

rf-SQUID-based traveling-wave parametric amplifier with input saturation power of -84 dBm across more than one octave in bandwidth

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Traveling-wave parametric amplifiers (TWPAs) have become an essential tool for the readout of quantum circuits and the search for dark matter. We report on the implementation of an rf-superconducting quantum interference device (SQUID)-based Josephson TWPA with an average saturation power of -84 dBm while providing an average power gain of 20 dB from 3.5 to 8.5 GHz. This wide bandwidth is enabled by reducing the curvature of the dispersion in the signal band while suppressing detrimental mixing processes. With 2393 rf-SQUIDs, our device has a comparable number of nonlinear elements but achieves ten times higher saturation power than previously reported devices. This shows that using rf-SQUIDs results in a more favorable trade-off between device length and saturation power. Harnessing wideband characterization techniques, we determine the TWPA's excess noise above the quantum limit to be 0.8 to 1.5 photons. In addition, we validate the signal-to-noise ratio improvement as a practical measure for tuning the TWPA to minimize the total system noise and demonstrate that its optimum does not coincide with the highest gain.

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I. INTRODUCTION

Over recent years, near-quantum-limited parametric amplifiers have gained widespread use across various fields of scientific research. By significantly improving the signal-to-noise ratio of experiments involving microwave signals with powers as low as a few femtowatts, they have advanced the technical capabilities of experimental setups. Such amplifiers have demonstrated exceptional performance in experiments targeting dark-matter detection [1–6], reading out signals from microwave superconducting quantum interference device (SQUID) multiplexers [7], and enhancing the sensitivity of microwave single-photon detectors [8,9] or phonon detectors [10]. Moreover, parametric amplifiers have become indispensable tools in the fields of quantum sensing and quantum information [11–16], where their ability to amplify weak signals with minimal added noise [17] is crucial.

Parametric amplifiers can be divided into two groups: resonator-based parametric amplifiers [13,18–21] and traveling-wave parametric amplifiers (TWPAs) [8,22–43].

The former generally exhibit lower noise, while the latter typically provide bandwidths of 1 GHz or more. This comes at the cost of increased circuit complexity, such as the need for sophisticated dispersion engineering to enable phase-matching and suppress unwanted frequency mixing. The wider bandwidth enables greater capacity for frequency multiplexing in applications such as qubit or detector readout. This feature makes TWPAs particularly advantageous for scaling up quantum computers, where simultaneous readout of many qubits is necessary. Their number is further constrained by the saturation power of the amplifier chain. Future qubit designs resilient to higher readout powers might necessitate amplifiers with higher power-handling capability [44].

Josephson traveling-wave parametric amplifiers (JTWPAs) typically have a saturation power of the order of -100 dBm at a gain of 20 dB [22–26]. TWPAs based on high-kinetic-inductance films [8,33–37] can achieve higher saturation power (-63 to -47 dBm), but they operate at pump powers 4–5 orders of magnitude higher than those of JTWPAs. Such strong pump tones require more isolation to shield qubits, and they also necessitate pump-cancellation techniques to avoid saturation of the next amplification stages, both of which add extra complexity to the experimental setup [45].

Further, intermodulation distortion must be considered when more than one signal is amplified. This arises from undesired mixing processes between the signals and increases drastically as the saturation power of the

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amplifier is approached. This effect can result in significant crosstalk between amplified signals, potentially increasing readout errors within the system [46]. One proposed mitigation strategy for this is to use a large detuning between the pump and signal frequencies and to increase the saturation power of the TWPA.

To address these issues, we have designed and fabricated a TWPA based on rf-SQUIDs [47]. This operates in the three-wave-mixing (3WM) regime, resulting in a large separation between the pump and signal frequencies. The dispersive and nonlinear properties are tailored to simultaneously achieve wide bandwidth and high saturation power. We engineer the dispersion by harnessing a multiperiodic variation of the capacitances to ground, enabling simultaneous phase matching of parametric amplification while suppressing unwanted mixing products [48]. In contrast to other 3WM-TWPAs, which typically rely on a low plasma frequency to achieve the latter [25,35], our approach allows for lower dispersion in the signal band, thereby increasing the attainable amplification bandwidth [35]. We demonstrate an average gain of 20 dB between 3.5 and 8.5 GHz, which exceeds one octave and is, to the best of our knowledge, the widest relative bandwidth achieved in TWPAs.

Furthermore, our device achieves a saturation power of -84 dBm, more than 1 order of magnitude higher than reported for other JTWPAs. To achieve this, we leverage the fact that the linear and nonlinear properties of rf-SQUIDs can be effectively decoupled. Hence, we design the rf-SQUIDs to be weakly nonlinear, aiming for higher pump powers, which consequently leads to proportionally increased saturation power. In this way, we break the trade-off between saturation power and device length that typically governs the design of JTWPAs based on Josephson junctions or dc-SQUIDs [49]. Indeed, with 2393 rf-SQUIDs, our device has a comparable number of nonlinear elements to those reported in Refs. [22,23,26], but it achieves a tenfold increase in saturation power.

II. DEVICE DESIGN AND FABRICATION

Figure 1(a) shows the circuit diagram of the rf-SQUID-based TWPA. The device comprises a chain of $N = 2393$ unit cells, each consisting of an rf-SQUID and a capacitor to ground C_g . Each rf-SQUID, formed by a meandering inductor L_m shunted by a Josephson junction with critical current I_c and junction capacitance C_J , is nominally identical and nonhysteretic, i.e., the screening parameter is $\beta_L = L_m I_c / \varphi_0 < 1$ [50]. Here, $\varphi_0 = \Phi_0 / 2\pi = \hbar / 2e$ is the reduced magnetic flux quantum.

Considering an injected ac current with a small amplitude $|i| \ll \Phi_0 / L_m$ and the corresponding phase oscillation ϕ , we obtain the current-phase relation for the rf-SQUID

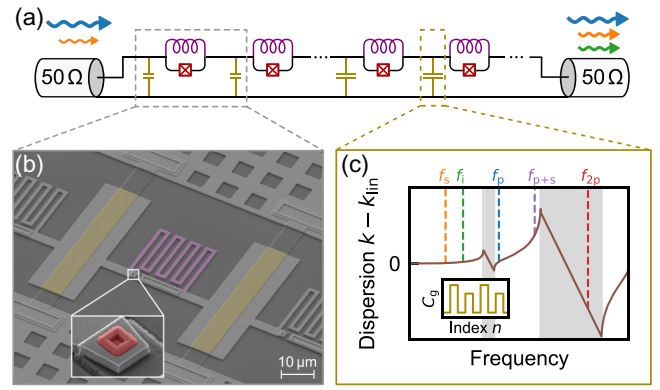


FIG. 1. Circuit diagram of the rf-SQUID-based TWPA, experimental implementation, and dispersion relation. (a) A strong pump tone (blue arrow) with frequency f_p and a weak signal tone (orange arrow) with frequency f_s propagate along the amplifier and interact via the nonlinearity of the rf-SQUIDs, resulting in energy transfer from the pump to the signal and the creation of an idler tone (green arrow) at frequency $f_i = f_p - f_s$. The rf-SQUIDs are formed by a Josephson junction (red) shunted by a linear inductor (violet). We periodically vary the capacitance to ground C_g (yellow) to realize dispersion engineering [see panel (c)]. (b) False-colored scanning electron microscopy image of an rf-SQUID between two capacitors to ground [colors according to panel (a)]. The enlarged view shows a junction of size $1 \times 1 \mu\text{m}^2$. (c) Sketched dispersion relation, obtained using multiperiodic capacitance variation (mPCV), which results in two stopbands (gray shaded regions) [48]. The second stopband mismatches unwanted mixing processes, such as $f_{p+s} = f_p + f_s$ (violet), $f_{p+i} = f_p + f_i$ (not shown), and second-harmonic generation $f_{2p} = 2f_p$ (red), while the first stopband enables phase-matching between f_p , f_s , and f_i to allow amplification. The inset shows the mPCV scheme using three values for C_g (see main text).

as follows [47]:

$$i(\phi) = \frac{\Phi_0}{2\pi L_{\text{SQ}}(\phi_{\text{dc}})} [\phi - \beta\phi^2 - \gamma\phi^3 + \dots], \quad (1)$$

where ϕ_{dc} is the solution of the transcendental equation $\phi_{\text{dc}} = \Phi_{\text{ext}}/\varphi_0 - \beta_L \sin \phi_{\text{dc}}$ for an external magnetic flux bias Φ_{ext} [51]. Here, β and γ are the coefficients of second- and third-order (Kerr) nonlinearity,

$$\beta = \frac{\beta_L}{2} \frac{\sin \phi_{\text{dc}}}{(1 + \beta_L \cos \phi_{\text{dc}})}, \quad (2)$$

$$\gamma = \frac{\beta_L}{6} \frac{\cos \phi_{\text{dc}}}{(1 + \beta_L \cos \phi_{\text{dc}})}. \quad (3)$$

To operate in the 3WM regime while minimizing unwanted Kerr effects, such as cross-phase modulation (XPM) and self-phase modulation (SPM) [52], we flux bias our device near the point $\phi_{\text{dc}} \approx \pi/2$. The flux dependence

of the rf-SQUID inductance is given by

$$L_{\text{SQ}}(\phi_{\text{dc}}) = L_{\text{m}}/(1 + \beta_L \cos \phi_{\text{dc}}), \quad (4)$$

and this renders the characteristic impedance and the wave number of the transmission line flux dependent. For frequencies much lower than the cutoff frequency, they can be expressed as $Z(\phi_{\text{dc}}) \approx \sqrt{L_{\text{SQ}}(\phi_{\text{dc}})/C_{\text{g}}}$ and $k_{\text{lin}}(\phi_{\text{dc}}) \approx \omega a^{-1} \sqrt{L_{\text{SQ}}(\phi_{\text{dc}})C_{\text{g}}}$, respectively. Here, $\omega = 2\pi f$ is the angular frequency and a is the length of the unit cell.

The 3WM process is governed by the energy-conservation law $f_{\text{p}} = f_{\text{s}} + f_{\text{i}}$ and the phase-matching condition $k_{\text{p}} - k_{\text{s}} - k_{\text{i}} \ll \pi/N$. Here, f_j and k_j , with $j \in \{\text{p}, \text{s}, \text{i}\}$, are the frequencies and wave numbers of the pump, signal, and idler, respectively. In contrast to four-wave-mixing (4WM), in a dispersion-free medium ($k_{\text{lin}} \propto \omega$), 3WM is naturally phase matched over a wide frequency range; however, this also leads to the phase matching of unwanted mixing processes, such as upconversion (e.g., $f_{\text{p}+\text{s}} = f_{\text{p}} + f_{\text{s}}$) and second-harmonic generation ($f_{2\text{p}} = 2f_{\text{p}}$), limiting the gain [53]. To address this, the primary objective of dispersion engineering in 3WM TWPAs is to suppress unwanted mixing processes while maintaining phase matching for the amplification [48,54].

Here, we realize this using a multiperiodic variation of the ground capacitances C_{g} . By varying the ground capacitance between three values [see inset of Fig. 1(c)], we create two stopbands, as can be seen in Fig. 1(c) [48]. Phase matching for parametric amplification is achieved by placing the pump frequency slightly above the first stopband, while undesired tones are suppressed due to large phase mismatches or cannot propagate in the stopband. Note that the dispersion for frequencies below the first stopband is flat, while it increases sharply above the first stopband. This allows us to fulfill the phase-matching condition over a wider frequency range compared to approaches that rely on a lowered plasma frequency to mismatch unwanted processes [25,29,30,35,55].

To estimate what saturation power can be attained with the TWPA, we assume a simplified case with perfectly phase-matched pure 3WM parametric amplification and suppression of unwanted processes. Then, the signal power gain in the center of the signal band, $f_{\text{s}} \approx f_{\text{p}}/2$, can be expressed as

$$G = \cosh^2(|\beta|k_{\text{p}}\phi_{\text{p}}Na/4), \quad (5)$$

where $\phi_{\text{p}} = L_{\text{SQ}}I_{\text{p}}/\phi_0$ denotes the amplitude of the pump phase oscillation associated with the pump current I_{p} [47, 56]. The incident pump power is $P_{\text{p}} \approx 0.5I_{\text{p}}^2Z$. Next, we suppose that pump depletion is the only cause of gain saturation. The input-referred saturation power, conventionally stated as the 1-dB compression point, is approximately

found by

$$P_{1\text{dB}} \approx \frac{P_{\text{p}}}{4G_0}, \quad (6)$$

or $P_{1\text{dB}}^{\text{dBm}} \approx P_{\text{p},\text{in}}^{\text{dBm}} - G^{\text{dB}} - 6 \text{ dB}$ (see Appendix A). In a 4WM-TWPA, the Kerr nonlinearity both facilitates parametric amplification and causes SPM and XPM. In this case,

$$P_{1\text{dB}} \approx \frac{P_{\text{p}}}{8G_0} \quad (7)$$

is smaller by a factor of 2 due to the additional signal-power-induced phase mismatch due to SPM and XPM [57].

Since the saturation power is proportional to the pump power, it is limited by the nonlinear element. In a Josephson-junction-based TWPA, this limit is set by the critical current; however, since increasing the critical current decreases the inductance and nonlinearity, for the same gain, proportionally more junctions are needed [49]. This establishes a trade-off between saturation power and device length. For an rf-SQUID-based TWPA, this trade-off is more favorable for two reasons. First, the rf-SQUID has two degrees of freedom, which allow design of the nonlinear properties [Eqs. (2) and (3)] independently from the linear one [Eq. (4)], decoupling k_{p} from β in Eq. (5). Second, thanks to the superconducting shunt L_{m} , there is no hard limit for the pump current at which the rf-SQUIDs switch to the voltage state (see Fig. 2). This allows pumping the TWPA with high amplitudes, limited only by the onset of detrimental effects caused by higher-order nonlinearities.

Figure 1(b) shows a scanning electron microscope (SEM) image of an rf-SQUID between two capacitors to ground. The device is fabricated in Nb trilayer technology using superconductor-insulator-superconductor junctions (see Appendix B), on an intrinsic Si wafer with 200 nm of thermal SiO₂. The Josephson junctions have a critical current of $I_{\text{c}} = 0.9 \mu\text{A}$. Together with the planned shunting inductance $L_{\text{m}} = 60 \text{ pH}$, this yields $\beta_L \approx 0.16$. The capacitors to ground are realized through plate capacitors using 240-nm-thick SiO_x as the dielectric and are designed to have the three values $C_1 = 10.5 \text{ fF}$, $C_2 = 68.2 \text{ fF}$, and $C_3 = 50.4 \text{ fF}$. Groups of six unit cells each have identical capacitances to ground, resulting in a periodicity of 24 unit cells.

III. CHARACTERIZATION

We characterize the device in a commercial dilution refrigerator at approximately 20 mK. Our experimental setup has a pair of coaxial switches to perform *in situ* forward calibration with a female-female SMA adapter as the reference. All cables used between the ports of the switches

are nominally of the same length. To set the operation point, we apply a dc current I_{dc} to the central conductor of the TWPA transmission line via two bias tees, resulting in an external flux $\Phi_{ext} = L_m I_{dc}$ in the rf-SQUID loops. For the details of the experimental setup, see Appendix C.

A. Pump-off measurements

Figures 2(a)–2(c) show the transmission of the TWPA versus probe-tone power for three flux-bias points. The dark regions around 11 GHz indicate the first stopband, the position of which varies depending on the flux bias. At the flux-bias points $\Phi_{ext} \approx 0$ and $\Phi_0/2$, the Kerr-nonlinearity coefficient γ has its extrema, and this results in a power-dependent stopband position due to SPM of the probe tone. While this is clearly visible for Fig. 2(c), in Fig. 2(a), it is not obvious. At $\Phi_{ext} \approx 0.3\Phi_0$, the Kerr

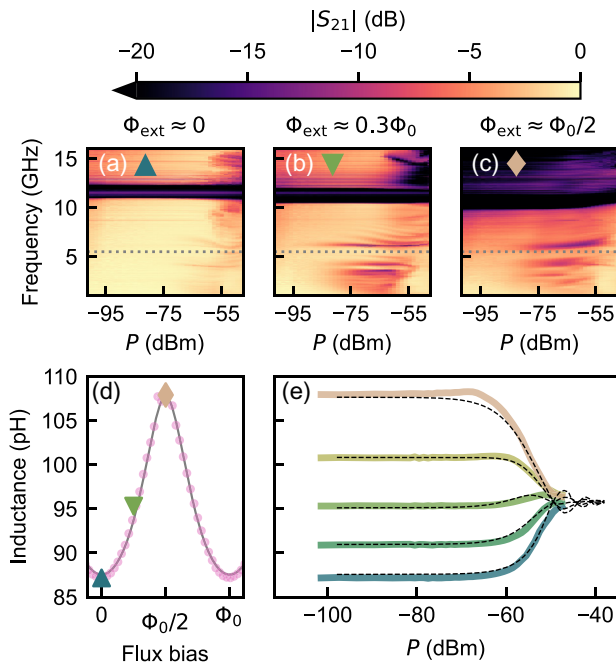


FIG. 2. Power dependence of transmission and rf-SQUID inductance. (a)–(c) Power dependence of $|S_{21}|$ measured at three relevant flux-bias points. The stopband position (dark region near 11 GHz) changes as a function of flux due to a change in rf-SQUID inductance [see panel (d)]. At higher powers, the stopband shows a shift caused by self-phase modulation of the probe tone. The direction of the shift reflects the Kerr-nonlinearity coefficient, which is (a) positive, (b) negligible, and (c) negative. For powers larger than -80 dBm in (b), we observe lower transmission between 1 and 6 GHz, which we attribute to second-harmonic generation. (d) Cell inductance L_{cell} as a function of flux bias. The pink points show L_{cell} extracted from the experimental data. The gray curve is a fit to the model of the TWPA unit cell (see Appendix D). (e) Power dependence of the rf-SQUID inductance extracted from the phase of S_{21} at 5.5 GHz [gray dotted lines in panels (a)–(c)]. The dashed lines represent WRSPICE simulations of the TWPA unit cell.

nonlinearity is negligible, and no power-dependent shift of the stopband is observed; the second-order nonlinearity coefficient β is close to its maximum and causes second-harmonic generation. This manifests in transmission loss, visible in the darker region for powers ranging from -80 to -50 dBm across frequencies of 1–6 GHz. Similarly, the lossy regions in Fig. 2(c) are attributed to third-harmonic generation, as well as residual second-harmonic generation. Notably, near the region denoted by the dotted line, this process is suppressed, as the second harmonic falls into the stopband.

Figure 2(d) shows the rf-SQUID inductance as a function of external flux. By fitting a cascaded transfer matrices (CTM) model [58] to the flux-dependent S_{21} spectra for a signal power of -100 dBm, we obtain the inductance values for different external flux values (pink points). Compared to the simplified circuit diagram [see Fig. 1(a)], the practical implementation of the rf-SQUID TWPA unit cell has additional elements—e.g., the inductance of the line connecting two neighboring rf-SQUIDs—that have to be accounted for. For our model of the flux-dependent rf-SQUID inductance, we take these additional elements into consideration to extract $I_c = 0.93 \mu\text{A}$ and $L_m = 58.6 \text{ pH}$ (see Appendix D).

Figure 2(e) shows a comparison of the experimentally determined power dependence of the cell inductance L_{cell} at different flux-bias points with that of WRSPICE simulations [59]. Note that the rf-SQUID inductance remains almost constant even for high power levels above -60 dBm for $\Phi_{ext} \approx 0.3\Phi_0$, allowing for high pump powers without detrimental SPM and XPM effects. To take advantage of this rf-SQUID feature, we operate our TWPA close to this setpoint. In comparison, a Josephson junction with the same inductance of 95 pH already switches to the voltage state at a power of -65 dBm. The cell inductance is extracted as a function of power from experimental data by analyzing the power-dependent phase of S_{21} at a fixed frequency of $5.5 \text{ GHz} \approx f_{stopband}/2$, where power leakage to the second harmonic is suppressed because of the stopband (see Appendix E). In WRSPICE, we simulate the behavior of a single unit cell of our TWPA according to the model described above and also extract the power-dependent inductance for different flux-bias points (see Appendix E). Finally, we align the experimental data to the simulation results, providing an input-power calibration at 5.5 GHz and yielding an input-line attenuation of 69 dB.

B. Pump-on measurements

To tune up the TWPA, we vary the flux bias, the pump power, and the pump frequency and measure the transmission compared to a through reference. Figure 3(a) shows the optimized power gain as a function of signal frequency using the flux-bias point $\Phi_{ext} \approx 0.33\Phi_0$, pump frequency $f_p = 12.08 \text{ GHz}$, and pump power $P_p \approx$

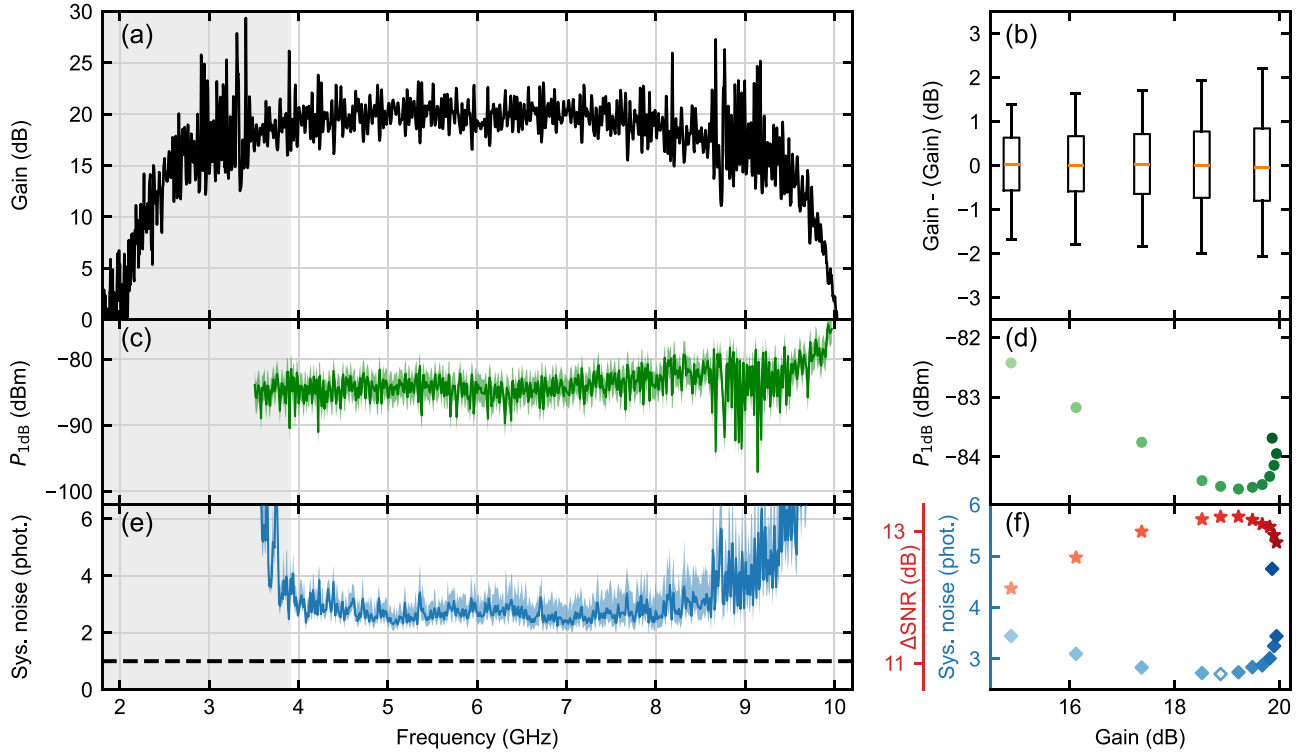


FIG. 3. Gain profile, saturation power, and system noise. (a) We demonstrate an average gain of 20 dB from 4 to 8 GHz relative to a through reference. The gray shaded region indicates frequencies where the isolator is out of band (see Appendix C). (b) Boxplots showing amplifier gain distribution for different mean gain values for frequencies from 4 to 8 GHz. Whiskers (boxes) represent the 5th (25th) and 95th (75th) percentiles. The orange line indicates the median value. (c) TWPA input saturation power (1-dB compression point P_{1dB}) as a function of frequency for the gain profile shown in panel (a). Our device reaches an average saturation power of -84 dBm. The shaded region indicates the uncertainty of the power calibration. (d) Dependence of average P_{1dB} on average gain across the 4–8 GHz frequency range. Darker color denotes higher pump power. Higher pump power increases the gain, resulting in a decrease of saturation power until the TWPA reaches its maximum gain. A further increase of pump power increases the saturation power but also increases the total system noise [see panel (f)]. (e) Using the TWPA as the first amplifier in the readout chain yields a total system noise between 2 and 4 photons at the gain value optimized for lowest average system noise [19 dB; see empty marker in panel (f)]. The shaded region indicates the uncertainty in the noise evaluation (see main text). (f) Dependence of average total system noise (blue diamonds) and signal-to-noise ratio improvement ΔSNR (red stars) on average power gain across the 4–8 GHz frequency range. Darker color denotes higher pump power. The minimal total system noise (and maximal ΔSNR) is observed at an average gain of 19 dB.

-56 dBm. To measure the gain, we use a signal power $P_s \approx -126$ dBm to avoid saturating losses. Therefore, we get a lower bound on the gain, which increases by up to 2 dB for higher signal powers (see Appendix F). Using the optimized pump parameters, 90% of gain values in the 4–8 GHz band, sampled with 5-MHz frequency spacing, fall within the range of (19.8 ± 2.4) dB. As shown in Fig. 3(b), lower gain results in lower ripple.

Gain ripple in TWPAs originates in the interference of multireflections of the signal and idler waves. This effect can be enhanced by backward amplification due to a reflected pump wave. Therefore, to decrease ripple, improved impedance matching is required for both the signal band and the pump frequency. We investigate the effect of the components following the TWPA by placing a 10-dB attenuator at its output, which decreases gain ripple,

resulting in a gain of (19.8 ± 1.5) dB in the 4 to 8 GHz band. All the sampled values between 3.6 and 8.3 GHz exceed 17 dB of gain (see Appendix G), emphasizing the importance of well-matched external circuitry.

We also characterize the phase-sensitive gain of the TWPA by measuring the signal gain at $f_s = f_p/2$ as a function of the phase difference between the signal and pump waves. The maximum ratio between amplification and deamplification, also called the phase-sensitive extinction ratio, is 58 dB (see Appendix H).

Figure 3(c) illustrates the power-handling capability of our TWPA by showing the input-referred 1-dB compression point P_{1dB} . Across the 4 to 8 GHz range, the device achieves an average saturation power of (-84 ± 2) dBm. To calibrate the absolute power at the TWPA input across the whole signal band, we use a wideband technique

(see Appendix J), which yields an average attenuation of (66 ± 2) dB in the line, close to the previously quoted attenuation of 69 dB at 5.5 GHz. Using $P_p \approx -56$ dBm calibrated in the same way, Eq. (6) predicts -82 dBm, in good agreement with the experimental value. Figure 3(d) illustrates the relationship between the average saturation power and the average gain. Increasing the pump power results in higher gain but simultaneously reduces the saturation power until the TWPA reaches its maximum gain. Increasing the pump power further does not lead to significantly higher gain, but it does lead to a slight increase in $P_{1\text{dB}}$.

IV. NOISE PERFORMANCE

The noise is characterized with a modified variant of the commonly used Y-factor method. A variable-temperature 50- Ω load terminating a superconducting coaxial cable connected to the TWPA input serves as a calibrated noise source (see Appendix C). For noise modeling and evaluation of the measured data, we use a two-mode model following the approach described in Refs. [35,60]. To minimize a systematic error in the evaluation, TPWAs should, in principle, be modeled as multimode amplifiers. Such a model not only accounts for the conversion of idler input noise to signal output noise but also for the noise at sideband frequencies [56,61]; however, experimentally quantifying the contribution of the sidebands is challenging because it requires absolute power calibration at the TWPA input for frequencies above the pump frequency. Here, we neglect the impact of sidebands on the output noise due to their strong suppression thanks to the multiperiodic capacitance variation (see Appendix L).

The two-mode noise model of the amplification chain used for fitting the experimental data is

$$N_{\text{out}}^s(T) = G_{\text{sys}} \left(N_{\text{in}}^s(T) + \frac{G_{\text{si}}}{G_{\text{ss}}} N_{\text{in}}^i(T) + N_{\text{sys,exc}} \right), \quad (8)$$

where the output noise is expressed as a number of photons $N_{\text{out}} = P_{\text{out}}/hf_s B$ at frequency f_s , and P_{out} is the noise power measured with a spectrum analyzer (SA) in the resolution bandwidth B . By varying the temperature T of the load, we change its Johnson-Nyquist noise $N_{\text{in}}^{s,i} = 0.5 \coth(hf_{s,i}/2k_B T)$, where k_B is the Boltzmann constant. Thus, we vary the noise incident to the TWPA at both the signal and idler frequencies. The conversion gain of the idler input noise to the signal output noise is expressed by G_{si} , which is ideally equal to the signal power gain $G_{\text{ss}} \gg 1$. Fitting Eq. (8) to the experimental data (P_{out}) yields the total system gain G_{sys} and system excess noise $N_{\text{sys,exc}}$, the latter of which describes the noise above the standard quantum limit of one photon.

Figure 3(e) shows the total system noise of our read-out chain using the TWPA as the first amplifier, setting the

gain asymmetry $G_{\text{si}}/G_{\text{ss}} = 1$ [see Eq. (8)]. Within the 4 to 8 GHz range, the total system noise is between 2.2 and 3.8 photons. To estimate the uncertainty of the system noise (blue shaded area) we take into account a power-level uncertainty of 0.1 dB in the SA, a temperature stability of ± 3 mK on top of the uncertainty provided by the calibration of the thermometer, and a gain asymmetry of ± 1 dB.

Figure 3(f) shows how the average system noise and signal-to-noise-ratio improvement $\Delta\text{SNR} = \text{SNR}_{\text{pump on}}/\text{SNR}_{\text{pump off}}$ change with the average TWPA gain. The operational point with the lowest system noise corresponds to the maximum ΔSNR , but this does not align with the TWPA's maximum gain. In addition, we compare the ΔSNR calculated using the results of the Y-factor measurement to the directly measured ΔSNR and find a root-mean-square (rms) deviation of less than 0.5 dB (see Appendix K). While ΔSNR is a relative measure and quantifies neither the system noise nor the TWPA's added noise, it serves as a practical metric for optimizing the TWPA's operation point, especially in measurement setups where no calibrated noise source is available. Therefore, for practical applications, TPWAs should be tuned for lowest noise performance rather than targeting the highest possible gain.

Having characterized the overall system noise, next, we disentangle its different contributions. Input-referred system noise includes contributions from losses in components before the TWPA, in the TWPA itself, and in components after it. To evaluate the contributions of the elements preceding the TWPA, we take into account that the noise source is separated from the TWPA input by a bias tee and a directional coupler (see Appendix C). From their insertion loss A measured separately at 4.2 K, we calculate the transmission efficiency $\eta_1 = 10^{-A/10}$. In addition, considering the noise of the elements after the TWPA, N_2 , and assuming vacuum noise $N_{\text{vac}} = 0.5$ at the TWPA input, from Eq. (8), we obtain the total system noise

$$N_{\text{sys}} = 2N_{\text{vac}} + 2\frac{1-\eta_1}{\eta_1}N_{\text{vac}} + \frac{N_{\text{T,exc}}}{\eta_1} + \frac{N_2}{\eta_1 G_{\text{ss}}} \quad (9)$$

for $G_{\text{si}}/G_{\text{ss}} = 1$. The first term, $2N_{\text{vac}} = 1$ photon, represents the standard quantum limit, while $N_{\text{T,exc}}$ captures the TWPA's excess noise above the quantum limit. The noise of elements following the TWPA, N_2 , is divided by the TWPA's gain G_{ss} . We measure N_2 using the Y-factor method, bypassing the TWPA and considering a single-mode model [Eq. (J1) in Appendix J].

All four contributions are depicted in Fig. 4 as a stacked plot. The total system noise is dominated by the TWPA's excess noise of 0.8 to 1.5 photons, which we attribute to losses in the SiO_x of the plate capacitors [22] and the generation of sidebands [61]. The former could be reduced by using a less dissipative dielectric, such as SiN_x or

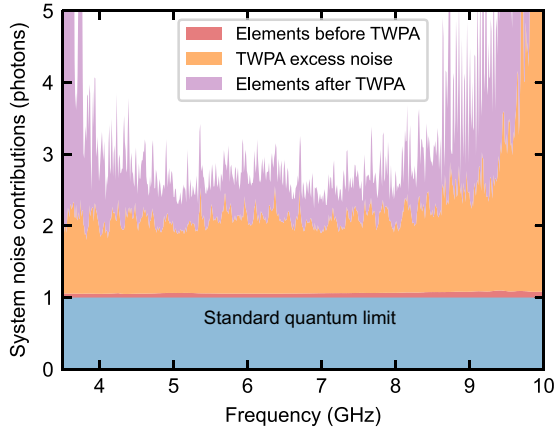


FIG. 4. Contributions to the total system noise as a function of frequency for a TWPA gain of 19 dB. The TWPA excess noise is between 0.8 and 1.5 added photons, which we attribute to dielectric loss in the SiO_x of plate capacitors to ground [22] and the generation of sidebands [61]. The added noise from the elements after the TWPA contributes 0.6 photons on average. The noise contribution of elements before the TWPA is less than 0.1 photons.

hydrogenated a-Si [62–64]. The noise contribution from elements after the TWPA, on average 0.6 photons, could be reduced by decreasing losses between the TWPA and the second amplifier, increasing TWPA gain without increasing its added noise [cf. Fig. 3(f)], or decreasing the added noise of the second amplifier. In our case, the contribution of the components before the TWPA is small (less than 0.1 photons), while this may not be the case for typical readout chains.

V. CONCLUSION

We designed, fabricated, and characterized a Josephson traveling-wave parametric amplifier based on rf-SQUIDs. To operate efficiently in the 3WM regime, we implemented dispersion engineering by multiperiodic capacitance variation, suppressing unwanted mixing processes while maintaining amplification over a wide frequency range. Our device attained an average gain of 20 dB from 3.5 to 8.5 GHz, thus exceeding one octave bandwidth. To also characterize saturation power and noise over that same frequency range, we applied wideband techniques based on a calibrated noise source. We demonstrated an average saturation power of -84 dBm, an order of magnitude larger than reported for JTWPAs [22–26]. This was achieved for a comparable number of nonlinear elements thanks to the more favorable trade-off between saturation power and device length when using rf-SQUIDs. We explicitly analyzed noise contributions from different circuit components, determining that our total system noise was dominated by the TWPA’s excess noise of 0.8 to 1.5 photons. While this was already on par with existing works, it could be straightforwardly reduced by decreasing the dielectric

loss in the parallel plate capacitors. Notably, we showed that the lowest system noise did not coincide with the TWPA’s maximum gain, emphasizing that TWPAs should be tuned to minimize the system noise instead of maximizing the gain. We also showed that the minimum of total system noise aligned with the maximum of signal-to-noise ratio improvement ΔSNR , validating the latter as a more practical tune-up approach for users.

In scaling up quantum computing, the high saturation power of our device should allow for either an increase in the number of qubits per readout line or a decrease in intermodulation distortion between frequency-multiplexed qubits through a larger overhead to the saturation power [46]. We believe that our device, with its wide bandwidth and high phase-sensitive extinction ratio of 58 dB, has great potential for single-mode and two-mode squeezing and the generation of broadband entangled microwave radiation [25,26,65,66], with applications in quantum sensing and continuous-variable quantum computing. We are convinced that rf-SQUID-based TWPAs can be further developed to be nonreciprocal devices featuring backward isolation [67,68]. This would potentially remove the need for bulky magnetic isolators between qubit devices and the TWPA, which, together with the demonstrated high saturation power, would aid the efforts to scale up superconducting-circuit-based quantum computers.

ACKNOWLEDGMENTS

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V.G. and C.K. contributed equally to this work. V.G. and C.K. designed the device and measured and analyzed the data, with input from L.G. V.G. fabricated the device. C.K. developed the experimental methods and theoretical models. V.G. and L.G. wrote the initial manuscript, which was refined by all authors. L.G. supervised the project.

DATA AVAILABILITY

The data that support the findings of this article are openly available [70].

APPENDIX A: GAIN SATURATION

The effect of pump depletion due to conversion of pump power to signal and idler power has been studied analytically for a 3WM-TWPA in the absence of phase mismatch and loss in Ref. [47]. The resulting signal power gain, depending on the signal input power P_s^{in} , is

$$G(P_s^{\text{in}}) = \frac{1}{\text{dn}^2(gN, k)}, \quad (\text{A1})$$

where $g = (1/4)|\beta|k_p a \phi_p \sqrt{1 - \delta^2}$ denotes the exponential gain coefficient for a dimensionless detuning $\delta = |f_s - f_p/2|/(f_p/2)$, and with the Jacobi elliptic function $\text{dn}(u, k)$ (see for example Ref. [71]) for the modulus

$$k = \frac{1}{\sqrt{1 + f_p P_s^{\text{in}}/f_s P_p^{\text{in}}}}, \quad (\text{A2})$$

where we use an upright symbol to distinguish the modulus k from the wave number k .

We propose the following approximation to Eq. (A1) to obtain a more illustrative expression for the signal-power dependence of the gain, where we assume large initial gain $G_0 = G(P_s^{\text{in}} \rightarrow 0) = \cosh^2 gN \gg 1$ and $k \approx 1$, i.e., $P_s^{\text{in}} \ll P_p^{\text{in}}$. According to Ref. [71], in this limit, the Jacobi elliptic function can be approximated to

$$\text{dn}(u, k) \approx \frac{1}{\cosh(u)} + \frac{k'^2}{4} \frac{\tanh(u)}{\cosh(u)} [\sinh(u) \cosh(u) + u], \quad (\text{A3})$$

where $k' := \sqrt{1 - k^2}$ is the complement modulus of k . For $\sinh(u) \cosh(u) \gg u$ and $\sinh^2(u) \approx \cosh^2(u)$, we can further simplify Eq. (A3) to

$$\text{dn}(u, k) \approx \frac{1}{\cosh(u)} \left[1 + \frac{k'^2}{4} \cosh^2(u) \right], \quad (\text{A4})$$

and find, after inserting Eq. (A4) into Eq. (A1) and replacing $\cosh^2 gN = G_0$,

$$G(P_s^{\text{in}}) \approx \frac{G_0}{1 + G_0 f_p P_s / 2 f_s P_p + (G_0 f_p P_s / 4 f_s P_p)^2}, \quad (\text{A5})$$

where powers denote input powers, omitting the superscript for brevity. Neglecting the quadratic term, and considering frequencies in the center of the gain band, $f_p \approx 2f_s$,

we obtain a simple expression for the gain saturation due to pump depletion,

$$G(P_s^{\text{in}}) \approx \frac{G_0}{1 + G_0 P_s / P_p}. \quad (\text{A6})$$

With this, the signal input power causing 1 dB gain compression, $G = 10^{-1/10} G_0$, can be found as

$$P_{\text{1dB}}^{\text{dBm}} \approx \frac{P_p}{4G_0} = P_p^{\text{dBm}} - G_0^{\text{dB}} - 6 \text{ dB}. \quad (\text{A7})$$

An expression similar to Eq. (A6) was derived in Ref. [57] for a 4WM optical-fiber TWPA,

$$G(P_s^{\text{in}}) = \frac{G_0}{1 + 2G_0 P_s / P_p}. \quad (\text{A8})$$

This equation covers pump depletion as well as signal-power-caused SPM and XPM. The resulting 1-dB compression point,

$$P_{\text{1dB}}^{\text{dBm}} \approx \frac{P_p}{8G_0} = P_p^{\text{dBm}} - G_0^{\text{dB}} - 9 \text{ dB}, \quad (\text{A9})$$

is 3 dB lower than that of a Kerr-free 3WM-TWPA. During the drafting of our manuscript, Le Gal *et al.* published a preprint dedicated to gain compression in 4WM TWPAs [72].

APPENDIX B: FABRICATION

The TWPA is fabricated at the cleanroom center of Physikalisch-Technische Bundesanstalt (PTB) using Nb trilayer technology based on the process outlined in Ref. [73]. On a high-resistivity ($\rho = 5 - 50 \text{ k}\Omega \text{ cm}$) 3-inch (76.2-mm) intrinsic Si wafer with 200 nm of thermal SiO_2 , we deposit 30 nm of Al_2O_3 , which serves as an etch-stop layer. This is followed by *in situ* deposition of a Nb/Al- AlO_x /Nb trilayer using dc magnetron sputtering. The base and counter Nb layers have thicknesses of 160 and 140 nm, while the Al layer is 10 nm thick. To achieve a critical current density of $j_c = 130 \text{ A/cm}^2$, we oxidize the Al layer at 160 mbar of pure oxygen for 32 min, including 12 min required to reach the target pressure in the chamber. After deposition of the trilayer, the wafer is covered with a 50-nm-thick SiO_x layer deposited using plasma-enhanced chemical vapor deposition (PECVD) to protect the Josephson junctions from anodic oxidation later in the process.

The Josephson junctions are patterned using electron-beam lithography. After development of the negative resist, we perform two sequential dry etching steps: reactive ion etching (RIE) with a CHF_3/Ar mixture to remove the SiO_x layer, followed by inductively coupled plasma reactive ion etching (ICP-RIE) in an SF_6 plasma to etch

Nb. The Al-AlO_x layer is wet etched in a solution of 65% HNO₃, 100% CH₃COOH, 85% H₃PO₄ and H₂O, mixed at a ratio of 1:1:16:2, as described in Ref. [74]. Following the etching steps, we anodize the base Nb layer. In the second lithography step, the base Nb electrode is patterned and etched using the same ICP-RIE process as for the counter electrode.

The entire wafer is then covered with 250 nm of PECVD-deposited SiO_x, accounting for approximately 10 nm loss during Ar precleaning of the wafer before the deposition of the wiring Nb layer. To enable contact between the Josephson junctions and the wiring layer, we etch windows into the SiO_x down to the counter electrode of the Josephson junctions. These windows are etched using ICP-RIE in a CHF₃ plasma.

The final step in fabrication is the deposition of the 400-nm-thick Nb wiring layer. After the exposure and development of the resist, the wiring layer is patterned using RIE with a SF₆/O₂ mixture at a process pressure of 20 Pa with the substrate temperature maintained at 40 °C. These etching parameters help prevent the formation of parasitic conductive structures at the edges of etched features sometimes referred to as fences.

After fabrication, we measure the tunnel resistances at room temperature R_t of 112 junctions incorporated into a cross-bridge Kelvin resistor structure as a proxy for the critical currents of the Josephson junctions [75]. This provides insight into the spread of the critical currents, which can be quantified as a coefficient of variation $\sigma(I_c)/\langle I_c \rangle$. For the wafer of the TWPA presented in the main text, this variation is unexpectedly large, exceeding 30%. Thanks to our design concept, the device performance is robust even against unusually large I_c spread [76], and it still exhibits a gain of 20 dB.

APPENDIX C: MEASUREMENT SETUP

In Fig. 5, we present a schematic of the experimental setup used in this work. The pump tone is coupled to a signal tone at room temperature. The signal tone is generated either by a vector network analyzer (VNA) for gain measurements or by a signal generator for signal-to-noise ratio (SNR) measurements. To avoid ground loops, we use inner and outer dc blocks at the input and the output, as shown in Fig. 5. The TWPA is protected against photons with sufficient energy for pair breaking with low-insertion-loss infrared filters [69] and a light-tight enclosure. The dc-current source is connected to the TWPA via bias tees and Thermocoax cables, the latter of which provide lowpass filtering (LPF) with a cutoff frequency of around 1 MHz and a stopband extending to the range [77]. The noise source consists of a 50 Ω load, a heater, and an individually calibrated ruthenium oxide temperature sensor, thermally anchored to a variable-temperature platform. The load terminates a superconducting coaxial cable made of NbTi,

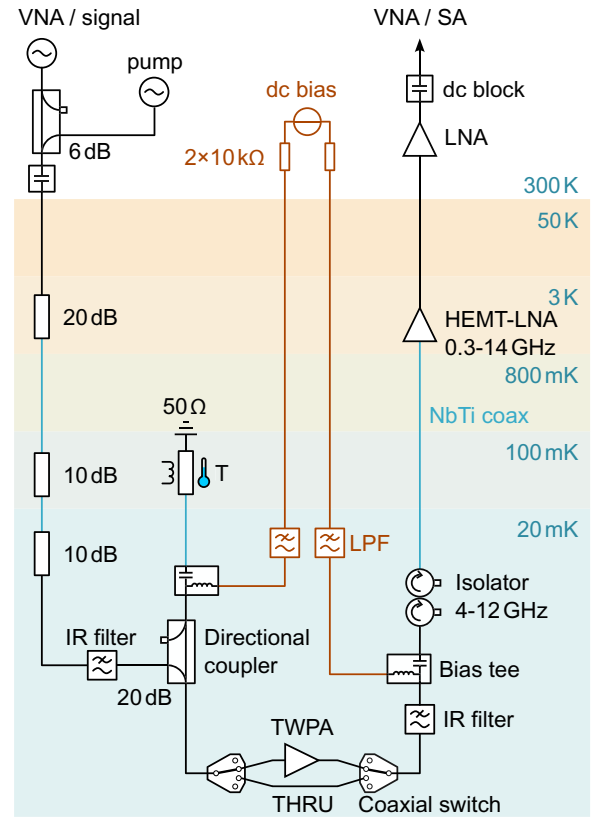


FIG. 5. Schematic of the experimental setup. Blue lines denote superconducting NbTi coaxial cables. Brown lines denote Thermocoax[®] cables serving as a lowpass filter for current biasing of our device. The TWPA is surrounded by a mu-metal shield, which protects it from external magnetic fields.

which thermally isolates the noise source from the rest of the setup. An SP6T coaxial switch allows for *in situ* VNA forward calibration and enables system noise (N_2) measurements of the chain with the TWPA bypassed (see main text).

APPENDIX D: MODEL OF THE rf-SQUID IMPLEMENTATION

To account for deviations between the idealized model [Fig. 1(a)] and the practical implementation of the TWPA unit cell, additional circuit elements are introduced to the model. Figure 6(a) shows a false-colored SEM image of the rf-SQUID with the corresponding circuit schematic [Fig. 1(b)]. The branch with the Josephson junction contains a parasitic linear inductance L_p . Apart from that, in our implementation, we use a Josephson junction as a via, connecting the base Nb with the counter Nb layer. This junction is 40 times larger than the Josephson junction with critical current I_c . While the latter dominates the nonlinear behavior of the rf-SQUID, the former merely adds quasilinear inductance $L_{J0,L} = \varphi_0/I_{c,L}$, with $I_{c,L} \approx 40I_c$. All the inductances contribute to the loop inductance,

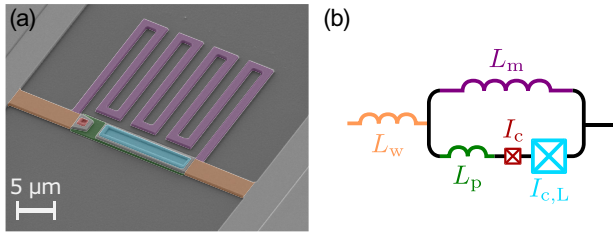


FIG. 6. Practical implementation of the rf-SQUID. (a) False-colored scanning electron microscopy image of an rf-SQUID [colors according to panel (b)]. (b) Corresponding circuit schematic, containing wire inductance L_w , meander inductance L_m , parasitic inductance L_p , critical current I_c , and critical current of the large junction $I_{c,L} \approx 40I_c$. The large junction serves as a via connecting the two metal layers.

modifying the formula for the screening parameter,

$$\beta_L = \frac{I_c L_{\text{loop}}}{\varphi_0} = \frac{(L_m + L_p + L_{J0,L})}{\varphi_0}. \quad (\text{D1})$$

and the total rf-SQUID inductance,

$$L_{\text{SQ}} = L_m \frac{1 + \alpha_p \beta_L \cos \phi_{\text{dc}}}{1 + \beta_L \cos \phi_{\text{dc}}}, \quad (\text{D2})$$

introducing the coefficient $\alpha_p = (L_p + L_{J0,L}) / (L_p + L_{J0,L} + L_m)$, which describes the ratio of parasitic inductance to the total loop inductance. Further, two neighboring rf-SQUIDs are connected to each other with a wire having inductance L_w , contributing to the unit-cell inductance $L_{\text{cell}} = L_w + L_{\text{SQ}}$.

Fitting $L_{\text{cell}}(\phi_{\text{dc}})$ to experimental data [shown in Fig. 2(d)] yields the values $L_m = 58.6$ pH, $L_w = 37.0$ pH, $L_p + L_{J0,L} = 8.9$ pH, and $I_c = 0.93$ μA . Furthermore, we determined the specific capacitance by measurements on test structures at 4.2 K. As a result, at $\phi_{\text{dc}} = \pi/2$, the characteristic impedance of the TWPA transmission line is 52 Ω .

See Ref. [56] for further details of the model for the rf-SQUID implementation used here.

APPENDIX E: POWER-DEPENDENT INDUCTANCE OF THE rf-SQUID

As the power of propagating waves in the TWPA increases, the rf-SQUID behavior deviates from the small-signal case. This manifests in the S_{21} spectra [see Figs. 2(a)–2(c)] as power-dependent frequency shifts of the stopband due to the power dependence of the rf-SQUID inductance. We use this effect for power calibration by aligning experimental data with simulation data.

1. Experiment

To extract the rf-SQUID inductance from experimental data, we measure the phase of S_{21} at 5.5 GHz, where the

second-harmonic generation is suppressed due to the stopband, which would otherwise cause an additional phase shift. From the phase data, we calculate the wave number

$$k = -\frac{\arg(S_{21})}{aN} \quad (\text{E1})$$

with the number of cells N , assuming $|S_{11}| \ll |S_{21}|$. When the power is increased from P_0 to P , the wave number changes by

$$\delta k = -\frac{\arg(S_{21}(P)) - \arg(S_{21}(P_0))}{aN}. \quad (\text{E2})$$

The cell inductance $L_{\text{cell}}(P_0)$ is extracted from the fit of a CTM model to S_{21} [see Fig. 2(e)]. At frequencies far below the first gap, the chromatic dispersion of the TWPA is small, so k is approximately linear and can be expressed via the phase velocity v_{ph} as

$$k_{\text{lin}}(P) = \frac{\omega}{v_{\text{ph}}} = \omega a^{-1} \sqrt{L_{\text{cell}}(P) \bar{C}_g}, \quad (\text{E3})$$

where $\bar{C}_g$ denotes the average ground capacitance. Rearranging Eq. (E3) and inserting $k_{\text{lin}}(P) = k_{\text{lin}}(P_0) + \delta k$ then yields

$$L_{\text{cell}}(P) = \frac{1}{\bar{C}_g} \left(\frac{k_{\text{lin}}(P_0) + \delta k(P)}{\omega a^{-1}} \right)^2. \quad (\text{E4})$$

2. Simulation

To obtain the power-dependent inductance of the rf-SQUID model shown in Fig. 6(b), we carry out WRSPICE simulations of the model, using the circuit parameters from Appendix D. The impedance of the rf-SQUID, biased at I_{dc} and driven by a complex ac current amplitude I_{ac} , can be found as

$$Z_{\text{cell}} = R_{\text{cell}} + i\omega L_{\text{cell}} = V_{\text{SQ}}/I_{\text{ac}}, \quad (\text{E5})$$

where V_{SQ} is the complex voltage drop across the SQUID. The resulting SQUID inductance is

$$L_{\text{cell}}(I_{\text{ac}}) = \frac{\text{Im} Z_{\text{cell}}(I_{\text{ac}})}{\omega}. \quad (\text{E6})$$

In the TWPA, the current I_{ac} flowing through the rf-SQUID corresponds to an incident power $P \approx 0.5|I_{\text{ac}}|^2 Z(\phi_{\text{dc}}) \approx 0.5|I_{\text{ac}}|^2 Z_0$ to the TWPA for frequencies much lower than the cutoff frequency of the TWPA transmission line [56].

As stated in the main text, using this method, we extract an input-line attenuation of 69 dB, which is 3 dB more than from the power calibration using the HEMT-LNA noise measurement (see Appendix J). We attribute the observed discrepancy to the fact that the experimentally determined

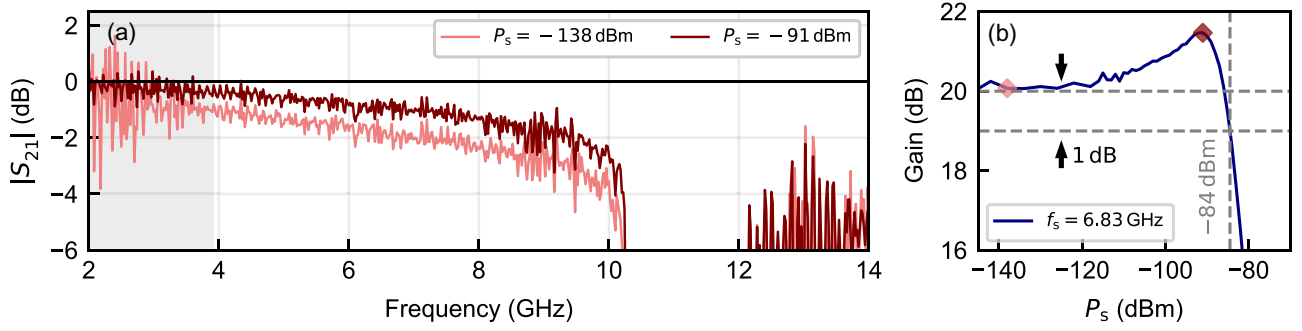


FIG. 7. Linear response of the TWPA and gain dependence on the signal input power. (a) S_{21} transmission of the unpumped TWPA at 20 mK for two different signal powers. The gray shaded region indicates frequencies where the isolator is out of band. The transmission dip at frequencies from 10.2 to 12 GHz is the designed stopband. We attribute the loss to two-level states (TLSs) [78], which saturate at higher powers. (b) Power gain as a function of the input signal power at signal frequency of 6.83 GHz. We attribute the increase of gain values to a gradual saturation of TLS loss. The two diamonds correspond to the signal powers shown in panel (a) with the same colors. Throughout the paper and, in particular, for the definition of the 1-dB compression point, we consider the gain at low powers.

rf-SQUID inductance $L_{\text{cell}}(P)$ should be seen as an average inductance of all unit cells of the TWPA. Moreover, accounting for the insertion loss of the TWPA, the input power is higher than the average power inside the TWPA. Ignoring the insertion loss therefore causes an underestimation of the input power, or an overestimation of the attenuation of the input line. Further, we note that the experimental extraction of the rf-SQUID circuit parameters involves two fits, which are additional sources of uncertainty.

APPENDIX F: POWER-DEPENDENT TRANSMISSION LOSS

Figure 7(a) shows the normalized S_{21} transmission of the TWPA, biased at $\Phi_{\text{ext}} \approx 0.33\Phi_0$ without a pump, measured at two different probe powers. The insertion loss of the device decreases by up to 1.5 dB at 9.8 GHz when increasing the probe power to -91 dBm. We attribute this effect to the saturation of TLS loss [62,78]. This power-dependent change of insertion loss impacts gain measurements and must be accounted for.

Figure 7(b) shows a typical saturation power measurement. Before the gain begins to drop due to pump depletion, we observe an increase by 1–2 dB, an effect that has been observed by other groups and is attributed to TLS saturation, among other effects [23]. For typical qubit read-out applications, powers are often below -120 dBm. To account for such applications, all gains reported in this paper are measured at powers below -120 dBm. Similarly, for saturation-power measurements, we take the gain value in the flat, low-power regime as a reference.

Note that we use the conventional definition for gain saturation as the signal power at which the gain decreases by 1 dB; however, the maximal signal power for which the amplifier operates linearly (e.g., the gain is constant within

± 1 dB) can be smaller than the saturation power, when the gain increases by more than 1 dB before it drops.

APPENDIX G: GAIN RIPPLE

To reduce gain ripple, reflections inside and outside the TWPA have to be decreased. These reflections originate from multiple impedance mismatches, including the TWPA transmission line itself, stopband sidelobes resulting from our dispersion engineering technique [48], TWPA

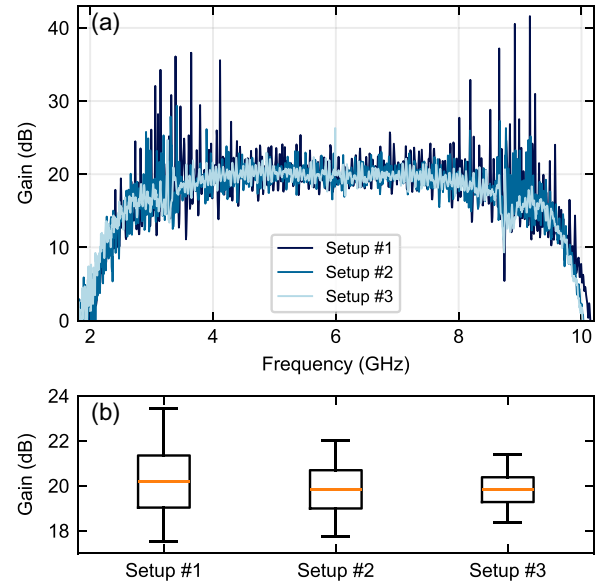


FIG. 8. Gain profiles of the TWPA using different peripheral components. (a) Each gain profile is obtained after tuning the TWPA to have the smallest gain ripple, while having an average gain of 20 dB. (b) Boxplots showing amplifier gain distribution for frequencies from 4 to 8 GHz for different setups. Whiskers (boxes) represent the 5th (25th) and 95th (75th) percentiles. The orange line indicates the median value.

packaging elements such as bond wires, and external circuitry. It is crucial to reduce reflections not only within the signal band but also at the pump frequency. Reflections at the pump frequency lead to backward gain, which further increases gain ripple. Since pump frequencies are at the edge of the rated bandwidth of the isolator in our setup, reflections of the pump wave are expected.

Figure 8(a) shows the TWPA gain profile measured in three cooldowns with different experimental setups. Each setup variant has different components that influence ripples of the resulting gain profile. In all cases, the TWPA is tuned to minimize gain ripple while providing an average gain of 20 dB. The boxplots in Fig. 8(b) illustrate the distribution of the gain values across the 4 to 8 GHz band. In Setup #1, 90% of the gain values fall within the range $20.3^{+3.2}_{-2.7}$ dB. In Setup #2, the SMA bulkheads of the TWPA enclosure were replaced with better-impedance-matched components, and this setup is used for the results reported in the main text. Here, 90% of the gain values fall within the range (19.8 ± 2.4) dB. To investigate the effect of reduced reflections, we use a 10-dB attenuator at the TWPA output in Setup #3. This modification reduced the spread of gain values to (19.8 ± 1.5) dB. Although this setup is not suitable for readout applications due to degraded SNR, it shows that using better-matched external circuitry has the potential to significantly reduce gain ripple. The same applies to components before the TWPA.

APPENDIX H: PHASE-SENSITIVE PARAMETRIC AMPLIFICATION

A special case in parametric amplification occurs when the signal and idler frequencies coincide, $f_s = f_i$, also called degenerate parametric amplification. In this regime, parametric amplification is phase sensitive, i.e., the signal gain depends on the phase of the signal wave relative to that of the pump wave, while for other frequencies, the gain is independent of phase (phase preserving). Phase-sensitive gain enables interesting effects such as single-mode squeezing and amplification with noise below the standard quantum limit [25,26].

In a 3WM TWPA, phase-sensitive parametric amplification occurs at $f_s = f_i = f_p/2$, in the center of the gain profile. This is in contrast to (single-pump) 4WM TWPAs, where it coincides with the pump frequency, $f_s = f_i = f_p$, and is therefore not accessible. To measure the dependence of the gain on the signal phase, we vary the phase of the signal generator relative to that of the pump generator. The signal and pump generators and the SA are synchronized. The phase-sensitive gain is measured at $f_s = f_p/2$, while the phase-preserving gain is measured at a frequency detuned by 10 kHz from $f_p/2$.

Figure 9 shows the phase-sensitive signal gain in comparison to the phase-preserving gain, both in dependence on the signal phase. While the phase-preserving gain does

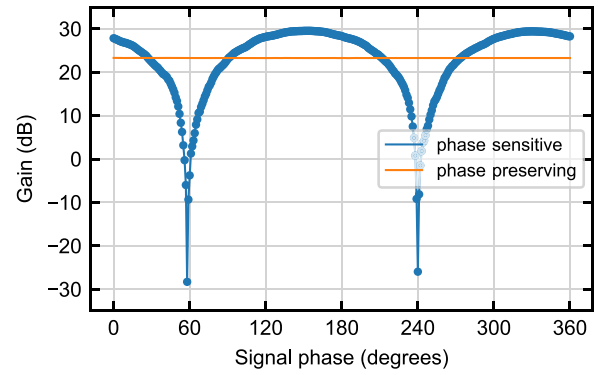


FIG. 9. Phase-sensitive and phase-preserving parametric amplification. The blue line shows the signal gain measured at $f_s = f_i = f_p/2$, while for the orange line, the signal frequency is detuned by 10 kHz from that condition. The phase of the signal generator is varied relative to that of the pump generator. The phase-sensitive gain ranges across 58 dB between maximum amplification and maximum deamplification. The maximum phase-sensitive gain is ca. 6 dB higher than the phase-preserving gain. Both gains are measured as the ratio of the signal transmission with the pump on and off.

not show any dependence on the signal phase, the phase-sensitive gain varies periodically between amplification and deamplification. The maximum amplification is ca. 6 dB above the phase-preserving gain due to the constructive interference of equal-amplitude signal and idler waves, and the maximum deamplification reaches more than 25 dB. The ratio of the two, also referred to as the phase-sensitive extinction ratio, is 58 dB, which compares to 48 dB reported for a resonator-based Josephson parametric amplifier [79] and 56 dB demonstrated with the dual-pump 4WM TWPA reported in Ref. [26].

We further observed pump subharmonic generation, appearing as a peak at $f_p/2$ in the output spectrum when no signal was applied to the TWPA. This effect is also known as half-tone generation or oscillation period doubling [80,81]. Subharmonic generation of the pump was not observed for the 4WM operating points shown in Fig. 10.

APPENDIX I: FOUR-WAVE-MIXING REGIME

Our rf-SQUID-based TWPA can also be operated in the 4WM regime by biasing it appropriately. In this case, amplification is enabled by the Kerr nonlinearity and is governed by the phase-matching condition $2k_p = k_s + k_i$ along with the energy-conservation law $2f_p = f_s + f_i$. Taking into account rf-SQUID inductance values of ca. 86–108 pH and an average capacitance of 36 fF determined from test-structure measurements, we obtain an average characteristic impedance ranging from 49 to 55 Ω . Therefore, the low-flux tunability of the inductance leads to reasonable impedance matching for all flux-bias points,

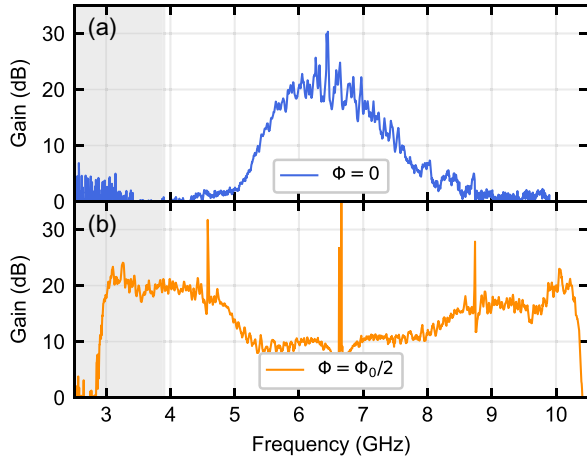


FIG. 10. 4WM gain profiles of the TWPA. (a) At a flux bias of $\Phi_{\text{ext}} = 0$, the second-order nonlinearity vanishes and Kerr nonlinearity is used for amplification. With the pump at $f_p = 10.56$ GHz and $P_p \approx -54$ dBm, phase matching is achieved around 6.5 GHz, and the device provides gain of ca. 20 dB. A second gain lobe is located around 17 GHz (not shown). (b) At a flux bias of $\Phi_{\text{ext}} = \Phi_0/2$, the Kerr-nonlinearity coefficient is negative and the gain profile has an M shape similar to Ref. [24]; the pump parameters are $f_p = 6.66$ GHz and $P_p \approx -56$ dBm. Two sharp peaks are caused by a resonance in the sample package at 8.8 GHz, which additionally creates an image peak at 4.52 GHz.

probably limited by the sidelobes of the stopband [48] and not by the flux-dependent average impedance of the transmission line.

Figure 10 shows the gain profiles at two flux-bias points, where the Kerr nonlinearity is maximized in each case. In Fig. 10(a), the flux bias is $\Phi_{\text{ext}} = 0$, and the corresponding Kerr-nonlinearity coefficient γ is positive. Setting the pump to 10.56 GHz, close to the lower edge of the stopband, phase matching is achieved, similarly to Refs. [22,23,27,33]. The device exhibits an average gain of 20 dB from 6 to 7 GHz. A second gain band lies at approximately 17 GHz, which is outside of the isolator bandwidth and is not measured.

Figure 10(b) shows the gain profile at the flux bias $\Phi_{\text{ext}} = \Phi_0/2$, where the Kerr-nonlinearity coefficient is negative. Phase matching can be achieved when the phase shifts caused by SPM and XPM compensate the chromatic dispersion, as used in Ref. [24]. An M-shaped gain profile is observed. The positions of the two gain lobes can be tuned via the pump frequency across several GHz.

The measurements were performed with a 10-dB attenuator at the TWPA output to avoid potential reflections caused by the impedance mismatch (see Appendix G). For appropriate calibration, the through line used for calibration was also equipped with a 10-dB attenuator of the same type.

APPENDIX J: WIDEBAND POWER CALIBRATION

Accurately determining the saturation power of parametric amplifiers operating at mK temperatures is a challenging task and requires power calibration at a cryogenic reference plane. To address this challenge, we leverage the noise measurements of the amplifier chain without the TWPA, where the HEMT-LNA is the first amplifier in the chain. The single-mode model of the output noise of the amplification chain is

$$N_{\text{out}}(T) = \eta_1 G_2 \left(N_{\text{in}}(T) + \frac{(1 - \eta_1)}{\eta_1} N_{\text{vac}} + \frac{N_2}{\eta_1} \right). \quad (\text{J1})$$

Fitting Eq. (J1) to the experimental data, we obtain the gain G_2 between the output of the TWPA and the output of the chain over a broad frequency range [purple curve in Fig. 11(a)]. Importantly, to correctly measure the noise power with the SA, an rms detector must be used, as other detector settings may introduce offsets in the displayed noise power level. While these offsets do not affect the fitting result N_2 , they lead to an incorrect system gain G_2 by the same offset.

In addition, we measure the roundtrip transmission $S_{21, \text{roundtrip}}$ of the setup in the same frequency range. For this measurement, the VNA is calibrated to have a reference plane at its own ports. Combining these two measurements, we can estimate the insertion loss of the input line as

$$IL^{\text{dB}} = G_2^{\text{dB}} - S_{21, \text{roundtrip}}^{\text{dB}} \quad (\text{J2})$$

across the whole signal frequency range. Repeating this procedure over several cooldowns and with different HEMT-LNAs, we found the extracted input-line insertion loss to be consistent within less than 2 dB.

APPENDIX K: Y-FACTOR METHOD AND SNR IMPROVEMENT

In this section, we demonstrate that ΔSNR , measured directly, can be independently reproduced using the results of the system noise measurements obtained with the modified Y-factor method. This serves as a consistency check for our noise characterization.

1. Y factor

Figures 11(a) and 11(b) show the system gain and noise of the TWPA-based amplifier chain, G_{sys} and N_{sys} (teal curves), as well as the gain and noise of the elements after the TWPA, G_2 and N_2 (purple curves). From these quantities, the TWPA gain can be calculated as $G_{\text{ss}} = G_{\text{sys}}/\eta_1 G_2$, shown in Fig. 11(a) as a gray curve. This is consistent with the corresponding VNA measurement of the TWPA gain (gold curve) within an rms error of 0.6 dB in the 4 to 8 GHz range.

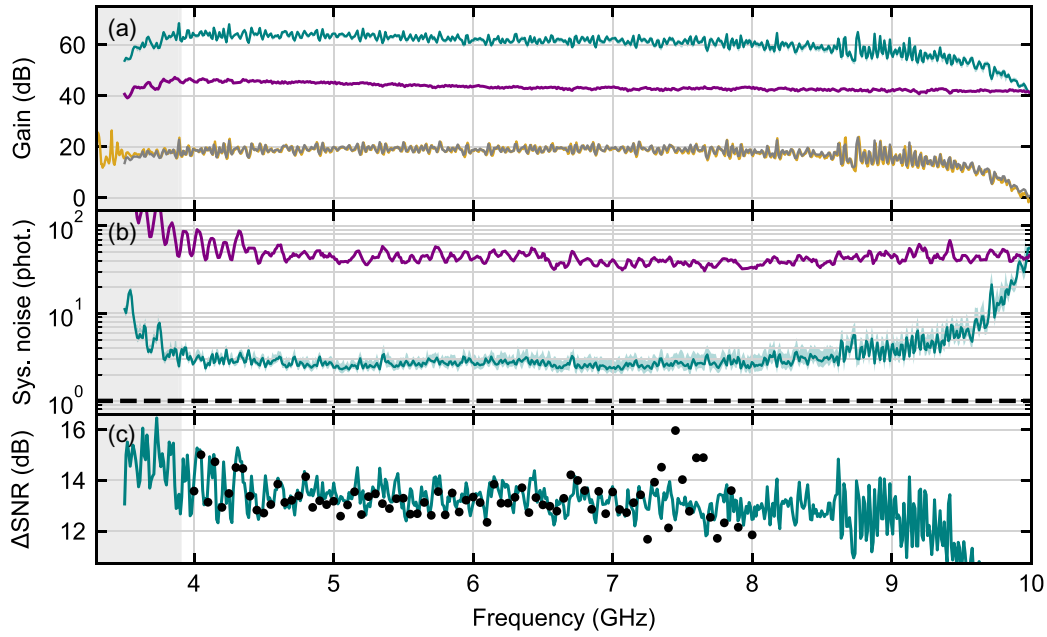


FIG. 11. System gain, system noise, and SNR improvement. (a) The teal and purple curves denote the system gain of the amplifier chains including the TWPA (G_{sys}) and bypassing it (G_2), respectively. The difference between the two curves results in the TWPA gain profile (gray curve), which agrees with the one measured with the VNA, showing an rms error of 0.6 dB (gold curve, cf. Fig. 3). (b) Comparison of the total system noise for two amplifier chains: one including the TWPA (teal) and one bypassing it (purple). (c) The teal curve shows ΔSNR analytically obtained from the total system noise measurements. Scatter dots indicate directly measured ΔSNR . From 4 to 7 GHz, the rms error is 0.5 dB, with a relative error of approximately 4%. The deviation between the measurements and analytical predictions above 7 GHz is under investigation.

2. SNR improvement

To calculate ΔSNR at a frequency f , one needs to know the TWPA's added noise $N_T = N_{\text{vac}} + N_{T,\text{exc}}$, power gain G_{ss} , and transmission efficiency $\eta_T = 10^{S_{21}^{\text{dB}}/10}$ (which accounts for signal attenuation by the TWPA when the pump is off), and the total system noise of the chain following the TWPA, N_2 . With these quantities, the SNR improvement can be found as

$$\Delta\text{SNR} = \frac{N_{\text{in}} + \frac{1-\eta_T}{\eta_T} N_{\text{vac}} + \frac{N_2}{\eta_T}}{N_{\text{in}} + N_T + \frac{N_2}{G_{\text{ss}}}}. \quad (\text{K1})$$

Figure 11(c) compares ΔSNR obtained using two different methods. The teal curve represents the calculated ΔSNR using Eq. (K1) with the results of the system noise measurements. The scatter dots show the ΔSNR obtained with the SA. To obtain the SNR, we send a probe tone from the signal generator at an incident power of approximately -124 dBm and measure the output spectrum with and without the pump. The parameters of the SA are a resolution bandwidth of 100 Hz, a video bandwidth of 1 Hz, and a span of 20 kHz. For the signal power, we take the maximum value in the measured span, while the noise power is given by the average of points outside the peak. The SNR improvement is defined as $\Delta\text{SNR} = \text{SNR}_{\text{pump on}}/\text{SNR}_{\text{pump off}}$.

From 4 to 7 GHz, we find an rms error of 0.5 dB, reinforcing the reliability of our measurement. We emphasize that it is not possible to obtain TWPA added noise from SNR measurements alone without additional measurements.

APPENDIX L: SIDEBAND-INDUCED EXCESS NOISE

Due to its broadband behavior, a TWPA converts not only the input noise at the idler frequency but also the noise at higher sideband frequencies into output signal noise, ultimately leading to an added noise above the quantum limit. Furthermore, noise incident on the output of the TWPA can convert into output signal noise, either via parametric forward-backward coupling or via reflections and subsequent parametric forward-forward coupling [61]. To quantify these contributions to the added noise of the TWPA, we adopt the approach outlined in Ref. [82], which we implement using WRSPICE simulations of a circuit model of the TWPA.

The input noise N_{in}^m at frequency f_m , $m \in \{s, i, p+s, p+i, 2p+s, \dots\}$, is converted to signal output noise $N_{\text{out}}^s = G_{s,m} N_{\text{in}}^m$ via the corresponding photon-conversion gain $G_{s,m}$. Similarly, noise at frequency f_m incident to the output of the TWPA is converted to signal output noise via the backward-forward photon-conversion gain $G_{s,m}^{\leftarrow}$. To

obtain these conversion gains, the input (output) is driven at frequency f_m , and the output response at f_s is measured:

$$G_{s,m} = \frac{|b_{2,s}|^2 f_m}{|a_{1,m}|^2 f_s}, \quad (\text{L1})$$

$$G_{s,m}^{\leftarrow} = \frac{|b_{2,s}|^2 f_m}{|a_{2,m}|^2 f_s}, \quad (\text{L2})$$

where $a_{q,j}$ and $b_{q,j}$ are the incident and outgoing power-wave quantities at port q and frequency f_j . The coefficient f_m/f_s accounts for different photon energies. With the noise $N_{\text{in},q}^m$ incident to the TWPA at port q and the contribution of internal loss $N_{\text{exc},\text{loss}}$, the output signal noise is

$$N_{\text{out}}^s = \sum_m G_{s,m} N_{\text{in},1}^m + \sum_m G_{s,m}^{\leftarrow} N_{\text{in},2}^m + G_{s,s} N_{\text{exc},\text{loss}} \quad (\text{L3})$$

$$= G_{s,s} \left(N_{\text{in}}^s + \frac{G_{s,i}}{G_{s,s}} N_{\text{in}}^i + N_{\text{exc,sideband}} + N_{\text{exc,loss}} \right), \quad (\text{L4})$$

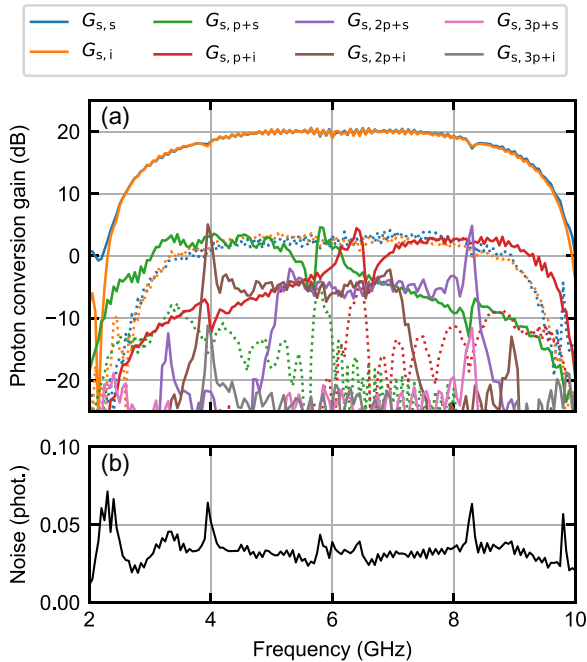


FIG. 12. Simulated conversion gains and sideband-induced added noise. (a) Photon-energy-normalized conversion gains quantifying the conversion of incident noise at frequency f_m ($m \in \{s, i, p+s, \dots\}$) to signal output noise at f_s . Solid (dotted) lines indicate forward-forward (backward-forward) coupling, i.e., the tone f_m is incident to the input (output) of the TWPA. (b) Contribution of the sidebands to the added noise of the TWPA, comprising noise from forward-forward-coupled higher sideband frequencies $f_{p+s}, f_{p+i}, f_{2p+s}, \dots$, as well as from all backward-forward couplings, up to the third order of the pump, and considering vacuum noise incident to both the input and output of the TWPA.

[cf. Eqs. (8) and (9), where $N_{T,\text{exc}} = N_{\text{exc,sideband}} + N_{\text{exc,loss}}$]. Assuming the TWPA is in a well-thermalized environment, and $k_B T \ll \hbar f_m$, the noise incident to the TWPA port q is $N_{\text{in},q}^m = 1/2$ photon. Then, the sideband-induced excess noise (including forward-backward coupling) is

$$N_{\text{exc,sideband}} = \frac{1}{2} \left(\sum_{m \in \{p+s, p+i, 2p+s, \dots\}} G_{s,m} + \sum_{m \in \{s, i, p+s, \dots\}} G_{s,m}^{\leftarrow} \right). \quad (\text{L5})$$

Figure 12(a) shows the simulated photon-conversion gains up to third order. The simulation suggests a gain asymmetry $G_{si}/G_{ss} < \pm 0.3$ dB in the range 3 to 9 GHz. Remarkably, all conversion gains of higher sidebands and backward-forward-coupled modes are at least 10 to 15 dB below the signal power gain G_{ss} , indicating that they are strongly suppressed due to the multiperiodic capacitance variation. The corresponding sideband excess noise $N_{\text{exc,sideband}}$ [Fig. 12(b)] remains well below 0.1 photon across the frequency range. This suggests that the contributions of sideband generation and backward-forward coupling to the added noise of the TWPA are rather small compared to those from internal loss. The peaks in the sideband-induced excess noise, accompanied by slight dips in the signal gain, can be attributed to favorable sideband-generation conditions in the vicinity of higher-order stopbands. Although this effect is not very pronounced, it is undesirable and could, in principle, be mitigated by using a sinusoidal rather than a discrete multiperiodic capacitance profile.

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