

Mixed-state surface impedance of Nb-Ti: Flux flow, pinning, and creep

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We present a thorough study of the surface impedance Z_s in the mixed state of sputtered Nb-Ti coatings on insulating substrates, with the aim to assess and quantify the mechanisms governing vortex motion. Measurements are performed by means of a dual mode dielectric-loaded resonator allowing the simultaneous determination of Z_s at 16.5 and 26.7 GHz, which enable to derive the various relevant vortex dynamics parameters. A static magnetic field $\mu_0 H \leq 1.25$ T is applied normally to the sample surface, in the $T = 5$ K $- T_c$ temperature range. The measured pinning frequency is ≈ 4.5 GHz at 5 K and 1 T, with a pronounced decrease with field. A scaling analysis of the pinning constant $k_p(H, T)$ dependence shows that pinning is dominated by point pins. Thermal creep is negligibly small at the lowest T , possibly becoming finite only near T_c . Flux flow resistivity is found to be consistent with the time-dependent Ginzburg-Landau predictions for a moderately dirty superconductor.

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I. INTRODUCTION

High-frequency (rf) vortex dynamics [1] dominates the surface impedance of superconductors in the mixed state. The electrodynamics is completely different from the BCS well-known theory [2], relevant in the Meissner state. Indeed, vortices set in motion by the microwave currents [3] determine the dissipation through the vortex core quasiparticles [4]. The process is contrasted by material defects, which act as preferential sites (pinning centers) for vortices and hinder their motion with a recall force contrasting vortex displacement. The effectiveness of this process depends on the type and dimensionality of the pins and on the vortex line and flux lattice elasticity [5]. Moreover, pinning effects are contrasted by creep processes, thermally activated vortex jumps between pinning sites. The electric field induced by moving vortices couples back with rf currents [1] giving rise to a complex

response in which various length scales, the London/quasiparticle/vortex motion penetration depths concur in determining the penetration of electromagnetic (EM) fields in a superconductor and, ultimately, the dissipation. Whereas for long-time vortex dynamics in technological superconductors has been predominantly studied in view of dc applications, at microwaves, on the other hand, the Meissner state has been the main focus because of the very low surface resistance R_s , beneficial in the realization of superconducting radio frequency (srf) accelerating cavities for particle physics experiments. In recent years, type II superconductors have been investigated to extend their exploitation also in high-frequency applications in intense static magnetic fields. Indeed, in the framework of the hunt for dark matter, in a dc magnetic field B galactic axions are predicted to convert into microwave photons [6]. Resonant cavities with high quality factor $Q \geq 10^6$, operating in strong fields, are needed to increase the power conversion $\propto QB^2$ [7] for detection of halos' axions (whence the name “haloscopes”). Superconductors can yield high Q by exploiting the small R_s of superconductors [8–12], although the need to operate in the vortex state is a relatively new requirement for applications of rf superconductivity. The first successful superconductors used in haloscope realization have been Nb-Ti and NbTiN, given the simplicity of their deposition by sputtering on the

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curved surfaces of the designed cylindrical haloscopes [8,9,13], with good preliminary results: at 4.2 K, 5 T, $Q \simeq 2.6 \times 10^5$ at approximately 14.5 GHz [8] and $Q \simeq 3 \times 10^5$ at approximately 9 GHz [9] were obtained, with a QB^2 product 10 times higher than with Cu. Despite the potential role of Nb-Ti in rf superconductivity [9,13], its microwave vortex dynamics is little known and controversial, with the quantitative underlying physics giving very different results. The analysis of the vortex motion parameters as extracted from the haloscope response yielded very high-pinning frequencies (several tens of GHz) [8], at odds with the available characterizations on Nb-Ti wires (below 1 GHz at 2.2 K [14]), and thin films (4 to 8 GHz, field dependent, in the 0.5 to 4 T range) [15]. Other Nb-based superconductors gave values of 5 GHz in Nb thin films at $\mu_0 H = 1$ T and $T = 6$ K [16], and 7 GHz in Nb₃Sn polycrystalline bulks at $\mu_0 H = 2$ T and $T = 6$ K [17]. Additional applications of high-frequency superconductivity in strong dc magnetic fields include the beam screen for the next-generation particle accelerator, the CERN Future Circular Collider (FCC) [18,19], and the muon cooling system for the International Muon Collider (IMC) [20], that is envisioned to use rf cavities. Despite different regimes, with haloscopes in the linear response and FCC and IMC applications requiring very large rf fields, a renewed interest in studying high-frequency vortex dynamics in high magnetic fields arose [21]. The presence of strong magnetic fields B involves a complete change of paradigm with respect to conventional rf superconductivity for srf cavities. Vortex motion induced by rf EM fields represents by far the dominant dissipation source. Quasiparticles and impurities dissipation, leading to the well-known BCS surface resistance [2] and its additional terms, of large relevance for srf cavities, is negligible. Indeed, material imperfections acting as pinning centers are expected to be beneficial to reduce dissipation, as opposed to the effect in the Meissner state. A reconsideration of vortex dynamics at high frequency is then useful even for superconducting materials well known in other contexts, but very few experimental data exist for the microwave surface resistance of Nb-based superconductors in dc magnetic fields in the Tesla range, even less so in Nb-Ti [15]. With a striking difference with respect to the surface resistance in zero dc field, as relevant to srf cavities, the physics of the microwave vortex dynamics in Nb-Ti is little explored. In this work, the vortex dynamics of Nb-Ti films is extensively investigated for fields up to 1.2 T and in a temperature range (5 K, T_c) in order to validate high-frequency vortex motion models and determine their vortex parameters, together with their field and temperature dependencies. The paper is organized as follows. Section II introduces the model for vortex dynamics, the experimental methods, and the sample preparation and characterization. Section III describes the surface impedance measurements. The extracted

vortex parameters are analyzed in Sec. IV. A short summary and conclusions are provided in Sec. V.

II. MODEL, METHODS, MATERIALS

A. Vortex dynamics model

Due to the very small vortex oscillations at microwave frequencies, a mean field approach is appropriate to describe the vortex dynamics [3,22]. The average force per unit length acting on a single vortex is given by the force balance equation:

$$\mathbf{J}_{\text{mw}} \times (\hat{\mathbf{n}}\Phi_0) - \eta\mathbf{v} - k_p\mathbf{u} + \mathbf{F}_{\text{thermal}} = 0, \quad (1)$$

where vortices bear a magnetic flux quantum Φ_0 , with the magnetic field oriented along $\hat{\mathbf{n}}$, $\mathbf{J}_{\text{mw}}$ is the microwave current, $\mathbf{u}$ and $\mathbf{v}$ are the vortex displacement and velocity, respectively. Vortices experience the stochastic and Lorentz force per unit length, $\mathbf{F}_{\text{thermal}}$ and $\mathbf{J}_{\text{mw}} \times (\hat{\mathbf{n}}\Phi_0)$. The response of the superconductor is given by the so-called viscosity η (drag coefficient) and pinning constant k_p (spring coefficient of the elastic recall). All quantities are per unit length. The vortex mass term is usually neglected up to the THz range [4,23]. Following the Coffey-Clem (CC) [1] or Brandt [24] approaches to take into account pinning and thermal effects [25], Eq. (1) translates to the complex vortex motion resistivity:

$$\rho_{\text{vm}} = \rho_{\text{vm},1} + i\rho_{\text{vm},2} = c_{\text{ff}}\rho_{\text{ff}} \frac{\chi + i\nu/\nu_c}{1 + i\nu/\nu_c}, \quad (2)$$

where the following quantities appear:

1. the flux flow resistivity $\rho_{\text{ff}} = \Phi_0 B / \eta = \alpha \rho_n B / B_{c2}$, where ρ_n is the normal state resistivity, B_{c2} is the upper critical field, and $\alpha(T, B)$ captures the detailed dependence of ρ_{ff} , as can be worked out in the time-dependent Ginzburg-Landau (TDGL) theory [26,27] in various limits. In the simple Bardeen-Stephen (BS) model [28], $\alpha = 1$ and $\rho_{\text{ff}} \propto B$;
2. the adimensional creep factor $\chi \in [0, 1]$ represents the weight of thermal creep effect. The values $\chi = 0$ and $\chi = 1$ indicate zero and maximum creep, respectively; with $\chi = 1$, one has $\rho_{\text{vm}} = \rho_{\text{ff}}$;
3. the characteristic frequency ν_c , which marks the crossover from the low-frequency-low-dissipation regime to the high-frequency-high dissipation regime (see Fig. 1). In the limit of zero creep, $\nu_c \rightarrow \nu_p$, where $\nu_p = k_p / (2\pi\eta)$ is the pinning frequency (also widely known as depinning frequency) and one recovers the Gittleman-Rosenblum (GR) model [29];
4. the tilting parameter c_{ff} takes into account the relative orientation between the vortex lines, nominally parallel to the applied magnetic field, and the

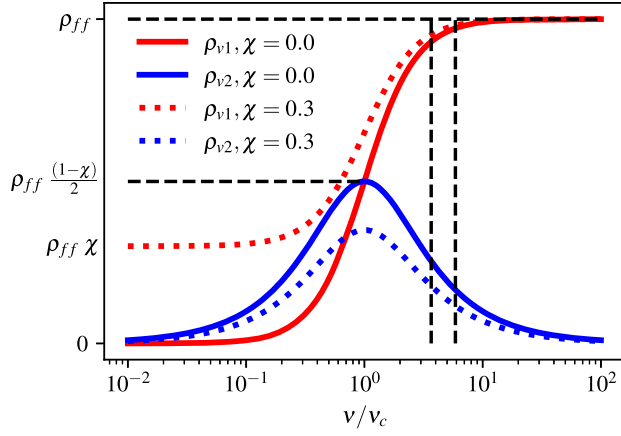


FIG. 1. Coffey-Clem model [1] for the frequency ν dependence of ρ_{vm} , illustrated for selected values of the creep factor χ . The vertical lines represent the experimental measuring frequencies referred to $\nu_c = 4.5$ GHz (see Secs. III and IV).

microwave currents [30–32]. For $\mathbf{J} \perp \mathbf{B}$ (maximum Lorentz force configuration), $c_{ff} = 1$.

The overall complex resistivity $\tilde{\rho}$ includes the two-fluid contribution σ_{2f} [1]:

$$\tilde{\rho} = \frac{\rho_{vm} + i/\sigma_2}{1 + i\sigma_1/\sigma_2} \simeq \rho_{vm} + i\frac{1}{\sigma_2} = \rho_{vm} + i\omega\mu_0\lambda^2, \quad (3)$$

where the approximate equality holds for T not too close to $T_c(B)$, where $\sigma_1/\sigma_2 \ll 1$, and the two-fluid conductivity σ_{2f} is given by

$$\sigma_{2f} = \sigma_1 - i\sigma_2 = \frac{x_n}{\rho_n} - i\frac{x_s}{\omega\mu_0\lambda_0^2}, \quad (4a)$$

$$x_n(T, B) + x_s(T, B) = 1, \quad (4b)$$

$$x_s(T, B) = (1 - t^4)(1 - b), \quad (4c)$$

$$\lambda(T, B) = \lambda_0/\sqrt{x_s(T, B)}, \quad (4d)$$

$$t = T/T_c, \quad (4e)$$

$$b = B/B_{c2}. \quad (4f)$$

Here, $T_c = T_c(B = 0)$ is the zero-field critical temperature, λ_0 and B_{c2} are the zero-temperature penetration depth and temperature-dependent upper critical field, respectively. The standard temperature dependence of the superconducting x_s fraction [33,34] has been confirmed experimentally in Nb-Ti films [15]. The field dependence of x_s captures the pair-breaking effect [1,34].

B. Surface impedance models

At microwave frequencies, the response function is the surface impedance $Z_s = E_{||}/H_{||}$, where $E_{||}$ and $H_{||}$ are the

microwave electric and magnetic field components tangential to the sample surface, respectively. In the local limit and in bulk samples, i.e., of thickness t_s much larger than the EM field penetration depth, the surface impedance is [35]:

$$Z_s = Z_{s,bulk} = \sqrt{i2\pi\nu\mu_0\tilde{\rho}}. \quad (5)$$

From Z_s , one derives the complex penetration depth as $\tilde{\lambda} = 1/(ik_s)$, the complex wave propagation constant k_s , and the characteristic length for the exponential attenuation of the EM fields, δ_{eff} :

$$Z_{s,bulk} = i2\pi\nu\mu_0\tilde{\lambda}, \quad (6a)$$

$$k_s = 2\pi\nu\mu_0/Z_{s,bulk}, \quad (6b)$$

$$\delta_{eff} = -1/\text{Im}(k_s) = |\tilde{\lambda}|^2/\text{Re}(\tilde{\lambda}). \quad (6c)$$

In the Meissner state, $\delta_{eff} = \lambda$, with negligible normal fluid contribution apart from $T \simeq T_c$.

For finite thickness samples and multilayers, the expression for the (effective) surface impedance is [35,36]:

$$Z_s = Z_{s,eff} = Z_{s,bulk} \frac{Z_d + iZ_{s,bulk} \tan(k_s t_s)}{Z_{s,bulk} + iZ_d \tan(k_s t_s)}, \quad (7)$$

where Z_d is the effective impedance of the structure underlying the film. In the present case, see Fig. 2(a), the entire structure is formed by the superconducting film, of surface impedance $Z_{s,bulk}$, and the insulating substrate and the backing metal support, of surface impedance Z_d .

For thin samples $t_s \leq \delta_{eff}$, the above expression simplifies to (“thin film” regime):

$$Z_{s,film} = \frac{\tilde{\rho}}{t_s}, \quad (8)$$

and $Z_s \simeq Z_{s,film}$ within a% for $\delta_{eff} > t_s/3$ [36]. Although the measured quantity is Z_s , it should be stressed that the connection with the intrinsic physics and to the material parameters depends on the EM regime. In the present study, our samples cross from bulk to effective to thin film regime depending on the (T, B) region explored, see Sec. III, so that the vortex motion parameters should be derived with care.

C. Experimental method

The experimental technique exploits a dielectric-loaded cylindrical resonator used in the so-called surface-perturbation approach [37]: the flat sample under investigation is placed on one of the bases, a mask with a hole exposes to the probing microwaves a circular portion of the sample surface. For each resonant mode, the changes in the quality factor Q and resonant frequency ν_r of the resonator

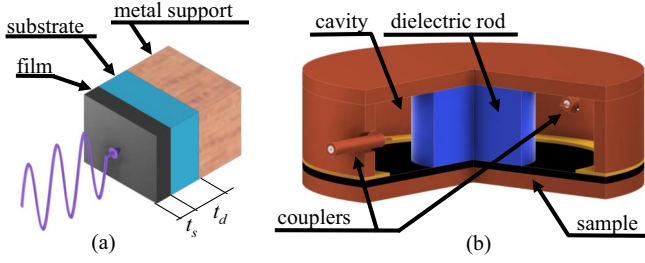


FIG. 2. (a) sample layered structure with thin film over an insulating substrate, backed by a bulk metal; (b) sketch of the dielectric-loaded resonator used as measurement cell.

induced by the application of the static magnetic field H yield the changes in the surface impedance [37]:

$$\Delta Z_s(H) = Z_s(H) - Z_s(0) = G \left[\frac{1}{Q(H)} - \frac{1}{Q(0)} - 2i \frac{\nu_r(H) - \nu_r(0)}{\nu_r(0)} \right], \quad (9)$$

where G is a mode and geometry dependent, numerically computed geometrical factor. Similarly, the temperature dependence of the surface resistance at zero field is

$$R_s(T) = \frac{G}{Q(T)} - \text{bkg}(T) \approx G \left[\frac{1}{Q(T)} - \frac{1}{Q(T_{\min})} \right], \quad (10)$$

where “bkg(T)” denotes the resonator contribution to the losses, and it depends in general on T but not on H since the resonator is built with nonmagnetic materials (for the same reasons no background terms appear in Eq. (9)). In limited T ranges as those involved in the present measurements, the background term has negligible T variation and thus can be estimated with $Q(T_{\min})$, since at the minimum measurement temperature $T_{\min}$ the superconductor $R_s(T_{\min}) \approx 0$ within the measurement uncertainty. The specific resonator used [sketch drawn in Fig. 2(b)] is made of oxygen-free high conductivity copper loaded with a sapphire dielectric rod having diameter 7.13 mm and height 4.50 mm. It is operated in transmission at two distinct transverse electric modes TE_{011} and TE_{021} , with resonant frequencies $\nu_{r,1} \approx 16.5$ GHz and $\nu_{r,2} \approx 26.7$ GHz, respectively. Both induce circular current patterns on planes parallel to the sample. The mask hole has diameter $2R_{\text{mask}} = 17.0$ mm, hence the geometrical factors $G_1 = 563 \Omega$ and $G_2 = 1944 \Omega$ for the TE_{011} and TE_{021} modes, respectively. Measurements are performed in a He-flow cryostat with a conventional electromagnet yielding fields up to 1.25 T, applied perpendicularly to the sample surface. $\mathbf{H}$ and $\mathbf{J}_{\text{mw}}$ are perpendicular, resulting in the maximum Lorentz force configuration. The resonator parameters Q and ν_r are obtained through fits of the complex scattering coefficients $S_{ij}(\nu)$ of the resonator, measured with a vector network analyzer (VNA). The VNA output power was set to 1 mW, and the expected dissipated power inside the resonator is

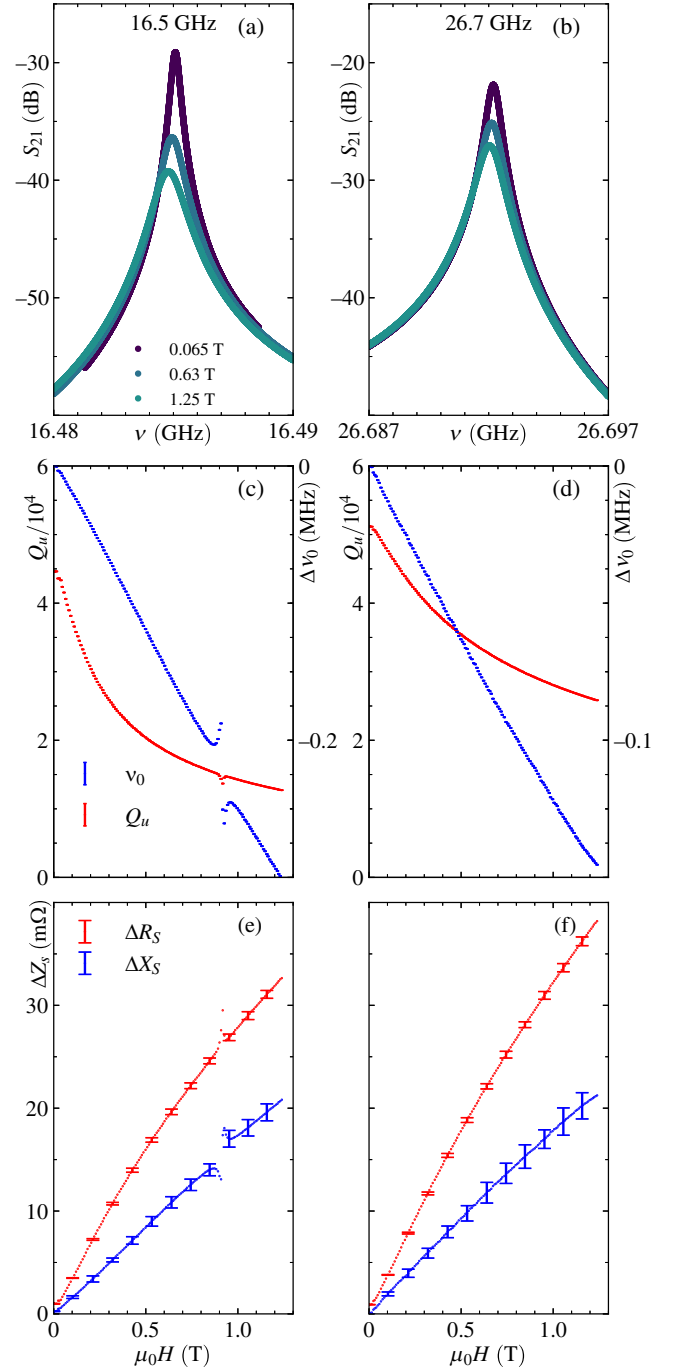


FIG. 3. Panels (a) and (b): $|S_{21}|$ vs ν at selected fields for the two resonant modes, respectively. Panels (c) and (d): field-induced quality factors and resonant frequency shifts with respect to zero-field values $\nu_{0,1} = 16.48508$ GHz and $\nu_{0,2} = 26.69221$ GHz, respectively. The corresponding surface impedance variations vs $\mu_0 H$ are shown in panels (e) and (f). Measurements are taken at $T = 5.03$ K. Error bars represent standard deviations with $K = 1$ (see text).

approximately 8 μW , thus yielding a peak current density at 16.5 GHz of $J_{\text{rf}} \sim 100 \text{ A cm}^{-2}$, which is definitively consistent with the linear regime vortex displacement

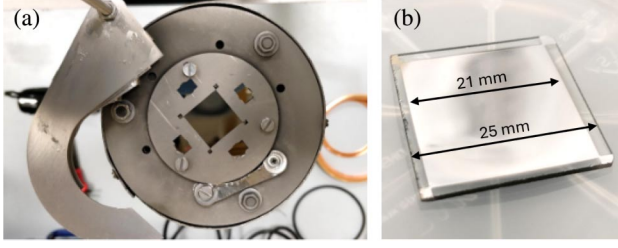


FIG. 4. (a) Sample holder with fan-shaped shutter in the open position. An AISI304 mask was used to clamp the substrate to the sample holder and produce a 21 mm square film (b) with four 2.0 mm squares in the corners, used for the electrical connection for dc measurements.

assumption. Nonidealities, such as partial calibration of the microwave line and cross couplings between the resonator ports, are carefully taken into account [38]. The uncertainty on Q and ν_r is dominated by such effects, with the following estimates for the maximum relative errors: $\text{err}_Q = 0.02$ for Q and $\text{err}_{\nu_r} = 3 \times 10^{-6}$ on ν_r [39]. The derivation of the uncertainty of $\Delta Z = Z(H) - Z(0)$ has been obtained by taking into account the correlation, with maximum correlation for resonant curves with similar $\{Q, \nu_r\}$, the statistical distribution for G with relative standard deviation of 1% [39], and making use of extensive Monte Carlo simulations. The full procedure will be the subject of a separate publication [40]. Figure 3 shows graphically the various steps of the process.

D. Sample preparation and characterization

Square films of side 21 mm have been grown by magnetron sputtering on square quartz substrates of side length 25 mm and thickness $t_d = 1.2$ mm. The samples were prepared using the same setup and deposition parameters used in the production of the Nb-Ti superconducting haloscopes of QUAX experiment [9,13]. Specifically, the following deposition parameters were used: target composition $\text{Nb}_{0.38}\text{Ti}_{0.62}$, process current of 1 A, process voltage between 410 and 450 V, process gas (Ar) pressure of 8.0×10^{-3} mbar, process temperature of 550 °C, target-substrate distance of 9.0 cm. The target is conditioned for 1 min before starting the Nb-Ti coating process, protecting the substrate with a fan-shaped shutter (see Fig. 4). The shutter is then closed while keeping the discharge on, and sputtering is performed for 30 min. The coating composition analysis by EDS shows that the samples have a 46% wt Ti (62.3% at. Ti), identical to target composition and in line with optimal alloys for cables [41] yielding maximum upper critical field [42]. Cross-sectional scanning electron microscopy (SEM) reported in Fig. 5 gave sample thickness $t_s = 2.2(2)$ μm , with inhomogeneity in the thickness profile within approximately 10%. Two identically prepared samples were devoted to microwaves characterization.

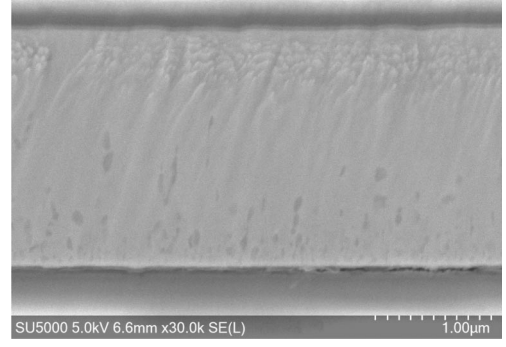


FIG. 5. SEM cross section of Nb-Ti sample, where the sample thickness is obtained.

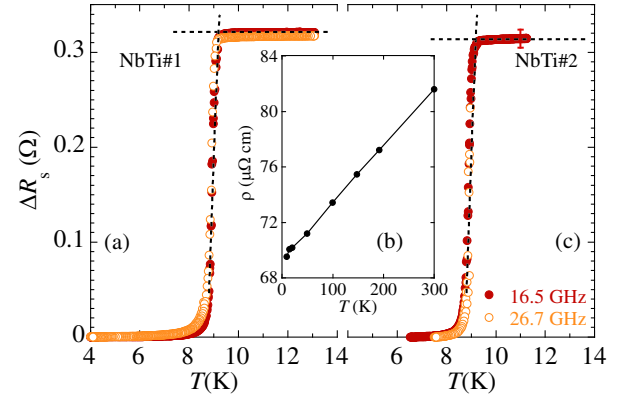


FIG. 6. Superconducting transitions at 16.5 and 26.7 GHz in zero dc magnetic field. (a) Sample Nb-Ti#1. (c) Sample Nb-Ti#2. (b) dc normal state resistivity ρ_{dc} vs T , on sample Nb-Ti#1. Dashed lines exemplify the determination of T_c .

The preparation procedure yields very reproducible results, as demonstrated by the microwave superconducting transitions in Fig. 6, and by the vortex motion parameters reported in Fig. 12(b) and in the inset of Fig. 16. In the following sections, we focus the discussion on sample Nb-Ti#1, being the behavior of the second sample nearly identical. The superconducting transition was checked by measurements of the surface resistance $\Delta R_s = R_s(T) - R_s(T_{\min}) \simeq R_s(T)$ (with $T_{\min} = 4.2$ K) at two frequencies, at zero field, as reported in Fig. 6. We find $T_c = 9.15$ K in both samples, and the normal state surface resistance $R_n = R_s(T \gtrsim T_c) \simeq 0.32(1)$ Ω and $R_n = R_s(T \gtrsim T_c) \simeq 0.31(1)$ Ω , for samples Nb-Ti#1 and Nb-Ti#2, respectively, independent from ν , thus confirming that the sample is in the thin film regime for $T > T_c$. From Eq. (8), we obtain $\rho_n(T_c^+) = 70(6)$ $\mu\Omega\text{cm}$ and $68(6)$ $\mu\Omega\text{cm}$, with the uncertainty dominated by the uncertainty on the film thickness. The corresponding normal state skin depth is $\delta_n = 3.2(2)$ $\mu\text{m} > t_s$ at $\nu_{r,1} = 16.5$ GHz. Dc resistivity measurements have been performed between 9.5 and 300 K using the van der Pauw four-point method, see Fig. 6(b). The obtained $R_n(9.5\text{ K}) = 0.31605(1)$ Ω , where

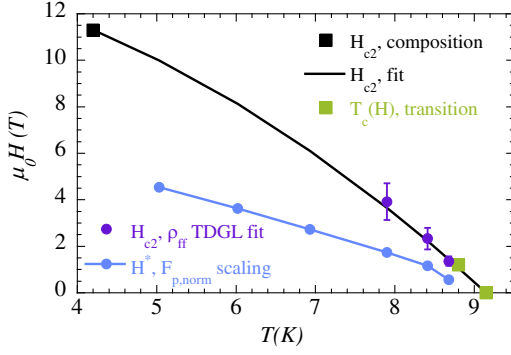


FIG. 7. Characteristic fields vs T for sample Nb-Ti#1. Black square: H_{c2} value for the sample composition as determined from T_c and ρ_n ; full line: H_{c2} extrapolation according to Eq. (11); green squares: $T_c(H)$ as determined from the transitions at $\mu_0 H = 1.2$ T (field cooling) and $\mu_0 H = 0$ T [Fig. 6(a)]. Circles data points are obtained from the scaling of the vortex parameters reported in Secs. IV B and IV C.

$\rho_{dc}(9.5 \text{ K}) = 69(6) \mu\Omega \text{ cm}$, is in full agreement with microwave measurements. The obtained $\text{RRR} = 1.17$ is well consistent with the literature data for alloys of same compositions [43]. T_c and ρ_n are in very good agreement with data reported in Ref. [42], attributed to the same composition here considered. The corresponding upper critical field $B_{c2}(4.2 \text{ K}) = 11.3 \text{ T}$ will be used for later elaborations, together with the empirical field dependence [44]:

$$B_{c2}(T) = B_{c2}(0)(1 - t^2), \quad (11)$$

where $t = T/T_c$ is the reduced temperature. $B_{c2}(T)$ as obtained from our data in Sec. IV B is reported in Fig. 7. The zero-temperature London penetration depth is calculated from T_c and ρ_n according to conventional superconductor electrodynamics [34,45]. In particular, in Nb-Ti $\lambda \propto \sqrt{\rho_n/T_c}$ [45,46]. The obtained value $\lambda_0 = 270 \text{ nm}$ is consistent with the literature, taking into account also compositional differences [15,43,47,48].

III. SURFACE IMPEDANCE MEASUREMENTS

Surface impedance variations induced by a static magnetic field H , $\Delta Z_s(H)$, have been measured in the T range between 5 K and T_c . H was applied perpendicular to the sample surface. The desired T was reached in zero-field cooling (ZFC). Selected measurements are reported in Fig. 8. Standard features can be recognized: ΔZ_s grows with H (except for ΔX_s near T_c), confirming a dominating vortex motion contribution; at a fixed field, ΔR_s increases with T , indicating increasing dissipation approaching T_c ; ΔR_s increases with the measuring frequency ν , consistently with high-frequency models for vortex dynamics as it can be derived from Eqs. (2) and (3). A small anomaly appears in ΔZ_s at $\mu_0 H \simeq 0.9 \text{ T}$ and $\nu = \nu_{r,1}$ due to an electron paramagnetic resonance (EPR) of paramagnetic

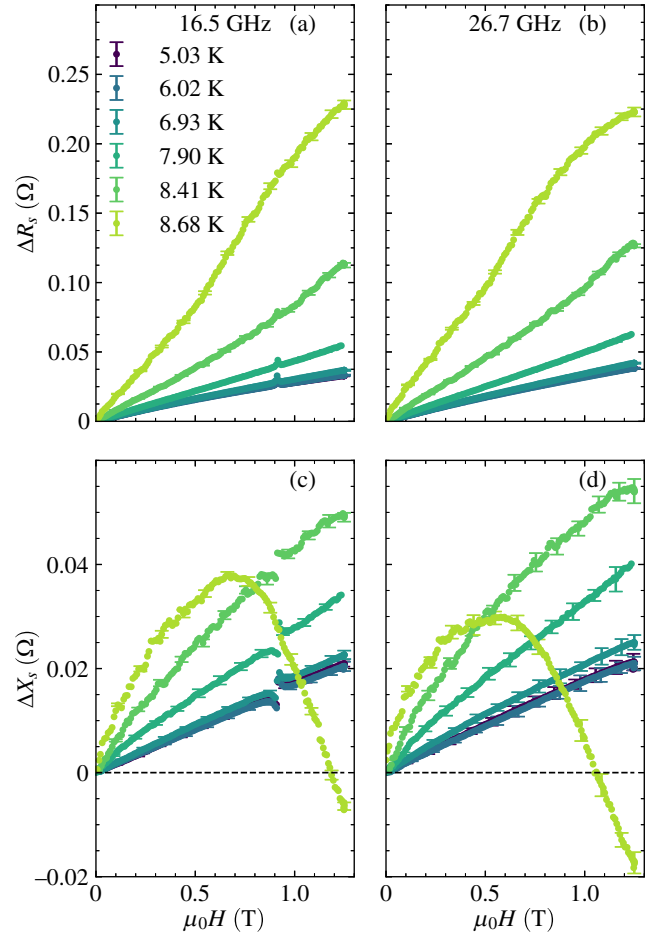


FIG. 8. $\Delta Z_s(H)$ at selected T at the two measurement frequencies. The anomaly at 16.5 GHz around 1 T is due to a paramagnetic resonance in the sapphire rod.

impurities in the sapphire rod, which can be ignored. In the subsequent elaboration of Sec. IV, the corresponding data will be grayed, with the purpose of reminding that the data analysis for those data point is not reliable. The H dependence of $\Delta Z_s(H)$ evolves with T . The sublinear $\Delta Z_s(H)$ at low T is reminiscent of $\Delta Z_s \propto \sqrt{B}$, i.e., the bulk expression with $\rho_{ff} \propto B$ as the dominant field dependence ($B = \mu_0 H$ in the London approximation). Near T_c , ΔR_s is almost $\propto H$ while ΔX_s is nonmonotonic and negative at high fields: this is typical of the thin film regime ($\Delta Z_s \propto \tilde{\rho}$) with a pair-breaking induced increase of λ and thus a decrease of ΔX_s . This is a first important feature of the layered structure of the sample in the T, ν range explored at finite fields. At low H and T , the EM field penetration depth $\delta_{\text{eff}} \ll t_s$, but at higher fields and/or temperatures a crossover to thin film behavior ($\delta_{\text{eff}} \gg t_s$) takes place. The vortex state penetration depth δ_{eff} can be computed through Eq. (6a): we use ΔZ_s from the experiment, and $Z_s(H=0)$ calculated from Eq. (5) to obtain $Z_s = \Delta Z + Z_s(H=0)$. The result is shown in Fig. 9, where $\delta_{\text{eff}}(H, T)$ is compared with t_s and $t_s/3$, see

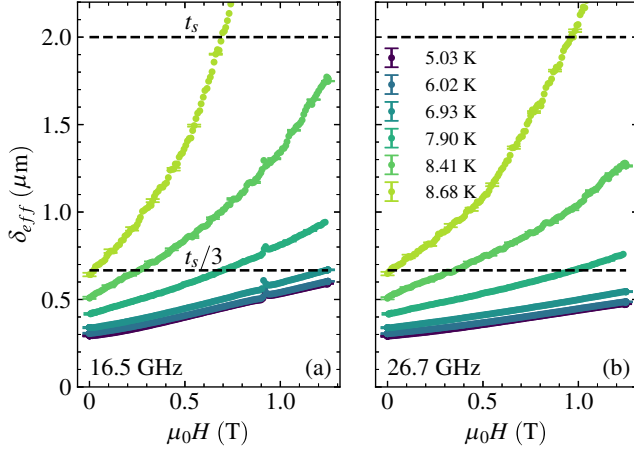


FIG. 9. Effective skin penetration depth δ_{eff} vs H at various T . (a) calculated at 16.5 GHz; (b) calculated at 26.7 GHz. Reference values t_s and $t_s/3$ are shown.

comment to Eq. (8). Figure 9 exemplifies how the EM regime crosses from electromagnetically thick sample (bulk) (fully attenuating the EM field within the sample thickness), $\delta_{\text{eff}} < t_s/3$, to thin film $\delta_{\text{eff}} > t_s$ (thus allowing the EM field to reach the underlying substrate), with increasing (H, T). Finally, we mention that a small but detectable hysteresis is observed at low fields, below 0.5 T, and at low temperatures, below 7 K, see Fig. 10. This is due to the building up of a critical state, with nonuniform field penetration. An extensive treatment of the hysteresis will be given elsewhere. The hysteresis can be well described by a Bean model for the penetration of the field.

IV. THE VORTEX PARAMETERS

The raw experimental data $\Delta Z_s(H)$ can be sometimes sufficient for specific applications at the same frequency, but the underlying physics is expressed by the vortex parameters: ρ_{ff} , ν_p , k_p , and χ . The derivation of the vortex parameters from $\Delta Z_s(H)$ requires the use of the electromagnetic and resistivity models discussed in Sec. II [(Eqs. (2), (3), (7) and ancillary equations)]. The derivation of three vortex parameters, $\{\rho_{\text{ff}}, \nu_c, \chi\}$, from four observables (ΔR_s and ΔX_s at two frequencies) using Eqs. (2), (3), and (7) is a nonlinear problem that cannot be analytically solved: a numerical fit must be performed, whose stability and reliability must be carefully controlled and evaluated. The procedure is shortly described in the Appendix, and the full details will be presented elsewhere [40]. The obtained fluxon parameters as a function of the field $\mu_0 H \simeq B$ (London approximation) and for all the temperatures T are reported and discussed in the following subsections.

A. Creep

As reported in the Appendix, the data provide the creep values for each T , along with their maximum error, as

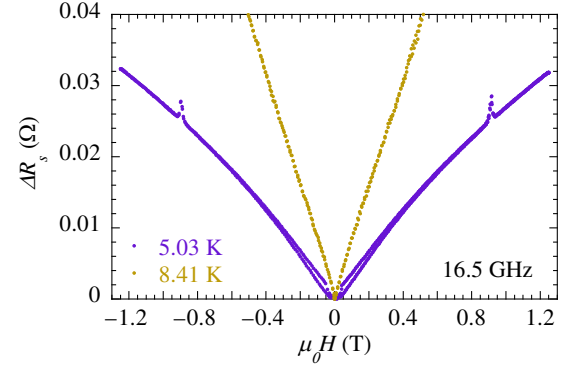


FIG. 10. $\Delta R_s(H)$, at 5.03 K and 8.41 K in sample #1, measured by cycling $\mu_0 H$ from 0 T up to +1.25 T, then down to -1.25 T, and then to 0 T again. Hysteresis is detectable at the lower T but not at the higher T .

reported in Fig. 11, across the entire range of measured magnetic fields. The procedure is not able to capture a potential field dependence. Such dependence, if existing, is contained within the reported uncertainty ranges. Negligible/zero creep is observed at the lowest T , consistently with what expected in low- T_c superconductors, even in the microwave regime. It is noticeable that creep at microwave frequencies is always present in the high- T_c superconductors, instead [49–52]. The appearance of a possibly finite creep, above 7 K, is signaled by the disappearance of the magnetic hysteresis at low fields. As expected, creep increases with increasing T . From our data, we can conclude that creep is negligible below 7 K, but can be finite above that temperature, that is for $t = T/T_c > 0.75$. This is, to our knowledge, the first report of a possible effect of flux creep at microwave frequencies in Nb-Ti. As in high- T_c superconductors, although at larger reduced temperatures, creep effects are detectable in a wider temperature range than in dc measurements. This result can be explained in terms of a pinning landscape made up of many small pinning wells, of reduced height, superimposed to larger pinning wells. Microwave frequencies imply very small oscillations induced by the Lorentz force, of less than a nm, while dc measurements imply large displacements on long timescales. Thus, microwave currents are more sensitive, for creep effects, to smaller pinning wells. This process is nonetheless less pronounced in Nb-Ti than in, e.g., $\text{YBa}_2\text{Cu}_3\text{O}_7$ [49–52] or $\text{Fe}(\text{Se}, \text{Te})$ [51], and it appears at higher t , possibly pointing to a more uniform pinning landscape.

B. Flux flow resistivity

The field dependence of ρ_{ff} gives valuable insight in the microscopic state. The temperature evolution of $\rho_{\text{ff}}(H)$ is reported in Fig. 12(a). At low temperatures, $\rho_{\text{ff}} \propto H$. With increasing T , starting from 7.9 K, an upward curvature develops, consistently with experimental observations at

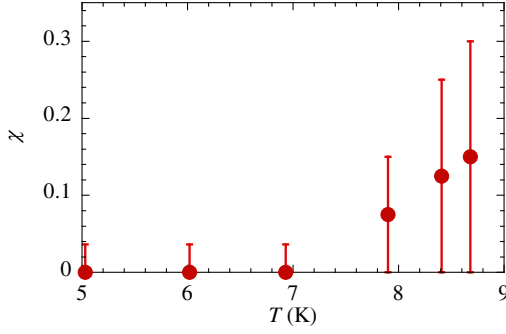


FIG. 11. Creep factor vs temperature T , with uncertainty bars indicating maximum error. Any potential field dependence is delimited within the uncertainty range provided.

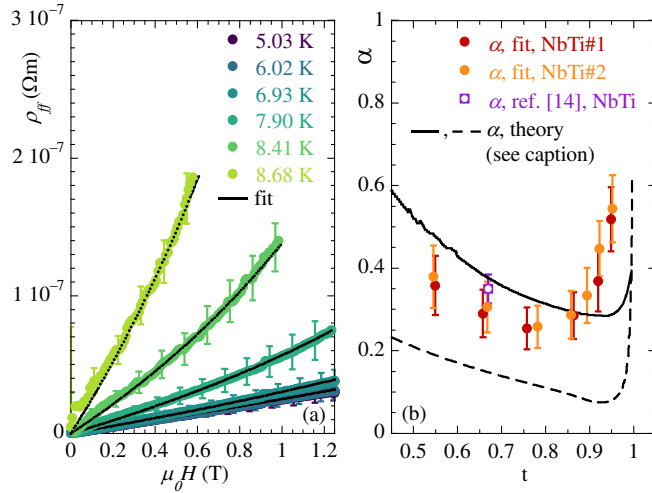


FIG. 12. Panel (a): flux flow resistivity ρ_{ff} vs H at various T in sample Nb-Ti#1 (symbols) and corresponding TDGL fits (continuous lines) according to Eq. (12). Panel (b): fit parameter α vs T/T_c in samples Nb-Ti#1 and Nb-Ti#2, experimental datum of a Nb-Ti with different composition (Ti 69% at.) [15], and comparison with theoretical models (continuous and dashed lines, [27] and [55], respectively). $B_{c2}(T)$ values are reported in Fig. 7.

increasing $b = B/B_{c2}$ [15,53,54], and with the predictions of the TDGL theory [26,27] in the appropriate $T - B$ ranges. The TDGL theory yields the following expression [53,56] for the flux flow resistivity:

$$\rho_{ff} = \rho_n \frac{\alpha B}{(\alpha - 1)B + B_{c2}},$$

$$\rho_{ff} \simeq \rho_n \alpha \frac{B}{B_{c2}} \quad \text{for } B/B_{c2} \ll 1, \quad (12)$$

where α depends on the microscopic mechanisms governing dissipation due to vortex motion. In the simplest Bardeen and Stephen regime [28], $\alpha = 1$. By fitting the experimental data to Eq. (12), it is possible to reliably extract both α and B_{c2} only at the highest temperatures,

from 7.9 K. At lower temperatures, where $\rho_{ff}(H)$ is almost linear in H , we use the theoretical values for $B_{c2}(t)$ according to Eq. (11), see Fig. 7, and use α as the only fitting parameter. Fit results are shown in Fig. 12(a) as black continuous lines, and α and B_{c2} are reported in Figs. 12(b) and 7, respectively. It can be seen that the directly available values (fitted and experimental) for B_{c2} nicely follow the theoretical curve. The coefficient α exhibits a non-monotonic temperature dependence, with a minimum at $T/T_c \approx 0.8$. In the temperature range $T/T_c = 0.5$ to 0.95, the attained numerical values around 0.4 correspond to the theoretical estimate for the intermediate T range, irrespective to the material considered [53]. In the first studies, the flux flow resistivity in Nb-Ti [14] minima in $\rho_{ff}(T)$ was observed at $t = 0.65$. These results had a tentative theoretical explanation within a model which foresaw an additional contribution to the viscosity $\eta = \Phi_0 B / \rho_{ff}$, explained in terms of a dissipation channel connected to entropy currents excited by moving vortices [57,58]. However, this result came from a model for an extremely dirty superconductor, with $\xi_0/\ell \gg 1$ (with ξ_0 and ℓ the coherence length and mean free path, respectively), to yield a significant excess η [57]. This is not the present case: the samples considered here are in the moderately dirty regime, as can be inferred from the estimate $\xi_0/\ell = (\rho_n/\lambda_0^2)(0.18\hbar)/(\mu_0 k_B T_c) \sim 1$, obtained using the BCS expression $\xi_0 = 0.18\hbar v_F/(k_B T_c)$ (being v_F the Fermi velocity, k_B the Boltzmann constant, and $\hbar$ the reduced Planck constant), $\rho_n = m/(ne^2\tau)$ (with m , n and e the charge carrier mass, density and charge, respectively; $\tau = \ell/v_F$ the scattering time), $\lambda_0^2 = m/(ne^2\mu_0)$ [34]. The field and temperature dependence of ρ_{ff} has been the subject of an extensive theoretical work for superconductors in the dirty limit. While many regimes were explored (e.g., $T \rightarrow 0$, $T \rightarrow T_c$, intermediate T range, magnetic or non-magnetic scatterers) [26,27,55,59,60], no unified picture emerged. When the theoretical predictions are expressed in terms of α , in the intermediate T range interpolating expressions have been worked out [27], showing a marked minimum in $\alpha(T)$ at a temperature T^* , such as $(1 - T^*/T_c) \sim \hbar\tau_{ph}^{-1}/(k_B T_c)$, with τ_{ph} the inelastic electron-phonon scattering time. More accurate evaluations of the crossover between the two regimes gave a less pronounced minimum in $\alpha(T)$ [55]. The theoretical curves for those models are plotted in Fig. 12(b) as continuous [27] and dashed [55] lines, showing that both models approach the experimental behavior. From the experimental temperature for the minimum in α , $T^*/T_c = 0.82(4)$, we estimate $\tau_{ph} \sim 0.5 \times 10^{-11}$ s. This result is consistent with the values derived in [55] for experimental data in Nb [61,62]. It is worthwhile to observe that the determination of the relaxation time of QP in the vortex state is recently subjected to intense research efforts for the assessment of vortex motion speed limits for single photon detectors [63]. The present analysis can be an alternative way to flux flow instabilities

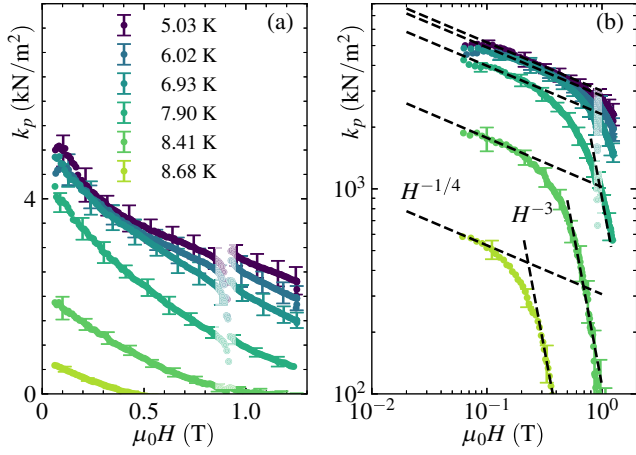


FIG. 13. Pinning constant k_p vs $\mu_0 H$ at various T , linear and log-log scales in panels (a) and (b), respectively. Low field data ($\mu_0 H \leq 0.1$ T) are affected by a large uncertainty due to hysteresis, and they are not reported.

measurements [64] for the determination of τ_{ph} . Summarizing, the H and the T dependence of ρ_{ff} as measured in the Nb-Ti sample have been determined to be in excellent agreement with the existing theoretical models, and an estimate of the inelastic electron-phonon scattering time τ_{ph} has been given on the basis of the experimental results for $\rho_{ff}(H, T)$. In the following subsection, we present and discuss the results on the pinning constant.

C. Pinning constant

The data for the pinning constant k_p are shown in Fig. 13, as a function of the applied field. We focus first on the temperature T and field H dependencies, and then, we comment on absolute values. At all T , k_p decreases with H . A field dependence of k_p implies a dependence on the flux density, that is fluxon-fluxon interactions play a dominant role. Thus, our data rule out an individual-vortex-pinning regime, where vortex pinning is due solely to the interaction between the vortex and uniformly distributed dense pins [3], even at low fields, contrary to what observed in high- T_c superconductors with second phase additions [52,65–69]. The actual field dependence arises from a complex combined effect of several factors: (i) the geometry and density of pins, (ii) the density of vortices $\propto B$, (iii) the specific vortex-pin and vortex-vortex interactions, influenced by the flux line lattice elasticity. The modeling of the process is the so-called summation problem [70–72]. A guide to understanding the pinning processes in place is to find power laws $k_p \propto H^{p'}$. The log-log plot of Fig. 13(b) shows $p' = -1/4$ at low fields, with a crossover to $p' = -3$ at high fields. The large high field exponent is different from $p' = -1/2$, as expected for a periodic, sinusoidal pinning potential [3,73]. It should be noted that

the field dependence of k_p is not in general addressed by the few models for the short-range (≤ 1 nm) oscillating motion of vortices driven by microwave currents [3,49,74,75]. In order to put our findings within a theoretical framework, we relate k_p to the critical current density J_c . In so doing, we recall that k_p is the elastic recall constant for short-range vortex oscillations, and thus, a measure of the curvature of the pinning potential minima, while J_c is the current density yielding a Lorentz force capable of overcoming the maximum pinning force and to set the flux line lattice (FLL) into motion, thus related to the height of the pinning potential. Assuming a pinning force linear with the vortex displacement from the pinning sites up to a cutoff, maximum pinning range r_p , one can estimate a maximum force (per vortex per unit length) $f_p = k_p r_p$ before depinning, to equate to $f_p = J_c \Phi_0$ [76]. With a pinning range $r_p \sim \xi$ [76] one gets:

$$k_p = \frac{J_c \Phi_0}{\xi}. \quad (13)$$

Equation (13) yields an estimate of J_c from microwave k_p data, $J_{c,mw}(1 \text{ T}, 5.03 \text{ K}) = 8.6 \text{ kA/mm}^2$, whereas from magnetization measurements in Nb-Ti strands one has $J_c(1 \text{ T}, 4.2 \text{ K}) = 9.2 \text{ kA/mm}^2$ [77]. These values are well consistent, despite slightly different T and differences in the samples. Thus, Eq. (13) can be used to reasonably describe the pinning properties of Nb-Ti. The class of models for J_c that can be invoked should take into account the different dynamic regimes involved. For example, J_c limited by the FLL breaking down due to excessive shear stress, as in [70], cannot be a mechanism acting in the microwave regime where only short-range vortex displacements occur. Even if the FLL does not break down, its nonlocal elasticity [72] can yield different elastic responses in the two distinct dynamical regimes which occur at dc and microwaves. Instead, models dealing mainly with energy balance considerations between pinned and unpinned states, such as the one provided in the seminal paper [71], can be tested with more confidence also at microwaves. To do so, it is useful to cast J_c into the volume pinning force $F_p = J_c B$, so that:

$$F_p(T, B) = \frac{k_p(T, B) \xi(T) B}{\Phi_0}. \quad (14)$$

With a single type of pinning mechanism, only one field scale dictates the behavior of $F_p(H)$, and it takes the well-known, empirical scaling form [71,78]:

$$F_p(T, H) = A H_{c2}^n (h)^p (1 - h)^q, \quad (15)$$

where A is a constant dependent on the material characteristics, H_{c2}^n sets an overall scale for the pinning strength related to the condensation energy, $n = 2 - 3$ [71], and

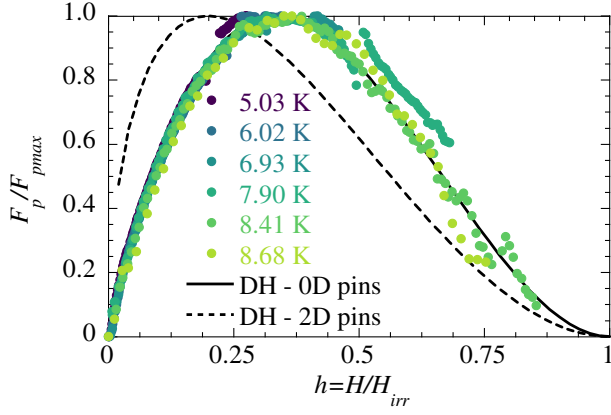


FIG. 14. Normalized pinning force $F_p/F_{p,\max}$ vs normalized field $h = H/H^*$.

$h = H/H^*$ is a suitably reduced magnetic field such that $h = 1$ sets the disappearance of vortex pinning, $F_p = 0$. In low- T_c , metallic superconductors, one often has $H^* = H_{c2}$, but this finding is not universal and $H^* < H_{c2}$ is found [79]. In high- T_c superconductors one has $H^* = H_{\text{irr}} \ll H_{c2}$, being H_{irr} the irreversibility field. The pinning mechanism is captured by the exponents p and q . In the DH model [71], it is possible to isolate core pinning or $\Delta\kappa$ pinning (κ is the Ginzburg-Landau parameter), that is pinning by normal/insulating regions or by inhomogeneities in the material parameters, respectively, and pins dimensionality (volume, surface, point). For example, and relevant for the remainder of the discussion, $p = \{0, 0.5, 1\}$ with $q = 2$ correspond to core pinning by volume, surface and point pins, respectively. In Fig. 14, we present the entire set of data scaled together. A satisfactory scaling takes place, with $H^* < H_{c2}$, see Fig. 7. In Nb-Ti, an irreversibility field H_{irr} smaller than H_{c2} has been observed by reversible magnetization measurements [79], with $H_{\text{irr}}/H_{c2} = 0.7\text{--}0.8$ [80] near T_c , reasonably consistent with the present data (see Fig. 7). A quantitative fitting of the scaled data by Eq. (15) yields $p = 1$ and $q = 2$, continuous curve in Fig. 14. Such fitting points to a dominant role of point pinning: point pins are most likely due to α -Ti precipitates, known to form at the present 46% wt Ti concentration [81], analogously to what observed in similar Nb-Ti samples [15]. The consistency of the microwave data with the microstructure rules out a relevant role of grain boundaries as planar pinning centers effective at microwave frequencies for the present samples in the low-medium T range. This feature can be understood either with excellent connection of superconducting grains that leads to a negligible role of grain boundaries as opposed to point pins, or to very fine and small grains with well coupled grain boundaries, smaller or of the order of the coherence length that are thus seen as point pins by the vortex cores. The low temperature (5 K to 7 K), low field data could also be scaled onto a different curve (dashed

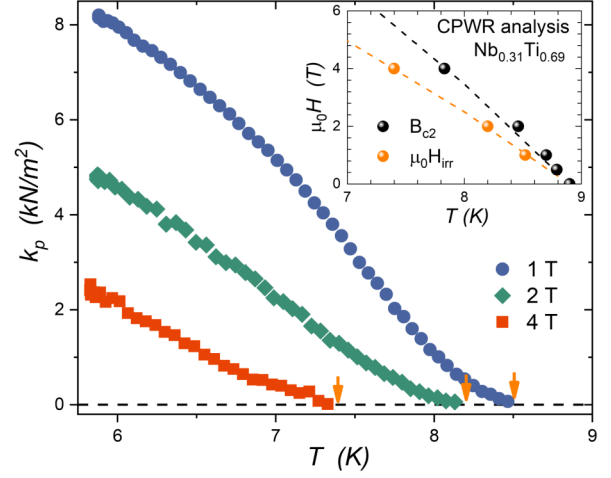


FIG. 15. Pinning constant as a function of temperature as obtained for a Nb-Ti film with Ti 69% at. by a coplanar waveguide resonator analysis, at different values of the applied dc field, listed in the legend. Arrows indicate the temperatures where $k_p \rightarrow 0$, useful to deduce the irreversibility line $\mu_0 H_{\text{irr}}(T)$ shown in the inset (orange points). In the inset, the upper critical field $B_{c2}(T)$ for the same sample is also reported (black points); dashed lines follow a $[1 - (T/T_c)^2]$ dependence.

curve in Fig. 14) with $p = 0.5$ (2D surface pins), which would require a nonphysical large scaling field, exceeding H_{c2} .

Finally, as a further confirmation of the observed $H_{\text{irr}} < H_{c2}$, we report additional microwave results obtained on a Nb-Ti film of different composition (Ti 69% at.), grown in the same laboratory and with the same technique as the ones discussed here (full details are in Ref. [15]). Measurements were performed through a coplanar waveguide resonator (CPWR) technique [82] at 11 GHz. The irreversibility line $\mu_0 H_{\text{irr}}(T)$ is obtained from $k_p(T)$ curves at selected applied fields (Fig. 15), by identifying the temperatures where k_p goes to zero. The results, depicted in the inset of Fig. 15, clearly show an irreversibility line distinct from $B_{c2}(T)$. The dashed lines in the inset follow a $\sim[1 - (T/T_c)^2]$ temperature dependence, analogous to the one reported in Fig. 7 for $B_{c2}(T)$.

D. Characteristic frequency

We conclude the discussion of the vortex parameters with the characteristic frequency ν_c (relevant for applications) and the pinning frequency ν_p (more directly relevant for the physics). The data are plotted in Fig. 16. The characteristic frequencies ν_c and ν_p are quite similar. This is a consequence of a small creep factor, since $\nu_c \sim \nu_p$ for $\chi \ll 1$ [1,24,25]. The field and temperature dependence of ν_c and ν_p arise from a combination of $k_p(H, T)$ and $\rho_{\text{ff}}(H, T)$ dependencies, see Sec. II. Since k_p and ρ_{ff} have different temperature evolution, nonmonotonous $\nu_c(T)$

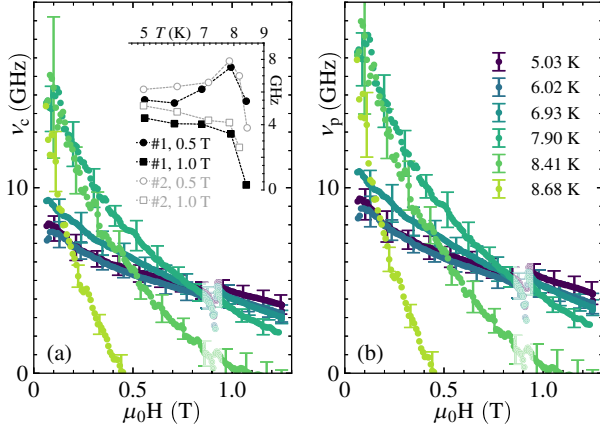


FIG. 16. (a) Characteristic frequency ν_c and (b) pinning frequency (ν_p) vs μ_0H at various T , for sample Nb-Ti#1. Inset: ν_c at fixed fields and as a function of T for samples Nb-Ti#1 (full symbols) and Nb-Ti#2 (open symbols).

and $\nu_p(T)$ can be observed, as reported in the inset of Fig. 16 for both samples examined. Regarding the absolute values, $\nu_p \simeq \nu_c \sim 4.5$ GHz is observed at 1 T and 5 K, consistent with $\nu_p(1 \text{ T}; 6 \text{ K}) \simeq 8.0$ GHz observed in sputtered films of Nb_{0.31}Ti_{0.69} at. %, with a different composition [15]. This is, however, in contrast with the estimation made in [14,83] for Nb-Ti wires, $\nu_p < 1$ GHz at 2.2 K. Apart from the different material involved, such discrepancies may come from very different pinning centers, and to the refined analysis here reported.

V. CONCLUSIONS

In this work, multifrequency measurements of the surface impedance of Nb-Ti thin films in the mixed state have been thoroughly analyzed within high-frequency vortex dynamics models, taking into account finite thickness effects and field-induced pair breaking. From the perspective of fundamental vortex physics, a complete and comprehensive understanding of the microwave properties of this compound in the mixed state emerged. The various mechanisms governing the mixed state response of Nb-Ti have been identified. The flux flow resistivity has been found to behave according to established theories. Flux creep, which appears to be ubiquitous in high- T_c superconductors, has been instead shown to be negligible apart from temperatures close to T_c . The quantitative study of the pinning constant with the help of a scaling function has shown that pinning, in the low-medium T range, is predominantly due to point pinning, presumably due to α -Ti precipitates. The role of grain boundaries is qualitatively possible only as very thin boundaries between fine grains, so that the resulting grain boundaries are smaller than the coherence length and thus appear as point pins. The values for the pinning frequency are approximately 6 GHz, with a marked field suppression. The values here

determined correct initial estimations in the historical literature and confirm recent results in similar samples [15]. Finally, the complete data set reveals that vortex motion dominates both dissipation and the increased penetration of the electromagnetic field in a Nb-Ti coating, outweighing pair breaking at fields $H \ll H_{c2}$. This must be considered when selecting coatings thickness for haloscopes. This study fills a shortage of data in the literature and gives quantitative data for the vortex parameters. Such data will form a founding part in the knowledge base to design devices, such as haloscopes, based on Nb-Ti technology.

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DATA AVAILABILITY

The data that support the findings of this article are not publicly available upon publication because it is not technically feasible and/or the cost of preparing, depositing, and hosting the data would be prohibitive within the terms of this research project. The data are available from the authors upon reasonable request.

APPENDIX: DERIVATION OF VORTEX PARAMETERS AND THEIR UNCERTAINTIES

In the fitting process of $\Delta Z_s(H)$, we use values of t_s and λ_0 as given in Sec. II D. We perform a fit of the whole set of data (four observables at each temperature and field) using the CC model for ρ_{vm} , Eq. (2), which requires the three fit parameters $\{\rho_{ff}, \nu_c, \chi\}$. The fit is performed by means of the nonlinear least squares Levenberg-Marquardt algorithm (Python libraries [84]). We have performed a thorough study of the behavior of the fit objective function (Cartesian distance between the experimental and computed ΔZ_s for both frequencies, for each field value), whose details are outside of the scope of this paper and they will be reported elsewhere [40]. The main outcome is that the objective function has two distinct minima: a “low-pinning–low-creep,” with zero creep at low T and moderate ν_c of a few GHz, and a “high-pinning–high-creep” solution, with large $\chi \sim 0.5$ and a high ν_c of several tens of GHz. An accurate, lengthy numerical study allows to infer that the “low-pinning–low-creep” is to be preferred, but in the present case the same conclusion can be drawn more directly by observing that the finite hysteresis at low T is not compatible with a large creep factor on physical grounds: $\chi \sim 0.5$ implies nearly free fluxons, thus ruling out a significant hysteresis. We are then led to exclude the “high-pinning–high-creep” solution and obtain our set of parameters $\{\rho_{ff}, \nu_c, \chi\}$.

To evaluate the uncertainty on the vortex parameters, we must take into account the possibility that one or more of the experimental curves at each temperature is affected by some unknown error (this is particularly delicate for measurements of the surface reactance that involve tiny variations of the resonant frequency, see Eq. (9)). We then chose a round-robin method: we grouped the four curves in four groups, each time discarding one of the curves. A fit for each group was done. We obtained four values for the vortex parameters, dispersed around average values consistent with those obtained through the whole four-curves fit. The semi-dispersion of the fitting vortex parameters can thus be taken as a measure of the uncertainty due to the systematic error. It is found that ρ_{ff} is affected by a $\pm 4\%$ maximum error, consistently with the previous remarks about the sensitivity of the measured data on this quantity, whereas ν_c and k_p by a $\pm 20\%$ maximum error at all T and H . Such uncertainties, of systematic origin, affect basically the observable by a scale factor and do not change neither the field dependence nor, to a slightly lesser degree, the T dependence. Hence, the considerations about the H and T dependencies proposed in Sec. IV are unaffected, and the corresponding plots do not include this uncertainty contribution. Finally, the error bars in the vortex parameters are drawn to represent compound uncertainties with coverage factor $K = 1$. The uncertainty on χ basically masks any possible field dependence. Thus, it is reported as a function of the temperature only; the possible field dependence is contained in the uncertainty bars.

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