

Numerical investigation of mechanical instabilities in superconducting rf cavities: Lorentz force detuning and microphonics

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Dynamic detuning, encompassing dynamic Lorentz force detuning (LFD) and microphonics, significantly restricts the operational stability of superconducting radio-frequency (SRF) cavity modules. This paper proposes a novel semianalytical method for calculating the dynamic detuning of SRF cavities, reducing computation time by several orders of magnitude compared to transient finite element analysis (FEA). Leveraging this computational efficiency, genetic algorithms (GA) are employed to determine the optimal parameters for active compensation. This work provides a high-efficiency simulation framework for dynamic detuning research, with potential for expansion into integrated low-level radio frequency (LLRF) system models. In this study, the coupling between mechanical loads (including ground vibrations, pressure fluctuations, Lorentz forces, and fast tuning) and dynamic detuning is analyzed to develop a modified mechanical model for SRF cavities. A key advancement of this model is the incorporation of a correction term for high-frequency mechanical modes, which enhances simulation accuracy relative to conventional approaches that neglect them. The proposed method was first validated using a 1.3 GHz TESLA cavity, demonstrating excellent agreement with transient FEA solvers and published experimental data. Subsequently, the method was applied to evaluate microphonics in the 499.8 MHz cavity of the Hefei Advanced Light Facility (HALF), specifically characterizing the impact of ground vibrations and pressure fluctuations. Regarding active compensation, an optimized half-sinusoidal pulse suppressed the dynamic LFD of the TESLA cavity from 220 Hz to less than 5 Hz during the beam flattop, and from 600 Hz to less than 40 Hz during the entire rf-on region. The global optimum for this three-parameter optimization problem was effectively identified within a broad search space. Furthermore, an optimized PI controller reduced microphonics of the HALF SRF module induced by stochastic ground vibrations from ± 400 to ± 50 Hz, thereby improving the open-loop phase and amplitude stability from $\pm 2^\circ$ and $\pm 13\%$ to $\pm 0.2^\circ$ and $\pm 1.3\%$, respectively.

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I. INTRODUCTION

Superconducting radio-frequency (SRF) cavities are core components of modern particle accelerators, with applications spanning materials science, nuclear physics, nuclear astrophysics, and high-energy physics [1]. These cavities exhibit surface resistances several orders of magnitude lower than that of copper, enabling intrinsic quality factors (Q_0) in the range of $10^9 - 10^{10}$ [2]. In linear accelerators, high accelerating gradients (E_{acc}) of up to 34 MV/m have been achieved in 1.3 GHz TESLA-type cavities at the

FLASH facility [3,4]. SRF technology has also been widely adopted for storage rings; for instance, the Hefei Advanced Light Facility (HALF) is a fourth-generation soft x-ray and vacuum ultraviolet synchrotron radiation source designed by the National Synchrotron Radiation Laboratory [5,6]. A 499.8 MHz SRF cavity is planned for the HALF storage ring to compensate for electron energy loss due to synchrotron radiation. This cavity is designed to provide an accelerating voltage of 1.5 MV and deliver 140 kW of power to the beam. To date, the bare cavity has been fabricated and has undergone vertical testing [7], while its fundamental power coupler (FPC) and higher-order mode (HOM) absorbers have been extensively investigated [8–10].

Maintaining the stability of the phase and amplitude of the electromagnetic fields within SRF cavities remains a significant operational challenge [11]. This instability primarily arises from dynamic detuning, which is induced by various mechanical loads, including Lorentz forces, ground vibrations, mechanical noise from auxiliary equipment (e.g., vacuum pumps), and rapid pressure fluctuations

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within the cryogenic system [12]. Detuning specifically caused by the radiation pressure of the electromagnetic field is defined as Lorentz force detuning (LFD), while disturbances from external sources are collectively termed microphonics. Due to the thin-walled niobium construction of SRF cavities (typically a few millimeters thick) [13–15], even minute mechanical disturbances can induce detuning ranging from tens to hundreds of hertz in the absence of passive damping or active control [16,17]. Conventionally, low-level rf (LLRF) systems counteract detuning by dynamically adjusting the forward rf power. However, several prominent accelerator facilities now operate with high loaded quality factors (Q_L) between 10^7 and 10^8 , corresponding to extremely narrow cavity bandwidths of only 1–10 Hz [18,19]. Under such conditions, the power required to compensate for detuning can significantly exceed the capacity of the installed rf sources. Consequently, it is imperative to suppress these detuning effects before resorting to rf power compensation.

Strategies for detuning mitigation generally fall into four categories: (i) cavity stiffening and cryomodule reinforcement to shift mechanical resonances [20–22]; (ii) vibration isolation to decouple the cavity from external sources [23]; (iii) passive dampers to dissipate vibrational energy [24,25]; and (iv) active compensation using fast tuners. Residual detuning persisting after passive mitigation can be actively suppressed via control algorithms. Piezo-based fast tuners, which emerged as a key active component in the early 2000s [16,26,27], have become indispensable in modern superconducting linacs [28,29]. The precision of active compensation design depends heavily on an accurate understanding of the cavity’s mechanical properties.

Pioneering theoretical studies on dynamic detuning were first conducted for normal-conducting cavities in the 1960s [30] and were later extended to SRF cavities by Schulze [31]. In the mid-1970s, Delayen analyzed the behavior of an SRF cavity operating in self-excited loops subject to dynamic LFD and microphonics [32]. These works established a mechanical model formulated as a set of ordinary differential equations (ODEs) representing mechanical eigenmodes driven by external loads [11]. However, accurately predicting dynamic detuning using a truncated set of modes remains challenging [33,34].

Alternatively, high-fidelity numerical approaches utilize finite element analysis (FEA) software, such as ANSYS. While multiphysics transient solvers offer high accuracy, their prohibitive computational cost often limits simulations to simplified, highly symmetric 2D or 3D models [35–37]. Such simplifications frequently fail to capture asymmetric influences arising from the complex geometry of the cavity or its auxiliary components through boundary conditions.

In this work, the classical ODE-based mechanical model [11] is modified and applied to investigate dynamic detuning effects, including dynamic LFD, microphonics, and their active compensation. This semianalytical

approach derives mechanical eigenmode parameters from 3D FEA and incorporates them into a system of ODEs featuring a novel correction term for high-frequency modes. The resulting model is coupled with the cavity’s electrical model to perform detuning simulations within the MATLAB-Simulink environment.

This paper is organized as follows. Section II describes the dynamic electrical and mechanical models for SRF cavities, detailing the extraction of eigenmode parameters and the physical modifications introduced. Section III analyzes dynamic LFD in a 1.3 GHz TESLA cavity. Section IV investigates microphonics in the 499.8 MHz HALF cavity-vessel-tuner assembly. Finally, Sec. V concludes the paper.

II. SRF CAVITY MODELS

A. Dynamic electrical model

To analyze the phase and amplitude instabilities induced by dynamic detuning, an electrical model is established based on the differential equations governing the cavity voltage [38]. These equations are derived from the equivalent RLC circuit representation of an SRF cavity, under the adiabatic and small-detuning approximations:

$$\begin{aligned} \dot{V}_r + \omega_{1/2} V_r + \Delta\omega V_i &= R_L \omega_{1/2} I_r \\ \dot{V}_i + \omega_{1/2} V_i - \Delta\omega V_r &= R_L \omega_{1/2} I_i \end{aligned} \quad (1)$$

where V_r , V_i , I_r , and I_i represent the real and imaginary components of the cavity voltage and current, respectively. The term $\omega_{1/2}$ denotes the angular half-bandwidth of the cavity, $\Delta\omega$ is the angular detuning, and R_L is the loaded impedance. The phase and amplitude information are implicitly contained within these quadrature components.

B. Dynamic mechanical model

The mechanical dynamics of an SRF cavity are strongly coupled to its electromagnetic characteristics. Forces exerted on the cavity walls, whether originating from electromagnetic radiation pressure, tuner mechanisms, or external vibrations, deform the cavity geometry, thereby shifting its resonant frequency. Given that these wall deformations typically exhibit micrometer-scale amplitudes and weak mechanical damping, the cavity can be modeled as a linear elastic continuum [35,39]. For such a system, modal decomposition is an effective approach for analyzing the vibratory response. The core principle of the dynamic mechanical model is to represent the physical deformation as a superposition of orthogonal mechanical eigenmodes [11].

Let the mechanical eigenmodes of an SRF cavity be defined by a complete set of orthonormal basis functions ϕ_μ . The time-dependent cavity wall displacement ξ and the applied mechanical load F can be expanded as

$$\vec{\xi}(x, y, z, t) = \sum_{\mu} q_{\mu}(t) \vec{\phi}_{\mu}(x, y, z), \quad (2)$$

$$\vec{F}(x, y, z, t) = \sum_{\mu} F_{\mu}(t) \vec{\phi}_{\mu}(x, y, z), \quad (3)$$

where $q_{\mu}(t)$ and $F_{\mu}(t)$ denote the modal displacement and the modal load amplitude for the μ th eigenmode, respectively. Derived from the Lagrangian formulation, the equation of motion for each mode μ is expressed as

$$\ddot{q}_{\mu}(t) + \frac{1}{\tau_{\mu}} \dot{q}_{\mu}(t) + \Omega_{\mu}^2 q_{\mu}(t) = \frac{\Omega_{\mu}^2}{c_{\mu}} F_{\mu}(t), \quad (4)$$

where τ_{μ} is the modal decay time, Ω_{μ} is the angular eigenfrequency, and c_{μ} is the elastic constant.

According to Slater's perturbation theory [40], the rf frequency shift $\Delta\omega_{\mu}$ is proportional to the modal displacement q_{μ} for small perturbations. Thus, the equation of motion can be reformulated in terms of detuning:

$$\begin{aligned} \Delta\ddot{q}_{\mu}(t) + \frac{1}{\tau_{\mu}} \Delta\dot{q}_{\mu}(t) + \Omega_{\mu}^2 \Delta q_{\mu}(t) \\ = k_{\mu} \Omega_{\mu}^2 F_{\text{Lorentz}}(t) + n_{\mu} \Omega_{\mu}^2 F_{\text{Tuning}}(t) + \sum_i m_{\mu,i} \Omega_{\mu}^2 F_i(t). \end{aligned} \quad (5)$$

Here, the total load is decomposed into specific sources, with F_{Lorentz} representing the Lorentz force and F_{Tuning} the tuning load. The coefficients k_{μ} , n_{μ} , and $m_{\mu,i}$ are the modal coupling factors, which incorporate the elastic constant c_{μ} and the conversion from the total loads $F_i(t)$ to the modal loads $F_{\mu,i}(t)$. The actual dynamic detuning $\Delta\omega(t)$ is the summation of all modal contributions:

$$\Delta\omega(t) = \sum_{\mu} \Delta\omega_{\mu}(t). \quad (6)$$

In the steady state (denoted by the subscript 0), the detuning contribution of mode μ follows from Eq. (5):

$$\Delta\omega_{\mu,0} = k_{\mu} F_{\text{Lorentz},0} + n_{\mu} F_{\text{Tuning},0} + \sum_i m_{\mu,i} F_{i,0}. \quad (7)$$

The actual steady-state detuning is thus:

$$\Delta\omega_0 = \sum_{\mu} \Delta\omega_{\mu,0}. \quad (8)$$

The static coupling coefficients, defined as the detuning per unit load, are related to the modal coupling factors by

$$k = \sum_{\mu} k_{\mu}, n = \sum_{\mu} n_{\mu}, m_i = \sum_{\mu} m_{\mu,i}, \quad (9)$$

where k is the static LFD coefficient, n is the static tuning coefficient, and m_i represents the static coefficient for other specific loads, such as pressure fluctuations or ground vibrations.

C. Parameter extraction for mechanical eigenmodes

The mechanical model described by Eq. (5) requires the determination of three sets of parameters: the eigenmode angular frequencies Ω_{μ} , the modal coupling factors (k_{μ} , n_{μ} , $m_{\mu,i}$), and the modal decay times τ_{μ} . Methodologies for characterizing these parameters have been established by Devanz *et al.* [41], Kononenko *et al.* [42], and Parise [43]. Our computational approach is summarized as follows:

First, the eigenfrequencies Ω_{μ} and the corresponding displacement eigenfunctions ϕ_{μ} are obtained through FEA-based mechanical modal analysis. Subsequently, the coupling factors are evaluated for each load case through the following procedure: (1) Calculation of modal amplitudes: Under a static external load $F_{i,0}$, the cavity wall deformation $\xi_{i,0}$ is calculated using a structural FEA solver. Following Eq. (2), the modal amplitudes $q_{\mu,i,0}$ are determined via orthogonal projection using 3D Gaussian integration:

$$q_{\mu,i,0} = \frac{\int_{V_s} \xi_{i,0}(x, y, z) \vec{\phi}_{\mu}(x, y, z) dV_s}{\int_{V_s} \vec{\phi}_{\mu}(x, y, z) \vec{\phi}_{\mu}(x, y, z) dV_s}, \quad (10)$$

where the denominator is the norm of ϕ_{μ} , and V_s denotes the structural solution domain.

(2) Determination of modal sensitivity: The modal sensitivity S_{μ} is defined as the frequency shift per unit modal displacement of the μ th. It quantifies the relationship between the steady-state modal detuning $\Delta\omega_{\mu,i,0}$ and the modal amplitude $q_{\mu,i,0}$:

$$\Delta\omega_{\mu,i,0} = S_{\mu} \times q_{\mu,i,0}. \quad (11)$$

Based on Slater's perturbation theory [40], the frequency shift due to a small surface deformation is given by

$$\begin{aligned} \frac{\Delta\omega}{\omega_0} &= \frac{\omega_0 - \omega}{\omega_0} = \frac{\int_{\Delta V} \left(\frac{\mu_0}{4} |\vec{H}_0|^2 - \frac{\epsilon_0}{4} |\vec{E}_0|^2 \right) dV}{\int_V \left(\frac{\mu_0}{4} |\vec{H}_0|^2 + \frac{\epsilon_0}{4} |\vec{E}_0|^2 \right) dV} \\ &= \frac{\int_{\Delta V} P_{\text{Lorentz}} dV}{W_{\text{tot}}}, \end{aligned} \quad (12)$$

where ω_0 is the unperturbed angular resonant frequency, ΔV denotes the volume change at the inner wall surface, $\vec{H}_0$ and $\vec{E}_0$ are the unperturbed magnetic and electric fields, P_{Lorentz} is the radiation pressure distribution, and W_{tot} is the time-averaged stored electromagnetic energy.

In FEA software, the 3D structure is discretized into elements defined by nodal points. If sufficiently small

elements are used, a good estimation of the displacement and electromagnetic fields at each point can be obtained. The discretized form of Eq. (12) is

$$\Delta\omega = \frac{\omega_0}{W_{\text{tot}}} \sum_{j=1}^N P_{\text{Lorentz},j} \Delta V_j, \quad (13)$$

where $P_{\text{Lorentz},j}$ and ΔV_j represent the radiation pressure and the volume change associated with the j th surface element, respectively. N is the total number of elements. In this study, the FEA model was developed in COMSOL using a mesh of quadratic tetrahedral elements. The inner cavity surface is represented by triangular elements, each with three nodes.

A specialized algorithm for evaluating Eq. (13), adapted from Ref. [43], was implemented in MATLAB with parallel computing support. The algorithm extracts surface mesh coordinates before and after deformation. $P_{\text{Lorentz},j}$ is computed using the arithmetic mean of the field values at the vertices of each triangular element obtained from an unperturbed rf eigenmode FEA (ω_0 and W_{tot} are obtained simultaneously). The volume change ΔV_j is accurately estimated by decomposing the displacement of each triangular element into three tetrahedra. The modal sensitivity S_μ is then determined by substituting the eigenfunction ϕ_μ (normalized to a maximum displacement of 1 μm) into Eq. (13).

(3) Extraction of coupling factors: With $q_{\mu,i,0}$ and S_μ known, the modal coupling factors are derived by normalizing the modal detuning to the applied load. Different coupling factors are obtained through individual analysis:

$$k_\mu = \frac{\Delta\omega_{\mu,\text{Lorentz},0}}{F_{\text{Lorentz},0}}, \quad n_\mu = \frac{\Delta\omega_{\mu,\text{Tuning},0}}{F_{\text{Tuning},0}}, \quad m_{\mu,i} = \frac{\Delta\omega_{\mu,i,0}}{F_{i,0}}. \quad (14)$$

Finally, to account for mechanical damping, we adopt the Rayleigh proportional damping model, which expresses the damping matrix C as a linear combination of the mass matrix M and stiffness matrix K [44]:

$$C = \alpha M + \beta K, \quad (15)$$

where the coefficients α and β are defined by the damping ratios ξ of two sampled modes m and n :

$$\alpha = 2\omega_m\omega_n \frac{\xi_m\omega_m - \xi_n\omega_n}{\omega_m^2 - \omega_n^2}, \quad \beta = 2 \frac{\xi_m\omega_m - \xi_n\omega_n}{\omega_m^2 - \omega_n^2}. \quad (16)$$

Here, ω_m and ω_n are the angular frequencies of the two sampled modes. The specific determination of these damping parameters for different cavity configurations is discussed in Secs. III and IV.

D. Physical modifications of mechanical model

Theoretically, a complete mechanical model requires an infinite summation of eigenmodes, as implied by Eq. (6), which is computationally intractable. Previous studies typically adopt a simplification by retaining only the first few dominant modes to facilitate implementation on field-programmable gate arrays [34]. However, such truncation can lead to inconsistencies with the static detuning requirements of Eq. (9), resulting in significant deviations from experimental observations, particularly under nonzero loads. In this work, we introduce two modifications to resolve these limitations and enhance computational fidelity.

First, selection criteria are established based on the relative detuning contribution of each mode. During parameter characterization, many eigenmodes exhibit negligible coupling factors, thus contributing minimally to the total detuning $\Delta\omega(t)$. Accordingly, we apply a threshold to retain only those modes with a significant impact on the cavity dynamics. In this study, the threshold is defined as 1% of the maximum absolute value among the modal coupling coefficients. Because the set of dominant modes is often mechanism-specific, modes that contribute negligibly across all detuning mechanisms are excluded from the dynamic model.

Second, a cutoff frequency f_c is defined based on the mechanical damping characteristics, as detailed in Secs. III and IV. This approach leverages the relationship between a mode's decay time τ_μ , quality factor Q_μ , and damping ratio ξ_μ :

$$\frac{1}{2} \tau_\mu \Omega_\mu = \frac{1}{2\xi_\mu} = Q_\mu. \quad (17)$$

In SRF cavities, higher-frequency mechanical modes generally exhibit stronger damping (lower Q_μ), leading to significantly shorter decay times [39]. For instance, assuming a constant damping ratio of $\xi_\mu \equiv 0.3\%$ [36], the decay time decreases from 0.531 s at 100 Hz to 0.021 s at 2500 Hz. This implies that high-frequency modes respond to the load input $F_i(t)$ almost instantaneously, following a quasistatic behavior. To account for the aggregate contribution of all mechanical modes above the cutoff frequency without explicitly solving their differential equations, we introduce an effective residual coupling coefficient $m_{i,\text{eff}}$. This coefficient is defined as the difference between the total static coefficient and the sum of the coefficients of the dynamically modeled modes:

$$m_{i,\text{eff}} = m_i - \sum_{\mu: f < f_c} m_{i,\mu}. \quad (18)$$

Consequently, the total dynamic detuning in Eq. (6) is reformulated as

$$\Delta\omega(t) = \sum_i \sum_{\mu, f < f_c} \Delta\omega_\mu(t) + \sum_i m_{i,\text{eff}} F_i(t). \quad (19)$$

Through these modifications, the mechanical model maintains high accuracy by combining a finite set of dynamic eigenmode ODEs with algebraic correction terms that account for the high-frequency modes.

III. LFD INVESTIGATION IN A TESLA CAVITY

Extensive experimental studies on the transient response to LFD have been reported in the literature, providing a robust basis for model validation. In this section, the dynamic LFD in a 1.3 GHz TESLA-type cavity at an accelerating gradient (E_{acc}) of 25 MV/m is analyzed. To simplify the computational framework, a unidirectional coupling scheme is implemented: the accelerating gradient pulse is transferred from the electrical model to the mechanical model without a feedback loop. Detuning caused by the Lorentz force and tuning loads is simulated independently and benchmarked against transient FEA solvers and published experimental data. Furthermore, active LFD compensation is investigated by applying a half-sinusoidal pulse to the fast piezo tuner, with pulse parameters optimized using a genetic algorithm (GA).

A. FEA model of the TESLA cavity

The 1.3 GHz TESLA nine-cell SRF cavity, developed in the early 2000s [13], has been widely deployed in modern electron linear accelerators [3,42,45]. A high-fidelity FEA model was constructed for dynamic LFD analysis.

The geometric parameters for the FEA model were sourced from Ref. [46]. Material properties were assigned as follows: the stiffening rings, end discs, and cavity walls were modeled as niobium (Young's modulus $E = 88.5$ GPa, Poisson's ratio $\sigma = 0.38$, and density $\rho = 8586$ kg/m³) [47], while the side flanges were modeled as a Nb-Ti 45 alloy ($E = 68.3$ GPa, $\sigma = 0.34$, and $\rho = 6360$ kg/m³) [48]. The internal cavity volume was treated as a vacuum for the rf eigenmode analysis. In the experimental setup, the helium vessel is rigidly connected to the cavity on one side and coupled via a bellows on the other. The tuner, serving as the load-transfer mechanism, is attached across the bellows. Based on the documented stiffness of this assembly (85 kN/mm [21]), a combined boundary condition was applied at the side flanges. This consisted of a prescribed displacement for transverse constraints and a spring foundation for the axial constraint (Fig. 1). A uniform thickness of 2.5 mm was assigned to the cavity walls and stiffening rings. Finally, a structured, high-symmetry mesh was generated to minimize discretization errors (Fig. 2).

The boundary conditions were validated by comparing the calculated static Lorentz force coefficients (k) with published results. Two numerical approaches were employed: (1) Multiphysics FEA using COMSOL: Initially, the electromagnetic fields of the accelerating mode were computed via an rf eigenfrequency analysis and scaled to an accelerating gradient of 25 MV/m. The resulting Lorentz force distribution on the inner surface was then applied as a static load in a structural mechanics module, coupled with a moving mesh to calculate the wall

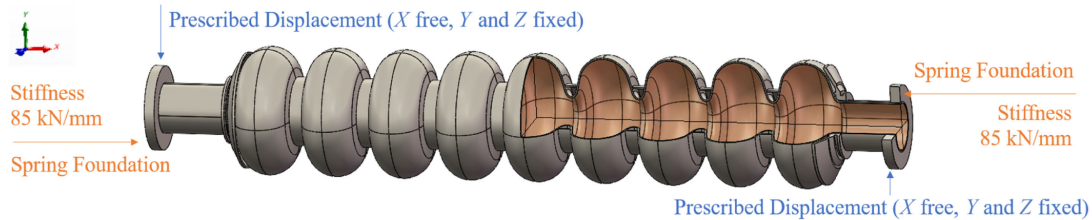


FIG. 1. FEA model of the 1.3 GHz TESLA cavity with sections.

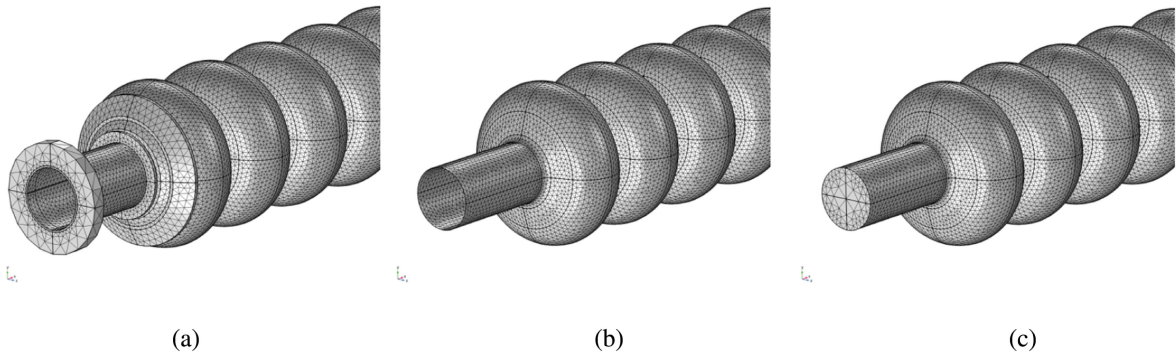


FIG. 2. Mesh configuration. (a) Structure domain, (b) multiphysics interface, (c) rf domain.

TABLE I. Static Lorentz force coefficient.

Unit	COMSOL	MATLAB	Reference
$k[\text{Hz}/(\text{MV}/\text{m})^2]$	-0.8802	-0.8718	-0.9

deformation. Finally, the deformed geometry was used in a subsequent rf eigenfrequency analysis to determine the static detuning $\Delta\omega_0$. (2) Slater's perturbation method implemented in MATLAB: This approach used the radiation pressure and surface deformation data obtained from the aforementioned FEA as inputs. The static detuning $\Delta\omega_0$ was then calculated using the discretized integral in Eq. (13).

For both methods, the static Lorentz force coefficient k was obtained using:

$$k = \frac{\Delta\omega_0}{F_{\text{Lorentz},0}}. \quad (20)$$

The results are summarized in Table I. The coefficients derived from both methods show excellent agreement with the experimental values reported in the literature [13].

B. Characterization of the mechanical eigenmodes

This section details the parameter characterization process used to establish the modified mechanical model for the TESLA-type cavity. Since the cavity deformation is primarily driven by the Lorentz force and active tuning during pulsed-mode operation, the coupling factor calculations were focused on these two contributions. As outlined in Sec. II, a suite of interrelated FEA simulations was performed, including rf eigenmode, mechanical modal, and static deformation analyses for both Lorentz force and tuning loads. To ensure numerical consistency and eliminate interpolation errors between different simulation stages, a single, high-fidelity mesh configuration was maintained throughout all analyses.

The characterization began with an rf eigenfrequency analysis to compute the electromagnetic fields of the

TM₀₁₀ π mode [Figs. 3(a) and 3(b)]. From these field distributions, the corresponding Lorentz force distribution was derived [Fig. 3(c)]. This distribution was subsequently applied as a static load in a structural analysis at an accelerating gradient of $E_{\text{acc}} = 25 \text{ MV}/\text{m}$. The resulting peak displacement was $0.41 \mu\text{m}$. Figure 3(d) illustrates the static deformation $\xi_{\text{Lorentz},0}$ with a magnification factor of 30,000, superimposed on the undeformed cavity profile.

Next, a static structural analysis was performed to characterize the mechanical response to the tuner. To simulate the tuner action, a longitudinal force of 100 N was applied to the left-side flange, resulting in a maximum displacement of $1.14 \mu\text{m}$. The corresponding deformation profile $\xi_{\text{Tuning},0}$ is visualized in Fig. 4 with a magnification factor of 15,000.

The static deformation profiles, $\xi_{\text{Lorentz},0}$ and $\xi_{\text{Tuning},0}$, were then decomposed onto the basis of the cavity's mechanical eigenfunctions using Eq. (10). This orthonormal basis, comprising the first 100 mechanical modes, was obtained from a separate mechanical modal analysis. For each mode, the angular eigenfrequency Ω_μ was recorded, and the deformation function ϕ_μ was normalized to a maximum amplitude of $1 \mu\text{m}$. The resulting modal amplitudes, $q_{\mu,\text{Lorentz},0}$ and $q_{\mu,\text{Tuning},0}$, are shown in Fig. 5. The decomposition reveals that the first longitudinal mode is the dominant contributor to both displacement profiles.

Subsequently, the modal sensitivity S_μ was determined by substituting the eigenfunctions ϕ_μ into Eq. (13). This analysis confirms that longitudinal modes exhibit significantly higher detuning sensitivities than transverse modes (Fig. 6).

Utilizing the modal amplitudes and sensitivities, the modal coupling factors k_μ and n_μ were derived via Eqs. (11) and (14) for $F_{\text{Lorentz},0} = 625 (\text{MV}/\text{m})^2$ and $F_{\text{Tuning},0} = 100 \text{ N}$. The cumulative sum of these modal factors, compared against the total static coefficient, is illustrated in Fig. 7. It is evident that the summation of the first 100 modes does not fully converge to the static coefficient calculated by the direct FEA method, thereby failing to satisfy Eq. (9). This discrepancy underscores the necessity of the correction terms introduced in Eq. (19).

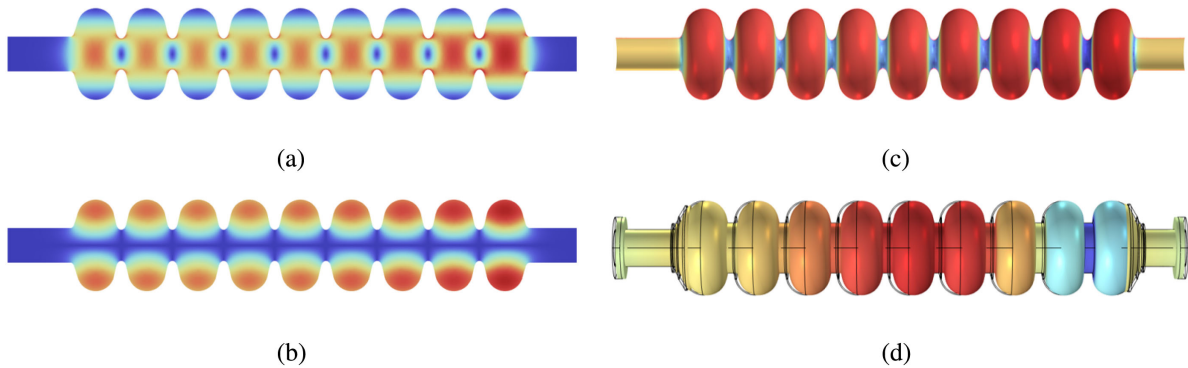
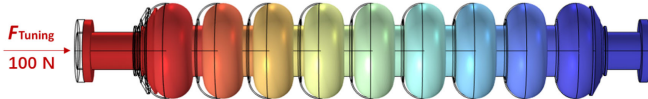
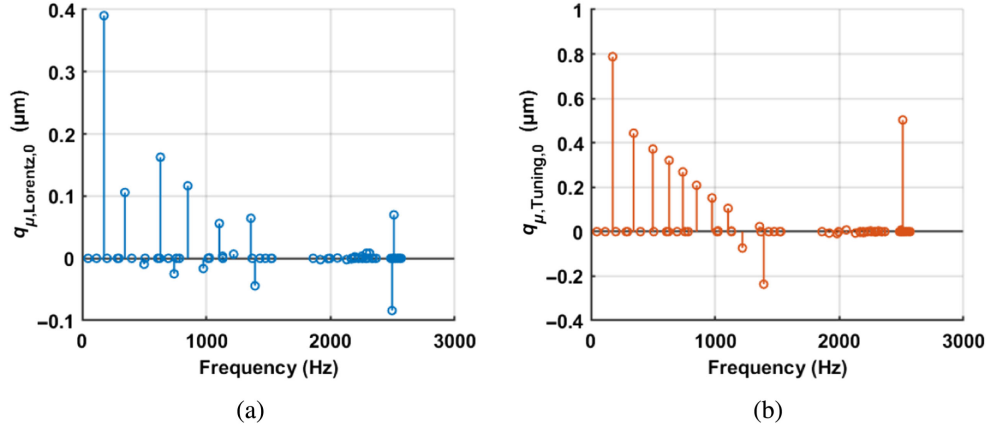
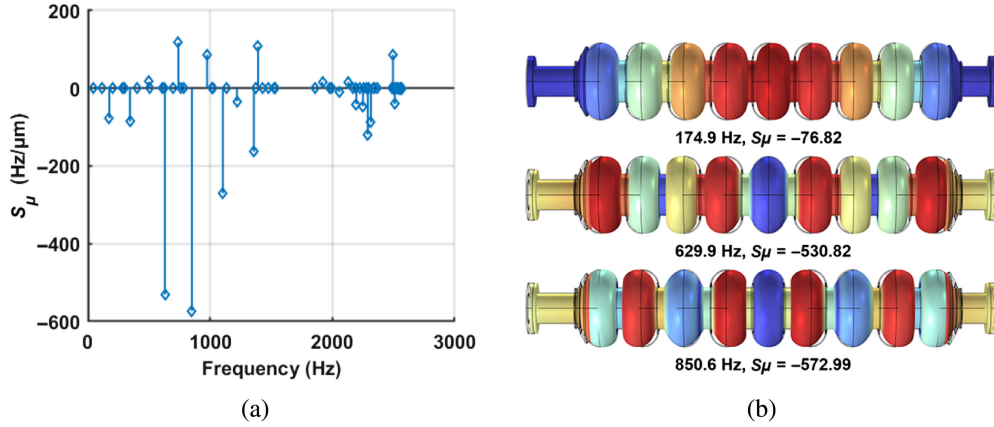
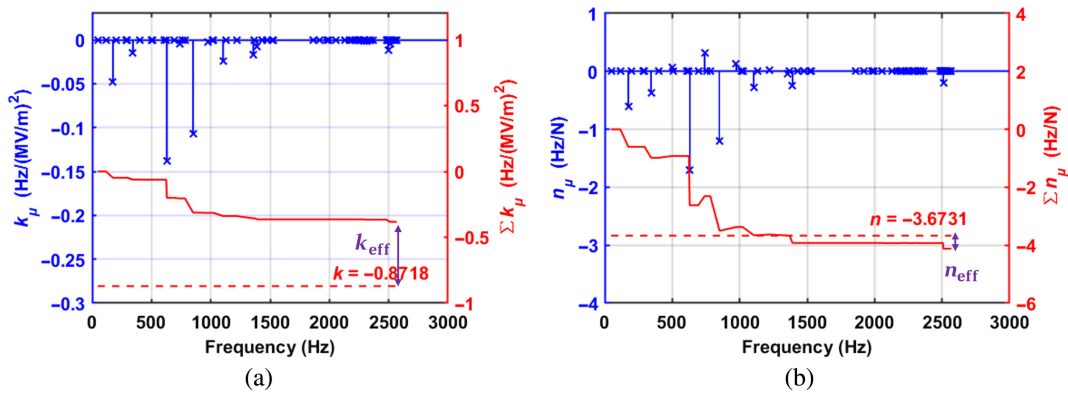


FIG. 3. Static LFD analysis. (a) Electric field, (b) magnetic field, (c) Lorentz force distribution P_{Lorentz} , and (d) static deformation $\xi_{\text{Lorentz},0}$.

FIG. 4. Static tuning deformation $\xi_{\text{Tuning},0}$.

Finally, the modal decay times τ_μ were determined using the Rayleigh damping model. The first two longitudinal modes were selected as reference points, with frequencies $\omega_m = 2\pi \times 174.92$ Hz and $\omega_n = 2\pi \times 343.92$ Hz. The damping ratios for these modes were both set to $\xi_m = \xi_n = 0.3\%$ [35]. Using Eq. (16), the mass and

FIG. 5. The displacement amplitudes. (a) $q_{\mu,\text{Lorentz},0}$ and (b) $q_{\mu,\text{Tuning},0}$.FIG. 6. Mode sensitivity analysis, (a) detuning sensitivity S_μ , (b) ϕ_μ of sensitive longitudinal modes.FIG. 7. The coupling factors. (a) k_μ and (b) n_μ .

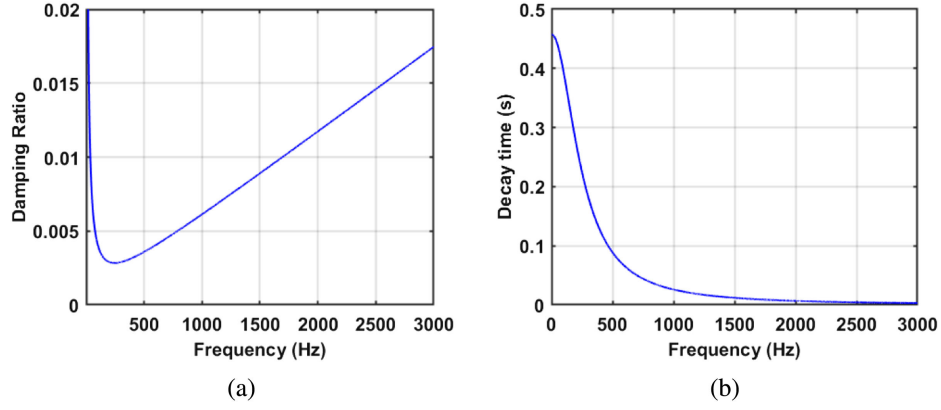


FIG. 8. Rayleigh damping model of TESLA cavity. (a) Damping ratio ξ and (b) decay time τ .

stiffness coefficients were calculated as $\alpha = 4.37$ and $\beta = 1.84 \times 10^{-6}$, respectively. The resulting damping ratios ξ_μ and decay time τ_μ for all modes up to 3000 Hz are presented in Fig. 8.

The complete parameter set for the modified mechanical model is summarized in Appendix A. Following the selection criteria defined in Sec. II, eigenmodes with negligible coupling factors ($k_\mu < 0.001$ and $n_\mu < 0.01$) were excluded. With the cutoff frequency f_c set to the 100th modal frequency, a total of 15 modes were retained for the dynamic simulation.

C. Dynamic LFD investigations

During operation, the cavity field is stabilized by a LLRF system employing feedback and feedforward control. To simplify the dynamic analysis, a unidirectional coupling scheme is implemented, wherein the accelerating gradient pulse is transferred from the electrical model to the mechanical model without a feedback loop. A schematic of this simulation framework is presented in Fig. 9.

In this framework, P_g and P_{ref} denote the forward and reflected power, respectively; I_b represents the beam current; and E_{acc} is the accelerating gradient. The electrical parameters, including $r/Q = 1038 \Omega$, $Q_L = 3 \times 10^6$, $\omega_{1/2} = 2\pi \times 216.67$ rad/s, and $R_L = 1.554 \times 10^9 \Omega$, were adopted from Refs. [38,46]. The mechanical modal parameters are detailed in Table V of Appendix A.

First, a convergence study was performed to identify an optimal time step that balances computational efficiency and numerical accuracy. Fixed time steps of $T = 100$, 10 and 1 μs were evaluated. While the 100 μs step introduced non-negligible errors, the results for 10 and 1 μs were nearly identical (Fig. 10). Consequently, a time step of $T = 10 \mu\text{s}$ was selected for all subsequent simulations. Two primary load cases were investigated: (1) LFD analysis: To isolate Lorentz force effects, the tuning input was disabled ($F_{\text{Tuning}} \equiv 0$). A periodic 25 MV/m accelerating gradient profile was generated by the electrical model and transferred to the mechanical model to simulate dynamic detuning. (2) Tuning response analysis: Conversely, the Lorentz force was deactivated ($P_g \equiv I_b \equiv 0$). Dynamic detuning was excited solely by a sinusoidal test pulse (amplitude $A = 100$ N, frequency $f = 174.92$ Hz) applied as the tuning input.

For method validation, parallel simulations were conducted using the transient FEA solver in COMSOL. In these benchmarks, cavity deformation data were exported at regular intervals and postprocessed in MATLAB to compute the dynamic detuning profiles.

As shown in Figs. 11 and 12, the dynamic detuning predicted by the modified model is in excellent agreement with the transient FEA results. Furthermore, the LFD results are consistent with published experimental data, which reported a maximum detuning of 220 Hz during the beam

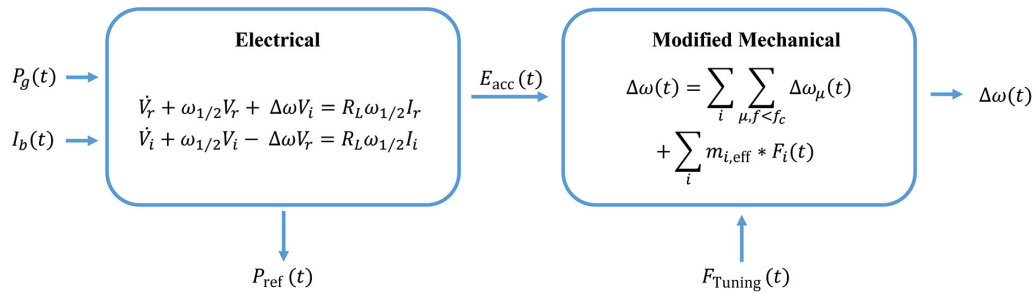


FIG. 9. LFD unidirectional coupling simulation.

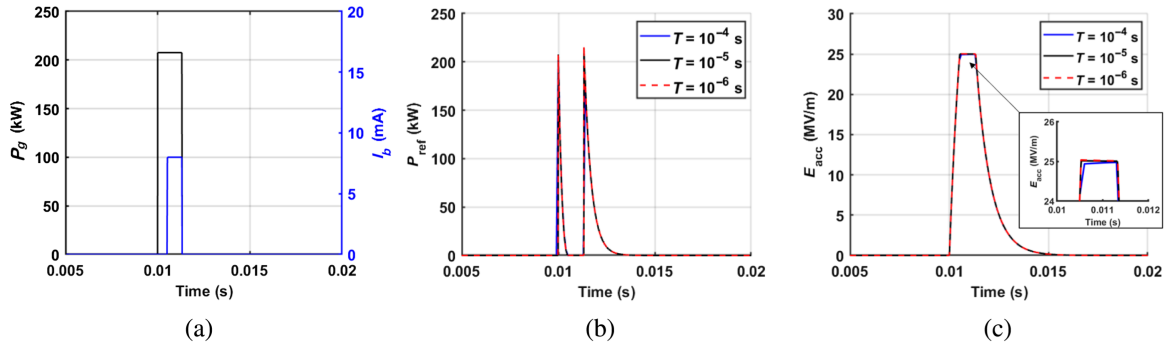
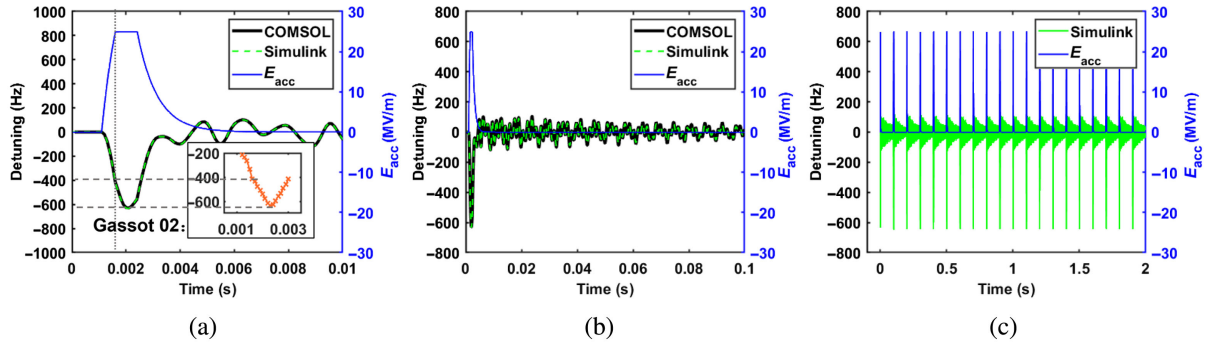
FIG. 10. Electrical model input and results. (a) P_g and I_b , (b) P_{ref} , and (c) E_{acc} .

FIG. 11. Dynamic LFD simulation. (a) 0–0.01 s, compared with the measured data form Ref. [21], (b) 0–0.1 s, and (c) 0–2 s.

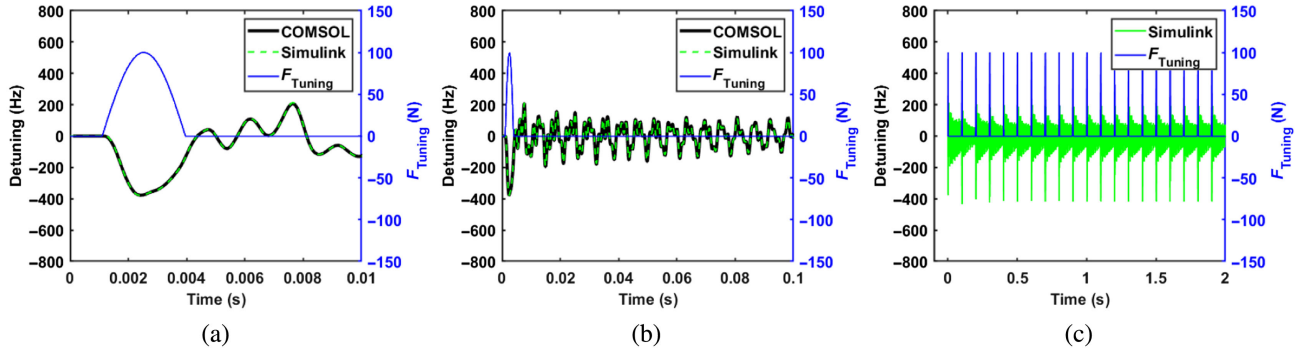


FIG. 12. Fast tuning simulation. (a) 0–0.01 s, (b) 0–0.1 s, and (c) 0–2 s.

flatop [21]. Additionally, the simulations indicate that the detuning response reaches a periodic steady state approximately 2 s after the initial excitation for both Lorentz force and tuning inputs.

Figure 13 provides a detailed comparison between the simulated and measured dynamic LFD under various accelerating gradients. In the simulation, each E_{acc} pulse maintained a constant filling time of 500 μ s and a flatop duration of 800 μ s. To facilitate comparison, a static pre-detuning was introduced such that the detuning at $t = 500 \mu$ s was zero. The simulated and measured results exhibit similar magnitudes and waveforms. Minor discrepancies may arise from slight

variations in the FEA boundary conditions or external disturbances present during experimental measurements.

The significance of the correction term introduced in Eq. (19) is highlighted in Fig. 14. Without the correction term, the simulation results diverge significantly from the baseline for nonzero inputs, demonstrating that accounting for high-frequency modal residuals is essential for accurate detuning prediction.

A key advantage of the proposed modified model is its high computational efficiency. For an FEA model with 500,000 tetrahedral elements, a transient simulation spanning 0.1 s required approximately 8 h of computation time.

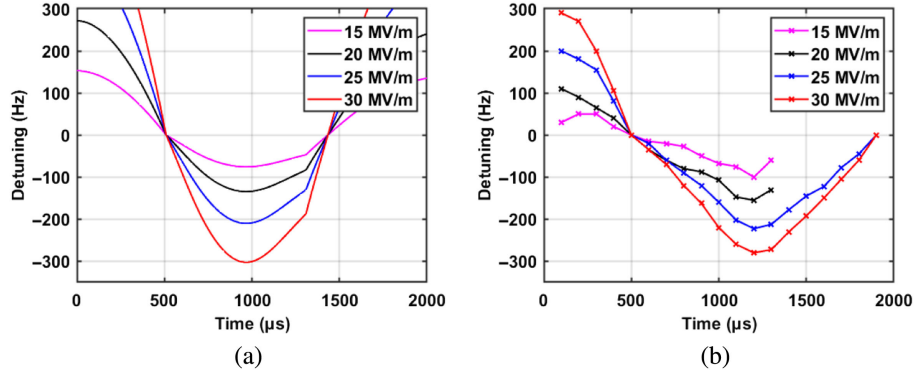


FIG. 13. Model verification at different E_{acc} . (a) Calculation results and (b) measurement results (data from [21]).

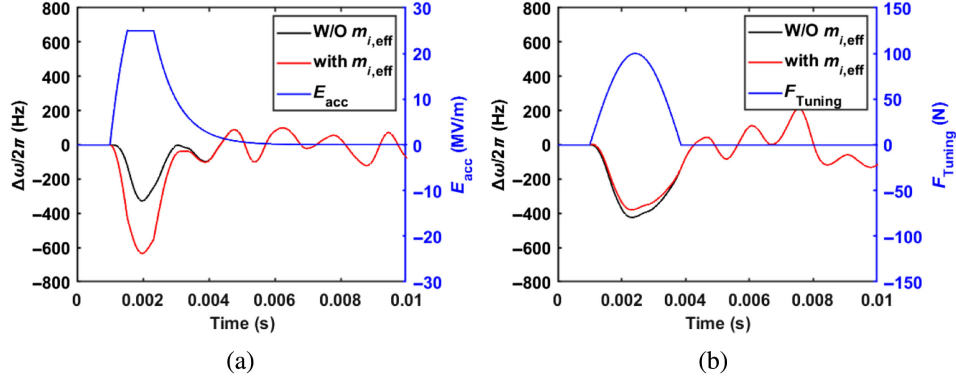


FIG. 14. Influence of correction term. (a) LFD analysis and (b) tuning analysis.

In contrast, the modified model completes a 2 s analysis in less than 5 s. This reduction in computational cost by several orders of magnitude enables large-scale optimization studies for active compensation.

We subsequently investigated the active compensation of dynamic LFD. Given its strong periodicity, LFD is well suited for feedforward compensation. A common strategy

involves applying a periodic, half-sinusoidal pulse to the fast tuner [49,50]. As illustrated in Fig. 15(a), the compensation pulse is defined by its amplitude A , frequency F , and delay time T_{de} . A negative T_{de} indicates that the tuning pulse precedes the E_{acc} pulse.

This multiparameter optimization problem was solved using a GA, with the configuration summarized in Table II.

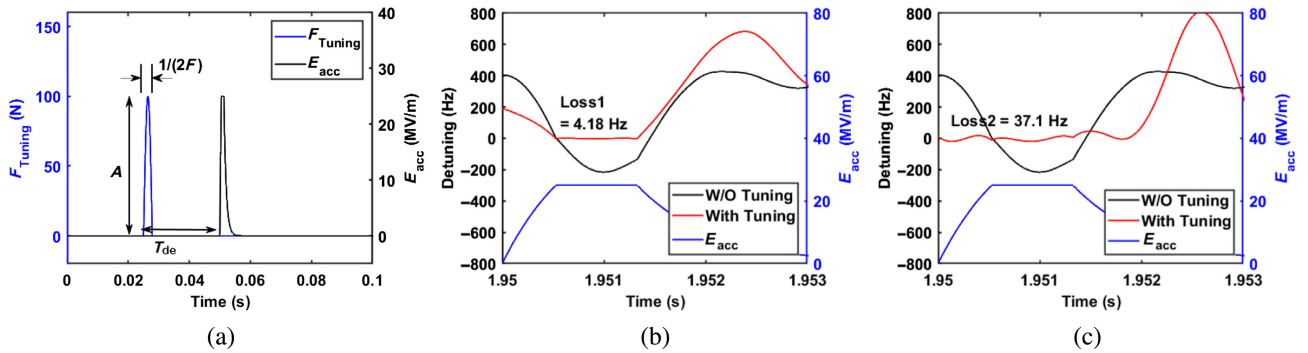


FIG. 15. LFD compensation. (a) Half sine wave, (b) only flattop, and (c) power injection + flattop.

TABLE II. GA parameters.

Parameters	Value
Population size	50
Crossover fraction	0.8
Max generations	200
Elite count	2
Function tolerance	5×10^{-3}
Max stall generations	10
GA repetitions	100

To ensure a global search, broad parameter ranges were defined: $A \in [-500 \text{ N}, 500 \text{ N}]$, $F \in [10 \text{ Hz}, 1000 \text{ Hz}]$, and $T_{de} \in [-0.4999 \text{ s}, 0.4999 \text{ s}]$. The objective function (loss) was defined as the peak-to-peak detuning within a specified time window:

$$\text{Loss} = (|\text{Max}(\Delta\omega(t)/2\pi)| - |\text{Min}(\Delta\omega(t)/2\pi)|)|_{t \in [\text{window}]} \quad (21)$$

The loss was evaluated during the final period (1.9 to 2.0 s) to ensure the system had reached a periodic steady state.

Two optimization objectives were studied. The first, Loss1, focused only on the 800 μs beam flat-top. Under this optimization, LFD was suppressed from 220 Hz to below 5 Hz [Fig. 15(b)]. The second, Loss2, encompassed the entire rf-on duration (510 μs filling time plus 800 μs flat-top), reducing the maximum LFD from 620 Hz to below 40 Hz [Fig. 15(c)]. These results are comparable to experimental benchmarks [49,50]. Notably, the cavity detuning at the beginning of the flat-top was adjusted to zero by introducing constant tuning inputs.

The GA optimization utilized parallel computing to further enhance efficiency. A typical run of 10 GA repetitions evaluated over 5000 parameter sets in approximately 1 h. A total of 100 GA repetitions were performed for both objectives, and the top 5 parameter sets are listed in Table III. The results demonstrate that various half-sinusoidal pulse configurations can effectively reduce LFD to levels negligible relative to the cavity half-bandwidth of 217 Hz.

IV. MICROPHONICS INVESTIGATION IN THE HALF CAVITY

Given that mechanical noise is not negligible in SRF cavities under heavy beam loading conditions [51], a primary objective of this study was to analyze microphonics in the 499.8 MHz single-cell cavity-vessel-tuner assembly of HALF prior to its commissioning. The open-loop phase and amplitude instabilities induced by microphonics were evaluated using a unidirectional coupling scheme, where detuning profiles from the mechanical model were integrated into the electrical model. This section investigates the effects of ground vibrations and rapid pressure fluctuations. Furthermore, active compensation of stochastic microphonics using a proportional-integral (PI) controller is presented.

A. FEA model of the HALF cavity-vessel-tuner assembly

The primary sources of microphonics include ground vibrations, mechanical noise from auxiliary equipment (e.g., vacuum pumps), and rapid pressure fluctuations within the cryogenic system, such as thermoacoustic oscillations (TAO) [12]. These disturbances are typically transmitted to the SRF cavity through its supporting structures, including support posts, beam pipes, and the helium vessel. This section describes the high-fidelity FEA model of the HALF cavity developed in COMSOL, which serves as the foundation for the subsequent analyses.

Figure 16 illustrates the FEA model with material assignments. The niobium cavity and its associated flanges (Nb-Ti 45 alloy) are highlighted in white. The helium vessel, flanges, and tuner mechanism are constructed from 316-grade stainless steel (SS316, shown in gray), while the beam pipes and the simplified higher-order mode (HOM) absorbers consist of oxygen-free copper (Cu-OF, shown in red). The transparent blue volumes represent the vacuum regions. The material properties used in this analysis are detailed in Table IV [47,48,52]. The thermal environment is accounted for through specific boundary conditions: components within the helium vessel and the cavity itself are maintained at 4.2 K; the beam pipes are thermally anchored to an 80 K thermal shield; and the HOM absorbers and tuner assembly are modeled at room temperature (293 K).

TABLE III. LFD GA results.

A	F	T_{de}	Loss1	A	F	T_{de}	Loss2
(N)	(Hz)	(s)	(Hz)	(N)	(Hz)	(s)	(Hz)
261.3	295.1	0.01154	4.18	-237.1	582.8	-0.04269	37.09
-138.3	257.0	-0.03527	4.26	144.4	581.0	-0.02437	46.48
-49.1	488.6	-0.02354	4.63	-228.8	439.0	-0.04286	51.27
381.5	94.7	-0.02831	5.22	447.4	227.2	-0.03130	61.47
211.3	286.3	0.02903	5.70	-188.4	785.6	-0.02828	64.02

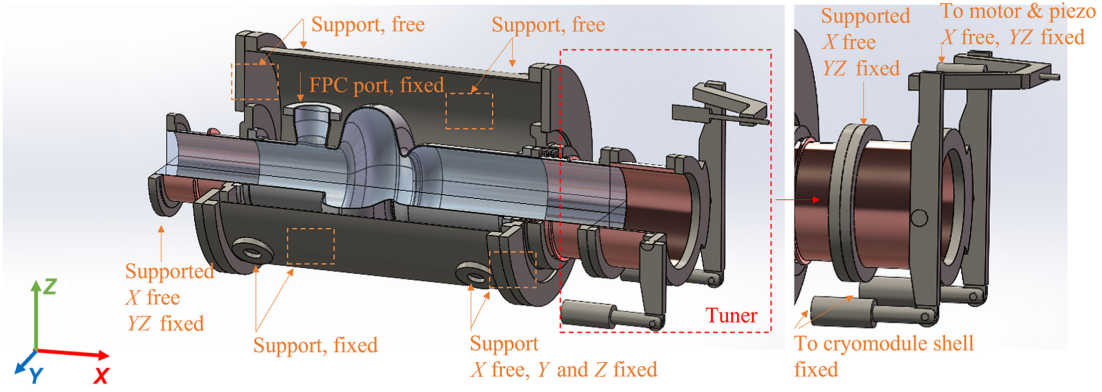


FIG. 16. FEA model of the HALF cavity-vessel-tuner system (with sections).

A modeling challenge involved the tuner's hinged connections (between the HOM absorbers, tuner arms, and supports), which were initially simplified as fixed joints in COMSOL. To assess the impact of this simplification, a comparative analysis was performed in ANSYS. The results indicated that while a larger tuning force was required in the fixed-joint model to overcome spurious torque, the difference in the calculated structural stiffness was marginal (74 kN/mm for the fixed model vs 70 kN/mm for the hinged model), as shown in Fig. 17. This indicates that the fixed-joint simplification is acceptable for the detuning analysis.

B. Characterization of the mechanical eigenmodes

As the HALF facility and its SRF cavities are currently under construction, *in situ* measurements of the mechanical environment are not yet available. Consequently, this study relies on numerical characterization of LFD, tuning, ground vibrations, and helium pressure fluctuations to establish a predictive model. Following the methodology described in Sec. III A, the characterization begins with an rf eigenfrequency simulation to determine the electromagnetic field distributions of the fundamental TM_{010} mode (Fig. 18), from which the radiation pressure distribution P_{Lorentz} is derived.

Subsequently, a mechanical modal analysis was performed to extract the eigenfrequencies Ω_μ and normalized eigenfunctions ϕ_μ for the first 200 modes (normalized to a

maximum displacement of $1\mu\text{m}$). The modal detuning sensitivities S_μ were then calculated using Eq. (13), as shown in Fig. 19.

To evaluate the modal coupling factors k_μ for LFD, a static deformation $\xi_{\text{Lorentz},0}$ was simulated at an accelerating gradient of $E_{\text{acc}} = 3\text{ MV/m}$. This deformation profile was decomposed into modal amplitudes $q_{\mu,\text{Lorentz},0}$ via Eq. (10), and the coupling factors were determined using Eqs. (11) and (14), as illustrated in Fig. 20.

To avoid numerical artifacts associated with the fixed-joint simplification in the tuner assembly, the tuning load was introduced via a prescribed displacement boundary condition. A static tuning deformation $\xi_{\text{Tuning},0}$ was calculated for a longitudinal displacement of 0.1 mm, with the resulting modal coupling factors n_μ presented in Fig. 21.

Helium pressure fluctuations were modeled by applying a uniform pressure boundary condition to the outer surface of the cavity and the inner surface of the vessel. For a static pressure of 10 mbar, the resulting deformation $\xi_{\text{Pressure},0}$ and corresponding modal coupling factors $m_{\mu,\text{He}}$ are shown in Fig. 22.

In the HALF cryomodule, the helium vessel is supported by eight posts (see Fig. 16). Ground vibrations were simulated by applying prescribed displacements to the support interfaces at the tuner and the small beam-pipe flange. The analysis indicates that the resonant frequency is most sensitive to longitudinal displacements (X direction, Fig. 23), while exhibiting minimal sensitivity to transverse horizontal motion (Y direction, Fig. 24). A moderate sensitivity to vertical displacement (Z direction, Fig. 25) was observed, which can be attributed to the structural asymmetry introduced by the fixed FPC port.

Overall, longitudinal loads (X direction) yield larger coupling factors compared to transverse loads (Y and Z directions). The first longitudinal eigenmode at 105.8 Hz provides the dominant detuning contribution under longitudinal excitation. A mesh convergence study is presented in Figs. 20–25, comparing cumulative coupling factors across three mesh densities: 1.1, 1.9, and 3.1 M elements. While results from the coarsest mesh show slight

TABLE IV. Material property.

Property	Young's modulus E (GPa)	Poisson's ratio σ	Density ρ (kg/m ³)
Niobium (4.2 K)	88.5	0.38	8586
Nb-Ti 45 (4.2 K)	68.3	0.34	6360
Cu-OF (80 K)	136	0.339	8960
Cu-OF (293 K)	128	0.346	8960
SS316 (4.2 K)	208.1	0.29	8000
SS316 (293 K)	195.1	0.29	8000

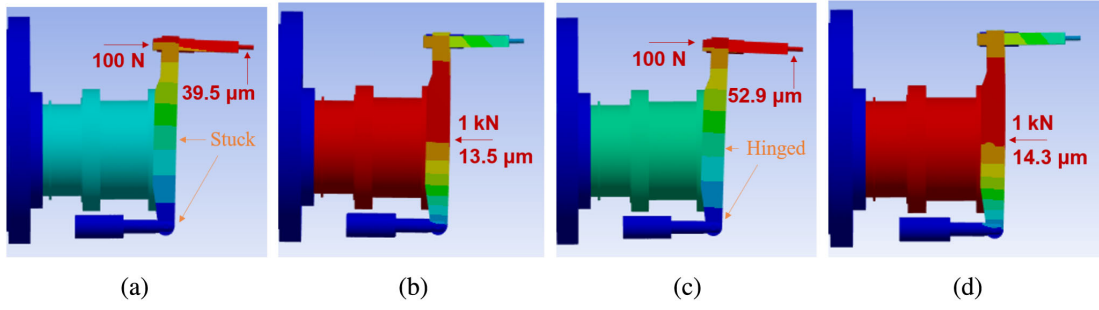


FIG. 17. Tuner analysis using ANSYS. (a) Stuck model, (b) stuck model stiffness (74 kN/mm), (c) hinged model, and (d) hinged model stiffness (70 kN/mm).



FIG. 18. HALF rf fields. (a) Electric field and (b) magnetic field.

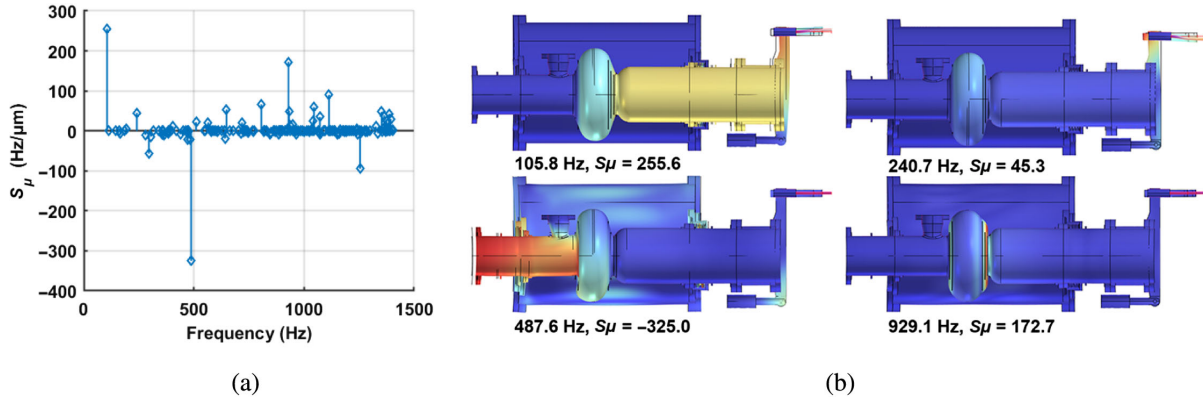


FIG. 19. HALF modal analysis, (a) Detuning sensitivity S_μ and (b) high sensitivity modes ϕ_μ .

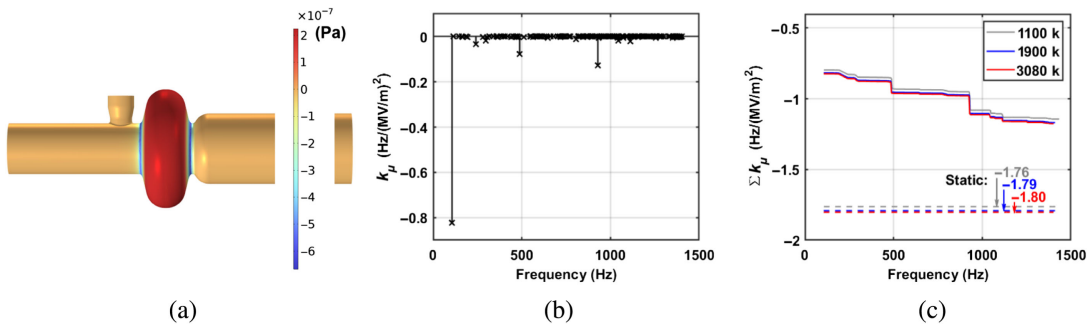
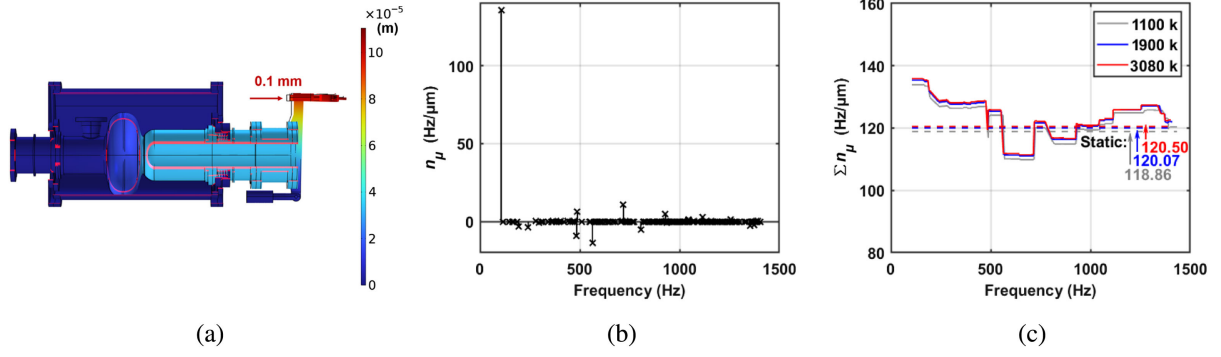
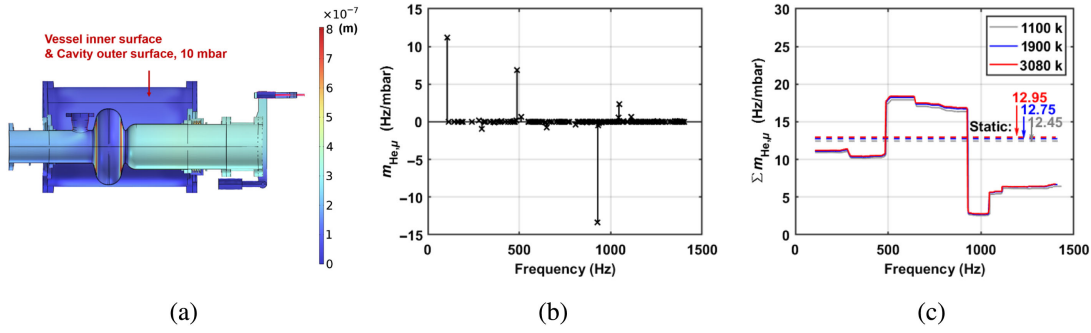


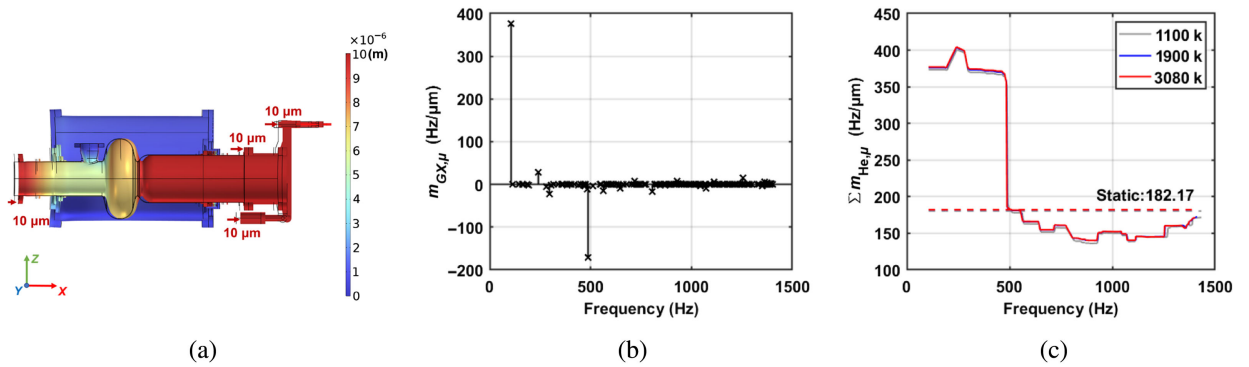
FIG. 20. HALF LFD analysis. (a) Lorentz force distribution P_{Lorentz} , (b) k_μ , and (c) Σk_μ .

FIG. 21. HALF tuning analysis. (a) Tuning displacement $\xi_{\text{Tuning},0}$, (b) n_μ , and (c) Σn_μ .FIG. 22. HALF pressure analysis. (a) Pressure displacement $\xi_{\text{Pressure},0}$, (b) $m_{\mu,\text{He}}$, and (c) $\Sigma m_{\mu,\text{He}}$.

deviations, the 1.9 and 3.1 M element meshes demonstrate good agreement, confirming numerical convergence. Subsequent analyses utilize the 3.1 M element mesh.

The system's damping characteristics were determined by applying the Rayleigh damping model to the first two dominant mechanical modes ($\omega_m = 2\pi \times 105.79$ Hz and $\omega_n = 2\pi \times 240.67$ Hz), with a uniform damping ratio of $\xi_m = \xi_n = 0.3\%$. The damping coefficients were derived as $\alpha = 2.77$ and $\beta = 2.75 \times 10^{-6}$. The resulting damping ratios ξ_μ and decay times τ_μ for modes up to 1500 Hz are shown in Fig. 26.

The dynamic analysis of the HALF cavity incorporates the first 200 mechanical modes, with the highest frequency at 1402.8 Hz. Modes above this frequency exhibit decay times shorter than 0.01 s; thus, the cutoff frequency f_c was set at 1402.8 Hz. Since microphonic loads are predominantly low-frequency phenomena (typically below 100 Hz) [53], the contributions of higher-frequency modes are accurately represented by correction terms. A summary of the parameters for the complete assembly is provided in Appendix B (Table VI). For this modified model, a total of 48 modes were selected based on the following coupling factor thresholds:

FIG. 23. HALF ground motion analysis (X direction). (a) Displacement $\xi_{\text{GroundX},0}$, (b) $m_{\mu,\text{GX}}$, (c) $\Sigma m_{\mu,\text{GX}}$.

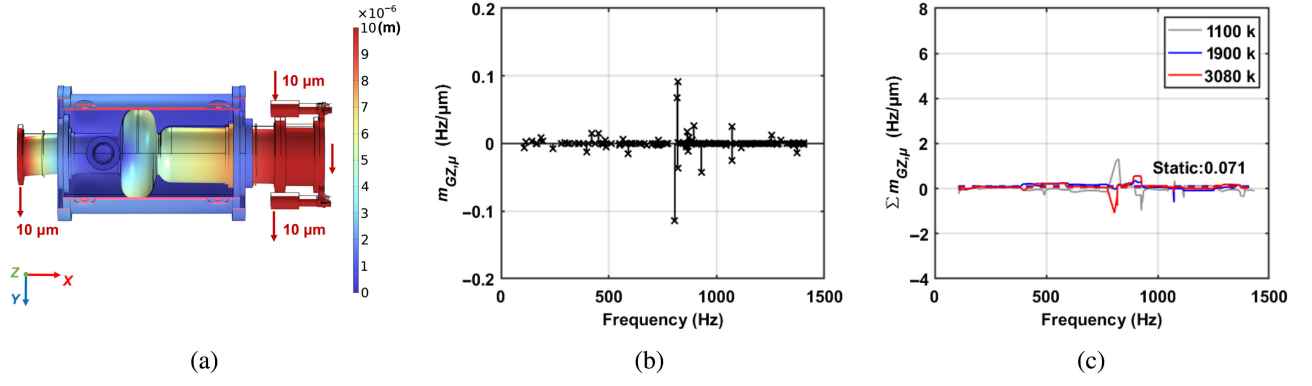


FIG. 24. HALF ground motion analysis (Y direction). (a) Displacement $\xi_{\text{Ground}Y,0}$, (b) $m_{\mu,GY}$, and (c) $\Sigma m_{\mu,GY}$.

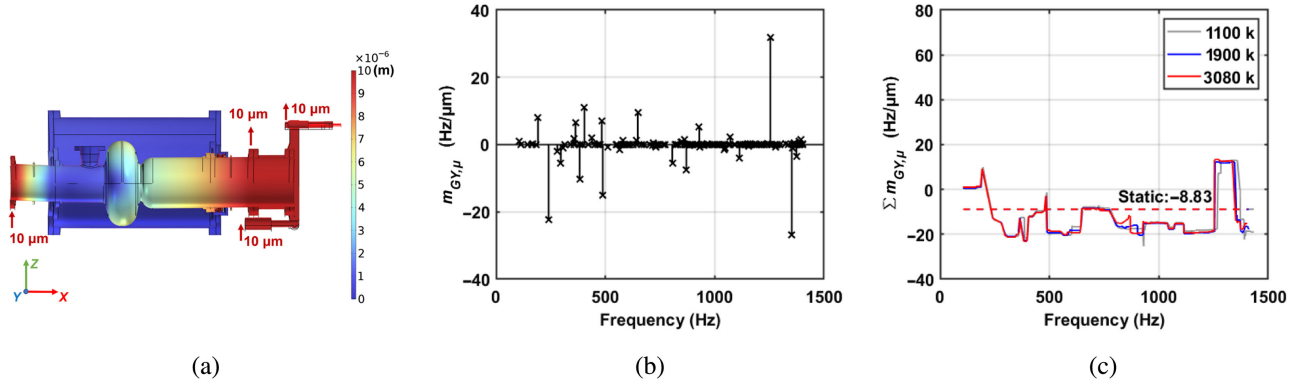


FIG. 25. HALF ground motion analysis (Z direction). (a) Displacement $\xi_{\text{Ground}Z,0}$, (b) $m_{\mu,GZ}$, (c) $\Sigma m_{\mu,GZ}$.

$k_{\mu} > 0.01$, $n_{\mu} > 1$, $m_{\mu,\text{He}} > 0.1$, $m_{\mu,GX} > 1$, $m_{\mu,GY} > 0.1$, or $m_{\mu,GZ} > 1$.

C. Microphonics investigations

The HALF cavity is designed for continuous-wave (cw) operation at a nominal accelerating voltage of 1.5 MV. Under these conditions, dynamic LFD effects are negligible. Consequently, this section focuses on the open-loop phase

and amplitude instabilities induced by microphonics. A unidirectional coupling scheme is employed to transfer detuning profiles from the mechanical model to the electrical model (Fig. 27). The electrical model incorporates HALF-specific parameters: $r/Q = 89.9 \Omega$, $Q_{L,\text{opt}} = 1.7877 \times 10^5$, $f_{\text{rf}} = 499.8 \text{ MHz}$, $P_g = 140 \text{ kW}$ and $I_b = 350 \text{ mA}$. The mechanical model comprises 48 selected modes (Table VI) and the high-frequency correction terms.

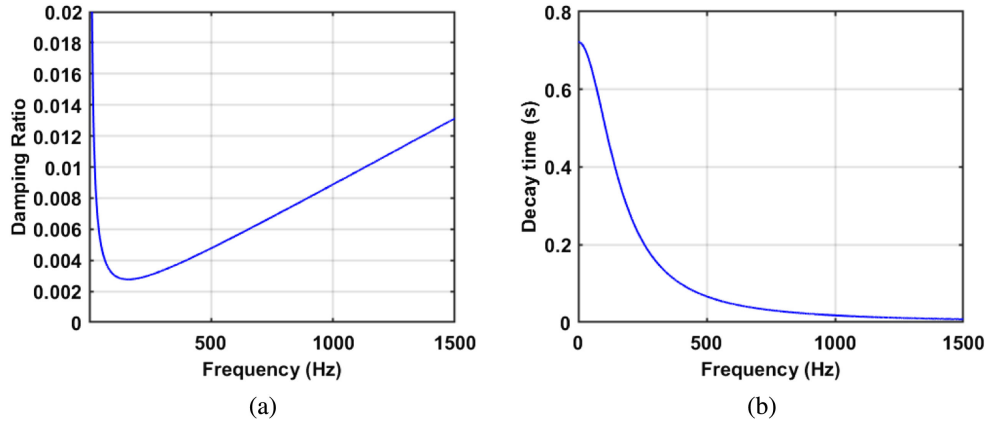


FIG. 26. HALF damping model ($\alpha = 2.77$, $\beta = 2.75 \times 10^{-6}$). (a) Damping ratio ξ and (b) decay time τ .

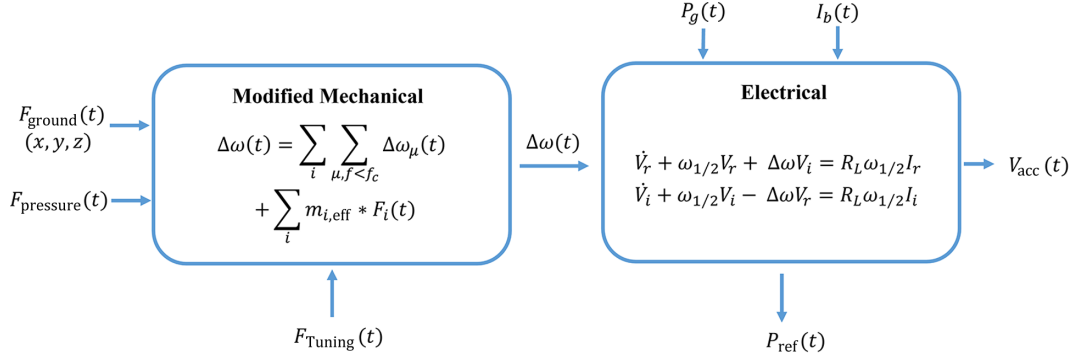


FIG. 27. Microphonics unidirectional coupling simulation.

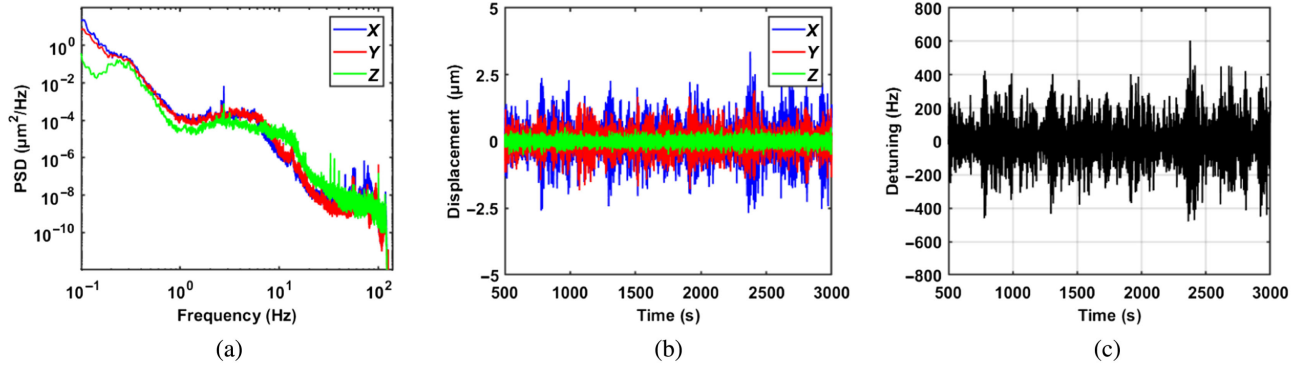


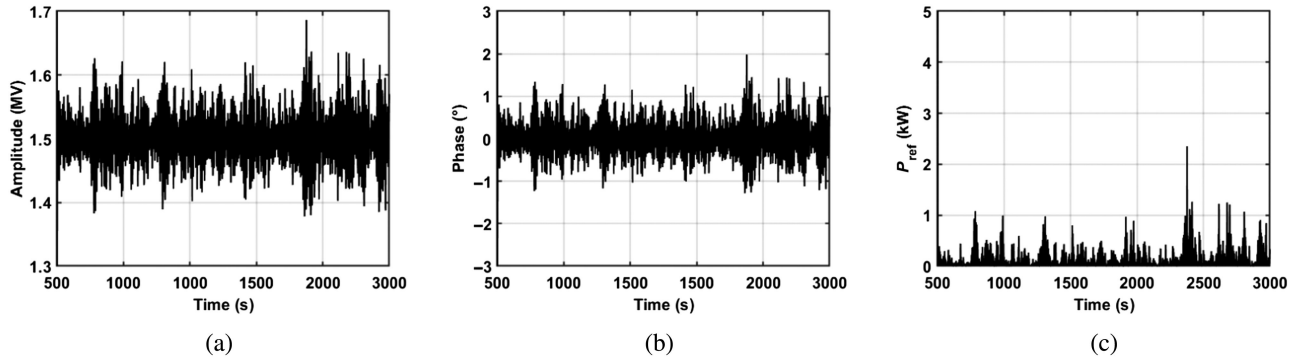
FIG. 28. Microphonics source measurement of HALF cavity (stochastic ground motion). (a) PSD, (b) ground displacements, and (c) detuning simulation.

Ground vibrations can be categorized into two distinct components: deterministic harmonic excitations originating from specific machinery (e.g., pumps or motors) and a stochastic component characterized by ambient environmental noise [11].

The stochastic component was characterized using a seismometer installed on the HALF storage ring floor. The raw velocity data, sampled at 250 Hz, were processed in MATLAB using a 0.1 Hz high-pass filter and subsequently integrated to obtain ground displacement. The resulting

power spectral density (PSD) is shown in Fig. 28, where X, Y, and Z denote the longitudinal, horizontal, and vertical directions, respectively. In line with the previously identified sensitivity to longitudinal motion, the detuning pattern is dominated by the X direction vibration characteristics.

The calculated detuning was simultaneously integrated into the electrical model for stability analysis. The results indicate a peak reflected power of $P_{\text{ref}} < 3$ kW (Fig. 29), which is well within the compensation overhead of the installed rf power sources.

FIG. 29. Simulation results for stability of HALF cavity (stochastic ground vibration). (a) Amplitude, (b) phase, and (c) P_{ref} .

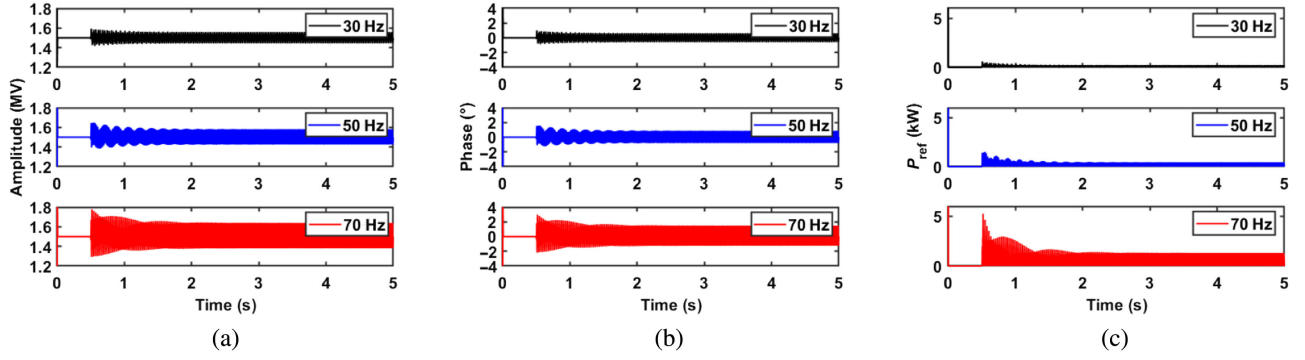


FIG. 30. Stability of HALF cavity (sinusoids ground vibration). (a) Amplitude, (b) phase, and (c) P_{ref} .

The deterministic component of ground motion was modeled as longitudinal (X direction) sinusoidal excitations with a $1 \mu\text{m}$ amplitude at discrete frequencies of 30, 50, 70, and 100 Hz. These frequencies were selected based on the assumption that most significant vibration sources reside below 100 Hz [54]. Since the lowest mechanical eigenfrequency of the HALF module is 105 Hz, most microphonic excitations are not expected to trigger severe mechanical resonances.

In these simulations, excitations were initiated 0.5 s after rf power-up. Results for 30, 50 and 70 Hz showed no significant resonant behavior, with reflected power levels remaining within acceptable limits (Fig. 30). However, as the 100 Hz excitation frequency approaches the 105.8 Hz modal frequency, the resulting detuning and subsequent open-loop voltage instabilities reach excessive levels (Fig. 31).

Rapid pressure fluctuations in the cryogenic system, particularly during TAO events, can reach amplitudes of $\pm 100 \text{ mbar}$ ($\pm 1.5 \text{ psig}$) at frequencies typically between 10 and 100 Hz [55]. To evaluate this impact, sinusoidal pressure loads of 100 mbar amplitude were simulated at the same discrete frequencies. The results indicate that the reflected power can exceed 20 kW even under nonresonant conditions (Fig. 32). This underscores the critical importance of both avoiding mechanical resonance (Fig. 33) and mitigating severe TAO events to ensure operational stability.

To enhance operational reliability, detuning effects should be suppressed to minimize the demand on the rf power system. Microphonics mitigation typically involves a combination of passive damping and active compensation. In active control schemes, stochastic low-frequency disturbances are commonly addressed using PI feedback loops, while deterministic components often require adaptive feedforward strategies [54]. In this study, a PI feedback loop was integrated into the model, with control parameters optimized via a GA.

To prevent the feedback loop from exciting high-frequency mechanical modes, a 100 Hz low-pass filter was applied to the detuning signal. The tuning force F_{Tuning} was then calculated based on the filtered detuning $\Delta\omega$ according to the PI control law:

$$F_{\text{Tuning}} = k_p \frac{\Delta\omega}{2\pi} + k_I \int \frac{\Delta\omega}{2\pi} dt. \quad (22)$$

The optimization targeted the controller gains k_I and k_p . To improve computational efficiency, a 50-s segment (from 500 to 550 s) of measured ground displacement data was utilized as input. The objective function (loss) was defined as the total peak-to-peak detuning magnitude evaluated within a window from 510 to 550 s, allowing the system 10 s to reach a steady state:

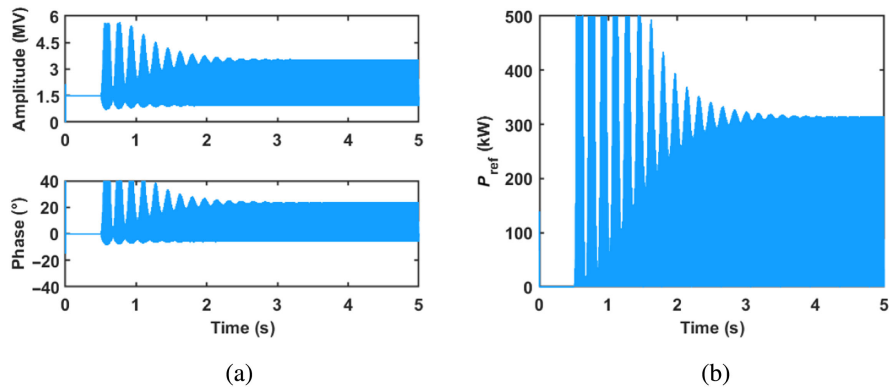
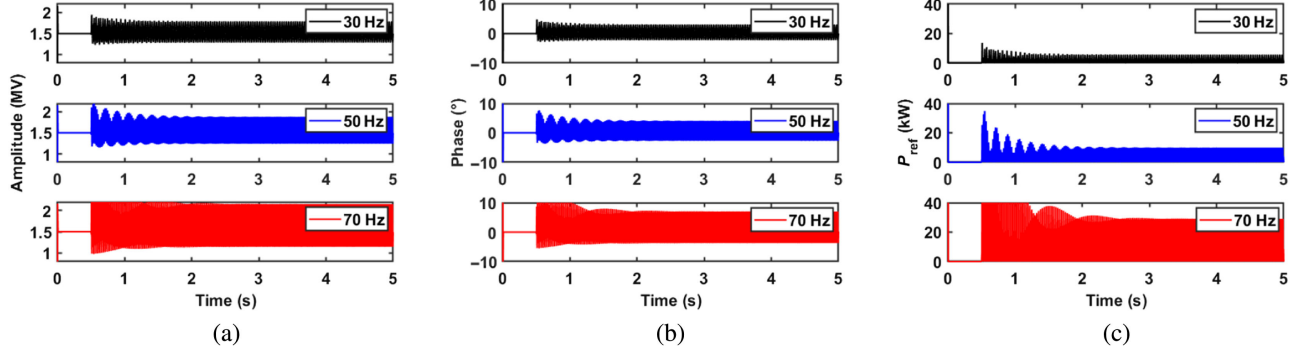
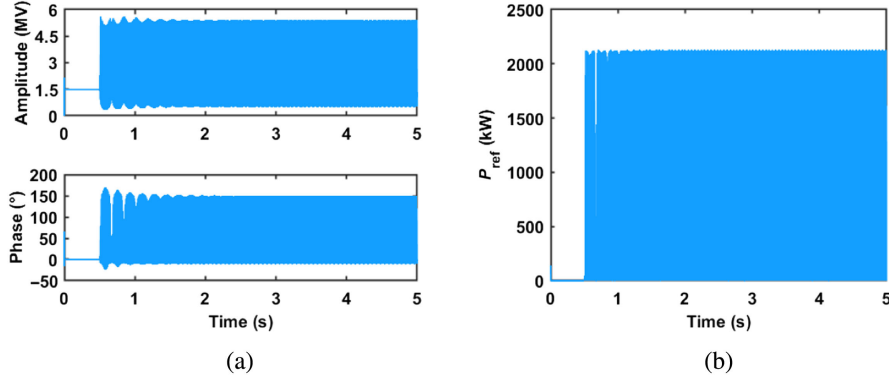


FIG. 31. Instability at 100 Hz resonance (sinusoids ground vibration). (a) Amplitude and phase and (b) P_{ref} .

FIG. 32. Stability of HALF cavity (sinusoids pressure fluctuation). (a) Amplitude, (b) phase, and (c) P_{ref} .FIG. 33. Instability at 100 Hz resonance (sinusoids pressure fluctuation). (a) Amplitude and phase and (b) P_{ref} .

$$\text{Loss} = (|\text{Max}(\Delta\omega(t)/2\pi)| + |\text{Min}(\Delta\omega(t)/2\pi)|)|_{t \in [510 \text{ s}, 550 \text{ s}]} \cdot \quad (23)$$

The search ranges were defined as $k_I \in [-10/120.5, 10/120.5]$ and $k_P \in [-0.01/120.5, 0.01/120.5]$, where 120.5 Hz/ μm is the static tuning coefficient, and GA parameters were consistent with Table II. The optimization yielded optimal gains of $k_I = -0.07805$ and

$k_P = 8.30 \times 10^{-5}$. Performance comparisons with and without PI control are presented in Fig. 34.

With the optimized PI controller, detuning induced by stochastic ground motion was suppressed from ± 400 to ± 50 Hz. Consequently, open-loop phase and amplitude instabilities were reduced from $\pm 2^\circ$ and $\pm 13\%$ to $\pm 0.2^\circ$ and $\pm 1.3\%$, respectively. Further stability improvements are expected upon integration with rf power compensation and advanced control algorithms.

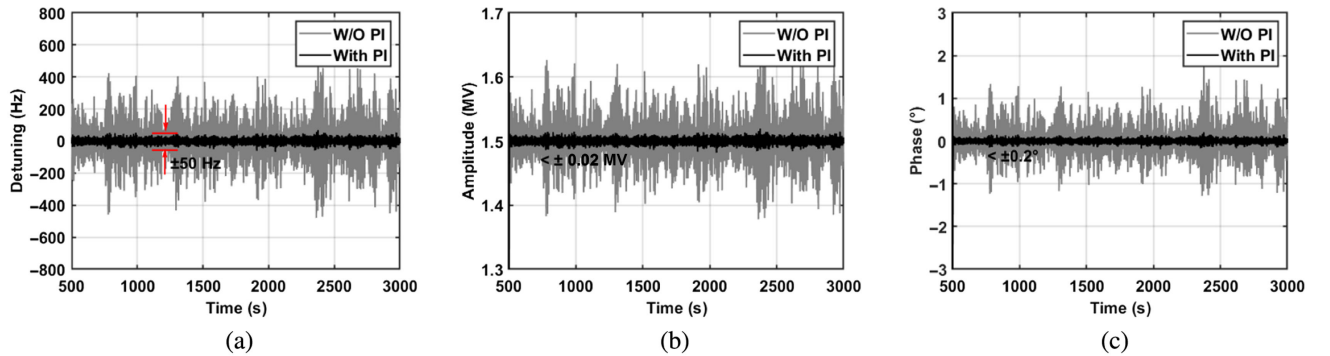


FIG. 34. Simulation results of microphonics control for the HALF cavity, (a) Detuning, (b) amplitude, and (c) phase.

V. CONCLUSION

A novel numerical method for analyzing dynamic detuning in SRF cavities is presented based on a modified mechanical model. Eigenmode parameters are extracted from 3D FEA and incorporated into unidirectional coupling frameworks implemented in MATLAB-Simulink. The model fidelity is enhanced by introducing correction terms to account for the aggregate response of high-frequency mechanical modes. This approach was applied to investigate dynamic LFD in pulsed 1.3 GHz TESLA-type cavities and microphonics in the 499.8 MHz single-cell elliptical cavity for the HALF under cw operation. Dynamic LFD simulations for the TESLA-type cavity demonstrate excellent agreement with both transient FEA benchmarks and published experimental data, validating the method and the TESLA cavity model. Leveraging the high computational efficiency of this semianalytical framework, which reduces simulation time by several orders of magnitude compared to full transient FEA, active LFD compensation was optimized using a GA. Specifically, dynamic LFD during the beam flattop was suppressed from 220 Hz to less than 5 Hz using an optimized half-sinusoidal tuning pulse. In the microphonics analysis of the HALF cavity, the impacts of various disturbance sources, including ground vibrations and pressure fluctuations, were evaluated. The results highlight that resonant excitations

near 100 Hz and rapid pressure fluctuations during TAO events pose significant operational risks. By implementing an optimized PI feedback controller for fast tuning, simulation results show that stochastic microphonics were suppressed from ± 400 Hz to within ± 50 Hz, representing an eightfold reduction. Consequently, the open-loop phase and amplitude instabilities were reduced from $\pm 2^\circ$ and $\pm 13\%$ to $\pm 0.2^\circ$ and $\pm 1.3\%$, respectively. The developed model is readily applicable to various superconducting cavity geometries and provides a powerful, high-efficiency simulation tool for the design, optimization, and operational stability analysis of SRF cavities.

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DATA AVAILABILITY

The data that support the findings of this article are not publicly available upon publication because it is not technically feasible and/or the cost of preparing, depositing, and hosting the data would be prohibitive within the terms of this research project. The data are available from the authors upon reasonable request.

APPENDIX A: TESLA MECHANICAL PARAMETERS

TABLE V. TESLA mode parameters.

Mode number	Frequency (Hz)	k_μ [Hz/(MV/m) ²]	n_μ (Hz/N)	τ_μ (s)
1	174.92	-0.04797	-0.60514	0.30334
2	343.92	-0.01448	-0.37917	0.15427
3	498.80	-0.00026	0.06428	0.08910
4	629.91	-0.13814	-1.70335	0.06024
5	739.95	-0.00467	0.31599	0.04530
6	850.56	-0.10689	-1.19712	0.03513
7	975.13	-0.00226	0.13115	0.02723
8	1104.27	-0.02416	-0.28344	0.02151
9	1219.68	-0.00036	0.02459	0.01778
10	1358.14	-0.01677	-0.03718	0.01445
11	1391.60	-0.00759	-0.25369	0.01379
12	2289.87	-0.00156	0.00257	0.00519
13	2314.59	-0.00115	-0.00238	0.00508
14	2495.39	-0.01164	0.00604	0.00438
15	2510.52	-0.00455	-0.20462	0.00433
$\sum m_{\mu,i}$	...	-0.36627	-3.92290	...
Steady state	...	-0.87184	-3.67306	...
$m_{i,\text{eff}}$	...	-0.50557	0.24985	...

APPENDIX B: HALF MECHANICAL PARAMETERS

TABLE VI. HALF mode parameters.

	Frequency	k_μ	n_μ	$m_{\mu,\text{He}}$	$m_{\mu,GX}$	$m_{\mu,GY}$	$m_{\mu,GZ}$	τ_μ
Mode number	Hz	Hz(MV/m) ²	Hz/ μm	Hz/mbar	Hz/ μm	Hz/ μm	Hz/ μm	s
1	105.79	-0.8230	135.81	11.214	376.98	-0.0686	0.986	0.502
2	192.06	-0.0011	-3.16	0.010	-0.84	0.0432	8.033	0.295
3	240.67	-0.0322	-3.49	-0.011	28.29	-0.0779	-22.220	0.220
4	279.86	-0.0011	0.28	0.205	-5.03	-0.0024	-2.003	0.177
5	296.07	-0.0166	-0.88	-0.931	-22.87	0.0113	-5.419	0.162
6	305.98	-0.0015	-0.04	-0.039	-2.31	0.0010	-1.049	0.154
7	356.68	-0.0003	0.06	0.008	-0.15	-0.0013	1.767	0.120
8	363.55	-0.0006	0.21	-0.049	-0.04	-0.0022	6.428	0.117
9	382.25	-0.0002	-0.31	0.036	-1.36	-0.0029	-10.276	0.107
10	396.89	0.0000	0.00	0.000	0.00	-0.1231	0.000	0.100
11	403.89	-0.0002	0.39	0.063	-0.09	-0.0007	10.925	0.097
12	420.54	0.0000	0.00	0.000	0.00	0.1516	0.000	0.091
13	438.30	-0.0005	0.20	0.001	-1.09	-0.0018	2.063	0.084
14	451.71	0.0000	0.00	0.000	0.00	0.1464	0.000	0.080
15	471.33	-0.0014	0.15	0.101	-3.11	-0.0020	0.237	0.074
16	483.70	-0.0031	-9.28	0.226	-11.55	0.0458	7.074	0.071
17	487.59	-0.0777	6.44	6.872	-171.12	-0.0518	-15.006	0.070
18	509.80	-0.0017	-0.15	0.689	-4.08	0.0015	-0.847	0.064
19	564.36	-0.0007	-13.98	-0.004	-15.36	0.0651	-1.554	0.053
20	581.57	0.0000	-0.06	0.001	-0.03	0.0001	1.219	0.051
21	590.50	0.0000	0.00	0.000	0.00	-0.1554	0.000	0.049
22	641.53	-0.0007	-0.03	-0.135	-1.65	-0.0030	1.197	0.042
23	647.91	-0.0038	-0.27	-0.751	-9.58	0.0003	9.460	0.041
24	718.40	-0.0003	10.67	-0.015	6.96	-0.0121	0.411	0.034
25	806.08	-0.0101	-5.34	-0.356	-17.46	-1.1386	-5.514	0.027
26	817.62	0.0000	0.00	0.000	-0.01	0.6792	-0.001	0.026
27	819.07	0.0000	-0.07	-0.002	-0.08	-0.3593	-0.018	0.026
28	819.14	0.0000	-0.01	0.000	-0.02	0.9120	-0.004	0.026
29	861.37	-0.0001	0.03	0.003	-0.09	-0.0473	1.509	0.024
30	864.70	0.0000	0.00	0.000	0.00	0.1734	0.004	0.024
31	869.12	-0.0002	-0.01	-0.009	-0.13	-0.1124	-0.017	0.024
32	870.25	-0.0006	-0.20	-0.073	-0.96	0.0940	-7.449	0.023
33	893.78	0.0000	0.00	0.000	0.00	0.2623	0.000	0.022
34	929.13	-0.1274	4.73	-13.338	8.70	-0.4220	5.289	0.021
35	931.93	-0.0039	0.25	-0.462	1.97	-0.0174	-0.722	0.021
36	935.23	-0.0017	-0.49	-0.193	0.08	-0.0011	0.002	0.020
37	950.69	-0.0002	0.03	-0.060	1.51	0.0002	-0.216	0.020
38	1042.29	-0.0041	0.34	0.525	0.62	-0.0057	-1.568	0.017
39	1045.00	-0.0174	1.42	2.335	-3.64	-0.0205	-0.920	0.016
40	1071.68	-0.0044	0.29	0.100	-8.47	-0.2452	2.189	0.016
41	1071.76	-0.0001	0.01	0.003	-0.22	0.2479	0.057	0.016
42	1113.37	-0.0208	2.84	0.644	5.20	-0.0602	-4.135	0.015
43	1254.36	-0.0065	1.38	0.088	15.13	0.1223	31.788	0.011
44	1352.25	-0.0040	-2.65	0.054	-3.80	-0.0147	-26.826	0.010
45	1363.20	-0.0011	0.02	0.110	5.21	-0.0057	0.585	0.010
46	1373.62	-0.0022	-2.29	0.029	4.47	-0.1356	-3.427	0.010
47	1386.66	-0.0006	0.10	0.046	2.20	0.0054	1.093	0.009
48	1392.86	-0.0004	0.10	0.011	1.56	0.0027	1.538	0.009
$\sum m_{\mu,i}$	...	-1.1726	123.07	6.947	173.78	-0.1254	-15.338	...
Static	...	-1.8022	120.50	12.955	182.18	0.0712	-8.824	...
$m_{i,\text{eff}}$	...	-0.62967	-2.56292	6.00833	8.40059	0.19659	6.51405	...

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