

rf quantum capacitance parametric amplifier

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(Received 27 January 2026; revised 7 July 2026; accepted 28 August 2026; published 19 October 2026)

We report an rf parametric amplifier that exploits the gate-tunable quantum capacitance of a semiconductor two-dimensional electron gas at cryogenic temperatures. The prototype amplifier exhibits a gain approximately 30 dB up to an input power of -66 dBm and a noise temperature $T_N \sim 2$ K at approximately 370 MHz while dissipating approximately a microwatt of power. Operable at tesla-scale magnetic fields and temperatures ranging from millikelvin to a few kelvin, the quantum capacitance parametric amplifier may find utility in the readout circuit of spin qubits. Here, a quantum dot system is used to verify amplifier performance, improving the signal-to-noise ratio of the readout by approximately 9 dB.

DOI: 10.1103/3146-fgdw

I. INTRODUCTION

Technologies spanning quantum computing [1], radio astronomy [2,3], space communications [4], particle physics detectors [5], and medical imaging systems [6] are united in a common quest to extract maximum information from inherently weak radio frequency signals, ensuring that every available photon contributes to a measurement. Central to this quest are the attributes of the first-stage amplifier, which, in terms of its gain, noise, bandwidth, and power dissipation, largely dominate the performance of the readout subsystem or receiver. In quantum computing, for instance, sensitive readout enables fast qubit bring-up and tuning, as well as reducing state preparation and measurement errors during quantum circuit operations. Improving the performance of the readout subsystem leads to higher fidelity, faster measurements, and efficient use of hardware resources, directly impacting the entire quantum computing platform [1,7–9].

Since the first-stage amplifier in the readout chain has a dramatic impact on the overall system performance,

significant efforts have been made to develop amplifiers that add only the bare minimum noise required by quantum mechanics [10,11]. To this end, the fluctuation-dissipation theorem [12] intuitively implies that one should look to nondissipative devices exhibiting superconductivity as a means of realizing amplifiers with the lowest noise. Indeed, various families of superconducting amplifiers have been developed over the last decades, including those based on superconducting quantum interference devices [13,14], bistable junctions [15], and parametric approaches [16–23]. Despite the success of these JJ-based implementations, challenges remain, including their limited frequency of operation, low input-power compression, and difficulty operating in even modest magnetic fields. Recently, kinetic inductance-based superconducting amplifiers [20,21,24–27] have outperformed JJ amplifiers in these respects but require a much larger area [28]. In contrast, semiconductor amplifiers, such as those based on high electron mobility transistors (HEMTs) [29] or silicon-germanium (SiGe) heterojunction bipolar transistors (HBTs) [30], address these challenges but at the cost of adding much more noise and significant power dissipation.

Here, we report a rf parametric amplifier based on the modulation of a low-loss “quantum capacitance” associated with an ultrahigh mobility two-dimensional electron gas (2DEG) in a GaAs/AlGaAs semiconductor heterostructure grown using molecular beam epitaxy (MBE). With an rf pump voltage applied to a surface gate, the density of the 2DEG is modulated, leading to an oscillating

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TABLE I. Low-noise amplifier (LNA) specifications representing common state-of-the-art technologies, including the historical varactor paramp and the quantum capacitance parametric amplifier (QCPA) reported here.

Specs	JPA	KTWPA	HEMT	Junction varactor paramp	QCPA
Gain	20–30 dB	20 dB	40 dB	20–30 dB	20–30 dB
Lowest frequency	Approximately equal to 100 MHz	500 MHz	100 s MHz	100 s MHz	100 s MHz
Bandwidth	Octave	Octave	Decade	Octave	Narrow (1–2 MHz)
Noise temperature	SQL	Near SQL	1–2 K	10 K	0.7–2.2 K
Physical temperature	< 1 K	< 5 K	< 20 K	300–4 K	< 10 K
Power consumption	Order of nW	Order of μ W	> 20 mW	> 20 mW	Order of μ W
Gain compression	–100 dBm	–60 dBm	–10 dBm	–10 dBm	–66 dBm
Magnetic field compatible	Minimal	Yes	Yes	Yes	Yes
Linearity	Moderate	Moderate	High	Moderate	Moderate
Directional	No	No	Yes	No	No
Technology	Josephson junction (JJ)	Kinetic inductance	III–V heterostructure	Abrupt PN junction	III–V heterostructure

quantum capacitance and parametric gain as the 2DEG is periodically depleted and accumulated. Very recently, the quantum capacitance of a quantum dot was also used to yield a small parametric gain based on the tunneling of electrons [31]. In contrast, 2D heterostructures in the ballistic transport regime can exhibit a much larger modulated quantum capacitance (relative to the total capacitance), leading to large gain operation. In such systems, dissipation can be suppressed dramatically [32], with carrier mean free paths in excess of 100 s of microns now relatively commonplace. In the limit where dissipation is reduced to zero, the QCPA acts as a dual of the superconducting parametric amplifier [33].

Short of this limit, the presence of moderate dissipation in the QCPA still yields specifications that fill a gap between superconducting para-amps and semiconductor approaches based on cryo-cooled transistors. Operating in the ultrahigh-frequency band typically used for semiconductor qubit readout and radio astronomy (0.3–3 GHz), the QCPA exhibits a gain of > 30 dB, 1-dB compression at an input power of –60 dBm, and, in the prototype presented here, a noise temperature comparable to state-of-the-art transistor amplifiers based on HEMTs (approximately 2 K). Unlike HEMT or HBT amplifiers, however, where the lowest achievable noise is constrained by their (milliwatt) self-heating [34], our prototype QCPA operates at millikelvin temperatures with microwatt power dissipation, suggesting that its noise can likely be further reduced by improved thermalization and suppressing ohmic losses in the gates, 2DEG, and contacts.

The use of variable capacitors to construct parametric amplifiers is, of course, well known, with a rich history dating back to the late 1950s [35], where their use in radio astronomy [36,37] and radar systems was fundamental [38]. We provide a discussion of some of this history as

supplemental material, giving context and motivation for the present work. A comparison of our QCPA to other cryogenic amplifiers is summarized in Table I. Below we report the performance of the QCPA, first in isolation and then further in enabling readout of a quantum dot device. Integrated with the quantum device at 20 mK the amplifier is shown to improve the signal-to-noise ratio (SNR) by approximately 9 dB, when pump power is applied.

II. THE QCPA

The QCPA is realized via modulating the electron density of a 2DEG formed 91 nm below the surface, at the interface of a GaAs/AlGaAs heterostructure grown using molecular beam epitaxy (ungated density = $1.8 \times 10^{11} \text{ cm}^{-2}$ and mobility $4.4 \times 10^6 \text{ cm}^2 \text{ V}^{-1} \text{ s}^{-1}$). The variable quantum capacitance is associated with the 2DEG acting as the bottom plate of the capacitor and a 200 nm niobium titanium nitride (NbTiN) metal gate layer acting as the top plate. A micrograph of the device is shown in Fig. 1(a). The top plate of the capacitor is fabricated as a set of five finger gates. Electrical connection to the 2DEG is formed via an array of parallel ohmic contacts comprising an alloy of NiGeAu, with the dc resistance of a single contact being of order 100 Ω (see Sec. VIII for an illustration of the device and further details of device fabrication). A large NbTiN ground plane surrounds the finger gates.

The QCPA is built as follows. The dc bias inductor, 1000 nH, is used to bias the gate of the GaAs nonlinear capacitance from an external source. This inductor has a self-resonance frequency at approximately 300 MHz but maintains a reasonably high impedance still at the frequencies of interest (on the order of 5 k Ω). It acts as an rf choke, preventing the resonant signal from appearing at the dc bias

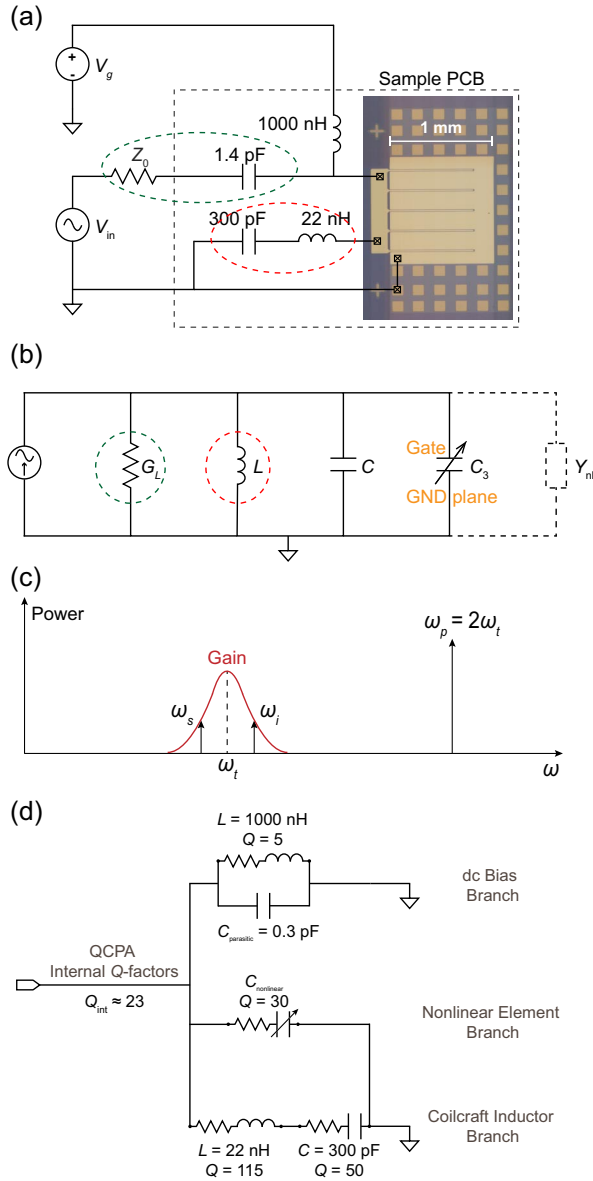


FIG. 1. The QCPA using a gate-configurable 2DEG device. (a) Shows a micrograph of the device with the components that form the QCPA circuit on the circuit board highlighted by the dashed box. The waves symbolize the incident source and pump tones and the reflected amplified source and idler tones. In (b), the QCPA circuit is transformed into a parallel tank circuit for gain analysis. Here, the capacitance is divided into the parasitic capacitance C and the variable part C_3 . The external inductor (22 nH) is in parallel with the active device and forms the second leg of the QCPA tank circuit L in (b). The coupling capacitance (1.4 pF) transforms the input and load impedances of the circuit to a larger value (series-to-parallel conversion [39]). The resultant loaded losses of the tank and the source and load are annotated as G_L in (b). (c) illustrates three-wave mixing giving nondegenerate gain with the source and idler signals detuned from the center $\omega_t = 1/\sqrt{LC}$ by $\mp \omega_x$, respectively. (d) Internal QCPA circuit for Q-factor calculation showing each branch separately. The parasitic capacitance of the dc bias inductor is shown to be 0.3 pF. The dc source is considered an rf ground for the purpose of the Q-factor estimate.

port of the circuit. The QCPA resonator is formed of the GaAs device's nonlinear gate capacitance and an air-core spring inductor of 22 nH, forming a parallel tank circuit in the frequency of range 360–370 MHz. The operating frequency of the amplifier was chosen to align with the tank frequency used for optimal spin qubit readout based on single-electron transistors or quantum point contacts (QPCs). To prevent the dc bias voltage of the nonlinear component from shorting to ground through the parallel 22 nH inductance, a dc block capacitance of 300 pF is added in series to the 22 nH. At the operation frequency of the QCPA, this capacitance effectively shifts the 22 nH slightly down to 21.3 nH. The value of the dc blocking capacitor is set to present a low-impedance path at frequencies above 100 MHz. The coupling capacitance (1.4 pF) controls the coupling regime of the QCPA to the port impedance of the circuit. An equivalent parallel circuit implementation is shown in Fig. 1(b) to aid in circuit analysis [39].

The nondegenerate gain G_p of the amplifier as a function of frequency can then be written as [40]:

$$G_p = \frac{G_{nl}^2 + x^2[G_{nl} + G_L(1 + x^2)]^2}{[-G_{nl} + G_L(1 + x^2)]^2 + x^2[G_{nl} + G_L(1 + x^2)]^2}, \quad (1)$$

with G_{nl} being the effective negative conductance due to the pumped variable capacitance at the tank resonance frequency $f_t = \omega_t/2\pi$, G_L the total loaded losses, Q_t the loaded quality factor of the amplifier, and $x = 2Q_t\omega_x/\omega_t$ representing the frequency detuning ω_x of the source signal from the tank resonance ω_t , as illustrated in Fig. 1(c). Furthermore, at high gain, the amplifier has a linear gain-bandwidth relationship:

$$\sqrt{G} \times B = \frac{f_t}{2Q_t} \quad (2)$$

with G being the maximum nondegenerate gain of the amplifier at the center frequency and B is the bandwidth [40]. A detailed derivation of the gain expression is provided in the Appendix.

The loss budget of the QCPA circuit is a critical analysis that directly influences both the design methodology and overall performance. The Q-factor of the circuit components at the resonance frequency of the amplifier is detailed in Fig. 1(d). The circuit is effectively divided into three branches. The rf choke inductor (1000 nH) has a Q of order 5, a self-resonance at 290 MHz, and a participation ratio of 0.009%. Because of its high impedance, this bias inductor, although possessing a low Q, stores a negligible amount of the resonator energy and hence does not participate effectively in loading the overall internal Q-factor of the QCPA. The Coilcraft spring

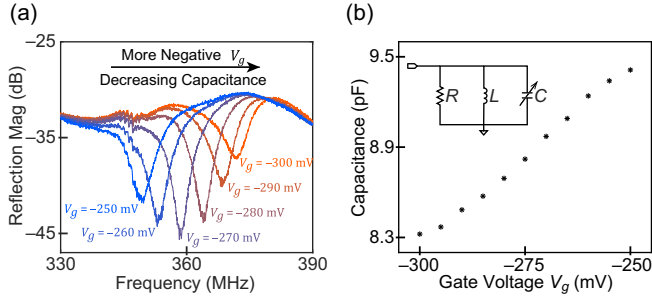


FIG. 2. The passive frequency response of the QCPA is characterized as a function of gate voltage, V_g , using a vector network analyzer to apply a low-power input tone. (a) The reflection coefficient magnitude is plotted for V_g ranging between -250 mV and -300 mV. (b) The gate capacitance is extracted from the shift in resonance frequency of the tank circuit (shown in the inset).

inductor (22 nH) has a Q of order 115 in series with the dc block capacitors with Q of order 50. The participation ratio of this inductive branch is computed to be 51.367%. The resultant estimated Q -factor of the 2DEG GaAs/AlGaAs nonlinear device is estimated to be near 30 with a participation ratio of 48.624%, which forms the dominant loss mechanism of the QCPA circuit. Projected from the measurements in Fig. 2 and the circuit parameters, the internal quality factor of the circuit is estimated to be approximately 23. With the relatively low internal quality factor of the QCPA, the circuit is undercoupled with an external quality factor of approximately 40. In future work, it is possible to adjust this coupling to improve the sensitivity of the amplifier for qubit readout and optimize the nonlinear element fabrication to aim for a higher internal quality factor.

III. QCPA CHARACTERIZATION

Measurements are carried out over a series of cool-downs with the amplifier at the base of a dilution refrigerator. Attenuators on the source line are chosen to reduce its noise temperature to a few millikelvin. Filtering on the pump and the dc ports removes their respective noise contributions at the source frequency (360–370 MHz). The pump power is not attenuated since amplitudes of order 10 s of millivolts are needed at the gate.

The reflection coefficient of the QCPA is measured as a function of frequency at low source power (-100 dBm), as the dc voltage applied to the gate is made more negative, as shown in Fig. 2(a). The shift in resonance frequency with gate voltage indicates a tunable capacitance, which can be determined from the resonance frequency using $f_i = 1/2\pi\sqrt{LC}$, as shown in the simplified tank circuit in the inset of Fig. 2(b) ($L = 22$ nH). The total capacitance is plotted as a function of gate voltage varying by 1.2 pF on a

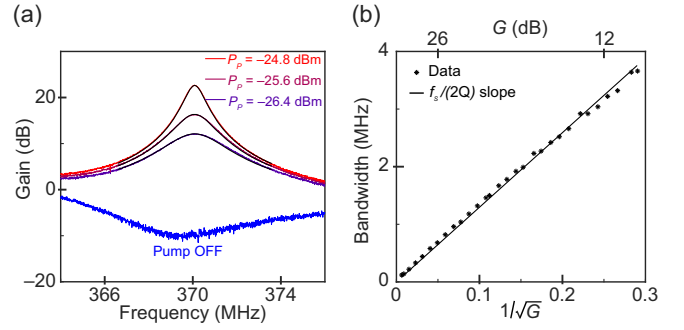


FIG. 3. The gain of the QCPA measured using a network analyzer while the device is pumped at $\omega_p = 740.17$ MHz. (a) The gain is plotted as a function of frequency and pump power, $P_p = -26.4$ dBm, -25.6 dBm, -24.8 dBm, respectively. The overlaid black curves show the gain predicted by the theoretical model of Eq. (1). The gain model is fit using $Q_i = 14.3$ and a pumped capacitance range of $C_3 = 0.51$ – 0.57 pF. (b) The bandwidth of the amplifier is extracted and plotted as a function of the inverse square root of the gain, verifying the relationship in Eq. (2).

background capacitance $C = 8.9$ pF. The slope of the C - V characteristic gives a lever arm of 26.7 pF/V.

We next look for amplification by varying the pump power and sweeping the source frequency. The gain of the amplifier manifests as an increase in reflected signal power when pumping, relative to the idle state (when the pump is off, and dc gate bias is set to $V_g = 0$ V). Measured gain as a function of frequency is plotted in Fig. 3(a), for different pump powers. Here the tank circuit center frequency is 370.085 MHz, and the pump is at 2×370.085 MHz = 740.170 MHz. We fit the gain dependence using Eq. (1), with $Q_i = 14.3$ and a pumped capacitance of $C_3 = 0.51$ – 0.57 pF. Plotting the amplifier bandwidth versus $1/\sqrt{G}$ leads to the linear dependence shown in Fig. 3(b), typical of parametric operation [40,41], as described by Eq. (2). Measuring the input power at which the amplifier saturates, Figs. 4(a) and 4(b) indicate 1-dB gain compression is reached at input powers of -66 dBm and -56 dBm at gains of 20.6 dB and 10.8 dB, respectively.

The operation of the QCPA in regimes of elevated temperature and strong magnetic field is significant for integration with spin qubits. Raising the cryostat temperature from 30 mK to 4 K, we observe a reduction in the maximum gain for a given pump power, as shown in Fig. 5(a). The gain can be recovered, however, by increasing the pump power at elevated temperatures. Although increased power will lead to increased dissipation, the cooling power of the cryostat is also significantly increased at elevated temperatures.

Given that resistive losses are largely temperature independent below 4 K, the reduction in gain (for a given pump power) is likely due to a decrease in the

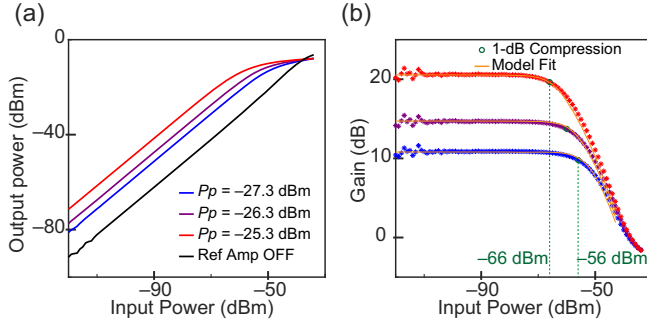


FIG. 4. The QCPA nondegenerate gain is measured as a function of source signal power. Panel (a) shows the output power measured at the spectrum analyzer as a function of the input power referred to the input of the QCPA. A reference curve is plotted in black for the case when the amplifier is OFF. (b) The gain of the QCPA is plotted as a function of increasing input power with the theoretical saturation behavior overlaid (gold colored lines).

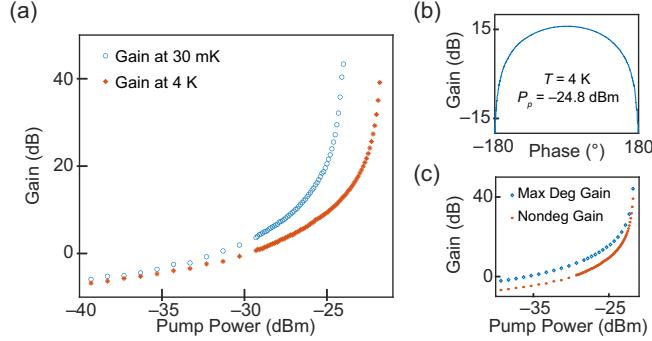


FIG. 5. (a) The QCPA is tested at $T \sim 4$ K, with gain shown as a function of pump power. The $T \sim 0.03$ K gain profile of the amplifier is shown for comparison. (b) The degenerate gain of the parametric amplifier was characterized at 4 K, plotted as a function of the pump phase. Panel (c) shows a comparison between the maximum attainable degenerate gain and nondegenerate gain.

transconductance with thermal broadening and partly by the loss of superconductivity of the aluminum (Al) bondwires. Nevertheless, we note that the QCPA yields a gain of 40 dB at elevated temperature, enabling troubleshooting and deployment of a readout subsystem at 4 K (or even higher temperatures).

We further characterize the amplifier at 4 K by comparing degenerate and nondegenerate operation. Degenerate phase-dependent behavior occurs when the source and idler tones are positioned at the same frequency. The maximum gain attained by the QCPA is the degenerate gain where $f_s = f_i = f_p/2 = f_t$ at a specific phase arrangement between the pump and the source tones in which the nonlinear capacitance presents the most negative admittance for a given pump power. With a 90° shift from the ideal degenerate phase relationship, however,

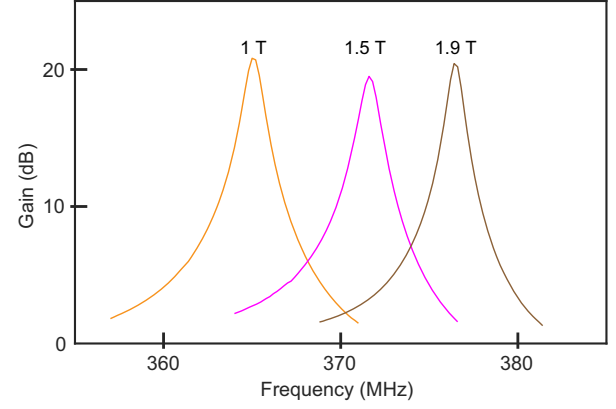


FIG. 6. The QCPA gain versus frequency, plotted at three magnetic field values (1.0, 1.5, and 1.9 T). The field is oriented in-plane of the QCPA active device. The amplifier achieves 20 dB gain with the operation point optimized accordingly; the pump power is set to -23.6 dBm, -17.4 dBm, and -12.3 dBm, respectively.

the nonlinear element assumes a real and positive value, which then acts to deamplify the source signal. Thus, the operation of the amplifier in the degenerate case is phase-dependent and can swing from high amplification to deep deamplification, as shown in Fig. 5(b). The characterization and comparison of the maximum degenerate and nondegenerate gain as a function of pump power at 4 K is shown in Fig. 5(c).

Turning to the behavior of the amplifier in a magnetic field, we note that various qubit platforms require tesla-scale fields for operation, usually oriented in-plane with the 2DEG (as in the setup of the experiment in Fig. 6). With the amplifier at 30 mK, we continue to observe moderate gain (> 20 dB) up to fields of several tesla (oriented in-plane with the QCPA chip). Interestingly, the data also reveal a small reduction in the quantum capacitance of the 2DEG with parallel field, which leads to an increase in resonance frequency. This change in capacitance is likely related to the well-known dependence of the 2D density of states on the field [42].

IV. NOISE ANALYSIS

To estimate the noise performance of the amplifier, we make use of the well-known Y-factor method, leveraging a commercial cryogenic noise source [43] (see Appendix B for details). This is made difficult in our setup by the small noise power of the amplifier and the presence of a significant insertion loss from the ferrite circulators operating at cryogenic temperatures. Measuring the integrated noise power at the readout port as a function of noise input power then yields the offset noise in the limit where the input is zero. Using this (Y-factor) method, the noise temperature of the system is determined to be

$T_{\text{sys}} = 6.49^{+1.06}_{-0.62}$ K, with the gain of the QCPA ~ 34 dB. By independently measuring the insertion loss from the circulators, we are able to estimate the intrinsic noise temperature of the QCPA to be $T_{\text{QCPA}} \sim 2.6$ K. Including the idler contribution to the noise results in twice the noise temperature compared with values where the idler is subtracted, i.e., T_{QCPA} (with idler) = 2.6 K or T_{QCPA} (without idler) = 1.3 K. Below we compare this estimate to values extracted using an alternate approach based on readout of a quantum dot device.

Likely, the origin of this measured noise is thermal contributions from ohmic elements associated with the amplifier and supporting components (in particular the copper inductor comprising the resonant circuit). In this regard, it is interesting to measure the noise temperature of the QCPA as a function of its physical temperature by heating the refrigerator. In doing this, we find T_{QCPA} only increases when the physical temperature exceeds approximately 1 K, possibly indicating that self-heating and poor thermalization limit the physical temperature and thus the performance of the amplifier in the present setup that requires a high pump power. Above approximately 1 K losses from the Al bond-wires in the normal rather than superconducting state may also contribute.

V. QUBIT DEVICE READOUT

We next move to evaluate the amplifier in the context of reading out a quantum device. Beyond demonstrating the utility of the amplifier for this application, reading out a quantum device serves to probe unwanted back-action effects, such as noise and heating, that can degrade qubit performance. Further, by directly comparing the SNR improvement from the amplifier, we are able to further estimate its noise performance.

Our quantum device comprises a gate-defined double quantum dot in a GaAs heterostructure with a proximal charge sensor. Here we operate the sensor as an rf-QPC [44]. The double dot is defined using surface gates shown in yellow in Fig. 7(a) and is part of a larger five-dot device structure. An rf LC tank circuit is used to transform the impedance of the QPC toward $Z_0 = 50 \Omega$ at the resonance frequency, $f_0 \sim 367$ MHz [44].

The QCPA is then connected via semirigid coaxial cable to the rf port of the quantum device and tuned such that the frequency of degenerate operation falls several 10 s of kHz away from the resonance frequency of the rf-QPC, ensuring the amplified readout tone is non-degenerate and phase-independent. The frequency response of the rf-QPC with and without the amplifier is shown in Fig. 7(b).

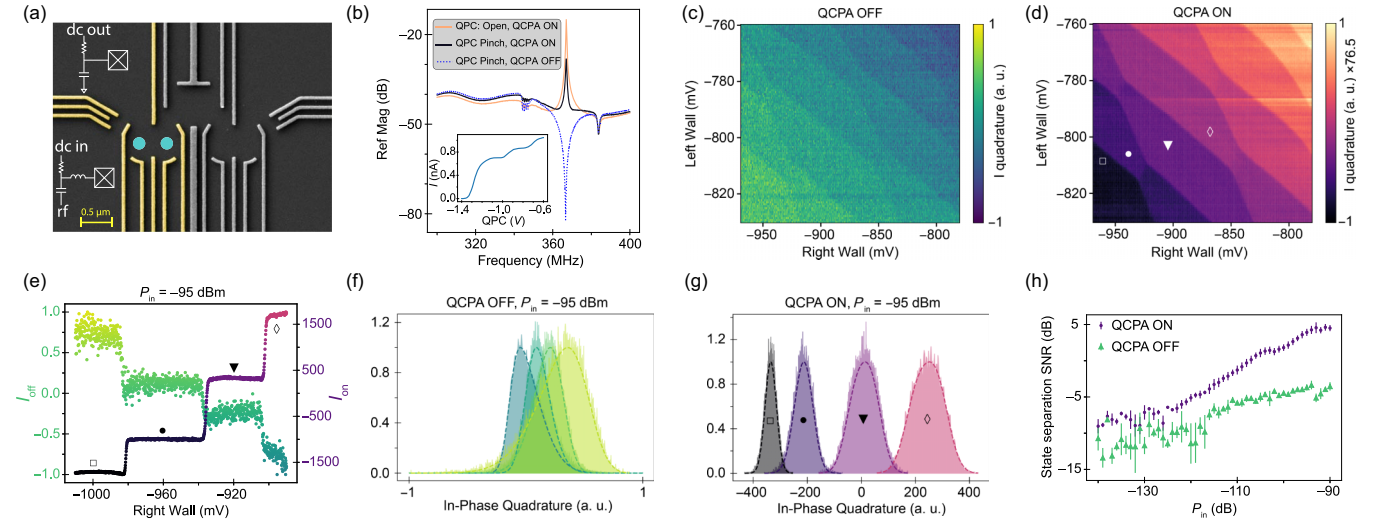


FIG. 7. Double quantum dot readout experiment using the QCPA as the first-stage LNA. (a) The quantum dot device is shown with the relevant gates colored. The readout is done through an rf quantum point contact (rf-QPC). (b) The QCPA and the readout rf-QPC resonance are matched in frequency. The QCPA flips the readout resonance into a gain peak (dc QPC characteristics extracted using a separate transport measurement). (c),(d) Charge stability diagram in the few-electron regime measured with the QCPA OFF in (c) and ON in (d). The data are measured in the I quadrature and normalized against the magnitudes observed in the QCPA OFF state. (e) The charge states as a function of bias are read with the QCPA ON (right axis) and OFF states (left axis). The states are labeled with the markers on the map in (c). (f),(g) The statistical distributions of the readout of the labeled charge states with the QCPA OFF in (f) and the QCPA ON in (g), both collected at an input power of -95 dBm at the input of the double quantum dot. (h) The SNR of the charge-state readout of the double dot extracted from the separation between the statistical distributions as a function of rf power at the input of the double-dot device.

Configuring the double dot to the few-electron regime yields the familiar charge-stability diagram for a two-electron system. Here the transition features correspond to changes in occupancy of the double dot by a single electron. We first look to see if the amplifier produces any apparent back-action shift in the position of these transitions with respect to the dc gate voltages. No shift in the transitions is observed within the uncertainty of repeated datasets, as shown in Figs. 7(c)–7(e). We further compare both inter- and intradot transitions with the amplifier on or off, noting that the width of these charge transitions serves as a millikelvin thermometer and also a sensitive probe of back-action noise. Histograms of the sensing signal are shown for the various charge states in Figs. 7(f) and 7(g). Again, we do not observe additional broadening of any charge transition when the amplifier is operational. It is worthwhile noting that more sensitive experiments with qubit systems may require further isolation than used in the current setup.

Already in the data discussed above, it is apparent that the amplifier improves the SNR of the readout response. We now turn to quantify the improvement to SNR and the associated noise temperature of the QCPA when it is embedded in the readout setup. Here, an estimate of the noise contribution from the QCPA is possible by comparing it to the noise generated by a commercial cryogenic (SiGe) amplifier with known specifications. Accurately knowing the insertion losses in the circuit and the noise temperature of the second-stage SiGe amplifier (CITLF2), the noise contribution from the QCPA can be inferred from the difference in SNR when the amplifier is operating. Cascading the noise from the first- to second-stage amplifier in the readout chain, the ratio of the SNRs (the noise visibility ratio [45]) can be shown to be equivalent to the ratio of the insertion loss in the path preceding the second amplifier, relative to the loss in the path preceding the QCPA:

$$\frac{\text{SNR}_{\text{QCPA,ON}}}{\text{SNR}_{\text{QCPA,OFF}}} = \frac{L_{\text{CITLF2}} \times T_{n,\text{CITLF2}}}{L_{\text{QCPA}} \times T_{n,\text{QCPA}}}. \quad (3)$$

The readout circuit insertion losses are characterized and measured in a separate calibrated setup with the ratio of the two paths approximately 4.4 dB [the ratio of losses is also the insertion loss of the part of the circuit highlighted with a dashed line in Fig. 8(c)]. Knowing the readout circuit losses and the noise temperature of the second amplifier (approximately 2 K at the readout frequency), we estimate the noise temperature of the QCPA to be 0.7–2.2 K (1.45 ± 0.75 K with idler contribution and 0.73 ± 0.38 K with the idler noise subtracted). Mindful that we improved the thermalization of the amplifier in the readout experiment, this measurement of the QCPA noise temperature is broadly consistent with the previously discussed Y-factor measurement. The small discrepancy

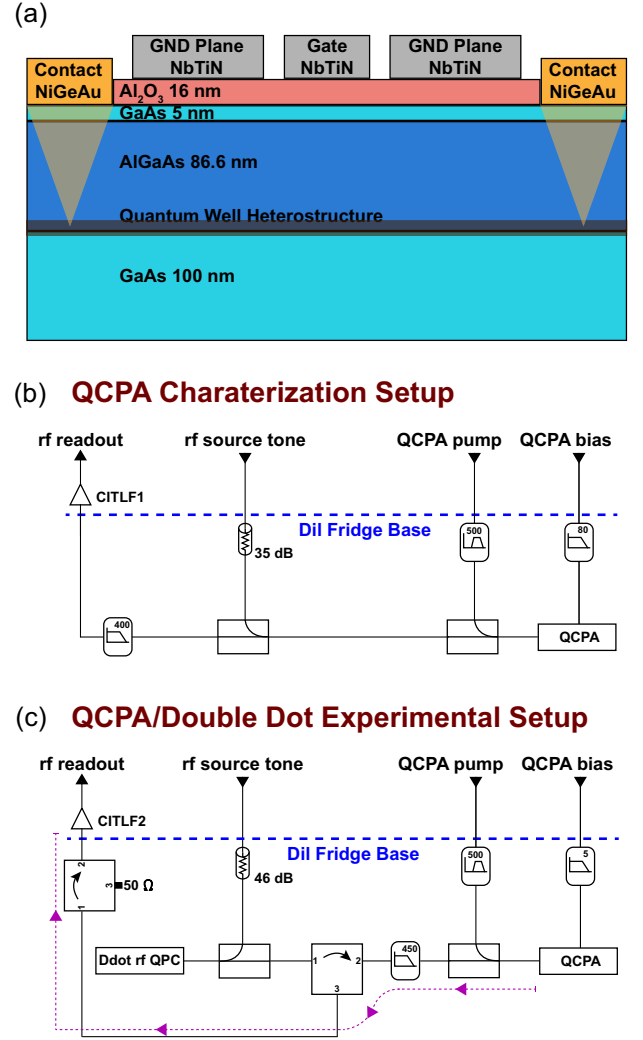


FIG. 8. (a) An illustration of the device cross section (dimensions not to scale). (b) The experimental setup used to characterize the QCPA. The source signal port is heavily attenuated to reduce the noise temperature in the band of the amplifier to the mK range. The pump port is filtered to pass only the pump signal and sharply reject signals in the input and idler bands. This is to prevent noise in the pump signal at the source frequency from corrupting the system. The dc bias port is low-pass filtered at 80 MHz. The readout signal is amplified at 4 K plate of the refrigerator with a commercial silicon-germanium low-noise amplifier CITLF1. A 400 MHz low-pass filter prevents the high-powered pump signal from saturating the 4 K LNA. (c) The double-dot QCPA readout experimental setup. The bias and pump signals are filtered as in (b). The rf probe tone is attenuated and then coupled into the rf-QPC of the double dot. The QCPA readout system employs a circulator to provide directionality and input-output separation. The second-stage amplifier is a CITLF2 (SiGe) with a circulator to isolate the back-action on the quantum system. The purple pathway indicates the readout circuit losses relevant to the noise temperature estimate of the QCPA.

in noise temperatures likely arises from differences in thermalization between the quantum dot and Y-factor measurement setup. Using the QCPA as a first-stage

amplifier improves the SNR by approximately 4–10 dB [see Fig. 7(h)]. We note that the SNR obtained in the QCPA OFF state has been comparable to that measured when the amplifier is replaced with an open-circuit rf standard. As such, dissipation from the QCPA in the OFF state is negligible.

VI. DISCUSSION

Having presented experimental results benchmarking our prototype device, we now discuss means of improving its performance and touching on additional parameters of the amplifier that are critical for its use in a scaled-up quantum computing or receiver system.

Starting with the opportunities to lower the noise temperature of the amplifier, we note that this is likely limited by the large (mV) pump amplitude and the presence of lossy circuit components, which together can lead to self-heating and a correspondingly high physical temperature in the present configuration. Reducing the pump power by more than 20 dB appears relatively straightforward via the use of a tank circuit to provide impedance matching on the pump port [46]. Further reduction in pump power can be achieved by increasing the C - V lever-arm by moving the 2DEG closer to the surface gate during the growth of the quantum well.

There are also several pathways to lowering dissipation. For the present device, losses in the normal metal (off-chip) inductor and losses in the ohmic contacts likely dominate dissipation. Both of these contributions can be improved by increasing the thickness and width of the metal and optimizing alloying and annealing conditions. Alternatively, as noted earlier, moving from GaAs to InAs (or similar III-V compounds) would lift the challenge of making low-resistance ohmic contacts since Fermi-level pinning in such materials leads to the absence of a Schottky barrier at the metal-semiconductor interface. On the other hand, GaAs will likely remain superior in terms of its accessible mobility and ease of fabrication. A further step in reducing dissipation could also come from incorporating superconductivity in the contacts [47] or in the 2DEG itself via the proximity effect [48]. Ultimately, the noise temperature will be limited by the electron temperature of the 2DEG in the limit of high pump power (likely a few 100 mK).

Although these improvements can reduce the noise temperature by lowering the physical temperature, we note that power dissipation in the present device is already sufficiently small to enable operation at millikelvin temperatures. Operating the QCPA at base temperature, we observe no significant impact on the mixing chamber temperature of the dilution fridge or neighboring quantum device. We estimate that this dissipation is of order a few 100 nW or lower, commensurate with 100 s of readout subsystems operating within the cooling power of a commercial dilution refrigerator at 100 mK.

It is also worth considering the current size of our prototype device, which is on the order a few mm². This footprint can likely be reduced significantly via the use of shallower 2DEG to maintain a sizeable gate capacitance. In such a configuration, 100 s of amplifiers can be fabricated on a single centimeter-scale chip that is either monolithically integrated with semiconductor qubits via nonreciprocal quantum hall circulators [49] or positioned proximal to the qubit plane using heterogeneous chip-to-chip interconnect approaches. We also draw attention to the likelihood that the QCPA can be configured as a traveling-wave device, where the quantum capacitance element is integrated in a matched coplanar waveguide to achieve wideband operation, similar to what has been achieved in superconducting implementations [17,20–22].

Stepping back from the specific details of our implementation, the use of a voltage-controllable, low-loss capacitance as a parametric element in which to realize amplification is a general result that applies broadly to a spectrum of nanoscale devices exhibiting a sizable quantum capacitance. Examples include other 2D systems based on silicon MOS structures [50], silicon-germanium heterostructures [51,52], van der Waals heterostructures and graphene [53], or even 1D devices [54,55]. Considering that many qubit platforms already leverage the dispersive response of an rf resonator for readout [50,56], there is also the potential to combine these readout resonators [57] with parametric capacitive elements that provide *in situ* amplification directly.

VII. CONCLUSION

In conclusion, we have realized a prototype parametric amplifier based on gate-modulation of a high-mobility 2DEG in a semiconductor heterostructure. With considerable opportunity for further refinement, the specifications of our amplifier likely fill a gap between the lowest-noise superconducting amplifiers and high-power transistor amplifiers. We anticipate the QCPA may find utility in realizing low power, integrated readout subsystems for semiconductor qubits, deep-space receivers, and radio astronomy applications.

VIII. METHODS

A. Sample fabrication

NiGeAu contacts are deposited first and annealed at 450 °C. The active area of the device is defined by a mesa etch with a subsequent 16-nm-thick layer of Al₂O₃ grown by atomic layer deposition. This layer enables operation beyond the gate bias defined by the Schottky barrier, passivating the semiconductor surface and suppressing leakage. In the last step of the fabrication, a metallic gate and ground plane (NbTiN) structure is patterned with optical lithography. An illustration of the device stack is shown in Fig. 8(a).

B. Dissipation and characterization

Losses are mitigated via two approaches. First, a large number of ohmic contacts are used in parallel (with resistance measured to be of the order of $100\ \Omega$ per contact at dc). The relatively large contacts ($100 \times 100\ \mu\text{m}^2$) will also couple capacitively to the 2DEG, reducing the impedance significantly at radio frequencies. Second, the ground plane is deposited relatively close to the gate ($7.5\ \mu\text{m}$ away) and serves to provide a low-impedance path to ground at high frequencies by forming a large capacitance (order of nF) with the active layer of the 2DEG.

The variable capacitance of the active device is extracted as a function of gate voltage, as shown in Fig. 2(b). The overall capacitance change is approximately 280 fF over a voltage range of 10 mV. For the purpose of the QCPA acting as a three-wave mixing circuit, the desirable C - V characteristic is one with a linear component in voltage. A linear dependence of the active capacitance on voltage introduces the mixing of the pump with the idler and source signals, resulting in their amplification. To enhance the linearity of the C - V curve, we chose a gate layout with an extensive perimeter.

C. The QCPA circuit

The device is fixed to a printed circuit board using silver paste and bonded to the circuit shown in Fig. 1(a), forming the QCPA tank. The ohmic contacts and the ground plane of the active device are bonded to the ground plane of the printed circuit board (PCB). The tank inductance is an air-core spring inductor of 22 nH connected in series with a 300 pF capacitance to ground. This capacitance acts as a dc block for the gate bias but is negligible at high frequency. The gate bias is applied through an rf choke inductance of $1\ \mu\text{H}$. The coupling capacitance of 1.4 pF serves to transform the impedance seen by the amplifier at the load from $50\ \Omega$ to $1.9\ \text{k}\Omega$ at the frequency of interest (370 MHz). This effectively raises the loaded Q-factor of the QCPA roughly by a factor of 19 times. It also forms a reasonably good match with the amplifier tank, so its resonance can be easily seen as in Fig. 2(a), allowing further characterization of the device. The coupling capacitor, along with the choke inductor, forms a bias tee on the PCB. The experimental circuit implemented on the PCB is then equivalent to a parallel tank parametric amplifier circuit [39], as shown in Fig. 1(b), which can be analytically solved for the gain.

ACKNOWLEDGMENTS

We thank S. Pauka, J. Colless, J. Hornibrook, M-C. Jarratt, D. Govender, S. Waddy, J. Witt, A. Jouan, and K. Zuo for useful conversations and technical contributions. This research was supported by the Microsoft Corporation and the Australian Research Council Centre of Excellence for Engineered Quantum Systems (EQUS, CE170100009).

The authors acknowledge the facilities and the scientific and technical assistance of the Research & Prototype Foundry Core Research Facility at the University of Sydney, part of the Australian National Fabrication Facility.

DATA AVAILABILITY

The data that support the findings of this article are openly available upon reasonable request; embargo periods may apply.

APPENDIX A: BACKGROUND AND FURTHER CONTEXT

The use of variable capacitors to construct parametric amplifiers is, of course, well known, with a rich history dating back to the late 1950s [35], where their use in radio astronomy [36,37] and radar systems was fundamental [38]. The early “paramps” relied on diode varactors to provide the variable reactance responsible for the parametric amplification [58]. Since the diode is reverse-biased, however, thermal noise (rather than shot noise) dominates their performance, with most low-noise applications requiring cryogenic cooling in liquid helium. At such temperatures, the performance of varactor-based paramps was theoretically projected to rival the maser [59,60]. In practice, however, pumping at the required power levels of 10 s of mW at microwave frequencies causes the junction temperature to rise locally, well above the ambient temperature of the liquid helium coolant [61]. Consequently, this self-heating yielded noise temperatures in the vicinity of 10 K [62]. Eventually, heterostructure-based FET devices arrived (in the 1980s) and mostly superseded the original varactor paramps by offering an affordable low-noise alternative [63] with the simplicity of dc bias rather than a microwave pump, a preferred unilateral directionality, and the advent of large-scale integration with high yield [64].

At present, state-of-the-art cryogenic HEMT amplifiers are commercially available with $> 30\ \text{dB}$ gain, a decade of bandwidth, and a noise temperature $T_N \sim 1\text{--}2\ \text{K}$, in the few 100 MHz [65]. These amplifiers typically dissipate $> 10\ \text{mW}$ of power, however, which leads to self-heating and a limit on the lowest achievable physical temperature that then also limits T_N . Recent work has identified this self-heating as a fundamental challenge since the mW power that is dissipated in the active area of the transistor cannot be removed, even via the use of superfluid helium [34]. Although lower temperatures can be achieved by reducing the transistor bias, this also reduces the transconductance and leads to the amplifier becoming limited by the shot noise of the drain current [40]. It is interesting that despite the continued improvements in carrier mobility in these heterostructure systems [32], parametric

operation has remained largely unexplored until the present work.

Perhaps one reason high-mobility semiconductor par-amps have not been developed relates to the advent of superconducting microwave amplifiers, which have largely addressed the noise limitations imposed by dissipation in hot transistor approaches [66]. The most popular for reading out the state of qubits are those devices based on variable inductive elements, such as JJs, or superconductors with large kinetic inductance [16–19,22,23,33,67]. The power handling capabilities of JJ-based amplifiers are limited, however, by the critical current of the junction, making frequency multiplexing challenging since the total input power is the sum of all frequency channels. Recent efforts may overcome this via the use of kinetic inductance-based traveling-wave parametric amplifiers made from NbTiN [20,21], although for semiconductor qubit readout, operation in the few 100 MHz range may be more difficult. Significant, of course, is that superconducting approaches generally make use of shielding from magnetic fields and operation below their critical temperature.

Comparing HEMTs and superconducting amplifiers, we identify a gap in performance between these platforms for applications that require submicrowatt power dissipation for millikelvin operation, high-power handling, and inherent compatibility with magnetic fields (see Table I). Indeed, a tightly integrated readout subsystem for semiconductor qubits is such an application, motivating the development of the QCPA, which largely fills this gap. Although the noise performance of the QCPA is unlikely to ever rival superconducting approaches, we suggest there is considerable scope to further reduce dissipation, leading to improved noise performance.

APPENDIX B: NOISE TEMPERATURE MEASUREMENT

For the noise performance of the QCPA, we modify the setup to incorporate a commercial cryogenic noise source [43], as shown in Fig. 9(a). The pump signal is coupled to the input of the amplifier, and a circulator routes the reflected signal to the readout line. A cryogenic switch is also added to the setup in order to compare and normalize the amplifier noise response relative to the response of an open circuit [labeled “OC” in Fig. 9(a)]. Measuring the integrated noise power at the readout port as a function of noise input power then yields the offset noise in the limit where the input is zero. Using this (Y-factor) method, the noise temperature of the system is determined to be $T_{\text{sys}} = 6.49^{+1.06}_{-0.62}$ K, with QCPA gain $G_{\text{QCPA}} = 33.86$ dB. The bounds on T_{sys} are given by the extrema of the Y-factor fitting within the measured error bars of the noise floor.

In comparison, with the switch [Fig. 9(a)] positioned to the open-circuit port, the system noise temperature is

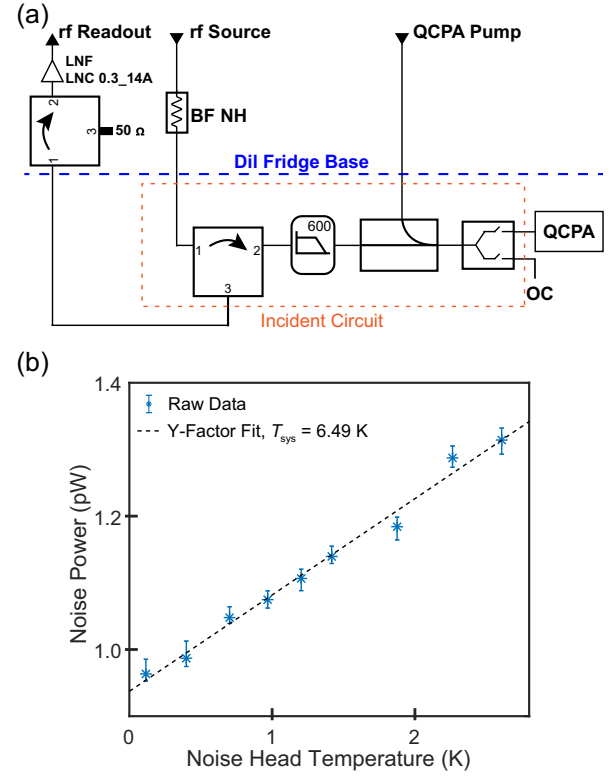


FIG. 9. The setup used for noise characterization and measurement. Panel (a) shows the circuit of the QCPA below the mixing chamber plate of the dilution refrigerator. (b) The measured system noise power as a function of noise source temperature, along with the corresponding linear fit.

measured to be $48.73 \text{ K}/G_{\text{QCPA}} \sim 20 \text{ mK}$, thus the additional noise beyond the QCPA is negligible and $T_{\text{sys}} \sim T_{\text{QCPA}}$. Finally, it is necessary to account for the insertion loss between the noise source and the input of the amplifier—labeled as the incident circuit in Fig. 9(a)—which serves to increase the effective measured system noise [see Eq.(B1)]. We find these losses by independently measuring the calibrated transmission path of the incident circuit in a separate cooldown. Owing to the poor performance of the ferrite circulator at these temperatures, we find the transmission losses to be approximately 4.01 dB. Correcting for these losses, we estimate the intrinsic noise temperature of the QCPA to be $T_{\text{QCPA}} = 2.58^{+0.42}_{-0.25}$ K.

1. Noise temperature correction and calibration

In order to extract the noise temperature of the QCPA individually, the experiment consists of measuring the spectra of the QCPA and its reference open-circuit termination with a sweep of the cryogenic noise source physical temperature. To ensure accurate reading of the noise floor, we set the spectrum analyzer (Keysight N9030B) to capture with a resolution bandwidth of 200 Hz, a video

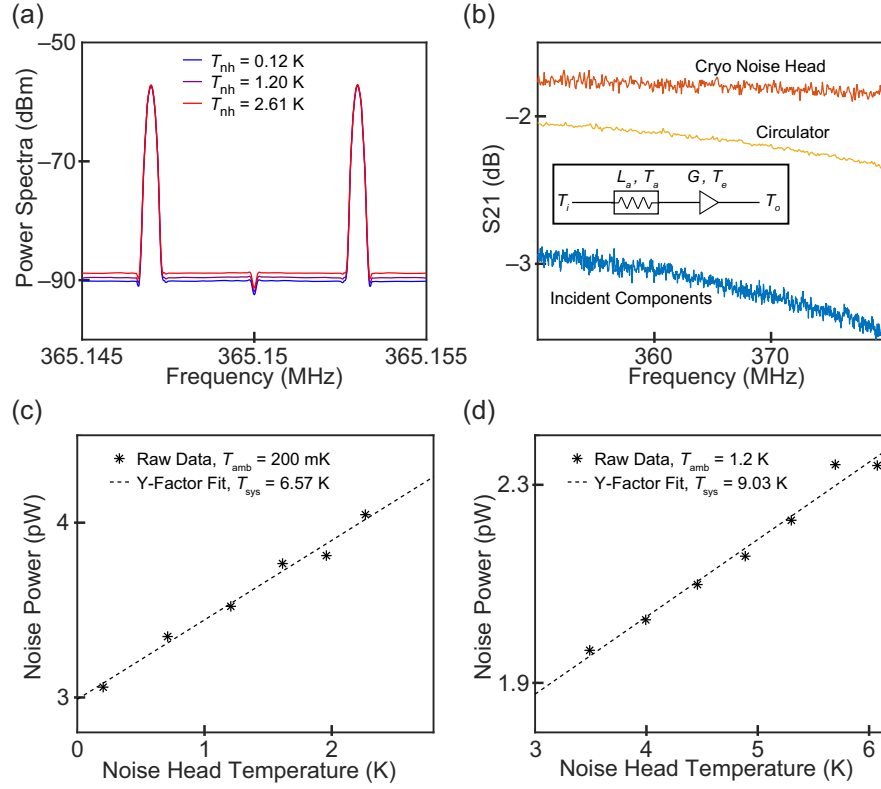


FIG. 10. (a) A sample of the raw spectra used for the computation of the noise temperature at $T_{amb} = 12$ mK. The legend shows the physical temperature of the noise head for each curve. (b) The calibrated S_{21} measurements of the incident components (circulator, the low-pass filter, the directional coupler, and the switch) are all kept in the same assembly as used in the main noise experiment of Fig. 9, the cryo noise head excess (above 20 dB) insertion loss, and the individual circulator QCY-M350400. (c),(d) The QCPA system noise temperature is characterized at higher ambient temperatures by applying heat to the mixing chamber stage: (c) $T_{amb} = 200$ mK. (d) $T_{amb} = 1.2$ K.

bandwidth of 1 Hz, and video averaging of 500 curves. This ensures that the error bars in the noise measurement are kept within reasonable margins. The frequency span is 10 kHz centered around the degeneracy-center frequency of the amplifier. An example of the raw spectrum data at base temperature $T = 12$ mK is shown in Fig. 10(a), where the different curves correspond to increasing physical temperatures of the noise source. The noise power is extracted as the median value of the noise floor in the source and idler sidebands of the amplifier, and the Y-factor method is used to extract the noise temperature of the corresponding system.

Furthermore, the incident circuit between the noise head and the QCPA must be characterized. We define the incident circuit as the collection of components that constitute the signal path for the wave traveling from the output of the noise head (a controllable heated attenuator) down to the input port of the QCPA sample PCB. We place this circuit in a separate dedicated short-open-load-through (SOLT) calibration setup in a 5 K environment. The SOLT calibration setup consists of a Fairview kit connected between 2 rf Radiall single-pole 6-throw (SP6T) R594F43617 relay switches, where the device under test is

connected with the same-electrical-length cables as the calibration standards. The entire setup is mounted on the 5 K stage of a cryogenic system.

The calibration setup is also used to measure the loss in the lengthy cables of the noise head assembly individually. In the main noise experiment, the noise head is used as a well-calibrated 20 dB attenuator whose temperature can be controlled via a heater and a temperature sensor. It allows “through” measurements where the signal enters the noise head, gets attenuated, and also acquires a noise temperature equivalent to the physical temperature of the noise head. Because of its potentially elevated body temperature, the noise head is mounted at the 100 mK cold plate (just above the mixing chamber in a BlueFors LD fridge) and is connected to the setup via lengthy superconducting cables to limit thermal conduction. The calibration setup allows measurement of the losses in the superconducting cables. The extra insertion loss above 20 dB is plotted in Fig. 10(b). We attribute this loss to the cables, where it is divided equally between the input and output sections. The incident circuit includes the output cable of the noise head, whose loss is then found to be 0.9 dB at the frequency of interest.

Next, we measure the rest of the incident circuit [highlighted in Fig. 9(a)], i.e., the circulator, the low-pass filter, the directional coupler, and the switch, all kept in the same assembly as used in the main noise experiment. The S_{21} transmission curve is shown in Fig. 10(b). At the frequency of interest, we find that the insertion loss of the above components is 3.1 dB. This elevated insertion loss is due to subpar performance of the ferrite circulator in transmission. The total insertion loss of the incident circuit is then found to be 4 dB.

The correction of the system noise temperature due to the incident circuit losses is detailed here. Consider the schematic in the inset of Fig. 10(b), where the input noise temperature is T_i , the amplifier is characterized by a gain G and noise temperature T_e , and the lossy component is characterized by an attenuation L_a and physical temperature T_a . Through this circuit, we can investigate the effect of the existence of attenuation L_a on the noise temperature of the system. The characteristic noise temperature of the lossy component is related to its physical temperature and defined as $T_a(L_a - 1)$. The noise temperature at the output of the attenuator is then written as:

$$T_{o,a} = \frac{T_i + T_a(L_a - 1)}{L_a}. \quad (\text{B1})$$

The output noise temperature T_o of the whole system can be expressed as:

$$T_o = \frac{G}{L_a} T_i + \frac{G}{L_a} T_a(L_a - 1) + G T_e. \quad (\text{B2})$$

Defining the total gain of the system as $G_t = G/L_a$, the system noise temperature is $T_{\text{sys}} = T_o/G_t - T_i$:

$$T_{\text{sys}} = T_a(L_a - 1) + L_a T_e. \quad (\text{B3})$$

In the system of the QCPA noise measurement experiment where $L_a = 2.52$ (4 dB) and assuming the temperature $T_a = 100$ mK, the term $T_a(L_a - 1) = 152$ mK is a very small correction compared with the measured $T_{\text{sys}} = 6.49$ K. However, the more pronounced effect of L_a , which we aim to correct for, is the term $L_a T_e$, where it multiplies the noise temperature of the QCPA by the losses of the preceding incident circuit, causing the system noise temperature to increase by 2.52 times.

APPENDIX C: GAIN VERSUS FREQUENCY

The derivation of the gain expression of the QCPA takes into consideration the parallel tank circuit shown in Fig. 1(b) of the main text and follows a similar approach as in Ref. [68]. The currents flowing into the circuit's single input-output port are the source and pump currents $i_{\text{in}} = I_s \sin(\omega_s t) + I_p \sin(\omega_p t + \phi_p)$, where I_s and I_p are the source and pump current amplitudes, ω_s and ω_p are the

source and pump angular frequencies, respectively, and ϕ_p is the pump's phase angle. The pump frequency is set to double the tank's resonance frequency $\omega_p = 2\omega_r$. The source and idler frequencies are symmetric around half the pump frequency, which is the tank's resonance, while still satisfying $\omega_p = \omega_s + \omega_i$, $\omega_s = \omega_t - \omega_x$, and $\omega_i = \omega_t + \omega_x$, where ω_x is defined as the source and idler frequencies detuning from the center of the tank. Analyzing the nonlinear part of the circuit, we can write the current in the variable capacitor as:

$$i_{\text{nl}} = \frac{dC_v v_t}{dt} = C_c \frac{dv_t^2}{dt}, \quad (\text{C1})$$

where C_v is the voltage-controlled capacitance assumed to have a linear dependence on voltage in the form of $C_v = C_c v_t$ and C_c is the constant slope of the variable capacitance versus voltage in (F/V). We should clarify that while the capacitance is taken as a linear function of voltage, the current in the variable capacitor is nonlinear in voltage; hence the nomenclature of nonlinear current i_{nl} . The voltage v_t of the tank is composed of source, idler, and pump sinusoidal terms:

$$v_t = V_1 \sin(\omega_s t + \phi_1) + V_2 \sin(\omega_i t + \phi_2) + V_3 \sin(\omega_p t + \phi_3), \quad (\text{C2})$$

where V_1, ϕ_1 are the amplitude and phase at the source frequency, V_2, ϕ_2 are the amplitude and phase at the idler frequency, and V_3, ϕ_3 are the amplitude and phase at the pump frequency. The current i_{nl} at the source and idler frequencies are then expressed as:

$$i_{\text{nl}}(\omega_s) = -C_c \omega_s V_2 V_3 \sin(\omega_s t + \phi_3 - \phi_2), \quad (\text{C3})$$

$$i_{\text{nl}}(\omega_i) = -C_c \omega_i V_1 V_3 \sin(\omega_i t + \phi_3 - \phi_1). \quad (\text{C4})$$

At low input source power, no pump depletion occurs from the source and idler components, and V_3 is only linearly dependent on the pump current $I_p \sin(\omega_p t + \phi_p)$ and the magnitude of the tank's impedance at the pump frequency; hence, we can take the term $C_c V_3 = C_3$ as a constant independent of the source and idler voltages. In subsequent steps, we transform the analysis to the phasor domain with the following notation $v_t \rightarrow V_t$ and $i_{\text{nl}} \rightarrow I_{\text{nl}}$:

$$I_{\text{nl}}(\omega_s) = j C_3 \omega_s V_2 e^{j(\phi_3 - \phi_2)}, \quad (\text{C5})$$

$$I_{\text{nl}}(\omega_i) = j C_3 \omega_i V_1 e^{j(\phi_3 - \phi_1)}. \quad (\text{C6})$$

The current I_l in the passive part of the tank admittance at the idler frequency and due to the tank voltage V_t is

$$I_l = -j V_2 e^{j\phi_2} Y_i, \quad (\text{C7})$$

where Y_i is the tank's admittance at the idler frequency. We then solve the equation $I_{nl} = -I_l$ using Eqs. (C6) and (C7) at the idler frequency to find an expression of V_2 :

$$V_2 = \frac{C_3 V_1 \omega_i e^{j(\phi_3 - \phi_2 - \phi_1)}}{Y_i}. \quad (C8)$$

Replacing the complex conjugate of $V_2^* = V_2$ from Eq. (C8) in Eq. (C5):

$$I_{nl}(\omega_s) = jC_3^2 \omega_s \omega_i \frac{V_1 e^{j\phi_1}}{Y_i^*}. \quad (C9)$$

The effective admittance of the nonlinear reactance at the source frequency, Y_{nl} :

$$Y_{nl} = \frac{I_{nl}(\omega_s)}{-jV_1 e^{j\phi_1}} = \frac{-\omega_s \omega_i C_3^2}{Y_i^*}. \quad (C10)$$

Notice that the tank voltages' phases are not present in the above admittance, which signifies the phase-independence of the nondegenerate behavior. The amplified source voltage at the tank is given as:

$$V_1 e^{j\phi_1} = \frac{I_s}{Y_s + Y_{nl}}, \quad (C11)$$

where Y_s is the tank's linear admittance at the source frequency. Using input-output theory as in Refs. [69,70] and assuming the amplifier is matched to the load conductance both at $G_L/2$, the amplified source voltage reflected at the load V_{ref} is then:

$$V_{ref} = -jV_1 e^{j\phi_1} - V_{in}, \quad (C12)$$

where $V_{in} = -jI_s/G_L$ is the input voltage due to the source alone at the source conductance (without amplification). The voltage gain is

$$G_v = \left| \frac{V_{ref}}{V_{in}} \right| = \left| -1 + \frac{G_L}{Y_s + Y_{nl}} \right|. \quad (C13)$$

The power gain is then given as $G_p = G_v^2$:

$$G_p = \left| -1 + \frac{G_L}{Y_s + Y_{nl}} \right|^2. \quad (C14)$$

At a frequency very close to the tank resonance ($\omega_s \approx \omega_t \approx \omega_i$, but still avoiding the center degeneracy), the idler and source admittances are approximated to be purely real $Y_s \approx Y_i \approx G_L$, and the effective nonlinear admittance of Eq. (C10) is written as:

$$Y_{nl,c} = -G_{nl,c} = \frac{-\omega_i^2 C_3^2}{G_L}. \quad (C15)$$

The nonlinear capacitance admittance reduces to a real number ($-G_{nl,c}$), the effective negative conductance in parallel to the tank. Therefore, we can rewrite Eq. (C14) as:

$$G_{p,c} = \left| -1 + \frac{G_L}{G_L - G_{nl,c}} \right|^2. \quad (C16)$$

The gain expression in Eq. (C16) tends to 0 if $G_{nl} \rightarrow 0$, satisfying the assumption that the amplifier's tank is matched to the source. The gain tends to infinity as G_{nl} approaches G_L , signifying the onset of an unstable oscillating tank circuit. Practically, the circuit will oscillate at half the pump frequency as the gain is highest at that point. Equation (C16) can be used to make an estimate of the maximum nondegenerate gain at the center frequency of an amplifier with certain characteristics.

Equation (C14) embeds the frequency dependence of the gain within the expressions of the source and the effective nonlinear admittances Y_s and Y_{nl} . The tank's admittance at the source frequency is

$$Y_s = G_L + j\omega_s C + \frac{1}{j\omega_s L}. \quad (C17)$$

Using $\omega_s = \omega_t - \omega_x$ and $\omega_t L = \frac{1}{G_L Q_t}$, where Q_t is the quality factor of the loaded tank circuit, then we can write:

$$Y_s = G_L - j \frac{\omega_x G_L Q_t}{\omega_t} \left(\frac{2 - \frac{\omega_x}{\omega_t}}{1 - \frac{\omega_x}{\omega_t}} \right). \quad (C18)$$

We can approximate the right-hand factor as $\frac{2 - \frac{\omega_x}{\omega_t}}{1 - \frac{\omega_x}{\omega_t}} \approx 2$, and the approximation is off by less than 0.7% for $\omega_x = 2\pi(5 \text{ MHz})$ and $\omega_t = 2\pi(370 \text{ MHz})$. Then the admittance Y_s is simplified to:

$$Y_s \approx G_L - j2G_L Q_t \frac{\omega_x}{\omega_t}. \quad (C19)$$

Similarly, the tank's admittance at the idler frequency is found to be

$$Y_i \approx G_L + j2G_L Q_t \frac{\omega_x}{\omega_t}. \quad (C20)$$

Now, looking at the expression of the nonlinear admittance from Eq. (C10) and replacing the idler admittance of Eq. (C20):

$$Y_{nl} = \frac{-\omega_s \omega_i C_3^2}{G_L - j \frac{2\omega_x G_L Q_t}{\omega_t}}. \quad (C21)$$

Factorizing $-G_{nl} = -\frac{\omega_s \omega_i C_3^2}{G_L}$ out of the expression:

$$Y_{nl} = -G_{nl} \frac{1 + j2Q_t \frac{\omega_x}{\omega_t}}{1 + (2Q_t \frac{\omega_x}{\omega_t})^2}. \quad (C22)$$

Notice that the conductance $G_{nl} = \frac{(\omega_t^2 - \omega_x^2)C_3^2}{G_L}$ is dependent on the detuning frequency ω_x . It is equal to the term defined earlier $G_{nl,c} = G_{nl}(\omega_x = 0)$ at zero detuning. However, the deviation of G_{nl} from $G_{nl,c}$ is indeed negligible for any detuning frequency of interest. The error of approximating $G_{nl} \approx G_{nl,c}$ is less than 0.02% for $\frac{\omega_x}{\omega_t} = \frac{2\pi(5 \text{ MHz})}{2\pi(370 \text{ MHz})}$. We will continue to use the conductance G_{nl} for the sake of generality of the gain expression derived below, but the aforementioned approximation will come in handy when we derive the gain-bandwidth relationship. Now, we replace Y_s and Y_{nl} in Eq. (C14) while also performing a variable transformation $x = 2Q_t\omega_x/\omega_t$, we arrive at the general gain expression:

$$G_p = \frac{G_{nl}^2 + x^2[G_{nl} + G_L(1 + x^2)]^2}{[-G_{nl} + G_L(1 + x^2)]^2 + x^2[G_{nl} + G_L(1 + x^2)]^2}. \quad (C23)$$

The general expression of the gain in Eq. (C23), also cited in the main text as Eq. (1), is equal to the peak gain at the center frequency $G_{p,c}$ for $x = 0$. Rewriting the expression of $G_{p,c}$ as:

$$G_{p,c} = \frac{G_{nl,c}^2}{(G_L - G_{nl,c})^2}. \quad (C24)$$

The roll-off of the power gain in Eq. (C23) versus frequency is mostly due to an effective imaginary admittance in the form of $jx(G_{nl} + G_L(1 + x^2))$, adding to the total admittance in the denominator of the gain expression (x being unitless). This gain in Eq. (C23) is a reasonably good analytical solution of the analyzed circuit and can be used to fit or predict the experimentally measured gain curves given the circuit parameters such as G_L , Q_t , f_t , and C_3 .

APPENDIX D: GAIN-BANDWIDTH RELATIONSHIP

It is well known that parametric amplifiers exhibit a specific gain-bandwidth relationship ($\sqrt{G_{p,c}} \times BW \approx \text{constant}$) for moderate to high gain levels [41,68,69]. In the following, we extract the gain-bandwidth relationship of the QCPA, starting with an approximation to simplify the gain expression of Eq. (C23). We approximate the term $(1 + x^2) \approx 1$ to unity. This is acceptable because for a tank circuit with a certain Q_t , we need only a negligible frequency detuning from the center of the band to observe the gain trend versus frequency at high-gain settings. Consequently, we can take $x^2 \ll 1$ with an approximation error of 7.3% for a detuning of $\pm 2Q_t \frac{3.5 \text{ MHz}}{370 \text{ MHz}}$. The approximation error is much smaller within the limits of the bandwidth in the high-gain regime. The approximation

becomes inaccurate with lower gain such that the gain-bandwidth linear profile is not a valid representation anymore. We also approximate the term $G_{nl} \approx G_{nl,c}$, as discussed previously. The gain expression is then rewritten as:

$$G_p = \frac{\frac{G_{nl,c}^2}{(G_L - G_{nl,c})^2} + x^2 \frac{(G_{nl,c} + G_L)^2}{(G_L - G_{nl,c})^2}}{1 + x^2 \frac{(G_{nl,c} + G_L)^2}{(G_L - G_{nl,c})^2}}. \quad (D1)$$

Notice that $G_{p,c} = \frac{G_{nl,c}^2}{(G_L - G_{nl,c})^2}$ is the left term of the numerator, and relabeling the second term as $c = \frac{(G_{nl,c} + G_L)^2}{(G_L - G_{nl,c})^2}$, the gain expression then assumes the following rational polynomial function:

$$G_p(x) = \frac{G_{p,c} + cx^2}{1 + cx^2}. \quad (D2)$$

The gain expression in Eq. (D2) highlights the Lorentzian profile of the gain curve as a function of frequency and can also be used to fit and predict the experimental gain curves given the correct set of circuit parameters. The gain tends to unity for sufficiently large detuning frequencies. The gain also has a maximum at $G_{p,c}$ for $x = 0$. To calculate the bandwidth, we set $G_p = 0.5G_{p,c}$ and find the detuning x_b :

$$x_b = \pm \sqrt{\frac{G_{p,c}}{c(G_{p,c} - 2)}}. \quad (D3)$$

The bandwidth in Hz is then given as:

$$BW = 2 \times \sqrt{\frac{G_{p,c}}{c(G_{p,c} - 2)}} \frac{f_t}{2Q_t}. \quad (D4)$$

Multiplying Eq. (D4) by the square root of the peak gain at the center frequency $G_{p,c}$, we obtain:

$$\sqrt{G_{p,c}} \times BW = \frac{2G_{p,c}}{\sqrt{c(G_{p,c} - 2)}} \frac{f_t}{2Q_t}. \quad (D5)$$

At moderate to high gain levels, the negative conductance is numerically close to the loaded losses of the circuit $G_{nl} \approx G_L$, which makes the constant $c \approx 4G_{p,c}$. Thus, the term $\frac{2G_{p,c}}{\sqrt{c(G_{p,c} - 2)}} \lesssim 1$. In practice, the term $\frac{2G_{p,c}}{\sqrt{c(G_{p,c} - 2)}}$ is slightly smaller than unity and, to fit the data as in Fig. 3(b) in the main text, we introduce the factor $a = 1.077$ in the expression $\sqrt{G_{p,c}} \times BW = \frac{f_t}{2aQ_t}$.

APPENDIX E: DEGENERATE GAIN

The parametric amplifier is said to be in the degenerate case when the tones' frequencies are arranged in a way that the idler and the source have the same frequency and are essentially the same tone $f_s = f_i = f_p/2 = f_t$. Looking back at the circuit of Fig. 1(b), the tank voltage is limited to just two tones, the source and the pump:

$$v_t = V_1 \sin(\omega_s t + \phi_1) + V_3 \sin(\omega_p t + \phi_3). \quad (\text{E1})$$

The current in the nonlinear capacitor at the source frequency is

$$i_{\text{nl}}(\omega_s) = -C_3 \omega_s V_1 \sin(\omega_s t + \phi_3 - \phi_1). \quad (\text{E2})$$

In phasor form,

$$I_{\text{nl}}(\omega_s) = jC_3 \omega_s V_1 e^{j(\phi_3 - \phi_1)}. \quad (\text{E3})$$

The nonlinear admittance at the source frequency is

$$Y_{\text{nl}} = \frac{I_{\text{nl}}}{-jV_1 e^{j\phi_1}} = -\omega_s C_3 e^{j(\phi_3 - 2\phi_1)}. \quad (\text{E4})$$

The analytical solution of the degenerate amplifier is more complicated than the nondegenerate case and will not be investigated further. However, we can still find the expression of the maximum gain occurring when $(\phi_3 - 2\phi_1 = 0)$ is satisfied. In that case, Y_{nl} is purely real and negative, $Y_{\text{nl}} = -G_{\text{nl}} = -\omega_s C_3$ and the gain is written as:

$$G_{p,\text{max}} = \frac{(\omega_s C_3)^2}{(G_L - \omega_s C_3)^2}. \quad (\text{E5})$$

It is worth noting that the gain in Eq. (E5) is the highest gain the parametric amplifier can achieve for a certain pump amplitude and is higher than the maximum center-frequency gain of the nondegenerate amplifier for the same circuit. This means that as the negative conductance increases with pump amplitude, it eventually becomes equal and negates the tank losses, thus causing the amplifier tank to enter an oscillatory mode. Being the point of highest gain, the oscillation occurs predominantly at the frequency $f_p/2$, and the amplifier is said to be unstable. On the other hand, with a different phase arrangement, Y_{nl} could take a real positive value, which then acts as an attenuator of the source signal. Thus, the operation of the amplifier in the degenerate case is phase-dependent and can swing from high amplification to deep attenuation, as shown in the main text.

APPENDIX F: NOISE SOURCES IN THE SINGLE TANK CIRCUIT

Taking after the analysis in Ref. [68], we will briefly introduce the various sources of noise in the single tank circuit under study. We will express the noise as current sources, and we will find the corresponding expressions of the amplified voltage that appears on the terminals of the parametric amplifier tank. We also assume the source and idler frequencies are very close to the tank's center frequency for simplicity.

First, we take into account the noise in the tank at the source frequency $i_{n,s}$. This includes thermal noise from resistive components in the tank at ω_s . This noise source is treated exactly like the source current to be amplified by the amplifier, and its associated tank voltage at the source frequency ω_s is

$$v_{n,s,s} = \frac{i_{n,s}}{G_L - G_{\text{nl}}}. \quad (\text{F1})$$

Second, consider the noise in the tank at the idler frequency $i_{n,i}$. This includes thermal noise from resistive components in the tank at ω_i . This noise source is amplified at the idler frequency as:

$$v_{n,i,i} = \frac{i_{n,i}}{G_L - G_{\text{nl}}}. \quad (\text{F2})$$

Then it is converted to the source frequency via the nonlinear reactance mixing the idler signal with the pump:

$$v_{n,s,i} = v_{n,i,i} \frac{\omega_s C_3}{G_L}. \quad (\text{F3})$$

Note that the conversion coefficient $\omega_s C_3 / G_L \approx 1$ for moderate to high-gain settings, so we end up with the idler noise folding over to the source frequency with almost the same efficiency as that of the source noise. Note that the source and idler noise currents could have large contributions from the powerful physical pump source sending noise at a wide range of frequencies. We, practically, found that the amplifier's noise behavior benefits a lot from filtering the pump port at the source and idler frequencies.

Finally, the noise at the pump frequency can be modeled as a random variation around the voltage V_3 . We call this source $V_{n,3}$ with a random phase $\phi_{n,3}$. We then translate this noise source to a noisy capacitance component $C_{n,3} = C_c V_{n,3}$. One can think about the noisy capacitor as a small random capacitance change over the main capacitance swing amplitude C_3 . This noise component mixes down to the source frequency via the idler signal and vice versa. The current in the nonlinear reactance due to $C_{n,3}$ at ω_s and ω_i , respectively, is

$$I_{\text{nl},n,s}(\omega_s) = j\omega_s C_{n,3} V_2 e^{j(\phi_{n,3} - \phi_2)}, \quad (\text{F4})$$

$$I_{nl,n,i}(\omega_i) = j\omega_i C_{n,3} V_1 e^{j(\phi_{n,3}-\phi_1)}. \quad (F5)$$

These expressions end up as two correlated noise contributions $v_{n,s,ps}$ and $v_{n,s,pi}$ at the source frequency:

$$v_{n,s,ps} = \frac{j\omega_s C_{n,3} V_2 e^{j(\phi_{n,3}-\phi_2)}}{G_L - G_{nl}}, \quad (F6)$$

$$v_{n,s,pi} = \frac{\omega_s C_3}{G_L} \frac{j\omega_i C_{n,3} V_1 e^{j(\phi_{n,3}-\phi_1)}}{G_L - G_{nl}}. \quad (F7)$$

Again, the idler component is converted to the source frequency with the same efficiency as the source-frequency noise. Notice that the noise contribution from this particular source is dependent on the source and idler signal levels through V_1 and V_2 , as it is physically down-converted through them into the amplifier's band.

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