

Startup regime of high-efficiency tapering-enhanced FEL oscillator

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In this paper, we present a design of a high-efficiency high-gain free-electron laser oscillator based on the use of a strongly tapered undulator for extracting energy from high-brightness electron beams. We provide an analytical model of the setup followed by numerical simulations for lasing at the wavelength of 13.5 nm. We discuss the optimization of the system in steady state and the conditions necessary for the pass-per-pass buildup of the power from shot noise level. We propose the use of fast phase shifters as a way to accelerate the buildup. Finally, we present time-dependent simulations of the oscillator and discuss the role of spectral filtering. The optimized working point yields a total energy conversion efficiency from the electron beam to output radiation above 1% at the wavelength of 13.5 nm.

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I. INTRODUCTION

Following more than a decade of astounding scientific successes [1], a new generation of x-ray free-electron lasers (FEL) [2–4] is being developed and coming online characterized by high-repetition-rate pulses enabled by superconducting radio-frequency (SRF) accelerators with the goal of greatly increasing the average power of the radiation and supporting a larger number of scientific users. Combining the MHz repetition rates and the high beam brightness available at these machines with the development of high-reflectivity mirrors at short wavelengths paves the way for innovative x-ray source opportunities based on the oscillator configuration. In these systems, the FEL amplifier is embedded in an optical cavity so that a fraction of the output radiation is recirculated and used as a seed to the subsequent electron beams. There are several advantages of cavity-based x-ray FELs over the conventional single-pass topology, including enhanced spectral brightness and improved output stability [5]. Multiple passes of the radiation pulse in the

wavelength-selective cavity result in increased longitudinal coherence with extremely narrowband pulses that have orders of magnitude higher spectral brightness than conventional single-pass XFELs [5]. This operation mode can also bring significant advantages in terms of energy extraction efficiency, as spectral filtering suppresses sideband instabilities that can otherwise limit power extraction [6].

Depending on the gain per pass, we can distinguish two regimes of operation within cavity-based x-ray FELs: the XFEL oscillator (XFELo) and the x-ray regenerative amplifier FEL (RAFEL). In XFELo designs [7,8], highly reflective ($> 99\%$) Bragg reflector mirrors and low-efficiency outcouplers (4%) are implemented to support modest (0.3–2) gain per pass. XFELos can reach an x-ray brightness 2 orders of magnitude higher than that of state-of-the-art single-pass x-ray FELs but are characterized by a relatively low output efficiency ($< 0.1\%$) [9]. At the same time, recent advances in high-brightness beam dynamics—including lower thermal emittance photocathodes, higher gradient electron guns, control of coherent synchrotron radiation-induced emittance growth in bends—together with a generally improved computationally-assisted optimization of the beam transport [10–14], have shown that high-repetition-rate SRF-based linacs can deliver electron beams with kilo-Ampere class peak currents and less than 1 mm rad normalized emittance suitable for high-gain amplification at short radiation wavelengths. X-ray RAFEL configurations [15–18] leverage this higher gain per pass, offering an attractive option to extend the

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reach of cavity-based FELs. For example, in order to realize a cavity-based FEL in the EUV or soft x-ray range, where mirror reflectivities are below 80% [19–21], an order-of-magnitude increase in the single-pass gain is needed to compensate round-trip losses, allowing for power buildup and enabling efficient energy extraction. A higher gain would also allow for larger outcoupling fractions (above 20%), increasing the power available to users while decreasing the power stored in the cavity and thereby mitigating the risk of damage to the optical components.

In this framework, it becomes natural to ask a general question on how to maximize the energy extraction efficiency, which is the fraction of the relativistic beam kinetic energy converted into coherent radiation [22–24]. In single-pass systems, tapering (i.e., varying) the parameters of the undulator has been shown to extend the resonant interaction for the decelerating electron beam far beyond the FEL saturation point, maximizing power extraction from the electron beam and the overall efficiency of the system [6,25]. At the same time, it was described early on [26,27] that using a tapered undulator in an oscillator at long wavelengths where mirrors have broadband reflectivity would simply lead to amplification at a red-shifted resonant wavelength. In fact, counterintuitively, due to the shape of the low-gain detuning curve, reverse tapering (where the undulator strength K grows along the undulator) has been found to improve the oscillator efficiency [28]. Even when narrowband mirrors are used to select the amplification bandwidth, it is not obvious how to start the oscillator from noise and reach sufficient steady-state recirculating power for the beam to follow a predetermined undulator tapering curve.

Strong undulator tapering in combination with intense radiation seed pulses and beam prebunching has been proposed to run the FEL in the so-called tapering-enhanced stimulated superradiant amplification (TESSA) regime, where the system is dominated by the stimulated emission dynamics [29,30]. This was demonstrated in the initial proof-of-principle experiment at the Accelerator Test Facility (ATF) at the Brookhaven National Laboratory with up to 30% energy conversion at 10 μm wavelength [31], followed by the application of the TESSA scheme in the THz regime with a 10% extraction efficiency using a group velocity compensating waveguide at UCLA Pegasus Laboratory [32]. Shorter wavelength demonstrations are planned [33]. One of the main ingredients of the TESSA regime is a very intense radiation pulse that provides the initial field to enable high-gradient particle deceleration. At wavelengths where such pulses are not readily available, a natural solution would be to reuse a part of the radiation to seed subsequent passes [34]. In such a configuration, the system becomes essentially equivalent to a RAFEL with high single-pass gain and very high efficiency.

In this paper, we study an FEL oscillator with a strongly tapered undulator in the TESSA regime, with the aim

of maximizing the energy conversion efficiency at the technologically relevant EUV lithography wavelength of 13.5 nm. While the design presented in the following is based on the parameters available at the FLASH FEL facility (DESY, Germany) [35], it is kept as general as possible so that it can be quickly adapted to other wavelengths and machine parameters. In Sec. II, we review the single-pass FEL amplifier efficiency for the case of a strong seed and extend the original 1D analysis from [36] (which combined prebunching and strong seeding) to incorporate 3D effects, reconciling it with the 3D analysis for the unseeded FEL amplifier from [37]. In Sec. II A, we derive handy semi-analytical formulas for estimating the optimal efficiency in a strongly tapered seeded FEL amplifier in the high-gain TESSA regime. Self-consistent 3D numerical simulations carried out in GENESIS 1.3, version 4 [38] are used to benchmark the theory and verify the key assumption that the bunching factor remains constant during the interaction.

After this, we apply the model (Sec. II B) to the steady state of an oscillator setup, including the effect of losses and outcoupling on the total efficiency [34], and use this prediction as a basis for the design (Sec. III) and numerical simulations of an EUV oscillator setup (Secs. IV A and IV B). Recognizing that intense seed pulses might not be available to start the oscillator at very high power [39], we use simulations (Sec. IV C) to study the power buildup in the oscillator over many passes starting from the shot noise until the value yielding the desired high efficiency is reached. In particular, we discuss how undulator tapering limits the interaction length at low seed powers and adversely affects the system performance, setting up a potential barrier (or “death valley”) that may prevent the buildup of oscillator power to very high levels. We propose ways to speed up the buildup process by dynamically adjusting undulator system properties and possibly the beam energy (see Sec. IV D). Finally, in Sec. IV E, we extend the simulations of the steady state and the buildup of the oscillator to the time-dependent case, discussing the need for a bandwidth-limiting monochromator and its influence on the radiation properties. Note that while we specify all the requirements of the outcoupling and spectral filtering elements in the oscillator cavity, we leave the technical details of their implementation to future investigations.

II. HIGH EFFICIENCY TAPERING-ENHANCED FEL OSCILLATOR THEORY

The setup we consider for our analytical study is shown schematically in Fig. 1 and is composed of a long planar undulator (amplifier) without break sections enclosed within a cavity. Although helical undulators maximize coupling [29] and can yield an advantage in terms of efficiency, the choice of planar undulators reduces the experimental complexities associated with a very enclosed geometry and circularly polarized light. In order to simplify

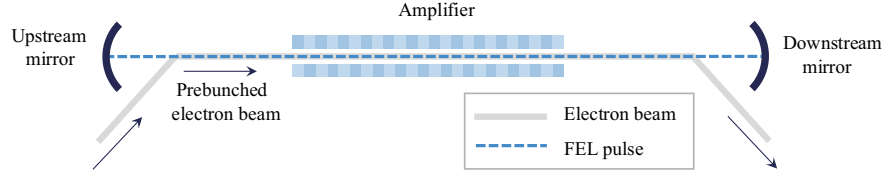


FIG. 1. Scheme of the setup discussed analytically: a long undulator (amplifier) enclosed in a cavity. In the amplifier, the prebunched electron beam is decelerated by the intense seed pulse (FEL pulse stored in the cavity), resulting in a large fraction of the beam power being converted into coherent radiation.

this first modeling step, we also assume the electron beam entering the amplifier to be already prebunched at the wavelength of interest.

The final steady-state circulating power in such a system depends on many factors, such as the incoming beam brightness, the optical system design, the outcoupling fraction, and the undulator tapering profile. The latter is at the center of our discussion as it determines the achievable gain per pass and, therefore, acceptable losses, including the fraction of energy that can be outcoupled and delivered to users. In the strong saturation FEL regime, the optimal tapering profile is the result of a compromise between a more aggressive taper that favors a quick energy extraction from the electron beam and a milder taper aimed at maintaining a larger number of electrons participating in the interaction. There are a variety of numerical optimization methods for finding that optimum when the input seed field is known [25,40–42]. However, in the oscillator design, the input field is a dynamic variable of the system and is not *a priori* known. Therefore, an analytical estimate of the power gain in a tapered undulator as a function of the input seed power is critical to guide the optimization process in an efficient way.

In order to simplify the highly nonlinear FEL system and obtain a reasonable prediction for the single-pass efficiency in a tapered FEL amplifier, we adopt the constant bunching approximation. The assumption is that the undulator field is tapered in such a way that a fraction of the beam particles is trapped in the FEL ponderomotive potential bucket throughout the entire interaction length, so the complex bunching factor b remains constant in amplitude and phase [6]. We will *a posteriori* verify the validity of this assumption.

In this model, the energy lost by the electron beam interacting with an electromagnetic wave of wavelength λ in the undulator of period λ_u can be estimated using the resonant particle approximation [6]. That is, (i) all trapped electrons behave similarly to one particle that maintains a constant resonant phase ψ_r in the FEL ponderomotive potential and has the resonant energy $\gamma_r(z) = \sqrt{\frac{\lambda_u(1+K(z)^2/2)}{2\lambda}}$, determined by the tapering of the undulator strength $K(z)$ and (ii) the untrapped electrons do not contribute to the energy transfer. The efficiency of the

FEL amplifier, which, by energy conservation is the relative energy loss of the beam, can then be written as

$$\eta = f_t \frac{|\Delta\gamma_r|}{\gamma_0} \cong \frac{e}{\gamma_0 m c^2} \left(\frac{K_0 J J_f \sin \psi_r}{2\gamma_0} \right) \int_0^{L_u} E(z) dz, \quad (1)$$

where f_t is the fraction of particles participating in the coherent radiation emission (trapped particles), and γ_0 is the input beam relativistic factor, in the following simply referred to as beam energy. The energy loss $\Delta\gamma_r$ is evaluated as an integral over the total undulator length L_u of the equation which gives the evolution of the resonant energy $\partial\gamma_r/\partial z$. The electron charge and rest mass are denoted as e and m , respectively, and the speed of light in vacuum is c . For a constant-period undulator, we have also assumed that the magnitude of the transverse velocity in the undulator remains constant along the interaction, i.e., $K(z)/\gamma_r(z) = K_0/\gamma_0$, which is valid as long as $K(z)$ remains much larger than 1 and the resonant condition remains satisfied. $JJ = J_0[K^2/(4+2K^2)] - J_1[K^2/(4+2K^2)]$ is the usual Bessel function term for planar undulators that takes into account the longitudinal velocity oscillations. If the electric field amplitude evolution $E(z)$ along the undulator is known, Eq. (1) can be used to obtain an expression for efficiency and hence the emitted power.

A. Single-pass efficiency for strongly seeded FEL amplifiers including 3D effects

In general, we can distinguish two contributions to the electromagnetic field evolution in the undulator: the seed and the radiation emitted by the particles. In the 1D case, neglecting finite transverse size and diffraction effects, these two terms can be written as a constant input field and a linearly growing field amplitude proportional to the beam current density (i.e., inversely proportional to the transverse cross section of the electron beam) [36]. Once 3D effects are taken into consideration, while the qualitative behavior remains similar, important differences arise. For example, tightly focusing the beam in the undulator turns out to be counterproductive, as the divergence in the diffracting radiation reduces the on-axis field intensity and ultimately decreases the power extraction.

In the constant bunching approximation, Schneidmiller and Yurkov [37] obtained an expression for the complex field amplitude generated by a bunched electron beam with a Gaussian transverse distribution of size σ :

$$E_{\text{self}}(z, r) = \chi \frac{K}{\gamma} J J b \int_0^z \frac{-k\sigma^2 dz'}{z - z' - ik\sigma^2} \times \exp\left[\frac{ikr^2}{2(z - z' - ik\sigma^2)}\right] e^{-iCz'}, \quad (2)$$

where $\chi = Z_0 I / 4\pi\sigma^2$ with Z_0 being the vacuum impedance, I is the beam current (assumed constant in time), $k = \omega/c = 2\pi/\lambda$, and $C = (k_u + k - \omega/v_z)$ is the detuning parameter, where k_u is the undulator wave vector and v_z is the average longitudinal beam velocity. A well-designed taper profile ($C = 0$) implies that the detuning is adjusted along the undulator to compensate for the change in energy of the particles, so that the FEL resonance is maintained throughout the interaction.

It is helpful to introduce the diffraction parameter $N = k\sigma^2/L_u$ for an undulator of length L_u to understand the field dependence in different diffraction regimes. For $N < 0.1$, strong diffraction causes the majority of electrons to sample fields approximately equal to the on-axis maximum, suggesting the approximation $E_{\text{self}}(z, r) \approx E_{\text{self}}(z, 0)$, with r being the radial coordinate. In this case, the evolution of the field can be integrated analytically as

$$E_{\text{self}}(z) = i\chi \frac{K}{\gamma} J J b f(z), \quad (3)$$

with

$$f(z) = k\sigma^2 \left[\arctan\left(\frac{z}{k\sigma^2}\right) - \frac{i}{2} \ln\left(1 + \frac{z^2}{k^2\sigma^4}\right) \right].$$

The diffraction effects are isolated in the function $f(z)$, which simplifies to $f(z) \approx z$ in the 1D limit. For systems where diffraction is less dominant, a correction factor can be used to account for the reduction of the field experienced by the off-axis particles.

In the oscillator system, the other important contribution to the field in the undulator is the recirculating seed pulse, which, in the first approximation, can be modeled as a TEM₀₀ Gaussian mode with waist size w_0 , waist position z_w , and amplitude E_0 :

$$E_{\text{seed}}(z) = E_0 \frac{w_0}{w(z)} e^{-i\theta(z)}, \quad (4)$$

where $\theta(z) = \arctan\left(\frac{z-z_w}{z_r}\right)$ is the Gouy phase, $w(z) = w_0 \sqrt{1 + (z - z_w)^2/z_r^2}$, and $z_r = kw_0^2/2$ is the Rayleigh length. The peak electric field E_0 in the waist position

can be expressed as $E_0 = \sqrt{4PZ_0/\pi w_0^2}$ as a function of the seed power P .

We can now plug the total field, i.e., the sum of the expressions from Eqs. (3) and (4), into the formula for the efficiency [Eq. (1)] and integrate it to estimate the single-pass conversion efficiency. In the limit of a very large electron beam and radiation spot size, and neglecting the phase difference between the emitted and seed radiation, we recover the 1D result from [36] modified for a planar undulator as

$$\eta_{1D}(z) = \frac{e}{\gamma_0 mc^2} \left(\frac{K_0 J J f_t \sin \psi_r}{2\gamma_0} \right) \times \left[E_0 z + \chi \left(\frac{K_0 J J f_t \sin \psi_r}{2\gamma_0} \right) z^2 \right], \quad (5)$$

where the linear and quadratic terms in z are proportional to the seed amplitude and the beam current density, respectively. In this picture, we identify $f_t e^{-i\psi_r}$ as the complex bunching factor b , which remains constant throughout the interaction.

In the more general case, including 3D effects, we need to carry out the integrals in

$$\eta = \frac{e}{\gamma_0 mc^2} \left(\frac{K_0 J J f_t \sin \psi_r}{2\gamma_0} \right) \int_0^z |E_{\text{seed}}(z') + E_{\text{self}}(z')| dz',$$

where it is important to note the relative phase $\psi_r - \pi/2$ between the two contributions associated with the fact that the field is generated by the particles with a $\pi/2$ phase offset. This expression can be approximated and rewritten to highlight the same qualitative behavior of Eq. (5) in terms of 3D efficiency functions that result from the energy loss integration as

$$\eta_{3D}(z) \cong \frac{e}{\gamma_0 mc^2} \left(\frac{K_0 J J f_t \sin \psi_r}{2\gamma_0} \right) \left[E_0 f_{3D,\text{seed}}(z) z + \chi^* \left(\frac{K_0 J J f_t \sin \psi_r}{\gamma_0} \right) f_{3D,\text{self}}(z) z \right], \quad (6)$$

where we modified $\chi^* = \chi/(1 + \eta_{MX})$, with the Xie correction factor η_{MX} to take into account emittance and energy spread effects [43]. The term $f_{3D,\text{seed}} =$

$(1/z) \int_0^z \frac{1}{\sqrt{1 + (z' - z_w)^2/z_r^2}} \frac{w(z')^2}{w(z)^2 + 2\sigma^2} dz'$ quantifies the energy lost due to the diffracting seed pulse, with a correction term to take into account particles sampling off-axis fields, which is obtained by convolving the density of the Gaussian electron beam with the radial dependence of the seed. For very large waist sizes, $f_{3D,\text{seed}} \rightarrow 1$ and the seed is responsible for a linear energy loss along the undulator. The energy loss to coherent radiation emission

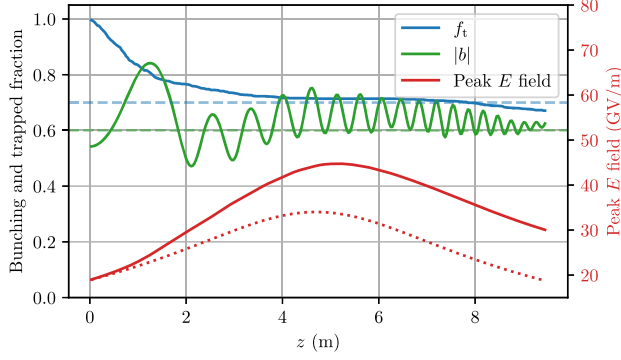


FIG. 2. Simulated bunching (green) and trapped fraction (blue) as functions of undulator length. Constant values are shown in dashed lines to highlight the validity of the approximation. The peak on-axis field from the simulation along the undulator is plotted in solid red. The dotted red line gives the analytical evolution of the Gaussian seed field, which peaks at the undulator center with a Rayleigh length of about 1.9 m. The difference between the solid and dotted curves quantifies the additional contribution from coherent emission by the prebunched beam.

depends on $f_{3D, \text{self}} = k\sigma^2 \left(\arctan \frac{z}{2k\sigma^2} + \frac{k\sigma^2}{z} \ln \frac{4k^2\sigma^4}{z^2 + 4k^2\sigma^4} \right)$. When the seed power is zero, the radiation power (beam power times efficiency) yields back the expression from [37]. For wide beams, $f_{3D, \text{self}} \rightarrow z/2$, and we recover the 1D efficiency limit, while for thin beams, the emitted power scales only linearly with z . For shorter undulators and relatively high seed powers (as can be the case in a high-gain oscillator) the first term dominates and can significantly increase the achievable efficiency.

The validity of the constant bunching approximation can be assessed by examining Fig. 2, which shows the complex bunching and radiation phases from a GENESIS 1.3 simulation using the parameters in Table I, with the analytic

TABLE I. FEL oscillator reference parameters.

Parameter	Notation	Value
Repetition rate	f_{rep}	3 MHz
Peak current	I	1 kA
Normalized emittance	ϵ_n	1 mm rad
Beam energy	γ_0	2348
Beam energy spread	$\sigma_{\gamma,0}$	0.2
Seed waist size (2σ)	w_0	90 μm
Wavelength	λ	13.5 nm
Seed power	P_{in}	30 GW
Modulator period	$\lambda_{u, \text{mod}}$	3.14 cm
Number of modulator periods	$N_{u, \text{mod}}$	8
Chicane longitudinal dispersion	R_{56}	3.56 μm
Amplifier period	λ_u	3.14 cm
Number of amplifier periods	N_u	300
Initial amplifier K value	K	2.735
Breaks between amplifier sections	L_b	0.35 m
Average β function	$\langle \beta \rangle$	5.1 m

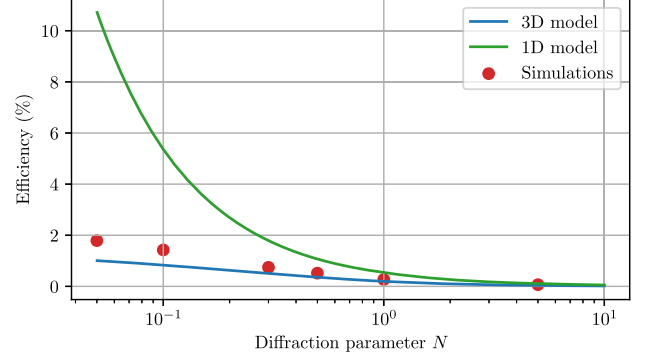


FIG. 3. Single-pass efficiency as a function of the diffraction parameter ($N = k\sigma^2/L_u$) for an unseeded amplifier using parameters from Table I while varying the electron beam spot size. The 1D model (green) predicts a $1/N$ -scaling for the efficiency at $N \rightarrow 0$. The 3D model (blue) includes diffraction and shows slower growth, which appears as approximately linear in the semilogarithmic plot. Red circles denote GENESIS 1.3 simulations, which follow the 3D trend and slightly exceed the analytical curve at small N due to the use of an optimized taper profile.

predictions shown as dotted lines. Because the seed radiation was defined as a fixed Gaussian beam in the simulation, it exactly matches Eq. (4). The amplitudes of the bunching and emitted radiation agree well with the analytical predictions, except for an initial increase in the bunching magnitude caused by synchrotron oscillations.

Figure 3 compares the semianalytical expressions for the efficiency of the single-pass unseeded FEL amplifier as a function of the diffraction parameter N to highlight the impact of 3D effects on efficiency. In the 1D approximation (green curve), the efficiency is simply inversely proportional to N . Looking at the simulation results (red circles), this behavior starts to break down for $N \leq 1$ at efficiencies beyond 1%. In the $N \rightarrow 0$ limit, the predicted 3D efficiency (blue curve) grows more slowly and better matches the simulations, as at smaller spot sizes the finite emittance leads to increased angular divergence and an effective energy spread that limits the fraction of trapped electrons.

B. Oscillator efficiency in constant bunching approximation

We applied these estimates to the discussion of the oscillator efficiency. Starting from the single-pass efficiency, it is possible to derive semianalytical expressions for the steady state of the oscillator. Referring to Fig. 1, a seed pulse with power P_n at pass n gains $\eta_{3D}(P_n)P_{\text{beam}}$, where $P_{\text{beam}} = Imc^2\gamma_0/e$ is the beam power and $\eta_{3D}(P_n)$ is the single-pass efficiency [Eq. (6)] for a seed power P_n at $z = L_u$.

Defining R as the recirculation fraction, i.e., the fraction of the power left in the cavity after the outcoupling and the losses on the mirrors, the seed power for the pass $n + 1$ can be written as

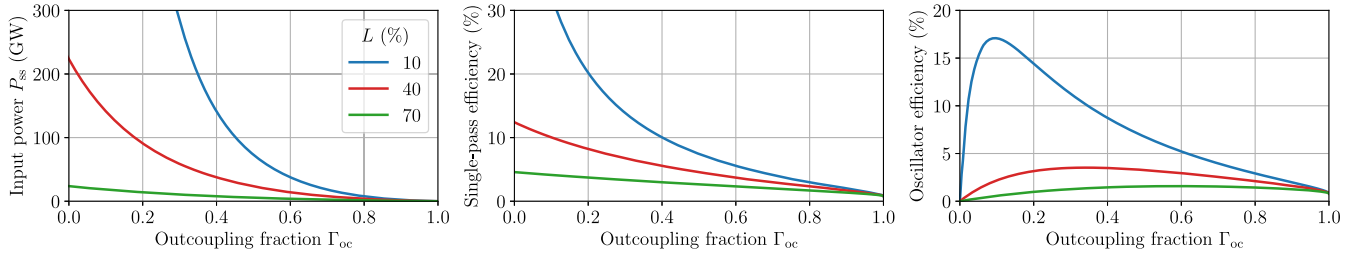


FIG. 4. Steady state input power P_{ss} (left), single-pass efficiency η_{3D} (middle), and oscillator efficiency η_{osc} (right) as functions of the outcoupling fraction Γ_{oc} for different values of the loss fraction L . At $\Gamma_{oc} = 1$, the predicted efficiency is nonzero as it corresponds to the single-pass emission of a prebunched beam, which does not lead to a buildup.

$$P_{n+1} = (P_n + \eta_{3D}(P_n)P_{beam})R. \quad (7)$$

The equation can be solved for the condition $P_{n+1} = P_n$ to find the steady-state input power P_{ss} for the system. Separating the different contributions to the recirculation fraction, we can write it as

$$R = (1 - L)(1 - \Gamma_{oc}), \quad (8)$$

where Γ_{oc} is the outcoupled radiation fraction and L represents the cavity losses. For the case of a two-mirror cavity, we can write $L = 1 - R_m^2$, with R_m being the reflectivity of a single mirror. Generally, L also includes the additional losses due to slippage and monochromatization. We assume these to be negligible for now, but we will add them in Sec. IV E, where time-dependent effects are considered.

The oscillator efficiency η_{osc} is defined as the ratio of the outcoupled power $P_{oc} = \Gamma_{oc}P_{ss}/R$ and the beam power P_{beam} , i.e.,

$$\eta_{osc} = \frac{\Gamma_{oc}P_{ss}}{P_{beam}R}. \quad (9)$$

From Eqs. (8) and (9) one might read that the efficiency η_{osc} increases with $\Gamma_{oc} \rightarrow 1$ and $R \rightarrow 0$. However, for a steady state, the low R values have to be compensated by a high FEL gain. The gain saturation with increasing input power

P_{ss} ultimately limits the outcoupling efficiency and Eq. (9) should be interpreted within this constraint.

For a given undulator and a specific cavity loss factor L , steady-state power is heavily influenced by the outcoupling fraction Γ_{oc} , which can be optimized to maximize the efficiency. This is illustrated in Fig. 4, where steady-state input power P_{ss} , single-pass efficiency η_{3D} , and oscillator efficiency η_{osc} are shown as functions of Γ_{oc} , for three different values of L and the set of parameters listed in Table I, which are used in the rest of the paper.

III. DESIGN CONSIDERATIONS

A. General setup

The setup we investigated with numerical simulations (Sec. IV) is shown in Fig. 5 and builds on the idealized setup considered analytically in Sec. II (see Fig. 1). The distance between the end mirrors is 50 m, consistent with a 3 MHz repetition rate of electron bunches.

A buncher, consisting of a short undulator (modulator) and a magnetic chicane, is used to prebunch the electron beam before it enters the amplifier. The continuously tapered amplifier section comprises four undulator modules separated by drift sections. The center of the amplifier coincides with the middle of the cavity. Given the high-repetition-rate of 3 MHz, in our modeling we consider the tapering profile to be fixed, that is, not modifiable on a pass-to-pass basis. Quadrupole magnets between the

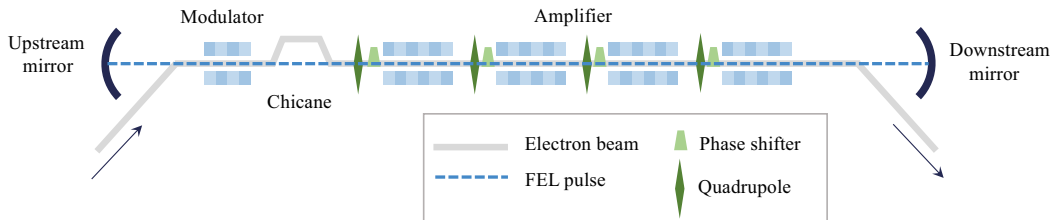


FIG. 5. Scheme of the FEL cavity. Four undulator modules (amplifier) are placed symmetrically between two curved end mirrors. Upstream of the amplifier there is a buncher section consisting of a modulator and a chicane. The same FEL beam circulating in the cavity interacts with the electron beam both in the modulator and the amplifier. In the modulator, an energy modulation of the electron beam is induced, which then leads to a density modulation (microbunching) after the chicane. In the amplifier, the microbunched beam is decelerated by the intense seed pulse, resulting in a large fraction of the beam power being converted into coherent radiation.

undulator modules form an ideal FODO lattice, optimized for a minimal average electron beam size in the undulators. Each undulator module is preceded by a phase shifter, a small magnetic delay line, which is used to adjust the relative phase between the bunching and the radiation. Phase shifters are critical for compensating the phase accumulated in drifts between undulators, including the longer beam transport between the modulator and the amplifier [44].

We assume an input electron beam with Gaussian transverse density distributions and parameters consistent with a FLASH-like injector as reported in Table I.

B. Choice of steady-state power

For our wavelength of interest of 13.5 nm, we consider the reflectivity of a single mirror up to $R_m = 0.75$ at normal incidence (more than 71% reached in [20]), giving $L = 1 - R_m^2 = 0.4375$. Following the discussion in Sec. II B (Fig. 4), the optimal working point is found at $P_{ss} \approx 36$ GW, yielding the single-pass efficiency of 5.5% and oscillator efficiency of 3.17%. Anticipating extra losses in the cavity due to slippage, as well as a lower amplification than predicted by the model—e.g., due to the addition of breaks in the undulator line—we choose, for the further numerical optimization, a slightly lower value of $P_{ss} = 30$ GW with a 5% predicted single-pass efficiency, corresponding to $R = 33.3\%$, $\Gamma_{oc} = 40\%$, and an oscillator efficiency of 3.06%. Note that for an FEL oscillator with an untapered amplifier of our chosen length, theory predicts a single-pass efficiency on the order of 0.14% in low-gain regime [45], while in high-gain regime the single-pass efficiency is comparable to the FEL parameter [46], yielding about 0.2% for our settings. The oscillator efficiency in both cases is lower due to losses and outcoupling.

C. Radiation spot size and optical damage

The optical design of the cavity is the result of careful consideration of different effects. A smaller waist size for the FEL beam, on the one hand, provides desirable higher peak intensity, and, on the other hand, can lead to detrapping due to strong intensity variations transversely across the electron bunch. In the low-gain case, the optimization of the waist position and the waist size is discussed in [34] and suggests to focus the radiation in the middle of the undulator with a Rayleigh range 15%–20% of the total undulator length.

For our oscillator setup, an additional advantage of a smaller waist size is the increased spot size on the end mirrors, which is critical to reducing the heat load. For a Rayleigh length of about 1.9 m, the $1/e^2$ radius of the FEL beam on the mirror 25 m downstream is 1.2 mm. Assuming an average duty cycle for the pulse train of 0.001, the average power density is then below 7 kW/cm², while the

single-shot fluence is about 0.24 J/cm². That is above the threshold reported in [47] for femtosecond pulses, which, however, does not consider the latest developments in multilayer mirrors [48]. A further reduction of the fluence could be achieved by increasing the drift length from the undulator to the downstream mirror, which would have to be accompanied with an appropriate adjustment of mirror radii of curvature.

D. Buncher design

The energy modulation induced by the seed in the modulator can be quantified by the ratio of the induced modulation amplitude and the initial energy spread, which for the parameters from Table I evaluates to 11.7 [49].

The R_{56} of the chicane is then optimized to maximize the bunching, resulting in a bunching factor of 55% at the amplifier entrance. The high energy modulation allows keeping the chicane R_{56} at a relatively low value of 3.56 μ m, which (in the time-dependent case) is critical to ensure good temporal overlap between the seed and the electron beam in both the modulator and the amplifier.

IV. SIMULATIONS

In our numerical study, we distinguish between the use cases of time-independent and time-dependent simulations. In the former case, only a single, one-wavelength-long slice of the electron beam is simulated, which is computationally lighter and convenient for the initial optimization.

We also distinguish between single-pass and multipass simulations. In the latter case, the radiation from each pass is propagated through the full cavity and fed back into the buncher-amplifier line, where it interacts with a new electron bunch on the next pass. This process is repeated for tens of passes or more, until a steady-state condition is reached.

In our case, only the portion of the setup where the electron beam is relevant (buncher and amplifier) is simulated in GENESIS 1.3. In the rest of the cavity, the radiation is propagated using the transfer-matrix formalism as described in [50]. During each pass through the cavity, the intensity of the field is attenuated with the factor R , the recirculation fraction. The propagation through the cavity also includes focusing and, optionally, monochromatization (in time-dependent simulations).

A. Tapering optimization in single-pass, time-independent simulations

The first major step in the numerical analysis is the optimization of the undulator tapering profile, which is performed in single-pass time-independent simulations. For these, we use an idealized input seed field with a transverse Gaussian profile, waist in the middle of the amplifier, and parameters listed in Table I, in order to emulate the radiation circulating in the cavity at steady state.

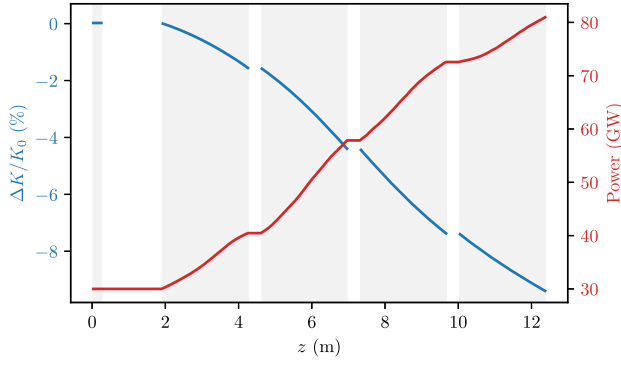


FIG. 6. Optimization results for the continuous tapering profile: relative change in undulator strength $\Delta K/K_0$ (blue) and radiation power (red) as functions of the longitudinal coordinate z along the undulators. Positions of the modulator and the four amplifier modules are highlighted in gray.

Assuming that the evolution of the central energy of the bucket is $\gamma(z) = \gamma_0 - \Delta\gamma(z)$, we can use the efficiency estimate from Eq. (6) to generate an initial guess for $\Delta\gamma(z)$ and the tapering function $K(z)$ along the amplifier, which can be used as basis for further numerical optimization.

The numerical optimization of the continuous tapering profile was carried out using time-independent GENESIS 1.3 simulations, maximizing the output power. First, this was applied to the strongly simplified case of a perfectly collimated electron beam in a long amplifier, where the breaks in-between sections were removed, which is closer to the setup discussed analytically (Fig. 1). The optimal taper found in the first step was then used as a starting point for subsequent optimization of the full setup, which includes realistic focusing of the beam, breaks between amplifier sections, and a phase shifter preceding each section. The results of the optimization are reported in Fig. 6, where we plot the relative change in the undulator strength $\Delta K(z)/K_0$ (blue) together with the power (red) as functions of the longitudinal coordinate z . The nearly linear increase of power with undulator length reflects that the seed radiation has the largest contribution to the beam deceleration in our design. In fact, taking the ratio of the two terms in Eq. (6), we find that the first term contributes to more than 80% of the final radiation power.

We note the significance of having an already established taper profile, which allows the optimizer to quickly tune the phase shifters and correct for the phases accumulated in the drifts. When running the optimizer on the full setup while skipping the first step, the solution typically converges to a less performing profile, since phase shifts can, to some degree, compensate for the tapering [44]. We denote the set of four phases optimized for the steady state by Φ_{ss} .

The simulated single-pass efficiency after optimization is $\eta = 4.25\%$ compared to the theoretical prediction of the model (see Sec. III) of 5%. We explain the discrepancy with the diffraction in the break sections (not present in the

model), the evolving spot size of the electron beam due to the quadrupole focusing, and the differences in the assumed phase-space distribution.

B. Multipass steady state, time-independent simulations

In the next step, we simulated the steady state of the oscillator, where the gain exactly compensates for the losses. This was done by recirculating the radiation from the first pass started from the nominal input field, and modeling the losses via a constant recirculation fraction R . The radii of curvature of the mirrors are adjusted to compensate for the shift in the effective source point of the outgoing radiation from the middle of the undulator resulting from the amplification process. This shift is expected as the contributions of the seed and the generated radiation to the output power are comparable and it requires using different curvatures at the end mirrors for most efficient coupling. The optimum is found from a 2D scan of both curvatures with the figure of merit being the circulating power in the cavity in steady state. Further interpolation gave us the values of $r_{c,d} = 48.5$ m and $r_{c,u} = 53.25$ m for the radii of curvature of the downstream and upstream mirrors, respectively. In case of optimal coupling, the oscillator is found to converge to a steady state where the single-pass amplification $A = P_{out}/P_{in}$ fully compensates for the losses in the cavity given by $R = 1/A \approx 37.8\%$. The discrepancy to the predicted $R \approx 33.3\%$ (see Sec. III B) comes from the lower single-pass efficiency in simulations, discussed in Sec. IV A.

Note that the transverse mode profile to which the oscillator converges in steady state differs from a pure Gaussian one and has a shifted focus position. The root-mean-square (rms) field size along the beamline is shown in Fig. 7 and compared to an ideal Gaussian field with nominal parameters (Table I). The inset shows the transverse field profile at the waist.

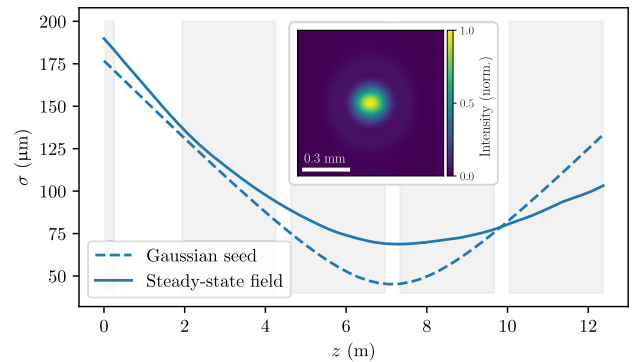


FIG. 7. rms field size along the beamline in steady state (i.e., after 50 passes). The dashed line corresponds to an ideal Gaussian seed, the solid line to the field to which the oscillator converges in steady state. The inset shows the intensity profile of the oscillator field in the waist.

We also note that in multipass simulations, the influence of the initial seed quickly fades, i.e., in a simulation started from a Gaussian beam, after several passes, the radiation mode is shaped by the FEL dynamics. We attribute this to the low cavity recirculation fraction, which allows the gain to dominate the mode evolution.

C. Startup, time-independent simulations

Since no high-power seed sources at 13.5 nm wavelength are available, we investigate the startup of the oscillator starting from shot noise. As mentioned in Sec. III, we assume the undulator tapering to be fixed, so the taper designed for the high steady-state power is also used during the buildup, where it hinders the FEL process, since at low input power P_{in} the electron beam cannot follow the designed energy loss rate.

Additionally, at low power, the buncher does not perform as intended, as the induced energy modulation is vanishingly low. We investigated whether the steady state is reachable under these constraints.

To develop a better understanding of the system, we performed single-pass, time-independent simulations for different P_{in} values ranging from the shot noise power up to the steady-state design value (30 GW). This allows us to assess the performance of the amplifier at different power levels experienced by the oscillator during the buildup in order to isolate the effects of the change in input power from other changes potentially occurring in the buildup in multipass simulations. The results are summarized in Fig. 8, where we show the resulting amplification $A = P_{\text{out}}/P_{\text{in}}$ as a function of P_{in} for the seed focused in the middle of the undulator line (blue curve).

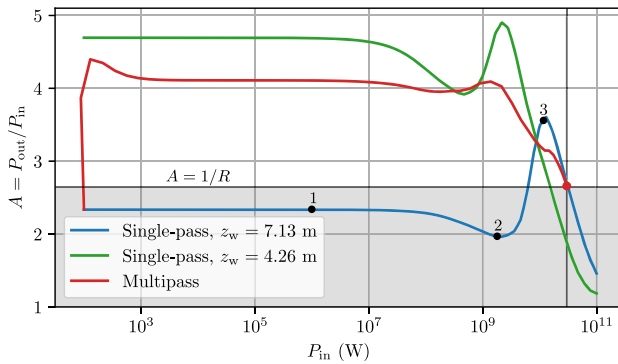


FIG. 8. Amplification as a function of input power from single-pass simulations with the input field waist position in the middle of the amplifier undulator line ($z = 7.13$ m, blue curve), after the first undulator ($z = 4.26$ m, green curve), and from multipass simulations (red curve), with the red circle marking the steady state reference value. The gray shaded area shows the region with $A < 1/R$ (for the design value of $R = 0.378$), in which the buildup is not possible. The gray vertical line marks the reference steady-state power.

Knowledge of $A(P_{\text{in}})$ as shown in Fig. 8 enabled us to create a simple model of the oscillator buildup as follows. If at pass number n the power at the amplifier entrance is $P_{\text{in}} = P_n$, then on the next pass it will be

$$P_{n+1} = P_n A(P_n) R. \quad (10)$$

The necessary condition for the buildup can be deduced from Eq. (10) as

$$A(P_{\text{in}}) > 1/R. \quad (11)$$

In Fig. 8, the shaded area corresponds to $A < 1/R$, where no buildup is possible, for our reference $R = 0.378$. The steady state is found at the crossing point between the $1/R$ line and the amplification curve and is marked with a red circle.

During the actual buildup, the field properties (transverse mode, waist position) change, so the state of the oscillator in early passes cannot be perfectly described with the seed parameters corresponding to the steady state, which can have dramatic impact on the amplification. To illustrate the strong dependence of the amplification on the transverse properties of the field, in Fig. 8 we additionally plot the amplification as a function of seed power for a seed with the waist shifted upstream to the end of the first undulator (green curve).

Comparing the curves with respect to the buildup condition [Eq. (11)], the model based on the reference seed (blue curve) predicts no buildup, while the seed with a shifted focus (green curve) would allow for buildup but not reach the target steady-state power.

Although significantly different in values, the two curves for different seed waist positions (blue and green) show similar qualitative features. Both exhibit constant amplification over a range in P_{in} which spans many orders of magnitude, followed by a dip, which we will refer to as the death valley (DV), after which the amplification recovers and decreases again, finally reaching the design value for the case where the waist position is in the middle of the amplifier (blue curve).

Further insights are obtained from Fig. 9(a), which shows the power evolution along the amplifier for three different power levels (1 MW, 1.8 GW, 11.6 GW), corresponding to the enumerated points on the blue curve in Fig. 8: before the dip, at the lowest point (DV), and at the subsequent peak. In Figs. 9(b)–9(d), we report the corresponding longitudinal phase-space distributions for a one-wavelength-long slice of the electron beam at the end of the first undulator module (U1). In the first case (before DV, blue curve), all particles are detrapped by the end of U1 due to the strong tapering. However, the energy spread is still very small. The phase oscillations of the free particles result in periodic energy exchange between the particles and the field, which can be seen in Fig. 9(a), blue curve for $z > 5$ m. The decrease in the period of these oscillations

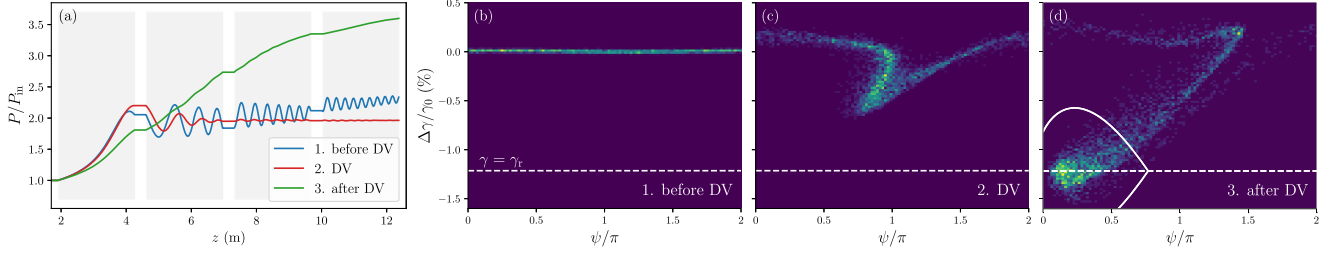


FIG. 9. (a) Power (normalized to P_{in}) along the amplifier ($z = 0$ corresponding to the beginning of the modulator) for three different values of P_{in} , corresponding to the points before, in, and after the death valley (DV), as marked on Fig. 8. (b)–(d) Electron beam phase space (single slice) at the end of the first undulator (U1), spanned by the phase of the particles ψ/π relative to the field and the relative energy deviation $\Delta\gamma/\gamma_0$. When calculating ψ , the on-axis phase of the field was used. The dashed horizontal line marks the resonant energy, while the solid line in (d) shows the separatrix based on the on-axis field.

is due to the taper. While the exchange would generally have zero average, the phase jumps induced by the phase shifters lead to a small net increase in field energy. In the second case (DV, orange curve), the particles are also detrapped by the end of U1, but the energy spread is already very high, owing to the increased prebunching and the stronger FEL interaction in the first undulator module. Due to this increased energy spread, there is barely any net energy exchange in the following undulator sections. In the last case (after DV, green curve), the energy loss of the particles is better matched to the resonance given by the tapering profile, so a significant fraction of the particles remain trapped.

We further investigated the buildup using multipass simulations in time-independent mode, starting with an input field at $P_{in} = 100$ W comparable to the shot noise power. The buildup at the nominal $R = 0.378$ does take place, and by extracting the amplification and input power on each pass, we obtain the red curve shown in Fig. 8. Neglecting the first few passes, where the transverse mode of the pulse transitions from a perfect Gaussian to the cavity

mode, we observe that the overall shape of the curve is comparable to the curves obtained in single-pass simulations. In terms of absolute values, the amplification in the multipass case (red) initially follows the green single-pass curve more closely, but as P_{in} increases toward the steady-state region, it approaches the values of the blue curve. We explain this with changes in field properties that were not accounted for in single-pass simulations with fixed parameters of the input field. As an example, in Fig. 10, we show the position of the radiation waist (blue) and the waist size (red, defined as 2σ of a Gaussian fit) in the multipass simulation as functions of P_{in} . In Fig. 10, the waist position is observed to shift downstream with increasing power, which can be explained as follows. While at steady-state power level the entire amplifier length (four undulators) contributes to the emission, in the early, low-power passes it is mainly the first undulator that contributes (see the blue curve in Fig. 9), and the radiation source point imaged by the cavity mirrors on the next pass is correspondingly shifted upstream. In the buildup stage this upstream shift of the focus position leads to a better coupling and higher gain.

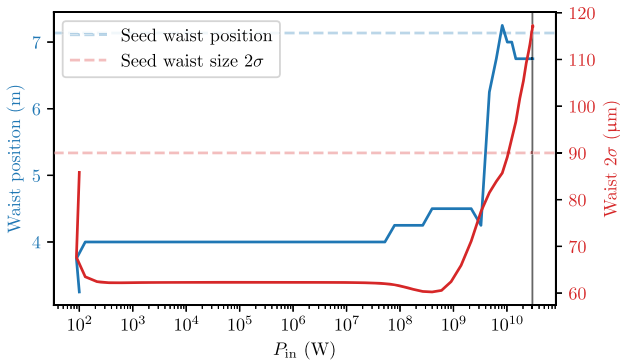


FIG. 10. Waist position along the beamline (blue) as well as waist size (red) of the field in multipass, single-slice simulations as a function of input power. To generate this plot, the field from the beginning of each pass was propagated in free space (no gain) and the field size was calculated at each point as double the sigma of a Gaussian fit. The dashed lines show the reference values used for the ideal Gaussian seed.

D. Assisted startup, time-independent simulations

We have seen that buildup is possible with our set of parameters without adapting them throughout the buildup, but in the general case, one might need to assist the system to reach the steady-state power levels. For example, it might occur that the death valley is too deep for the target recirculation fraction R and one would need to decrease the outcoupling fraction, thus sacrificing the overall efficiency to allow for buildup. Additionally, if the accelerator runs in burst mode [35], the limited number of bunches per train adds an important constraint on how fast the buildup has to be. In general, it is in our interest to have the buildup happen as fast as possible, as during the buildup the source is not stable and does not meet the reference parameters, so that the electron beams spent on the buildup do not contribute to the delivery of light to users. Additionally, for a given design steady-state power, we want to operate at the lowest possible R , which corresponds to the highest

outcoupling fraction Γ_{oc} , and therefore maximizes the power available to users.

We investigated the possibility of dynamically tuning key parameters of the system to control the evolution of the amplification. In order to be effective, the variation of these parameters will have to occur within a few passes, which at 3 MHz corresponds to a few microseconds, limiting the available tuning knobs. In the following, we discuss the option of using fast electromagnetic phase shifters.

The phase-shifter phases giving the highest amplification depend on the input field and therefore change from pass to pass during the buildup, while until now we have used a fixed set of phases Φ_{ss} optimized for the steady state. However, continuous adaptation of phases during the buildup is technically challenging and requires elaborate optimization, so we propose a single switch between two sets. This is motivated by the fact that the amplification is constant during most of the buildup (Fig. 8), and the conditions in this regime barely change, suggesting that a single set of phase-shifter phases could lead to improved performance in the whole constant amplification range and lead to faster buildup.

Following this ansatz, we optimized the phase shifters for maximum amplification at $P_{in} = 1$ MW in single-pass simulations with an approach similar to the one followed when optimizing the taper (see Sec. IV A). We denote this new set of buildup-optimized phases as Φ_b . Figure 11 shows the amplification curve from a multipass simulation with phases Φ_b (red) and Φ_{ss} (blue). As intended, the phases optimized for low seed power in single-pass simulations perform much better in the initial stage of buildup in multipass simulation. At $P_{in} \approx 1.3 \times 10^9$ W, the amplification curves cross, after which Φ_b perform worse than Φ_{ss} , ultimately leading to a lower steady-state power. The intuitive idea is then to use the first set of phases for the

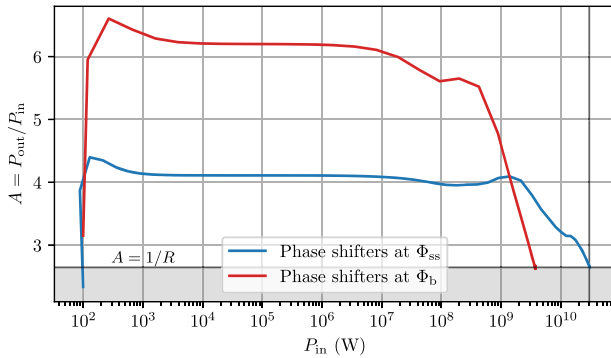


FIG. 11. Amplification $A = P_{out}/P_{in}$ as a function of input power P_{in} from multipass simulations with phase-shifter phases optimized for the power in buildup (Φ_b , red) and steady state (Φ_{ss} , blue). The gray horizontal and vertical lines mark the reference steady-state amplification $A = 1/R$ and input power $P_{in} = 30$ GW. Gray shading is applied to the area where the amplification is insufficient for the buildup.

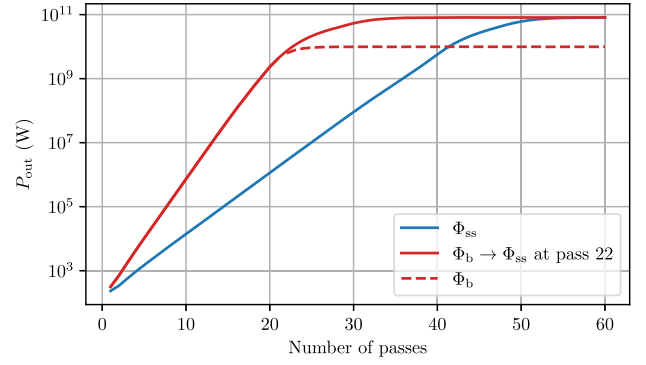


FIG. 12. Power as a function of number of passes for different phase shifter configurations: phases Φ_{ss} optimized for steady state fixed throughout the whole buildup (blue) and phases Φ_b optimized for buildup used the first part of buildup and switched to Φ_{ss} after (red, solid). The red dashed line shows how the system converges to a lower steady state if the switch is not done, so only phases Φ_b are used.

buildup and then switch to the new set at the crossing point. We approximate this in multipass simulations under the assumption that the switch is fast enough to occur between two consecutive shots. The resulting buildup curve is shown in the solid red line in Fig. 12, along with the cases where only a single set of phases is used. One can see that changing the phases allows the oscillator to reach the designed steady state in about 30 passes as opposed to 50, shortening the buildup by almost a factor of 2.

Finally, we consider the possibility of changing the electron beam energy during buildup. This change is possible by appropriately preshaping the accelerating radio-frequency pulse, which defines the beam energy along the pulse train [51]. Due to the strong taper, at low seed power the choice of the beam energy influences which part of the amplifier lases at the wavelength of interest and therefore affects the source point position.

E. Time-dependent simulations

In this section of the paper, we extend the simulations to the time-dependent case to include the effect of slippage. Assuming that the FEL pulse and the electron bunch are perfectly overlapped at the entrance of the modulator, by the end of the beamline (amplifier exit) the FEL pulse is shifted toward the head of the electron bunch by the total slippage length, which includes slippage in the undulators, as well as the delays introduced in the drifts and the chicane. The cavity length has to be adjusted (cavity detuning) to compensate for the slippage and ensure a consistent overlap between the electron beam and the radiation on the next pass. This is implemented in the simulations by applying a longitudinal shift Δs to the field in each pass.

In the time-dependent regime, the finite slippage between the electron bunch and the radiation pulse as well as the sideband instability affect the spectral width of the

circulating field, eventually preventing stable operation. We introduce a spectral monochromator in the cavity which acts as a passive stabilizing element by limiting the spectral bandwidth of the radiation. In our simulations, this element is modeled as a Gaussian spectral filter, whose width determines the steady-state bandwidth and indirectly the pulse duration. In the following, we will use the term monochromator bandwidth to refer to the relative rms bandwidth of this filter, i.e., $\sigma_{\lambda,m}/\lambda$. Monochromatization suppresses sideband growth naturally included in the GENESIS 1.3 time-dependent model, leading to a well-defined, reproducible pulse in steady state. In this sense, the monochromator balances the slippage effects, enhances the spectral longitudinal coherence and prevents super-radiant broadening, ensuring convergence to a stable narrowband steady state. Since the spectral losses in the monochromator decrease the outcoupling efficiency, one has to strike a delicate balance between having a bandwidth too narrow for effective outcoupling and too large to counteract time-dependent effects. Although the implementation of narrow-bandwidth filters at these wavelengths is technically demanding, a cavity-based grating monochromator could, in principle, serve the dual role of a spectral filter and an outcoupler using the 0th order reflection, a concept that merits further investigation beyond the scope of the present theoretical study. The suppression of the sideband instability justifies our choice of time-independent simulations for tapering optimization [42].

In the first step of time-dependent optimization, we found the optimal cavity length by performing a series of multipass simulations at different Δs . The simulations were started with a 30 GW input field and the peak powers were compared after about 30 passes, when the steady state was reached for all cases. The optimum was found to be at $\Delta s \approx -5.5 \mu\text{m}$. This is comparable to the total slippage expected in the undulators and the additional contribution from the magnetic chicane. The cavity length scan was repeated for three different monochromator bandwidths ($\sigma_{\lambda,m}/\lambda = 2 \times 10^{-4}, 3 \times 10^{-4}, 5 \times 10^{-4}$) and, as expected, yielded the same optimal Δs . However, the sensitivity of the output power to changes in cavity length (i.e., for nonoptimal Δs) was stronger at larger monochromator bandwidths.

The last step in our design is the choice of the optimal monochromator bandwidth, which depends on the bunch length. Longer bunch lengths reduce the relative impact of slippage and allow operation with lower spectral losses in the monochromator and hence a higher outcoupling fraction. In Fig. 13, we report the outcoupling fraction (blue) as a function of the monochromator bandwidth for electron bunches with lengths of 30, 60, and 90 μm and constant current of 1 kA. For each point, R was adjusted by tuning Γ_{oc} to bring the steady-state input power to the design value of 30 GW (within 0.2%). In all the reported conditions, increasing the monochromator bandwidth decreases the

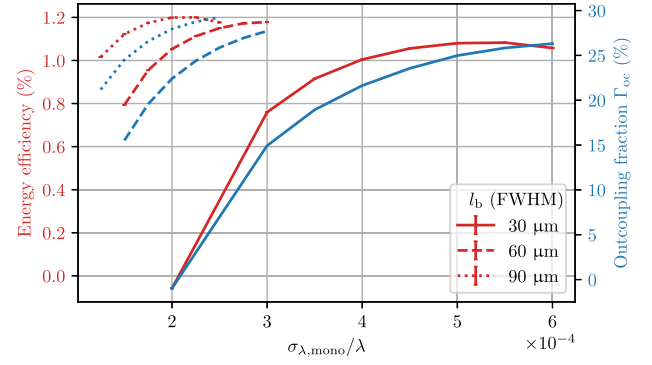


FIG. 13. Oscillator energy efficiency (red) and outcoupling fraction (blue) as functions of the monochromator bandwidth for three different electron bunch lengths. The optimal efficiency is the compromise between the increasing outcoupling fraction and decreasing pulse energy due to shortening of the pulse.

spectral losses and allows for a higher outcoupling fraction Γ_{oc} and therefore an overall increased peak efficiency.

For time-dependent simulations, it is more meaningful to consider the full energy of the outcoupled pulse, rather than only the peak power as a figure of merit. We can then define the oscillator energy efficiency (integrated efficiency) as the ratio of the outcoupled radiation energy to the integrated electron beam power ($W_{oc}/(\int P_{beam}(t)dt)$). In Fig. 13, this quantity is plotted in red and shows a maximum at intermediate monochromator bandwidths, suggesting an optimal operation point. This behavior is due to the decrease in pulse length (and therefore total energy) with increasing monochromator bandwidth.

Finally, we present the time-dependent simulations of the startup using the optimal parameters of $\sigma_{\lambda,m}/\lambda = 2.0 \times 10^{-4}$, $R = 40.53\%$ for the 90 μm -long electron beam, which ultimately yield an energy efficiency of about 1.2%. The higher recirculation fraction is necessary to compensate for spectral losses. The upper plot (a) in Fig. 14 shows the peak power in the cavity (red) and the rms pulse length (blue) along the buildup. Below, we report the power profiles (b)–(d) and normalized spectral intensities (e)–(g) for passes 10, 60, and 100, together with the corresponding temporal and spectral phases (red). The final FWHM relative spectral width is about $\approx 1.59 \times 10^{-4}$, almost 3 times narrower than the monochromator bandwidth, while the temporal phase in steady state shows a quadratic chirp. This chirp is a consequence of the phase lag incurred by the electrons that experience lower driving field amplitudes in different regions of the radiation pulse. Only at the design value for the peak power is the phase of the trapped electrons matched to the resonant phase. Whenever the power is lower, the stable region in the ponderomotive phase shifts in phase, pulling the radiation field, leading to a natural development of a quadratic chirp of the output phase over many amplification passes. This illustrates another benefit arising from spectral filtering—in steady state the monochromator effectively compensates for the chirp-induced in amplification in a single pass.

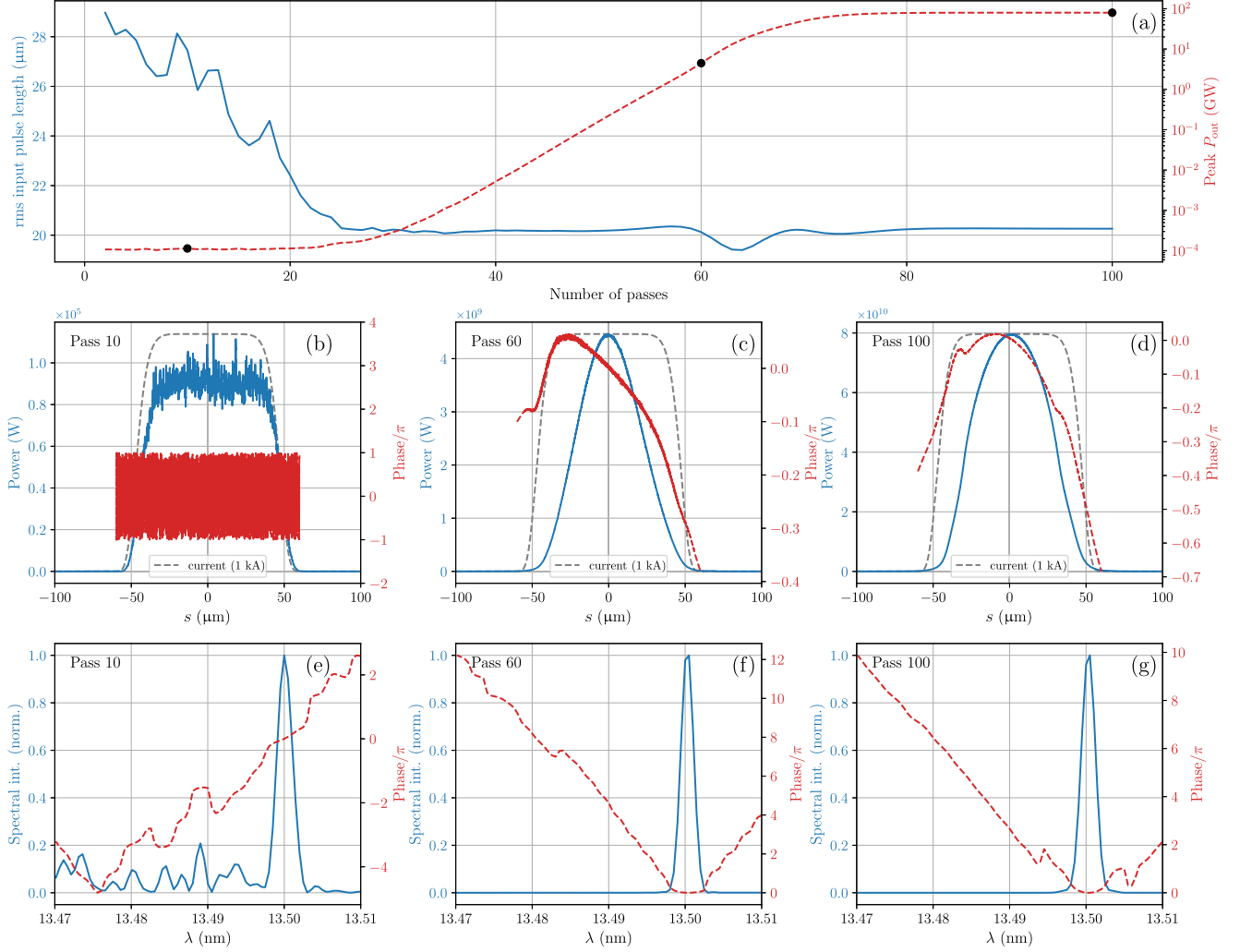


FIG. 14. Time-dependent simulations of the oscillator startup from shot noise for the optimal monochromator bandwidth of $\sigma_{\lambda,m}/\lambda = 2.0 \times 10^{-4}$, the recirculation fraction $R = 40.53\%$ (outcoupling fraction $\Gamma_{oc} = 27.9\%$), and a $90 \mu\text{m}$ electron beam (other parameters as in Table I). Plot (a) shows the peak power (red) and rms pulse length (blue) as functions of the number of passes. The black circles mark the passes 10, 60, and 100, for which we show the power profiles [(b)–(d), blue] and normalized spectral intensities [(e)–(g), blue], together with corresponding temporal and spectral phases (red).

V. CONCLUSION

In summary, we investigated a strongly tapered FEL oscillator with a prebuncher, aimed at an increased energy conversion efficiency. We provided a 3D analytical model of a single-pass strongly tapered FEL amplifier, which was then applied to the case of a high-power seed and a strongly prebunched beam. The resulting equation gave an estimate of achievable efficiency and a recipe for the optimal tapering design. The model was applied to the case of an EUV FEL oscillator at 13.5 nm wavelength and compared to numerical simulations with GENESIS 1.3. We described the step-by-step optimization of the setup, starting from the analytically predicted tapering profile guess. We then used time-independent simulations to study the power buildup in the oscillator from the shot noise, since seeding sources with adequate power are missing at the

wavelength of interest. Furthermore, we discussed ways to facilitate the buildup. Finally, we extended the simulations to incorporate time-dependent effects and confirmed that with the use of spectral filtering the startup from shot noise takes place. We discussed the necessary optimization of the cavity length and monochromator bandwidth. At the optimized working point, the system reached peak intra-cavity power of about 80 GW corresponding to 15.6 mJ pulse energy, and an overall outcoupling energy efficiency of 1.2% .

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DATA AVAILABILITY

The data that support the findings of this article are not publicly available upon publication because it is not technically feasible and/or the cost of preparing, depositing, and hosting the data would be prohibitive within the terms of this research project. The data are available from the authors upon reasonable request.

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