

Resource-state generation for a multispin register in a hybrid matter-photon quantum information processor

Yu Liu^{*} and Martin B. Plenio[†]

Institut für Theoretische Physik und IQST, Universität Ulm, Albert-Einstein-Allee 11, D-89081 Ulm, Germany



(Received 6 September 2024; revised 3 December 2025; accepted 7 January 2026; published 2 February 2026)

Hybrid quantum architectures that integrate matter and photonic degrees of freedom present a promising pathway toward scalable fault-tolerant quantum computing. This approach needs to combine well-established entangling operations between distant registers using photonic degrees of freedom with direct interactions between matter qubits within a solid-state register. The high-fidelity control of such a register, however, poses significant challenges. In this work, we address these challenges with pulsed control sequences that modulate all interspin interactions to preserve the nearest-neighbor couplings while eliminating unwanted long-range interactions. We derive pulse sequences, including broadband and selective gates, using composite-pulse and shaped-pulse techniques as well as optimal-control methods. This ensures a general pulse sequence in the presence of spin-position bias, robustness against static offset detunings, and Rabi-frequency fluctuations of the control fields. The control techniques developed here apply well beyond the present setting to a broad range of physical platforms. We demonstrate the efficacy of our methods for the resource-state generation for fusion-based quantum computing in four- and six-spin systems encoded in the electronic ground states of nitrogen-vacancy centers or other molecular solid-state qubits. We also outline other elements of the proposed architecture, highlighting its potential for advancing quantum computing technology.

DOI: [10.1103/185z-nxt1](https://doi.org/10.1103/185z-nxt1)

I. INTRODUCTION

Quantum computation is formulated in terms of either the gate-array model or the measurement-based model. In the gate-array model, one- and two-qubit quantum gates, described as unitary operators, are used to build up a desired state transformation on a multiqubit device [1,2]. In contrast, in the measurement-based gate implementations [3,4], Bell states are used to enable the synthesis of quantum gates based on local operations. More generally, a multipartite entangled resource state, such as the cluster state, can be prepared [5–7] and then generic quantum computation can be realized via single-qubit measurements and single-qubit rotations [6–9].

The measurement-based approach is particularly attractive for optical quantum information processing, where a universal quantum gate set is difficult to implement directly, as this would require large nonlinearities. Photon detection, however, is fast, readily available, and in

fact essential, as photons have short lifetimes in integrated photonic structures and need to be measured before they are absorbed. A detailed measurement-based approach to optical quantum computing was first formulated in Ref. [10] and later refined into more resource-efficient designs within the language of the measurement-based approach based on cluster states [6,11,12]. However, the preparation of a large cluster state in advance is challenging due to environmental noise and the limitations of current control techniques.

Recently, as an alternative, fusion-based quantum computation (FBQC) [13,14], a highly modular architecture, has been formulated. Here, only small graph states (the so-called four- and six-ring states) are required as resource states, which are then connected by entangling measurements [11,15] between pairs of such resource states in a fusion network “on the fly,” while simultaneously propagating the quantum computation. This all-optical approach requires highly multiplexed photonic down-conversion sources and fast optical switches to produce the required four- and six-ring resource states quasideterministically. Multiplexed sources, switches, and photon detectors need to be operated in a cryogenic environment to achieve high photon-detection efficiency [16].

These daunting requirements have motivated the search for alternative deterministic photon sources, e.g., based on quantum dots [17–20], where the internal spin degree of

^{*}Contact author: yu.liu@uni-ulm.de

[†]Contact author: martin.plenio@uni-ulm.de

Published by the American Physical Society under the terms of the [Creative Commons Attribution 4.0 International](https://creativecommons.org/licenses/by/4.0/) license. Further distribution of this work must maintain attribution to the author(s) and the published article's title, journal citation, and DOI.

freedom can be used to control the generation of entangled photon states. The use of solid-state emitters with an internal degree of freedom suggests a further shift of paradigm toward hybrid schemes in which quantum information is held on matter degrees of freedom (stationary qubits) and connected via photonic degrees of freedom using entangling measurements [15]. This represents an alternative approach that implements spin-optical quantum computing by generating source states in a spin-based system and performing fusion measurements through photonic methods. Such a hybrid architecture effectively integrates established matter-based and photonic based techniques, establishing a potential platform to implement fault-tolerant quantum computing (for a recent summary of such an approach, see, e.g., Ref. [21]).

While one might use entangling measurements on photons to build a cluster state and then execute measurement-based quantum computation, it seems tempting to transfer the concept of fusion-based quantum computing to a hybrid matter-photonic system. There are now two possible routes toward this goal. In the first approach, individual optically active spin qubits are each placed in a separate optical resonator, which is then connected to a photonic network that allows for the implementation of entangling operations based on Ref. [15] or reflection and detection of photons [22], or here we take an approach in which each resonator contains a small number of optically active spin qubits that are magnetically or otherwise coupled in order to create the desired resource states (e.g., four- and six-ring cluster states) locally on the matter qubits. The latter may offer additional opportunities, as matter qubits exhibit greater versatility and flexibility in their mutual interaction, which can be used to create desired quantum states and may therefore offer advantages.

In this work, we are placing our emphasis on the matter-based quantum register in a fusion-based hybrid matter-photon quantum computer with the objective of demonstrating the potential for generating the desired resource state. The spin-based quantum information processor in a solid-state system employs the spin states of electrons or nuclei in solid materials to encode, manipulate, and process quantum information. However, the preparation of complex multipartite quantum states in these systems remains a challenge. Especially in solid-state quantum registers based on magnetic interactions, the nearest-neighbor (NN) couplings dominate but, nevertheless, beyond-NN interactions are still relevant when aiming to achieve the extremely high fidelities that are necessary for fault-tolerant quantum error correction. In conventional schemes capable of eliminating long-range spin interactions, the NN ones tend to be suppressed as well. Moreover, Ref. [23] proposes a general dynamical-decoupling-based protocol that appropriately applies π -pulse sequences on selected qubits to average out next-nearest-neighbor (NNN) couplings while preserving the

desired NN ones, enabling high-fidelity preparation of one-dimensional cluster states. The evolution is divided into several short intervals and spin-selective π pulses are applied at suitable timing. Both xy and zz interactions can be refocused while NNN terms average over one dynamic cycle, leaving an effective Hamiltonian dominated by NN ones. The protocol is, however, formulated specifically for the one-dimensional spin chain and does not explicitly address how control sequences modify longer-range interactions, thereby limiting its direct applicability to more general architectures with extended couplings.

In addition, conventional schemes typically assume an idealized scenario in which all spins are precisely positioned, usually forming a perfect one-dimensional spin chain or two-dimensional square lattice. In practice, however, deviations in the positions of the spins due, e.g., to the implantation of color centers or those associated with nuclear spins, are common. The resulting variations in the mutual distances lead to inhomogeneities in the coupling strengths that, in turn, can significantly affect the effectiveness of most state-preparation schemes.

Finally, in solid-state systems, strong interactions typically emerge when the spins are closely spaced, which poses significant challenges for performing local manipulations and measurements. Specifically, in electron-spin or nuclear-spin systems, sufficiently strong direct magnetic dipole interactions occur on the scale of tens down to a few nanometers. Consequently, it is crucial to investigate methods for quantum state preparation that are feasible with only a global drive. Note that the ability to manipulate spins without individual control is a fundamental and common necessity for quantum spin systems.

In our work, we present a new approach for generating cluster states using a carefully designed sequence of selective π pulses. This sequence is designed to modify the interspin couplings in a nonuniform manner across different segments, ensuring the preservation of NN interactions while eliminating long-range interactions. Our scheme is illustrated in Fig. 1(a), where a quantum spin system is driven by a global control field (green light) and the solid and dashed lines between the spins denote the NN and long-range couplings. In Fig. 1(b), we show the pulse sequence, which is composed of m segments. $R_{\pi}^x(\mathbf{i})$ is a selective π pulse about the x axis that flips all the spins indicated by an index vector $\mathbf{i}$, and the single selective π pulse [the inset in Fig. 1(b)] is implemented by a composite pulse that only flips the resonant spin, while leaving the others unaffected. To implement our protocol, we develop a general framework of a frequency-selective pulse using composite-pulse and shaped-pulse techniques. These pulses, optimized using optimal-control methods, are robust to possible imperfections in the global driving field.

Our paper is organized as follows. In Sec. II, we introduce the general framework of the cluster-state preparation

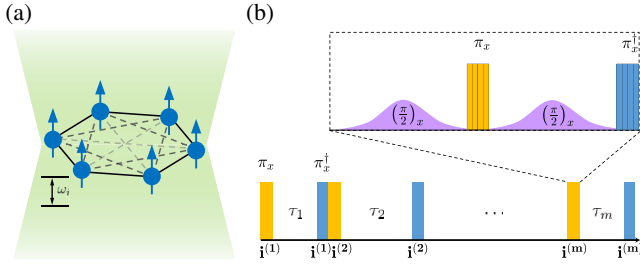


FIG. 1. (a) The scheme of the pulsed-cluster preparation in a quantum multispin system, where all spins are driven by the global field (light green). The solid lines indicate the NN couplings that have to be preserved, while the dashed lines represent the unwanted long-range couplings. ω_i indicates the unique energy splittings of spin i . (b) The pulsed dynamical sequence to eliminate long-range couplings. The whole process is divided into m segments and each segment consists of a free-evolution time τ_k sandwiched by two selective π_x and $\pi_x^\dagger$ gates, denoted as $R_{\pi}^x(\mathbf{i}^{(k)})$ and $R_{\pi}^{-x}(\mathbf{i}^{(k)})$. The index vector $\mathbf{i}^{(k)}$ indicates the spins flipped by the π_x pulses in the k th segment. The foundation of the scheme lies in a selective single π_x pulse, which selectively flips the resonant spin while leaving the remainder unchanged. As we discuss in Secs. IV C and IV D, the selective π_x and $\pi_x^\dagger$ pulses could be implemented by a sequence of two shaped $(\pi/2)_x$ pulses (purple curves) and two composite π_x pulses (yellow and blue blocks), depicted within the dashed box. The generation of all pulses is attributed to the global driving field due to limitations in the spatial resolution of the control field.

utilizing a pulsed control sequence. We then present several examples by giving a few multispin systems and discuss the extension to larger systems in Sec. III. In Sec. IV, we demonstrate the feasibility of implementing the proposed scheme in nitrogen-vacancy- (N-V) center systems [24,25], with the development of selective pulses and the numerical simulation of preparation fidelity. A brief conclusion of our proposal and an outlook toward the hybrid matter-photon quantum information processor are given in Sec. V.

II. THEORETICAL FRAMEWORK

To generate a cluster state in a fully connected spin system, we first initialize all spins into a product state $|\psi_0\rangle = \otimes_i |+\rangle_i$ and then the cluster state $|\psi\rangle_C$ is prepared by applying an NN controlled-Z gate $S = \prod_{\langle i,j \rangle} S_{ij}$, where

$$S_{ij} = |0\rangle_{ii} \langle 0| \otimes \mathbb{1}_j + |1\rangle_{ii} \langle 1| \otimes \sigma_j^z, \quad (1)$$

and $\langle i,j \rangle$ denotes NN spins. S is a joint controlled-Z gate and is typically generated using an Ising-like zz -coupling Hamiltonian $H_C/\hbar = \sum_{\langle i,j \rangle} g(1 - \sigma_i^z)(1 - \sigma_j^z)$ with the required evolution time $t = \pi/4g$ [8]. S can be decomposed into a product of three basic gates: $S = \prod_{\langle i,j \rangle} U_{\pi}^{zz}(i,j) R_{\pi/2}^z(i) R_{\pi/2}^z(j)$. Here and subsequently, $R_{\alpha}^z(\mathbf{i})$ indicates a joint α rotation along the z axis of

spins denoted by the index vector $\mathbf{i}$, while $U_{\pi}^{zz}(i,j) = \exp(-i\pi\sigma_i^z\sigma_j^z/4)$ acts between spin i and j . Because all elements in S commute, S can be written as

$$S = \prod_i R_{\pi}^{\varphi_i}(i) \prod_{\langle i,j \rangle} U_{\pi}^{zz}(i,j), \quad (2)$$

where $\varphi_i = n_i\pi/2$ and n_i indicates the number of NN pairs in which spin i is involved. We obtain a target unitary U_t to prepare the cluster state after dropping the first local transforms

$$U_t = \prod_{\langle i,j \rangle} e^{-i\frac{\pi}{4}\sigma_i^z\sigma_j^z}. \quad (3)$$

This necessitates an engineered evolution process that ensures that the phases of the NN coupling terms are equal to π while simultaneously eliminating the inherent long-range couplings. However, as the size of the spin system increases, the number of two-body interactions increases quadratically with the number of spins. This makes mitigation of long-range interactions more challenging. To explain our proposal to eliminate the long-range zz interaction, we consider a fully connected N -spin Hamiltonian ($\hbar = 1$)

$$H_{zz} = \sum_{i \neq j} \frac{g_{ij}}{4} \sigma_i^z \sigma_j^z. \quad (4)$$

For simplicity, we assume that the coupling terms g_{ij} are arranged in descending order based on their corresponding strengths rather than the indices.

The coupling strength g_{ij} is typically determined by the relative position between spin i and j . In practice, possible variations in spin positions lead to inhomogeneities in interaction strengths, causing differences in the accumulated phase (denoted as $\theta_{ij} = g_{ij}t$) for each interaction term during the evolution $U_{zz}(t) = \exp(-iH_{zz}t)$. The inhomogeneity results in unique phase shifts for each spin pair, affecting the overall quantum state dynamics. To make the phases accumulated by all NN interactions equal to π while eliminating long-range couplings, we introduce a pulsed dynamical decoupling sequence that selectively reverses specific interaction strengths. As illustrated in Fig. 1(b), an elementary segment in the sequence consists of two multispin π pulses $R_{\pi}^x(\mathbf{i}^{(k)})$ and $R_{\pi}^{-x}(\mathbf{i}^{(k)})$ before and after a free-evolution period of a length τ_k . This segment selectively inverts the coupling terms [23]. Here,

$$R_{\pi}^{\phi}(\mathbf{i}) = \otimes_{j=1}^{n_i} \exp\left(-i\frac{\pi}{2}\sigma_{ij}^{\phi}\right), \quad (5)$$

where index vector $\mathbf{i} = (i_1, i_2, \dots, i_{n_i})$, indicating which spins are flipped are arranged in increasing order; and $\sigma_{ij}^{\phi} = \cos(\phi)\sigma_{ij}^x + \sin(\phi)\sigma_{ij}^y$ denotes the rotation operator

of the i_j th spin. The evolution of the k th segment is given by

$$U(\mathbf{i}^{(k)}, \tau_k) = R_\pi^{-x}(\mathbf{i}^{(k)}) U_{zz}(\tau_k) R_\pi^x(\mathbf{i}^{(k)}) \\ = \exp \left(-i \sum_{i \neq j} f_{ij}(k) \frac{g_{ij} \tau_k}{4} \sigma_i^z \sigma_j^z \right), \quad (6)$$

where the modulation factor $f_{ij}(k) = (-1)^{\mathcal{N}_{ij}(k)}$ and $\mathcal{N}_{ij}(k) = |\{i, j\} \cap \mathbf{i}^{(k)}|$ indicates the number of spins flipped by $R_\pi^x(\mathbf{i}^{(k)})$ in spin $\{i, j\}$. Here, τ_k denotes the duration of the k th segment, as π pulses are assumed to be instantaneous in the analysis. If only one of the spins i and j is affected by the $R_\pi^x(\mathbf{i}^{(k)})$ pulse, the effective coupling strength during this evolution is inverted, $g_{ij} \rightarrow -g_{ij}$. If none or both of them are involved, the sign of the coupling strength g_{ij} remains unchanged.

Therefore, we are able to tailor the accumulated phases by applying a pulse sequence. The sequence includes m elementary segments with corresponding indices $\mathcal{S} = \{\mathbf{i}^{(1)}, \mathbf{i}^{(2)}, \dots, \mathbf{i}^{(m)}\}$ and corresponding duration $\vec{\tau} = (\tau_1, \tau_2, \dots, \tau_m)^T$. The whole evolution unitary is

$$U(T_c) = \prod_{k=1}^m U(\mathbf{i}^{(k)}, \tau_k) = \exp \left(-i \sum_{i \neq j} \frac{\theta_{ij}}{4} \sigma_i^z \sigma_j^z \right), \quad (7)$$

where the accumulated phases and total duration are

$$\theta_{ij} = g_{ij} \sum_{k=1}^m f_{ij}(k) \tau_k, \quad T_c = \sum_{k=1}^m \tau_k. \quad (8)$$

These two equations naturally form a linear equation

$$M \cdot \vec{\tau} = \vec{\alpha}, \quad (9)$$

with the factor matrix $M \in \mathbb{R}^{(n_g+1) \times m}$ and target phase vector $\vec{\alpha} \in \mathbb{R}^{(n_g+1) \times 1}$. The first $n_g + 1$ elements of $\vec{\alpha}$ are θ_{ij}/g_{ij} and $\alpha_{(n_g+1)} = T_c$. Here, $n_g = N(N+1)/2$ is the number of coupling terms. It is apparent that the form of the pulse sequence is determined by solving the linear equation. The solution of the equation is general and does not require a perfect lattice. After fabrication, all couplings are fixed, thereby enabling the determination of the corresponding pulse sequence.

In the next section, we will provide solutions in the small spin-ring systems employed in FBQC as examples to explain our procedure. Note that these decoupling pulses modify the interaction strengths while they do not change their forms. Hence, the order of applying the pulses can be arbitrary, as the unitary evolution operators of all segments mutually commute. In addition, each pulse can be applied at most once, as multiple segments with the same pulses can be moved together and merged.

III. RESOURCE-STATE GENERATION

We proceed with discussing the specific forms of our pulsed-cluster-state-preparation protocol in different systems. First, we consider two small spin-ring systems that are fully solvable and are required for the implementation of FBQC. In these systems, we consider both the cases with and without uncertainties in the positions. We then investigate an ideal larger spin-ring system, utilizing its spatial symmetry, and demonstrate that our proposal could be extended to a larger quantum system with about 20 spins.

A. Spin-ring system

Our discussion initially focuses on two resource-state generators associated with four- and six-spin-ring systems. These two systems are fully solvable, whereby the invertibility of the matrix M and the relevance of the semi-positive solution can be guaranteed by selecting $n_g + 1$ suitable pulses

$$\vec{\tau} = M^{-1} \cdot \vec{\alpha} \geq 0. \quad (10)$$

In the four-spin system, as shown in Fig. 2, the spin index sequence is $\mathcal{S}_4 = \{(0), (1), (1, 2), (2), (2, 3), (3), (4)\}$ where (0) means that no pulses are applied in the first segment and (i, j) means that pulses are applied to spins i and j . The corresponding time-duration solution is

$$\vec{\tau} = \frac{1}{4} \begin{pmatrix} \alpha_{12} + \alpha_{23} + \alpha_{34} + \alpha_{14} \\ T_c - \alpha_{12} - \alpha_{14} + \alpha_{24} \\ \alpha_{12} + \alpha_{34} - \alpha_{13} - \alpha_{24} \\ T_c - \alpha_{12} - \alpha_{23} + \alpha_{13} \\ \alpha_{23} + \alpha_{14} - \alpha_{13} - \alpha_{24} \\ T_c - \alpha_{23} - \alpha_{34} + \alpha_{24} \\ T_c - \alpha_{34} - \alpha_{14} + \alpha_{13} \end{pmatrix}, \quad (11)$$

where the first $n_g = 6$ elements of $\vec{\alpha}$ are defined as $\alpha_{ij} = \theta_{ij}/g_{ij}$.

This is a general solution of a four-spin system and the sequence $\mathcal{S}_4$ is chosen to minimize the number of single-spin pulses N_π after canceling the pulses between adjacent segments, resulting in a pulse sequence $[\tau_1 - \pi_1 - \tau_2 - \pi_2 - \tau_3 - \pi_1^\dagger - \tau_4 - \pi_3 - \tau_5 - \pi_2^\dagger - \tau_6 - \pi_3^\dagger - \tau_4 - \pi_7 - \pi_4^\dagger]$. In the expression for the pulse sequence, the duration $\tau_i^\dagger$ represents the i th free-evolution process, while π_i and $\pi_i^\dagger$ are abbreviations for $R_\pi^x(i)$ and $R_\pi^{-x}(i)$ in this section. A cluster state connecting NN pairs is generated by setting two phases of the long-range terms $\alpha_{13} = \alpha_{24} = 0$. The evolution of the accumulated phases θ_{ij} and the pulse sequence are shown in Fig. 2(c), where the phases of the NN couplings and the NNN coupling, respectively, are π and 0 in the end.

In the ideal case, all spins are located on the square-lattice sites, denoted by the solid green dots in Fig. 2(a).

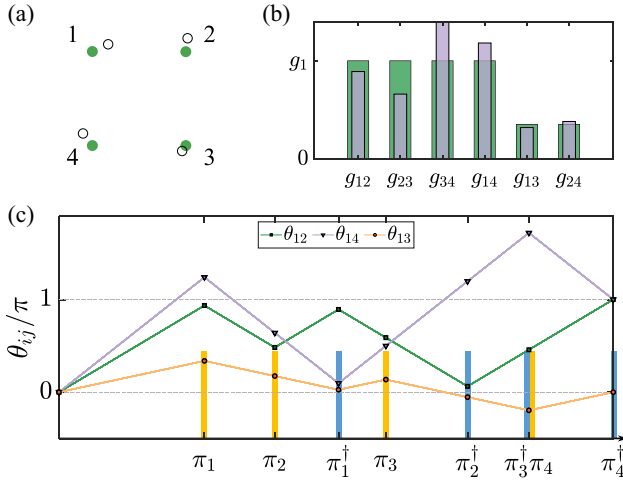


FIG. 2. (a),(b) The preparation of a cluster state in a four-spin system, where the square lattices without and with position errors are represented by green dots and black circles in (a), respectively. The corresponding coupling strengths g_{ij} are represented by green and purple bars in (b). g_1 is the NN coupling strength in the ideal case. (c) The evolution of the accumulated phase θ_{ij} with respect to the NN couplings $g_{12,14}$ and one NNN coupling g_{13} are represented. In each segment, the derivatives of the phases with respect to time are determined by the corresponding modulation factors. The sequence of the selective π pulses employed is $\mathcal{S}_4 = \{(0), (1), (1, 2), (2), (2, 3), (3), (4)\}$, resulting in a pulse sequence $[\tau_1 - \pi_1 - \tau_2 - \pi_2 - \tau_3 - \pi_1^\dagger - \tau_4 - \pi_3 - \tau_5 - \pi_2^\dagger - \tau_6 - \pi_3^\dagger \pi_4 - \tau_7 - \pi_4^\dagger]$ —the notation and meaning of τ_i and π_i are introduced in the text. The parameters are identical to those presented in Sec. IVE within a solid-state system.

Thus, the elements in vector $\vec{\alpha}$ associated with NN and NNN couplings are identical to π/g_1 and 0. Here, g_1 is the strength of the NN coupling. The minimal length of the sequence is $T_c = 2\pi/g_1$ and the length of each segment is

$$\tau_1 = \frac{\pi}{g_1}, \quad \tau_{2,4,6,7} = 0, \quad \tau_{3,5} = \frac{\pi}{2g_1}. \quad (12)$$

Thereby, the corresponding index sequence is $\mathcal{S}_4 = \{(0), (1, 2), (2, 3)\}$ and only six pulses remain, as the pulse sequence is $[\tau_1 - \pi_1 \pi_2 - \tau_3 - \pi_1^\dagger \pi_3 - \tau_5 - \pi_2^\dagger \pi_3^\dagger]$ after the cancellation.

Similarly to the four-spin case, solving Eq. (10), a cluster state could be prepared by a sequence of two-body pulses in a six-spin system. We obtain a sequence of the pulse indices $\mathcal{S}_6 = \{(0), (1, 2), (2, 3), (3, 4), (4, 5), (5, 6), (1, 6), (1, 3), (1, 4), (4, 6), (2, 4), (2, 6), (3, 6), (3, 5), (1, 5), (2, 5)\}$, which means that the required number of π pulses is $N_\pi = 32$. The six spins are labeled clockwise and the general pulse sequence is $[\tau_1 - \pi_1 \pi_2 - \tau_2 - \pi_1^\dagger \pi_3 - \tau_3 - \pi_2^\dagger \pi_4 - \tau_4 - \pi_3^\dagger \pi_5 - \tau_5 - \pi_4^\dagger \pi_6 - \tau_6 - \pi_5^\dagger \pi_1 - \tau_7 - \pi_6^\dagger \pi_1 - \tau_8 - \pi_3^\dagger \pi_4 - \tau_9 - \pi_1^\dagger \pi_6 - \tau_{10} - \pi_6^\dagger \pi_2 - \tau_{11} - \pi_4^\dagger \pi_6 - \tau_{12} - \pi_2^\dagger \pi_3 - \tau_{13} - \pi_6^\dagger \pi_5 - \tau_{14} - \pi_3^\dagger \pi_1 - \tau_{15} - \pi_1^\dagger \pi_2 - \tau_{16} - \pi_2^\dagger \pi_5^\dagger]$. In the ideal case, the corresponding length of each

segment is

$$\begin{aligned} \tau_1 &= \frac{5\pi}{8g_1}, & \tau_{2,3,5,6} &= \frac{\pi}{g_1}, & \tau_4 &= \frac{3\pi}{8g_1}, \\ \tau_{8,10,14 \rightarrow 16} &= \frac{\pi}{8g_1}, & \tau_{9,10,12,13} &= 0, \end{aligned} \quad (13)$$

with the total duration of the sequence $T_c = 2\pi/g_1$. More details and sequences of quantum spin systems are provided in Appendix A. It is essential to note that for a multispin system, our proposal enables the generation of cluster states with diverse geometries and connectivities by appropriately choosing the target phase vector $\vec{\alpha}$. In particular, the same framework can be used either to effectively realize higher-dimensional cluster states or to isolate a single desired coupling by suppressing all other interactions, as illustrated by the two representative examples in Appendix A 2.

B. Larger spin-ring system

As illustrated in the preceding section, the state-generation proposals employ up to two-spin pulses in a small spin system. Inspired by this, we aim to explore the preparation of the cluster state in larger spin-ring systems. Here, we assume that N spins are equally spaced and the periodic condition $\sigma_1^z = \sigma_{N+1}^z$ leads to a spatial symmetry that is the key in our proposal. First, we label the two-spin couplings with respect to the distance between them. Thus, all NN couplings are denoted as $\mathcal{C}^{(1)}$, the NNN couplings are labeled as $\mathcal{C}^{(2)}$, and the l th order of the collective coupling is

$$\mathcal{C}^{(l)} = \sum_{i=1}^{n_l} \frac{1}{4} \sigma_i^z \sigma_{i+l}^z, \quad (14)$$

where n_l is the number of elements in $\mathcal{C}^{(l)}$. The Hamiltonian in Eq. (4) can be rewritten as $H_{zz} = \sum_{l=1}^L g_l \mathcal{C}^{(l)}$, where g_l is the l th coupling strength and L indicates the number of collective couplings. The target unitary [see Eq. (3)] in a spin-ring system is now

$$U_t = \exp(-i\pi \mathcal{C}^{(1)}), \quad (15)$$

which requires the preservation of the first collective coupling. In a system with an odd number of spins, the number of collective couplings is given as $L = (N - 1)/2$. In the l th collective coupling, the number of two-spin interactions is $n_l = N$. In contrast, in an even spin system, the number of collective couplings is $L = N/2$. Each coupling contains $n_l = N$ elements, except that the number of elements in the L th coupling is $n_L = N/2$.

Similarly, the k th two-spin collective pulse, $\mathcal{G}^{(k)}$, is defined to include all possible pulses applied on two spins with a specific distance $\mathcal{G}^{(k)} = \{R_\pi^x(1, 1 + k), R_\pi^x(2, 2 +$

$k), \dots\}$. The number of elements in $\mathcal{G}^{(k)}$ is n_k , which is identical to the number of coupling terms in the k th collective coupling $\mathcal{C}^{(k)}$. All the two-spin pulses in $\mathcal{G}^{(k)}$ are applied before and after a unitary U_{zz} sequentially, the evolution in the k th segment being

$$\mathcal{G}^{(k)}[H_{zz}, \tau_k] = \prod_{i=1}^{n_k} R_{\pi}^{-x}(i, i+k) U_{zz}(\tau_k) R_{\pi}^x(i, i+k). \quad (16)$$

Due to the symmetry of the system, the unitary operator can be expanded as

$$\mathcal{G}^{(k)}[H_{zz}, \tau_k] = \exp\left(-i \sum_{l=1}^L f'_{lk} g_l \mathcal{C}^{(l)} \tau_k\right), \quad (17)$$

where f'_{lk} is the modulation factor of the l th coupling $\mathcal{C}^{(l)}$ modified by the k th collective gate $\mathcal{G}^{(k)}$. The values of f'_{lk} are listed in Table I and the last two columns are relevant to f'_{ll} in the odd and even spin systems, respectively.

We take an example $C_{a,a+l} = \sigma_a^z \sigma_{a+l}^z$, an element in coupling $\mathcal{C}^{(l)}$, to explain the value of f'_{lk} . The involved spins a and $(a+l)$ are only affected by four pulses, $R_{\pi}^x(a-k, a)$, $R_{\pi}^x(a, a+k)$, $R_{\pi}^x(a+l-k, a+l)$, and $R_{\pi}^x(a+l, a+l+k)$, in collective pulse $\mathcal{G}^{(k)}$. Thus, if $l \neq k$, all four pulses inverse the coupling $C_{a,a+l}$ and the rest of the $n_k - 4$ elements in $\mathcal{G}^{(k)}$ do not affect the coupling, which gives the modulation factor of $n_k - 8$. If $l = k$, only two unique pulses, $R_{\pi}^x(a-k, a)$ and $R_{\pi}^x(a+k, a+2k)$, remain and invert the coupling, resulting in a modulation factor of $n_k - 4$. One special case is the performance of the last collective pulse, $\mathcal{G}^{(L)}$, in the even spin system $N = 2L$. If $l \neq N/2$, $R_{\pi}^x(a-L, a)$, $R_{\pi}^x(a, a+L)$, $R_{\pi}^x(a+l-L, a+l)$, and $R_{\pi}^x(a+l, a+l+L)$ reduce to two pulses, $R_{\pi}^x(a-L, a)$, and $R_{\pi}^x(a+l-L, a+l)$, due to the symmetric property $a-L = a+L$. Thus, the modulation factor here is $N/2 - 4$. If $l = L$, neither of the two pulses inverts coupling $C_{a,a+l}$, giving a factor of $N/2$.

As demonstrated in Table I, a collective pulse $\mathcal{G}^{(k)}$ modulates all collective couplings in the same manner, with the exception of $\mathcal{C}^{(k)}$. Thus, we are able to preserve first-order interactions while nullifying others by choosing only the first two pulses, $\mathcal{G}^{(0)}$ and $\mathcal{G}^{(1)}$, with the corresponding time intervals $(\tau_0, \tau_1) = (8-N, 1)\tau_1$. The evolution is

$$\mathcal{G}^{(1)}[H_{zz}, \tau_1] \mathcal{G}^{(0)}[H_{zz}, \tau_0] = \exp(-i 4g_1 \tau_1 \mathcal{C}^{(1)}). \quad (18)$$

We obtain the solution $(\tau_0, \tau_1) = (8-N, 1)\pi/4g_1$ by setting $4g_1 \tau_1 = \pi$. The total duration of the sequence is

$$T_c = \tau_0 + n_1 \tau_1 = \frac{2\pi}{g_1}. \quad (19)$$

The corresponding preparation sequence of the indices is $\mathcal{S}_N = \{(0), (1, 2), (2, 3), \dots, (N, 1)\}$ and the total number

of single-spin π pulses (including π and $\pi^\dagger$) is $2N + 2$. This two-spin cluster-state-preparation scheme is consistent with the sequence $\mathcal{S}_6$ that we have derived in Sec. III A. It should be noted that this proposal is valid when $N \leq 8$, to ensure that $8 - N$ is a positive integer. The larger spin system will be discussed in the following text.

Here, we extend our protocol to the generation of cluster states in a larger spin-ring system with $N > 8$. The principle of the preparation requires multispin (up to $\mathcal{N}_g$ spins) pulses to increase the variety of coupling modulations, as opposed to relying solely on two-spin pulses. Similarly, we define the k th collective pulse $\mathcal{G}_{m_k}^{(k)} = \{R_{\pi}^x(\mathbf{i}_k^{(1)}), R_{\pi}^x(\mathbf{i}_k^{(2)}), \dots, R_{\pi}^x(\mathbf{i}_k^{(N_k)})\}$ as a set of m_k -spin pulses possessing the k th smallest internal distance $\mathcal{D}_k$. Here, $\mathcal{D} = \sum_m |\vec{r}_{i_{m+1}} - \vec{r}_{i_m}|$ is the sum of the distance between every two nearby spins involved by a selective π pulse $R_{\pi}^x(\mathbf{i})$, with all indices in the vector $\mathbf{i} = \{i_1, i_2, \dots\}$ listed in ascending order.

Due to the symmetry of the quantum spin-ring system, the collective pulse modifies the free evolution in the k th segment as

$$\begin{aligned} \mathcal{G}_{m_k}^{(k)}[H_{zz}, \tau_k] &= \prod_{i=1}^{N_k} R_{\pi}^{-x}(\mathbf{i}_k^{(i)}) U_{zz}(\tau_k) R_{\pi}^x(\mathbf{i}_k^{(i)}) \\ &= \exp\left(-i \sum_{l=1}^L \tilde{f}_{lk} g_l \mathcal{C}^{(l)} \tau_k\right). \end{aligned} \quad (20)$$

The factor $\tilde{f}_{lk}$ quantifies the modulation achieved by the collective gate $\mathcal{G}_{m_k}^{(k)}$ to the l th collective coupling. We apply the collective pulses sequentially, with the corresponding sequence being $(k_1, k_2, \dots, k_{n_c})$, resulting in an overall unitary $U(T_c) = \prod_{j=1}^{n_c} \mathcal{G}_{m_{k_j}}^{(k_j)}[H_{zz}, \tau_j]$ with the duration $T_c = \sum_{j=1}^{n_c} N_{k_j} \tau_j$. Here, N_{k_j} denotes the number of m_{k_j} -spin pulses included in $\mathcal{G}_{m_{k_j}}^{(k_j)}$ and τ_j is the corresponding time interval. Similar to Eq. (9), the linear equation is

$$\tilde{M} \cdot \vec{\tau} = \vec{\alpha}. \quad (21)$$

Here, $\tilde{M} \in \mathbb{R}^{o \times n_c}$ represents the modulation factors corresponding to the n_c collective pulses employed; $\vec{\tau} \in \mathbb{R}^{n_c \times 1}$ indicates the time interval within each segment and the target phases $\vec{\alpha} = 0$ except for the first element $\alpha(1) = \pi/g_1$.

While the utilization of collective gates to modify the coupling has notably simplified the computational complexity, the increase of the system size presents challenges. As the size of the system increases, the number of long-range couplings and multispin pulses increases quadratically and polynomially, respectively. Although serendipity may lead to a few solutions for quantum multispin systems, these solutions are not inherently optimal. Ideally,

TABLE I. The modulation factor of the l th collective coupling $\mathcal{C}^{(l)}$ modified by the k th two-spin collective pulse $\mathcal{G}^{(k)}$. The last two columns present f_{lk}' in the system with odd and even spins, respectively. The k th ($k > 0$) collective pulse modulates all the collective couplings in a unit manner, with the exception of the k th coupling.

f_{lk}'	$\mathcal{G}^{(0)}$	$\mathcal{G}^{(1)}$	$\mathcal{G}^{(2)}$	$\mathcal{G}^{(3)}$	$\dots$	$\mathcal{G}_{\text{odd}}^{(L)}$	$\mathcal{G}_{\text{even}}^{(L)}$
$\mathcal{C}^{(1)}$	1	$N - 4$	$N - 8$	$N - 8$	$\dots$	$N - 8$	$N/2 - 4$
$\mathcal{C}^{(2)}$	1	$N - 8$	$N - 4$	$N - 8$	$\dots$	$N - 8$	$N/2 - 4$
$\mathcal{C}^{(3)}$	1	$N - 8$	$N - 8$	$N - 4$	$\dots$	$N - 8$	$N/2 - 4$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\vdots$
$\mathcal{C}^{(L)}$	1	$N - 8$	$N - 8$	$N - 8$	$\dots$	$N - 4$	$N/2$

the number of single-spin pulses involved should be minimized and the total evolution time should be kept to a minimum.

The key challenge lies in developing an efficient search algorithm to address the scalability of our cluster-state-preparation proposal. In this context, we propose a feasible numerical solution: initially selecting $n_c = L - 1$ elements from all collective pulses randomly, then finding the least-squares solution (LSS) $\vec{\tau}_{\text{LSS}}$ of Eq. (21), which minimizes the relative residual $\sigma = \|\vec{M} \cdot \vec{\tau}_{\text{LSS}} - \vec{\alpha}\|_2 / \|\vec{\alpha}\|_2$ ($\|\vec{x}\|_2 = \sqrt{\vec{x}^T \vec{x}}$). Finally, from all possible solutions, we identify the ones that satisfy the following criteria: (i) positive time intervals; (ii) a low residual, with a value no greater than a specified threshold σ_0 ; and (iii) a limited duration, with a maximum value of a specified limit T_0 . The number of single-spin pulses can be further reduced by following two steps. First, the segments the time intervals of which are less than a certain value τ_{min} , are removed. Second, all the multispin pulses involved are arranged to cancel the pulses between two adjacent segments as much as possible. In Fig. 3(a), we present the solutions with high preparation fidelity ($F_C > 0.99999$) that we obtain in the ten-spin system, as well as the number of single-spin pulses and the duration T_c . Furthermore, we show the minimal number of N_π as a function of the size of the system in Fig. 3(b), with the parameters $\sigma_0 = 0.1$, $T_0 = 10T_1$, and $\tau_{\text{min}} = 10^{-5}T_1$, with $T_1 = 2\pi/g_1$.

IV. POTENTIAL EXPERIMENTAL IMPLEMENTATIONS

In this section, we present a potential implementation of our proposal for preparing a pulsed-cluster state, using an N- V center system as an illustrative example. In principle, our proposal is well suited to entangling the quantum spin system using the inherent mutual zz interactions as the resource. The N- V center, an artificial defect in the diamond, represents a promising platform due to the desired zz couplings arising from mutual direct magnetic dipole interactions. Moreover, nearby nuclear spins could be involved in the same interaction framework.

We begin this section by briefly introducing the N- V center, outlining relevant practical challenges, and deriving the interaction Hamiltonian in Eq. (4). Then, we develop the broadband and selective pulses necessary for our proposal, employing optimal-control methods as well as composite and shaped pulses. Finally, we conduct the numerical simulation of our proposal in the N- V center system.

A. N- V centers in diamond

N- V centers in diamond are well-studied point defects with electronic spin-1 ground states that can be manipulated using microwave fields and initialized and read out optically. The electronic spin properties combined with the ability to couple to nearby nuclear spins make the N- V center a promising candidate for scalable quantum networks and computing architectures [24,26–30]. However, several practical challenges must be addressed when developing a matter-photon quantum information processor based on N- V centers.

First, achieving the desired Hamiltonian [see Eq. (4)] relies on the magnetic dipole-dipole interaction between N- V centers, requiring nanometer-scale precision control of the positions of defects to achieve appropriate coupling strengths. In particular, the deterministic preparation of

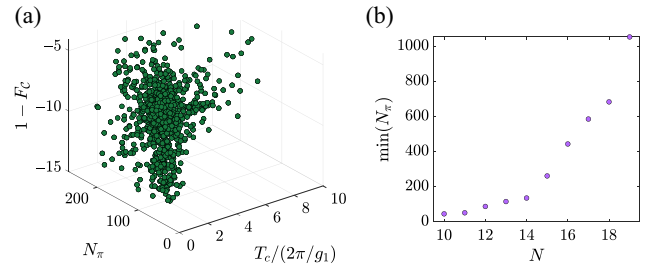


FIG. 3. The preparation of cluster states in the larger spin-ring system. (a) The fidelity of the state preparation of the solutions obtained within a ten-spin system, along with the corresponding number of single-spin π_x pulses and the duration of the control field, T_c . (b) The minimum number of single-spin π_x pulses required to prepare a cluster state in a multispin system increases with the system size.

color centers in large numbers represents a significant challenge for the realization of scalable quantum computing. This constraint is somewhat relaxed in FBQC, where at most six spins are required in a single resource-state generator (RSG). For scalable solid-state spin registers in diamond, a key requirement is the ability to position a small number of N- V centers at nanometer-scale separations. A well-established method for this purpose is the nitrogen implantation process, where nitrogen ions are accelerated and implanted into diamond through a nanohole to create dipolar-coupled N- V centers. By optimizing the mask geometry and implantation parameters, interacting N- V pairs with separations down to a few nanometers have been demonstrated [31]. Moreover, tailored ion beams and deterministic single-ion sources provide additional control over extraction voltage, beam focusing, and beam monochromaticity, further improving the spatial accuracy and implantation yield for the creation of color centers [32–34]. The number of coupled N- V centers could be further increased by replacing the ion beam with ionized nitrogen-rich molecules. Consequently, the positions of N- V centers are determined by the intrinsic structure of the molecule and implantation parameters [30,35], offering a potential route toward the fabrication of regular spin arrays. A recent development in the field involves the fabrication of a system comprising four coupled N- V centers, with a separation distance of approximately 9 nm [36]. Moreover, coupled N- V center pairs exhibit to be entangled with a reported fidelity of 96% in Ref. [30], and the separation of the N- V pair can be rapidly determined optically, as demonstrated in Ref. [37].

Second, the implementation of selective pulses is another issue to be addressed. The ability to selectively address, manipulate, and read out individual spins is fundamental to the development of matter-based quantum information processors. Constrained by the distance between N- V centers, frequency-selective operations offer a viable alternative. Meanwhile, the coupling strengths can be extracted using the double electron-electron resonance technique [38,39], which is essential for determining the preparation sequence.

Third, the preparation proposal must be robust against noise, typically being consistent with the present methods to suppress the decoherence, such as the dynamical decoupling. The last challenge relates to increasing the low photonic connection efficiency between spatially separated registers, which is constrained by the low zero-phonon-line (ZPL) emission efficiency and the low collection efficiency. Techniques such as lowering the temperature, placing the N- V center in an optical resonant cavity, and fabricating diamond nanostructures can enhance the probability of detecting ZPL photons.

These challenges are specific to an N- V center-based quantum information processor. In our work, focusing on state preparation, the main concern is to realize the

selectivity to suppress the long-range interactions. Based on the designed broadband and selective pulses, the pulsed-cluster proposal demonstrates robustness against uncertainties in spin positions, imperfections of the control fields, and the dephasing noise.

B. Hamiltonian

We explore the preparation of a cluster state in a multi-spin system, where each spin is encoded in the electronic ground state of the N- V center. Ideally, these N- V centers are positioned close together, equidistant from one another, and aligned in the same plane, with their orientations perpendicular to this plane. Each N- V center represents a spin $S = 1$ system with three ground states that can be initialized, manipulated, and read out under ambient conditions.

The Hamiltonian of the system is

$$H = \sum_i H_i^0 + \sum_{ij} H_{ij}^{\text{dd}} + H_c(t), \quad (22)$$

where H_i^0 is the Hamiltonian for the N- V center at site i under an external static magnetic field B_z applied along z to lift the degeneracy of the spin states $m_s = \pm 1$,

$$H_i^0 = (D_0 + \delta_i^D)(S_i^z)^2 - (\gamma_e B_z + \delta_i^z)S_i^z, \quad (23)$$

where $D_0 = (2\pi)2.87$ GHz is the zero-field splitting and γ_e is the electronic spin gyromagnetic ratio. δ_i^D and δ_i^z represent the energy disorder of site i , primarily attributed to lattice strain, the positioning of the N- V centers, and the inclusion of scattered paramagnetic impurities [40,41]. A time-dependent global driving, $H_c(t) = \sum_i \sqrt{2}\Omega(t) \cos[\omega_c t + \phi(t)]S_i^x$, is applied to the system, which generates all the control pulses required. Finally, H_{ij}^{dd} is the magnetic dipole-dipole interaction between spins i and j .

In our system, each spin-1/2 is encoded in the ground state of the N- V center as $|0(1)\rangle = |m_s = +1(0)\rangle$. Now moving to the interaction picture with respect to $H_0 = \sum_i \omega_c \sigma_i^z/2$ and applying the rotating-wave approximation (RWA), we obtain the effective Hamiltonian [42]

$$H_I = \sum_i \frac{\Delta_i}{2} \sigma_i^z + \sum_{ij} \frac{g_{ij}}{4} \sigma_i^z \sigma_j^z - \frac{g_{ij}}{2} (\sigma_i^+ \sigma_j^- + \sigma_i^- \sigma_j^+) + \frac{\Omega(t)}{2} \sum_i [\cos \phi(t) \sigma_i^x + \sin \phi(t) \sigma_i^y], \quad (24)$$

where the detuning $\Delta_i = \omega_i - \omega + \sum_{k \neq i} g_{ik}/4$ results from the disorder and $\Omega(t)$ and $\phi(t)$ are the amplitude and phase of the global field.

Although the presence of unwanted detunings Δ_i typically leads to deviation in evolution, it also offers an opportunity for individual spin control, provided that the spins

acquire distinct energy splittings $\vec{\omega} = (\omega_1, \omega_2, \dots, \omega_N)$ that exhibit sufficiently large differences. Selectivity is achieved by applying a resonant narrow-band π pulse, which is composed of two shaped pulses and two broadband π pulses. Further details can be found in Sec. IV D. This ability to manipulate individual spins in nanoscale N- V arrays paves the way for the realization of scalable quantum information processors [43].

In the Hamiltonian, the zz -coupling term as delineated in Eq. (4) naturally emerges. Thus, a carefully designed evolution process becomes essential to mitigate the presence of undesired flip-flops and detuning terms. The disorder of the spin resonant transition and optical transition of each N- V center is intrinsic and can be modified by the application of external fields [44,45]. We establish a lower bound on the energy-splitting difference for every two spins, set as $\Delta_{\min} = (2\pi)4$ MHz. This is defined as $\forall(i, j), |\omega_i - \omega_j| = |\Delta_i - \Delta_j| \geq \Delta_{\min}$. With neighboring spins at a distance of 30 nm, the resulting coupling strength is $g_{ij} = (2\pi)1.924$ kHz. Thus, the flip-flop effect induced by $\sigma_i^+ \sigma_j^- + \sigma_i^- \sigma_j^+$ is highly suppressed due to the large detuning difference ($g_{ij} \ll \Delta_{\min}$). In addition, the unique energy splitting enables the manipulation of individual spin (see Sec. IV D). Therefore, we can write the effective Hamiltonian in a free-evolution process as [26]

$$H_z = \sum_i \frac{\Delta_i}{2} \sigma_i^z + \sum_{ij} \frac{g_{ij}}{4} \sigma_i^z \sigma_j^z. \quad (25)$$

To eliminate the detuning term, we design a basic pulse sequence that consists of a free evolution $U_z(t) = \exp(-iH_z t)$ and a following equal-length free evolution sandwiched by two broadband π pulses, $R_\pi^x(\mathbf{i}_N)$ and $R_\pi^{-x}(\mathbf{i}_N)$. Here, indices $\mathbf{i}_N = (1, 2, \dots, N)$ indicate all the spins, which means that all spins will be flipped simultaneously. A unitary governed by the zz Hamiltonian [see Eq. (4)] is realized as

$$\begin{aligned} U_{zz}(2t) &= R_\pi^{-x}(\mathbf{i}_N) U(t) R_\pi^x(\mathbf{i}_N) U(t) \\ &= \exp \left(-i \sum_{i \neq j} \frac{g_{ij} t}{2} \sigma_i^z \sigma_j^z \right). \end{aligned} \quad (26)$$

The design of the broadband pulse is explained in Sec. IV C. Note that this process is typically an application of pulsed dynamical decoupling in a multispin system [46], thereby inheriting the property of robustness against the dephasing noise.

In principle, the decoupling pulse could be replaced by more efficient pulses, such as the Carr-Purcell-Meiboom-Gill (CPMG) [47,48] sequence or other periodic pulses [49–51], to further extend the coherence time. Previous studies have reported coherence times of approximately 1 s [52], which is a crucial prerequisite for the applicability of our proposal to solid-state systems.

In addition, during the whole process, we have to deal with the inherent imperfections of the control field $H_c(t)$ [53]. The errors that we consider are the offset error δ and the Rabi-frequency error ϵ . The Hamiltonian with these two errors is

$$\begin{aligned} H_c(\epsilon, \delta, t) &= \frac{\delta \Omega_0}{2} \sigma_z \\ &+ \frac{\Omega(t)(1 + \epsilon)}{2} [\cos \phi(t) \sigma_x + \sin \phi(t) \sigma_y], \end{aligned} \quad (27)$$

where $\Omega_0 = \max |\Omega(t)|$ is the peak value of Rabi frequency $\Omega(t)$. In the following section, we develop general frameworks for generating broadband and narrow-band pulses, which are crucial to the experimental implementation, utilizing the global field $H_c(t)$ and the quantum optimal-control method. This approach integrates composite-pulse and shaped-pulse techniques to achieve the desired pulse characteristics, while also demonstrating robustness against the errors of the control field. In the simulation of the construction of the pulses, the weak-interaction zz terms are ignored, since their coupling strengths are much weaker than the Rabi frequency (approximately megahertz) of the driving field. For simplicity, we denote the single-spin pulses as $\alpha_\phi = \exp(-i\alpha\sigma_\phi/2)$ and $\alpha_z = \exp(-i\alpha\sigma_z/2)$.

C. Broadband pulses

We first design the broadband pulses π_x and $(\pi/2)_x$ (i.e., π_0 and $(\pi/2)_0$) employed to flip and initialize all spins simultaneously. To this end, we develop a composite pulse that is robust against the detuning error. The composite pulse is an easily implemented technique to generate a robust quantum pulse using a series of optimally chosen pulses to cancel out imperfections [54–58]. The pulses inside the composite pulse are realized by a constant Rabi frequency (i.e., rectangular pulses) with the Hamiltonian $H = (\delta\Omega_b/2)\sigma_z + [\Omega_b(1 + \epsilon)/2]\sigma_\phi$. The composite pulse is designed to be robust against the detuning error with a broad high-fidelity ($1 - F < 10^{-5}$) range $|\delta| < \delta_0$ that covers all spins,

$$2\delta_0\Omega_b > \omega_{\max} - \omega_{\min}, \quad (28)$$

where $\omega_{\max(\min)} = \max(\min)\{\vec{\omega}\}$. The frequency of the driving field is chosen to be $\omega_c = (\omega_{\max} + \omega_{\min})/2$. We construct a π_x gate with a symmetric pulse sequence $U(\vec{\phi}_{n_\phi}) = \pi_{\phi_1} \pi_{\phi_2} \dots \pi_{\phi_n} \dots \pi_{\phi_2} \pi_{\phi_1}$, where the phases $\vec{\phi}_{n_\phi} = (\phi_1 \dots \phi_n)$ are the parameters to be optimized. To develop a broadband π_x gate, we define a collective cost function $G(\vec{\phi}_{n_\phi}) = \sum_i a_i g(\delta_i, \vec{\phi}_{n_\phi})$, where a_i is the weight of the contribution g at working point $\delta = \delta_i$, and a single

component $g(\delta_i, \vec{\phi}_{n_\phi})$ is

$$g(\delta_i, \vec{\phi}_{n_\phi}) = 1 - F_i + D_i(\delta), \quad (29)$$

where the fidelity is $F_i = \text{Tr}[\pi_x U^\dagger(\delta_i, \vec{\phi}_{n_\phi})]/2$. The derivative is

$$D_i(\delta) = \sum_{k=1}^{k_i} \left\| c^{-k} \frac{\partial^k U(\delta_i, \vec{\phi}_{n_\phi})}{\partial \delta^k} \right\|_{\delta=\delta_i} \Big|_F, \quad (30)$$

where k_i is the derivative order with respect to δ at each working point δ_i , the coefficient c^{-k} is the weight of each order of derivative, and we usually set $c = 10$; $\|X\|_F = \sqrt{\sum_{ij} |X_{ij}|^2}$ is the Frobenius norm. By choosing the parameters and minimizing the cost function, a series of broadband composite pulses are generated (see Fig. 7). In addition, a broadband $(\pi/2)_x$ is developed by using a different symmetric pulse sequence, $U(\alpha, \vec{\phi}_{n_\phi}) = \alpha_{\phi_1} \pi_{\phi_2} \pi_{\phi_3} \cdots \pi_{\phi_{n_\phi}} \cdots \pi_{\phi_3} \pi_{\phi_2} \alpha_{\phi_1}$ [59], where α and $\vec{\phi}_{n_\phi}$ are the parameters to be optimized.

We present numerical simulations illustrating the infidelities of the optimal composite broadband pulses π_x and $(\pi/2)_x$ in Figs. 4(a) and 4(c), respectively. These simulation results demonstrate the robustness of composite pulses against both detuning and amplitude errors. The red dotted boxes in the figures highlight regions of high fidelity, where the relative biases of detuning and control amplitude errors are tolerable, with values of $\delta_0 = 0.35$ and $\epsilon_0 = 0.03$, respectively. For a large Rabi frequency of $\Omega_b = (2\pi)40$ MHz, seven spins at most could be manipulated with high fidelity simultaneously. In Figs. 4(b) and 4(d), we display vertical and horizontal slices of the infidelities along $\delta = 0$ and $\epsilon = 0$, respectively. Note that by adding a global phase Φ to the optimal phase $\vec{\phi}_{n_\phi}$, arbitrary broadband gates π_Φ and $(\pi/2)_\Phi$ are obtained. These various broadband pulses π and $\pi/2$ enable the implementation of pulse sequences such as the *XY*-series pulse [60–62].

D. Selective pulses

We now introduce the approach to implementing the selective pulses that are the building blocks of our proposal. These tailored pulses ensure precise control over the state of the targeted spin while minimizing disruption to the states of other spins. Within our framework, the unique energy splitting exhibited by each spin allows the manipulation of spin j at the spatial site to be equated with the application of a resonant narrow-band pulse that closely matches the intrinsic frequency ω_j . In the absence of spin interactions, the system Hamiltonian in Eq. (24) simplifies

to the single-spin case,

$$H_s(t) = \frac{\delta\Omega_0}{2}\sigma_z + \frac{\Omega_x(t)}{2}\sigma_x + \frac{\Omega_y(t)}{2}\sigma_y, \quad (31)$$

resulting in a unitary $U_s = \mathcal{T}e^{-i\int H_s(\tau)d\tau}$.

The third diagram depicted in Fig. 5(b) illustrates the application of a desired selective α_x (i.e., $U_s = \alpha_x$) to the spin in the resonant region (RR), defined as $|\delta| \leq \delta_r$. This RR exhibits a finite width, which serves to maintain the accuracy of a selective pulse in the presence of an off-set error. On the other side, the states of spins within the far-detuning regions (DRs), defined as $|\delta| \geq \delta_d$, remain unperturbed. Thus, to implement a selective α_x to a specific spin, it is necessary to ensure that the driving is nearly resonant to the target spin, and that the detunings of the rest of the spins are strictly within the DR. This condition is naturally satisfied by setting the minimum disorder value greater than the TR width ($\Delta_{\min} \geq \delta_d\Omega_0$).

In Fig. 5(a), we illustrate the scheme of the composite selective pulse α_x , which is composed of two distinct processes. The first is a single $(\alpha/2)_x$ pulse, while the second is a $(\alpha/2)_x$ pulse sandwiched between two broadband π_x and $\pi_x^\dagger$ pulses. In Fig. 5(b), we demonstrate the details of the selective pulse in the frequency domain, which is divided into five ranges by δ_r and δ_d . In the RR, the evolution of the entire process is $\pi_x^\dagger (\alpha/2)_x \pi_x (\alpha/2)_x = \alpha_x$, which is the desired gate. However, within the DR, the offset term $\delta\Omega_0\sigma_z/2$ dominates the Hamiltonian, resulting in a nearly pure z rotation. Thus the evolution in the DR is $\pi_x^\dagger \theta_z \pi_x \theta_z = \mathbb{1}$, where $\theta = \delta\Omega_0 T_s/2$. Lastly, the evolution operator of the shaped pulse in the transition region (TR), defined as $|\delta| \in (\delta_s, \delta_d)$, is of no consequence.

At this point, the objective is to design the pulses to implement the α_x and identities within the RR and the DR, respectively. The values of δ_d and δ_r characterize the selectivity in the frequency domain and robustness against the detuning, respectively. Therefore, it is recommended to obtain a relatively low value of δ_d to achieve high selectivity, while a larger value of δ_d is required to ensure robustness against detuning in the optimization process. We introduce a shaped-pulse control field to achieve these objectives, enabling additional robustness against the Rabi-frequency fluctuation error ϵ . The two components of the shaped pulse are both the sum of half-sine functions that are modulated by a Gaussian envelope [63]:

$$\tilde{\Omega}_{x(y)}(t) = \sum_{k=1}^{N_{a(b)}} a(b)_k \sin[(2k-1)\omega_s t] e^{-\frac{(t-T_s/4)^2}{2\sigma^2}}, \quad (32)$$

where $\omega_s = 2\pi/T_s$, $\vec{a}$, $\vec{b}$, and σ are the parameters to generate a smooth pulse. Gaussian-shaped functions in the time domain usually result in selectivity in the frequency domain, which has been widely used [64,65]. Employing

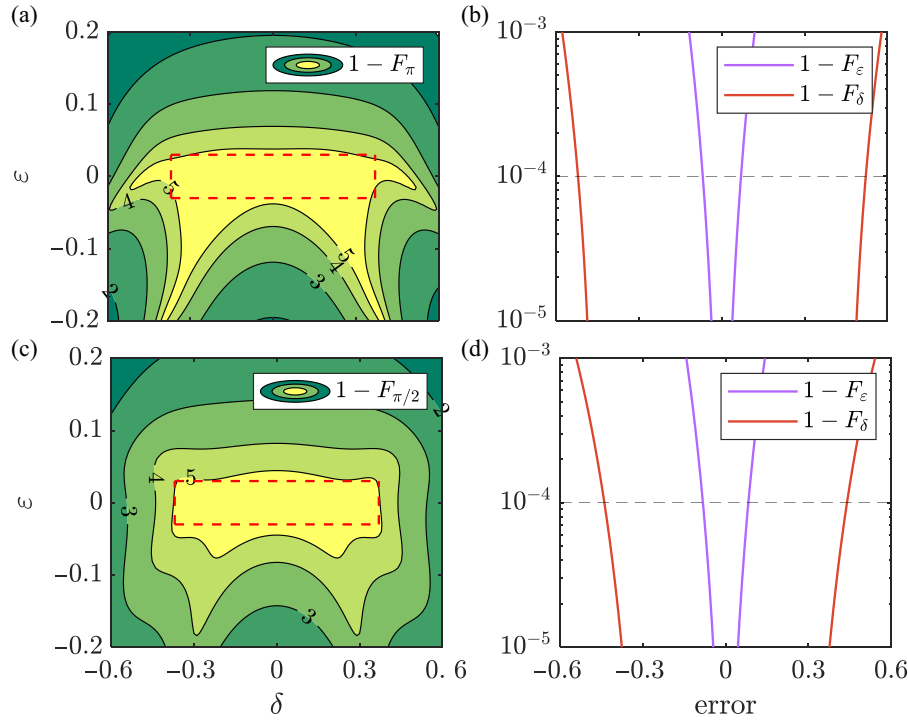


FIG. 4. Simulated infidelities between the optimal composite pulses and the target pulses. (a) The infidelity of the composite pulse π_x versus the static detuning and control-amplitude error with $n_\phi = 10$. (b) The vertical and horizontal cut along $\delta = 0$ and $\epsilon = 0$, respectively. (c),(d) The same with (a) and (b) but for an optimal $(\pi/2)_x$ gate and the corresponding parameter $n_\phi = 8$. The integers m in (a) and (c) indicate the infidelity 10^{-m} . Additionally, two regions of high fidelity ($F > 1 - 10^{-5}$) are identified by dashed red boxes, bounded by $|\delta| \leq 0.35$ and $|\epsilon| \leq 0.03$.

sine components guarantees that the Rabi frequency $\tilde{\Omega}_{x(y)}$ begins and ends with zero values. To limit the magnitude of the Rabi frequency to the value Ω_0 , both components are adjusted relative to a normalization factor, $\Omega_{x(y)}(t) = r\tilde{\Omega}_{x(y)}(t)$, where $r = \Omega_0 / \max(\sqrt{\tilde{\Omega}_x^2(t) + \tilde{\Omega}_y^2(t)})$. In the presence of errors δ and ϵ , we define a cost function

$$G = \bar{I}_I + \bar{I}_\alpha + c\bar{D}(\epsilon), \quad (33)$$

where the quantities $\bar{I}_\alpha$ and $\bar{I}_I$ represent the average infidelities associated with the desired α_x and the identity gate within the RR and the DR, respectively. The third term is the average derivative with respect to ϵ in the RR and c is the weight.

By minimizing the cost function, the obtained optimal pulse shapes are determined by the parameters $\vec{a}$ and $\vec{b}$. The shaped pulses and the resulting infidelity $1 - F_\pi$ associated with π_x near the RR are shown in Figs. 5(c) and 5(d). We highlight the high-fidelity area with the dotted red box that is bounded by $|\delta| \leq \delta_r = 0.05$ and $|\epsilon| \leq 0.03$. The horizontal cut of F_π along $\epsilon = 0$ is shown in Fig. 5(e). Furthermore, we show the infidelity associated with the identity operator in Fig. 5(f). The corresponding boundary of the DR, denoted by the dotted red lines, is located

at $\delta_d = 1.75$. The narrow peaks of fidelity F_π and infidelity I_I in the vicinity of $\delta = 0$ illustrate the frequency selectivity of the optimal pulse, which allows the realization of the cluster-state-preparation proposal. More details and parameters can be found in Appendix C. In addition, an optimal selective $(\pi/2)_x$ pulse is presented in Fig. 8. It is worth noting that we use composite pulses to generate selective π -pulse coupling modulation, to demonstrate the importance of selectivity more intuitively. However, a simpler modulation method, with only the Gaussian-shaped pulse in Eq. (32) to implement a π_x pulse within the RR, regardless of z rotations within the DR, is achievable, given that all these local z rotations commute with the evolution of each segment.

E. Numerical simulation of the preparation protocol

We complete this section with the numerical simulations for both the four- and six-spin systems using the broadband and selective pulses we have developed. The systems that we consider here are both spin-ring systems, in which the position of each spin shifts slightly from the desired position. Thus, the position-dependent coupling strengths are determined, giving us the details of the time intervals $\vec{\tau}$. Using the pulse sequence $\mathcal{S}_4$ and $\mathcal{S}_6$ that we have developed, we obtain the total evolution process in the presence

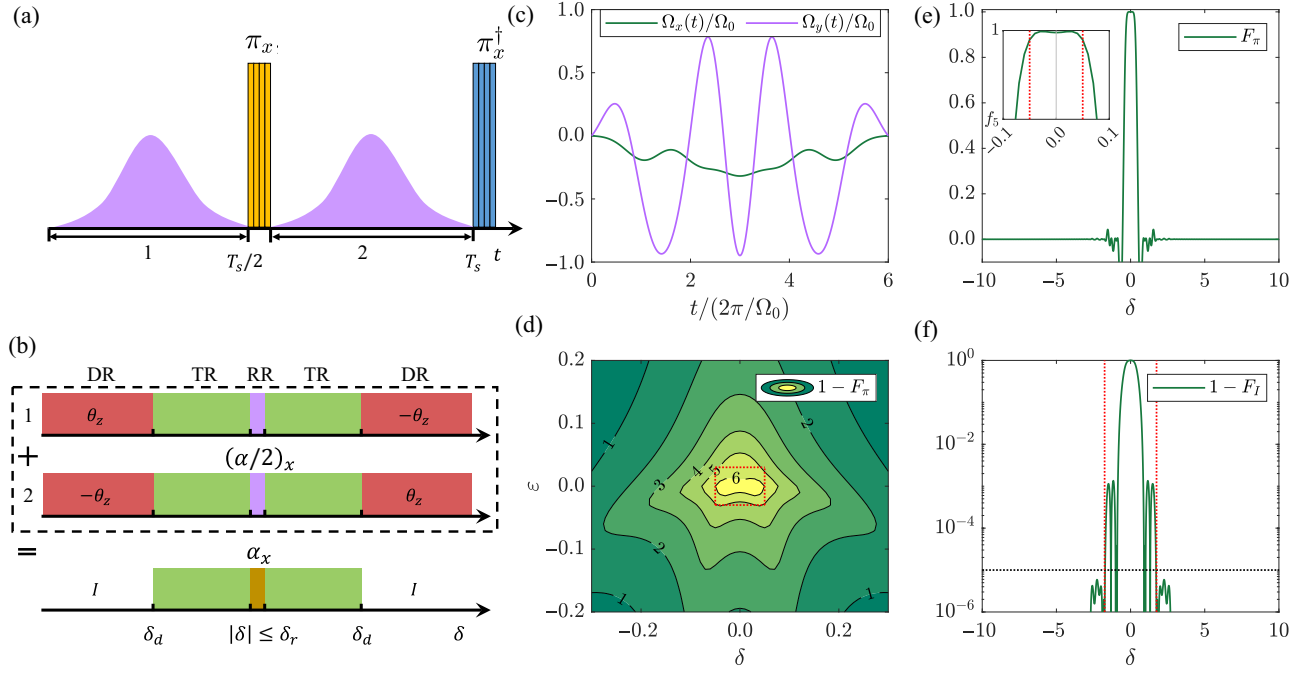


FIG. 5. The scheme of the composite selective pulse α_x . (a) The time domain of the selective pulse α_x . The pulse is composed of two sequential shaped $(\alpha/2)_x$ (purple curves) and two broadband π_x and $\pi_x^\dagger$ pulses (yellow and blue blocks) applied at $t = T_s/2$ and T_s , respectively. (b) The frequency domain of the pulse α_x . The first two diagrams represent the performance of two pulses in the time domain with the corresponding unitaries $(\alpha/2)_x$ and $\pi_x^\dagger (\alpha/2)_x \pi_x$, respectively. In the far-detuning region (DR), defined as $|\delta| \geq \delta_d$, the large-detuning term $\delta\Omega_0\sigma_z/2$ dominates and results in two nearly pure z pulses $\pm\theta_z$, where $\theta = \delta\Omega_0 T_s$. As shown in the third diagram, the two z pulses ultimately cancel, yielding an identity operator. Meanwhile, in the resonant region (RR), $|\delta| \leq \delta_r$, both of the two unitary operators remain $(\alpha/2)_x$ (purple blocks), resulting in a desired α_x pulse. The transition region (TR) is given by $|\delta| \in (\delta_s, \delta_d)$, where the evolution (green blocks) is usually quite different from the desired gate. (c) The Gaussian-shaped pulses $\Omega_{x(y)}$ to implement a single $(\alpha/2)_x$ pulse. (d) Contour plots of the simulated infidelity of the selective π_x pulse versus δ and ε . The high-fidelity area is highlighted by the red box bounded by $|\delta| = 0.05$ and $|\varepsilon| = 0.03$. (e) The fidelity between the composite pulse and the ideal π_x gate as a function of the detuning. The inset shows the fidelity in the RR and $f_5 = 0.99999$. The resonant region is indicated by two red dotted lines $\delta = \pm 0.05$. (f) The infidelity between the composite pulse and the identity gate as a function of the detuning, the DRs being defined as $|\delta| > \delta_d = 1.75$ here.

of errors as

$$U_C(\vec{\tau}, \delta, \varepsilon) = \prod_k R_\pi^x(\mathbf{i}^{(k)}) e^{-iH_{zz}\tau_k} R_\pi^{-x}(\mathbf{i}^{(k)}). \quad (34)$$

In our simulation, the selective π pulse $R_\pi^x(\mathbf{i}^{(k)})$ is applied sequentially as $R_\pi^x(\mathbf{i}^{(k)}) = R_\pi^x(i_{n_k}^{(k)}) \cdots R_\pi^x(i_2^{(k)}) R_\pi^x(i_1^{(k)})$ to each spin involved in $\mathbf{i}^{(k)}$. We choose the maximum value of the selective pulses as $\Omega_0 = \Delta_{\min}/\delta_d = (2\pi) 2.28$ MHz, resulting in $T_s = 6.2132 \mu\text{s}$. The Rabi frequency in the broadband pulse is $\Omega_b/2\pi = 30$ (40) MHz for the four-(six-) spin system. The achievable fidelity of the preparation $F_C = |\langle \psi_0 | U_C^\dagger U_I | \psi_0 \rangle|^2$ depends on the fidelity of each selective pulse, $R_\pi^x(\mathbf{i}^{(k)})$, usually constrained by the presence of zz crosstalk. In this context, we propose an optimization procedure to mitigate the impact of zz coupling through adjustments in the time intervals $\vec{\tau}_o$. The optimal procedure is demonstrated under the nonerror

conditions

$$1 - F_{\text{op}} = \min_{\vec{\tau}} \frac{1}{2^N} \text{Tr} \left\{ U_I U_C^\dagger(\vec{\tau}, \delta = 0, \varepsilon = 0) \right\}, \quad (35)$$

where F_{op} is the optimal cluster-state-preparation fidelity. It is pertinent to note that the fidelity of preparation in the spin-lattice system should be comparable to that in the spin-ring system, given that they both require identical pulse sequences, albeit with differing intervals. Moreover, the fidelity of the cluster-state preparation can be further enhanced by applying a series of proper local unitary operations on each spin.

Here, we show the simulated optimal fidelities within four- and six-spin-ring systems in Fig. 6. In Figs. 6(a) and 6(c), the green dots and black circles denote the positions of individual spins and their corresponding ideal positions, respectively. By choosing the total time of the cluster-state preparation $T_c = 4\pi/g_1 = 1.0395$ ms, the optimal

time intervals $\bar{\tau}_o$ are determined by demonstrating the optimization process in Eq. (35); the corresponding contour plots of the preparation fidelity are shown in Figs. 6(b) and 6(d), in which a high-fidelity area over 0.999 (0.99) exists in the four- (six-) spin system. The values of the fidelity without error are 0.9997 and 0.9937, respectively. More details of the simulations are given in Appendix C. Moreover, in Appendix D, we perform numerical evaluations of the impact of possible error sources during the preparation process. The results indicate that the preparation scheme maintains high fidelity across the range of experimentally feasible parameters.

F. Toward hybrid quantum computing

We conclude this section by discussing the potential approaches for developing a hybrid FBQC using spin-ring systems as RSGs. Once the resource states are prepared, two main steps are required to complete the entangling fusion measurement. First, the target spins within each RSG must be entangled with photonic qubits. Then, the photons from two RSGs are collected and interfered at a beam splitter to carry out the required fusion measurements.

Several schemes have been proposed and implemented with the objective of entangling photons and spins [66–69]. These schemes are based on the principle that certain degrees of freedom inherent to photons exhibit responses conditional on spin states and transitions. Typically, the spin systems are embedded in separate optical cavities to enhance the spin-photon interference and improve the photon-collecting efficiency. In our approach, the selective pulses that we have developed are used to populate the state of the target spin to the far-detuned third $|a\rangle = |m_s = -1\rangle$ level. This enables selective entanglement of the target spin with photons, owing to the significantly different optical transition of the target spin. Additionally, we assume that the energy splittings between $|1\rangle$ and $|a\rangle$ of each spin in an RSG exhibit sufficiently large differences, which is essential to ensure the effectiveness of the selective pulses.

One entanglement method starts by applying a selective π pulse to flip the target spin from $|1\rangle = |m_s = 0\rangle$ to $|a\rangle$. Subsequently, the state $|a\rangle|v\rangle$ couples with $|1\rangle|f\rangle$ (i.e., the vacuum Rabi oscillation) when the cavity mode is resonant with the transition between $|1\rangle$ and $|a\rangle$. Here, $|v\rangle$ represents the vacuum state of the cavity mode and $|f\rangle$ denotes the single-photon Fock state in the cavity [15,70,71]. An alternative entanglement protocol involves photon emission from an excited state. For example, we could use a short laser pulse to excite $|1\rangle$ to the excited state $|E_{x/y}\rangle$ presenting the same spin projection. The spontaneous emission back to $|1\rangle$ then directly entangles the spin with the number of photons [72–74].

Once the spin-photon entangled states have been prepared, time must be allowed for the photons to leak from the cavities. The required entangling measurements are then performed using linear quantum optical methods [75,76]. A common technique is “path erasure,” in which photons are directed to interfere on a beam splitter to remove the which-path information [15]. The detection of a single photon in this setup probabilistically projects the spins in the two RSGs into an entangled state. The success of path erasure depends on the photons being indistinguishable in all their degrees of freedom, including the frequency, spatial wave packet, polarization, and arrival time. To ensure that photons from different cavities share identical frequencies, the ability to tune optical transitions with local external electromagnetic fields is essential [72,73].

In the spin-photon entanglement scheme based on conditional-amplitude reflection [22,77], satisfying the indistinguishability condition may be less challenging, because the photons are generated from the same coherent source. In this scheme, the photon is reflected by a high-cooperativity cavity when the transition of the target spin is resonant with the cavity. This mechanism entangles the spin state and the photon numbers. The required high cooperativity not only facilitates entanglement generation but also demonstrates robustness against detuning. The cooperativity $C \sim 1$ [78,79] achieved thus far by coupling a single N-V center with an optical cavity is significantly lower than the requisite level of high cooperativity. However, it is possible to achieve high entanglement fidelity by sacrificing entanglement probability using the reflected or transmitted photons as resources. Reference [80] presents an entanglement protocol based on reflection from two remote N-V cavity nodes. A single photon splits in a Mach-Zehnder interferometer and is reflected from both nodes; the conditional detection in one output port then heralds an entangled state between the two N-V centers. By appropriately choosing the cavity decay rate and the mirror asymmetry, the scheme ensures high-fidelity heralded entanglement with moderate cooperativity. Reference [81] proposes another entangling scheme based on the transmitted photon through a symmetric cavity, operating within the bad-cavity regime and with a homogeneously broadened matter spin. Moreover, the frequency-bin encoding scheme provides an alternative method for spin and photon entanglement without employing indistinguishable photons. In this approach, the target spins in two RSGs are resonant with the corresponding cavity and the photonic qubit is encoded in two distinct frequencies, typically enabling a potential platform for the photonic quantum information process [82,83].

The atomlike properties of the N-V centers enable the application of photonic connection schemes originally developed on the basis of a cavity with single atoms and optical photons. However, in our configuration, a basic

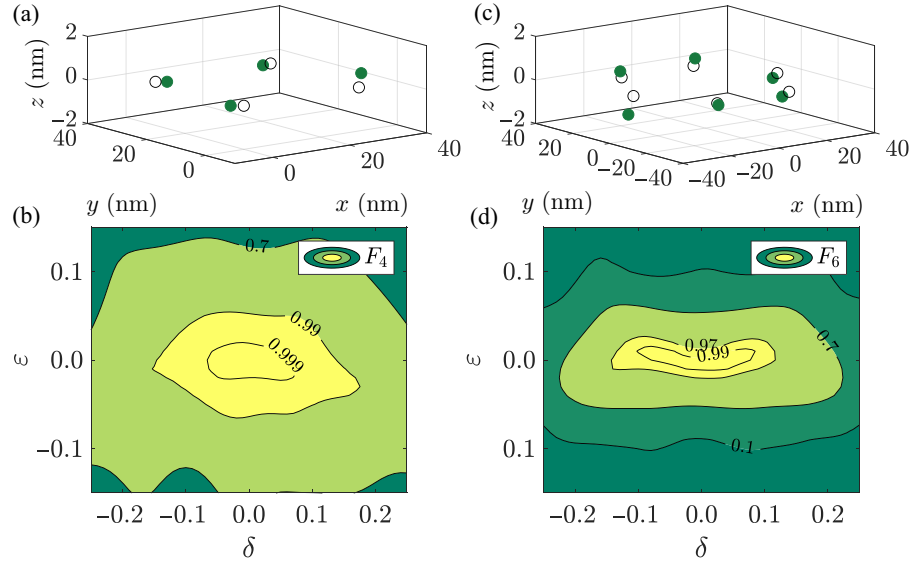


FIG. 6. The simulated fidelities of the cluster-state preparation in four- and six-spin systems. (a),(c) The solid green circles represent the positions of the spin, while the open black circles denote the ideal positions. (b),(d) Contour plots of the state-preparation fidelities F_{op} versus the detuning error δ and the Rabi-frequency error ϵ . The values of the fidelities in the four- and six-spin systems are 0.9997 and 0.9937 in nonerror cases, respectively.

register consists of a spin-ring system placed in a cavity. Therefore, existing connection schemes need to be modified to achieve selective connectivity associated with the target spin. One advantage of this configuration is that it requires fewer resources to construct a quantum network, as it demands fewer optical connections.

Finally, additional optimizations or improvements may be necessary to enhance the probability of establishing a successful connection. These may include implementing a repeat-until-success protocol with a timing-encoding bin, improving the photon detection efficiency, and addressing photon loss. The RSG methodology and photonic connection techniques suggest the possibility of developing a hybrid quantum computing technology that leverages both matter spins and light.

V. CONCLUSIONS AND OUTLOOK

In conclusion, we establish a scheme to generate cluster states within the quantum spin system, where spins are spatially localized and separated. The principle of our proposal is to design an optimal pulse-control sequence that exhibits nonuniform modulation among two-spin couplings in each segment, eventually preserving the NN couplings while eliminating all long-range ones. Our proposal applies to systems of varying configurations, including one-dimensional and even high-dimensional systems. We theoretically derive the corresponding pulse sequences for few-spin systems, accounting for the possible positional errors of each spin, and explore the feasibility of extending this approach to larger systems. The present preparation

proposal is a versatile tool and can be extended to more general systems with similar forms of interaction, such as other single defects and molecular spin systems, etc. To illustrate our proposal in a solid-state system, we develop the broadband and selective gates required in the pulse sequence using composite-pulse and shaped-pulse techniques with experimentally feasible parameters. All pulses are designed to be robust against static offset detuning and Rabi-frequency fluctuations to mitigate inaccuracies caused by these control imperfections.

Our preparation scheme facilitates the realization of various quantum information scenarios. The first point is the construction of the hybrid matter-photon quantum computing architecture. The preparation of four- and six-ring cluster states in N-V center systems represents a crucial first step toward FBQC, serving as RSGs [14]. The second step involves implementing an entangling fusion measurement between two spins located in distinct resource states. This is achieved by placing each in an optical cavity and employing probabilistic linear-optical operations [22]. Furthermore, the proposed scheme offers a candidate quantum control method for the direct interaction-based spin systems [84], with the potential to advance quantum information processing in spin arrays [85].

The second potential application is that the capacity to exert selective control over the system represents a possibility for building a quantum register within the solid-state spin system. First, the obtained solutions of the linear equation [see Eq. (9)] allow the preparation of different entangled states with the various target phase vectors $\vec{\alpha}$. This approach is expected to facilitate the generation

of the entangled states, such as the weighted cluster state and the Greenberger-Horne-Zeilinger states, and even provides the possibility of simulating high-dimensional entangled states within a lower-dimensional system [86]. Second, the selective pulses that we have developed enable three basic single-spin gates to be implemented, as $\theta_x = (\pi/2)_{\pi/2}\pi_{(\theta-\pi)/2}(\pi/2)_{\pi/2}$, $\theta_y = (\pi/2)_{\pi}\pi_{\theta/2}(\pi/2)_{\pi}$, and $\theta_z = \pi_{\pi+\theta/2}\pi_0$ [87]. Also, a selective controlled-Z gate could be implemented by setting one element, α_{ij} , equal to π . Consequently, a set of universal quantum gates is implemented [88]. Lastly, the sizes of the system could be extended by involving the proximal nuclear spins (e.g., N and ^{13}C) [42,89–91].

We would like to stress that our approach is versatile and can be translated directly to other systems, such as molecular color centers in suitable organic molecules. As an example, the optically addressable chromium-ion- (Cr^{4+}) based molecular qubit exhibits a spin-1 ground state, which can be manipulated by microwaves and tuned by strong-field (aryl) ligands in a high-symmetry configuration [92]. The established synthetic chemical method allows for precise control over the positions of molecular qubits, thereby ensuring the emergence of dipole interactions [93]. Furthermore, the coherence time of the molecular qubit can be significantly extended by the symmetry control over the host-crystal surroundings [94]. Due to its excellent spin properties and rich degrees of control, it offers an alternative platform for implementing our cluster-state-preparation scheme and, more importantly, it is a crucial platform for realizing spin-based quantum information processing [43]. Moreover, in the cluster-state-preparation scheme proposed in other quantum systems, such as trapped ions, external controls are employed to engineer the interactions and thereby adjust their accumulated phases [95,96]. Our approach, leveraging pulsed-coupling modulation, may introduce an additional and auxiliary means that reduces the required control resources.

Finally, some other improvements to our proposal for preparation are worthy of further discussion and investigation. The first of these improvements is to develop an advanced and efficient algorithm to generate cluster states within larger systems, thus expanding the scope and scalability of our preparation proposal. In the N- V center system, this requires the development of more selective narrow-band pulses to lower the detuning difference demanded. The important steps remain the preparation of a large N- V center system with the same orientations and the appropriate manipulations, including the possible modulation of the energy splittings of electronic spins using an electric field through the electrode or applying a magnetic gradient [44,45]. The second is to use composite gates to implement a robust two-qubit zz gate, in which random fluctuations in the coupling strength are corrected [97,98]. Although this scheme is developed for a two-qubit system,

it suggests that residual errors arising from uncertainties in the coupling characterization can be mitigated by suitable composite-pulse techniques.

In addition, although our state-preparation scheme achieves high fidelity by suppressing long-range zz couplings during free evolution, the zz crosstalk during the implementation of selective pulses still constrains our fidelity. Therefore, we set $d = 30$ nm to maintain small coupling strengths. Several approaches have been proposed to suppress zz crosstalk. A widely used one is to introduce additional hardware (a tunable coupler [99–101]) or extra qubits (shunted flux qubits [102, 103]), to eliminate undesired zz -coupling terms, albeit at the expense of increased system complexity. Alternative approaches, such as adjusting the control fields [104] applied to spins or suppressing the impact of zz crosstalk through pulse and scheduling co-optimization [105] or pulsed dynamical decouplings [101,106], however, require the capability of individual control of spins. Thus, in solid-state spin systems, advancing high-fidelity selective pulses under the limitations of global control and nontunable coupling is imperative.

ACKNOWLEDGMENTS

We are grateful for the comments on the manuscript by Daniel Dulog and Jonas Breustedt. Discussions with Jan Haase at the early stages of this work are acknowledged. Y. L. thanks Qingyun Cao for useful discussions on the experimental aspects of the N- V center. This work is supported by the Bundesministerium für Bildung und Forschung (BMBF, the Federal Ministry of Education and Research), under the funding program “Quantum Technologies—From Basic Research To Market” in the “Spinning” project (Project No. 13N16215) and the “Competitive German Quantum Computer” (CoGeQ) project (Project No. 13N16101). This work was supported by the European Union (EU) “Capacitation of Quantum-Entangled NV-Center Sensing” (C-QuENS) project (Grant No. 101135359) and the EU “Spin Based Quantum Computer and Simulator” (SPINUS) project (Grant No. 101135699). We acknowledge support by the state of Baden-Württemberg through “High Performance Computing, Data Intensive Computing and Large Scale Scientific Data Management in Baden-Württemberg” (bwHPC) and the German Research Foundation (DFG) through Grant No. INST 40/575-1 FUGG (JUSTUS 2 cluster).

DATA AVAILABILITY

The data that support the findings of this paper are not publicly available. The data are available from the authors upon reasonable request.

APPENDIX A: ADDITIONAL INFORMATION ON SOLVABLE SYSTEMS

1. Solvable systems

In this appendix, we are going to present more details about the solutions in the four-, five-, and six-spin systems. In a four-qubit system, the index sequence of spin is $\mathcal{S}_4 = \{(0), (1), (1, 2), (2), (2, 3), (3), (4)\}$, with the corresponding matrix

$$M_4 = \begin{pmatrix} 1 & -1 & 1 & -1 & -1 & 1 & 1 \\ 1 & 1 & -1 & -1 & 1 & -1 & 1 \\ 1 & 1 & 1 & 1 & -1 & -1 & 1 \\ 1 & -1 & -1 & 1 & 1 & 1 & -1 \\ 1 & -1 & -1 & 1 & -1 & -1 & 1 \\ 1 & 1 & -1 & -1 & -1 & 1 & -1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix} \quad (\text{A1})$$

and the corresponding inverse matrix

$$M_4^{-1} = \frac{1}{4} \begin{pmatrix} 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ -1 & 0 & 0 & -1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & -1 & -1 & 0 \\ -1 & -1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & -1 & -1 & 0 \\ 0 & -1 & -1 & 0 & 0 & 1 & 1 \\ 0 & 0 & -1 & -1 & 1 & 0 & 1 \end{pmatrix}. \quad (\text{A2})$$

Here, $\vec{\alpha} = (\alpha_{12}, \alpha_{23}, \alpha_{34}, \alpha_{14}, \alpha_{13}, \alpha_{24}, T_c)^T$ is the target vector.

In a five-spin system, we identify two distinct sequences of spin indices. The first sequence is $\mathcal{S}_5 = \{(0), (1), (1, 2), (2), (2, 3), (3), (3, 4), (4), (4, 5), (5), (1, 5)\}$ and the corresponding inverse matrix is

$$M_5^{-1} = \frac{1}{4} \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & -1 \\ -1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 1 & 1 \\ -1 & -1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & -1 & -1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & -1 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & -1 & -1 & 0 & 0 & 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & -1 & 0 & 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 & 0 & -1 & -1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & -1 & -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 & -1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & -1 & -1 & 0 \end{pmatrix}. \quad (\text{A3})$$

The second sequence is $\{(0), (1, 2), (2, 3), (3, 4), (4, 5), (1, 5), (1, 4), (2, 4), (2, 5), (3, 5), (1, 3)\}$, and the corresponding inverse matrix is

$$M_5^{-1} = \frac{1}{12} \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 2 \\ -1 & -1 & 2 & 2 & -1 & -1 & -1 & 2 & -1 & -1 & 1 \\ -1 & -1 & -1 & 2 & 2 & -1 & -1 & -1 & 2 & -1 & 1 \\ 2 & -1 & -1 & -1 & 2 & -1 & -1 & -1 & -1 & 2 & 1 \\ 2 & 2 & -1 & -1 & -1 & 2 & -1 & -1 & -1 & -1 & 1 \\ -1 & 2 & 2 & -1 & -1 & -1 & 2 & -1 & -1 & -1 & 1 \\ -1 & -1 & -1 & 2 & -1 & -1 & 2 & -1 & -1 & 2 & 1 \\ -1 & -1 & -1 & -1 & 2 & 2 & -1 & 2 & -1 & -1 & 1 \\ 2 & -1 & -1 & -1 & -1 & -1 & 2 & -1 & 2 & -1 & 1 \\ -1 & 2 & -1 & -1 & -1 & -1 & -1 & 2 & -1 & 2 & 1 \\ -1 & -1 & 2 & -1 & -1 & 2 & -1 & -1 & 2 & -1 & 1 \end{pmatrix}. \quad (\text{A4})$$

The solutions of the time intervals are obtained directly as $\vec{\tau} = M_5^{-1} \cdot \vec{\alpha}$, with the target vector

$$\vec{\alpha} = (\alpha_{12}, \alpha_{23}, \alpha_{34}, \alpha_{45}, \alpha_{15}, \alpha_{13}, \alpha_{24}, \alpha_{35}, \alpha_{14}, \alpha_{25}, T_c)^T. \quad (\text{A5})$$

The numbers of π pulses employed in each sequence are 12 and 22, respectively.

In the six-spin system, the sequence is $\mathcal{S}_6 = \{(0), (1, 2), (2, 3), (3, 4), (4, 5), (5, 6), (1, 6), (1, 3), (1, 4), (4, 6), (2, 4),$

$(2, 6), (3, 6), (3, 5), (1, 5), (2, 5)\}$, which means that the required number of π pulses is $N_\pi = 32$. The target vector is

$$\vec{\alpha} = (\alpha_{12}, \alpha_{23}, \alpha_{34}, \alpha_{45}, \alpha_{56}, \alpha_{16}, \alpha_{13}, \alpha_{24}, \alpha_{35}, \alpha_{46}, \alpha_{15}, \alpha_{26}, \alpha_{14}, \alpha_{25}, \alpha_{36}, T_c)^T. \quad (\text{A6})$$

The corresponding inverse matrix is

$$M_6^{-1} = \frac{1}{16} \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & 1 & 1 & -1 & -1 & -1 & 1 & 1 & -1 & -1 & -1 & -1 & 1 & 1 \\ -1 & 1 & -1 & 1 & 1 & 1 & -1 & -1 & -1 & 1 & 1 & -1 & 1 & -1 & -1 & 1 \\ 1 & -1 & 1 & -1 & 1 & 1 & -1 & -1 & -1 & -1 & 1 & 1 & -1 & 1 & -1 & 1 \\ 1 & 1 & -1 & 1 & -1 & 1 & 1 & -1 & -1 & -1 & -1 & 1 & -1 & -1 & 1 & 1 \\ 1 & 1 & 1 & -1 & 1 & -1 & 1 & 1 & -1 & -1 & -1 & -1 & 1 & -1 & -1 & 1 \\ -1 & 1 & 1 & 1 & -1 & 1 & -1 & 1 & 1 & -1 & -1 & -1 & -1 & 1 & -1 & 1 \\ -1 & -1 & -1 & 1 & 1 & -1 & 1 & 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 & 1 \\ -1 & 1 & -1 & -1 & 1 & -1 & -1 & -1 & -1 & 1 & -1 & -1 & 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 & -1 & -1 & 1 & -1 & 1 & 1 & 1 & -1 & -1 & 1 & -1 & 1 \\ -1 & -1 & -1 & -1 & 1 & 1 & 1 & 1 & 1 & -1 & 1 & -1 & -1 & -1 & 1 & 1 \\ -1 & -1 & 1 & 1 & -1 & -1 & 1 & -1 & 1 & -1 & 1 & 1 & 1 & -1 & -1 & 1 \\ 1 & -1 & -1 & 1 & -1 & -1 & -1 & 1 & -1 & -1 & 1 & -1 & 1 & 1 & 1 & 1 \\ 1 & -1 & -1 & -1 & -1 & 1 & -1 & 1 & 1 & 1 & -1 & 1 & 1 & -1 & -1 & 1 \\ -1 & 1 & 1 & -1 & -1 & -1 & -1 & 1 & -1 & 1 & 1 & 1 & -1 & -1 & 1 & 1 \\ -1 & -1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 & -1 & -1 & 1 & 1 & 1 & 1 \end{pmatrix}. \quad (\text{A7})$$

2. Two examples in the four-spin system

Here, we introduce two interesting examples with different target vectors $\vec{\alpha}$ of the four-spin system, demonstrating the broad applicability of our proposal. The first is to prepare a cluster state within a small three-dimensional system, where four spins are located on the vertices of a regular tetrahedron. We now set all phases to $\theta_{ij} = \pi$, resulting in $\alpha_{12} = \alpha_{23} = \alpha_{34} = \alpha_{14} = \pi/g_1$ and $\alpha_{13} = \alpha_{24} = \pi/(\sqrt{8}g_1)$, assuming that the coupling strength $g \propto r^{-3}$. The duration $T_c = (2 - 1/\sqrt{8})\pi/g_1$ and the length of each segment is

$$\tau_1 = \frac{\pi}{g_1}, \quad \tau_{2,4,6,7} = 0, \quad \tau_{3,5} = \left(1 - \frac{1}{\sqrt{8}}\right) \frac{\pi}{2g_1}. \quad (\text{A8})$$

The second example is to demonstrate an identity evolution, i.e., to eliminate all of the coupling terms. This is achieved by setting $\theta_{12} = 4\pi$ and the rest of the phases equal to 0, which results in a unitary $U(T_c) = \exp(-i\pi\sigma_1^z\sigma_2^z) = -\mathbb{1}$. The corresponding time duration

$$T_c = 4\pi/g_1 \text{ and}$$

$$\tau_{1,3,6,7} = \frac{\pi}{g_1}, \quad \tau_{2,4,5} = 0. \quad (\text{A9})$$

APPENDIX B: DETAILS OF STATE PREPARATION IN N-V CENTER SYSTEMS

First, we initialize all N-V centers to the ground state $|1\rangle = |m_s = 0\rangle$, and a product state $\otimes_i |+\rangle_i$ could be prepared by applying a broadband pulse $(\pi/2)_y$. In the four-spin system, the detuning of each N-V center that we set is $(2\pi)[-6, -2, 2, 6]$ MHz. As shown in Fig. 4, a wide high-fidelity range $2\delta_0\Omega_b = (2\pi)21$ MHz with $\Omega_b = (2\pi)30$ MHz allows one to rotate all N-V centers simultaneously. The corresponding positions of the four N-V centers are

$$\begin{aligned} \vec{r}_1 &= [-1.3077, 2.7694, -0.0631], \\ \vec{r}_2 &= [29.5664, -1.3499, 0.7147], \\ \vec{r}_3 &= [30.3426, 33.0349, -0.2050], \\ \vec{r}_4 &= [3.5784, 30.7254, -0.1241] \text{ nm}, \end{aligned} \quad (\text{B1})$$

denoted by the green dots in Fig. 6(a), which results in the coupling strengths

$$\begin{aligned} [g_{12}, g_{23}, g_{34}, g_{14}]_1 &= (2\pi)[1.7143, 1.2728, \\ &\quad 2.6796, 2.2727], \\ [g_{13}, g_{24}]_2 &= (2\pi)[0.6186, 0.7370] \text{ kHz}. \end{aligned} \quad (\text{B2})$$

The corresponding ideal NN and NNN coupling strengths are $g = (2\pi)[1.9241, 0.6802]$. Choosing a total evolution time $T_c = 4\pi/g_1$, the corresponding optimal time intervals are

$$\begin{aligned} \vec{\tau}_o &= [267.6897, 135.7387, 111.6563, 93.3421, \\ &\quad 149.3451, 119.6501, 163.0152] \mu\text{s}. \end{aligned} \quad (\text{B3})$$

In the six-spin system, the detunings are $(2\pi)[-10, -6, -2, 2, 6, 10]$ MHz and the Rabi frequency is $\Omega_b = (2\pi)40$ MHz. The spin positions are

$$\begin{aligned} \vec{r}_1 &= [29.2127, 1.4384, -0.2414], \\ \vec{r}_2 &= [15.8884, 26.3060, 0.3192], \\ \vec{r}_3 &= [-16.1471, 25.2258, 0.3129], \\ \vec{r}_4 &= [-31.0689, 1.3703, -0.8649], \\ \vec{r}_5 &= [-15.8095, -27.6923, -0.0301], \\ \vec{r}_6 &= [12.0557, -26.0830, -0.1649] \text{ nm}, \end{aligned} \quad (\text{B4})$$

which are indicated by the green dots in Fig. 6(c). These spins are slightly different from the ideal position represented by the black circles. The corresponding coupling strengths are

$$\begin{aligned} [g_{12}, g_{23}, g_{34}, g_{45}, g_{56}, g_{16}]_1 &= (2\pi)[2.3094, 1.5774, 2.3136, 1.4645, 2.3888, 1.5229], \\ [g_{13}, g_{24}, g_{35}, g_{46}, g_{26}, g_{15}]_2 &= (2\pi)[0.3864, 0.3449, 0.3505, 0.3885, 0.3583, 0.3369], \\ [g_{14}, g_{25}, g_{36}]_3 &= (2\pi)[0.2370, 0.2116, 0.2588] \text{ kHz}, \end{aligned} \quad (\text{B5})$$

and the ideal coupling strengths are $(2\pi)[1.9241, 0.3703, 0.2405]$, respectively. By choosing $T_c = 4\pi/g_1$, the optimal intervals are

$$\begin{aligned} \vec{\tau}_o &= [175.8969, 92.8740, 118.1285, 87.8812, \\ &\quad 122.2728, 88.8799, 117.1140, 31.2665, \\ &\quad 30.8702, 34.4913, 31.2770, 36.4844, \\ &\quad 38.8677, 35.8695, 35.4856, 35.2711] \mu\text{s}. \end{aligned} \quad (\text{B6})$$

APPENDIX C: DETAILS OF OPTIMIZATION

Here, we present details of the optimization process employed in the development of broadband and selective pulses. All optimizations are demonstrated in the MATLAB Optimization Toolbox [107] to minimize the corresponding cost functions.

1. Derivative of the optimal-control field

Achieving a robust and optimal-control field requires aligning $U(T, \vec{\lambda})$ closely with the target gate while minimizing its variations with respect to errors $\vec{\lambda}$. To obtain the derivatives $\partial U / \partial \vec{\lambda}$, we first expand the whole process using a Dyson series, with a general Hamiltonian

$$H(t) = H_0(t) + H_p(t), \quad (\text{C1})$$

where $H_p(t)$ represents a perturbation error. Thus the evolution is

$$\begin{aligned} U(T) &= U_0(T) \sum_{n=0}^{\infty} \int_0^T dt_1 \int_0^{t_1} dt_2 \int_0^{t_2} dt_3 \cdots \int_0^{t_{n-1}} dt_n \\ &\quad \times (-i)^n \tilde{H}_p(t_1) \tilde{H}_p(t_2) \tilde{H}_p(t_3) \cdots \tilde{H}_p(t_n), \end{aligned}$$

where $U_0(T) = \mathcal{T} \exp \left[-i \int_0^T d\tau H_0(\tau) \right]$ and $\tilde{H}_p(t) = U_0(t)^{-1} H_p(t) U_0(t)$.

In a single-error case, i.e., $\tilde{H}_p(t) = \varepsilon H_\varepsilon(t)$, the k th-order derivative is

$$\begin{aligned} \frac{d^k U(T)}{d\varepsilon^k} &= U_0(T) \sum_{n=k}^{\infty} (-i)^n \frac{n!}{(n-k)!} \varepsilon^{n-k} \\ &\quad \times \mathcal{D}(\tilde{H}_\varepsilon^{(1)}, \tilde{H}_\varepsilon^{(2)}, \dots, \tilde{H}_\varepsilon^{(n)}), \end{aligned} \quad (\text{C2})$$

where $\tilde{H}_\varepsilon^{(i)} = \tilde{H}_\varepsilon(t_i)$ and where $\mathcal{D}(\tilde{H}_\varepsilon^{(1)}, \tilde{H}_\varepsilon^{(2)}, \dots, \tilde{H}_\varepsilon^{(n)})$ denotes the multiple integrals in Eq. (C2) times ε^{-n} , which could be numerically calculated as an exponential of a matrix [108–112]:

$$L(t) = -i \begin{pmatrix} H_0(t) & A_1(t) & 0 & \cdots & 0 & 0 \\ 0 & H_0(t) & A_2(t) & \cdots & 0 & 0 \\ 0 & 0 & H_0(t) & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & H_0(t) & A_n(t) \\ 0 & 0 & 0 & \cdots & 0 & H_0(t) \end{pmatrix},$$

resulting in

$$\mathcal{T} \exp \left[\int_0^T d\tau L(\tau) \right] = \begin{pmatrix} U_0(T) & \mathcal{D}(\tilde{A}_1^{(1)}) & \cdots & \mathcal{D}(\tilde{A}_1^{(1)}, \dots, \tilde{A}_n^{(n)}) \\ 0 & U_0(T) & \cdots & \mathcal{D}(\tilde{A}_2^{(1)}, \dots, \tilde{A}_n^{(n-1)}) \\ 0 & 0 & \cdots & \mathcal{D}(\tilde{A}_3^{(1)}, \dots, \tilde{A}_n^{(n-2)}) \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \mathcal{D}(\tilde{A}_n^{(1)}) \\ 0 & 0 & \cdots & U_0(T) \end{pmatrix},$$

where $\tilde{A}_k^{(i)} = U_0^{-1}(t_i) A_k(t_i) U_0(t_i)$. Here, all the diagonal blocks are $U_0(T)$. By choosing all $A_k(t) = H_\varepsilon(t)$, we directly obtain all the orders of the derivatives. At the zero point $\varepsilon = 0$, the derivative is

$$\frac{d^k U(T)}{d\varepsilon^k} = U_0(T) (-i)^k k! \mathcal{D}(\tilde{H}_p^{(1)}, \tilde{H}_p^{(2)}, \dots, \tilde{H}_p^{(n)}). \quad (\text{C3})$$

In the two-error case, the perturbation Hamiltonian reads

$$H_p(t) = \varepsilon H_\varepsilon(t) + \delta H_\delta(t). \quad (\text{C4})$$

Expanding to the second order, the evolution operator is

$$U(T) = U_0(T) \left\{ \mathbb{1} - i \int_0^T dt_1 [\varepsilon \tilde{H}_\varepsilon(t_1) + \delta \tilde{H}_\delta(t_1)] - \int_0^T dt_1 \int_0^{t_1} dt_2 [\delta^2 \tilde{H}_\delta^{(1)} \tilde{H}_\delta^{(2)} + \varepsilon^2 \tilde{H}_\varepsilon^{(1)} \tilde{H}_\varepsilon^{(2)} + \delta \varepsilon (\tilde{H}_\delta^{(1)} \tilde{H}_\varepsilon^{(2)} + \tilde{H}_\varepsilon^{(1)} \tilde{H}_\delta^{(2)})] \right\} + \dots, \quad (\text{C5})$$

which gives a multiple derivative at $\delta = 0$ and $\varepsilon = 0$:

$$\frac{\partial}{\partial \varepsilon} \frac{\partial}{\partial \delta} U(T) = -U_0(T) \left[\mathcal{D}(\tilde{H}_\delta^{(1)}, \tilde{H}_\varepsilon^{(2)}) + \mathcal{D}(\tilde{H}_\varepsilon^{(1)}, \tilde{H}_\delta^{(2)}) \right].$$

In our simulation, the optimization is demonstrated on multiple working points δ_i . The required corresponding derivatives on these points are calculated by choosing a new Hamiltonian $\tilde{H}_0(t) = H_0(t) + \delta_i H_\delta(t)$. All the exponential functions of the time-dependent matrices are approximated numerically by the production of constant N_T slices of the total duration as

$$\mathcal{T} \exp \left[\int_0^T d\tau B(\tau) \right] = \prod_{k=1}^{N_T} \exp[B(t_j) \Delta t], \quad (\text{C6})$$

where $t_j = j \Delta t = jT/N_T$.

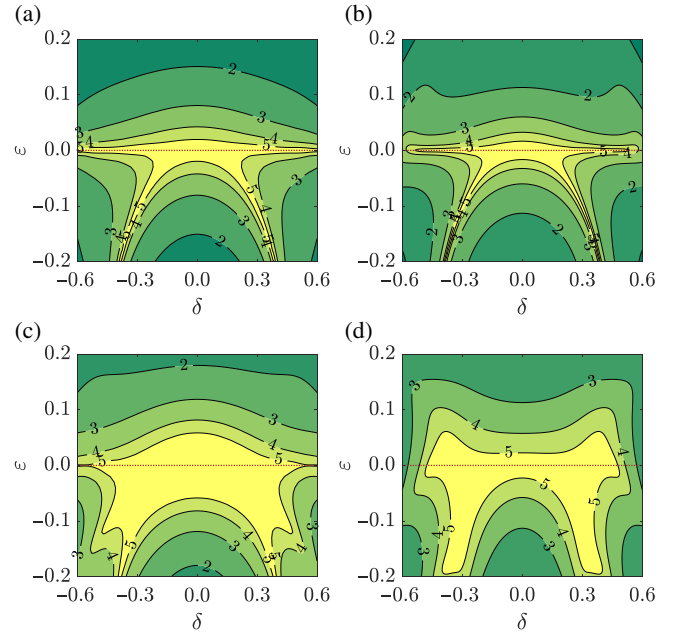


FIG. 7. The simulated infidelities the optimal broadband pulses π_x . (a),(b) Two optimal pulses with wide high-fidelity ranges $|\delta| \leq 0.6$, with the corresponding (a) $n_\phi = 8$ and (b) $n_\phi = 10$. (c),(d) Two pulses that are robust against both δ and ε , with (c) $n_\phi = 11$ and (d) $n_\phi = 10$.

2. Details of optimal broadband and selective pulses

In this section, we present further details on the development of broadband and selective pulses. To illustrate the practicality of our broadband pulse-optimization scheme, we present four additional optimal broadband pulses π_x in Fig. 7. Unlike the top two pulses, which are only robust against the detuning error, the two pulses at the bottom are robust against both errors. Furthermore, the parameters for the broadband pulse π_x in Fig. 4 are $\vec{\phi} = [0.4531, 1.1692, -0.2510, 0.8733, 0.9023, 0.7061, 0.6578, -0.3322, 1.3085]\pi$ and for $(\pi/2)_x$ are $(\alpha, \vec{\phi}) = [0.5355, 0.4645, 0.1322, 0.3789, 1.3156, 0.8444, 0.8222, 1.1370, -0.1073]\pi$.

In the development of selective pulses, the unitary of the basic segment $(\alpha/2)_x$ is governed by the Hamiltonian

$$H_s(\delta, \varepsilon, t) = \frac{\delta \Omega_0}{2} \sigma_z + \frac{(1 + \varepsilon)}{2} [\Omega_x(t) \sigma_x + \Omega_y(t) \sigma_y]. \quad (\text{C7})$$

In our simulation, the corresponding evolution is

$$U_s(\delta, \varepsilon, T_s/2) = \mathcal{T} e^{-i \int_0^{T_s/2} d\tau H(\delta, \varepsilon, \tau)} \quad (\text{C8})$$

and the total evolution of the composite pulse is

$$U(\delta, \varepsilon, T_s) = \pi_x^\dagger U_s(\delta, \varepsilon, T_s/2) \pi_x U_s(\delta, \varepsilon, T_s/2). \quad (\text{C9})$$

TABLE II. The parameters to implement selective π_x and $(\pi/2)_x$ pulses.

α	Parameters
π	$\vec{a} = [-2.5136 \ 1.6657, -1.1799, -0.1023, 0.53610.3596]$ $\vec{b} = [-2.8761 \ -3.1094 \ 4.1194 \ 9.4484 \ 1.9703 \ 2.0822]$ $\vec{\delta}_s = [0, 0.02, 0.04, 0.06], \vec{\delta}_l = [1, 2, \dots, 20]$
$\pi/2$	$\vec{a} = [0.5937 \ 2.3491 \ 0.2494 \ -0.9176 \ -0.4898 \ 0.1127]$ $\vec{b} = [0.8659 \ 3.9191 \ -1.3086 \ -3.6099 \ -3.1087 \ 0.8589]$ $\vec{\delta}_s = [0, 0.02, \dots, 1], \vec{\delta}_l = [1, 2, \dots, 20]$

Specifically, the cost function in Eq. (33) is defined as

$$G = c_1 \bar{I}_I(\vec{\delta}_l) + c_2 \bar{I}_\alpha(\vec{\delta}_s) + c_3 \bar{D}(\vec{\delta}'_s) \Big|_{\varepsilon=0}, \quad (\text{C10})$$

where $\vec{\delta}_l$ and $\vec{\delta}_s$ denote the discrete values sampled within the DR and the RR, respectively, and the weights are $\vec{c} = [1, 1, 10^{-4}]$. The average derivative with respect to ε is

$$\bar{D}(\vec{\delta}'_s) = \frac{1}{\tilde{n}_s} \sum_{k=1}^{\tilde{n}_s} \left\| \frac{\partial U_s(\vec{\delta}'_s(k), \varepsilon, T_s/2)}{\partial \varepsilon} \right\|_{\varepsilon=0} \Big|_F, \quad (\text{C11})$$

where $\vec{\delta}'_s = [0, 0.04]$ represents another discrete sampling in the RR, different from $\vec{\delta}_s$. The duration $T_s = 12(2\pi/g_1)$ and $\sigma = T_s/10$. In the numerical simulation of the evolution in a single $(\alpha/2)_x$ process, the number of slices is $N_T = 1500$. The rest of the parameters of the pulses, α_x , are listed in Table II. In Fig. 8, we show the composite selective $(\pi/2)_x$ pulse obtained by the optimization as described in the main text.

APPENDIX D: THE ERROR RESOURCES

In this appendix, we briefly review the additional sources of error in the preparation protocol. We identify three main sources of concern: deviations in the selective pulses due to interactions between N-V centers, the impact of the far-detuned third levels of the N-V centers, and the influence of local environmental factors such as ^{13}C nuclei or P_1 centers. The sources of error affect two main aspects of the preparation process, including the control pulses and the free-evolution process. To demonstrate the robustness of our protocol to these errors, we numerically evaluate the corresponding preparation fidelity within the range of experimentally feasible parameters.

1. Impact of N-V–N-V interaction

In the development of selective pulses, we have ignored the weak interactions between N-V centers. Although

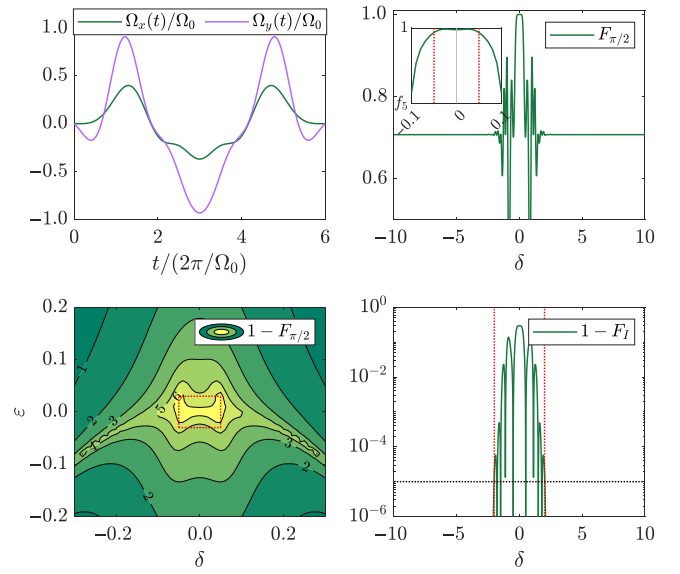


FIG. 8. The optimal composite $(\pi/2)_x$ pulse. The notation is identical to that used in Fig. 5.

Gaussian-shaped pulses enhance the frequency-domain selectivity, they also reduce the ability to suppress the interactions due to their relatively longer duration. On a microsecond timescale, the coupling strengths at the kilohertz level still have an impact on the extremely demanding fidelities that we are aiming to achieve here. To quantify this influence, we simulate the effect of varying the NN distance d in an ideal four-spin square-lattice system. In Fig. 9(a), we depict the average infidelities $\bar{I}_\pi$ of the selective pulses $U_s(j)$ that we have developed for four spins with $j = 1, 2, 3, 4$ compared to the ideal pulses π_j^x . In the presence of different driving imperfections, the infidelities are approximately 10^{-4} when $d = 30$ nm and become negligible compared to other sources of imperfections when $d > 30$ nm. For the sake of simplicity, we define the infidelity as

$$\bar{I}_\pi = 1 - \frac{1}{4} \sum_{j=1}^4 \frac{1}{2^4} \text{Tr} \left[\pi_j^x U_s^\dagger(j) \right]. \quad (\text{D1})$$

In the presence of interactions, the Hamiltonian to implement the desired selective pulse is

$$H(t) = H_s(t) + H_{\text{int}}, \quad (\text{D2})$$

where $H_s(t)$ represents the Gaussian-shaped pulse applied on all spins. According to time-dependent perturbation theory, the weak-interaction term H_{int} could be regarded as a perturbation term. Therefore, to implement a selective pulse on spin j , the corresponding unitary operator $U_s(T_s) = \mathcal{T} \exp \left[-i \int_0^{T_s} H(\tau) d\tau \right]$ is expanded up to first

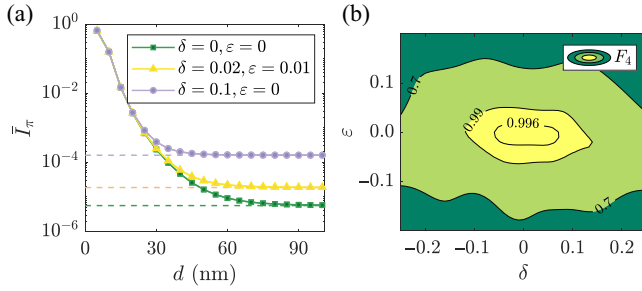


FIG. 9. (a) The simulated average infidelity $\bar{I}_\pi$ with respect to four selective π_x pulses. In this configuration, all four are located at the vertices of a square with sides of length d . The green squares, yellow triangles, and purple circles represent the average infidelities $\bar{I}_\pi$ with the presence of driving imperfections as $(\delta, \epsilon) = (0, 0)$, $(0.02, 0.01)$, $(0.1, 0)$, respectively. The dashed lines indicate the infidelities in the absence of interactions. (b) The fidelity of cluster-state preparation in the four-spin system without optimizing the length of the time intervals $\bar{\tau}$ [i.e., Eq. (10)]. All parameters are identical to those presented in Fig. 2(b).

order as

$$U(T_s) \simeq U_s(T_s)(1 - i\tilde{H}_{\text{int}}T_s) \simeq U_s(T_s) \exp(-i\tilde{H}_{\text{int}}T_s), \quad (\text{D3})$$

where the unitary $U_s(T_s) = \mathcal{T} \exp[-i \int_0^{T_s} H_s(\tau) d\tau]$ represents the high-fidelity selective pulse π_j^x that we have developed in the absence of interactions. Consequently, the interaction can be approximately expressed as $\tilde{H}_{\text{int}} \simeq (\pi_j^x)^\dagger H_{zz} \pi_j^x = H_{zz} - 2H_{zz}^j$. This arises because the zz couplings are the crucial components of the interaction Hamiltonian [see Eq. (24)]. Here, H_{zz}^j represents a subset of H_{zz} , which is the sum of the zz couplings involving spin j .

In our preparation pulses, a selective pulse is usually followed by an evolution $U_{zz}(\tau)$ [i.e., Eq. (26)] governed by H_{zz} . The bias induced by $\tilde{H}_{\text{int}}$ in Eq. (D3) typically affects the subsequent evolution, despite the fact that H_{zz} and $\tilde{H}_{\text{int}}$ are different in some certain components. Thus, it is possible and necessary to adjust the evolution time τ to compensate for the bias to some extent and thereby achieve higher fidelity. This is the reason why the optimization [i.e., Eq. (35)] over $\bar{\tau}$ is effective. To provide a point of comparison, we present the fidelity of cluster-state preparation associated with the original time length $\bar{\tau}$, as defined in Eq. (10), which has a maximum value of 0.9967 [see Fig. 9(b)].

2. Impact of the third N-V level

The N-V center contains ground-state triplet (^3A), in which the degeneracy of the states $|m_s = \pm 1\rangle$ is lifted by applying an external magnetic field B_z along the z axis. For the sake of simplicity, we denote the three states as $|0\rangle$ and $|\pm 1\rangle$ here. With a relatively strong B_z , all the

driving fields employed in our proposal are almost resonant with the transitions between $|0\rangle$ and $|+1\rangle$, while they are far detuned with those between $|0\rangle$ and $|-1\rangle$. Consequently, all the transitions to $|-1\rangle$ will be suppressed by the large detuning. In addition, there are other ways to remove the effects of the third level. At first, it is helpful to take the third level $|-1\rangle$ as an additional source of error in the development of the selective and broadband pulses. Second, it is also worthwhile noting that use of the circularly polarized microwave fields will help to reduce further the impact of the off-resonant third level in the N-V centers.

The Hamiltonian of the system is

$$H(t) = \sum_i H_i^0 + \sum_{ij} H_{ij}^{\text{dd}} + H_c(t). \quad (\text{D4})$$

We now derive the effective Hamiltonian $\tilde{H}(t) = U_0^\dagger(t)(H - H_0)U_0(t)$ in the interaction picture with respect to the unitary $U_0(t) = \exp(-iH_0t)$, with

$$H_0 = \sum_i \omega_c | +1 \rangle_i \langle +1 | + \omega_c | -1 \rangle_i \langle -1 |. \quad (\text{D5})$$

The two identical frequencies ω_c are assigned to ensure a time-independent interaction. This transforms the operators into

$$U_0^\dagger(t) |\pm 1\rangle_i \langle 0| U_0(t) = e^{i\omega_c t} |\pm 1\rangle_i \langle 0|, \quad (\text{D6})$$

which makes the spin operators S_i^x and S_i^y oscillate rapidly in time.

The first term of the local Hamiltonian becomes

$$\tilde{H}_i^0 = \Delta_i | +1 \rangle_i \langle +1 | + \Delta_i^{(-1)} | -1 \rangle_i \langle -1 |, \quad (\text{D7})$$

where the detunings $\Delta_i = D + \delta_i^D - \gamma_e B_z - \delta_i^z - \omega_c$ and $\Delta_i^{(-1)} = 2\gamma_e B_z + 2\delta_i^z + \Delta_i$. The Zeeman-splitting term $\omega_e = 2\gamma_e B_z$ dominates; thus a large detuning $\Delta_i^{(-1)}$ suppresses the transitions to state $|-1\rangle$. The control field becomes

$$\tilde{H}_c(t) = \sum_{i=1}^N \frac{1}{\sqrt{2}} \Omega(t) [\cos(\phi(t)) S_i^x + \sin(\phi(t)) S_i^y]. \quad (\text{D8})$$

The magnetic dipole-dipole interaction between spin i and spin j is

$$H_{ij}^{\text{dd}} = \vec{S}_i \cdot \vec{A}_{ij} \cdot \vec{S}_j, \quad (\text{D9})$$

and among the nine couplings in $\tilde{H}_{ij}^{\text{dd}} = U_0^\dagger(t) H_{ij}^{\text{dd}} U_0(t)$, the fast oscillating terms $S_i^x S_j^z$, $S_i^y S_j^z$, $S_i^z S_j^x$ and $S_i^z S_j^y$ are

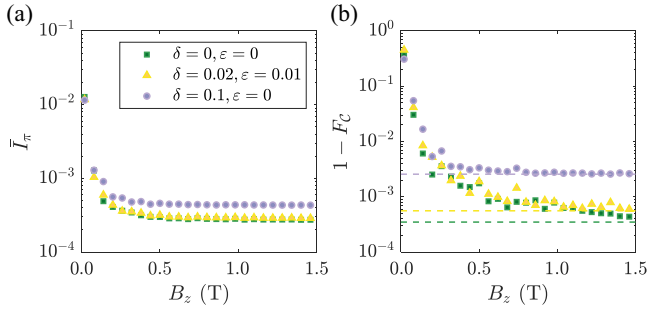


FIG. 10. The simulation in the triplet states. (a) The simulated average infidelity $\bar{I}_\pi$ with respect to all selective π_x pulses in the four-spin system as a function of B_z . (b) The infidelities of the cluster-state preparation within the triplet states. The dashed lines represent the infidelities in two-level systems.

ignored due to the RWA. Thus we derive the interaction Hamiltonian as

$$\begin{aligned} \tilde{H}_{\text{nn}} = & \sum_{i \neq j}^N A_{ij}^{xx} (c_{i,+} c_{j,+}^\dagger + \text{h.c.}) + A_{ij}^{yy} (c_{i,-} c_{j,-}^\dagger + \text{h.c.}) \\ & + A_{ij}^{xy} (i c_{i,-1} c_{j,+1}^\dagger - i c_{i,+1} c_{j,-1}^\dagger + \text{h.c.}) + A_{ij}^{zz} S_i^z S_j^z. \end{aligned} \quad (\text{D10})$$

Here, the operators are $c_{i,\pm} = |\pm\rangle_{ii} \langle 0|$ and $c_{i,\pm 1} = |\pm 1\rangle_{ii} \langle 0|$, with the states defined as $|\pm\rangle_i = (|+1\rangle \pm |-1\rangle)_i / \sqrt{2}$. $A_{ij}^{\alpha\beta}$ is the element of the coupling tensor $\vec{A}_{ij}$, and the coupling strength in the main text is $g_{ij} = A_{ij}^{zz}$. The Hamiltonian [see Eq. ((25))] could be naturally obtained with a projection $\mathcal{P} = \otimes_i P_i = \otimes_i (|+1\rangle_{ii} \langle +1| + |0\rangle_{ii} \langle 0|)$

$$H_I = \mathcal{P} \tilde{H}(t) \mathcal{P}. \quad (\text{D11})$$

In Fig. 10(a), we present the average infidelities $\bar{I}_\pi$ with respect to four selective π_x pulses as a function of the magnetic field B_z . The infidelity is defined as $1 - \text{Tr}[\pi_i^x \mathcal{P} U_s^\dagger(j) \mathcal{P}] / 2^4$, where $U_s^\dagger(j)$ is the selective pulse-flipping spin j within the triplet states. In Fig. 10(b), we show the cluster-state preparation with different driving imperfections [the labels are the same as those in Fig. 9(a)]. The three dashed lines indicate the infidelities of the preparation in the two-level system extracted from Fig. 6(b).

3. Effect of nearby nuclear spins

In this section, we aim to discuss the effect of the local environment on all N-V centers. The main factor contributing to dephasing is the interaction with other spins, particularly the P1 centers and the ^{13}C nuclear spin bath. To achieve long coherence times, our proposal emphasizes using highly pure samples to ensure that only a minimal

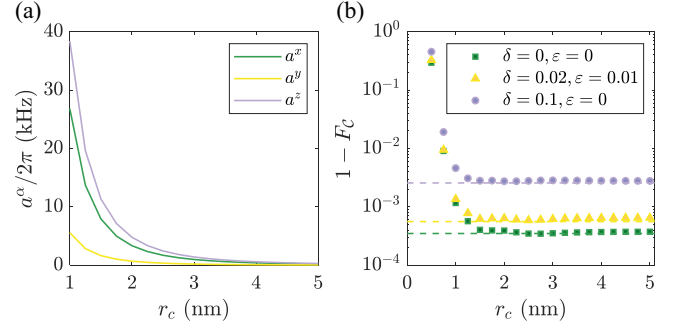


FIG. 11. The simulation in the four-spin system, with each N-V center having a nearby ^{13}C nuclear spin. (a) The maximum coupling strength a^α . (b) The simulated infidelity of the system including the nuclear spins as a function of the distances between the N-V centers and the nuclear spins. The imperfections of the driving field are also taken into account. For distances exceeding 1.5 nm, the influence of the nuclei becomes negligible compared to other sources of imperfection.

number of nuclear spins interact with the N-V center system. For simplicity, we assume that each N-V center is coupled to a nearby nuclear spin at a distance of a few nanometers. In this case, the total Hamiltonian is [62]

$$H_{\text{tot}} = H_{\text{N-V}} + H_C + H_{\text{hyp}}. \quad (\text{D12})$$

Here, $H_{\text{N-V}}$ is the H_z [see Eq. ((25))] of the N-V centers. The nuclear-spin Hamiltonian is $H_C = -\gamma_c \sum_i B_z \vec{I}_i$, where $\vec{I}_i = [I_i^x, I_i^y, I_i^z]$ are the spin-1/2 operators of the nuclear spin i . The hyperfine interaction is $H_{\text{hyp}} = \sum_i \vec{S}_i^z \vec{a}_i \cdot \vec{I}_i$, where $\vec{a}_i$ is the coupling-strength vector. Moving into the subspace $S_i^z \rightarrow (\sigma_i^z + \mathbb{1})/2$, the total Hamiltonian is

$$H_{\text{tot}} = H_z + \sum_i \left(-\gamma_c B_z I_i^z + \frac{1}{2} (\sigma_i^z + \mathbb{1}) \vec{a}_i \cdot \vec{I}_i \right). \quad (\text{D13})$$

Moving to the interaction picture with respect to $-\omega_z I_i^z$ with $\omega_z = \gamma_c B_z$, we could eliminate the fast-oscillating terms $I_i^x \rightarrow \cos(\omega_z t) I_i^x + \sin(\omega_z t) I_i^y$ and $I_i^y \rightarrow -\sin(\omega_z t) I_i^x + \cos(\omega_z t) I_i^y$ due to the RWA. We then obtain the effective Hamiltonian, which is

$$\tilde{H}_{\text{tot}} = H_z + \sum_i \left(\frac{\vec{a}_i^z}{2} \sigma_i^z I_i^z + \frac{\vec{a}_i^z}{2} I_i^z \right). \quad (\text{D14})$$

These three terms mutually commute and the $\sigma_i^z I_i^z$ interaction could be averaged together with the detuning error in Eq. (26). The above derivation is based on the fact that the N-V centers interact weakly with the nuclear spins.

In the numerical simulation, we choose the Hamiltonian in Eq. (D13) with $B_z = 1.5$ T to describe the system. To evaluate the influence of the couplings between the N-V centers and the nuclear spins, we assign the vector of the

nuclear spin i connecting the corresponding N- V center i as $\vec{r}_c = r_c \hat{e}_i$. The unit vectors are randomly chosen as

$$\begin{aligned}\hat{e}_1 &= [0.1817, 0.6198, -0.7634]^T, \\ \hat{e}_2 &= [0.5394, 0.1994, -0.8181]^T, \\ \hat{e}_3 &= [-0.1197, 0.0946, 0.9883]^T, \\ \hat{e}_4 &= [0.6404, -0.3121, 0.7018]^T.\end{aligned}\quad (\text{D15})$$

In Fig. 11(a), we show the coupling strengths a^α between the N- V centers and the corresponding nuclear spins. Here, $a^\alpha = \max(a_i^\alpha)$ ($i = 1, 2, 3, 4$) is the maximum value among four coupling strengths with $\alpha = x, y, z$. In Fig. 11(b), we show the preparation infidelities with different distances r_c . Since a new error source, the nuclear spins, is included, we optimize the time intervals as in Eq. (35) to obtain the result in Fig. 11(b). With a large magnetic field B_z , the effects of the nuclear spins are highly suppressed. At short distances, the nuclear spins still lead to significant infidelities in the preparation due to their large coupling strengths, which are tens of times larger than g_1 , and the long preparation time. A reported enriched ^{12}C (99.998%) sample [113] indicates that the probability of finding a single ^{13}C within 1.5 nm of the N- V center is about 5%. This low value ensures that the preparation process is free from the impact of nuclear spins.

- [1] D. Deutsch, Quantum theory, the Church-Turing principle and the universal quantum computer, *Proc. Roy. Soc. A* **400**, 97 (1985).
- [2] V. Vedral and M. B. Plenio, Basics of quantum computation, *Prog. Quantum Electron.* **22**, 1 (1998).
- [3] D. Gottesman and I. L. Chuang, Demonstrating the viability of universal quantum computation using teleportation and single-qubit operations, *Nature* **402**, 390 (1999).
- [4] J. Eisert, K. Jacobs, P. Papadopoulos, and M. B. Plenio, Optimal local implementation of nonlocal quantum gates, *Phys. Rev. A* **62**, 052317 (2000).
- [5] R. Raussendorf and H. J. Briegel, A one-way quantum computer, *Phys. Rev. Lett.* **86**, 5188 (2001).
- [6] M. A. Nielsen, Optical quantum computation using cluster states, *Phys. Rev. Lett.* **93**, 040503 (2004).
- [7] H. J. Briegel, D. E. Browne, W. Dür, R. Raussendorf, and M. Van den Nest, Measurement-based quantum computation, *Nat. Phys.* **5**, 19 (2009).
- [8] R. Raussendorf, D. E. Browne, and H. J. Briegel, Measurement-based quantum computation on cluster states, *Phys. Rev. A* **68**, 022312 (2003).
- [9] M. A. Nielsen, Cluster-state quantum computation, *Rep. Math. Phys.* **57**, 147 (2006).
- [10] E. Knill, R. Laflamme, and G. J. Milburn, A scheme for efficient quantum computation with linear optics, *Nature* **409**, 46 (2001).
- [11] D. E. Browne and T. Rudolph, Resource-efficient linear optical quantum computation, *Phys. Rev. Lett.* **95**, 010501 (2005).
- [12] M. A. Nielsen and C. M. Dawson, Fault-tolerant quantum computation with cluster states, *Phys. Rev. A* **71**, 042323 (2005).
- [13] H. Bombin, I. H. Kim, D. Litinski, N. Nickerson, M. Pant, F. Pastawski, S. Roberts, and T. Rudolph, Interleaving: Modular architectures for fault-tolerant photonic quantum computing, *arXiv:2103.08612* [quant-ph].
- [14] S. Bartolucci, P. Birchall, H. Bombin, H. Cable, C. Dawson, M. Gimenó-Segovia, E. Johnston, K. Kieling, N. Nickerson, M. Pant, F. Pastawski, T. Rudolph, and C. Sparrow, Fusion-based quantum computation, *Nat. Commun.* **14**, 912 (2023).
- [15] S. Bose, P. L. Knight, M. B. Plenio, and V. Vedral, Proposal for teleportation of an atomic state via cavity decay, *Phys. Rev. Lett.* **83**, 5158 (1999).
- [16] PsiQuantum team, A manufacturable platform for photonic quantum computing, *Nature* **641**, 876 (2025).
- [17] I. Schwartz, D. Cogan, E. R. Schmidgall, Y. Don, L. Gantz, O. Kenneth, N. H. Lindner, and D. Gershoni, Deterministic generation of a cluster state of entangled photons, *Science* **354**, 434 (2016).
- [18] M. V. Larsen, X. Guo, C. R. Breum, J. S. Neergaard-Nielsen, and U. L. Andersen, Deterministic generation of a two-dimensional cluster state, *Science* **366**, 369 (2019).
- [19] D. Istrati, Y. Pilnyak, J. C. Loredó, C. Antón, N. Somaschi, P. Hilaire, H. Ollivier, M. Esmann, L. Cohen, L. Vidro, C. Millet, A. Lemaître, I. Sagnes, A. Harouri, L. Lanco, P. Senellart, and H. S. Eisenberg, Sequential generation of linear cluster states from a single photon emitter, *Nat. Commun.* **11**, 5501 (2020).
- [20] V. S. Ferreira, G. Kim, A. Butler, H. Pichler, and O. Painter, Deterministic generation of multidimensional photonic cluster states with a single quantum emitter, *Nat. Phys.* **20**, 865 (2024).
- [21] G. de Glinasty, P. Hilaire, P.-E. Emeriau, S. C. Wein, A. Salavrakos, and S. Mansfield, A spin-optical quantum computing architecture, *Quantum* **8**, 1423 (2024).
- [22] K. Nemoto, M. Trupke, S. J. Devitt, A. M. Stephens, B. Scharfenberger, K. Buczak, T. Nöbauer, M. S. Everitt, J. Schmiedmayer, and W. J. Munro, Photonic architecture for scalable quantum information processing in diamond, *Phys. Rev. X* **4**, 031022 (2014).
- [23] J.-Y. Zhang, Z.-W. Zhou, and G.-C. Guo, Eliminating next-nearest-neighbor interactions in the preparation of cluster state, *Chin. Phys. Lett.* **28**, 050301 (2011).
- [24] M. W. Doherty, N. B. Manson, P. Delaney, F. Jelezko, J. Wrachtrup, and L. C. L. Hollenberg, The nitrogen-vacancy colour centre in diamond, *Phys. Rep.* **528**, 1 (2013).
- [25] Y. Wu, F. Jelezko, M. B. Plenio, and T. Weil, Diamond quantum devices in biology, *Angew. Chem. Int. Ed.* **55**, 6586 (2016).
- [26] F. Dolde, V. Bergholm, Y. Wang, I. Jakobi, B. Naydenov, S. Pezzagna, J. Meijer, F. Jelezko, P. Neumann, T. Schulte-Herbrügen, J. Biamonte, and J. Wrachtrup, High-fidelity spin entanglement using optimal control, *Nat. Commun.* **5**, 3371 (2014).

- [27] C. E. Bradley, J. Randall, M. H. Aboeib, R. C. Berrevoets, M. J. Degen, M. A. Bakker, M. Markham, D. J. Twitchen, and T. H. Taminiau, A ten-qubit solid-state spin register with quantum memory up to one minute, *Phys. Rev. X* **9**, 031045 (2019).
- [28] T. Xie, Z. Zhao, S. Xu, X. Kong, Z. Yang, M. Wang, Y. Wang, F. Shi, and J. Du, 99.92%-fidelity CNOT gates in solids by noise filtering, *Phys. Rev. Lett.* **130**, 030601 (2023).
- [29] H. P. Bartling, J. Yun, K. N. Schymik, M. van Riggelen, L. A. Enthoven, H. B. van Ommen, M. Babaie, F. Sebastiano, M. Markham, D. J. Twitchen, and T. H. Taminiau, Universal high-fidelity quantum gates for spin qubits in diamond, *Phys. Rev. Appl.* **23**, 034052 (2025).
- [30] T. Joas, F. Ferlemann, R. Sailer, P. J. Vetter, J. Zhang, R. S. Said, T. Teraji, S. Onoda, T. Calarco, G. Genov, M. M. Müller, and F. Jelezko, High-fidelity electron spin gates for scaling diamond quantum registers, *Phys. Rev. X* **15**, 021069 (2025).
- [31] I. Jakobi, S. A. Momenzadeh, F. Fávoro de Oliveira, J. Michl, F. Ziem, M. Schreck, P. Neumann, A. Denisenko, and J. Wrachtrup, Efficient creation of dipolar coupled nitrogen-vacancy spin qubits in diamond, *J. Phys.: Conf. Ser.* **752**, 012001 (2016).
- [32] A. Tobalina, C. Munuera-Javaloy, E. Torrontegui, J. G. Muga, and J. Casanova, Tailored ion beam for precise colour centre creation, *Philos. Trans. R. Soc., A* **380**, 20210271 (2022).
- [33] G. Jacob, K. Groot-Berning, S. Wolf, S. Ulm, L. Couturier, S. T. Dawkins, U. G. Poschinger, F. Schmidt-Kaler, and K. Singer, Transmission microscopy with nanometer resolution using a deterministic single ion source, *Phys. Rev. Lett.* **117**, 043001 (2016).
- [34] K. Groot-Berning, T. Kornher, G. Jacob, F. Stopp, S. T. Dawkins, R. Kolesov, J. Wrachtrup, K. Singer, and F. Schmidt-Kaler, Deterministic single-ion implantation of rare-earth ions for nanometer-resolution color-center generation, *Phys. Rev. Lett.* **123**, 106802 (2019).
- [35] M. Haruyama, S. Onoda, T. Higuchi, W. Kada, A. Chiba, Y. Hirano, T. Teraji, R. Igarashi, S. Kawai, H. Kawarada, Y. Ishii, R. Fukuda, T. Tani, J. Isoya, T. Ohshima, and O. Hanaizumi, Triple nitrogen-vacancy centre fabrication by $C_5N_4H_n$ ion implantation, *Nat. Commun.* **10**, 2664 (2019).
- [36] K. Kimura, S. Onoda, K. Yamada, W. Kada, T. Teraji, J. Isoya, O. Hanaizumi, and T. Ohshima, Creation of multiple NV centers by phthalocyanine ion implantation, *Appl. Phys. Express* **15**, 066501 (2022).
- [37] D. Reinhardt, T. Lühmann, P. Räcke, J. Heupel, M. Kieschnick, S. Mändl, C. Popov, J. B. Meijer, and R. Wunderlich, Subdiffraction distance measurement of dipolar emitting qubit pairs, *ACS Photonics* **11**, 1382 (2024).
- [38] B. Grotz, J. Beck, P. Neumann, B. Naydenov, R. Reuter, F. Reinhard, F. Jelezko, J. Wrachtrup, D. Schweinfurth, B. Sarkar, and P. Hemmer, Sensing external spins with nitrogen-vacancy diamond, *New J. Phys.* **13**, 055004 (2011).
- [39] J. Du, F. Shi, X. Kong, F. Jelezko, and J. Wrachtrup, Single-molecule scale magnetic resonance spectroscopy using quantum diamond sensors, *Rev. Mod. Phys.* **96**, 025001 (2024).
- [40] S. Choi, J. Choi, R. Landig, G. Kucsko, H. Zhou, J. Isoya, F. Jelezko, S. Onoda, H. Sumiya, V. Khemani, C. von Keyserlingk, N. Y. Yao, E. Demler, and M. D. Lukin, Observation of discrete time-crystalline order in a disordered dipolar many-body system, *Nature* **543**, 221 (2017).
- [41] G. Kucsko, S. Choi, J. Choi, P. C. Maurer, H. Zhou, R. Landig, H. Sumiya, S. Onoda, J. Isoya, F. Jelezko, E. Demler, N. Y. Yao, and M. D. Lukin, Critical thermalization of a disordered dipolar spin system in diamond, *Phys. Rev. Lett.* **121**, 023601 (2018).
- [42] J. Cai, A. Retzker, F. Jelezko, and M. B. Plenio, A large-scale quantum simulator on a diamond surface at room temperature, *Nat. Phys.* **9**, 168 (2013).
- [43] N. P. de Leon, K. M. Itoh, D. Kim, K. K. Mehta, T. E. Northup, H. Paik, B. S. Palmer, N. Samarth, S. Sangtawesin, and D. W. Steuerman, Materials challenges and opportunities for quantum computing hardware, *Science* **372**, eabb2823 (2021).
- [44] H. Zhang, K. Arai, C. Belthangady, J.-C. Jaskula, and R. L. Walsworth, Selective addressing of solid-state spins at the nanoscale via magnetic resonance frequency encoding, *npj Quantum Inf.* **3**, 31 (2017).
- [45] S. Bodenstedt, I. Jakobi, J. Michl, I. Gerhardt, P. Neumann, and J. Wrachtrup, Nanoscale spin manipulation with pulsed magnetic gradient fields from a hard disc drive writer, *Nano Lett.* **18**, 5389 (2018).
- [46] N. Bar-Gill, L. M. Pham, C. Belthangady, D. Le Sage, P. Cappellaro, J. R. Maze, M. D. Lukin, A. Yacoby, and R. Walsworth, Suppression of spin-bath dynamics for improved coherence of multi-spin-qubit systems, *Nat. Commun.* **3**, 858 (2012).
- [47] H. Y. Carr and E. M. Purcell, Effects of diffusion on free precession in nuclear magnetic resonance experiments, *Phys. Rev.* **94**, 630 (1954).
- [48] S. Meiboom and D. Gill, Modified spin-echo method for measuring nuclear relaxation times, *Rev. Sci. Instrum.* **29**, 688 (1958).
- [49] C. A. Ryan, J. S. Hodges, and D. G. Cory, Robust decoupling techniques to extend quantum coherence in diamond, *Phys. Rev. Lett.* **105**, 200402 (2010).
- [50] A. M. Souza, G. A. Álvarez, and D. Suter, Robust dynamical decoupling for quantum computing and quantum memory, *Phys. Rev. Lett.* **106**, 240501 (2011).
- [51] M. A. Ali Ahmed, G. A. Álvarez, and D. Suter, Robustness of dynamical decoupling sequences, *Phys. Rev. A* **87**, 042309 (2013).
- [52] M. H. Aboeib, J. Cramer, M. A. Bakker, N. Kalb, M. Markham, D. J. Twitchen, and T. H. Taminiau, One-second coherence for a single electron spin coupled to a multi-qubit nuclear-spin environment, *Nat. Commun.* **9**, 2552 (2018).
- [53] X. Rong, J. Geng, F. Shi, Y. Liu, K. Xu, W. Ma, F. Kong, Z. Jiang, Y. Wu, and J. Du, Experimental fault-tolerant universal quantum gates with solid-state spins under ambient conditions, *Nat. Commun.* **6**, 8748 (2015).
- [54] B. T. Torosov, B. W. Shore, and N. V. Vitanov, Coherent control techniques for two-state quantum systems: A comparative study, *Phys. Rev. A* **103**, 033110 (2021).

- [55] M. H. Levitt, Composite pulses, *Prog. Nucl. Magn. Reson. Spectrosc.* **18**, 61 (1986).
- [56] R. Tycko, Broadband population inversion, *Phys. Rev. Lett.* **51**, 775 (1983).
- [57] J. A. Jones, Composite pulses in NMR quantum computation, *J. Indian Inst. Sci.* **89**, 303 (2009).
- [58] J. A. Jones, Designing short robust NOT gates for quantum computation, *Phys. Rev. A* **87**, 052317 (2013).
- [59] H. L. Gevorgyan and N. V. Vitanov, Ultrahigh-fidelity composite rotational quantum gates, *Phys. Rev. A* **104**, 012609 (2021).
- [60] A. A. Maudsley, Modified Carr-Purcell-Meiboom-Gill sequence for NMR Fourier imaging applications, *J. Magn. Reson.* **69**, 488 (1986).
- [61] T. Gullion, D. B. Baker, and M. S. Conradi, New, compensated Carr-Purcell sequences, *J. Magn. Reson.* **89**, 479 (1990).
- [62] J. Casanova, Z.-Y. Wang, J. F. Haase, and M. B. Plenio, Robust dynamical decoupling sequences for individual-nuclear-spin addressing, *Phys. Rev. A* **92**, 042304 (2015).
- [63] M. Steffen and R. H. Koch, Shaped pulses for quantum computing, *Phys. Rev. A* **75**, 062326 (2007).
- [64] L. M. K. Vandersypen and I. L. Chuang, NMR techniques for quantum control and computation, *Rev. Mod. Phys.* **76**, 1037 (2005).
- [65] J. F. Haase, Z.-Y. Wang, J. Casanova, and M. B. Plenio, Soft quantum control for highly selective interactions among joint quantum systems, *Phys. Rev. Lett.* **121**, 050402 (2018).
- [66] S. Johnson, P. R. Dolan, and J. M. Smith, Diamond photonics for distributed quantum networks, *Prog. Quantum Electron.* **55**, 129 (2017).
- [67] I. Aharonovich, A. D. Greentree, and S. Praver, Diamond photonics, *Nat. Photonics* **5**, 397 (2011).
- [68] H. K. C. Beukers, M. Pasini, H. Choi, D. Englund, R. Hanson, and J. Borregaard, Remote-entanglement protocols for stationary qubits with photonic interfaces, *PRX Quantum* **5**, 010202 (2024).
- [69] M. Ruf, N. H. Wan, H. Choi, D. Englund, and R. Hanson, Quantum networks based on color centers in diamond, *J. Appl. Phys.* **130**, 070901 (2021).
- [70] D. E. Browne, M. B. Plenio, and S. F. Huelga, Robust creation of entanglement between ions in spatially separate cavities, *Phys. Rev. Lett.* **91**, 067901 (2003).
- [71] S. D. Barrett and P. Kok, Efficient high-fidelity quantum computation using matter qubits and linear optics, *Phys. Rev. A* **71**, 060310(R) (2005).
- [72] H. Bernien, B. Hensen, W. Pfaff, G. Koolstra, M. S. Blok, L. Robledo, T. H. Taminiau, M. Markham, D. J. Twitchen, L. Childress, and R. Hanson, Heralded entanglement between solid-state qubits separated by three metres, *Nature* **497**, 86 (2013).
- [73] B. Hensen, H. Bernien, A. E. Dréau, A. Reiserer, N. Kalb, M. S. Blok, J. Ruitenbergh, R. F. L. Vermeulen, R. N. Schouten, C. Abellán, W. Amaya, V. Pruneri, M. W. Mitchell, M. Markham, D. J. Twitchen, D. Elkouss, S. Wehner, T. H. Taminiau, and R. Hanson, Loophole-free Bell inequality violation using electron spins separated by 1.3 kilometres, *Nature* **526**, 682 (2015).
- [74] M. Pompili, S. L. N. Hermans, S. Baier, H. K. C. Beukers, P. C. Humphreys, R. N. Schouten, R. F. L. Vermeulen, M. J. Tiggelman, L. dos Santos Martins, B. Dirkse, S. Wehner, and R. Hanson, Realization of a multinode quantum network of remote solid-state qubits, *Science* **372**, 259 (2021).
- [75] P. Kok, W. J. Munro, K. Nemoto, T. C. Ralph, J. P. Dowling, and G. J. Milburn, Linear optical quantum computing with photonic qubits, *Rev. Mod. Phys.* **79**, 135 (2007).
- [76] A. Reiserer and G. Rempe, Cavity-based quantum networks with single atoms and optical photons, *Rev. Mod. Phys.* **87**, 1379 (2015).
- [77] M. K. Bhaskar, R. Riedinger, B. Machielse, D. S. Levonian, C. T. Nguyen, E. N. Knall, H. Park, D. Englund, M. Lončar, D. D. Sukachev, and M. D. Lukin, Experimental demonstration of memory-enhanced quantum communication, *Nature* **580**, 60 (2020).
- [78] D. Riedel, I. Söllner, B. J. Shields, S. Starosielec, P. Appel, E. Neu, P. Maletinsky, and R. J. Warburton, Deterministic enhancement of coherent photon generation from a nitrogen-vacancy center in ultrapure diamond, *Phys. Rev. X* **7**, 031040 (2017).
- [79] V. Yurgens, Y. Fontana, A. Corazza, B. J. Shields, P. Maletinsky, and R. J. Warburton, Cavity-assisted resonance fluorescence from a nitrogen-vacancy center in diamond, *npj Quantum Inf.* **10**, 112 (2024).
- [80] F. Omlor, B. Tissot, and G. Burkard, Entanglement generation using single-photon pulse reflection in realistic networks, *Phys. Rev. A* **111**, 012612 (2025).
- [81] K. Koshino and Y. Matsuzaki, Entangling homogeneously broadened matter qubits in the weak-coupling cavity-QED regime, *Phys. Rev. A* **86**, 020305(R) (2012).
- [82] J. M. Lukens and P. Lougovski, Frequency-encoded photonic qubits for scalable quantum information processing, *Optica* **4**, 8 (2017).
- [83] M. Clementi, F. A. Sabattoli, M. Borghi, L. Gianini, N. Tagliavacche, H. El Dirani, L. Youssef, N. Bergamasco, C. Petit-Etienne, E. Pargon, J. E. Sipe, M. Liscidini, C. Sciancalepore, M. Galli, and D. Bajoni, Programmable frequency-bin quantum states in a nano-engineered silicon device, *Nat. Commun.* **14**, 176 (2023).
- [84] T. V. Tschertbul, J. Ye, and A. M. Rey, Robust nuclear spin entanglement via dipolar interactions in polar molecules, *Phys. Rev. Lett.* **130**, 143002 (2023).
- [85] N. H. Le, M. Cykiert, and E. Ginossar, Scalable and robust quantum computing on qubit arrays with fixed coupling, *npj Quantum Inf.* **9**, 1 (2023).
- [86] F. Rajabi, S. Motlakunta, C.-Y. Shih, N. Kotibhaskar, Q. Quraishi, A. Ajoy, and R. Islam, Dynamical Hamiltonian engineering of 2D rectangular lattices in a one-dimensional ion chain, *npj Quantum Inf.* **5**, 1 (2019).
- [87] B. T. Torosov and N. V. Vitanov, High-fidelity error-resilient composite phase gates, *Phys. Rev. A* **90**, 012341 (2014).
- [88] A. Barenco, C. H. Bennett, R. Cleve, D. P. DiVincenzo, N. Margolus, P. Shor, T. Sleator, J. A. Smolin, and H. Woerner, Elementary gates for quantum computation, *Phys. Rev. A* **52**, 3457 (1995).
- [89] M. V. G. Dutt, L. Childress, L. Jiang, E. Togan, J. Maze, F. Jelezko, A. S. Zibrov, P. R. Hemmer, and M. D.

- Lukin, Quantum register based on individual electronic and nuclear spin qubits in diamond, *Science* **316**, 1312 (2007).
- [90] P. Neumann, R. Kolesov, B. Naydenov, J. Beck, F. Rempp, M. Steiner, V. Jacques, G. Balasubramanian, M. L. Markham, D. J. Twitchen, S. Pezzagna, J. Meijer, J. Twamley, F. Jelezko, and J. Wrachtrup, Quantum register based on coupled electron spins in a room-temperature solid, *Nat. Phys.* **6**, 249 (2010).
- [91] Y. Wu, Y. Wang, X. Qin, X. Rong, and J. Du, A programmable two-qubit solid-state quantum processor under ambient conditions, *npj Quantum Inf.* **5**, 9 (2019).
- [92] S. L. Bayliss, D. W. Laorenza, P. J. Mintun, B. D. Kovos, D. E. Freedman, and D. D. Awschalom, Optically addressable molecular spins for quantum information processing, *Science* **370**, 1309 (2020).
- [93] A. Gaita-Ariño, F. Luis, S. Hill, and E. Coronado, Molecular spins for quantum computation, *Nat. Chem.* **11**, 301 (2019).
- [94] S. L. Bayliss, P. Deb, D. W. Laorenza, M. Onizhuk, G. Galli, D. E. Freedman, and D. D. Awschalom, Enhancing spin coherence in optically addressable molecular qubits through host-matrix control, *Phys. Rev. X* **12**, 031028 (2022).
- [95] P. A. Ivanov, N. V. Vitanov, and M. B. Plenio, Creation of cluster states of trapped ions by collective addressing, *Phys. Rev. A* **78**, 012323 (2008).
- [96] H. Wunderlich, C. Wunderlich, K. Singer, and F. Schmidt-Kaler, Two-dimensional cluster-state preparation with linear ion traps, *Phys. Rev. A* **79**, 052324 (2009).
- [97] T. Ichikawa, U. Güngördü, M. Bando, Y. Kondo, and M. Nakahara, Minimal and robust composite two-qubit gates with Ising-type interaction, *Phys. Rev. A* **87**, 022323 (2013).
- [98] S. S. Ivanov and N. V. Vitanov, Composite two-qubit gates, *Phys. Rev. A* **92**, 022333 (2015).
- [99] A. O. Niskanen, K. Harrabi, F. Yoshihara, Y. Nakamura, S. Lloyd, and J. S. Tsai, Quantum coherent tunable coupling of superconducting qubits, *Science* **316**, 723 (2007).
- [100] Y. Sung, L. Ding, J. Braumüller, A. Vepsäläinen, B. Kannan, M. Kjaergaard, A. Greene, G. O. Samach, C. McNally, D. Kim, A. Melville, B. M. Niedzielski, M. E. Schwartz, J. L. Yoder, T. P. Orlando, S. Gustavsson, and W. D. Oliver, Realization of high-fidelity CZ and ZZ-free iSWAP gates with a tunable coupler, *Phys. Rev. X* **11**, 021058 (2021).
- [101] X. Li, T. Cai, H. Yan, Z. Wang, X. Pan, Y. Ma, W. Cai, J. Han, Z. Hua, X. Han, Y. Wu, H. Zhang, H. Wang, Y. Song, L. Duan, and L. Sun, Tunable coupler for realizing a controlled-phase gate with dynamically decoupled regime in a superconducting circuit, *Phys. Rev. Appl.* **14**, 024070 (2020).
- [102] P. Zhao, P. Xu, D. Lan, J. Chu, X. Tan, H. Yu, and Y. Yu, High-contrast ZZ interaction using superconducting qubits with opposite-sign anharmonicity, *Phys. Rev. Lett.* **125**, 200503 (2020).
- [103] J. Ku, X. Xu, M. Brink, D. C. McKay, J. B. Hertzberg, M. H. Ansari, and B. L. T. Plourde, Suppression of unwanted ZZ interactions in a hybrid two-qubit system, *Phys. Rev. Lett.* **125**, 200504 (2020).
- [104] Y. Liang, M.-J. Liang, S. Li, Z. D. Wang, and Z.-Y. Xue, Scalable protocol to mitigate ZZ crosstalk in universal quantum gates, *Phys. Rev. Appl.* **21**, 024016 (2024).
- [105] L. Xie, J. Zhai, Z. Zhang, J. Allcock, S. Zhang, and Y.-C. Zheng, Suppressing ZZ crosstalk of Quantum computers through pulse and scheduling co-optimization, in *ASPLOS '22: Proceedings of the 27th ACM International Conference on Architectural Support for Programming Languages and Operating Systems* (Association for Computing Machinery (ACM), New York, NY, USA, 2022), pp. 499–513.
- [106] V. Tripathi, H. Chen, M. Khezri, K.-W. Yip, E. M. Levenson-Falk, and D. A. Lidar, Suppression of crosstalk in superconducting qubits using dynamical decoupling, *Phys. Rev. Appl.* **18**, 024068 (2022).
- [107] MATLAB Optimization Toolbox (r2023a), the MathWorks, Natick, Massachusetts. <https://de.mathworks.com/products/optimization.html>.
- [108] C. Van Loan, Computing integrals involving the matrix exponential, *IEEE Trans. Autom. Control* **23**, 395 (1978).
- [109] I. Najfeld and T. F. Havel, Derivatives of the matrix exponential and their computation, *Adv. Appl. Math.* **16**, 321 (1995).
- [110] F. Carbonell, J. C. Jiménez, and L. M. Pedroso, Computing multiple integrals involving matrix exponentials, *J. Comput. Appl. Math.* **213**, 300 (2008).
- [111] D. L. Goodwin and I. Kuprov, Auxiliary matrix formalism for interaction representation transformations, optimal control, and spin relaxation theories, *J. Chem. Phys.* **143**, 084113 (2015).
- [112] H. Haas, D. Pizzuoli, F. Zhang, and D. G. Cory, Engineering effective Hamiltonians, *New J. Phys.* **21**, 103011 (2019).
- [113] E. D. Herbschleb, H. Kato, Y. Maruyama, T. Danjo, T. Makino, S. Yamasaki, I. Ohki, K. Hayashi, H. Morishita, M. Fujiwara, and N. Mizuochi, Ultra-long coherence times amongst room-temperature solid-state spins, *Nat. Commun.* **10**, 3766 (2019).