

Strong-coupling quantum thermodynamics using a superconducting flux qubit

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Thermodynamics in quantum circuits aims to find improved functionalities of thermal machines, highlight fundamental phenomena peculiar to quantum nature in thermodynamics, and point out limitations in quantum information processing due to the coupling of the system to its environment. An important aspect in achieving some of these goals is the regime of strong coupling, which has remained until now a domain of theoretical works only. Our aim is to demonstrate strong coupling features in heat transport using a superconducting flux qubit, which is capable of reaching strong- to deep-ultrastrong-coupling regimes, as shown in previous studies. Here, we show experimental evidence of strong coupling by observing a hybridized state of the qubit with two cavities coupled to it, leading to a triplet-like thermal transport via this combined system around the minimum energy of the qubit at power levels of tens of femtowatts, exceeding by an order of magnitude those in earlier experiments. We also demonstrate a nearly 100% on-off switching ratio of heat current mediated by photons by applying magnetic flux to the qubit. Our experiment opens a way towards testing debated questions in strong-coupling thermodynamics, such as what heat is in this regime. We also present a theoretical model that aligns with our experimental findings and explains the mechanism behind heat transport in our device. Furthermore, our experiment opens possibilities for quantum thermodynamics, aiming to realize true quantum heat engines and refrigerators with enhanced power and efficiency, by leveraging ultrastrong coupling between the system and its environment in future experiments.

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I. INTRODUCTION

Quantum thermodynamics (QTD) [1–4], or thermodynamics of quantum systems and processes, has strong roots in the theory of open quantum systems. Experimental work on QTD is limited, since access to true thermodynamic quantities, most importantly heat, is challenging due to the smallness of energies involved in these sensitive quantum systems [5–10]. Therefore, most experimental works are only indirect, in that they address observables not directly related to thermodynamics, like populations in the quantum systems themselves, and infer thermodynamic quantities like heat, work, and efficiencies based on the models believed to govern the system and its environment. Superconducting quantum circuits, combined with electronic on-chip reservoirs, form an exception in this respect,

since heat currents can be measured directly at low temperatures via thermometry, due to the efficient isolation and small heat capacities of the reservoirs [7–9]. Moreover, thanks to the rapid development of superconducting quantum circuits over the past few decades [11–14], they can be considered as almost ideal coherent quantum systems that then couple to the heat reservoirs; furthermore, accurate design and control of the circuits is achievable.

In order to realize true quantum heat engines and refrigerators [15–18], one needs to understand and optimize heat transport mediated by a quantum circuit, which is to be the working substance of the quantum thermal machine. Here the important aspect is the coupling between the quantum circuit, in most cases a qubit, and the reservoir [6,15]. The whole process of heat transport via the qubit is based on sequential excitation $|g\rangle \rightarrow |e\rangle$ and relaxation $|e\rangle \rightarrow |g\rangle$ events, where $|e\rangle$ and $|g\rangle$ represent excited and ground states of the qubit. Excitation removes heat equal to the qubit energy separation from the hot bath, and relaxation releases the same amount of heat to the cold bath [6,15]. The superconducting transmon [11,12,14,19] is the most studied qubit, but as in quantum information processing, it also has

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its limitations in thermal transport. For the transmon to retain its anharmonicity, an important characteristic of the qubit, it can be coupled only weakly to the remaining circuit [11,19]. On the other hand, the flux qubit [20–22], particularly in a galvanically coupled configuration [23–26], enables access to the strong- and ultrastrong-coupling regimes, as well as the deep-ultrastrong-coupling regime [27,28]. These coupling regimes open the way to increase power in thermal machines and, on the fundamental side, give the possibility of addressing what is called strong-coupling thermodynamics [15,29–39], but they have been out of reach of experiments up to now. From the conceptual point of view, basic questions, such as what heat is in the case of a strongly coupled system and reservoir, are important issues: in the weak-coupling theories, the energy associated with the coupling Hamiltonian can be ignored, but this is not possible for a strongly coupled system, such as the reported one.

In this work, we report on two devices: Device 1 and Device 2. Device 1 is a thermal device architecture that forms the core of the reported experiment. Device 2,

which plays a supporting role, is a spectroscopy-enabled device designed for performing qubit spectroscopy useful for complete understanding of the experiment. In Device 1, we experimentally investigate quantum heat transport between two reservoirs mediated by an interconnected artificial atom, which is a superconducting flux qubit. The concept of Device 1, illustrated in Fig. 1(a), depicts photons energized by Joule heating in a metallic reservoir transferring heat to a cooler region, but being blocked by a superconducting qubit, which, modulated by a weak magnetic field, acts as a controlled gate, allowing photon passage only at certain flux values. By varying the external magnetic field, we tune the qubit frequency accordingly with respect to that of the cavity, which allows us to control heat transport across the qubit [6,9]. The observed heat transport exhibits a triplet structure, with the central peak reaching its maximum at the degeneracy point, corresponding to half a flux quantum in a superconducting flux qubit, as depicted in Fig. 1(c). In an rf environment, utilizing a readout resonator, characteristic transmission across two ports of the spectroscopy-enabled device shown in

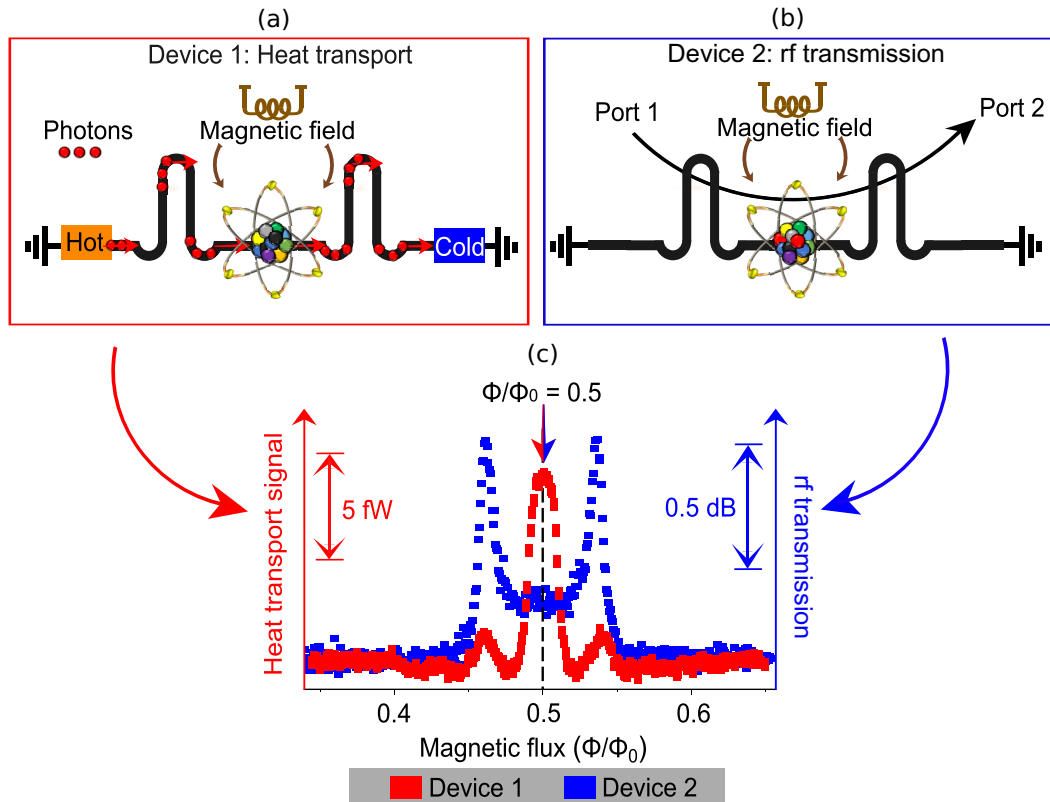


FIG. 1. Device concept. (a) A schematic showing thermal photons (red) generated by Joule heating in a metallic reservoir being blocked by an artificial atom as they travel toward the cold reservoir. The atom, tuned by a magnetic field, allows photon passage only close to half-flux quantum (indicated by red arrows). (b) Scheme for a two-port readout of a transmission signal in a spectroscopy-enabled device without any metallic reservoir. (c) A qualitative, comparative plot shows maximum heat transport signal (red) at half-flux quantum, with suppression when qubit and resonator frequencies align, as seen in the transmission signal (blue). The power scales on both x axes are provided for approximate reference and are not intended for direct comparison. Here, Φ is the applied flux and $\Phi_0 = h/2e$ is the flux quantum.

Fig. 1(b) is observed upon changing the magnetic field, as observed in Fig. 1(c). By overlaying these two signals, a preliminary understanding of the system's behavior under a varying magnetic field in both dc and rf environments is achieved, revealing the origin of the satellite peaks in the triplet heat transport signal when the qubit frequency aligns with the resonator frequency, outlined in Fig. 1(c). In Ref. [6], a weakly anharmonic transmon qubit was used to demonstrate the concept of such a measurement; however the device performance was limited by its weak qubit-resonator and resonator-reservoir coupling strengths. The reported flux qubit architecture enables access to ultrastrong- and even deep-ultrastrong-coupling regimes with minimal modifications to the qubit design. These regimes, which remain experimentally unexplored in the field of quantum thermodynamics, are largely inaccessible using transmon-type qubit architectures. Our alternative approach exploits the length-dependent mutual inductance of the coupling element, which enables further enhancement of the qubit-resonator coupling strength, pushing it into the ultrastrong-coupling regime and beyond [23,25–27]. A theoretical perspective towards high performance of such devices is addressed in Ref. [15], where the importance of couplings g and γ is discussed. In this work, within the framework of strong coupling between the reservoir, resonator, and qubit, our results demonstrate an order of magnitude increase in power modulation along with a nearly 100% on-off switching ratio of heat current in the photon-dominated heat transport regime. With a primary focus on experimental observations, this work also introduces a theoretical model that quantitatively captures the observed heat current and qualitatively explains the flux-dependent power. The device design is reported in Fig. 2, with detailed description provided in Sec. II.

II. DEVICE DESIGN

Since the primary experiment is performed using Device 1, we focus on its design in this section. The design details of Device 2 are discussed separately in Sec. IV. The device fabrication for both devices are discussed in Appendix A. The hybrid quantum system is composed of a flux qubit galvanically connected to two nominally identical superconducting half-wavelength coplanar resonators with a designed frequency of $f_r \approx 6.5$ GHz, which are in turn coupled to two copper reservoirs referred to as source (S) and drain (D) reservoirs. Thus, the device architecture can be succinctly described: source—Resonator 1—flux qubit—resonator 2—drain. For convenience, we will refer to the source reservoir as Reservoir 1 and the drain reservoir as Reservoir 2 throughout the article. The measured resistances of the reservoirs R are approximately 6Ω at helium temperatures. The reservoirs are coupled to their respective resonators with a coupling strength given by $\gamma/2\pi = f_r/Q_R \approx 500$ MHz. Here, Q_R is the

quality factor of the reservoir given by $Q_R = (\pi/2)Z_0/R$, where $Z_0 \approx 50 \Omega$ is the designed characteristic impedance of the resonator [6]. The measured device is shown in Fig. 2(a). The flux qubit in the middle is a superconducting loop interrupted by three Josephson junctions in series, of which, two are designed to be identical with the critical current $I_{C2} = I_{C3} = I_C$, while the area of the first junction is reduced by a factor of $\alpha \approx 0.6$ with critical current $I_{C1} = \alpha I_C$. The enlarged scanning electron microscope (SEM) image of the flux qubit in Fig. 2(a) shows the galvanic connections to Resonator 1 and Resonator 2 via the inductances L_1 and L_2 realized by a thin superconducting Al wire, facilitating couplings g_1 and g_2 with each resonator, respectively. The coupling constants are proportional to the corresponding inductances as $g_j \propto L_j$ (where $L_1 \approx L_2$) [23,24]. The fundamental frequencies of the resonators and the qubit-resonator coupling have been carefully designed to be identical on both sides of the qubit (side 1 and side 2) to maintain symmetry in the device. The estimated coupling strength $g_1/2\pi \simeq g_2/2\pi \simeq 200$ MHz was extracted by fitting spectroscopy data described in Sec. IV.

Each of the $\lambda/2$ resonators attached to the flux qubit is terminated at its end by a reservoir. They are shown in orange and black at the bottom of Fig. 2(a). Each reservoir has four normal metal-insulator-superconductor (NIS) junctions attached to it. The electronic temperatures of Reservoir 1, T_1 , and Reservoir 2, T_2 , can be controlled (by voltage biasing the NIS junctions so they act as heaters) and monitored (via SINIS junctions acting as thermometers), as shown in the SEM images of Fig. 2(a) [5]. Each reservoir is strongly coupled to its respective resonator. The thermal model of the device including both reservoirs, connected via a tuneable hybrid quantum system (Resonator 1—flux qubit—Resonator 2) and each of them furthermore connected to the phonon bath at temperature T_0 , is schematically shown in Fig. 2(b). Panel (c) of Fig. 2 shows the circuit diagram of a single reservoir coupled to the resonator-qubit system, which is used to obtain the noise spectrum $S_\Phi^{(r)}(\omega_{ij})$ for the theoretical analysis.

III. THERMAL TRANSPORT MEASUREMENTS

The heat is transported from Reservoir 1 to Reservoir 2 under steady-state conditions. Energy conservation for Reservoir 2 reads $P_{1 \rightarrow 2} = P_{\text{ep},2}$, where $P_{\text{ep},2} = \Sigma \mathcal{V} (T_2^n - T_0^n)$ is the conventional power between the electrons in Reservoir 2 at temperature T_2 and the phonon bath at temperature T_0 , see Fig. 2(b). Here, indices 1 and 2 denote the respective reservoirs, and “ep” refers to electron-phonon coupling [3]. Here, Σ represents the electron-phonon coupling constant, $\mathcal{V}$ the volume of the reservoir, and the phonon temperature T_0 is equal to the cryostat temperature. The constant value of n is material

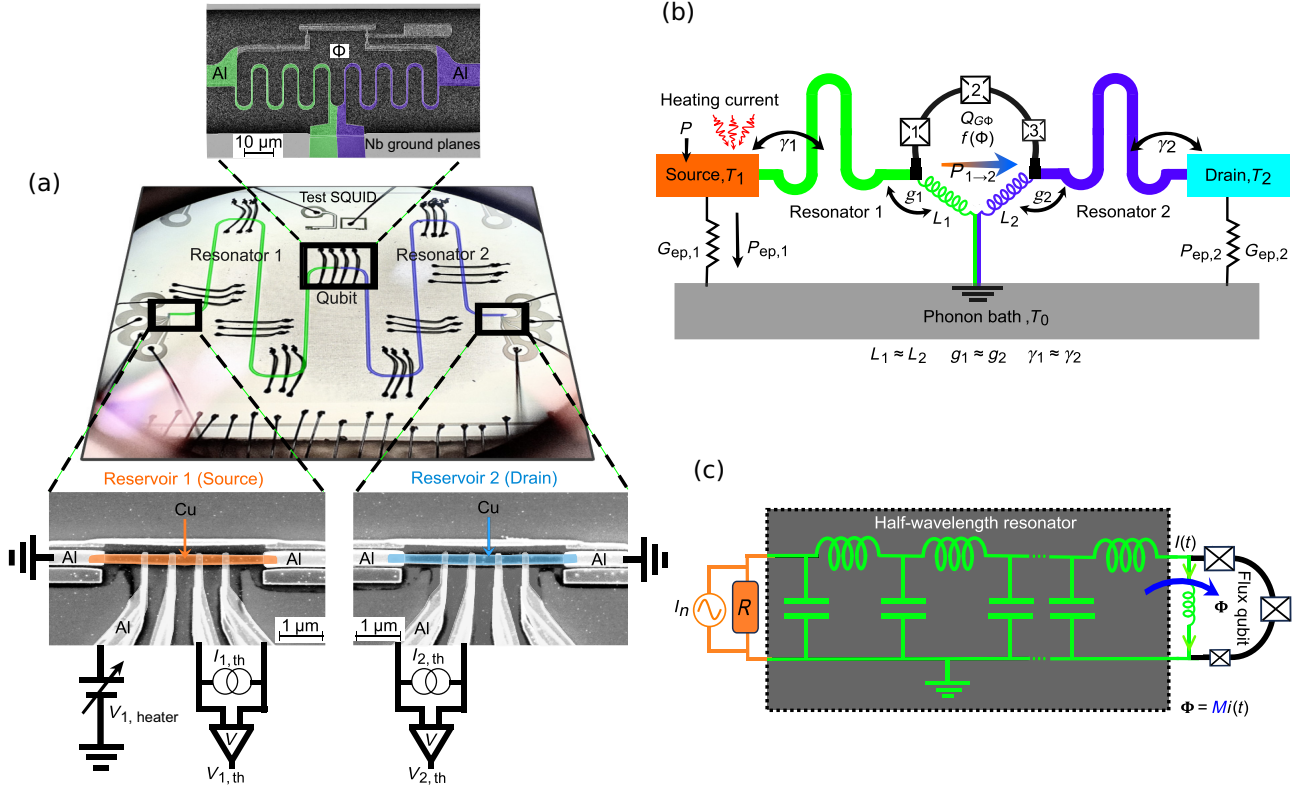


FIG. 2. Device design and circuit. (a) An optical image of the reported device. At the center, within the solid black box, is a superconducting qubit coupled to Resonator 1 (green) and Resonator 2 (blue) and connected to the ground plane. The lower sections display Reservoir 1 and Reservoir 2, along with a brief measurement scheme provided in detail in Appendix B. (b) Conceptual schematic showing both reservoirs, each connected to a phonon bath at temperature T_0 , with thermal conductance $G_{ep,1}$ and $G_{ep,2}$, respectively. The qubit's flux-dependent frequency $f(\Phi)$ controls heat transport $Q_{G\Phi}$, thus it acts as a tunable quantum heat switch. (c) Circuit diagram shows a resistor R connected to a half-wavelength resonator and qubit, with i_n as current noise and $\Phi = Mi(t)$ the input coil inductance. Reported scanning electron microscope images have been colored for clarity.

specific and depends on the type of disorder in normal metals. For a clean three-dimensional normal metal, $n = 5$ [40–42]. The equation can be linearized for small temperature differences to $P_{ep,2} = G_{ep,2} \Delta T$ with $G_{ep,2} = n \Sigma V T_2^n$ denoting the electron-phonon thermal conductance. For the reported experiment, ΣV values for Reservoir 1 and Reservoir 2 are estimated to be 9.35×10^{-10} and $11.44 \times 10^{-10} \text{ WK}^{-5}$, respectively. This can be calculated using the relation $\frac{1}{2} I^2 R_{heater} = \Sigma V (T_e^n - T_0^n)$, where I is the applied current, R_{heater} is the resistance of heater junction (which was measured to be around 20 k Ω), T_e is the electronic temperature, and the slope corresponds to ΣV [42].

We monitor T_2 when heating Reservoir 1 by Joule power applied via a single NIS junction [5]. The temperature T_2 is monitored via measurement of the thermal voltage across another pair of NIS junctions attached to it and T_1 is monitored similarly. The calibration process for converting the thermometer's voltage into temperature, and subsequently into absorbed power, is detailed in Appendix B. Another control parameter in the experiment is the magnetic flux on the qubit affecting the energy separation between the

ground and excited states. Schematics of the experiment are shown in Fig. 3(a).

While varying the magnetic flux with constant temperature bias between Reservoir 1 and Reservoir 2, the measured T_2 exhibits periodic variations in superconducting flux quantum $\Phi_0 = h/2e$, as plotted in Fig. 3(a) for Reservoir 1 temperature $T_1 = 295 \text{ mK}$, at phonon bath temperature $T_0 = 80 \text{ mK}$. The close-up of the main feature is plotted in panel (b) of Fig. 3 for Reservoir 1 temperatures $T_1 = 295, 270, 235$, and 70 mK .

The main features of the observed power on Reservoir 2 are as follows: (1) There is an absorption peak at half-integer flux positions (in units of Φ_0) of magnitude $P \sim 10 \text{ fW}$, which is more than an order of magnitude higher than that observed previously in a transmon system [6], while varying the magnetic field [see panel (c) of Fig. 3]. (2) The central peak is accompanied by two main satellite peaks, positioned at flux values where the qubit energy is degenerate with that of the resonators (see also Fig. 1). (3) Under a more careful measurement [Fig. 3(d)], we observe additional weaker side

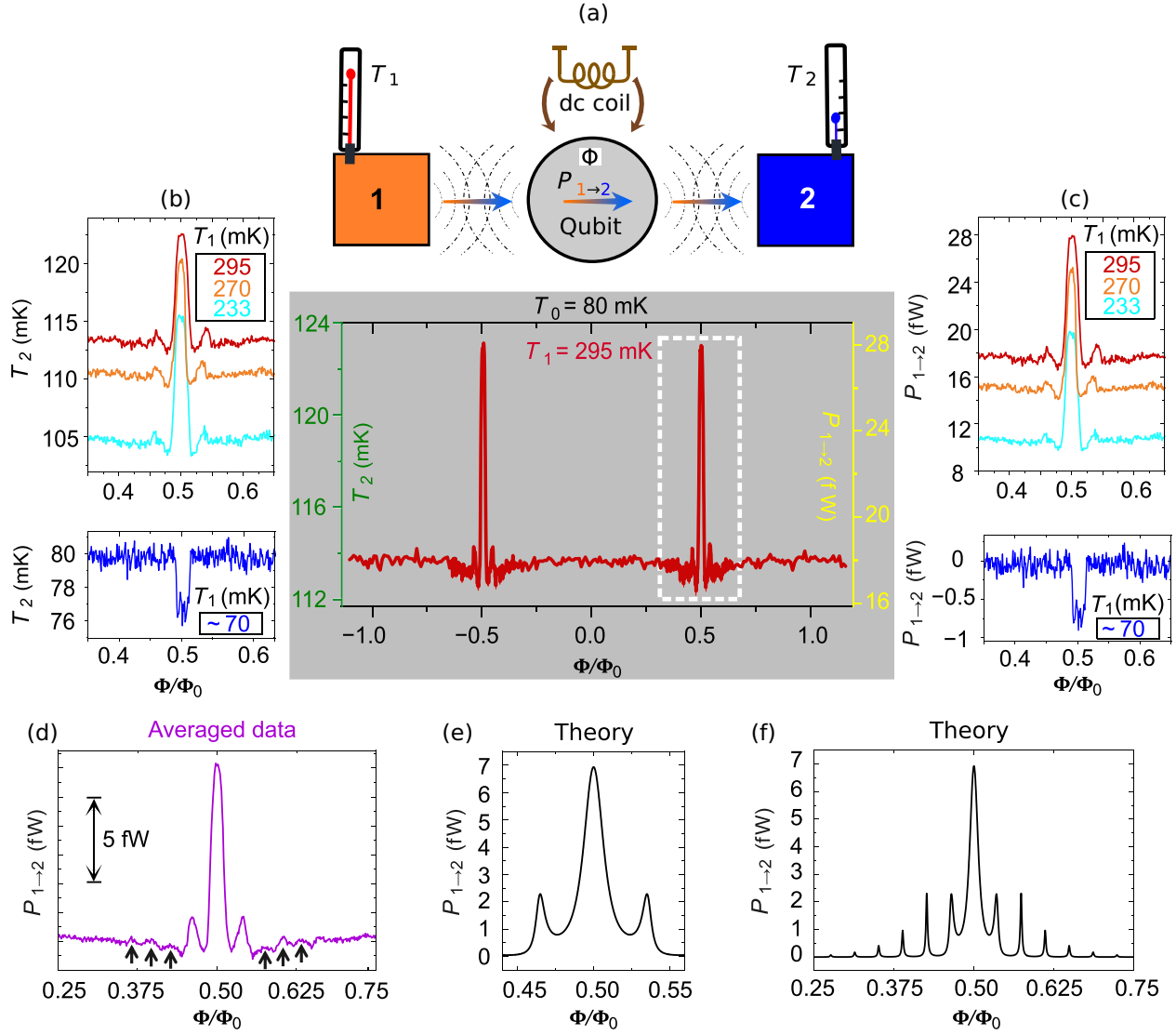


FIG. 3. Flux-dependent power transport. (a) Flux-dependent power transport $P_{1 \rightarrow 2}$ across the qubit, driven by a magnetic flux generated by the coil, with power from Reservoir 1 (red) absorbed by Reservoir 2 (blue), at $T_0 = 80$ mK. The bottom plot shows the modulation of Reservoir 2's temperature (left y axis) and transported power (right y axis) as functions of the applied magnetic field. The region within the dashed rectangle in the center of the image is enlarged in panels (b),(c). In panel (b), the y axis represents temperature, while in panel (c), it represents the power absorbed by Reservoir 2. (b) T_2 vs applied magnetic flux at different T_1 . (c) $P_{1 \rightarrow 2}$ vs applied magnetic flux at different T_1 . (d) Averaged data (pink curve) from multiple sweeps show small satellite peaks (black arrows). (e) $P_{1 \rightarrow 2}$ from the theoretical model. (f) Satellite peaks captured by the theoretical model. The average rms noise, calculated over the range $\pm 0.25\Phi/\Phi_0$ for the data shown in panel (a), is approximately 0.14 fW, which is considerably smaller than the amplitude of the signal of interest.

peaks outside the central triplet. (4) When lowering the temperature of Reservoir 1 below the bath temperature [43,44], we observe cooling of Reservoir 2, as expected [see the lowest trace in Fig. 3(b)]. Electronic cooling is achieved by voltage biasing the NIS junction at the optimal cooling point, Δ/e , allowing only high-energy (hot) electrons in the normal metal to tunnel into the superconductor, while the lower-energy electrons remain in the normal metal. As a consequence of the selective evacuation of high-energy electrons from the normal metal, electronic cooling is observed [5,44]. (5) There is

substantial “background” flux-independent heat transport between both reservoirs, which is most likely carried by the phonons (substrate).

The theoretical model of the system, described in Sec. V, can account for the observations (1)–(4), see panels (e),(f) of Fig. 3. The central peak at $\Phi/\Phi_0 = 0.5$ arises from the minimum energy of the qubit allowing for thermal transitions (excitations) from the ground to the excited state at this position and at a sufficiently high temperature of Reservoir 1. While moving away from this minimum energy position [see Fig. 4(b)], these excitations quickly

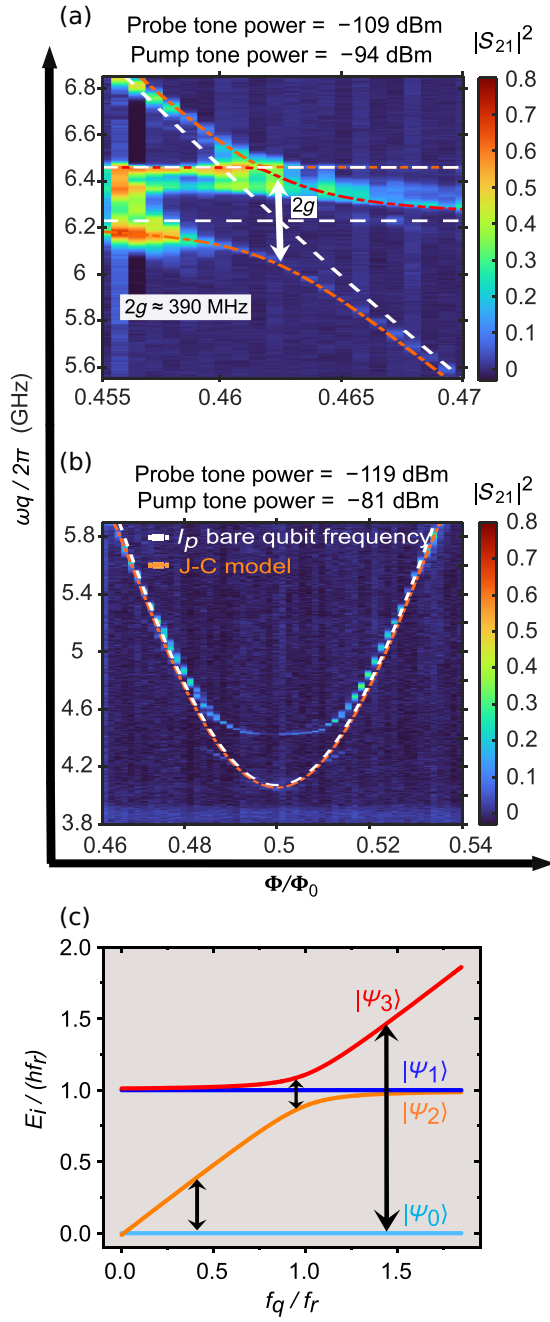


FIG. 4. Device spectroscopy. (a) Two-tone spectroscopy measurement probing the interaction of a strongly coupled qubit-resonator system via a readout resonator at the readout frequency $f_{\text{readout}} \approx 7$ GHz. The estimated coupling strength is $g/2\pi \approx 200$ MHz, obtained by fitting with the Jaynes-Cummings (J-C) model. (b) Qubit energy spectrum vs applied magnetic flux, with minimum frequency (approximately 4.0 GHz) at half-flux. The orange dashed line shows the J-C model fit; the white dashed line fits $\hbar\epsilon = 2I_p(\Phi - \Phi_0/2)$, with $I_p \approx 21$ nA. The avoided crossing with a stationary feature around 4.4 GHz arises from the interaction of the system with a parasitic resonance, which is disregarded in the analysis of the results. (c) Dominant transitions from the theoretical model responsible for heat transport across the qubit.

diminish, and the power drops back to the base level. But when the qubit becomes degenerate with the resonators, this opens a new transport channel in the hybridized qubit-resonator system leading to the side peaks in the triplet. Owing to the multiple modes of the resonator, there are several satellite peaks around the sweet spot, as shown in panel (d) of Fig. 3.

IV. QUBIT READOUT

Here, we report on Device 2, which enables us to perform rf spectroscopy on it to extract parameters such as the resonator frequency, qubit transition frequency, and their coupling strengths. Since the resonator frequency and coupling strength are merely geometry-dependent quantities, the spectroscopy replica gives estimates for these parameters. However, the qubit frequency is more susceptible to precise fabrication processes, and therefore variations between Devices 1 and 2 may be large. These parameters are essential for supporting the interpretation of the thermal transport measurements performed using Device 1. The Device 2 design schematic is shown in Fig. 5. It

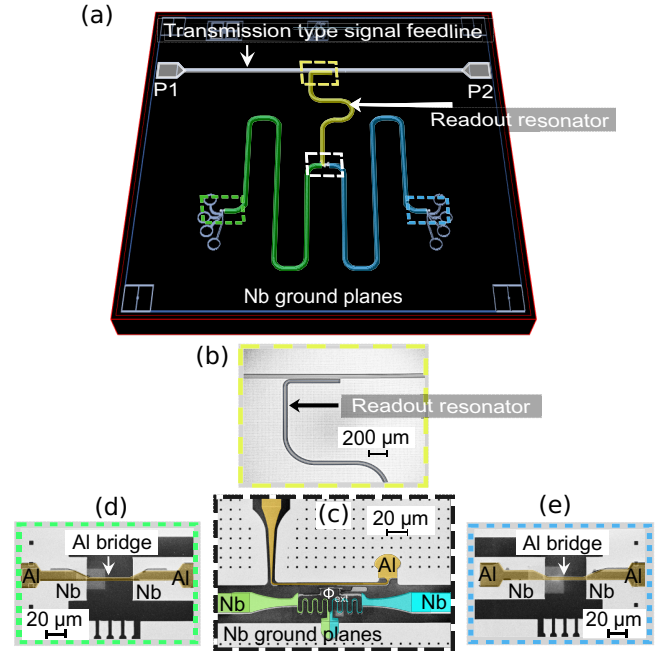


FIG. 5. Qubit spectroscopy device. (a) Device 2: Design layout of the measured spectroscopy device. Colored semisolid boxes represent different components of the device shown in panels (b)–(e). Here, P1 and P2 represents port 1 and port 2, respectively. (b) Scanning electron microscope (SEM) image showing capacitive coupling of the readout resonator to the transmission feedline. (c) Electron micrograph of the flux qubit. The narrow part of the readout resonator is made of Al. (d),(e) In Device 2, superconducting Al bridges are utilized to link Resonator 1 and Resonator 2 to the ground plane. The SEM images have been colored to enhance visibility of the features.

features a signal feedline that is capacitively coupled to one end of a quarter-wavelength readout resonator, with a designed readout frequency $f_{\text{readout}} = 7.0$ GHz, see panel (b) of Fig. 5. The opposite end of the readout resonator is positioned in close proximity to the flux qubit and is inductively terminated to the on-chip ground plane, as shown in Fig. 5(c). As reported in Ref. [45], the broadening of the resonance linewidth is expected with copper terminations. Therefore, for measurement convenience, in the spectroscopy device, copper terminations are not used. Instead, a superconducting aluminum bridge is employed to connect the resonator to the niobium ground plane, see panels (d) and (e) of Fig. 5. We use continuous wave transmission coefficient measurements to probe the device characteristics.

Details about the cryostat and the measurement setup are covered in Appendix B and they remain similar to the scheme employed in Ref. [26]. To characterize the qubit properties we perform single- and two-tone spectroscopy. The weakly coupled qubit-readout resonator interaction is detected by the change in the readout resonator frequency as a function of applied magnetic field. The qubit energy spectrum as a function of applied magnetic flux is measured using two-tone spectroscopy where the second tone is used to excite the qubit and the probe tone detects the change in qubit state. For more details on single- and two-tone spectroscopy techniques see Ref. [26]. In this article, the two-tone qubit spectroscopy data are presented in panels (a) and (b) of Fig. 4. The splitting visible in panel (a) highlights the strong interaction ($2g$) between the qubit frequency and the resonances observed in the device, while panel (b) reveals the qubit energy spectrum, which eventually reaches a minimum at approximately 3.9 GHz at the qubit's degeneracy point. To estimate the qubit-resonator interaction energy and the qubit's persistent current I_p of the device we fit the plots shown in panels (a) and (b) of Fig. 4 with the well-known Jaynes-Cummings model [46] and $\hbar\epsilon = 2I_p(\Phi - \Phi_0/2)$ [47], where $\omega = \sqrt{\epsilon^2 + \Delta^2}$. In particular, we may write $hf_q(\Phi) = hf_q^0 \sqrt{1 + \left(I_p/hf_q^0\right)^2 \left(\Phi/\Phi_0 - \frac{1}{2}\right)^2}$, where $f_q(\Phi)$ and f_q^0 represent the qubit frequencies at flux Φ and at $\Phi = 0$, respectively. For this particular sample, we obtain $I_p = 21$ nA.

In this section, we analyze the measured system theoretically to explain the origin of the observed heat transport characteristics of the device. The total Hamiltonian $\hat{\mathcal{H}}$ for the setup shown in Fig. 2(b), and derived in Appendix D, excluding Reservoir 1 and Reservoir 2, is given by

$$\hat{\mathcal{H}} = \hat{\mathcal{H}}_q + \sum_{i=1,2} \hat{\mathcal{H}}_i + \sum_{i=1,2} \hat{\mathcal{H}}_{\text{qci}} + \hat{\mathcal{H}}_{12}. \quad (1)$$

Here, $\hat{\mathcal{H}}_q = hf_q(\Phi)\hat{b}^\dagger\hat{b}$ is the Hamiltonian of the qubit, $\hat{\mathcal{H}}_i = \sum_{i=1,2} hf_i\hat{a}_i^\dagger\hat{a}_i$ that of each resonator, and we

approximate the coupling between the resonator i and the qubit as $\hat{\mathcal{H}}_{\text{qci}} = -\sum_{i=1,2} \hbar g_i (\hat{a}_i + \hat{a}_i^\dagger)(\hat{b} + \hat{b}^\dagger)$, and that between the two resonators as $\hat{\mathcal{H}}_{12} = -\hbar\gamma(\hat{a}_1 + \hat{a}_1^\dagger)(\hat{a}_2 + \hat{a}_2^\dagger)$ [48]. Here, $\hat{b}^\dagger$ and $\hat{b}$ represent the creation and annihilation operators for the qubit, and similarly, $\hat{a}_i^\dagger$ and $\hat{a}_i$ are for resonator i ; g_i denotes the coupling constant between the qubit and resonator i and γ is the coupling constant between the two resonators. We use the Hilbert space $\{|0\rangle \equiv |g00\rangle, |1\rangle \equiv |e00\rangle, |2\rangle \equiv |g10\rangle, \text{ and } |3\rangle \equiv |g01\rangle\}$, where the three entries in each state refer to the qubit that is either in the ground state $|g\rangle$ or excited state $|e\rangle$, Resonator 1, and Resonator 2, respectively. The nonvanishing transitions between the eigenstates, which contribute to the heat transport in our device, are spectroscopically examined and presented in panels (a) and (b) of Fig. 4. These transitions are modeled as illustrated in panel (c) of the same figure.

The transition rate induced by the reservoir r ($r = 1, 2$) between the normalized eigenstates $|\psi_i\rangle$ and $|\psi_j\rangle$ of Hamiltonian (D1) reads

$$\Gamma_{i \rightarrow j}^{(r)} = \frac{1}{\hbar^2} |\langle \psi_i | \frac{\partial \hat{\mathcal{H}}}{\partial \Phi} | \psi_j \rangle|^2 S_\Phi^{(r)}(\omega_{ij}), \quad (2)$$

where $|\psi_k\rangle = \sum_{i=0}^3 a_{ki} |i\rangle$, with the coefficients a_{ki} provided in Appendix C. Here, $S_\Phi(\omega_{ij})$ represents the spectral density of flux noise, which will be discussed later in this section. Further details on $\partial \hat{\mathcal{H}} / \partial \Phi$ and the matrix elements $|\langle \psi_i | \partial \hat{\mathcal{H}} / \partial \Phi | \psi_j \rangle|^2$ can be found in Appendix C.

In order to obtain the noise spectrum $S_\Phi^{(r)}(\omega_{ij})$ in the expression of the transition rates, $S_\Phi^{(r)}(\omega_{ij})$ in Eq. (D9), we consider the influence of each (mutually uncorrelated) reservoir separately, and in the following we therefore drop the reservoir indices in the corresponding quantities. We first note that the flux $\Phi(t)$ and current $I(t)$ through the input coil of inductance M of the flux qubit are related as $\Phi(t) = MI(t)$ for a directly coupled qubit such as ours [see Fig. 2(c)]. Then for the spectral density of noise we have $S_\Phi(\omega_{ij}) = M^2 S_I(\omega_{ij})$. Owing to the coupling via a $\lambda/2$ coplanar wave resonator with resonance frequency f_r , we have the flux noise on the qubit at frequency $f = \omega/2\pi$ in form $S_\Phi(\omega) = M^2 |t|^2 S_{I_n}(\omega)$, where $|t|^2$ is the transmission through the resonator, reading as

$$|t|^2 = \frac{1}{\cos^2\left(\pi \frac{f}{f_r}\right) + \left(\frac{Z_\infty}{R}\right)^2 \sin^2\left(\pi \frac{f}{f_r}\right)}. \quad (3)$$

Here, $Z_\infty \approx 50 \Omega$ is the designed impedance of the transmission line. The bare current noise spectrum of the resistor R at temperature $T = 1/(k_B\beta)$ is $S_{I_n}(\omega) = 2\hbar\omega/[R(1 - e^{-\beta\hbar\omega})]$. The diagonal elements of the density matrix under nondriven conditions are given by the

steady-state solution of

$$\sum_j \Gamma_{j \rightarrow i} \rho_{jj} - \sum_j \Gamma_{i \rightarrow j} \rho_{ii} = 0. \quad (4)$$

Here, $\Gamma_{i \rightarrow j} = \Gamma_{i \rightarrow j}^{r=1} + \Gamma_{i \rightarrow j}^{r=2}$. The power to reservoir r ($r = 1, 2$) then reads

$$P_{1 \rightarrow 2} = \sum_{k,l} \rho_k \hbar \omega_{kl} \Gamma_{k \rightarrow l}^{(r=2)}. \quad (5)$$

Figures 3(a)–3(d) and 3(f) show the flux dependence of the power in the experiment and that obtained by the model above, respectively. The parameters used in the calculations and for plotting the theoretical curves are as follows: $R = 6 \, \Omega$, $L = 0.8 \, \text{nH}$, $I_p = 30 \, \text{nA}$, the minimum energy of the qubit at half-flux $f_q^{(0)} = 2 \, \text{GHz}$, and $f_r = 7 \, \text{GHz}$. Our model has some simplifying assumptions. We restrict it to a single-excitation basis and use the Fermi golden rule transition rates. These formulations are fine as long as temperatures are sufficiently low and couplings are not too strong.

The spectroscopically observed values for Device 2 are $f_r \approx 6.4 \, \text{GHz}$, $f_q \approx 4.0 \, \text{GHz}$, and $I_p \approx 21 \, \text{nA}$. These parameters, although only approximate, support the theoretical modeling used to reproduce the experiments performed with Device 1, although they exhibit modest but noticeable deviations from the modeled values. These deviations may be attributed to the contributing factors outlined below. Architecturally, Device 1 adopts an N—resonator—flux qubit—resonator—N configuration, whereas Device 2 follows an S—resonator—flux qubit—resonator—S layout, substituting the normal metal terminal with an Al strip to facilitate spectroscopy. Both devices share fabrication steps up to qubit integration, after which Device 1 undergoes additional processing to incorporate heat reservoirs and NIS junctions, which may alter the Josephson junction characteristics. Furthermore, the readout resonator present in Device 2 [Fig. 5(c)] may influence the electromagnetic environment and contribute to frequency deviations. Finally, it is well known that precise frequency targeting in flux qubits with three Josephson junctions is experimentally challenging, particularly in research-grade fabrication environments.

V. CONCLUSION AND OUTLOOK

The flux qubit presented here offers several advantages for quantum thermodynamics experiments and devices as compared to the common transmon qubit. The main merits are the anharmonicity, which remains robust against strong coupling to the resonators and reservoirs, and the galvanic coupling of the qubit rendering efficient transport. Notably, the inductive coupling design in the reported device architecture facilitates access to the ultrastrong-coupling regime

by adjusting the length-dependent inductance of the coupling element [26]. This leads to the ability to transfer large heat currents via the qubit (proportional to g^2 for coupling strength g based on the Fermi golden rule arguments) without significant compromises; an order of magnitude enhancement is achieved here compared with earlier transmon experiments [6]. Secondly, weak nonreciprocity in heat transport has previously been demonstrated using a transmon qubit, but this is limited by the nearly equidistant energy levels, i.e., transmon is almost linear. A flux qubit would thus present a two-level qubit (maximal nonlinearity), with strong directionality of heat transport, exhibiting an efficient thermal diode [48]. Additionally, due to the sharply peaked heat transport in the femtowatt range around $\Phi = \Phi_0/2$, as demonstrated by our experiment, the system operates as a heat switch controlled by magnetic flux. Measured at the temperature of 80 mK, the on-off

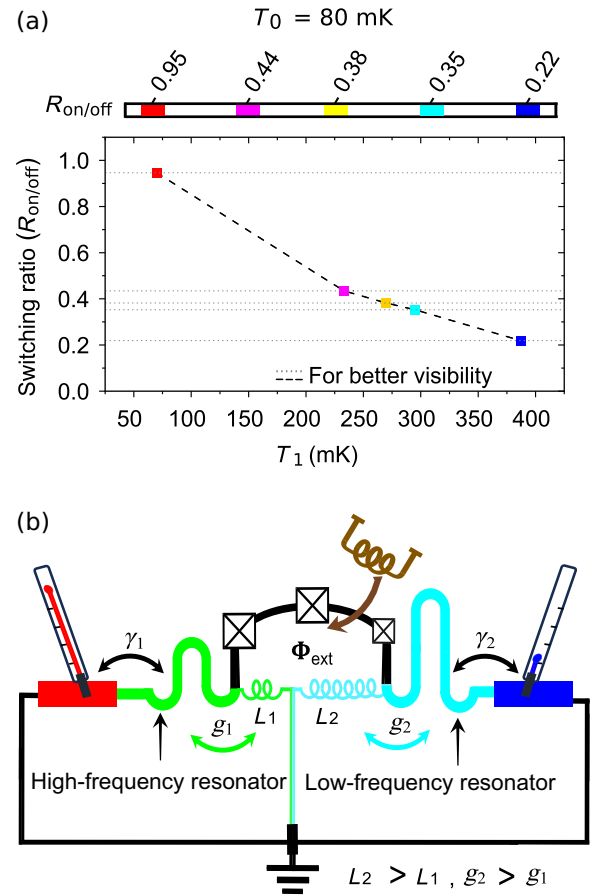


FIG. 6. Potential applications of the flux-qubit device. (a) Switching ratio from Eq. (6) vs Reservoir 1 temperatures at $T_0 = 80 \, \text{mK}$, with semisolid lines and horizontal grid lines for visibility, demonstrates a nearly 100% switching ratio. The color bar corresponds to the values of $R_{\text{on/off}}$ at different temperatures of Reservoir 1 (T_1). (b) Towards quantum heat diode and Otto refrigerator: Asymmetry introduced by tailoring resonator frequencies and qubit coupling.

switching ratio $R_{\text{on/off}}$ of the heat current approaches 100%, as estimated using the ratio of the corresponding powers as

$$R_{\text{on/off}} = \frac{P_{\text{on}} - P_{\text{off}}}{P_{\text{on}}}, \quad (6)$$

and shown in panel Fig. 6(a). Here the on and off conditions are defined as those at $\Phi = \Phi_0/2$ and $\Phi = 0$, respectively. When T_1 and T_2 are low, with T_1 even less than $T_2 = 80$ mK, which corresponds to the maximum cooling point of the NIS control junction, the magnitude of phonon heat transport is weakest, and photon transport dominates in system. As a result, we approach a switching ratio close to 100%. However, when T_1 is increased to 230 mK and above, the phonon contribution becomes dominant (since phonon heat conductance generally obeys a T^3 dependence [49]), and the switching ratio, determined by the overall power including both photons and phonons, diminishes.

Based on all the merits listed above, a flux qubit enables the development of efficient quantum thermal machines. Introducing asymmetry to the system in terms of the frequencies of the two superconducting resonators and tailoring qubit coupling inductances, one finds opportunities for developing not only quantum heat diodes [10,48] but also quantum refrigerators [15]. The required asymmetries are achieved by choosing the length of the corresponding element, resonator, or coupling inductance, as shown in Fig. 6(b). This refrigerator can pump heat under the application of alternating magnetic flux to the qubit from a cold reservoir via the lower-frequency resonator to the hot reservoir via the higher-frequency one.

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The device design and fabrication was done by R.U. The measurements were carried by R.U., Y-C.C., and D.S.

The data analysis was done by R.U., B.K., and C.D.S. The theoretical model was conceived by J.P.P. and B.K. The technical support was provided by D.S., Y-C.C., and J.T.P. The manuscript was written by R.U., J.P.P., and B.K., with important contributions from all other authors. The work was supported and supervised by J.P.P.

The authors declare no competing interests.

DATA AVAILABILITY

The data that support the plots within this article are available from the corresponding author upon reasonable request.

APPENDIX A: DEVICE FABRICATION

A 675- μm -thick highly resistive Si wafer is used as a substrate. On top of it, a 30-nm-thick dielectric film of Al_2O_3 is deposited using atomic layer deposition, followed by a 200-nm-thick deposition of niobium metal using dc magnetron sputtering. The resonator mask is patterned over a uniform positive *e*-beam resist using *e*-beam lithography. After development of patterns, the wafer is baked for a few minutes to strengthen the remaining resist and secure the main structures, avoiding their over-etching. Reactive ion etching is performed using SF_6 chemistry to etch the exposed Nb metal. In the last step, the remaining resist is removed using O_2 descum dry resist stripping.

To pattern the qubit, a bilayer resist stack of MMA/poly(methyl methacrylate) (PMMA) is spin-coated, and this then undergoes *e*-beam exposure. The bilayer stack is used to create an undercut useful for multi-angle evaporation. We use a solution of methyl isobutyl ketone:isopropanol alcohol (MIBK:IPA volume ratio, 1 : 3) to develop the PMMA top layer. The undercut profile is provided by a second development step using a methyl glycol-methanol solution with a volume ratio of 1 : 2. A high-vacuum *e*-beam evaporator with an *in situ* plasma-etching module is used for evaporation. Before evaporation, the sample is exposed to an argon beam for about 40 s to clean possible deposited native oxide, enabling a clean contact at Nb-Al interfaces. Using the Dolan bridge shadow evaporation technique [50], a 30-nm-thick Al layer is deposited at $+18^\circ$. After the first layer deposition, the devices are oxidized using an *in situ* oxidation module (4.3 mbar for 4 min), and this followed by another Al layer deposition at a second angle (-18°). Finally, the devices are submerged in a hot ($+52^\circ\text{C}$) acetone bath for about an hour to perform the metal lift-off.

For fabricating heat reservoirs and thermometers, which is the next step, we use the three-angle evaporation method. After spin-coating another bilayer resist stack of P(MMA-MMA) copolymer and PMMA, *e*-beam exposure, development, and native oxide milling, a 20-nm-thick

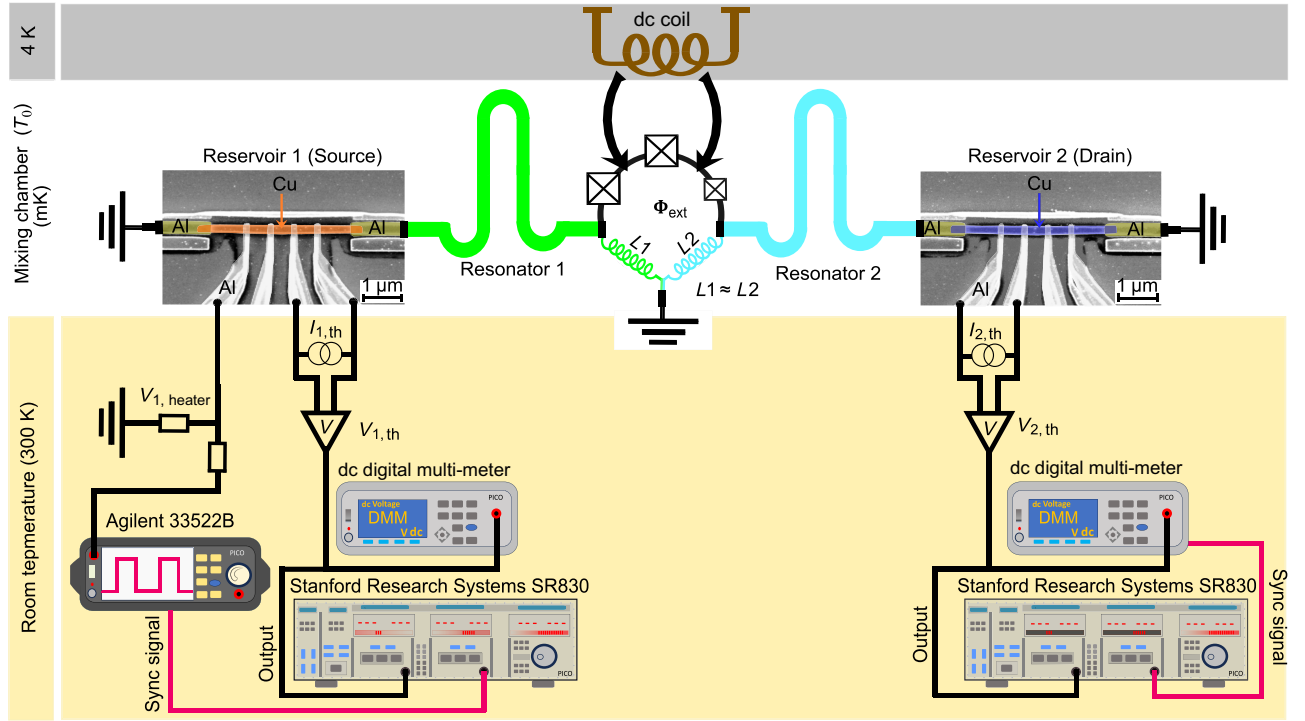


FIG. 7. Thermal conductance measurement setup of the reported device at various temperature stages. The top row, shown in gray, represents the 4-K stage where the global magnetic coil is placed. Below it, the middle row corresponds to the mixing chamber of the cryostat at temperature T_0 , where the device is located. The third and bottom-most row, highlighted in yellow, shows the configuration of the electronic equipment used in the experiments, which are at room temperature.

Al layer is deposited at $+39^\circ$, forming the superconducting (S) part of the NIS structures. The deposited layer is oxidized under 2.5-mbar chamber pressure for about 5 min, forming an insulating tunnel oxide barrier (I). Then a 2-nm-thick Al layer is deposited at -6.5° , and this is followed by the deposition of a 50-nm-thick normal metal Cu (N) at the same tilt angle. The 2-nm Al layer strengthens the adhesion of the Cu layer to the substrate. In the last evaporation step, a 90-nm-thick layer of Al is deposited at $+20^\circ$. This layer enables a clean contact between the NIS structures and the superconducting resonators.

The device is analyzed under SEM, and then it is diced and bonded for cryogenic measurements. Integrated within the same device, we have a replica of our flux qubit connected to two pads, allowing Al bonding wires to measure the normal state resistance of the SQUID (superconducting quantum interference device) loop. The measured resistance of the replica is approximately 3.4 k Ω at liquid-He temperatures.

APPENDIX B: MEASUREMENT SETUP

1. Thermal measurement setup

The sample is glued and wire-bonded ultrasonically to a brass sample stage. Encapsulated within two brass caps acting as radiation shields, the sample holder is then plugged into the mixing chamber (MXC) of a

custom-made plastic wet dilution refrigerator with a base temperature of approximately 70 mK. For readout purposes, we use conventional thermocoax-filtered cryogenic lines that connect the sample stage from the MXC to the room-temperature break-out box. Figure 7 reports the measurement setup employed for the thermal measurements for Device 1. For low-impedance loads, the effective signal bandwidth for the cryogenic filtered lines are between 0 and 10 kHz. For measurements, programmable voltage sources and Keysight digital multimeters are employed. For amplification of output voltages we use a room-temperature Femto DLPA-100-F-D voltage amplifier. An Agilent 33522B waveform generator is used as the bias voltage source. For reading the heat current modulation as a function of applied magnetic field we use an SRS SR830 DSP lock-in amplifier, which is synchronized to the square-wave modulation of the voltage bias from the Agilent 33522B waveform generator. We perform measurements at several MXC temperatures. The MXC temperatures are controlled by applying a voltage to a resistor that is connected to the MXC. For accurate analysis, the calibration of the integrated thermometers (realized as a SINIS configuration) are performed at various cryostat temperatures, ranging from 80 up to 360 mK. This is done by measuring the voltage drop across the SINIS configuration (set at a current bias of $I_{Th} = 50$ pA) for each cryostat temperature, with no heater bias voltage applied

($V_{1,\text{heater}} = 0$ V). After setting the MXC to the next higher desired temperature, we wait for over 20 min before performing any measurements. This ensures that the device is effectively thermalized to the MXC, allowing us to assume that the phonon temperature is in equilibrium with the MXC temperature. The MXC temperature is monitored using a ruthenium oxide thermometer calibrated against a Coulomb blockade thermometer. For measurements involving the modulation of the voltage drop across the reservoir thermometers (heat current) as a function of the applied magnetic field, the procedure is as follows: Heat-current measurements as a function of magnetic flux are initiated by stabilizing at the initial flux point for 5 s. After this, at each subsequent flux point, the system is allowed to stabilize for 2.1 s before data acquisition. As previously described, this process is carried out at various values of T_0 , with a waiting time of over 20 min after the cryostat temperature is increased to ensure proper thermalization.

2. Spectroscopy measurement setup

The spectroscopy device, which includes a signal feed-line, readout resonator, and a qubit galvanically coupled to both resonators (as shown in Fig. 5), is mounted on a gold-plated copper sample stage and placed in a dilution refrigerator with a base temperature of approximately 15 mK. Similar to the measurement scheme reported in Ref. [26], the sample holder is equipped with an external superconducting coil for the application of a dc magnetic field. Suppression of blackbody radiation is achieved by employing a series of impedance-matched cryogenic attenuators placed at different temperature stages inside the cryostat. Once attenuated, the input signal passes through the device and is directed through two isolators situated in the cryostat mixing chamber. Subsequently, it undergoes amplification by a 42-dB low-noise high-electron-mobility transistor amplifier at the 4-K stage, followed by a further amplification of 52 dB using two additional amplifiers positioned at room temperature. For single-tone spectroscopy, we utilize a commercially available vector network analyzer. For two-tone spectroscopy, we use a commercial microwave signal generator. The frequency range of interest, 4–8 GHz, is supported by the setup. The measurement setup and procedures for single-tone and two-tone spectroscopy are similar to those outlined in Ref. [26].

APPENDIX C: THERMAL TRANSPORT EXPERIMENTS

1. Thermal measurements

A custom-made plastic wet dilution refrigerator is used for thermal conductance measurements. These measurements are conducted over a MXC temperature range of 80–360 mK, which is controlled by applying voltage to a resistor located at the MXC of the cryostat. The

device achieves effective thermalization with the MXC, allowing us to assume that the phonon temperature, T_0 , is in equilibrium with that of the MXC. This temperature is accurately measured using a ruthenium oxide thermometer, which has been calibrated with a Coulomb blockade thermometer for precision. The detailed measurement setup configuration is provided in Appendix B. Reservoir 1 is locally heated using an NIS junction, which is biased by an external dc voltage source $V_{1,\text{heater}}$. The Reservoir 1 and Reservoir 2 thermometers are biased with a small current of $I_{Th} = 50$ pA, and the voltage drops across the thermometers ($V_{1,\text{th}}$ and $V_{2,\text{th}}$, respectively) are recorded at several cryostat temperatures T_0 . To determine the temperatures of Reservoir 1 (T_1) and Reservoir 2 (T_2), we measure the thermal voltage drop across both thermometers (set at a current bias of $I_{Th} = 50$ pA) for each cryostat temperature (T_0) with no heating bias voltage applied ($V_{1,\text{heater}} = 0$). The recorded thermal voltage is converted to its respective temperature using the calibration method described in Appendix B. Using the calibrated data, the power absorbed by Reservoir 2, $P_{1 \rightarrow 2}$, is then calculated using the expression $P_{\text{ep},2} = \Sigma \mathcal{V}(T_2^5 - T_0^5)$. The magnetic flux controlling the energy of the qubit is provided by an external superconducting coil at a temperature of 4 K. To observe the modulation in the voltage drop of the thermometers at Reservoir 2, $V_{2,\text{th}}$, we heat Reservoir 1 ($V_{1,\text{heater}} \neq 0$) while sweeping the magnetic field and record the change in $V_{2,\text{th}}$. After calibrating the data, we arrive at the figure presented in Fig. 3.

2. Thermal voltage to power conversion: Mechanism

Figure 8(a) presents the measurement of the thermometer voltage $V_{1,\text{th}}$ at a fixed applied current of $I_{1,\text{th}} = 50$ pA for various bath temperatures, $T_0 = 80$ –360 mK, with a temperature interval of 20 mK between each measurement, from top to bottom, against the voltage $V_{1,\text{heater}}$ across the NIS heater. The equilibrium values of $V_{1,\text{th}}$ at $V_{1,\text{heater}} = 0$, represented by symbols in Fig. 8(b) [with the position indicated by the black dashed line in panel (a)], are plotted as a function of bath temperature T_0 . These data points $V_{1,\text{th}}(V_{1,\text{heater}} = 0)$ serve as the calibration for the corresponding reservoir, here the Reservoir 1 temperature. The solid black line represents a fit to the measured data. At $80 \text{ mK} \leq T_1 < 135 \text{ mK}$, the fitting function is $T_1 = a + b \log_{10}(c - V_{1,\text{Th}})$, where a , b , and c are constants [51], and at $135 \leq T_1 < 370 \text{ mK}$ we use a standard third-order polynomial (nearly linear) function. Figures 8(c) and 8(d) display how the Reservoir 1 temperature T_1 is extracted. With the same procedure we can obtain the Reservoir 2 temperature T_2 . The measured temperatures are then converted using Eq. (1) described in the main text to obtain the power transmitted to Reservoir 2.

3. The ideal symmetric device

The architecture of the reported device is symmetric, which is also reflected by the thermal properties of the

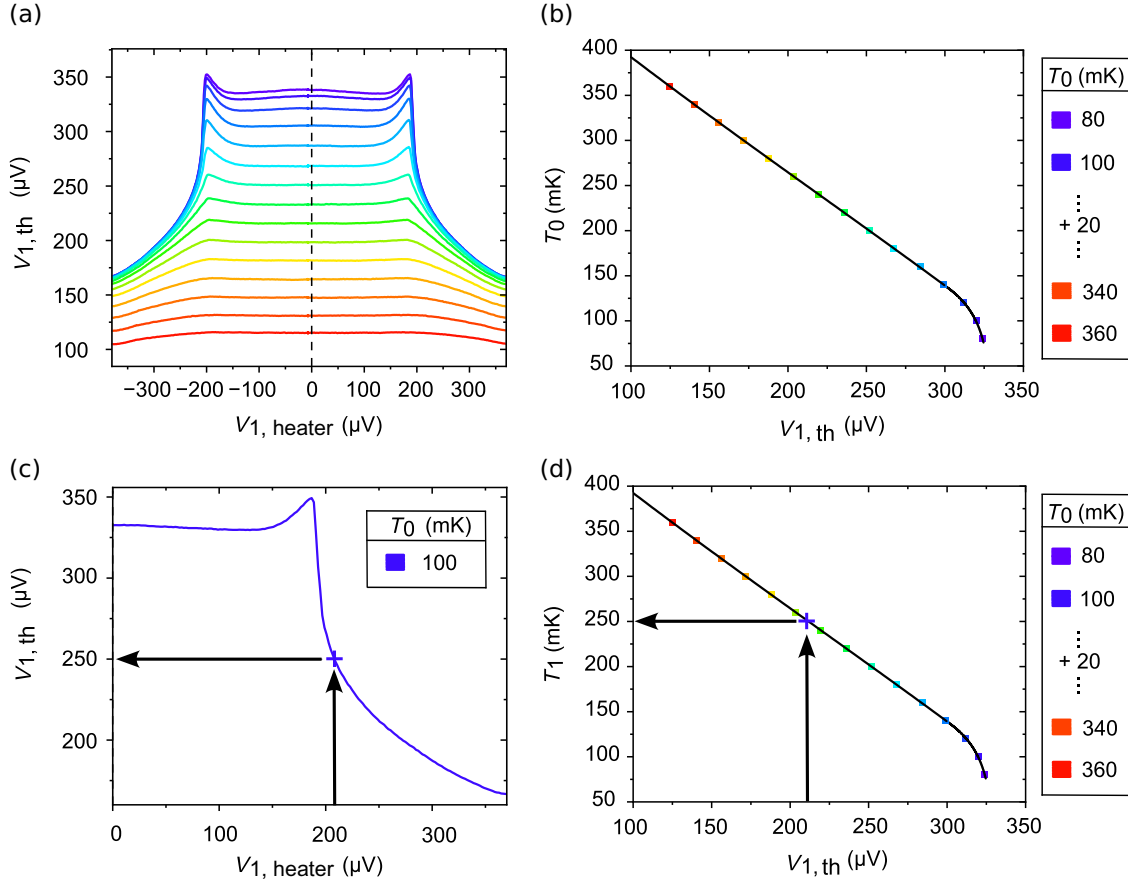


FIG. 8. Thermal voltage to temperature conversion. (a) Typical thermal voltage curve of the SINIS (S, superconductor; I, insulator; N, normal metal) thermometers when biased with varying heating voltages at different cryostat temperatures. (b) Reservoir 1 thermal voltage ($V_{1,heater}$), plotted against various cryostat temperatures, at $V_{1,heater} = 0$, indicated by the dashed line in panel (a). (c), (d) The SINIS thermal voltage curve at a cryostat temperature of 100 mK indicates that a voltage of approximately 210 μV corresponds to a temperature of 250 mK. Black solid arrows are used for better visibility.

device. After calibrating the thermal voltage curves of both reservoir islands and converting them to their respective temperatures, the data for both reservoirs, expressed in temperature, are plotted against each other. The plots are reported in Fig. 9. Panels (a), (b), (c), (d), and (e) in Fig. 9 correspond to the phonon temperatures of 80, 100, 120, 140, and 160 mK. In Fig. 9, we can observe that the temperature curves for both Reservoir 1 and Reservoir 2 are practically identical, as expected from the symmetry of the device. In panel (e) of Fig. 9, a slight difference becomes noticeable when the source reservoir temperature exceeds 200 mK. However, the difference between the two curves remains below 1 mK, which is remarkable.

APPENDIX D: THEORETICAL MODEL

1. Flux qubit hybridized with two resonators coupled to two reservoirs

The total Hamiltonian $\hat{H}$ for the setup shown in Fig. 10, excluding the resistors to the left and right, is given by

$$\hat{\mathcal{H}} = \hat{\mathcal{H}}_q + \sum_{i=1,2} \hat{\mathcal{H}}_i + \sum_{i=1,2} \hat{\mathcal{H}}_{ci} + \hat{\mathcal{H}}_{12}, \quad (D1)$$

where $\hat{\mathcal{H}}_q$ is the Hamiltonian of the qubit, $\hat{\mathcal{H}}_i$ that of each resonator, $\hat{\mathcal{H}}_{ci}$ the coupling between the resonator i and the qubit, and $\hat{\mathcal{H}}_{12}$ the coupling between the two resonators. We write Eq. (D1) in the form [48]

$$\begin{aligned} \hat{\mathcal{H}} = & hf_q(\Phi) \hat{b}^\dagger \hat{b} + \sum_{i=1,2} hf_i \hat{a}_i^\dagger \hat{a}_i - \sum_{i=1,2} hg_j (\hat{a}_j + \hat{a}_j^\dagger) (\hat{b} + \hat{b}^\dagger) \\ & - h\gamma (\hat{a}_1 + \hat{a}_1^\dagger) (\hat{a}_2 + \hat{a}_2^\dagger). \end{aligned} \quad (D2)$$

Here, $\hat{b}^\dagger$ and $\hat{b}$ represent the creation and annihilation operators for the qubit, and similarly, $\hat{a}_i^\dagger$ and $\hat{a}_i$ are for resonator i , g_i denotes the coupling constant between the qubit and resonator i , and γ that between the two resonators. $\hat{\mathcal{H}}_q$ can be written as

$$\hat{\mathcal{H}}_q = hf_q(\Phi) |e\rangle \langle e|. \quad (D3)$$

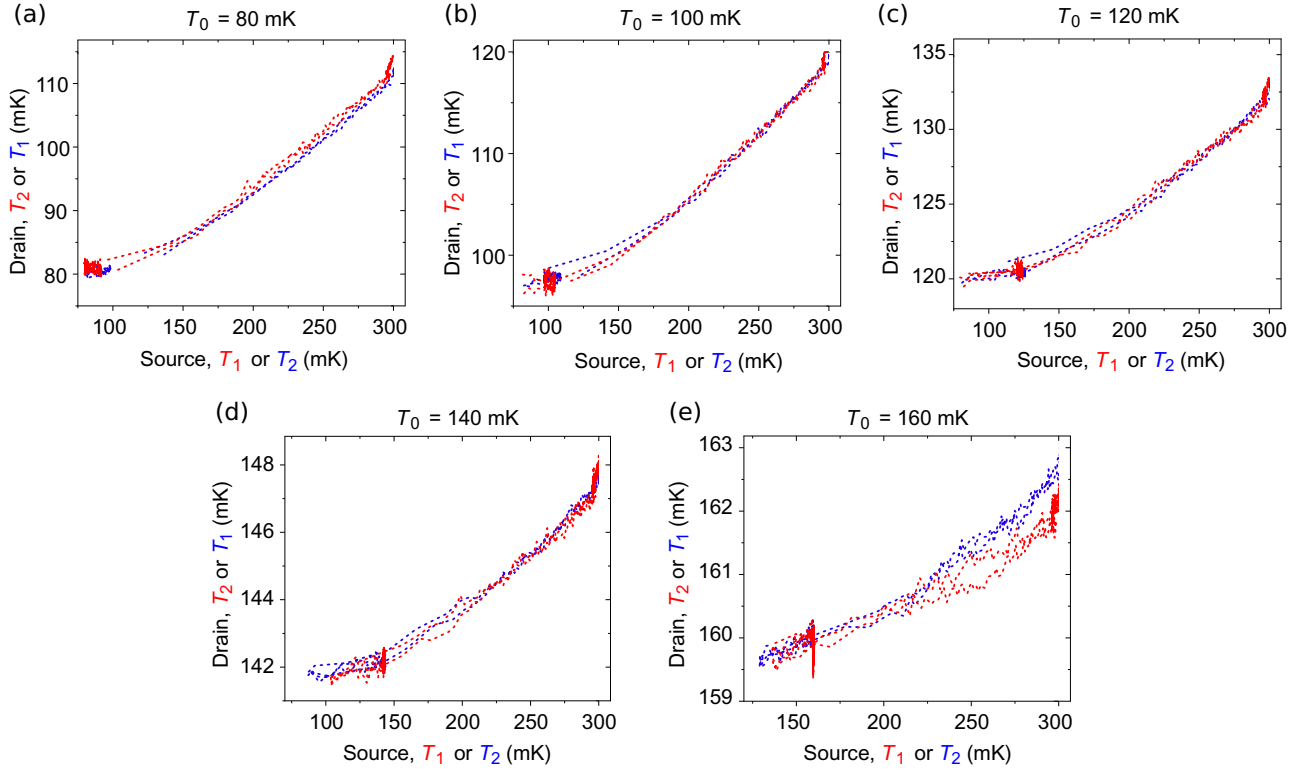


FIG. 9. The ideal symmetric device. (a) Correlation between the temperatures of both reservoirs at different phonon temperatures. The x axis represents the temperature of the heated reservoir (source), which is controlled using a normal metal-insulator-superconductor (NIS) junction, while the y axis shows the temperature of the drain reservoir. Each panel corresponds to a different phonon temperature T_0 : (a) 80 mK, (b) 100 mK, (c) 120 mK, (d) 140 mK, and (e) 160 mK.

In the four-level basis of $\{|0\rangle \equiv |g00\rangle, |1\rangle \equiv |e00\rangle, |2\rangle \equiv |g10\rangle, \text{ and } |3\rangle \equiv |g01\rangle\}$, where the entries in each state refer to the qubit that is either in the ground state $|g\rangle$ or excited state $|e\rangle$, the left resonator, and the right resonator, respectively; the matrix form of the Hamiltonian is

given by

$$\mathcal{H} = h \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & f_q & -g_1 & -g_2 \\ 0 & -g_1^* & f_1 & \gamma \\ 0 & -g_2^* & \gamma & f_2 \end{pmatrix}. \quad (\text{D4})$$

We assume that the frequencies of the two resonators and their couplings to the qubit are equal, i.e., $f_1 = f_2 \equiv f_r$ and $g_1 = g_2 \equiv g$. The eigenenergies λ_k of this Hamiltonian are

$$\begin{aligned} E_1 &= 0, \\ E_2 &= hf_r, \\ E_3 &= \frac{h}{2} \left[f_q + f_r - \sqrt{(f_q - f_r)^2 + 8g^2} \right], \\ E_4 &= \frac{h}{2} \left[f_q + f_r + \sqrt{(f_q - f_r)^2 + 8g^2} \right]. \end{aligned} \quad (\text{D5})$$

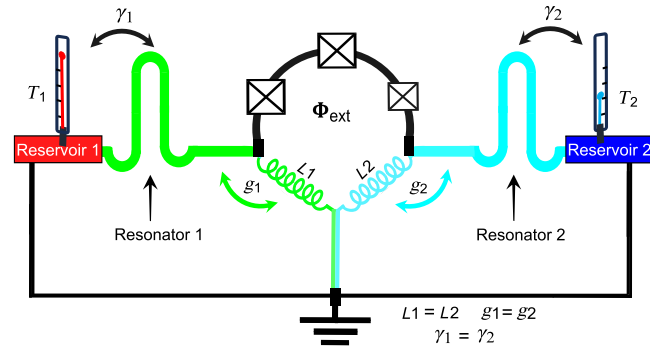


FIG. 10. Device circuit schematic. A system with a three-junction flux qubit coupled with two resonators, which are connected to two separate reservoirs.

The normalized eigenstates corresponding to these eigenenergies are

$$|\psi_0\rangle = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}, |\psi_1\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ 0 \\ -1 \\ 1 \end{pmatrix}, \quad (\text{D6})$$

$$|\psi_2\rangle = \frac{1}{\sqrt{8g^2 + [-f_q + f_r + \sqrt{(f_q - f_r)^2 + 8g^2}]^2}} \begin{pmatrix} 0 \\ -f_q + f_r + \sqrt{(f_q - f_r)^2 + 8g^2} \\ 2g \\ 2g \end{pmatrix}, \quad (\text{D7})$$

$$|\psi_3\rangle = \frac{1}{\sqrt{8g^2 + [f_q - f_r + \sqrt{(f_q - f_r)^2 + 8g^2}]^2}} \begin{pmatrix} 0 \\ -f_q + f_r - \sqrt{(f_q - f_r)^2 + 8g^2} \\ 2g \\ 2g \end{pmatrix}. \quad (\text{D8})$$

The transition rate between the states i and j reads

$$\Gamma_{ij} = \frac{1}{\hbar^2} |\langle \psi_i | \frac{\partial \hat{\mathcal{H}}}{\partial \Phi} | \psi_j \rangle|^2 S_{\Phi}(\omega_{ij}), \quad (\text{D9})$$

where $\hbar\omega_{ij}$ is the energy separation between states i and j . In the Hamiltonian of Eq. (D1), there are two contributions, $\hat{\mathcal{H}}_q$ and $\sum_{i=1,2} \hat{\mathcal{H}}_{\text{qci}}$, that depend on flux Φ . Their derivatives with respect to flux are presented below. For $\hat{\mathcal{H}}_q$ we have

$$\frac{\partial \hat{\mathcal{H}}_q}{\partial \Phi} = h \frac{\partial f_q(\Phi)}{\partial \Phi} |e\rangle \langle e| + \hbar f_q(\Phi) \left(\frac{\partial |e\rangle}{\partial \Phi} \langle e| + |e\rangle \frac{\partial \langle e|}{\partial \Phi} \right). \quad (\text{D10})$$

This equation can be written as

$$\frac{\partial \hat{\mathcal{H}}_q}{\partial \Phi} = 2I_p \sqrt{1 - \left(\frac{f_q^{(0)}}{f_q} \right)^2} \hat{b}^\dagger \hat{b} + \frac{f_q^{(0)}}{f_q} I_p (\hat{b} + \hat{b}^\dagger). \quad (\text{D11})$$

For the coupling Hamiltonians we have

$$\begin{aligned} \frac{\partial \hat{\mathcal{H}}_{\text{qci}}}{\partial \Phi} &= -\hbar g_i (\hat{a}_i + \hat{a}_i^\dagger) \left(\frac{\partial |e\rangle}{\partial \Phi} \langle g| + |e\rangle \frac{\partial \langle g|}{\partial \Phi} \right. \\ &\quad \left. + \frac{\partial |g\rangle}{\partial \Phi} \langle e| + |g\rangle \frac{\partial \langle e|}{\partial \Phi} \right), \end{aligned} \quad (\text{D12})$$

$$\frac{\partial \hat{\mathcal{H}}_{\text{qci}}}{\partial \Phi} = \frac{g_i}{\Phi_0} \frac{I_p}{f_q^{(0)}} \left(\frac{f_q^{(0)}}{f_q} \right)^2 (\hat{a}_i + \hat{a}_i^\dagger) (2\hat{b}^\dagger \hat{b} - 1). \quad (\text{D13})$$

Rewriting the states $|\psi_i\rangle$ in Eqs. (D6), (D7), and (D8) as

$$|\psi_k\rangle = \sum_{i=0}^3 a_{ki} |i\rangle, \quad (\text{D14})$$

the nonvanishing matrix elements in the transition rates given in Eq. (D9) are as follows:

$$M_{10} \equiv \langle \psi_1 | \frac{\partial \hat{\mathcal{H}}_{\text{qc1}}}{\partial \Phi} | \psi_0 \rangle + \langle \psi_1 | \frac{\partial \hat{\mathcal{H}}_{\text{qc2}}}{\partial \Phi} | \psi_0 \rangle = -a_{12} \frac{\hbar g_1}{\Phi_0} \frac{I_p}{ef_q^{(0)}} \left(\frac{f_q^{(0)}}{f_q} \right)^2 - a_{13} \frac{\hbar g_2}{\Phi_0} \frac{I_p}{ef_q^{(0)}} \left(\frac{f_q^{(0)}}{f_q} \right)^2 = M_{01}^*, \quad (\text{D15})$$

$$\begin{aligned} M_{02} &\equiv \langle \psi_0 | \frac{\partial \hat{\mathcal{H}}_q}{\partial \Phi} | \psi_2 \rangle + \langle \psi_0 | \frac{\partial \hat{\mathcal{H}}_{\text{qc1}}}{\partial \Phi} | \psi_2 \rangle + \langle \psi_0 | \frac{\partial \hat{\mathcal{H}}_{\text{qc2}}}{\partial \Phi} | \psi_2 \rangle \\ &= a_{21} \frac{f_q^{(0)}}{f_q} I_p - a_{22} \frac{\hbar g_1}{\Phi_0} \frac{I_p}{ef_q^{(0)}} \left(\frac{f_q^{(0)}}{f_q} \right)^2 - a_{23} \frac{\hbar g_2}{\Phi_0} \frac{I_p}{ef_q^{(0)}} \left(\frac{f_q^{(0)}}{f_q} \right)^2 = M_{20}^*, \end{aligned} \quad (\text{D16})$$

$$\begin{aligned} M_{03} &\equiv \langle \psi_0 | \frac{\partial \hat{\mathcal{H}}_q}{\partial \Phi} | \psi_3 \rangle + \langle \psi_0 | \frac{\partial \hat{\mathcal{H}}_{\text{qc1}}}{\partial \Phi} | \psi_3 \rangle + \langle \psi_0 | \frac{\partial \hat{\mathcal{H}}_{\text{qc2}}}{\partial \Phi} | \psi_3 \rangle \\ &= a_{31} \frac{f_q^{(0)}}{f_q} I_p - a_{32} \frac{\hbar g_1}{\Phi_0} \frac{I_p}{ef_q^{(0)}} \left(\frac{f_q^{(0)}}{f_q} \right)^2 - a_{33} \frac{\hbar g_2}{\Phi_0} \frac{I_p}{ef_q^{(0)}} \left(\frac{f_q^{(0)}}{f_q} \right)^2 = M_{30}^*, \end{aligned} \quad (\text{D17})$$

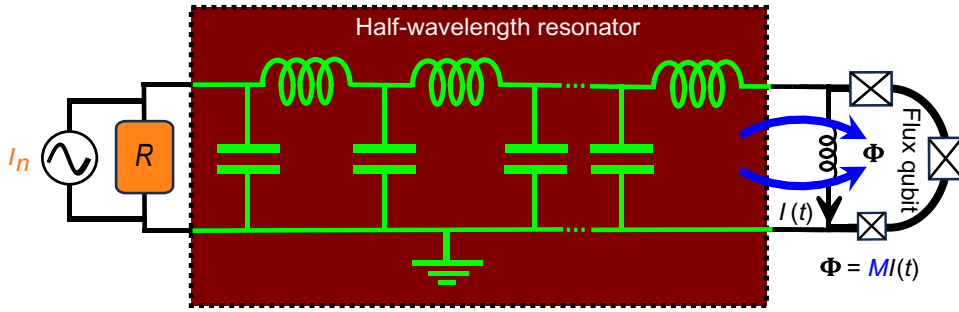


FIG. 11. Circuit representation of one side of the device reported in the main text.

$$M_{23} \equiv \langle \psi_2 | \frac{\partial \hat{\mathcal{H}}_q}{\partial \Phi} | \psi_3 \rangle = a_{21} a_{31} 2I_p \sqrt{1 - \left(\frac{f_q^{(0)}}{f_q} \right)^2} = M_{32}^*. \quad (\text{D18})$$

Since the couplings between each resonator and the qubit are equal, $g_1 = g_2 \equiv g$, the matrix elements $M_{10} = M_{01} = 0$. Then, $|\psi_1\rangle$ remains as a dark state with no transitions into or out from it. Panel (a) of Fig. 4 of the main text provides an overview of the nonvanishing transitions between eigenstates that contribute to heat transport in this setup.

2. Coupling of the noise to the flux qubit

In order to obtain the noise spectrum $S_\Phi(\omega_{ij})$ in the expression of the transition rates, $S_\Phi(\omega_{ij})$ in Eq. (D9), we first note that the flux $\Phi(t)$ and current $I_L(t)$ through the input coil of mutual inductance M of the flux qubit are related as

$$\Phi(t) = MI(t) \quad (\text{D19})$$

for a directly coupled qubit such as ours (see Fig. 11). Owing to the coupling via a $\lambda/2$ coplanar wave resonator with resonance frequency f_r , we have the flux noise on the qubit at frequency $f = \omega/2\pi$ in the form

$$S_\Phi(\omega) = M^2 |t|^2 S_I(\omega), \quad (\text{D20})$$

where $S_I(\omega)$ is the bare current noise produced by the resistor, and for this distributed LC resonator we have

$$|t|^2 = \frac{1}{\cos^2(\pi \frac{f}{f_r}) + \left(\frac{Z_\infty}{R}\right)^2 \sin^2\left(\pi \frac{f}{f_r}\right)}, \quad (\text{D21})$$

where $Z_\infty \approx 50 \, \Omega$ is the impedance of the transmission line. The bare current noise spectrum of the resistor R at inverse temperature β is

$$S_I(\omega) = \frac{2}{R} \frac{\hbar \omega}{1 - e^{-\beta \hbar \omega}}. \quad (\text{D22})$$

Based on the allowed transitions presented in Fig. 4(c) in the main text, the diagonal elements of the density matrix

under nondriven conditions are given by

$$\begin{aligned} \dot{\rho}_0 &= \Gamma_{2 \rightarrow 0} \rho_2 + \Gamma_{3 \rightarrow 0} \rho_3 - (\Gamma_{0 \rightarrow 2} + \Gamma_{0 \rightarrow 3}) \rho_0, \\ \dot{\rho}_1 &= 0, \\ \dot{\rho}_2 &= \Gamma_{0 \rightarrow 2} \rho_0 + \Gamma_{3 \rightarrow 2} \rho_3 - (\Gamma_{2 \rightarrow 0} + \Gamma_{2 \rightarrow 3}) \rho_2, \\ \dot{\rho}_3 &= \Gamma_{0 \rightarrow 3} \rho_0 + \Gamma_{2 \rightarrow 3} \rho_2 - (\Gamma_{3 \rightarrow 0} + \Gamma_{3 \rightarrow 2}) \rho_3, \end{aligned} \quad (\text{D23})$$

where $\Gamma_{i \rightarrow j} = \Gamma_{i \rightarrow j}^{r=1} + \Gamma_{i \rightarrow j}^{r=2}$, in which the superscript r refers to Reservoir 1 and Reservoir 2. For the steady state we put $\dot{\rho}_i = 0$ ($i = 0, \dots, 3$) and $\sum_{i=0}^3 \rho_i = 1$, which yield the populations of the levels ρ_i as

$$\begin{aligned} \rho_0 &= \frac{\Gamma_{2 \rightarrow 0} \Gamma_{3 \rightarrow 0} + \Gamma_{2 \rightarrow 3} \Gamma_{3 \rightarrow 0} + \Gamma_{2 \rightarrow 0} \Gamma_{3 \rightarrow 2}}{A + B + C} \\ &= \frac{A}{A + B + C}, \\ \rho_1 &= 0, \\ \rho_2 &= \frac{\Gamma_{0 \rightarrow 2} \Gamma_{3 \rightarrow 0} + \Gamma_{0 \rightarrow 2} \Gamma_{3 \rightarrow 2} + \Gamma_{0 \rightarrow 3} \Gamma_{3 \rightarrow 2}}{A + B + C} \\ &= \frac{B}{A + B + C}, \\ \rho_3 &= \frac{\Gamma_{0 \rightarrow 3} \Gamma_{2 \rightarrow 0} + \Gamma_{0 \rightarrow 2} \Gamma_{2 \rightarrow 3} + \Gamma_{0 \rightarrow 3} \Gamma_{2 \rightarrow 3}}{A + B + C} \\ &= \frac{C}{A + B + C}. \end{aligned} \quad (\text{D24})$$

The power transmitted from Reservoir 1 to Reservoir 2 then reads

$$P_{1 \rightarrow 2} = \sum_{k,l} \rho_k \hbar \omega_{kl} \Gamma_{k \rightarrow l}^{(r=2)}. \quad (\text{D25})$$

The calculated power $P_{1 \rightarrow 2} (= -P_{2 \rightarrow 1})$ is plotted in panel (a) of Fig. 12 together with experimental results.

3. Flux qubit coupled to a bare resistor

We can obtain the power nearly quantitatively at the symmetry point $\Phi = \Phi_0/2$, i.e., “the central peak,” by

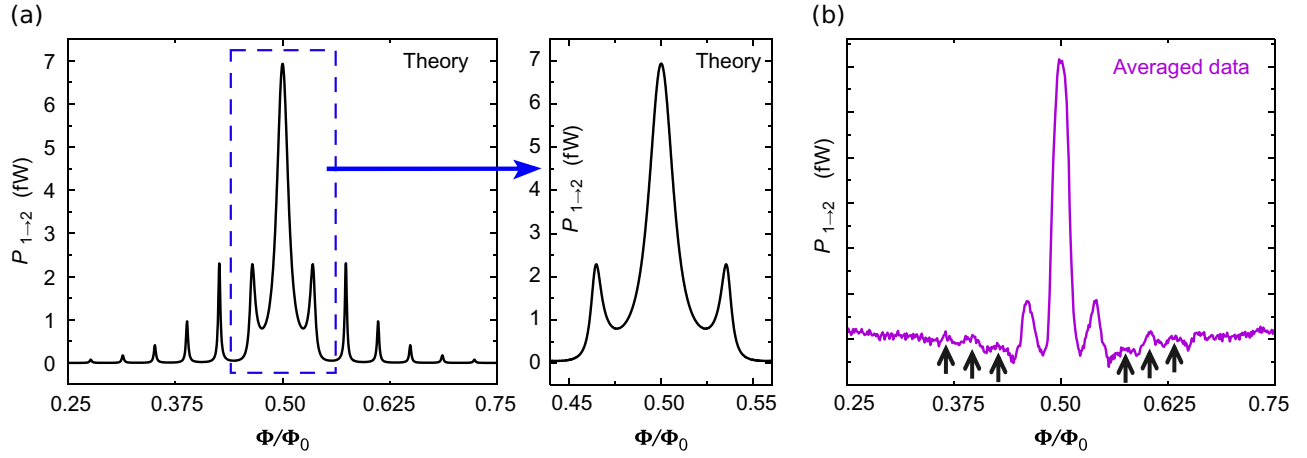


FIG. 12. Observations in theory and experimental data. (a) The experimental results (left) and the theoretical prediction (right) for the power transmitted in the setup. The parameters in the calculation are as follows: $R = 6 \, \Omega$, $L = 0.8 \, \text{nH}$, $I_p = 30 \, \text{nA}$, $f_q^{(0)} = 2 \, \text{GHz}$, and $f_r = 7 \, \text{GHz}$.

simply assuming that a bare resistor (reservoir) is coupled inductively to the qubit. The equivalent circuit representing a resistor with resistance R coupled via mutual inductance M to a flux qubit is shown in Fig. 13. Based on the Kirchhoff's circuit law, the current I reads

$$I = \frac{R}{R + i\omega L} I_n. \quad (\text{D26})$$

The noise spectral density of Eq. (D26) is then given by

$$S_I(\omega) = \frac{R^2}{R^2 + \omega^2 L^2} S_{I_n}(\omega). \quad (\text{D27})$$

Since for the bare resistor with temperature $T = (k_B \beta)^{-1}$, $S_{I_n}(\omega) = 2/R[\hbar\omega/(1 - e^{-\beta\hbar\omega})]$, we have

$$S_I(\omega) = \frac{2R}{R^2 + \omega^2 L^2} \frac{\hbar\omega}{1 - e^{-\beta\hbar\omega}}. \quad (\text{D28})$$

The flux into the qubit is $\Phi = MI$. Using Eq. (D28), the noise spectral density of flux, $S_\Phi = M^2 S_I$, reads

$$S_\Phi(\omega) = 2M^2 \frac{R}{R^2 + \omega^2 L^2} \frac{\hbar\omega}{1 - e^{-\beta\hbar\omega}}. \quad (\text{D29})$$

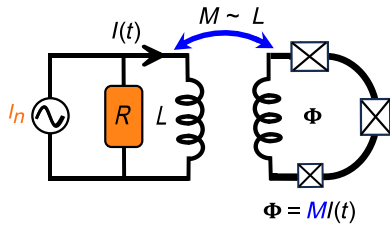


FIG. 13. Thermal noise of a resistor to a flux qubit via inductive coupling.

Based on the golden rule, the transition rate up $\Gamma_\uparrow$ between the two levels of the qubit (ground state $|g\rangle$ and excited state $|e\rangle$) due to the presence of the bath (resistor R) is given by

$$\Gamma_\uparrow = \frac{1}{\hbar^2} |\langle g | \frac{\partial H}{\partial \Phi} | e \rangle|^2 S_\Phi(-\omega_{01}), \quad (\text{D30})$$

where the Hamiltonian of the qubit is $\hat{\mathcal{H}}_q = -(\hbar/2)\varepsilon\sigma_z$, which becomes equivalent to $\hat{\mathcal{H}}_q = \hbar f_q(\Phi)\hat{b}^\dagger\hat{b}$ when restricted to the two lowest energy states. Here we have $\varepsilon = (2I_p/\hbar)(\Phi - \Phi_0/2)$, with I_p the persistent circulating current in the qubit loop. Since $\partial H/\partial \Phi = -I_p\sigma_z$, the transition rate is

$$\Gamma_\uparrow = \frac{I_p^2 M^2}{\hbar^2} \frac{2R}{R^2 + \omega_{01}^2 L^2} \frac{\hbar\omega_{01}}{1 - e^{-\beta\hbar\omega_{01}}}. \quad (\text{D31})$$

In order to obtain the power between the two baths at temperatures T_1 and T_2 , we need the diagonal elements of the density matrix. With no time-dependent driving we then have

$$\dot{\rho}_{ee} = \Gamma_{\Sigma,\uparrow} \rho_{gg} - \Gamma_{\Sigma,\downarrow} \rho_{ee}, \quad (\text{D32})$$

where $\Gamma_{\Sigma,\uparrow} = \Gamma_{1,\uparrow} + \Gamma_{2,\uparrow}$ and $\Gamma_{\Sigma,\downarrow} = \Gamma_{1,\downarrow} + \Gamma_{2,\downarrow}$. By applying steady-state condition $\dot{\rho}_{ee} = 0$, the population of the excited state reads

$$\rho_{ee} = \frac{\Gamma_{\Sigma,\uparrow}}{\Gamma_{\Sigma,\downarrow} + \Gamma_{\Sigma,\uparrow}}. \quad (\text{D33})$$

The power $\dot{Q}$ that goes out from the bath (resistor) is given by

$$\dot{Q} = \hbar\omega_{01} [\Gamma_{1,\uparrow} \rho_{gg} - \Gamma_{1,\downarrow} \rho_{ee}], \quad (\text{D34})$$

where $\rho_{gg} = 1 - \rho_{ee}$ denotes the population of the ground state of the qubit. If we assume a two-bath system with $T_2 \ll T_1$, then this leads to $\Gamma_{2,\uparrow} \simeq 0$ and $\Gamma_{2,\downarrow} \simeq \Gamma_{1,\downarrow}$. In this case power $\dot{Q}$ reads

$$\dot{Q} \simeq \frac{1}{2} \hbar \omega_{01} \Gamma_{1,\uparrow}. \quad (\text{D35})$$

We then have

$$\dot{Q} \simeq I_p^2 M^2 \frac{R}{R^2 + \omega_{01}^2 L^2} \frac{\omega_{01}^2}{e^{\beta_1 \hbar \omega_{01}} - 1}, \quad (\text{D36})$$

where $\beta_1 = (k_B T_1)^{-1}$ is the temperature of the hot bath (resistor). For the galvanic coupling we have here, $M \simeq L$. We then have

$$\dot{Q} \simeq I_p^2 R \frac{(\omega_{01} L)^2}{R^2 + (\omega_{01} L)^2} \frac{1}{e^{\beta_1 \hbar \omega_{01}} - 1}. \quad (\text{D37})$$

We see that for $L \ll R/\omega_{01}$, $\dot{Q} \propto L^2$ and it obtains its maximum value

$$\dot{Q}_{\max} = I_p^2 L \frac{\omega_{01}}{e^{\beta_1 \hbar \omega_{01}} - 1}, \quad (\text{D38})$$

when $L = R/\omega_{01}$. With the values used in the theoretical model, with the parameter values as stated above, we have from Eq. (D37), $\dot{Q} \simeq 7.5$ fW at $T_1 = 300$ mK, which remains in agreement with the experiment.

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