

Enhancing the speed of direct volumetric imaging through annular illumination and spatial coherence of light

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We propose and demonstrate a direct volumetric imaging method based on spatial coherence of light and sequential illumination from pointlike sources distributed along a ring. We prove that when the source geometry restricts illumination to high-angle modes matching the system numerical aperture, linear annular illumination achieves the same lateral resolution and similar axial sectioning as planar full-disk illumination, while offering the advantage of a threefold faster acquisition and an even lighter computational demand. The proposed approach offers a simplified and efficient alternative to conventional light-field and computational imaging, opening new paths for fast, cost-effective, real-time volumetric imaging and three-dimensional microscopy.

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I. INTRODUCTION

Wide-field microscopy is a cornerstone imaging technique for real-time visualization of biological and industrial specimens, prized for its direct imaging capabilities, low cost, and compatibility with live imaging [1]. Nonetheless, its performance is fundamentally constrained when extended to a three-dimensional (3D) object. The high resolution guaranteed by objectives with a large numerical aperture (NA) is accompanied by a limited depth of field (DOF); this forces a compromise between lateral resolution and axial coverage, requiring mechanical z scanning to retrieve volumetric information [2,3]. However, the scanning process is inherently slow and requires precise hardware, which limits its suitability for studying dynamic processes, high-throughput tasks, and cost-sensitive applications.

In response to these limitations, a range of scanning-free 3D imaging methods have been proposed. Among them, light-field microscopy (LFM) has gained traction for enabling single-shot acquisition of volumetric data by capturing both spatial and angular information via a microlens array placed in the image plane [4–6]. This configuration permits volumetric reconstruction from a single camera exposure, facilitating instantaneous three-dimensional

imaging of fast biological or physical phenomena [7,8]. The acquired four-dimensional (4D) dataset, called a “light-field function,” is computationally transformed, through wave optics–based deconvolution or synthetic refocusing algorithms [6], to retrieve 3D information about the shape of the object in the volume of interest. LFM offers exceptional temporal resolution and volumetric acquisition speed; however, it suffers from a strong trade-off between resolution and DOF: the acquired depth of field is gained at the expense of the spatial resolution, which is typically at least 2 times lower than the diffraction limit. This is largely due to its reliance on geometric optics, where defocusing is governed by the size of the NA-dependent circle of confusion (COC; a common feature of direct imaging techniques based on spatially *incoherent* illumination) [9]. LFM thus exhibits degraded resolution, which undermines its effectiveness for quantitative analysis at the microscale [10,11]. Recent work has integrated deep learning–based restoration and deconvolution methods [12,13] to compensate for aberrations and increasing spatial resolution toward near-isotropic performance.

A different class of techniques addresses the resolution versus DOF trade-off by leveraging the illumination strategy. These include methods such as ptychography, Fourier ptychography, and digital holography, which share the common goal of computationally retrieving either volumetric or high-resolution information by synthetically expanding the numerical aperture of the optical system through structured or angularly multiplexed illumination [14–16]. In this category, it is also worth mentioning structured illumination microscopy. Designed to increase resolution by illuminating the specimen with spatially

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structured light patterns and performing Fourier-space demodulation [17–19], structured illumination microscopy enables high-resolution volumetric imaging and dynamic sample analysis by applying interferometric and learning-based methods [20]. The aforementioned approaches typically require the sequential acquisition of multiple images under various illumination angles or phases. Computationally expensive inverse reconstructions typically relying on iterative optimization algorithms are then required to retrieve both the amplitude and the phase of the object. Although extremely powerful in resolution and flexibility, these computational illumination methods are not *direct* imaging techniques, as they require intensive postprocessing and are inherently slow due to the numerical reconstruction required for the full retrieval of object information. This limitation is particularly pronounced in real-time or high-throughput applications, where speed and simplicity are essential.

In contrast to these indirect approaches, recent advances have shown that the spatial coherence of light can be harnessed as a resource to enable high-resolution *direct* volumetric imaging [21]. In particular, away from the focused plane, the transverse resolution of *coherent* imaging (namely, imaging through spatially coherent illumination of the object) was shown to be completely independent of the NA of the imaging system, and to depend exclusively on the propagation of spatially coherent light; as a consequence, the resolution versus DOF trade-off is highly released and significantly higher resolution can be achieved. However, coherent imaging lacks the axial localization required for volumetric imaging; this is compensated by combining coherent imaging with sequential illumination from a two-dimensional (2D) array of sources with known transverse position: this approach was shown to entail the sharp NA-dependent axial sectioning capability typical of *incoherent* imaging, with a longitudinal resolution that depended exclusively on the dominating NA of the optical design [21]. The coherence-based direct volumetric imaging approach presented in Ref. [21] thus combines the high transverse resolution of coherent imaging with the high longitudinal resolution of incoherent imaging. This paradigm shift leverages the physics of spatial coherence—as opposed to geometric optics or full wave front recovery,—to enable the reconstruction of the same 4D light-field function typically retrieved in LFM, but without microlenses and with significantly improved 3D resolution and DOF. This approach thus opens up a new class of spatial coherence–assisted volumetric imaging modalities, which combines the speed and simplicity of wide-field imaging with the volumetric reach and resolution enhancement typical of indirect methods—yet without their computational burden.

In this work, we further develop this direction, and demonstrate that the sequential full-grid (2D) illumination used in the work reported in Ref. [21] can be effectively

replaced by a sequential *annular* (hence, linear) illumination, with the advantage of speeding up acquisition and elaboration, while fully preserving image resolution and axial sectioning capability. The underlying idea is that the information encoded within the measured light-field function (*four-dimensional*) might be partially redundant, as it is typically used to retrieve *three-dimensional* information about the object. Most of the information encoded by illumination of the scene from a 2D array will be shown to be not needed, as the properties of the reconstructed volumetric image are found to be defined by the outermost circular sector of the illumination grid. In particular, the maximum useful illumination radius is found to be defined by the angular acceptance of the imaging device: matching the radius of the annular illumination with the NA of the optical system enables maximization of both the lateral resolution in focus and the axial resolution of refocused images, as they are both defined by the dominant NA in the optical design. High-resolution volumetric reconstruction is thus found not to require accessing of the entire 4D light-field function: the annular distribution of pointlike sources limits illumination to the optical modes with the largest accepted angular divergence, and, still, preserves the resolution and DOF of the full-grid approach, while greatly improving the temporal performance of the technique.

While similar annular illumination strategies have been used in Fourier ptychography [22,23], they have thus far been limited to indirect imaging frameworks, relying on iterative reconstruction. In contrast, our method enables a *direct* imaging modality with substantial temporal advantages, as the volumetric image is obtained by appropriate summing of the collected dataset, as is: each pixel of the final image is obtained by simply summing pixels of the initial dataset [21,24] without the need for time-consuming wave front reconstruction or statistical inference typical of indirect imaging methods. The results presented thus pave the way for simplified, efficient, and real-time volumetric imaging and 3D microscopy.

This paper is organized as follows. In Sec. II, we introduce the principles and main features of coherence-based direct volumetric imaging based on sequential annular illumination. We start by recalling the analytical expression for the intensity distribution of defocused coherent images obtained by a single pointlike illuminator (Sec. II A). The volumetric reconstruction capability enabled by an array of sequentially illuminated point sources is reported in Sec. II B, where we highlight the effect of both spatial coherence and illumination geometry on transverse resolution, and dig into the underlying physical mechanism of the reconstruction algorithm. In Sec. II C, we demonstrate and discuss the physical origin of the axial resolution of the technique, relating it to geometric effects. In Sec. II D, we present a formal comparison of coherent imaging with full-disk illumination and with annular illumination. This analysis is then extended quantitatively in Sec. II E

through the use of an estimator, referred to as “image fidelity,” which enables a direct comparison of the axial and longitudinal resolution achieved by the two illumination strategies. In Sec. IV, we discuss the results and draw relevant conclusions.

II. THEORETICAL ANALYSIS

A. Analytical expression for defocused coherent images obtained with a single pointlike illuminator

We start by evaluating the intensity distribution that describes the coherent image of a two-dimensional object having complex field transmittance $A(\mathbf{r}_o)$, where $\mathbf{r}_o = (x_o, y_o)$ is the transverse coordinate on the object plane. As reported in Fig. 1, we suppose the object is illuminated by a pointlike source placed at a transverse coordinate $\mathbf{r}_s = (x_s, y_s)$, defined on the source plane, and the light transmitted by the object is collected by an imaging system reproducing an out-of-focus image on a high-resolution two-dimensional photosensor. The intensity at a transverse coordinate $\mathbf{r} = (x, y)$ of the sensor is given by

$$I(\mathbf{r}_s, \mathbf{r}) = S(\mathbf{r}_s) \left| \int \mathcal{L}(\mathbf{r}_s, \mathbf{r}_o) A(\mathbf{r}_o) \mathcal{P}_\delta(\mathbf{r}_o, \mathbf{r}) d^2 \mathbf{r}_o \right|^2, \quad (1)$$

where $\mathcal{L}(\mathbf{r}_s, \mathbf{r}_o)$ describes the propagation of the electric field from the illumination plane up to the object plane, $\mathcal{P}_\delta(\mathbf{r}_o, \mathbf{r})$ describes the propagation of the electric field from the object plane through the *defocused* optical device up to the photosensor [25,26], and $S(\mathbf{r}_s)$ is the optical intensity of the pointlike emitter at coordinate $\mathbf{r}_s$. The parameter δ , named the “defocusing distance,” indicates the distance between the transverse plane of the object and the plane optically conjugated to the sensor; δ is taken to be positive if the object is placed at a distance from the imaging device that is greater than the focusing distance, and to be negative otherwise. The form of Eq. (1) is explained and discussed in Appendix A.

As reported in Ref. [21], the field propagation through a defocused system can be decomposed into two terms: free-space propagation from the object plane to the conjugate plane of the sensor, followed by propagation through the focused imaging system, until the sensor. The Green’s function $\mathcal{P}_\delta$ can thus be expressed as

$$\mathcal{P}_\delta(\mathbf{r}_o, \mathbf{r}) = \int \exp \left[i \frac{k}{2\delta} |\mathbf{r}_o - \mathbf{r}_f|^2 \right] \mathcal{P}_0(\mathbf{r}_f, \mathbf{r}) d^2 \mathbf{r}_f. \quad (2)$$

The first, exponential function in the integrand describes free-space propagation of a monochromatic electric field, with wave number k , by a distance δ from the object plane to the conjugate plane of the sensor, with transverse coordinate $\mathbf{r}_f$. The second term describes the optical response of the imaging system to a spatial impulse located

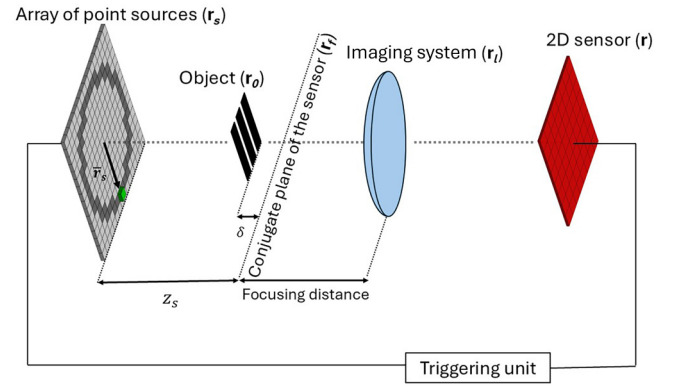


FIG. 1. Coherence-based direct volumetric imaging. Illumination is provided by an array of point sources that we turn on one at a time (as indicated by the green pixel depicted on the source array); depending on the desired illumination geometry, the array will be a ring or a full circle of radius $\bar{r}_0$. Light from each pointlike source illuminates the object. Transmitted light is then collected by an imaging system and detected by a 2D sensor. A triggering unit is used to acquire an image for each lit source. The conjugate plane of the 2D sensor, which is at the *focusing distance* from the imaging system, is at a distance z_s from the array of point sources and at a distance δ (referred to as the “defocusing distance”) from the object. The defocusing distance is chosen to be larger than the DOF of the (incoherently illuminated) imaging system. When all pointlike sources from the full circle or the ring of radius $\bar{r}_0$ are simultaneously ON (i.e., in conditions of incoherent illumination), the image of the object on the 2D sensor is out of focus, and hence blurred.

in the conjugate plane of the sensor; its squared modulus corresponds to the Airy disk.

Regardless of the spatial coherence properties of the illumination, the image resolution in *focused conditions* (namely, when the object is in the conjugate plane of the sensor) is defined by the Airy disk (hence, by the system NA). Conversely, recent work has shown that the quality of *defocused images* changes significantly according to whether the system is illuminated by spatially coherent or incoherent light [21,27,28]. For conventional (incoherent) imaging, the resolution of defocused images is well known to be defined by the NA-dependent COC—namely, the geometric projection of the limiting aperture of the imaging system onto the plane where the defocused image is observed. However, as demonstrated in Refs. [27,28] for coherent imaging, the transverse resolution of defocused images does not depend on the NA of the imaging system, and no COC-induced blurring occurs; rather, the loss of image quality with defocusing is dictated by free-space propagation of the field *diffracted* by the object.

In light of these considerations, we assume, for simplicity, ideality of the imaging system [$\mathcal{P}_0(\mathbf{r}_f, \mathbf{r}) =$

$\delta^{(2)}(\mathbf{r}_f - \mathbf{r}/M)$], so

$$\mathcal{P}_\delta(\mathbf{r}_o, \mathbf{r}) = \exp \left[i \frac{k}{2\delta} \left| \mathbf{r}_o - \frac{\mathbf{r}}{M} \right|^2 \right], \quad (3)$$

always bearing in mind that such a strong simplification is valid only for a coherent system, and only reasonably far away from its plane in focus (defocusing greater than the ratio between the wavelength and the squared NA). As we shall see, interesting insight into the features of coherent images is gained by use of Eq. (3) in the expression for the intensity at the detector.

From Eq. (1), we see that the intensity distribution of the coherent image depends not only on the focusing conditions (P_δ) but also on the illumination scheme ($\mathcal{L}$); as discussed in Ref. [21], the features of coherent images can be tuned by either changing the distance of the illumination plane from the object or using relay lenses, or even changing the size of the single illuminators. In this paper, we consider only the case where the object is separated by a distance z_s , in free space, from the illumination plane, and sources are pointlike. Hence, the illumination field at the object plane is given by

$$\mathcal{L}(\mathbf{r}_s, \mathbf{r}_o) = \exp \left[i \frac{k}{2(z_s - \delta)} |\mathbf{r}_o - \mathbf{r}_s|^2 \right]. \quad (4)$$

By plugging Eqs. (3) and (4) into Eq. (1), we find that the intensity distribution of the detected coherent image reduces to

$$I(\mathbf{r}_s, \mathbf{r}) \simeq S(\mathbf{r}_s) \left| \int A(\mathbf{r}_o) \exp \left[i \frac{k}{2[\delta^{-1} + (z_s - \delta)^{-1}]} \left| \mathbf{r}_o - \left(\frac{z_s - \delta}{z_s} \mathbf{r} + \frac{\delta}{z_s} \mathbf{r}_s \right) \right|^2 \right] d^2 \mathbf{r}_o \right|^2. \quad (5)$$

This result indicates that the coherent image produced by a point source is described by the diffraction pattern of the object, as observed from the *equivalent* defocusing distance $[\delta^{-1} + (z_s - \delta)^{-1}]$; this distance reduces to the real defocusing distance δ only in the case of plane-wave illumination ($z_s \rightarrow \infty$), in accordance with the findings of Ref. [21]. As theoretically and experimentally demonstrated in Ref. [21], one of the most interesting consequences of this result is that the transverse resolution of defocused coherent images has a square-root dependence on the defocusing distance, as is typical of coherent imaging; we come back to this point when discussing Fig. 5. Upon further inspection, Eq. (5) also indicates that the images produced by different single point sources from the illumination array are basically identical to one another, with the only difference being a slight δ -dependent magnification $[z_s/(z_s - \delta)]$ and a $\mathbf{r}_s$ -dependent shift on the sensor given by $\delta \mathbf{r}_s / z_s$, thus depending both on the defocusing and on the illumination distance.

B. Refocusing algorithm for coherence-based direct volumetric imaging

The identified δ dependence of the transverse shifts that defocused coherent images undergo when the illumination coordinate $\mathbf{r}_s$ is changed can be exploited for axially localizing the object or, equivalently, for isolating the “slice” of the volume where the planar object is placed [21]. This is done by means of the so-called *refocusing* or *reconstruction* algorithm, which realigns the intensities $I(\mathbf{r}_s, \mathbf{r})$ associated with a given axial coordinate δ and sums them together:

$$\Sigma_\delta(\mathbf{r}) = \int I \left(\mathbf{r}_s, -\frac{\delta}{z_s - \delta} M \mathbf{r}_s - \frac{z_s}{z_s - \delta} M \mathbf{r} \right) d^2 \mathbf{r}_s \propto I(0, M \mathbf{r}). \quad (6)$$

Refocusing is thus the result of a two-dimensional Radon transform: the plane at the axial coordinate δ is reconstructed by means of two Radon transformations of the acquired images $I(\mathbf{r}_s, \mathbf{r})$ at the same angle $\phi = \delta/z_s$, one in the (x_s, x) plane and the other one in the (y_s, y) plane [24]. Since obtaining a reconstruction from the dataset is a very easy computational operation, involving only pixel shifting and a sum, the required Radon transform is computationally nondemanding, and can be performed in real time by up-to-date graphics-processing unit architectures [29].

From Eqs. (5) and (6), we see that the reconstruction process retrieves the Fresnel diffraction pattern of the object after free-space propagation through the equivalent distance $[\delta^{-1} + (z_s - \delta)^{-1}]$ in Eq. (5). The transverse resolution of coherence-based direct volumetric imaging is thus expected to be predominantly defined by the way diffraction and free-space propagation affect the *individual* images produced by single pointlike sources, while the specific pattern of pointlike sources (i.e., their number, their positions, and the overall extension of the illumination array) is expected not to play a relevant role. The theoretical and experimental demonstration of this intuitive result is reported in the following sections.

We now study the role played by the numerical aperture of the imaging system regarding the retrieved defocused images in the ideal case of an infinitely extended uniform illumination plane [$S(\mathbf{r}_s) = 1$]. To this end, we report in Fig. 2(a) the 2D slices of the 4D dataset $I(x_s, y_s, x, y)$ corresponding to $y_s = y = 0$; the left-hand-side and right-hand-side slices correspond to two different values of the NA of the imaging system. The simulation is performed by our considering a Gaussian-shaped transmissive slit

$$A(\mathbf{r}_s) = \exp \left[-\frac{r_s^2}{4\sigma_s^2} \right],$$

with $\sigma_s = 100 \mu\text{m}$, and pointlike emitters placed at an illumination distance $z_s = 100 \text{ mm}$, and our shining green

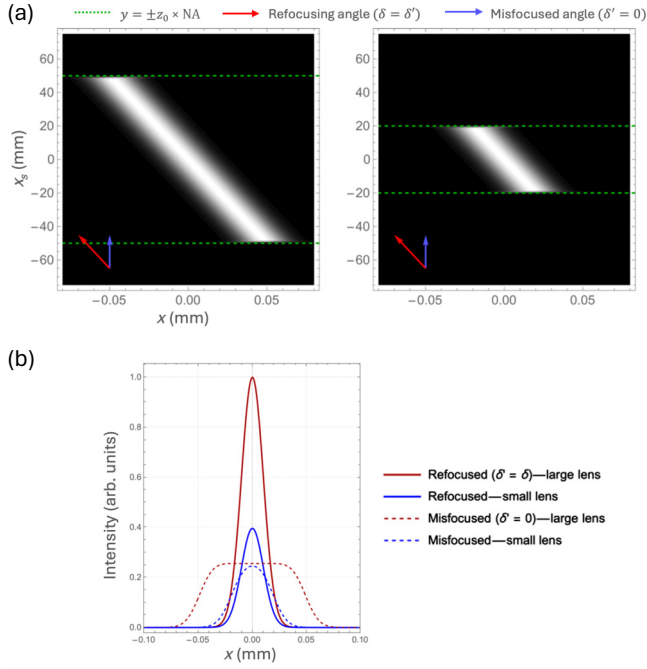


FIG. 2. (a) Simulation of 2D slices of the 4D dataset $I(x_s, y_s, x, y)$ corresponding to $y_s = y = 0$ for a Gaussian-shaped transmissive object and two different values of the NA of the imaging system, larger for the left plot and smaller for the right plot (see the text for details). (b) Quantitative comparison of background suppression, characterized via the COC, between high-NA (red) and low-NA (blue) configurations.

light ($\lambda = 532$ nm) on the sample. The simulated imaging device is a unit-magnification $4f$ system with focal length $f = 10$ mm, having an acceptance pupil of radius l : in the left plot in Fig. 2(a), $l = 5$ mm and $\text{NA} = 0.5$, while in the right plot, $l = 2$ mm and $\text{NA} = 0.2$. Both plots are based on Eq. (1), without our performing any of the approximations discussed so far. As can be easily seen, the most evident effect of the NA of the imaging system is to determine a cutoff in the space of the illumination array, so illumination from larger x_s coordinates does not produce detectable coherent images on the sensor. In other words, light from illumination angles greater than the NA of the optical system does not contribute to $I(x_s, y_s, x, y)$. Interestingly, this result can be accurately predicted through geometric optics. To see this, we apply a stationary phase approximation ($\lambda \rightarrow 0$) to the full expression as given in Eq. (1), obtaining

$$I^{(\text{ray})}(\mathbf{r}_s, \mathbf{r}) = \lim_{k \rightarrow \infty} I(\mathbf{r}_s, \mathbf{r}) = \left| A \left(\frac{\delta}{z_s} \mathbf{r}_s + \frac{z_s - \delta}{z_s} \frac{\mathbf{r}}{M} \right) \right|^2 P \left[-\frac{l}{z_s \times \text{NA}} \mathbf{r}_s + \left(1 + \frac{l}{z_s \times \text{NA}} \right) \frac{\mathbf{r}}{M} \right], \quad (7)$$

where the function $P(\mathbf{r}_l)$ is the dominant iris (of radius l) and determines the NA of the imaging system. The first factor in Eq. (7) is the defocused image of the object produced by the point source placed at the coordinate $\mathbf{r}_s$; the geometric approximation thus maps the object plane to a mixed transverse coordinate that depends on both the illumination and the detection coordinates:

$$\mathbf{r}_o \rightarrow \frac{\delta}{z_s} \mathbf{r}_s + \frac{z_s - \delta}{z_s} \frac{\mathbf{r}}{M}.$$

Comparison of Eqs. (7) and (5) clearly indicates that ray optics predicts the same shifting and magnification of the single out-of-focus coherent images, but is obviously incapable of describing the image degradation due to diffraction. As the object shows no degradation due to diffraction, the geometric limit incorrectly predicts no loss of resolution, regardless of the amount of defocusing. Nevertheless, the geometric optics framework is capable of predicting the largest illumination coordinate $\mathbf{r}_s$ giving rise to a coherent image on the sensor: on the basis of Eq. (7), we see that an object point in the center of the field of view (FOV) $\mathbf{r}_o = 0$ is mapped onto an infinite range of optical rays between the illumination plane and the detector plane, satisfying

$$\mathbf{r}_o = 0 = \frac{\delta}{z_s} \mathbf{r}_s + \frac{z_s - \delta}{z_s} \frac{\mathbf{r}}{M}.$$

Among those, only rays crossing the lens plane at coordinates $\mathbf{r}_l$ within the iris can contribute to image formation. This requirement can be deduced from Eq. (7), which requires that

$$\left| -\frac{l}{z_s \times \text{NA}} \mathbf{r}_s + \left(1 + \frac{l}{z_s \times \text{NA}} \right) \frac{\mathbf{r}}{M} \right| \leq l.$$

In the hypothesis of small defocusing ($\delta \ll z_s$), a point at the center of the FOV is thus projected onto a coordinate $\mathbf{r}_l$ obeying the mapping

$$\mathbf{r}_o = 0 \rightarrow \mathbf{r}_l \simeq \frac{l}{\text{NA}} \frac{\mathbf{r}_s}{z_s}.$$

Therefore, a sample coordinate near the center of the FOV is imaged properly only as long as $r_s l / (z_s \times \text{NA}) \lesssim l$ or, equivalently,

$$r_s \lesssim r_s^{(\text{max})} = z_s \times \text{NA}. \quad (8)$$

Equation (8) is easy to interpret, as it requires that the *illumination-side* numerical aperture

$$\text{NA}_s = \frac{\bar{r}_s}{z_s}, \quad (9)$$

with $\bar{r}_s$ the largest illumination radius, should at most match, but never exceed, the acceptance NA.

This result, which we remark is obtained purely from geometric arguments, is reported as dashed green lines $x_s = \pm z_s \times \text{NA}$ in the plots in Fig. 2(a). The plots clearly demonstrate the ability of the geometric argument to provide a good quantitative evaluation of the identified cutoff even outside the geometric context.

In summary, the NA of the imaging system has a negligible effect on the transverse resolution, but indeed limits the “useful” illumination coordinates ($\mathbf{r}_s$) in the illumination plane to angles compatible with the pupil geometric acceptance. This limits the possible illumination patterns $S(\mathbf{r}_s)$ to functions whose support is contained within the angular domain defined by the lens. We also highlight an important implication of the findings in this section. Given that lens pupils are typically circular, or can be reasonably approximated as such, the region on the illumination plane that matches the system angular acceptance is also circular. As a result, in architectures using illumination from a 2D grid of emitters matched to the system NA, only the sources located within a circle inscribed in the grid contribute effectively. For this reason, in the remainder of this paper, we restrict our analysis to a comparison between full, disk-shaped, illumination patterns and annular, ring-shaped, illumination patterns.

C. Effect of the illumination NA on the axial resolution

The limited support of $I(\mathbf{r}_s, \mathbf{r})$, imposed by the acceptance and/or illumination NA, implies that the integration domain of the reconstruction algorithm in Eq. (6) does not extend infinitely along the $\mathbf{r}_s$ axes even if $S(\mathbf{r}_s) = 1$ in Eq. (1). While we have shown that the transverse structure of the source does not influence the lateral resolution, in this section we demonstrate that the finite extent of the accessible $\mathbf{r}_s$ coordinates fully determines the axial resolution properties.

Let us consider the analytical expression for a reconstruction [Eq. (6)] at a generic axial coordinate δ' , which can in principle be different from the actual coordinate δ at which the object is placed. There are several reasons why one might be interested in a “misfocused” reconstruction. Primarily, this arises from the fact that the experimenter does not know *a priori* the axial position δ of the object. As a result, a (software) z scan is always required, where Eq. (6) is evaluated over a range of axial values. This is analogous to the conventional experimental approach of seeking the best focus by physically adjusting the imaging system (hardware z scan) and visually identifying the sharpest image. In this scanning procedure, the axial resolution of the technique manifests itself as the axial “thickness” of the reconstruction—namely, the range of δ' values over which the object can plausibly be located and confidently identified as being at position δ .

The analytical expression for the generic reconstruction in δ' is obtained by application of Eq. (6) with $\delta \rightarrow \delta'$ to

Eq. (5) and reads, in the $\delta \ll z_s$ approximation,

$$\Sigma_{\delta'}^{\text{disk}}(\mathbf{r}) \simeq \frac{1}{\pi \times \text{NA}^2 (\delta - \delta')^2} \times \int \int |\mathcal{A}_\delta(\mathbf{r}'_o)|^2 \chi_{\text{disk}} \left(\frac{1}{|\delta - \delta'| \times \text{NA}} (\mathbf{r} - \mathbf{r}'_o) \right) d^2 \mathbf{r}'_o, \quad (10)$$

where

$$\mathcal{A}_\delta(\mathbf{r}'_o) = \int A(\mathbf{r}_o) \exp \left[i \frac{k |\mathbf{r}_o - \mathbf{r}'_o|^2}{2 [\delta^{-1} + (z_s - \delta)^{-1}]^{-1}} \right] d^2 \mathbf{r}_o \quad (11)$$

is the Fresnel diffraction pattern generated by the object at the equivalent observation distance. χ_{disk} is a disk-shaped characteristic function, valued 1 whenever its argument is within a unitary radius, and 0 outside, and is obtained by assuming matched illumination NA and acceptance NA.

Equation (10) yields exactly the diffraction pattern of the object [Eq. (11)] only when $\delta' = \delta$; in all other cases, it entails a convolution with a disk-shaped COC. Such an effect is purely geometric, as is expected from a COC [21], as the same point-spread function (PSF) is obtained by application of a stationary phase approximation to Eq. (10). The radius of the disk-shaped COC is defined by the product $|\delta - \delta'| \times \text{NA}$, as is typical of defocused incoherent imaging systems [21].

The effect of the COC and its role in defining the axial resolution can be seen in Fig. 2(b), where we show a comparison between the x sections of the reconstruction of the Gaussian slit in Fig. 2(a). As discussed in Sec. II B, images refocused at the correct object position δ are virtually identical for both the larger lens (NA = 0.5, solid red line) and the smaller lens (NA = 0.2, solid blue line). The only evident difference is the overall brightness, which is reduced in the low-NA case due to decreased light collection. However, this effect can be easily corrected during post-processing, provided that a sufficiently high signal-to-noise is available.

A far more significant distinction emerges when we consider the suppression of out-of-focus planes. This is illustrated by the reconstructions at $\delta' = 0$, corresponding to a defocus of 100 μm relative to the true object plane at δ . The two dashed lines (red for high NA, blue for low NA) reveal that the lower-NA system produces a much smaller COC, which—under a moderate defocus—hampers the ability to clearly distinguish the in-focus plane. Moreover, the reduced performance in background suppression is reflected in the limited contrast, evaluated as the ratio between the peak intensities of the correctly refocused image and the misfocused image. In contrast, the high-NA system exhibits a significantly larger COC and superior suppression of background planes, as evidenced

by the higher contrast between in-focus and out-of-focus reconstructions.

D. Comparison of coherence-based volumetric imaging with annular illumination and with full-disk illumination

In this section, we show that the essential features of coherence-based direct volumetric imaging are retained when the full 2D illumination array is replaced by a *ring* of pointlike sources.

Let us consider the reconstruction obtained by illumination of the sample with an annular pattern, defined as the outermost ring of a disk in the illumination plane, matching the acceptance NA. As was done in Sec. II C for full-disk illumination, the reconstruction in δ' resulting from annular illumination reads

$$\Sigma_{\delta'}^{\text{ring}}(\mathbf{r}) = \frac{1}{2\pi \times \text{NA} |\delta - \delta'|} \times \int |\mathcal{A}_{\delta}(\mathbf{r}'_o)|^2 \chi_{\text{ring}} \left(\frac{1}{|\delta - \delta'| \times \text{NA}} |\mathbf{r} - \mathbf{r}'_o| \right) d^2 \mathbf{r}'_o, \quad (12)$$

where $\chi_{\text{ring}}(|\xi_1 - \xi_2|) = \delta^{(1)}(|\xi_1 - \xi_2| - 1)$ describes a ring-shaped PSF of radius 1 in the (ξ_1, ξ_2) space, while $\mathcal{A}_{\delta}$ is again defined as in Eq. (11).

Aside from differences in overall brightness, disk and annular illumination yield identical images under perfect reconstruction conditions. However, whenever $\delta' \neq \delta$, the

two produce distinct background responses: annular illumination results in ring-shaped blurring on out-of-focus planes, whereas disk illumination generates a ring-shaped PSF around the object details. A noteworthy observation is that the radii of the PSFs—whether full or annular—are identical. According to the results presented in Sec. II C, the axial sectioning performance of both illumination strategies is thus expected to be comparable, as it depends primarily on the size of the COC.

Annular illumination can therefore be leveraged to achieve the same high axial resolution enabled by a high-NA design, while suppressing redundant information associated with small-angle components of the illumination, which do not contribute to axial sectioning.

Figure 3 shows a comparison between the volumetric reconstructions obtained via software scans in the cases of annular illumination (upper row) and full-disk illumination (lower row). For ease of calculation, we simulated a Gaussian illumination pattern to obtain the full-disk illumination case—namely,

$$S(\mathbf{r}_s) = \exp \left[-\frac{r_s^2}{2\sigma_s^2} \right] = \exp \left[-\frac{r_s^2}{2(1.5\bar{r}_s)^2} \right], \quad (13)$$

where the width of the equivalent Gaussian has been set to $1.5\bar{r}_s$ to qualitatively match the axial performance of a full-disk illumination of radius $\bar{r}_s$. The simulated object is a 100- μm -wide Gaussian slit, placed at axial position $\delta = +200 \mu\text{m}$ for both illumination strategies.

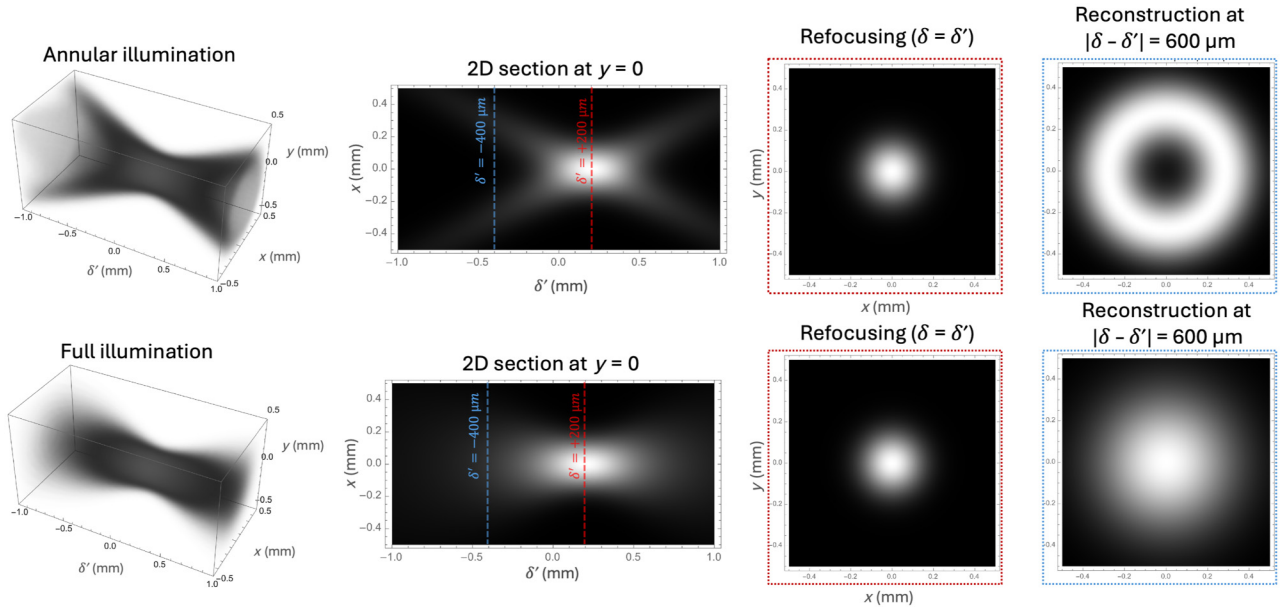


FIG. 3. Qualitative comparison between two illumination strategies. Images obtained with full Gaussian illumination (upper row) and with annular illumination (lower row). From left to right, the plots represent a whole 3D z scan obtained by the sweeping of δ' in the reconstructed images obtained with Eq. (6); a 2D section of the z scan along the (x, δ') plane, evaluated at $y = 0$; the sharpest reconstruction at $\delta' = \delta$; a misfocused reconstruction at $|\delta - \delta'| = 600 \mu\text{m}$.

As can be seen from the two pairs of plots on the left in Fig. 3 reporting, respectively, the 3D software z scans and their 2D slices at $y = 0$, both illumination strategies enable a very similar axial localization of the object around the correct refocusing position $\delta' \simeq \delta$. The software z scan is obtained by evaluating the reconstruction algorithm over multiple axial positions δ' , assuming the correct position of the object is not known *a priori*. The object position is identified as the axial location where the reconstructed image appears sharpest. It corresponds to the point at which the blur, quantified by the radius of the COC associated with the reconstruction procedure [Eqs. (10) and (12)], is minimum [24].

As shown by the two images highlighted by a dashed red outline, the performance of the two illumination geometries is identical when reconstruction is performed exactly on the object plane—namely, when $\delta' = \delta$; in the specific case of the reported simulation, this corresponds to considering the 2D slice of the z scan obtained at $\delta' = +100 \mu\text{m}$.

As clearly seen in the two rightmost images outlined by a dashed blue square, a significant difference between the two illumination strategies appears when we consider out-of-focus images; in the specific case, the reported images correspond to a misfocus of $600 \mu\text{m}$ from the position of the object. The images show that the illumination strategy based on Gaussian illumination induces a Gaussian-shaped COC (bottom right panel), whereas annular illumination results in a donut-shaped image (top right panel), having approximately the same radius as the Gaussian-shaped COC. This difference will clearly appear in the background planes of reconstructed images, as we see in Sec. III.

As a final remark, we discuss the speedup of acquisition and reconstruction associated with annular illumination versus full-disk illumination. For annular illumination, the reconstruction algorithm involves only one integration variable, as the angular coordinate is swept while the illumination radius is fixed. The acquired dataset thus becomes three-dimensional, rather than four-dimensional:

$$I(x_s, y_s, \mathbf{r}) \longrightarrow I(\bar{r}_s \cos \theta, \bar{r}_s \sin \theta, \mathbf{r}) =: \mathcal{I}(\theta, \mathbf{r}). \quad (14)$$

The reconstruction algorithm needs to be changed accordingly, as it now involves a single-axis integration:

$$\Sigma_{\delta'}^{\text{ring}}(x, y) = \int_0^{2\pi} \mathcal{I}\left(\theta, \frac{z_s}{z_s - \delta'} M \begin{bmatrix} x \\ y \end{bmatrix} - \frac{\delta'}{z_s - \delta'} M r_s \begin{bmatrix} \cos \theta \\ \sin \theta \end{bmatrix}\right) d\theta. \quad (15)$$

This function can be recognized as no longer consisting of two cascaded Radon transforms [as in Eq. (6)], as it involves shifting the coherent images along the θ axis in the (r_s, θ) plane. Interestingly, Eq. (15) represents the

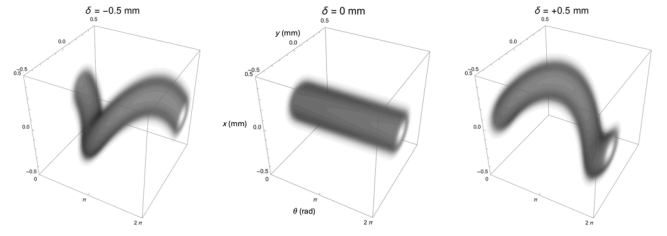


FIG. 4. Simulations of the 3D dataset $\mathcal{I}(\theta, \mathbf{r})$ [Eq. (14)] resulting from annular illumination of a Gaussian object, corresponding to three values of defocusing δ values (see the text for further details).

parametrization of a *helicoidal path integral* over the $(\theta, \mathbf{r})$ space; a single path integral over a helix thus replaces the double path integral over lines typical of the reconstruction algorithm with full illumination [Eq. (6)]. This can be seen in Fig. 4, where we report the 3D dataset $\mathcal{I}(\theta, \mathbf{r})$ obtained by illumination of the Gaussian sample corresponding to Fig. 2 with an annular pattern of radius $r_s = z_s \times \text{NA}$ in the cases of negative defocusing ($\delta = -0.5 \text{ mm}$, left panel), positive defocusing ($\delta = +0.5 \text{ mm}$, right panel) and no defocusing ($\delta = 0$, middle panel). As expected, the coherent image of the slit does not show any shift in the focused case. However, object points describe helices in the $(\theta, \mathbf{r})$ space, with axes parallel to the θ axis, chiralities defined by the sign of the defocusing, and whose radii depend on the absolute defocusing itself.

E. Quantitative comparison between full-disk illumination and annular illumination: Fidelity analysis

To quantitatively compare the resolution and axial sectioning capability of the two illumination strategies, we evaluate the so-called image fidelity, defined by the functional scalar product between the m -normalized reference function $|A(\mathbf{r})|^2$ and its reconstruction Σ (see Appendix B for details):

$$F^{(m)}(\{p\}) = \frac{\int |A(\mathbf{r})|^{\frac{m}{2}} |\Sigma(\mathbf{r}; \{p\})|^{\frac{m}{2}} d^2r}{\left(\int |A(\mathbf{r})|^m d^2r\right)^{\frac{1}{2}} \left(\int |\Sigma(\mathbf{r}; \{p\})|^m d^2r\right)^{\frac{1}{2}}}. \quad (16)$$

The generic n -ple $\{p\}$ represent the set of parameters on which the final image depends, including the object parameters (size, shape, defocusing, etc.), optical parameters (optical NA, illumination NA, defocusing) and the reconstruction parameter δ' . In particular, to assess the performance of the imaging technique, we evaluate the parametric curves

$$F(\sigma_o, \delta, \delta', \text{NA}_s) \geq c_{\text{th}}, \quad (17)$$

where c_{th} is a threshold above which the image is considered faithful [24,27,28]. We thus use the fidelity to evaluate

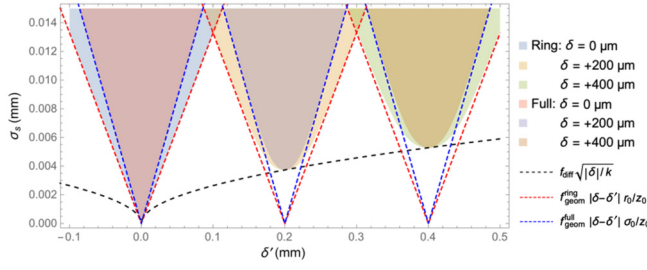


FIG. 5. Fidelity of the reconstructed images in the cases of full (Gaussian-shaped) illumination and annular illumination.

the quality of the reconstructed image over the 4D space of the parameters $(\sigma_o, \delta, \text{NA}_s, \delta')$ that determine the reconstructed image. The quality of the refocused image of an object of size σ_o depends not only on the distance between the object and the plane in focus (δ), as for conventional incoherent imaging, but also on the coordinate where the reconstruction is performed (δ'); the parameter NA_s is also be considered to achieve a quantitative and systematic confirmation of the role of the illumination aperture NA_s in determining the axial sectioning.

On the basis of the analytical expressions reported in Appendix B [Eqs. (B2) and (B5)], the fidelities of correctly refocused ($\delta' = \delta$) images are identical in the cases of full-disk illumination and annular illumination—namely, $F_{\text{geom}}^{\text{ring}}(\sigma_o, \delta, \delta' = \delta) = F_{\text{geom}}^{\text{full}}(\sigma_o, \delta, \delta' = \delta)$. This result reflects the fact that the transverse resolution obtained by the two illumination strategies is independent of the illumination aperture NA_s ; the effect of the illumination aperture on the fidelity is entirely contained within the term associated with the COC, as discussed in the previous section. The curves of best resolution for differing axial position of the object have the typical square-root trend of coherent imaging [21,27,28]:

$$F_{\text{diff}}^{\text{full, ring}}(\sigma_o, \delta, \delta' = \delta) \geq c_{\text{th}} \implies \sigma_o(\delta) \geq \sqrt{\frac{|\delta|}{k}} f_{\text{diff}}^{\text{ring}}(c_{\text{th}}), \quad (18)$$

where the expression for $f_{\text{diff}}(c)$ is given in Appendix C. This trend describes the diffraction-limited *transverse* resolution attained by coherence-based direct volumetric imaging as a function of defocusing, regardless of the illumination strategy (dashed black line in Fig. 5).

In Fig. 5, the colored regions of space serve as a complete qualitative reference of the behavior of the fidelity for full Gaussian illumination (pink, purple, and orange regions, corresponding to an object placed at $\delta = 0 \mu\text{m}$, $\delta = 200 \mu\text{m}$, and $\delta = 400 \mu\text{m}$, respectively) and for annular illumination (blue, yellow, and green regions, at the same defocusing values). The plots are obtained by our assuming $\text{NA}_s = \sigma_s/z_s = 0.1$ for full illumination, and

$\text{NA}_s = \bar{r}_s/z_s = 0.1$ for annular illumination, and illumination distance $z_s = 100 \text{ mm}$. The fidelity evaluation was performed with $c_{\text{th}} = 99\%$. Both illumination strategies clearly give rise to the typical NA-independent loss of resolution as the object is moved away from the focus; at the focus, the image has the best possible resolution and all vertices of the colored regions at high fidelity lie on the same square-root trend defined by coherence (dashed black line). However, far from the best resolution, the fidelity of the reconstructed image assumes the typical V-shaped trend of the COC-induced linear scaling of resolution versus DOF [21]. Although the widths of the V-shaped regions are slightly different for the two illumination strategies, the two techniques perform similarly, with full illumination apparently enabling slightly better axial sectioning (a point we address later).

We now study in a more quantitative way the axial performance obtained with the two illumination modalities. Being uniquely determined by the COC, the axial resolution is independent of diffraction-induced artifacts due to coherence [21]; we thus work in the limit of negligible diffraction (details are provided in Appendix B). The expressions for the fidelity in the absence of diffraction-induced artifacts read, respectively,

$$F_{\text{geom}}^{\text{full}}(\sigma_o, |\delta - \delta'|, \text{NA}_s) \geq c_{\text{th}} \implies \sigma_o \geq |\delta - \delta'| \times \text{NA}_s \times f_{\text{geom}}^{\text{full}}(c_{\text{th}}), \quad (19)$$

$$F_{\text{geom}}^{\text{ring}}(\sigma_o, |\delta - \delta'|, \text{NA}_s) \geq c_{\text{th}} \implies \sigma_o \gtrsim |\delta - \delta'| \times \text{NA}_s \times f_{\text{geom}}^{\text{ring}}(c_{\text{th}}), \quad (20)$$

where the explicit forms of the two threshold-dependent coefficients are provided in Appendix C. These two expressions can be recognized as determining the COC-induced resolution curves, having the same linear scaling as the COC induced by the acceptance pupil. They are similar, as expected, differing only in the multiplying threshold-dependent function f . The dashed red and blue lines in Fig. 5 represent Eqs. (20) and (19), respectively. Figure 5 shows that, far from the diffraction curve, the behavior of the fidelity can be completely explained by the expression obtained in the geometric approximation; the two expressions have the same direct proportionality with both the defocusing NA and the illumination NA, as expected by the COC. Figure 5, as already mentioned, also shows a better axial sectioning for full (Gaussian) illumination. This is because a Gaussian illumination pattern of width NA_s contains illumination angles much larger than NA_s , whereas the annular illumination is, by its nature, limited to the single angle defined by the illumination aperture. One always has $f_{\text{geom}}^{\text{ring}}(c_{\text{th}}) < f_{\text{geom}}^{\text{full}}(c_{\text{th}})$ for any fidelity threshold $c_{\text{th}} < 1$. To match the imaging performance of the two

illumination strategies, the full Gaussian illumination thus needs to be chosen with an *equivalent* numerical aperture given, for high fidelity ($c_{th} \simeq 1$), by

$$NA_s^{Gauss} = \frac{f_{geom}^{ring}(c_{th})}{f_{geom}^{full}(c_{th})} \times NA_s \simeq \frac{NA_s}{\sqrt{2}}.$$

III. EXPERIMENTAL RESULTS

A schematic representation of the setup used for experimental validation of the results presented is shown in Fig. 1. In the experiment, illumination was provided by an array of 2427 WS2812B red-green-blue light-emitting diodes (LEDs), although only green light was used for simplification of the proof-of-principle demonstration. The illumination patterns used were either a disk or a ring with a diameter of 12 LEDs, as shown in Fig. 6(a). The objects were elements 5 and 6 of group 5 of a U.S. Air Force (USAF) resolution test target, having linewidths 9.84 and 8.77 μm , respectively. The imaging system was an optical microscope consisting of an objective lens (Nikon Plan Apo λ 20 $\times$, NA = 0.75) and a tube lens (Thorlabs WFA4100), combined to give 10 $\times$ magnification. The 2D sensor was a Thorlabs Zelux CS165MU1 monochrome complementary metal-oxide semiconductor camera. An external microcontroller (Arduino UNO), depicted as the triggering unit in Fig. 1, was used to power each LED separately, and to simultaneously trigger the camera. The complete dataset $I(\mathbf{r}_s, \mathbf{r})$ was obtained by our lighting, one at a time, all LEDs of the ring or disk and acquiring the corresponding images produced on the camera.

The condition for coherent imaging was achieved by guaranteeing that, on the object plane, the coherence area of single LED illumination was larger than or comparable to the object details; to this end, the array was placed at a large enough distance ($z_s = 110$ mm) from the working distance of the microscope.

In Fig. 6(b), we report the experimental reconstructed images obtained by use of Eqs. (6) and (12), respectively, for annular illumination (left panel) and full-disk illumination (right panel) while δ' is swept in the interval $[-150 \mu\text{m}, +150 \mu\text{m}]$.

The plots enable comparison of the axial sectioning obtained by means of the two illumination strategies. In particular, the plots in Fig. 6(b) are obtained by our slicing the 3D reconstructed cube on the same y coordinate, resulting in a 2D (x, δ') axial section. The results indicate that both illumination geometries equally enable the axial localization of the USAF test target; both reconstructed images show, for instance, the tilting of the mask slide with respect to the z axis, as the two triple-slit elements (corresponding to elements 6 and 5) are not parallel to the x axis (i.e., are not refocused at the same δ'). This is in agreement with the results reported in Fig. 6(c), where we report reconstructed images at the coordinates $\delta' = 0 \mu\text{m}$,

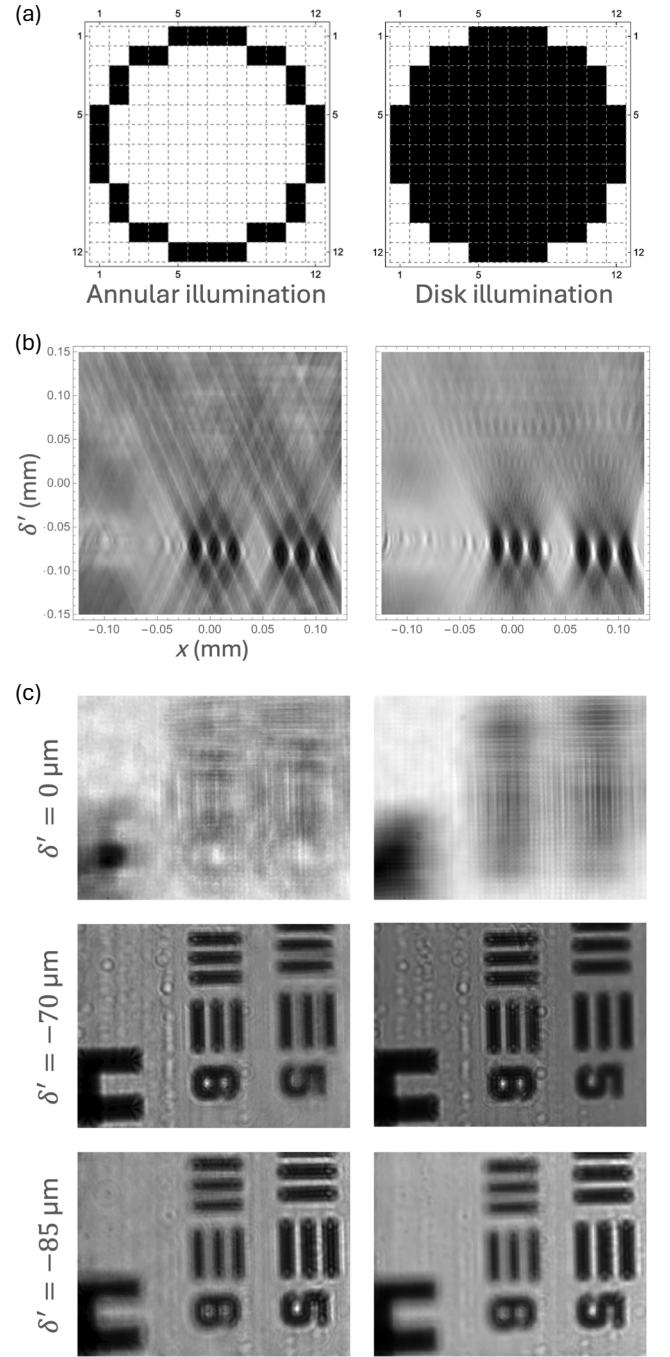


FIG. 6. (a) The two different illumination patterns used: annular and full in a 12-light-source-diameter ring. (b) 2D (x, δ') axial section obtained by slicing of the 3D reconstructed cube on the same y coordinate by means of the two illumination strategies. (c) Comparison of the reconstructed images of the USAF target at reconstruction coordinates $\delta' = 0 \mu\text{m}$, $\delta' = -70 \mu\text{m}$, and $\delta' = -85 \mu\text{m}$.

$\delta' = -70 \mu\text{m}$, and $\delta' = -85 \mu\text{m}$, from top to bottom: the reconstructed images reported in the second and third rows show that element 6 is reconstructed slightly close to the plane in focus ($\delta' = -70 \mu\text{m}$) compared with the element 5, which is reconstructed at $\delta' = -85 \mu\text{m}$. The

accuracy, quality, and features of the reconstructed images appear to be essentially unaffected by the illumination geometry, as expected. In contrast, the effect of the different illumination strategies appears in the reconstruction at $\delta' = 0$ [upper plots in Fig. 6(c)], where the features in the background planes of the reconstructed defocused images clearly change with the illumination geometry. This is due to the difference, presented in Sec. IID, between the COC of annular illumination and the COC of full-disk illumination, which is ring-shaped for the former (a typical result for defocused catadioptric lenses [30]) and smoother for the latter.

IV. DISCUSSION AND CONCLUSIONS

The optimal combination of NA-independent transverse resolution and NA-defined axial localization characterizing the coherence-based direct volumetric imaging technique has been shown to be fully maintained when the number of illumination sources, and hence the number of acquired coherent images, is reduced.

We have shown that the axial sectioning enabled by full illumination is *de facto* reproduced by an annular illumination pattern matching the acceptance numerical aperture of the imaging system. The matching between the illumination NA and the acceptance NA was found to be the best working condition: illumination angles larger than the acceptance NA are irrelevant in the image reconstruction process, as they cannot produce images on the camera; on the other hand, angles corresponding to lower illumination angles do not significantly contribute to the axial sectioning of the technique. Shining light from a densely populated array of pointlike emitters is thus not needed. This conclusion is perfectly in line with the purpose of this work, which aims not at improving the image quality but at a speedup of acquisition and elaboration that preserves 3D resolution and DOF.

To be more quantitative, we now prove that annular illumination yields an advantage that becomes significantly larger as the NA is increased and the illumination pitch decreased. By assuming that pointlike emitters are spaced by a quantity δr_s on the illumination plane, we find that the number of coherent images that must be acquired for full-grid illumination, the number of coherent images that must be acquired for full-disk illumination, and the number of coherent images that must be acquired for annular illumination are given by, respectively,

$$N_{\text{full grid}} = \frac{4}{\delta r_s^2} r_s^2, \quad N_{\text{disk}} \simeq \frac{\pi}{\delta r_s^2} r_s^2, \quad N_{\text{ring}} \simeq \frac{2\pi}{\delta r_s} r_s, \quad (21)$$

where r_s is the illumination radius matching the acceptance NA of the imaging system. On the basis of the number of acquired frames in the cases of full-disk illumination and

annular illumination (N_{disk} and N_{ring} , respectively), annular illumination guarantees a linearly increasing acquisition speedup as the illumination radius is increased; the circumference scales linearly with the radius of the circle, even for a large linear density of emitters $r_s/\delta r_s$. This advantage was leveraged in our experimental demonstration to reduce the number of LEDs from 144 for full-square-grid illumination or 112 for full-disk illumination to 32 for annular illumination; despite the modest LED density characterizing our array, this resulted in a 3.5-fold speedup.

Apart from their application significance, the reported results also carry a relevant fundamental weight. The direct volumetric imaging technique considered has profound analogies with so-called correlation plenoptic imaging (CPI) [31–33], which retrieves a 4D light-field function with extremely similar coherence-induced properties by means of photon correlation. Uncovering the possibility of performing CPI by considering light propagating at only the extreme angles of the optical setup and by reducing the dimensionality of the required dataset from 4 to 3 has the potential to positively impact the performance of correlation-based imaging techniques, where the number of involved modes has detrimental effects on the signal-to-noise ratio of the final images [34–37]. Significant developments of the proposed approach are thus expected in view of both a more relevant speedup than the one achieved in the present proof-of-principle demonstration and an extension of the presented working principle to more complex, e.g., correlation-based, 3D imaging scenarios.

Finally, it is worth remarking that by adopting the ideal imaging condition that leads to Eq. (3), we are ignoring not only the diffraction-limited resolution of the plane in focus but also effects that become relevant at large NA, such as aberrations. Our choice is motivated by the need to reduce the number of variables so as to highlight the essential features of the proposed imaging method by means of purely analytical results. We postpone the investigation of the large-NA effects to future research on the topic.

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DATA AVAILABILITY

The data that support the findings of this article are openly available [38].

APPENDIX A: FORMAL INTERPRETATION OF EQ. (1)

The propagation of an electric field in a vacuum can be described as a convolution between the transverse field profile at the input plane and the appropriate Green's function (or, equivalently, as a multiplication by the corresponding transfer function in the Fourier domain) [26]. When the electric field passes through an optical setup that includes a sample and lenses, the propagation must additionally account for the phase and amplitude modifications introduced by these components. For simplicity, we assume that both the sample and the lenses are suf-

ficiently thin, so that their effect can be modeled by a *complex transmittance* acting multiplicatively on the incident field. Under these assumptions, the complex electric field at the output of the optical system can be expressed as a cascade of integrals: each free-space segment contributes a convolution, while at every plane containing a thin sample or optical element the integrand is multiplied by the corresponding transmittance function.

In Eq. (1), we assume that the electric field at the source plane passes through some illumination-shaping optics before reaching the sample. In general, we can suppose that the illumination architecture is made up of M cascaded lenses, each separated by a distance z_j from the next optical component, with $j = 1, \dots, M - 1$. Therefore, the field on the sample (supposed to be placed at a distance z_M from the last optical component of the illumination chain) is described by the equation

$$E_{\text{sample}}(\mathbf{r}_0) = \int d^2 r_N \mathcal{V}_N(\mathbf{r}_0 - \mathbf{r}_N) P_N(\mathbf{r}_N) \cdots \int d^2 r_1 \mathcal{V}_1(\mathbf{r}_2 - \mathbf{r}_1) P_1(\mathbf{r}_1) \int d^2 r_s \mathcal{V}_0(\mathbf{r}_1 - \mathbf{r}_s) E_{\text{source}}(\mathbf{r}_s), \quad (\text{A1})$$

where

$$\mathcal{V}_j(\mathbf{r}) = \exp\left(ik \frac{r^2}{2z_j}\right) \quad (\text{A2})$$

is the paraxial vacuum Green's function, which causes propagation of the electric field by the distances z_j that separate the optical components, and $P_j(\mathbf{r}_j)$ is the transmittance of the j th optical component, including both the effect of the lenses on the field phase and the limitation of the wave fronts induced by irises. The electric field in Eq. (A1) clearly involves, in its most general form, one convolution and one multiplication at any plane occupied by an optical element. Although treating this expression analytically can become cumbersome for illumination strategies involving even a small number of lenses, one can always formally change the order of the integration variables and write

$$E_{\text{sample}}(\mathbf{r}_0) = \int d^2 r_s \mathcal{L}(\mathbf{r}_s, \mathbf{r}_0) E_{\text{source}}(\mathbf{r}_s), \quad (\text{A3})$$

where the function $\mathcal{L}(\mathbf{r}_s, \mathbf{r}_0)$ is the result of performing the N intermediate integrations on the surface of the optical components. It is easily verified, even in the simple case of a single lens, that $\mathcal{L}$ depends explicitly on both coordinates $\mathbf{r}_0$ and $\mathbf{r}_s$, rather than having an implicit dependence on their difference, as for the Green's function in Eq. (A2). Therefore, the equation does not typically represent a convolution. Incidentally, a case in which $\mathcal{L}$ degenerates into

a single-variable function, with explicit dependence on the difference between coordinates, is in the case study of this paper, where the sample is directly illuminated through free-space propagation [see Eq. (4)].

The same reasoning that allowed us to attribute a physical meaning to $\mathcal{L}$ can be applied to the propagation of the electric field from the sample plane to the plane of the photodetectors, through an arbitrarily complex imaging setup. If $E_{\text{sample}}(\mathbf{r}_0)$ is the electric field illuminating the sample plane, the electric field at the detector plane is given by

$$E_{\text{out}}(\mathbf{r}_s) = \int d^2 r_0 P_\delta(\mathbf{r}_0, \mathbf{r}) A(\mathbf{r}_0) E_{\text{sample}}(\mathbf{r}_0). \quad (\text{A4})$$

Also in this instance, $P_\delta(\mathbf{r}_0, \mathbf{r})$ does not generally depend on the coordinate difference, so the integral does not necessarily represent a convolution.

The expression in Eq. (1) is obtained by considering that the illumination scheme is a collection of pointlike emitters placed at transverse coordinates $\mathbf{r}'_s$ defined on the source plane, so that $E_{\text{source}}(\mathbf{r}_s) = \delta^{(2)}(\mathbf{r}_s - \mathbf{r}'_s)$. By performing this substitution in the equations above, and considering that intensity measurement is sensitive to the squared modulus of the output field, we can easily verify the formal soundness of Eq. (1). In addition to the terms describing electric field propagation, we have included the factor $S(\mathbf{r}'_s)$, which defines the support of the illumination and the brightness of a given pointlike emitter.

As a final remark, we emphasize that our formalism remains fully compatible with the established description

of incoherent imaging systems, in which the recorded intensity is given by the convolution between the object and the PSF of the optical system. The image of a sample illuminated by a spatially incoherent source [e.g., an extended lamp of intensity profile $S(\mathbf{r}_s)$] can be described by the incoherent superposition of all the coherent images produced by the single emitters placed at all possible angles; its analytical expression is thus given by the integral of Eq. (1) over $\mathbf{r}_s$, which gives

$$I_{\text{inc}}(\mathbf{r}) = \int d^2r_o d^2r'_o A(\mathbf{r}_o) A^*(\mathbf{r}'_o) P_\delta(\mathbf{r}_o, \mathbf{r}) P_\delta^*(\mathbf{r}'_o, \mathbf{r}) \times \int d^2r_s S(\mathbf{r}_s) \mathcal{L}(\mathbf{r}_s, \mathbf{r}_o) \mathcal{L}^*(\mathbf{r}_s, \mathbf{r}'_o). \quad (\text{A5})$$

For definiteness, let us consider the case of simple free space separating the emitter from the sample, in which case the integral in the first line degenerates into the Fourier transform of the source. By considering the typical case of uniform illumination ($S = 1$), we recover the relationship between the image and the PSF of the system—namely,

$$I_{\text{inc}}(\mathbf{r}) = \int d^2r_o |A(\mathbf{r}_o)|^2 |P_\delta(\mathbf{r}_o, \mathbf{r})|^2. \quad (\text{A6})$$

This relationship describes the convolution between the object and the system response function. For typical paraxial, aberration-free, simple imaging systems (e.g., a single imaging lens, a $4f$ confocal system), the only contributions not depending on the coordinate difference are in the phase. Since incoherent imaging depends only on the amplitude part of the function P_δ , characterized by the dependence $|P_\delta(\mathbf{r}_o, \mathbf{r})|^2 = |P_\delta(\mathbf{r}_o - \mathbf{r}/M)|^2$, our formalism is fully compatible with incoherent imaging [21].

APPENDIX B: EVALUATION OF THE IMAGE FIDELITY

The fidelity defined in Eq. (16) is a generalization of the expression so far used in the literature [21], which corresponds to the case $m = 1$. This is motivated by the fact that the analytical evaluation of the fidelity with annular illumination does not lead for $m = 1$ to an analytical form. Calculations were thus performed in the simpler case of $m = 2$, although throughout the entire paper, we have simplified the notation by dropping the superscript “(2)” for better readability.

By our choosing a Gaussian object centered in the FOV, the fidelity analysis developed in this paper depends only on five parameters: one related to the object (its width σ_o), one related to the algorithm (δ'), and three related to the optical design—namely, the illumination NA, the optical NA, and defocusing. However, a five-parameter study is cumbersome and not needed, so we chose to ignore the

parameter related to the optical NA. To quantify the performance of the illumination strategy, we thus evaluated the parametric curves in the limit of infinite NA—namely,

$$F_\infty(\sigma_o, \delta, \delta', \text{NA}_s) = \lim_{\text{NA} \rightarrow \infty} F(\sigma_o, \delta, \delta', \text{NA}_s, \text{NA}) = c_{\text{th}}. \quad (\text{B1})$$

The ignoring of the lens aperture ($\text{NA} \rightarrow \infty$) represents a working hypothesis of $\text{NA}_s \ll \text{NA}$. If we were working with a circular pupil, rather than the Gaussian pupil we used to obtain analytical forms, this hypothesis would not be required. As we showed in Sec. II B, the presence of a binary pupil imposes an abrupt transition between allowed illumination NA, for which the presence of a limiting pupil is negligible, and angles that are blocked. On the other hand, a Gaussian pupil behaves very differently, as it transmits rays at very large angles (although attenuated in the tails); as a consequence, the concurrent effect of both NA and NA_s (which is also a Gaussian) is never negligible. Our approximation of $\text{NA}_s \ll \text{NA}$ with Gaussian apertures thus corresponds to the more physically sound situation of $\text{NA}_s \lesssim \text{NA}$ with circular pupils. We expect the infinite-NA approximation to correctly describe the behavior of our system regardless of the illumination size, with the exception of the focused plane, where the image is entirely defined by NA. As a consequence, our analysis will predict pointlike resolution in focus, rather than the λ/NA limit defined by the Airy disk [21].

In the case of a Gaussian-shaped full illumination, and in the approximation $z_s \gg \delta$, the fidelity of Eq. (16), with $m = 2$, can easily be evaluated to be

$$F_\infty^{\text{full}}(\sigma_o, \delta, \delta', \text{NA}_s) \simeq 2 \frac{\sqrt{1 + \sigma_{\text{COC}}^2/\sigma_o^2 + \sigma_{\text{diff}}^2/\sigma_o^2}}{2 + \sigma_{\text{COC}}^2/\sigma_o^2 + \sigma_{\text{diff}}^2/\sigma_o^2}, \quad (\text{B2})$$

where

$$\sigma_{\text{COC}}(|\delta - \delta'|, \text{NA}_s) = |\delta - \delta'| \times \text{NA}_s = |\delta - \delta'| \frac{\sigma_s}{z_s}, \quad (\text{B3})$$

$$\sigma_{\text{diff}}(\delta, \sigma_o) = \frac{|\delta|}{2k\sigma_o}. \quad (\text{B4})$$

As anticipated, perfect fidelity ($F = 1$) is obtained at the focus ($\delta = \delta' = 0$), while, away from the focus, the fidelity of a coherent imaging system limited by the illumination NA (NA_s) has the same limitations as a system limited by the optical NA defined by the acceptance pupil.

In the case of annular illumination of radius r_s , the fidelity has a more complex analytical expression:

$$F_{\infty}^{\text{ring}}(\sigma_o, \delta, \delta', \text{NA}_s) \simeq 2 \frac{\exp \left[-\frac{r_{\text{COC}}^2/\sigma_o^2}{(1+\sigma_{\text{diff}}^2/\sigma_o^2)(2+\sigma_{\text{diff}}^2/\sigma_o^2)} \right] \sqrt{1 + \sigma_{\text{diff}}^2/\sigma_o^2}}{\sqrt{\mathcal{I}_0 \left(\frac{r_{\text{COC}}^2/\sigma_o^2}{2(1+\sigma_{\text{diff}}^2/\sigma_o^2)} \right)}}, \quad (\text{B5})$$

where $\mathcal{I}_0(t)$ is the zero-order modified Bessel function of the first kind and

$$r_{\text{COC}}(|\delta - \delta'|, \text{NA}_s) = |\delta - \delta'| \times \text{NA}_s = |\delta - \delta'| \frac{r_s}{z_s} \quad (\text{B6})$$

is the radius of the COC in the case of annular illumination.

The effects of the Airy disk at the focus are not included [$F_{\infty}^{\text{ring}}(\sigma_o, \delta = 0, \delta' = 0, \text{NA}_s) = 1$], as we have ignored the finite size of the acceptance pupil.

In the main text, we discussed how the effect of diffraction becomes rapidly irrelevant as one moves away from the diffraction-defined resolution limits at a given defocusing δ . Furthermore, the axial sectioning of the techniques is related to geometric optics phenomena (the COC), so the expression for the fidelity obtained by ignoring diffraction can be of interest. On the basis of the definition of σ_{diff} , this condition is met either when the ratio $\sigma_{\text{diff}}/\sigma_o$ is very small—namely, when one is working with object details much larger than the resolution limit of Eq. (18)—or when one is working in the geometric optics approximation. The diffraction-free expression for the fidelity is obtained by setting the ratio $\sigma_{\text{diff}}/\sigma_o$ to zero or, equivalently, by evaluating the limit $k \rightarrow \infty$. Either way, such simplification yields a simpler form for Eqs. (B2) and (B5). In particular, the expression related to full illumination has the advantage of having an explicit form in the $(\sigma_o, \delta' - \delta)$ plane, as reported in Eq. (19). The same does not hold for annular illumination, whose diffraction-free fidelity reads

$$F_{\text{geom}}^{\text{ring}}(\sigma_o, |\delta - \delta'|, \text{NA}_s) = \sqrt{\mathcal{I}_0 \left[\frac{1}{2} \left(\frac{|\delta - \delta'| \sigma_s}{z_s \sigma_o} \right)^2 \right]^{-1}} \geq c_{\text{th}} \quad (\text{B7})$$

and does not have an explicit form, so it cannot be directly compared with the full-illumination case. However, the comparison between the resolution limits is particularly simple when $\sigma_{\text{COC}} \ll \sigma_o$. Apart from the obvious requirement that the COC should be much smaller than the object itself, this condition can also be considered as the regime that determines the shape of the resolution curves far from the diffraction limit; the condition that the COC must be

smaller than the object is very easily satisfied, for the same defocusing, with larger objects that are away from the resolution limit (upper part of Fig. 5). If the condition holds, $\sqrt{I_0(x)^{-1}} \simeq 1 - x^2/8$, and Eq. (B7) assumes the explicit form reported in Eq. (20).

APPENDIX C: EXPRESSION FOR THE THRESHOLD-DEPENDENT FACTOR OF THE FIDELITY CURVES

In this appendix, we report the analytical form of the $f(c_{\text{th}})$ functions from Sec. II E. Such functions appear in the expression for the fidelity curves as multiplying factors, and are the only terms depending on the threshold value chosen arbitrarily to discriminate between faithful and unfaithful images.

For Eq. (18), we have

$$f_{\text{diff}}(c) = \left[\sqrt[4]{8 \left(c^{-2} - 1 + \sqrt{c^{-2} - 1} \right)} \right]^{-1}.$$

For Eqs. (19) and (20), the expressions for the coefficients read, respectively,

$$f_{\text{geom}}^{\text{full}}(c) = \sqrt{\left[\sqrt{(1 - c^2)^{-1}} - 1 \right] / 2},$$

$$f_{\text{geom}}^{\text{ring}}(c) = 1 / \sqrt[4]{32(1 - c)}.$$

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