude of S_{32} remains nearly constant over an input power range spanning more than 50 dBm. The circulator exhibits approximately 20 dB of nonreciprocity over an input power range

from -150 to -90 dBm, with an approximate bandwidth of 50 MHz [Fig. 2(i)]. This level of isolation and power handling is particularly significant for quantum information science, where protecting qubits from amplifier backaction is critical, and for axion dark matter searches like ADMX, which require ultralow-noise, broadband signal routing at cryogenic temperatures. The ability to achieve strong nonreciprocity at such low input powers and moderate bandwidths supports scalable readout chains and noise-resilient detection architectures in both domains [18,19].

The spectral response at varying input power levels for the LC-resonator-coupled device is presented in greater detail in Figs. 2(j)–2(l). At an input power of $P = -149$ dBm, S_{32} exhibits a Lorentzian-like resonance with the quality factor $Q = 21$, while S_{31} shows a subtle resonance peak [Fig. 2(j)]. Compared with the EMP circulator without coupling to an LC resonator, the LC-resonator-coupled configuration achieves significantly higher transmission, with an insertion loss of approximately 6 dB at the transmission peak (see Appendix D). This enhancement can be attributed to the impedance matching between the port impedance and that of the QAH edge state. Transmission can be enhanced to near-unity levels through optimized impedance matching and improved circuit fabrication.

The broadband nonreciprocal response in the 450–650 MHz frequency range, observed in both the LC-coupled and uncoupled circulators, arises from interference between the phase-sensitive plasmonic pathway and the parasitic capacitance pathway. For the shorter edge path (port 2 to 3), two LC resonators are capacitively coupled to the EMP resonator, leading to constructive interference with the parasitic transmission. In contrast, for the longer edge path (port 1 to 3), the same coupling occurs; however, the increased path length induces an out-of-phase interaction with the parasitic transmission. This phase mismatch results in destructive interference between the plasmonic and parasitic paths, thereby reducing overall transmission.

For the LC-coupled-circulator, we find that increasing the input microwave power leads to the gradual emergence of a sharp transmission dip near 543.8 MHz, which reaches its minimum at $P = -119$ dBm [Figs. 2(k) and 2(l)]. In addition to the broadband suppression of transmission from port 1 to port 3, this narrow resonance feature produces exceptionally high isolation, exceeding 50 dB within a bandwidth of approximately 5 MHz. In contrast, this pronounced and narrow transmission feature is absent when the circulator operates without coupling to the LC resonators [Figs. 2(a)–2(f)].

Next, we further characterize the nonreciprocal response at $f = 543.8$ MHz and investigate its physical origin. In particular, we examine how the transmission and phase evolution of S_{31} depend on the input power.

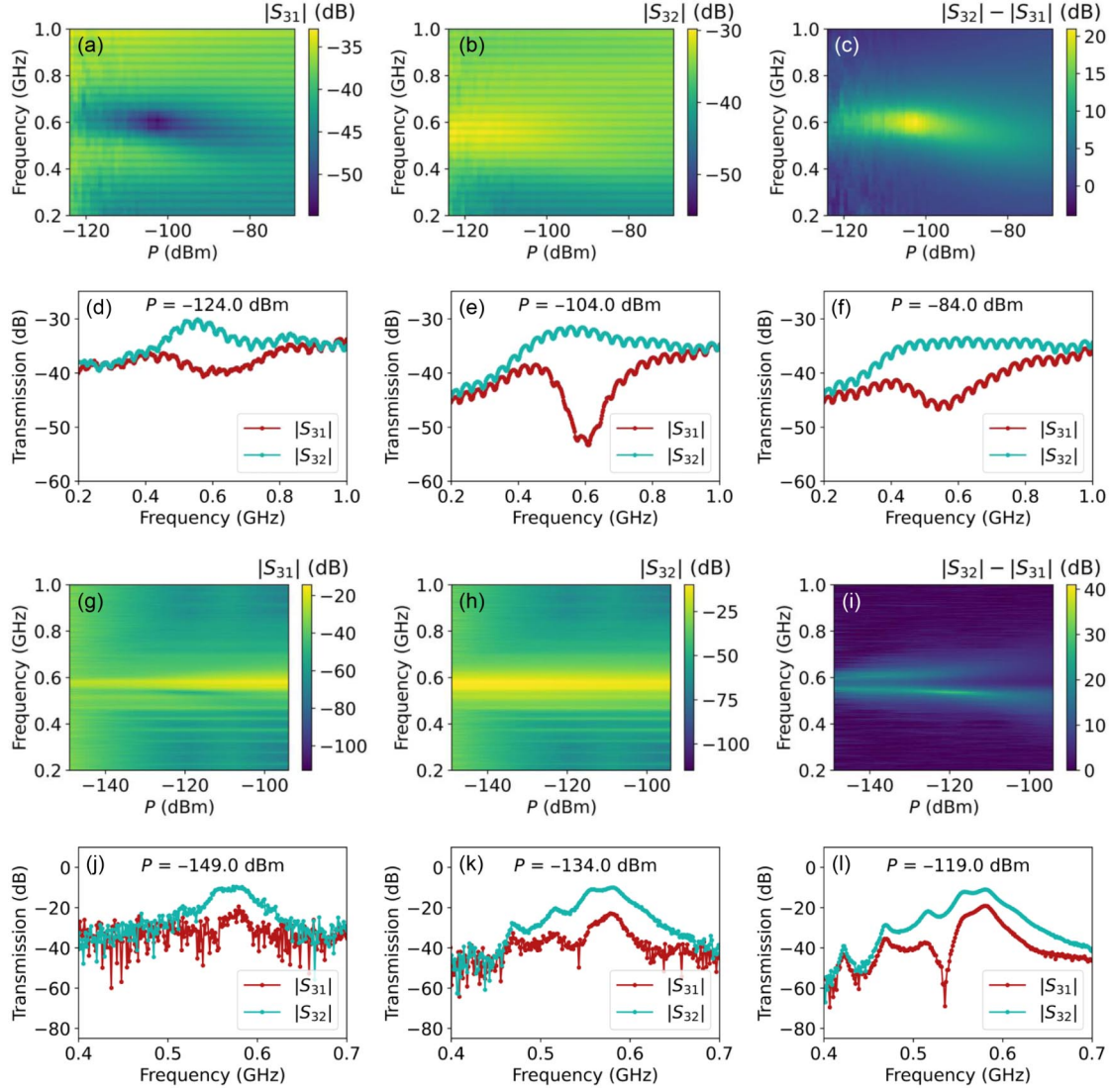


FIG. 2. Transmission response for circulators without and with coupling to LC resonators. (a)–(c) Power-dependent microwave transmission responses $|S_{31}|$, $|S_{32}|$, and $|S_{32}| - |S_{31}|$ for a 100- μm -radius circulator without coupling to LC resonators. The signal is excited from ports 1 and 2 and measured at port 3. (d)–(f) Frequency line cuts from panels a and b, shown for three representative input power levels. (g)–(i) Power-dependent microwave transmission responses $|S_{31}|$, $|S_{32}|$, and $|S_{32}| - |S_{31}|$ for a 100- μm -radius circulator coupled to LC resonators. (j)–(l) Frequency line cuts from panels g and h, shown for three representative input power levels.

Figures 3(a) and 3(b) show the magnitude and phase of S_{31} and S_{32} , respectively, over a narrow frequency range. Figures 3(c) and 3(d) illustrate the evolution of the $|S_{31}|$ and phase difference $\angle S_{31} - \angle S_{32}$ as a function of input power. A particularly striking observation is that the sharpest transmission contrast appears only within a narrow input power window [Fig. 3(c)]. Within this regime, S_{31} undergoes an abrupt phase transition from negative to positive, exhibiting an approximately 180° phase shift relative to S_{32} [Fig. 3(d), red curve]. This pronounced nonreciprocal transmission, accompanied by a corresponding π phase shift, is consistent with strongly asymmetric coupling between the two LC resonators,

where the effective coupling becomes highly suppressed in one propagation direction [20–22].

We attribute the emergence of the anomalous transmission dip to an effective, power-dependent phase shift in the EMP propagation pathway, which dynamically modulates the coupling between the two LC resonators. As the input microwave power increases, the EMP mode experiences a shift in both velocity and dissipation, altering the accumulated phase along the edge-transport path. The phase and dissipation shifts with increasing power may arise from the presence of charge puddles in proximity to the edge states. The photon-excited charge puddles could slow edge-plasmon propagation and provide dissipation

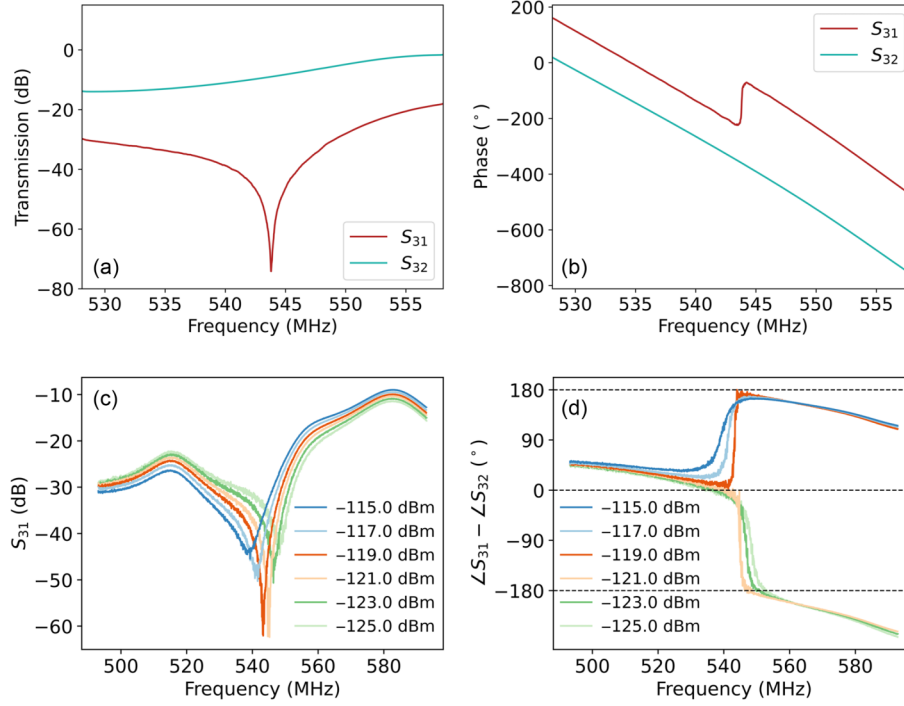


FIG. 3. Circulator transmission measurement. (a) and (b) Experimental results for the transmission magnitude and phase for S_{31} and S_{32} . The data were taken at an input power of -120 dBm, corresponding to the estimated rf power at the circulator input ports, where the strongest asymmetric transport behavior is observed. (c) and (d) Measured S_{31} and S_{32} magnitudes and their relative phase at input power levels below, near, and above the condition of maximum nonreciprocal transmission.

channels through the bulk states [23]. Because the EMP-mediated coupling and the parasitic capacitive coupling interfere either constructively or destructively depending on their relative phase, even a small power-induced change in the EMP phase can significantly modify the net interaction strength. At low powers, the plasmonic and parasitic pathways partially cancel, producing moderate suppression of transmission. As the power increases toward ($P \sim -119$ dBm), the phase accumulated along the longer EMP path shifts, such that the plasmonic signal becomes nearly out of phase with the parasitic contribution. This condition leads to strong destructive interference and substantial suppression of the effective coupling in one propagation direction. The resulting asymmetric transport behavior produces the sharp, unidirectional transmission dip observed in the measurements and is qualitatively consistent with the strongly asymmetric transport regime of the non-Hermitian coupled plasmon-photon system.

C. Simulation

We now turn to simulations of the transmission magnitude and the associated positive phase shift using coupled-mode theory. The system is modeled by the non-Hermitian coupled-mode matrix in Eq. (1), which assumes ideal nonreciprocity [7,14–16]. Given the almost identical resonance frequencies of the input and output LC resonators [Fig. 5(c)], we set $\omega_a = \omega_b = \omega_0$ and $\kappa_a = \kappa_b = \kappa_0$.

In the EMP resonator, asymmetric coupling emerges due to its chiral nature, leading to differences in propagation time and loss for the m_1 and m_2 modes (despite $\omega_{m_1} = \omega_{m_2} = \omega_m$). These differences depend on the path length and the input microwave power. By applying coupled-mode theory to the steady-state driven response of the non-Hermitian four-mode model in Eq. (1) and then eliminating the intermediate EMP modes, as described in Appendix E, the transmission spectrum can be expressed as,

$$t(\omega) = -\frac{\sqrt{2}\kappa_0\alpha_m g^2}{\alpha_0^2\alpha_m^2 - g^4}, \quad (3)$$

where $\alpha_0 \equiv -i\omega + i\omega_0 + \kappa_0$ characterizes the LC resonator and $\alpha_m \equiv -i\omega + i\omega_m + \kappa_m$ characterizes the EMP resonator. Since the longer propagation length of mode m_1 is approximately twice that of m_2 , we adopt the phenomenological approximation $\alpha_{m_1} \approx 2\alpha_{m_2}$ near the operating point of strongest asymmetry. This compactly captures the path-dependent phase accumulation and dissipation and gives $\alpha_m = \sqrt{\alpha_{m_1}\alpha_{m_2}} \approx \sqrt{2}\alpha_{m_2}$.

We find that the measured transmission spectra are qualitatively in agreement with the transmission response derived from the effective two-site Hatano–Nelson-type model, indicating that the circulation is consistent with a mechanism dominated by asymmetric coupling between

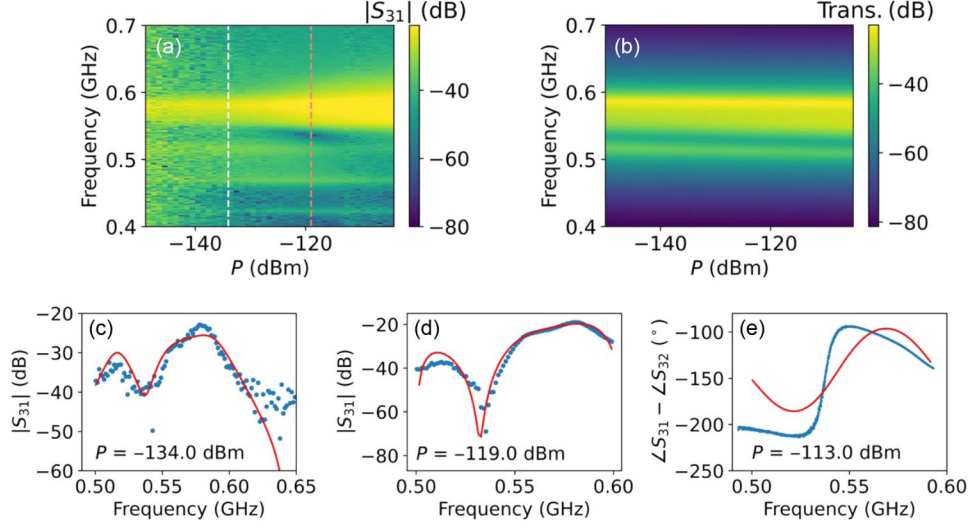


FIG. 4. Comparison of data and simulation. (a) and (b) Close-up views of the experimental (a) and simulated (b) spectral responses as a function of frequency and input power. The white (orange) dashed line indicates the input power corresponding to the measurements shown in c and d. (c) and (d) Measured $|S_{31}|$ magnitude (blue dots) as functions of frequency at excitation powers of $P = -134$ and -119 dBm, respectively. The solid red curves represent fits from the theoretical model in Eq. (3). (e) Measured (blue) phase difference between S_{31} and S_{32} at an excitation power of approximately $P = -113$ dBm and calculated (red) phase response based on Eq. (3) using parameters extracted from the fit presented in d. See text for details.

the two LC resonators mediated by the EMP mode [Figs. 4(a) and 4(b)]. At low excitation power, the measured transmission closely matches the model, with fitted parameters: $g/2\pi = 5.3$ MHz, $\kappa_0/2\pi = 3.0$ MHz, and $\kappa_m/2\pi = 1.0$ MHz [Fig. 4(c)]. As the input power increases, the fitted coupling strength g rises to 6.9 MHz, while $\kappa_0/2\pi$ remains at 3.0 MHz and $\kappa_m/2\pi$ increases to 3.3 MHz, respectively, at the point of maximum isolation [(Fig. 4(d))].

This effective model also captures the phase behavior observed in the experimental data, particularly in the regime of strongest asymmetric transport, as shown in Fig. 4(e). It is worth noting that the phase data in Figs. 4(a) and 4(b) are not sampled densely enough to enable a reliable comparison between experiment and simulation. To address this, we took a high-resolution data scan and plotted the phase response of S_{31} after subtracting the phase of S_{32} , thereby removing the line delay. This allows for a direct comparison with the theoretical result. The residual mismatch between experiment and theory at higher powers is likely due to power-dependent plasmon damping and coupling effects that lie outside the minimal model.

III. CONCLUSION

In this work, we demonstrated a circulation mechanism in a topological circulator comprising a QAH edge magnetoplasmonic resonator coupled to LC resonators. Within an input power range of -150 to -120 dBm, the circulator maintains a relatively stable bandwidth, pass-band center frequency, and isolation, indicating robust

performance under low-power operating conditions. This power independence suggests that the device is well suited for integration into superconducting quantum information science platforms and high-energy physics experiments. If fully harnessed, this mechanism could facilitate the realization of a topological circulator with near-unitary forward transmission and exceptionally high isolation in the reverse direction.

In the present device geometry, the operating frequency is primarily determined by the edge magnetoplasmon velocity and the dimensions of the topological insulator mesa. The demonstrated operation near 0.5 GHz serves as a proof of principle for realizing effective non-Hermitian coupling and nonreciprocal microwave transport using magnetic topological insulators. Furthermore, we note that compact on-chip nonreciprocal devices operating in the sub-GHz frequency range may find applications in superconducting quantum circuits that employ low-frequency signals for flux control, biasing, and tunable coupling. In particular, the frequency range demonstrated here overlaps with that of fluxonium-based architectures, where control-line-limited relaxation and flux-controlled interactions have recently attracted significant interest [24–27]. Looking forward, the operating frequency can be extended to higher microwave frequencies through device scaling and QAH material engineering of the edge-plasmon properties.

In conclusion, our results highlight the significant potential of chiral plasmons to provide enhanced control over signal transmission and isolation across a wide dynamic range.

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DATA AVAILABILITY

The data that support the findings of this article are not publicly available upon publication because it is not technically feasible and/or the cost of preparing, depositing, and hosting the data would be prohibitive within the terms of this research project. The data are available from the authors upon reasonable request.

APPENDIX A: EXPERIMENTAL CONFIGURATION AND DEVICE FABRICATION

The transmission of the circulator is probed by measuring the S_{21} transmission parameters for various powers. The experimental configuration is plotted in Fig. 5(a). The S_{21} measurements were performed on two ports of the circulator with an rf switch located at room temperature. The input microwave signals are transmitted through coaxial cables with a 20 dB attenuator at room temperature [not shown in Fig. 5(a)] and a 20 dB attenuator at 4 K. The total attenuation from the vector network analyzer down to

the sample is about -64 dB. The power values expressed in decibels in this article represent the estimated power at the input port of the circulator. The circulator output goes through a cryogenic isolator at the still stage and is amplified by a HEMT amplifier (LNF-LNC0.3-14A, Low Noise Factory) with a noise temperature of approximately 2 K at the 4 K stage. At room temperature, the transmission signal is further amplified by a low-noise room-temperature amplifier with a total gain of 20 dB (ZX60-4016E-S+, Mini-Circuits).

Topological insulators are materials with an insulating bulk and topologically protected conducting surface or edge states arising from strong spin-orbit coupling and a nontrivial electronic band topology [28–30]. Magnetic topological insulators are topological insulators in which magnetic order breaks time-reversal symmetry, leading to gapped surface states and enabling exotic phenomena such as the QAH effect. The six quintuple layers of the magnetic topological insulator $\text{Cr}_{0.12}(\text{Bi}_{0.26}\text{Sb}_{0.62})_2\text{Te}_3$ thin films were grown using ultrahigh-vacuum molecular beam epitaxy. Epi-ready semi-insulating ($\rho > 10^6 \Omega\text{cm}$) GaAs (111) B substrates were preannealed in the chamber at up to 600°C under Te-rich environment. During the coevaporated growth, high-purity Bi (99.9999%), Te (99.9999%), Cr (99.99%) were evaporated from Knudsen effusion cells, while Sb (99.999%) was evaporated from a cracker cell. Upon material growth, the circulator devices were fabricated via standard photolithography. The mesa disk was first patterned by photolithography and then etched by CF_4/Ar using reactive ion etching. The coplanar waveguides and ground planes were patterned by photolithography, and 10 nm of Ti and 100 nm of Au were deposited using electron-beam deposition, followed by a

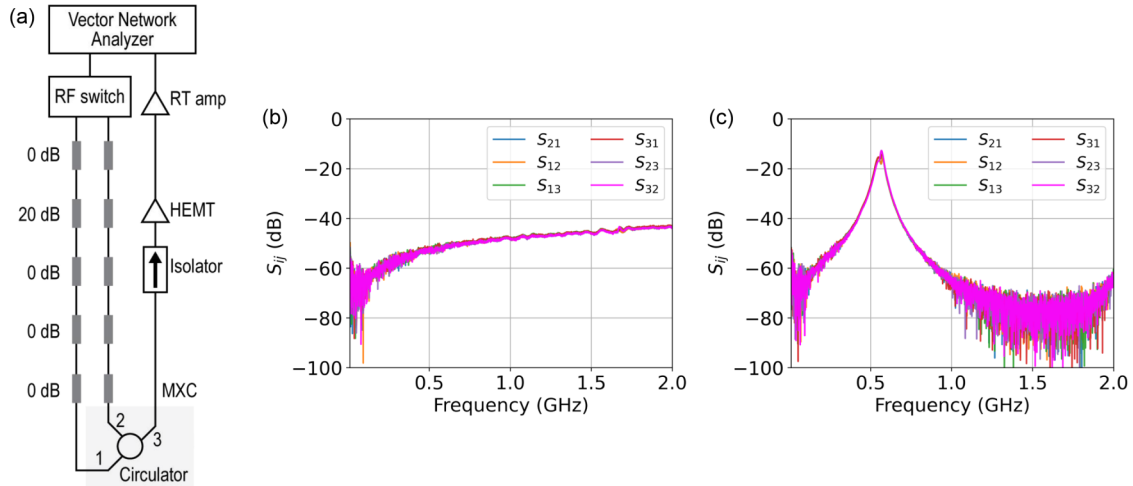


FIG. 5. Experimental configuration and the transmission spectra of the device without and with the inductors measured at room temperature. (a) Experimental setup showing the circulator mounted on the mixing-chamber plate of a dilution refrigerator, which cools the device to below 10 mK. (b) Complete six-way transmission responses of the 100- μm -radius circulator without inductors measured at room temperature. (c) The transmission spectra S_{ij} for the 100- μm -radius circulator with inductors, showing a frequency response resulting from the LC impedance-matching circuits.

lift-off process. The Hall characterization of the same batch of magnetic topological insulator wafer can be found in Ref. [31]. All measurements presented in this work were performed under an external magnetic field of approximately 40 mT, generated by attaching a permanent magnet plate underneath the QAH mesa.

APPENDIX B: ROOM-TEMPERATURE DEVICE CHARACTERIZATION

The circulator sample was initially characterized without a matching inductor, and its impedance was expected to be on the order of the Hall resistance, $\rho_{xy} = 25 \text{ k}\Omega$. Since the impedance of the measurement circuit was $50 \text{ }\Omega$, the lack of a matching circuit at the input results in approximately 99% of the power being reflected at the input port of the circulator. As a control, we first measured all microwave S parameters before mounting the inductors, observing that all ports are equivalent, with a transmission magnitude approximately -50 dB at 530 MHz, as shown in Fig. 5(b).

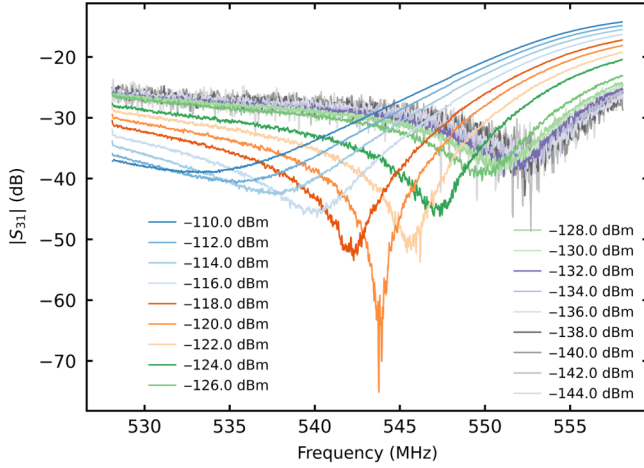


FIG. 6. Transmission spectra from ports 1 to 3 at various input power levels. The transmission dip shows a power-dependent frequency shift.

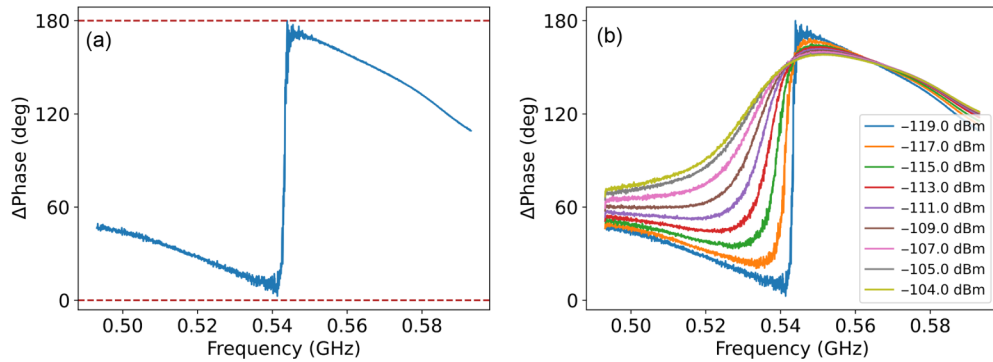


FIG. 7. The measured phase difference between S_{31} and S_{32} at various input power levels. (a) $\Delta\text{Phase} = \angle S_{31} - \angle S_{32}$ at an input power of $P = -119 \text{ dBm}$. (b) The phase difference at various input power levels above the EP condition.

We then implemented a matching network at the input ports of the circulator by utilizing three lumped-element 180 nH inductors in series [Fig. 1(a)]. The inductors, coupled with the effective capacitance of the circuit, serve as an impedance-matching network for the $50 \text{ }\Omega$ coaxial lines and the high impedance of the circulator. Physically, the matching process utilizes the resonance created by the inductors and the effective stray capacitance of the circuit. The resonance is tuned to coincide with the circulator's edge magnetoplasmonic resonance frequency, effectively increasing the energy coupled into the circulator. Increasing the energy coupled into the circuit increases the signal-to-noise ratio. In practice, better coupling improves the insertion loss associated with reflections arising from impedance mismatch. The appropriate inductor value was determined by testing different inductors on a similar sample and collecting transmission measurements at room temperature. Figures 5(b) and 5(c) present the six-way transmission spectra measured without and with the inductors, respectively. At room temperature, the LC network exhibits a resonant frequency of $f = 566 \text{ MHz}$ and a quality factor of $Q = 21$.

APPENDIX C: POWER-DEPENDENT TRANSMISSION MEASUREMENTS FOR S_{31}

Figure 6 shows the magnitude of S_{31} plotted as a function of frequency for various input power levels at temperatures below 10 mK. When the signal is applied at port 1, both the center frequency and bandwidth exhibit a clear dependence on excitation power. In contrast, transmission from port 2 shows no significant power dependence.

The phase difference between S_{31} and S_{32} at different excitation powers is illustrated in Fig. 7. At the maximum S_{31} transmission dip ($P = -119 \text{ dBm}$), a phase shift of approximately 180° is observed [Fig. 7(a)]. The slope of phase change decreases when P exceeds -119 dBm [Fig. 7(b)].

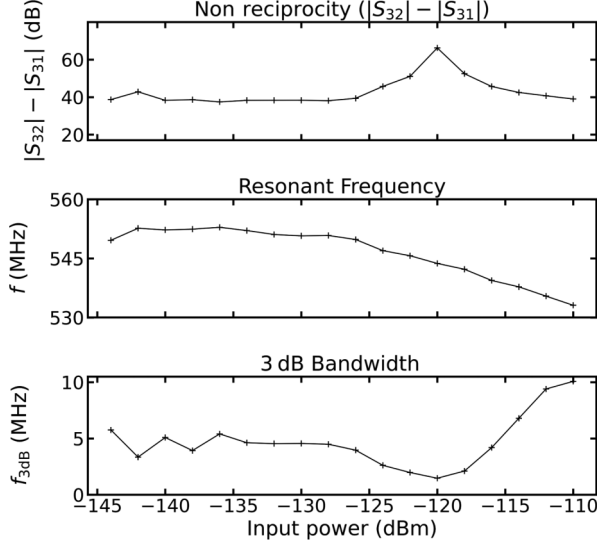


FIG. 8. Parameters extracted from the transmission spectra as a function of input power. The nonreciprocity, defined as the difference between $|S_{32}|$ and $|S_{31}|$, along with the frequency dip in $|S_{31}|$ and the 3 dB bandwidth, are plotted versus input power.

Figure 8 presents the nonreciprocity $|S_{32}| - |S_{31}|$, the resonant frequency shift, and the 3 dB bandwidth as functions of input power. Notably, within the input power range of -145 to -125 dBm, both the circulator's bandwidth and center frequency remain relatively stable, indicating a power-independent regime. This suggests that the circulator can operate reliably within power levels relevant to superconducting quantum information systems and high-energy physics experiments.

APPENDIX D: INSERTION LOSS CHARACTERIZATION

The insertion loss of the circulator was measured in a different Bluefors dilution refrigerator after the sample, with the LC resonators attached, had been stored for more than 2 years at ambient temperature in a low-vacuum desiccator. The characterization diagram is shown in Fig 9. The circulator was cooled to below 30 mK. A network analyzer was used to perform S_{21} transmission measurements through the system. To determine the insertion loss of the circulator, a transmission measurement was made through the transmission port (I_1) and the proportion of power transmitted

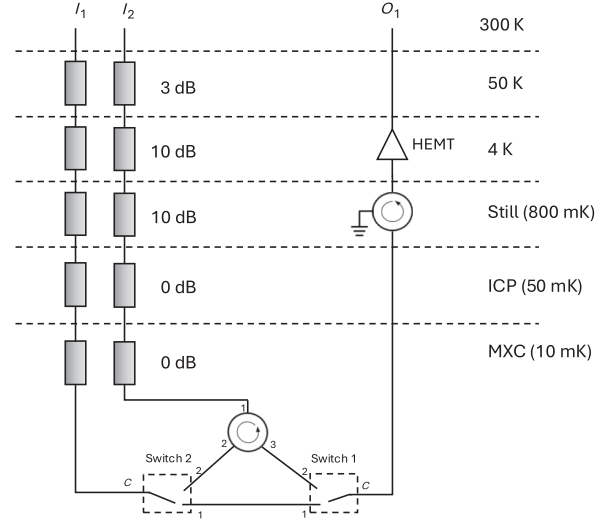


FIG. 9. Experimental setup for insertion loss measurements. The circulator is cooled to below 30 mK. A network analyzer is used to measure transmission through the receiver, and two rf switches enable bypassing the circulator. Measurement of transmission with and without the circulator enables a direct measurement of the insertion loss of the circulator.

through the receiver was measured at the output (O_1). Two cryogenic switches (Radial R585433210) enabled swapping between the circulator ports and a short cable. In addition, a second transmission line (I_2) was used to measure the isolation of the circulator. On the output line (O_1), the signal was amplified by a HEMT amplifier mounted to the 4 K stage, followed by a room-temperature amplifier. An isolator before the HEMT amplifier was used to reduce standing waves within the receiver. Broadband transmission measurements were made of both S_{32} and S_{31} of the circulator. At approximately 10 mK, we observe an approximately 2 dB difference in transmission between input lines I_1 and I_2 .

To determine the insertion loss of the circulator, transmission measurements were performed with the circulator both in the receiver circuit and bypassed, while applying a range of input powers. Any additional transmission losses observed when the circulator was switched in were attributed to its insertion loss. As shown in Fig. 10, the insertion loss of the circulator was found to be approximately 5.9 ± 2 dB with an isolation of approximately 20 dB in the low-power regime.

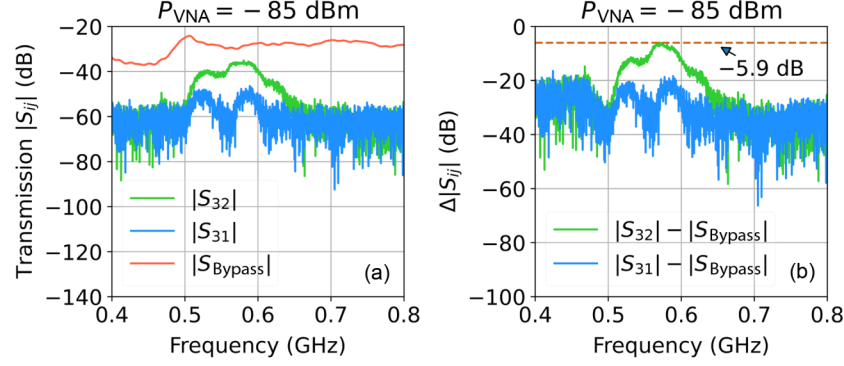


FIG. 10. Insertion loss of the circulator at approximately 10 mK. (a) Transmission of the circulator and the short bypass cable at the vector network analyzer (VNA) power of $P_{\text{VNA}} = -85$ dBm. (b) An insertion loss of 5.9 dB is observed at the transmission peak of $|S_{32}|$ with an isolation of approximately 20 dB.

APPENDIX E: EFFECTIVE HATANO–NELSON DESCRIPTION AND DERIVATION OF THE TRANSMISSION MODEL

In this appendix, we show how the four-mode model in Eq. (1) reduces to an effective two-resonator description with asymmetric directional couplings and how the same reduction yields the transmission formula in Eq. (3). Equation (1) is a minimal device-level coupled-mode model for the two LC resonators and the two directional EMP propagation channels.

We begin from the non-Hermitian four-mode matrix in the basis $\Psi = (a, m_1, b, m_2)^T$,

$$H_{\text{eff}}/\hbar = \begin{pmatrix} \omega_a - i\kappa_a & g & 0 & 0 \\ 0 & \omega_{m_1} - i\kappa_{m_1} & g & 0 \\ 0 & 0 & \omega_b - i\kappa_b & g \\ g & 0 & 0 & \omega_{m_2} - i\kappa_{m_2} \end{pmatrix}, \quad (\text{E1})$$

where a and b denote the two LC resonators, m_1 and m_2 denote the two directional EMP channels, and g is the LC-EMP coupling rate. In the experiment, the resonators are nearly identical, so we take $\omega_a = \omega_b = \omega_0$ and $\kappa_a = \kappa_b = \kappa_0$. The off-diagonal structure of Eq. (E1) describes the directional sequence:

$$a \rightarrow m_1 \rightarrow b \rightarrow m_2 \rightarrow a. \quad (\text{E2})$$

The two EMP channels correspond to inequivalent chiral propagation paths around the QAH mesa and therefore generally acquire different phase shifts and dissipation.

For a probe frequency ω , we define

$$\alpha_j(\omega) \equiv -i\omega + i\omega_j + \kappa_j, \quad j \in \{a, b, m_1, m_2\}. \quad (\text{E3})$$

It follows that the steady-state coupled-mode equations for a coherent drive, s_{in} , incident on resonator a are

$$\alpha_a a + igm_1 = s_{\text{in}}, \quad (\text{E4})$$

$$\alpha_{m_1} m_1 + igb = 0, \quad (\text{E5})$$

$$\alpha_b b + igm_2 = 0, \quad (\text{E6})$$

$$iga + \alpha_{m_2} m_2 = 0. \quad (\text{E7})$$

Eliminating the EMP amplitudes (m_1, m_2) gives

$$\alpha_a a + \frac{g^2}{\alpha_{m_1}} b = s_{\text{in}}, \quad (\text{E8})$$

$$\alpha_b b + \frac{g^2}{\alpha_{m_2}} a = 0. \quad (\text{E9})$$

The two resonators are therefore coupled by the effective directional couplings

$$\lambda_{ab}^{\text{eff}}(\omega) = \frac{g^2}{\alpha_{m_1}(\omega)}, \quad \lambda_{ba}^{\text{eff}}(\omega) = \frac{g^2}{\alpha_{m_2}(\omega)}, \quad (\text{E10})$$

and because the circulation paths have different lengths and dissipation, $\alpha_{m_1} \neq \alpha_{m_2}$, leading to asymmetric coupling. Hence, Eq. (1) reduces to a two-site Hatano–Nelson-type model with effective matrix:

$$M_{\text{eff}}(\omega) = \begin{pmatrix} \alpha_a(\omega) & \lambda_{ab}^{\text{eff}}(\omega) \\ \lambda_{ba}^{\text{eff}}(\omega) & \alpha_b(\omega) \end{pmatrix}. \quad (\text{E11})$$

The effective asymmetric coupling between the two resonators can be interpreted as a two-site reduction of the canonical Hatano–Nelson Hamiltonian,

$$H/\hbar = \sum_n (\omega_n \hat{a}_n^\dagger \hat{a}_n + \lambda_{ab} \hat{a}_{n+1}^\dagger \hat{a}_n + \lambda_{ba} \hat{a}_n^\dagger \hat{a}_{n+1}), \quad (\text{E12})$$

where ω_n denotes the resonant frequency of the n th resonator, which is taken to be identical for all resonators,

i.e., $\omega_n = \omega_0$; $\hat{a}_n^\dagger$ and $\hat{a}_n$ are the bosonic creation and annihilation operators at the n th resonator; and λ_{ab} and λ_{ba} denote non-Hermitian couplings between adjacent resonators. In our system, the effective couplings are set by the EMP propagation paths and are generally unequal, $\lambda_{ab} \neq \lambda_{ba}$. Thus, the device realizes the minimal two-site analogue of Hatano–Nelson physics in an open, driven, and dissipative system.

The same reduction also yields the transmission formula used to fit the data. Solving Eqs. (E8) and (E9) gives

$$\frac{b}{s_{\text{in}}} = -\frac{g^2 \alpha_{m_1}}{\alpha_a \alpha_b \alpha_{m_1} \alpha_{m_2} - g^4}. \quad (\text{E13})$$

For nearly identical LC resonators, $\alpha_a = \alpha_b \equiv \alpha_0$, where

$$\alpha_0 = -i\omega + i\omega_0 + \kappa_0, \quad (\text{E14})$$

so that

$$\frac{b}{s_{\text{in}}} = -\frac{g^2 \alpha_{m_1}}{\alpha_0^2 \alpha_{m_1} \alpha_{m_2} - g^4}. \quad (\text{E15})$$

Introducing α_m through

$$\alpha_m^2 \equiv \alpha_{m_1} \alpha_{m_2}, \quad (\text{E16})$$

and using $\alpha_{m_1} = 2\alpha_{m_2}$ near the operating point (see main text), we obtain Eq. (3);

$$t(\omega) = -\frac{\sqrt{2}\kappa_0 \alpha_m g^2}{\alpha_0^2 \alpha_m^2 - g^4}, \quad (\text{E17})$$

where the factor κ_0 comes from the canonical input-output transmission amplitude normalization for a resonator coupled to a transmission line.

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