

Compensation of emittance variation in a diffraction-limited storage ring

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In diffraction-limited storage rings (DLSRs), due to the comparable energy loss from bending magnets to that from insertion devices (IDs), the independent adjustments of ID gaps can introduce random emittance variations, which need to be compensated. Hefei Advanced Light Facility (HALF) is a soft x-ray DLSR with both 20 long dispersion-free and 20 short dispersive straight sections. Its original emittance compensation scheme is based on the radiation damping effect, achieved by adjusting the gaps of two damping wigglers (DWs) located in long straight sections. This paper proposes a novel emittance compensation scheme for HALF based on the quantum excitation effect. In this new scheme, a short variable-gap wiggler is installed in a dispersive straight section, with the two DWs operating at their minimum gaps. The proposed scheme achieves emittance compensation comparable to the original scheme in both cases with and without intrabeam scattering, even though the wiggler is significantly shorter. It also yields significantly smaller energy spread variations and consequently, very small effective emittance variations at short dispersive straight sections, with the effective emittance considered basically compensated. Moreover, fixing the minimum gaps of the two DWs can provide additional beamlines for users as well as enhanced radiation damping, which not only compensates for the emittance growth caused by the short wiggler due to quantum excitation but also reduces the damping times to suppress intrabeam scattering. For DLSRs without dispersive straight sections, the proposed quantum-excitation-based emittance compensation scheme can be applied by converting a dispersion-free straight section into a dispersive one simply using a dipole chicane.

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I. INTRODUCTION

In a diffraction-limited storage ring (DLSR), the beam emittance is reduced to a level comparable to the diffraction-limited emittance of emitted x-rays [1]. To achieve such ultralow emittances, multibend achromat (MBA) lattices [2] are used in the DLSR design [3–11], which utilize stronger transverse focusing and weaker dipole fields, compared with third-generation light source storage rings. Due to the lower average dipole field, the energy loss from bending magnets has a significant reduction and becomes comparable to that from insertion devices (IDs). Independent adjustment of ID gaps by users will change the radiation energy from IDs and thus introduce random emittance variations in a DLSR, which need to be compensated.

Several methods have been developed to compensate for ID-induced emittance variations in DLSRs [12–18].

The most common method is to use variable-gap wigglers in dispersion-free straight sections, utilizing the radiation damping effect to compensate for emittance [15,18,19]. By adjusting their gaps, the radiation energy loss of the wigglers can be controlled to counteract the change in energy loss caused by adjustments of ID gaps. A passive method studied for SPring-8-II works by leaking a small amount of dispersion into each ID straight section, which avoids the use of additional compensation wigglers [16,17]. This method minimizes variations in effective emittance by optimizing the value of the leaked dispersion. In Ref [14], a method using a variable dispersion bump inside a fixed-gap wiggler was proposed to compensate for emittance variations.

Hefei Advanced Light Facility (HALF) is a soft x-ray DLSR under construction in Hefei, China. The HALF storage ring employs a modified hybrid 6BA (H6BA) lattice and provides 20 long dispersion-free and 20 short dispersive straight sections [9,20]. The linear optics and magnet layout of the HALF H6BA lattice are shown in Fig. 1. The main parameters of the HALF storage ring are listed in Table I. The original scheme of emittance compensation for HALF adopts the most common method based on radiation damping. Two damping wigglers (DWs) located in long straight sections can serve as

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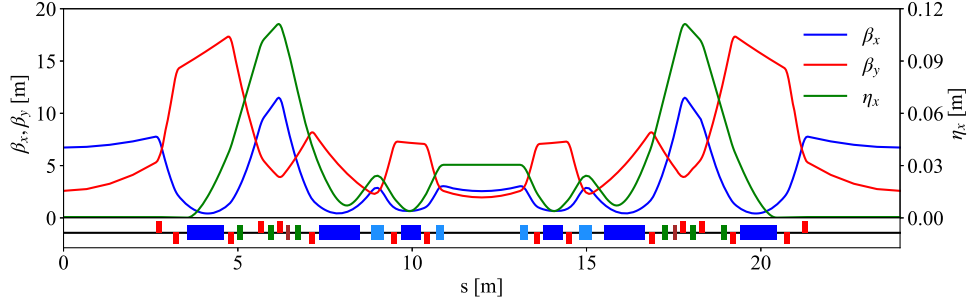


FIG. 1. Linear optics and magnet layout of the HALF H6BA lattice. In the magnet layout, main bends are shown in blue, reverse bends in cyan, quadrupoles in red, sextupoles in green, and octupoles in brown.

variable-gap wigglers for emittance compensation. This scheme introduces large energy spread variations during emittance compensation, leading to effective emittance variations at the short straight section. The emittance compensation for HALF needs to simultaneously address the emittance variations at both types of straight sections. Inspired by the passive method, in this paper, we propose a novel emittance compensation scheme for HALF based on the quantum excitation effect, where a short variable-gap wiggler is installed in a dispersive straight section and the two DWs are fixed at their minimum gaps. This new scheme significantly reduces the effective emittance variations at short straight sections while compensating for the emittance variations at long straight sections, offering multiple advantages that will be presented in this paper.

This paper is structured as follows. Section II presents the emittance compensation schemes for HALF, including the original radiation-damping-based scheme and the proposed new quantum-excitation-based scheme. Section III analyzes the emittance variations induced by the Phase-I IDs in HALF. The effectiveness of the proposed new scheme is evaluated through comparative analysis with the original scheme for the case without the intrabeam scattering (IBS) effect in Sec. IV and the case with IBS in Sec. V. Section VI investigates some influencing factors in

the proposed scheme. Section VII presents a preliminary study, by applying the proposed scheme, on the emittance compensation with the inclusion of assumed Phase-II IDs. Finally, a conclusion is given in Sec. VIII.

II. EMITTANCE COMPENSATION SCHEMES FOR HALF

The equilibrium emittances and energy spreads for different ID gaps can be calculated using synchrotron radiation integrals [21]:

$$\epsilon_x = C_q \gamma^2 \frac{I_5}{I_2 - I_4}, \quad \sigma_\delta = \sqrt{C_q \gamma^2 \frac{I_3}{2I_2 + I_4}}, \quad (1)$$

where γ is the Lorentz factor, $C_q = 3.382 \times 10^{-13}$ m, and I_2, I_3, I_4, I_5 are the synchrotron radiation integrals. The contribution of an ID to the radiation integrals is given by [15,22]

$$\begin{aligned} \Delta I_{2,\text{ID}} &= \frac{L_{\text{ID}}}{2\rho_{\text{ID}}^2}, \\ \Delta I_{3,\text{ID}} &= \frac{4L_{\text{ID}}}{3\pi\rho_{\text{ID}}^3}, \\ \Delta I_{4,\text{ID}} &\approx 0, \\ \Delta I_{5,\text{ID}} &\approx \frac{\langle \beta_x \rangle \lambda_{\text{ID}}^2 L_{\text{ID}}}{15\pi^3 \rho_{\text{ID}}^5} + \frac{4L_{\text{ID}} \mathcal{H}_{x0}}{3\pi\rho_{\text{ID}}^3}, \end{aligned} \quad (2)$$

where L_{ID} is the length of the ID, ρ_{ID} is the minimum curvature radius associated with the maximum vertical magnetic field, λ_{ID} is the period length of the ID, $\langle \beta_x \rangle$ is the average of the horizontal β function over L_{ID} , and $\mathcal{H}_{x0}$ is the $\mathcal{H}$ function at the center of the ID.

The above analysis is based on the uncoupled beam. In the presence of transverse coupling, there are two methods for calculating projected horizontal and vertical emittances. The conventional method first computes the uncoupled equilibrium emittance via Eqs. (1) and (2) and then derives the projected emittances from the uncoupled equilibrium emittance, transverse coupling ratio, and

TABLE I. Main parameters of the HALF storage ring.

Parameter	Value
Energy	2.2 GeV (retained up to 2.5 GeV)
Circumference	479.86 m
Number of cells	20
Natural emittance	85.9 pm · rad
Natural energy spread	0.61×10^{-3}
Betatron tunes (H/V)	48.15/17.15
Natural chromaticities (H/V)	-80.8/ -56.4
Momentum compaction factor	0.94×10^{-4}
Damping partition (H/V/L)	1.36/1.0/1.64
Natural damping times (H/V/L)	28.5/38.8/23.7 ms
Bend energy loss per turn	181.4 keV

horizontal and vertical damping partition numbers [21]. The other method, based on normal-mode decomposition of the one-turn transfer matrix, derives the beam sigma matrix and then extracts the projected emittances from the determinants of the 2×2 diagonal blocks [23]. We compared the projected emittances calculated using the two methods based on the HALF storage ring lattice with 20% and 99% coupling ratios. The relative difference between the two methods is less than 0.3%, which is negligible. For simplicity, the conventional method is adopted in the subsequent calculations.

To compensate for the emittance variations induced by IDs, the most straightforward method is to introduce a variable-gap compensation wiggler to counteract the variations in synchrotron radiation integrals. From Eqs. (1) and (2), it is known that the emittance compensation is primarily achieved by controlling I_2 and I_5 , which are associated with the radiation damping effect and the quantum excitation effect, respectively. Both effects can be utilized to compensate for emittance variations.

Installing variable-gap wigglers in dispersion-free straight sections of a DLSR can lead to a minor change in I_5 but a significant change in I_2 , which can be used for emittance compensation. This method works primarily based on the radiation damping effect. However, according to Eq. (1), this method also results in a substantial variation of beam energy spread during the emittance compensation [15,18]. For HALF, the energy spread variation will result in the variation of the effective emittance at the short dispersive straight section, which is calculated as [24]

$$\epsilon_{x,\text{eff}} = \sqrt{\epsilon_x^2 + \epsilon_x \eta_{x1}^2 \sigma_\delta^2 / \beta_{x1}}, \quad (3)$$

where η_{x1} and β_{x1} are the dispersion and horizontal β function at the center of the short straight section, respectively. Consequently, two sets of compensation wigglers are required to simultaneously compensate for both emittance and energy spread variations [18]. In the HALF storage ring, since the natural damping times are relatively long, as shown in Table I, two DWs located in long dispersion-free straight sections are employed to suppress IBS-induced emittance growth. To compensate for emittance variations, the two DWs can also serve as compensation wigglers by adjusting their gaps. This is the original emittance compensation scheme for HALF.

For some DLSRs with both dispersion-free and dispersive straight sections, such as HALF [9], Diamond-II [25], Elettra 2.0 [26], installing a variable-gap wiggler in a dispersive straight section can induce considerable changes in both I_2 and I_5 . If the dispersion is sufficiently large, the change in I_5 will have a dominant contribution to the emittance change. In this case, the variable-gap wiggler located in a dispersive straight section can be used for emittance compensation, which is mainly based on the

quantum excitation effect. However, this method will lead to relatively large compensated emittance due to quantum excitation. In the HALF storage ring, the 20th short straight section is unavailable for a user beamline due to spatial conflict with the injection, which can be used to install a variable-gap wiggler for quantum-excitation-based emittance compensation. The dispersion of the short straight section is 30 mm, as shown in Fig. 1. As will be shown, the short variable-gap wiggler is highly effective for emittance compensation; it also reduces the energy spread variations and, consequently, the effective emittance variations at short straight sections. Thus, the gaps of the two DWs can be fixed at their minimum values, which helps to better reduce damping times and compensate for the emittance growth caused by the wiggler in the 20th short straight section. This is the new emittance compensation scheme for HALF proposed in this paper.

III. EMITTANCE VARIATIONS INDUCED BY THE PHASE-I IDS IN HALF

The beam energy of the HALF storage ring is 2.2 GeV, with the potential to be increased to 2.5 GeV to mitigate the IBS-induced emittance growth. In the Phase-I operation, eleven undulators and two DWs will be installed in the HALF storage ring [27]. The undulators include one helical undulator (HU), two linearly polarized undulators (LPUs), six elliptically polarized undulators (EPUs), and two in-vacuum undulators (IVUs). Among these, one short IVU will be installed in a short straight section, while the others are located in long straight sections. The main parameters of the Phase-I IDs are summarized in Table II. The peak magnetic field of each ID increases as its gap is reduced, as shown for some undulators in Fig. 2. Independently adjusting the undulator gaps during user operation introduces random variations in beam emittance and energy spread.

To quantify the impact of independent adjustments of the Phase-I undulator gaps on beam emittance and energy spread, a statistical analysis similar to Ref. [13] was

TABLE II. Main parameters of the HALF Phase-I IDs.

ID Type	λ_{ID} (mm)	N_{period}	Max. K	Number
HU	115	35	8.97	1
EPU	120	33	11.26	1
EPU	63	65	5.77	1
EPU	43.5	36	2.94	2
EPU	46	90	3.29	1
EPU	41.5	99	2.67	1
IVU	20.7	182	2.10	1
IVU	20.7	52	2.10	1
LPU	38.2	108	2.83	1
LPU	38.5	108	3.36	1
DW	100	42	16.00	2

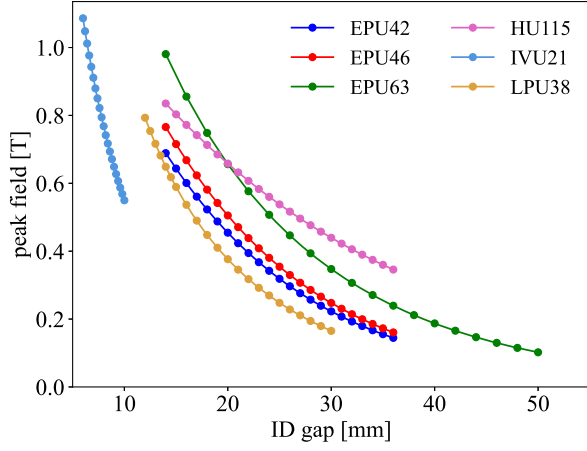


FIG. 2. Peak magnetic field as a function of gap for some undulators of HALF.

performed for the beam energy of 2.2 GeV. In the statistical analysis, it was assumed that all undulators operated in the horizontal polarization mode and that each operational gap value had an equal probability of being used. The calculated statistical results for undulator-induced emittance and energy spread variations are shown in Fig. 3. The average emittance is $78.2 \text{ pm} \cdot \text{rad}$, with a standard deviation of $2.3 \text{ pm} \cdot \text{rad}$. The emittance lies between 72.2 and $84.0 \text{ pm} \cdot \text{rad}$ for approximately 99% of operational cases, corresponding to a normalized variation of $\pm 7.5\%$ about the median. The energy spread is concentrated within the range from 5.88×10^{-4} to 6.04×10^{-4} .

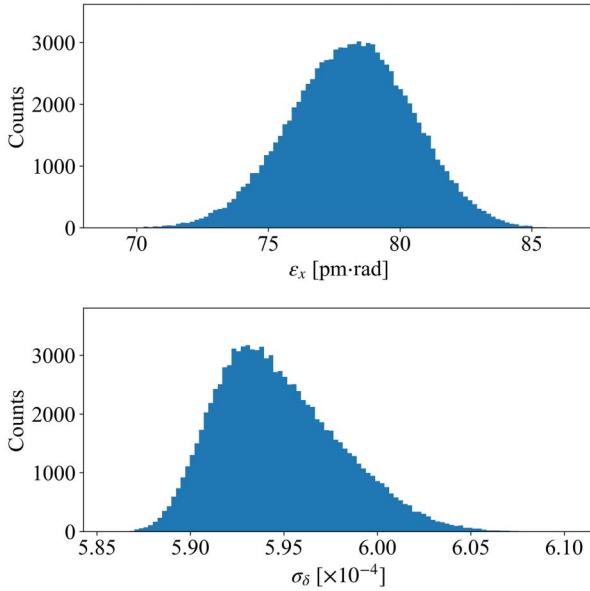


FIG. 3. Statistical analysis for undulator-induced emittance (upper) and energy spread (lower) variations due to independent adjustments of undulator gaps, performed on 100,000 randomly generated samples.

IV. COMPENSATION OF EMITTANCE VARIATION WITHOUT IBS

A comparative analysis in the absence of the IBS effect between the original radiation-damping-based compensation scheme and the proposed new quantum-excitation-based compensation scheme will be conducted in this section.

A. Radiation-damping-based emittance compensation scheme

The original radiation-damping-based emittance compensation scheme for HALF is to adjust the gaps of two DWs located in long dispersion-free straight sections. Our analysis of emittance compensation follows the method presented in Ref [15], as illustrated in Fig. 4. The green regions are the ranges of emittance and energy spread variations induced by random adjustments of the undulator gaps for approximately 99% operational configurations, and the emittance varies between 72.2 and $84.0 \text{ pm} \cdot \text{rad}$. For the user ID configuration corresponding to the upper limit $84.0 \text{ pm} \cdot \text{rad}$, as the magnetic fields of the two compensation wigglers increase from zero, the emittance

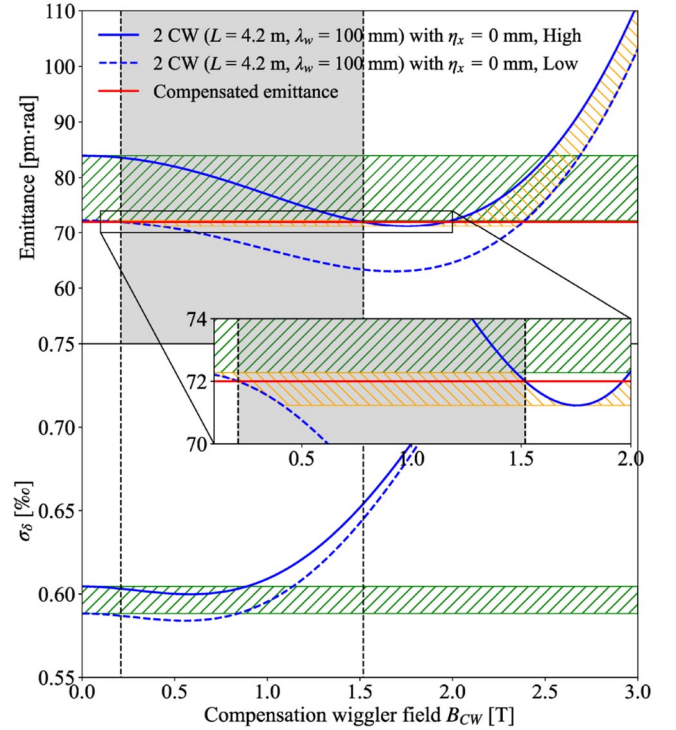


FIG. 4. Analysis of emittance compensation with two variable-gap wigglers in dispersion-free straight sections, under the condition without IBS. The green regions indicate the ranges of emittance and energy spread variations induced by user IDs. The orange region is the available range of emittance compensation, in which the red horizontal line is the compensated emittance, with the range of required wiggler field shown as the gray region.

follows the *High* curve and reaches a minimum value $71.2 \text{ pm} \cdot \text{rad}$ at 1.74 T . The behavior for the lower limit is analogous, as depicted by the *Low* curve. The energy spread exhibits a similar behavior, as shown by the two curves in the bottom panel of Fig. 4.

To maintain constant emittance, the compensated emittance must lie above the minimum value of the *High* curve, and below the lower limit $72.2 \text{ pm} \cdot \text{rad}$ when the magnetic field is lower than 1.74 T . The available range for compensated emittance is marked by the orange region in Fig. 4, and the lowest achievable compensated emittance occurs at the minimum of *High* curve. A reasonable choice of compensated emittance is $72.0 \text{ pm} \cdot \text{rad}$, marked by the red horizontal line. The gray region indicates the range of required magnetic field to achieve this compensated emittance, with a maximum magnetic field of 1.56 T , which is within the allowable field range for the HALF DWs. Achieving a lower compensated emittance would require a stronger magnetic field. It is also evident from Fig. 4 that the minimum emittance value of the *High* curve is very close to the lower limit of the emittance variation range. Therefore, two DWs are just sufficient for emittance compensation, leaving little margin.

This scheme offers the advantage of emittance reduction, with a compensated emittance below the emittance variation range. However, it comes with two notable drawbacks. First, as shown in the bottom panel of Fig. 4, it induces a relatively large variation in energy spread, ranging from 5.840×10^{-4} to 6.518×10^{-4} , corresponding to a normalized variation range of $\pm 5.5\%$. Since both DWs are dedicated to emittance control, there are no additional knobs to actively compensate for the energy spread variation. Second, the magnetic attraction force between the wiggler poles is strong, which imposes high requirement on the mechanical control for the adjustment of wiggler gaps.

B. Quantum-excitation-based emittance compensation scheme

In the quantum-excitation-based emittance compensation scheme, a variable-gap wiggler is installed in the 20th short dispersive straight section of the HALF storage ring, and the two DWs operate at their minimum gaps. In this section, the effective length and period length of this short compensation wiggler are 1.5 m and 100 mm , respectively. The influence of the wiggler parameters will be discussed in Sec. VI. With the two DWs operating at their minimum gaps, the emittance variation induced by user IDs ranges from 63.0 to $71.2 \text{ pm} \cdot \text{rad}$, shown as the green region in the top panel of Fig. 5.

In contrast to the radiation-damping-based compensation scheme, the beam emittance increases monotonically with the magnetic field of the compensation wiggler. To compensate for emittance variation, the compensated emittance must be no less than the upper limit $71.2 \text{ pm} \cdot \text{rad}$.

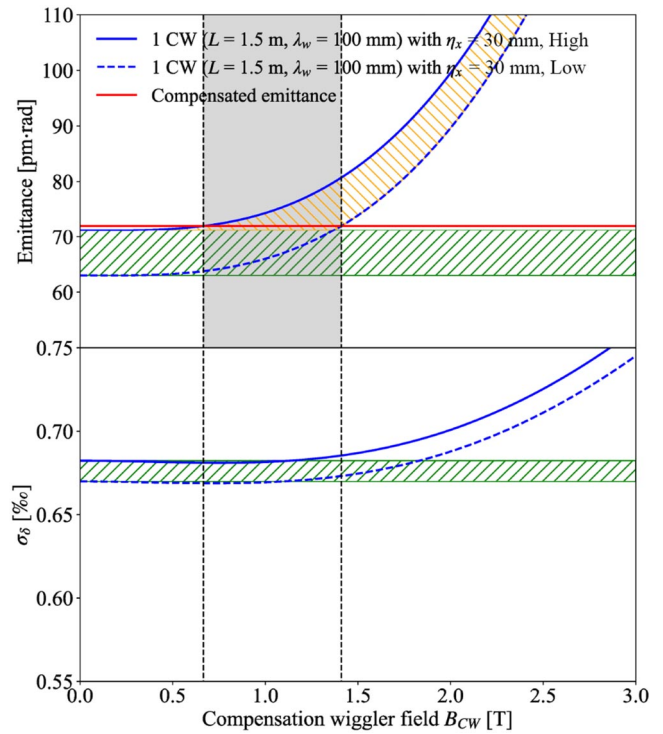


FIG. 5. Analysis of emittance compensation using one variable-gap wiggler in a short dispersive straight section, with the gaps of two DWs closed, under the condition without IBS. The green regions indicate the ranges of emittance and energy spread variations induced by user IDs. The orange region is the available range of emittance compensation, in which the red horizontal line is the compensated emittance, with the range of required wiggler field shown as the gray region.

The available range of compensated emittance is shown as the orange region in Fig. 5. Due to that a lower compensated emittance corresponds to a lower magnetic field of the compensation wiggler in this scheme, the best choice for the compensated emittance is $71.2 \text{ pm} \cdot \text{rad}$. Here, for better comparison, the compensated emittance is chosen as $72.0 \text{ pm} \cdot \text{rad}$, identical to that of the radiation-damping-based compensation scheme. The range of required magnetic field is $0.61\text{--}1.41 \text{ T}$. Compared with the radiation-damping-based scheme, the maximum magnetic field required is lower. Further considering that the compensation wiggler is significantly shorter, the quantum-excitation-based scheme is far more effective in emittance compensation. The magnetic attraction force between the wiggler poles is also significantly reduced.

From Figs. 4 and 5, it is clearly seen that the energy spread variation range in the quantum-excitation-based scheme is significantly smaller than that in the radiation-damping-based scheme. The energy spread variation in the quantum-excitation-based scheme is in a very narrow range from 6.688×10^{-4} to 6.854×10^{-4} , corresponding to a normalized variation range of $\pm 1.2\%$. For further comparison of the two schemes, we simulated the effects

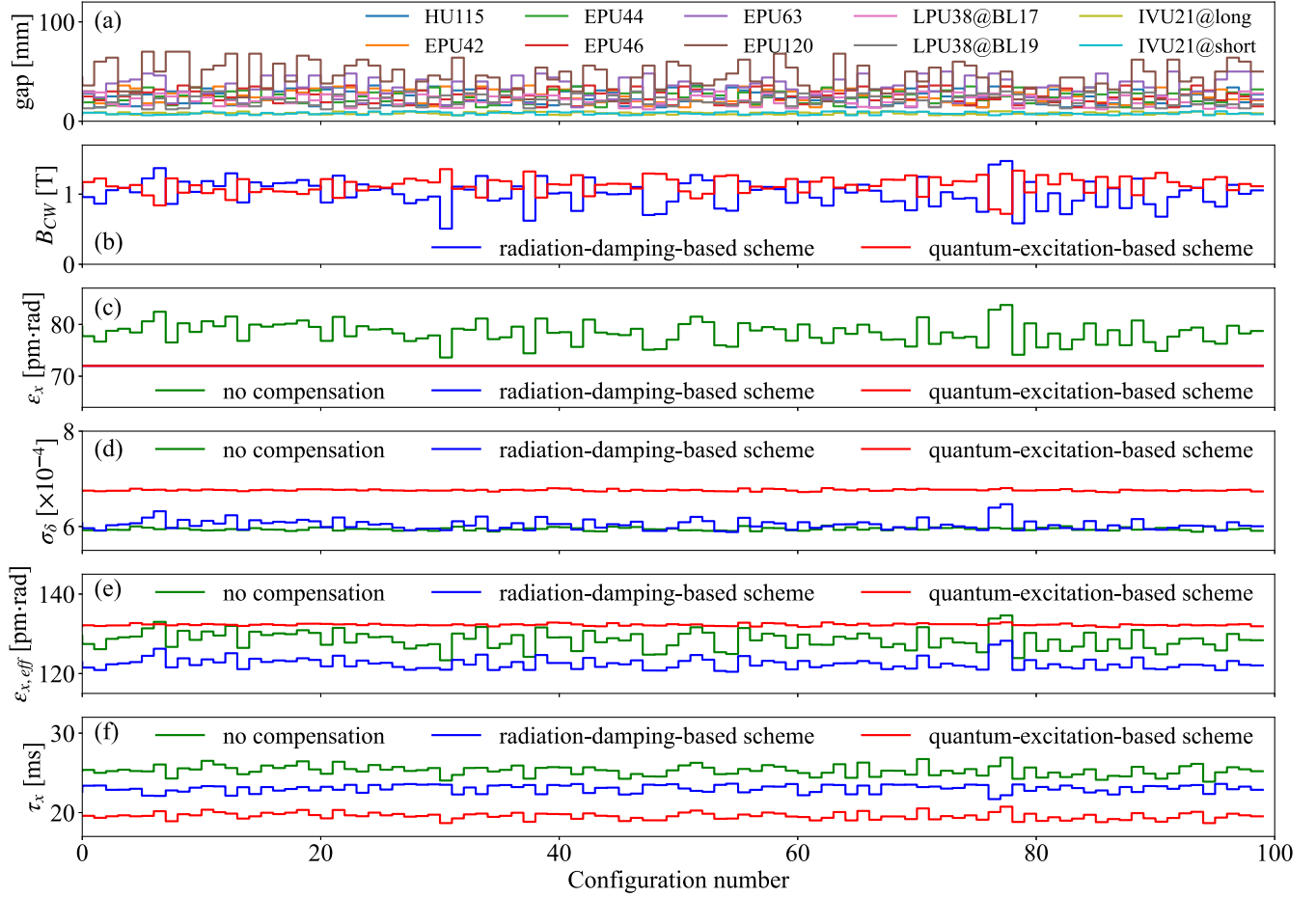


FIG. 6. Simulation of effects caused by random variations in the HALF user ID gaps for the radiation-damping-based and quantum-excitation-based emittance compensation schemes, under the condition without IBS. (a) Random adjustment of the ID gaps; (b) required magnetic fields of compensation wigglers; (c) emittance variations before and after the two emittance compensation schemes; (d) energy spread variations before and after the two schemes; (e) effective emittance variations at short dispersive straight sections before and after the two schemes; (f) horizontal damping time variations before and after the two schemes.

caused by random variations in the user ID gaps. In the simulation, the required magnetic fields of the compensation wigglers, along with the resulting variations in emittance, energy spread, effective emittance at short dispersive straight sections and damping time during emittance compensation, were calculated for 1000 random configurations of ID gaps. Simulation results for the first 100 configurations are shown in Fig. 6. It confirms the smaller energy spread variations in the quantum-excitation-based scheme. For the 1000 configurations of ID gaps, the standard deviation of energy spread variations before emittance compensation is 0.324×10^{-5} . After the compensation, the standard deviation of energy spread variations increases to 1.130×10^{-5} for the radiation-damping-based scheme, whereas it is reduced to 0.196×10^{-5} for the quantum-excitation-based scheme. As a result, the effective emittance variations at short dispersive straight sections are significantly reduced in the quantum-excitation-based scheme, as shown in the

panel (e) of Fig. 6. The normalized effective emittance variation range is $\pm 0.5\%$, and thus, the effective emittance can be considered basically compensated.

The reduced energy spread variation in the quantum-excitation-based scheme is mainly due to the partial cancellation between the energy spread variation induced by user IDs and that induced by the wiggler during the emittance compensation. The average magnetic field of user IDs is relatively weak, whereas that of the wiggler is strong. As a result, for most configurations, an increase in the average magnetic field of user IDs leads to decrease in both emittance and energy spread, while an increase in the wiggler field generally leads to growth in both emittance and energy spread. However, for most configurations in the radiation-damping-based scheme, the emittance and energy spread exhibit opposite changes when the fields of the two DWs vary, resulting in the amplified energy spread variation during emittance compensation.

Besides the reduced energy spread variation, the damping times are also shortened in the quantum-excitation-based scheme due to the two DWs operating at their minimum gaps, with the horizontal damping time shown in the panel (f) of Fig. 6. The average horizontal damping time is 25.25 ms before emittance compensation, which is reduced to 23.06 ms in the radiation-damping-based scheme and further reduced to 19.55 ms in the quantum-excitation-based scheme. The reduced damping time helps to suppress the emittance growth caused by IBS. Additionally, with fixed gaps, the two DWs can provide two additional beamlines, thereby enhancing the overall beamline supply capacity of the HALF storage ring. The only disadvantage of this quantum-excitation-based scheme is that it compensates for emittance variation by increasing emittance. However, since the two DWs operate at their minimum gaps, the emittance is significantly reduced, and thus, the compensated emittance can be nearly the same as that achieved in the radiation-damping-based scheme.

V. COMPENSATION OF EMITTANCE VARIATION WITH IBS

For the HALF storage ring, the diffraction-limited emittance and the relatively low beam energy lead to a significant IBS effect. The equilibrium emittance is determined by the balance among radiation damping, quantum excitation and IBS. To evaluate the performance of the two emittance compensation schemes with IBS, we will analyze the equilibrium emittance variation for two representative transverse coupling ratios, 20% and 100%, while assuming the bunch lengthened by a factor of 4 with harmonic cavity and the beam current of 400 mA.

A. Coupling ratio of 20%

Unlike the zero-current case, the emittance variation with IBS and 20% coupling is compensated using only one DW in the radiation-damping-based scheme, while the other DW operates at its minimum gap. This is attributed to the fact that the IBS-induced emittance growth is closely related to the damping time. Increasing the magnetic field of DW shortens the damping time, thereby mitigating the emittance growth induced by IBS. Therefore, in the presence of IBS, the radiation-damping-based scheme is more effective in controlling the emittance variation than in the absence of IBS. While for the quantum-excitation-based scheme, the change of damping time induced by the compensation wiggler is small due to the short wiggler length.

A comparative analysis between the two compensation schemes with IBS and 20% coupling is illustrated in panels (a) and (b) of Fig. 7. In the radiation-damping-based scheme, with one DW operating at the minimum gap,

the emittance variation before compensation ranges from 123.2 to 132.8 pm · rad. In the quantum-excitation-based scheme, with both DWs operating at their minimum gaps, the corresponding emittance variation ranges from 114.7 to 122.7 pm · rad. Therefore, both schemes can compensate for the emittance to 123.0 pm · rad. In the former scheme, the required magnetic field varies from 0.34 to 1.68 T; while in the latter, it ranges from 0.66 to 1.72 T. Although both schemes demand similar maximum magnetic fields, the quantum-excitation-based scheme still has significantly weaker magnetic attraction force due to its shorter wiggler length.

In the radiation-damping-based scheme, the energy spread ranges from 7.514×10^{-4} to 7.829×10^{-4} , corresponding to a normalized variation of $\pm 2.1\%$. In the quantum-excitation-based scheme, the energy spread ranges from 7.673×10^{-4} to 7.835×10^{-4} , with a normalized variation of $\pm 1.0\%$. Like the zero-current case, similar emittance compensation simulations with 1000 configurations of ID gaps were performed for the two schemes. The variations of energy spread and effective emittance at short dispersive straight sections for the first 100 configurations, corresponding to the ID gap configurations presented in the panel (a) of Fig. 6, are shown in Fig. 8. The standard deviation of the energy spread variations before compensation is 0.300×10^{-5} . After the emittance compensation, it rises to 0.643×10^{-5} for the radiation-damping-based scheme, whereas it is 0.305×10^{-5} for the quantum-excitation-based scheme. Therefore, the range of the normalized effective emittance variations at short dispersive straight sections, as shown in Fig. 8, is reduced to $\pm 0.9\%$ for the quantum-excitation-based scheme, versus $\pm 1.5\%$ for the radiation-damping-based scheme. Compared with the zero-current case, the effective emittance variations are reduced in the radiation-damping-based scheme but increased in the quantum-excitation-based scheme. Nevertheless, the latter remains significantly smaller than the former.

B. Coupling ratio of 100%

For the coupling ratio of 100%, the IBS effect is relatively weak compared with the 20% coupling case. A comparative analysis of the two compensation schemes is presented in the panels (c) and (d) of Fig. 7. In the radiation-damping-based scheme, also with one DW operating at the minimum gap, the range of emittance variation induced by IDs is between 66.3 and 72.7 pm · rad. The emittance is compensated to 66.2 pm · rad with the wiggler field ranging from 0.23 to 1.74 T. The maximum field 1.74 T slightly exceeds the capability of the HALF DWs. Therefore, a single DW cannot cover 99% of user ID configurations for emittance compensation. If two DWs are used for emittance compensation in the radiation-damping-based scheme, the compensated emittance is larger due to longer damping time. In the

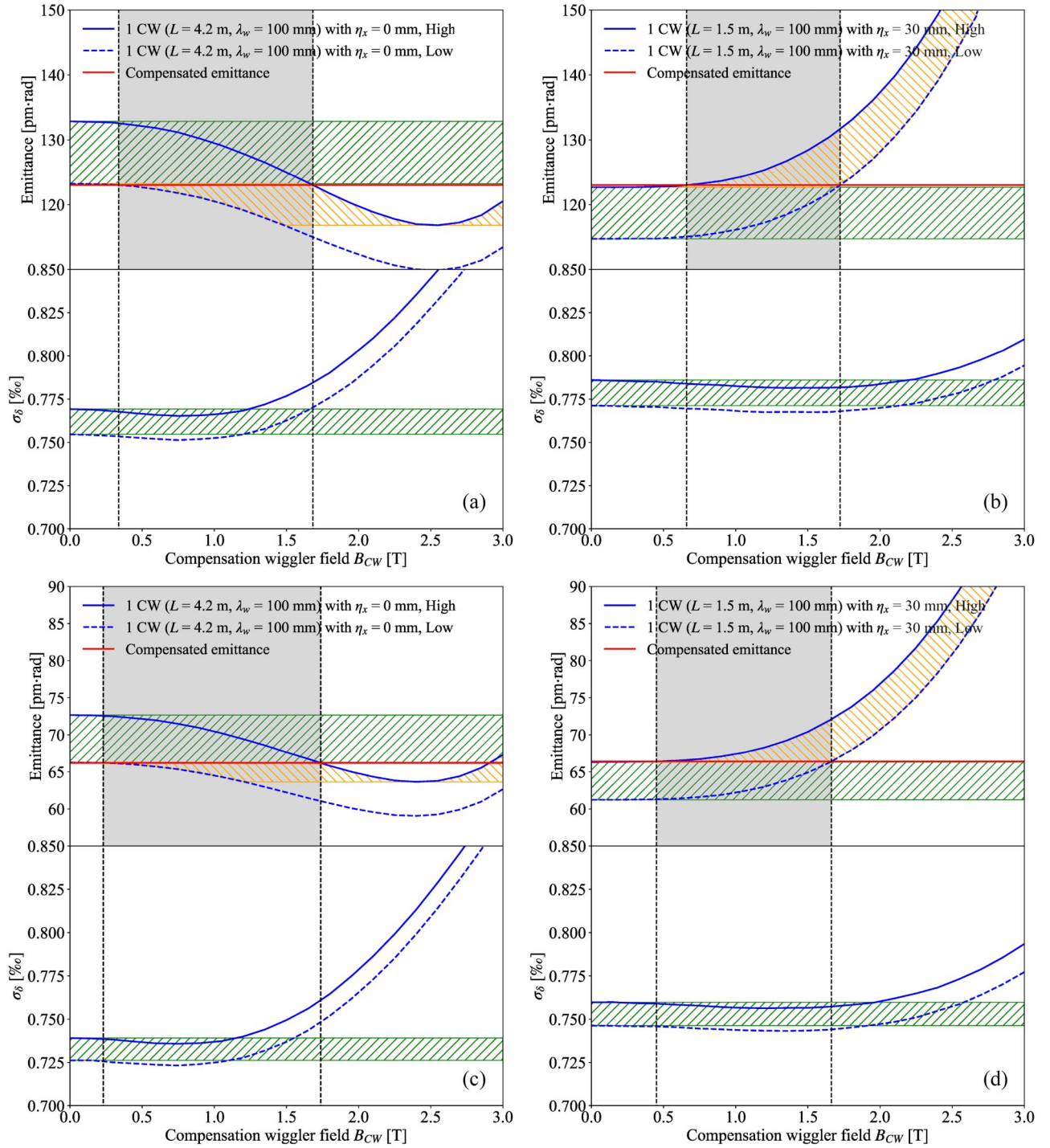


FIG. 7. Comparative analysis of the radiation-damping-based (left panels) and quantum-excitation-based (right panels) emittance compensation schemes with IBS. In the radiation-damping-based scheme, one DW is used for emittance compensation and the other operates at the minimum gap. The top and bottom panels illustrate the cases of 20% and 100% transverse coupling ratios, respectively. The green regions indicate the ranges of emittance and energy spread variations induced by user IDs, the orange regions the available ranges of emittance compensation, and the gray regions the ranges of the required wiggler fields. The compensated emittances for panels (a)–(d) are 123.0, 123.0, 66.2, and 66.4 pm · rad, respectively, shown as the red lines.

quantum-excitation-based scheme, with both DWs operating at their minimum gaps, the emittance variation range decreases to 61.2–66.3 pm · rad before compensation, and the emittance is compensated to 66.4 pm · rad by the short

wiggler, which requires a magnetic field ranging from 0.45 to 1.66 T. The maximum field is lower than that of the 20% coupling case, due to the reduced IBS effect. In the quantum-excitation-based scheme, the normalized

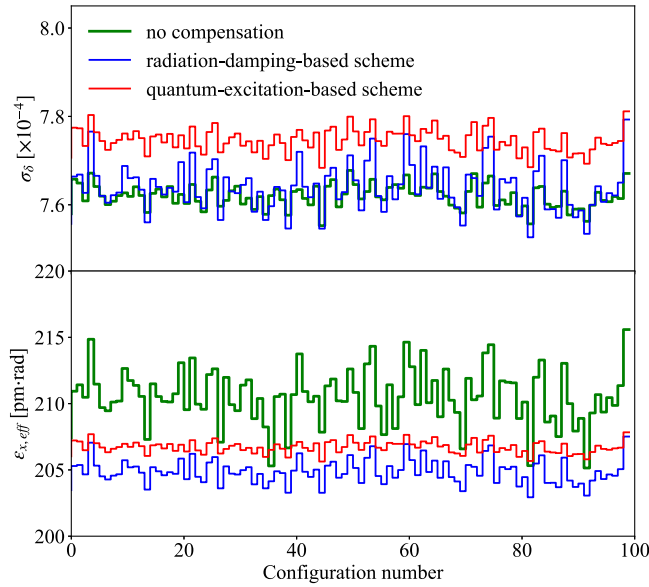


FIG. 8. Variations of energy spread and effective emittance at short straight sections during emittance compensation under the condition of IBS and 20% coupling, corresponding to the ID gap configurations shown in the panel (a) of Fig. 6.

effective emittance variation at short dispersive straight sections after the emittance compensation is $\pm 0.8\%$, still significantly lower than that of the radiation-damping-based scheme ($\pm 1.9\%$).

Based on the analysis above, when the IBS is considered, the quantum-excitation-based scheme retains its advantages in the zero-current case. With serious IBS, the radiation-damping-based scheme can achieve emittance compensation using only one DW, whereas in the quantum-excitation-based scheme, the wiggler requires a somewhat higher magnetic field, which is still within the available range.

VI. INFLUENCING FACTORS IN THE QUANTUM-EXCITATION-BASED EMITTANCE COMPENSATION SCHEME

A. Influence of compensation wiggler parameters

The influence of the wiggler's effective length and period length on the required maximum magnetic field for emittance compensation was examined under the condition of IBS and 20% coupling, as shown in Fig. 9. For the same compensated emittance, the required maximum magnetic field decreases primarily with increasing wiggler length, and increasing the period length also helps reduce it to some extent. However, a longer period length may lead to relatively large lattice optics distortion and thus dynamic aperture degradation. As a result, for the HALF storage ring, to reduce the required wiggler field in the quantum-excitation-based emittance compensation scheme, the wiggler length needs to be as long as possible

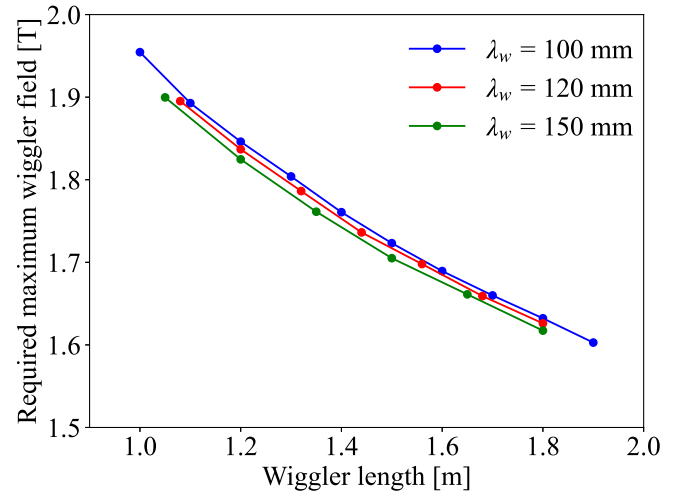


FIG. 9. Dependence of required maximum magnetic field on the wiggler length and period length for the same compensated emittance in the quantum-excitation-based scheme under the condition of IBS and 20% coupling.

and the period length is preferably in the range of 100–150 mm.

B. Influence of dispersion

The quantum-excitation-based scheme relies on the nonzero dispersion at the compensation wiggler. Figure 10 shows the dependence of the required maximum magnetic field for emittance compensation on the dispersion at the compensation wiggler under the condition of IBS and 20% coupling. As the dispersion increases, the required maximum field decreases significantly. If the dispersion is

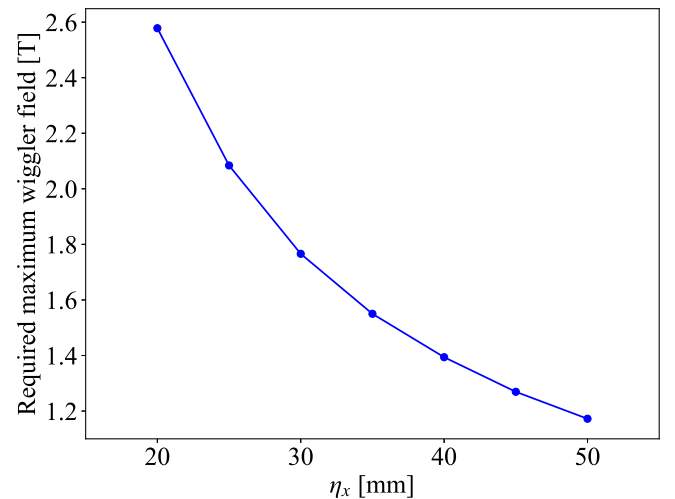


FIG. 10. Dependence of required maximum magnetic field on the dispersion at the compensation wiggler for the same compensated emittance in the quantum-excitation-based scheme under the condition of IBS and 20% coupling. The wiggler length and period length are 1.5 m and 100 mm, respectively.

lower than 25 mm, the maximum field is higher than 2.0 T, exceeding the capability of the compensation wiggler of HALF. If the HALF storage ring lattice is designed with higher dispersion in short straight sections, the emittance will increase significantly due to the quantum excitation from the IDs located in these straight sections. As a result, the dispersion in the short straight section of the HALF storage ring is preferably in the range of 30–35 mm. To compensate for larger emittance variations induced by more IDs in the future, a super-period lattice, in which only the dispersion in the 20th short straight section where the compensation wiggler is installed is increased, will be studied for HALF as a future work.

C. Influence of beam energy

The beam energy of the HALF storage ring is considered to be increased to 2.5 GeV in the future, which can mitigate the IBS-induced emittance growth. At 2.5 GeV, the natural emittance of the bare lattice becomes 111 pm · rad, and both the emittance variation caused by IDs and the required wiggler fields for emittance compensation will also change. Next, we will analyze the emittance compensation of the quantum-excitation-based scheme at 2.5 GeV, and for comparison, the radiation-damping-based scheme will also be analyzed.

For the zero-current case, in the radiation-damping-based scheme, the emittance variation induced by IDs ranges from 96.4 to 108.3 pm · rad, with a normalized emittance variation range of $\pm 5.8\%$, which is smaller than that at 2.2 GeV. The reason is that more energy loss from bending magnets reduces the relative contribution of IDs to the total energy loss. As shown in the panel (a) of Fig. 11, both DWs are necessary for the emittance compensation. The compensated emittance is 96.0 pm · rad. Due to the reduced emittance variation range, the maximum wiggler field decreases to 1.40 T compared to that at 2.2 GeV. During the emittance compensation, the normalized energy spread variation range is $\pm 3.0\%$, which is also smaller than that at 2.2 GeV due to reduced wiggler fields. In the quantum-excitation-based scheme, with both DWs operating at their minimum gaps, the natural emittance is 94.3 pm · rad. As shown in the panel (b) of Fig. 11, the range of emittance variation induced by IDs is 84.1–92.8 pm · rad. With the emittance compensated to 93.0 pm · rad, the required maximum wiggler field is 1.45 T and the normalized energy spread variation range is $\pm 1.1\%$, which are similar to those at 2.2 GeV.

For the IBS case with 400 mA current and 20% coupling, the radiation-damping-based scheme needs two DWs for emittance compensation, unlike the IBS case at 2.2 GeV, which uses only one DW. The emittance variation induced by IDs ranges from 119.6 to 130.5 pm · rad. The compensated emittance is 119.0 pm · rad, and the required maximum fields of two DWs are 1.26 T. Due to the reduced average wiggler field compared to that at

2.2 GeV, the normalized energy spread variation range is reduced to $\pm 1.5\%$. In the quantum-excitation-based scheme, with two DWs operating at their minimum gaps, the emittance variation induced by IDs ranges from 103.0 to 110.8 pm · rad. The compensated emittance is 111.0 pm · rad, lower than that of the radiation-damping-based scheme. The maximum wiggler field is 1.63 T, and the normalized energy spread variation range is $\pm 0.9\%$, which are also similar to those at 2.2 GeV. Therefore, the influence of beam energy on the quantum-excitation-based scheme is small for both the zero-current case and the IBS case.

VII. PRELIMINARY STUDY OF EMITTANCE COMPENSATION FOR THE FULL RING WITH BOTH THE PHASE-I AND PHASE-II IDS

The emittance compensation study for the HALF Phase-I IDs was conducted above. The addition of the Phase-II IDs will impose higher requirements on the emittance compensation wiggler. Therefore, based on the assumed parameters of the Phase-II IDs, we conducted a preliminary study on the emittance compensation for the full ring with both the Phase-I and Phase-II IDs. Considering the future energy increase of HALF, the study was carried out at a beam energy of 2.5 GeV. The natural emittance with and without two DWs at 2.5 GeV is 94.3 and 111.0 pm · rad, respectively. Apart from the straight sections occupied by the Phase-I IDs, the injection system, the rf system, two DWs, and an emittance compensation wiggler, there remain 7 long straight sections and 17 short straight sections. Considering the future installation of two superbend beamlines, the two short straight sections upstream of them are unavailable for user ID beamlines. This leaves 7 long and 15 short straight sections for the Phase-II IDs, of which 3 IVUs and 4 out-vacuum undulators (OVUs) are assumed to be placed in the long ones, and 2 IVUs and 13 OVUs in the short ones. These IVUs are assumed to have the same parameters as the Phase-I IVUs. The OVUs in the long straight sections adopt the Phase-I EPU46 parameters, while those in the short straight sections adopt the Phase-I EPU42 parameters but with a length of 1.162 m. In addition, the two short straight sections upstream of the superbends, if required, could be considered for placing emittance compensation wigglers, as they do not need beamline extraction.

Following the same procedure as above, we studied the emittance variations and compensation schemes for the full ring with the Phase-I and Phase-II user IDs, and the results are summarized in Table III. For the zero-current case, the normalized emittance variations before compensation are $\pm 6.6\%$ and $\pm 7.5\%$ with and without two DWs, respectively. As a comparison, the emittance variation before compensation for the Phase-I IDs is $\pm 5.8\%$ without the DWs, as presented in Sec. VI C. The increase in the emittance variation range after introducing the Phase-II

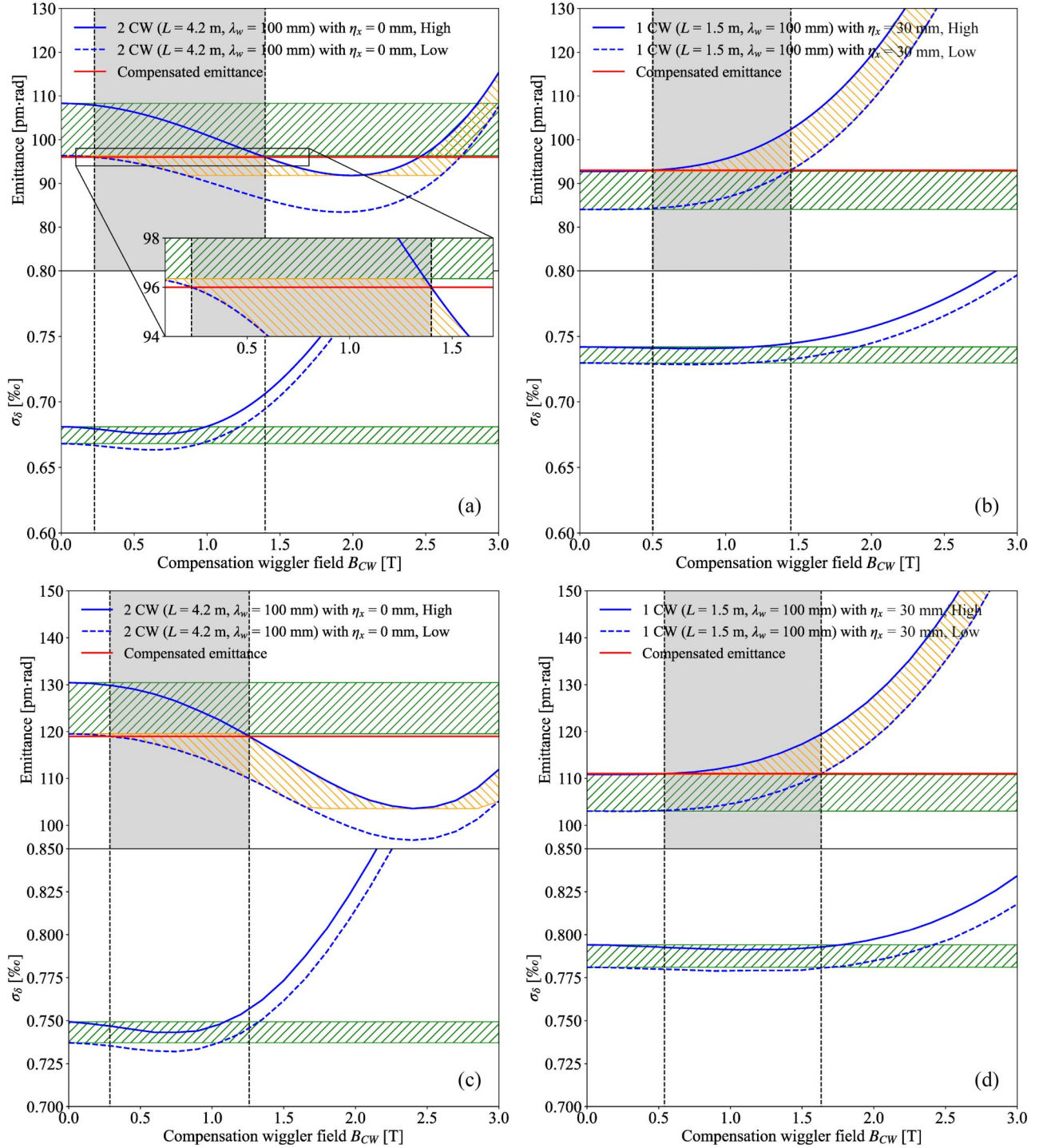


FIG. 11. Comparative analysis of the radiation-damping-based (left panels) and quantum-excitation-based (right panels) emittance compensation schemes at 2.5 GeV. The top panels illustrate the zero-current cases, and the bottom panels illustrate the IBS cases. The green regions indicate the ranges of emittance and energy spread variations induced by user IDs, the orange regions indicate the available ranges of emittance compensation, and the gray regions indicate the required wiggler fields. The compensated emittances for panels (a)–(d) are 96.0, 93.0, 119.0, and 110.0 pm · rad, respectively, shown as the red lines.

IDs is smaller than intuitively expected. This is because the IDs in the short straight sections are significantly shorter and have weaker fields, and the emittance variation caused by random ID gap adjustments does not scale linearly with

the number of IDs. Based on the DW parameters in Table II, the radiation-damping-based scheme requires three such 4.2-m wigglers to compensate for the emittance variation. If only two 4.2-m wigglers are employed,

TABLE III. Comparison of the radiation-damping-based and quantum-excitation-based emittance compensation schemes for the full ring with both the Phase-I and Phase-II IDs at 2.5 GeV. Three cases are considered: the zero-current case, the IBS case with 20% coupling, and the IBS case with 100% coupling.

	Zero-current		IBS and 20% coupling		IBS and 100% coupling	
Emittance compensation scheme	RD ^a	QE ^b	RD	QE	RD	QE
Emittance variation before compensation	$\pm 7.5\%$	$\pm 6.6\%$	$\pm 5.6\%$	$\pm 4.8\%$	$\pm 6.5\%$	$\pm 5.5\%$
Compensated emittance (pm · rad)	90.0	91.3	112.3	108.2	64.2	62.3
Total compensation wiggler length (m)	3×4.2	1.5	2×4.2	1.5	2×4.2	1.5
Maximum compensation wiggler field (T)	1.34	1.62	1.46	1.82	1.48	1.80
Energy spread variation after compensation	$\pm 3.5\%$	$\pm 1.4\%$	$\pm 2.4\%$	$\pm 1.1\%$	$\pm 2.7\%$	$\pm 1.2\%$
Effective emittance variation after compensation	$\pm 2.0\%$	$\pm 0.7\%$	$\pm 1.4\%$	$\pm 0.7\%$	$\pm 1.9\%$	$\pm 0.8\%$

^aRD represents the radiation-damping-based scheme.

^bQE represents the quantum-excitation-based scheme.

the required maximum field is 1.92 T, which exceeds the capability of the HALF DWs. With the three 4.2-m wigglers, the resulting compensated emittance decreases from 96.0 pm · rad for the Phase-I IDs to 90.0 pm · rad, while the normalized energy spread variation after emittance compensation increases from $\pm 3.0\%$ to $\pm 3.5\%$. The quantum-excitation-based scheme, with two DWs operating at their minimum gaps, requires one 1.5-m wiggler that was used for the emittance compensation of the Phase-I IDs, but with a higher maximum field of 1.62 T. The compensated emittance decreases from 93.0 to 91.3 pm · rad. The quantum-excitation-based scheme maintains its advantages, with only small increases in the energy spread variation and effective emittance variation compared with the Phase-I ID case. In contrast, if both the emittance and the effective emittance are to be compensated, the three 4.2-m wigglers will be insufficient in the radiation-damping-based scheme.

For the IBS case with 20% coupling, the radiation-damping-based scheme requires two 4.2-m wigglers. Compared with the same case for the Phase-I IDs, in the radiation-damping-based scheme, the compensated emittance is reduced from 119.0 to 112.3 pm · rad, the required maximum wiggler field increases from 1.26 to 1.46 T, and the normalized energy spread variation also increases from $\pm 1.5\%$ to $\pm 2.4\%$. For the quantum-excitation-based scheme, one 1.5-m wiggler with a maximum field of 1.82 T can compensate for the emittance variation. Compared with the same case for the Phase-I IDs, the compensated emittance is reduced to 108.2 pm · rad, and the normalized energy spread variation increases from $\pm 0.9\%$ to $\pm 1.1\%$. The normalized effective emittance variation increases from $\pm 0.5\%$ to $\pm 0.7\%$, which still can be considered basically compensated. For the IBS case with 100% coupling, the quantum-excitation-based scheme yields similar maximum wiggler field and effective emittance variation as those for the 20% coupling case.

The study here is based on some assumptions made for the Phase-II IDs. For the future IDs, the maximum field

required for the short wiggler in the quantum-excitation-based scheme may be higher. To address this potential issue, we have considered several options. First, we can increase the dispersion at the 20th short straight section. For example, based on the assumed Phase-II IDs, if this dispersion is moderately increased from the present 30 to 34 mm, the maximum field of the compensation wiggler will be reduced from 1.82 to 1.67 T. Second, we can reduce the dispersion at all short straight sections except the 20th to the optimal value required for the passive scheme proposed at Spring-8-II [16], leading to a hybrid scheme that combines the quantum-excitation-based scheme and the passive scheme. This can lower the requirement for the maximum field of the short wiggler. Both options require modifying the present lattice to a super-period lattice, which could degrade the nonlinear dynamics performance due to the periodicity breaking. A third option, which does not need to change the lattice, is to employ an additional short wiggler in a short straight section upstream of a superbend for emittance compensation.

VIII. CONCLUSION

This paper presents a novel emittance compensation scheme based on quantum excitation for the HALF storage ring with long dispersion-free and short dispersive straight sections. We conducted a comparative analysis, with and without IBS, between the proposed quantum-excitation-based scheme and the original radiation-damping-based scheme. The proposed scheme employs a short variable-gap wiggler in a dispersive straight section with two DWs at their minimum gaps, while the original scheme adjusts the gaps of two DWs in dispersion-free straight sections. Even with a significantly shorter wiggler, the proposed quantum-excitation-based scheme achieves similar emittance compensation performance to that of the radiation-damping-based scheme. This scheme offers several advantages: (1) significantly smaller energy spread variations, leading to very small effective emittance variations at short dispersive straight sections, so the effective

emittance can be considered basically compensated; (2) shorter damping times, which help suppress the IBS effect; (3) significantly weaker magnetic attractive force between the compensation wiggler poles; (4) two additional DW beamlines for users. The only drawback of this scheme is that it compensates for emittance variation at the cost of increased emittance due to quantum excitation. However, since the two DWs operate at their minimum gaps, the compensated emittance is nearly the same as or even lower than that of the radiation-damping-based scheme.

In the preliminary study of emittance compensation for the full ring with both the Phase-I and assumed Phase-II IDs, the quantum-excitation-based scheme maintains its advantages, and the required maximum field of the short compensation wiggler increases to about 1.8 T with the IBS effect included. In case the future Phase-II IDs necessitate a higher maximum field for the short compensation wiggler, three options have been proposed to address this: increasing the dispersion at the compensation wiggler, adopting a hybrid compensation scheme, or adding an additional short wiggler. For DLSRs without dispersive straight sections, one can convert a dispersion-free straight section into a dispersive one simply using a dipole chicane, and install a variable-gap wiggler, thereby enabling the quantum-excitation-based scheme to be applied for emittance compensation.

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DATA AVAILABILITY

The data that support the findings of this article are not publicly available. The data are available from the authors upon reasonable request.

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