

Muon injection and acceleration via ultraintense electron beam-driven plasma wakefield

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High-flux, high-energy muon beams hold transformative potential for particle physics, yet rapid acceleration to relativistic energies in plasma wakefields is limited by dephasing arising from their larger mass and lower velocity. Here, we propose a method for longitudinal injection and stable acceleration of low-energy muons using a plasma density up-ramp to separate the deceleration and acceleration regions. The nonlinear wakefield configuration required for muon injection was obtained analytically using Hamiltonian dynamics, and particle-in-cell (PIC) simulations showed that the initial energy and emittance conditions of injectable muons can be tuned. Driven by an ultraintense 23 GeV electron beam, muons with an initial energy of 300 MeV attain a final energy of 40 GeV, with an 18.85% energy spread and a normalized emittance of 8.86 mm mrad. These findings provide a feasible pathway toward the development of compact muon colliders, with potential to advance particle physics and enable the exploration of phenomena beyond the Standard Model.

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I. INTRODUCTION

The muon, a second-generation lepton with a rest mass of 105.7 MeV/c² and a unit electric charge, is an unstable particle exhibiting a mean lifetime of $\tau = 2.2 \mu\text{s}$ at rest [1]. With a mass 207 times that of the electron and pointlike particle characteristics, muons offer unique advantages for high-energy collider physics and neutrino research [2]. Future muon colliders [3,4] could probe Higgs boson couplings with unprecedented precision and explore multi-TeV energy scales that were inaccessible to the Large Hadron Collider [5]. Conventional muon production methods, such as hadronic shower processes [6] and direct production [7–11], face limitations in beam quality and efficiency. GeV-level electron beams generated via laser-plasma wakefield acceleration, when impinging on high-Z targets, can produce collimated muon beams with peak

energies of 200–300 MeV [12,13] and this has been experimentally verified [14]. A mm-scale ps-duration laser-ion-based muon source was obtained [15]. However, such muon sources still cannot meet the requirements of a muon collider. Although high-flux, high-energy muon beams hold transformative potential for muon science, critical challenges still remain. The low-energy muons suffer from rapid decay and their acceleration time is limited by the accelerating gradients, typically below 100 MV/m [16], offered by conventional radiofrequency accelerators because of material breakdown thresholds. Therefore, how to rapidly accelerate muons to high-energy states within short time-scales remains a significant challenge, necessitating innovative acceleration schemes to unlock the full potential of muon-based particle physics experiments.

Plasma wakefield acceleration driven by lasers (LWFA) [17] or charged particle beams (PWFA) [18] can achieve extremely high accelerating gradients of several GV/m or even tens of GV/m. Plasma wakefield acceleration, which enables next-generation compact high-energy electron beam sources [19–23], also holds significant potential for advancing muon source research [24]. Plasma wakefield acceleration of low-energy muons, due to the initial velocity of muons being less than the speed of light and the mass heavier than that of electrons, will face the problem of muon phase slippage, and there is a risk of being unable to accelerate over long distances. Meanwhile,

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capturing low-energy muons requires accelerating them to near-light speed on an extremely short timescale, making their injection into the wakefield challenging. Recent studies have demonstrated promising integrated schemes for muon generation and acceleration based on plasma wakefield acceleration [25–27]. In LWFA, a spatial distribution of muons according to their initial energies in the bubble can solve the phase slippage [25]. A combination of Gaussian lasers and flying-focus lasers [28] can achieve phase-locked and acceleration of low-energy muons. In addition, a scheme of LWFA ion acceleration with a transverse flying focus might also be utilized to accelerate muons [29]. For PWFA, ultrahigh gradients enable rapid muon acceleration to relativistic velocities [30]. Plasma density down-ramps can reduce wake phase velocity, thereby lowering the energy capture threshold for low-energy muons [31].

These current schemes do not comprehensively elucidate the muon injection process. The capture of low-energy muons necessitates their acceleration to near-light speed within an extremely short timescale, rendering their injection into the wakefield nearly unfeasible [32]. Transverse injection imposes exceedingly stringent phase-matching requirements, demanding precise injection into a specific phase on the micrometer and femtosecond scales [33], while low-energy muons rapidly dephase and fail to synchronize with the phase velocity of the wakefield. Additionally, in electron-driven plasma wakefields, the first half-cycle constitutes a decelerating field [34], further complicating longitudinal injection. Furthermore, the acceleration of muons from realistic sources, which possess an initial energy spread and emittance, and the achieved energy gains fall short of collider requirements [2].

In this article, we propose using an ultraintense Gaussian beam and a plasma density up-ramp to achieve precise

longitudinal injection of low-energy muons, a process whereby muons are captured from the front of the plasma bubble and slipped into the accelerating phase. Muons decelerate at a lower wakefield strength in the low-density region and then rapidly accelerate at an extremely high wakefield strength in the high-density region during muon longitudinal injection. By precisely controlling the center position of plasma density up-ramp, muons can transit from the deceleration phase to the acceleration phase at the optimal point. Furthermore, using quasi-three-dimensional PIC simulations, we found key injection-related factors, including the dependence of the minimum injectable muon energy on driver charge, and the correlation between the up-ramp center position and the injectable energy range. The injection process of muons with initial emittance was also conducted via three-dimensional PIC simulations and the final acceleration results of muon beams are presented.

II. THEORETICAL ANALYTIC

Under strong nonlinear effects, the electric field $E(\xi)$ can be approximately described as a triangular distribution [20,35], where $\xi = z - ct$ is the coordinate of muons in the moving frame.

$$E(\xi) = \begin{cases} 0, & \text{if } \xi < \xi_0, \\ -E_0 + E_0 \frac{(\xi - \xi_0)}{(\xi_1 - \xi_0)}, & \text{if } \xi_0 \leq \xi \leq \xi_1, \\ 0, & \text{if } \xi > \xi_1, \end{cases} \quad (1)$$

where ξ_0 and ξ_1 represent the longitudinal positions of the two ends of the accelerating field, respectively, E_0 is the peak amplitude of longitudinal electric field strength. Then the electric potential ϕ is calculated via $\phi(\xi) = \phi(-\infty) - \int_{-\infty}^{\xi} E(\xi) d\xi$ as

$$\phi(\xi) = \begin{cases} -\frac{E_0}{2}(\xi_1 - \xi_0), & \text{if } \xi < \xi_0, \\ E_0(\xi - \xi_0) - \frac{E_0}{2(\xi_1 - \xi_0)}(\xi - \xi_0)^2 - \frac{E_0}{2}(\xi_1 - \xi_0), & \text{if } \xi_0 \leq \xi \leq \xi_1, \\ 0, & \text{if } \xi > \xi_1. \end{cases} \quad (2)$$

Here, $\phi(\xi > \xi_1)$ is set to zero. The motion of a muon in the plasma wakefield is determined by the conserved Hamiltonian [36]

$$H(\xi, p_z) = \sqrt{1 + p_z^2} - p_z - \Phi(\xi), \quad (3)$$

where normalized electric potential $\Phi(\xi) = e\phi(\xi)/m_\mu c^2$, p_z is normalized to $m_\mu c$, $H(\xi, p_z)$ is normalized to $m_\mu c^2$. Utilizing Hamiltonian dynamics to analyze the trapping and acceleration of charged particles can be used to explore nonlinear plasma waves [32]. Without losing generality, the relevant parameters can be set as: $\xi_0 = -40 \mu\text{m}$,

$\xi_1 = 40 \mu\text{m}$, $E_0 = 1 \text{ TV/m}$, then the profile of $E_z(\xi)$ and potential $\Phi(\xi)$ are shown in Fig. 1(a). In the region $\xi > \xi_1$, where the potential is zero, the Hamiltonian is entirely governed by the initial longitudinal momentum p_z . On the other hand, when $p_z \gg 1$, the Hamiltonian is predominantly determined by the potential Φ . Figure 1(b) shows the contour plot corresponding to initial longitudinal momentum p_z ranging from 0 to 10, which represents muon trajectories with initial energies between 0 and 1.05 GeV. It can be observed that, under this idealized initial condition, muons with initial momenta greater than 1.15 (i.e., energies exceeding 121 MeV) can be captured by the wakefield and continuously accelerated to $p_z > 50$.

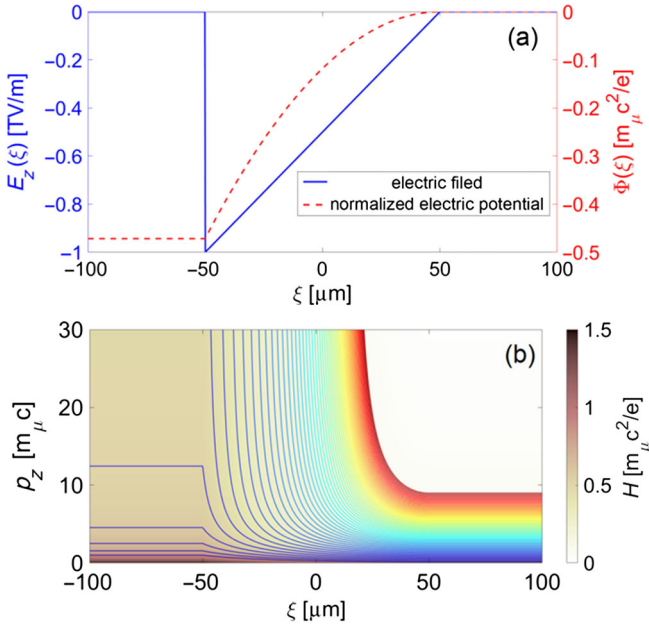


FIG. 1. (a) The red dashed line represents the normalized electric potential $\Phi(\xi)$ while the blue solid one illustrates the longitudinal electric field $E(\xi)$. (b) The background with brown color map renders the Hamiltonian value $H(\xi, p_z)$. The curves with the rainbow colorcode, representing contours of the Hamiltonian value, exhibit the evolution of muons in $\xi - p_z$ phase space for a range of initial longitudinal momentum p_z ranging from 0 to 10.

This result demonstrates that, when the peak gradient of the wakefield exceeds the TV/m level, the capture of low-energy muons is feasible. The capture and injection of muons are completed within tens of picoseconds, and muon decay does not affect the process.

Varying the peak accelerating field E_0 alters the minimum energy required for muon injection. We fix the initial muon energy at 300 MeV, corresponding to $p_z = 2.86$ and an initial Hamiltonian of 0.17. We consider that when the muon energy reaches 1.5 GeV (corresponding to $p_z = 14.3$), it can be injected if its position is located at least 2.5 μm ahead of the peak gradient. This condition can be expressed mathematically as

$$H(\xi = -35 \mu\text{m}, p_z = 14.3) > 0.17, \quad (4)$$

it follows that to guarantee the capture of muons with an initial energy of 300 MeV, the peak accelerating field E_0 must exceed 0.4 TV/m [37].

In practice, muons cannot be precisely injected at the front of the accelerating field and directly accelerated. For longitudinal injection, they typically experience the decelerating field in the first half of the bubble. Therefore, the residual energy of the muon after experiencing the decelerating field plays a critical role in determining whether it can be captured by the accelerating phase of the wakefield for further acceleration. According to Eq. (3), the initial

Hamiltonian of the muon is positive, so muon capture can only occur if the potential satisfies $\phi_{\min} < 0$. In PWFA, the transformer ratio R , defined as the ratio of the decelerating field experienced by the driver to the accelerating field experienced by the witness beam [38], is fundamentally limited to $R < 2$ [39] in the case of a longitudinally symmetric driver. We consider the case with a transformer ratio $R = 2$, where the decelerating field has a sinusoidal profile, with a length twice that of the accelerating field. This configuration is consistent with the typical wakefield structure in PWFA. The calculation shows that $\phi_{\min} > 0$, which implies that the muons could not be captured. Therefore, it is necessary to reduce the strength of the decelerating field in order to achieve $\phi_{\min} < 0$, enabling effective capture.

III. INJECTION DYNAMICS

In order to enable the capture and injection of low-energy muons with energies around 300 MeV produced by laser-target interactions [12–14], it is necessary to further reduce the energy loss experienced by muons in the decelerating region during the longitudinal injection. Unlike the traditional approaches that use a plasma density down-ramp to control the phase velocity of the wakefield and reduce the dephasing effect [40,41], our scheme employs a plasma density up-ramp to separate the deceleration and acceleration regions, where muons are first decelerated in the low-density region and then rapidly accelerated in the high-density region.

We conducted full three-dimensional PIC simulations using the PIC simulation software WarpX [42]. To eliminate the effects of numerical Cherenkov instability, we used the FDTD Maxwell solver and activated the FDTD numerical Cherenkov instability corrector [43]. The simulation box $x \times y \times z = 800 \mu\text{m} \times 800 \mu\text{m} \times 750 \mu\text{m}$ was divided into $256 \times 256 \times 512$ cells, with a moving window in the z direction. The axial coordinate ξ in the moving computational domain can be written as $\xi = z - ct$. A Gaussian electron driver with a peak density of $7 \times 10^{25} \text{m}^{-3}$, transverse size $\sigma_r = 8 \mu\text{m}$, longitudinal size $\sigma_z = 15 \mu\text{m}$, and centered at $\xi = 0 \mu\text{m}$ was employed. The driver parameters used in the simulations align with the current direction of cutting-edge technological progress and are within reach in a foreseeable future [44–46]. The 300 MeV muon beam with a density of $1 \times 10^{21} \text{m}^{-3}$ is uniformly distributed longitudinally at $z = 94 - 98 \mu\text{m}$. The plasma density remains constant at $1 \times 10^{23} \text{m}^{-3}$ in the region where z is less than 2.25 mm, increases linearly to $7 \times 10^{23} \text{m}^{-3}$ between 2.25 and 2.35 mm, and stays constant at $7 \times 10^{23} \text{m}^{-3}$ in the region where z is more than 2.35 mm, as illustrated by the black line in Fig. 3(a). It is worth pointing out that the muon beam does not contribute to the formation of strongly nonlinear wakefields.

Figure 2 shows the plasma bubble and longitudinal electric field in the low- and high-density regions. Under

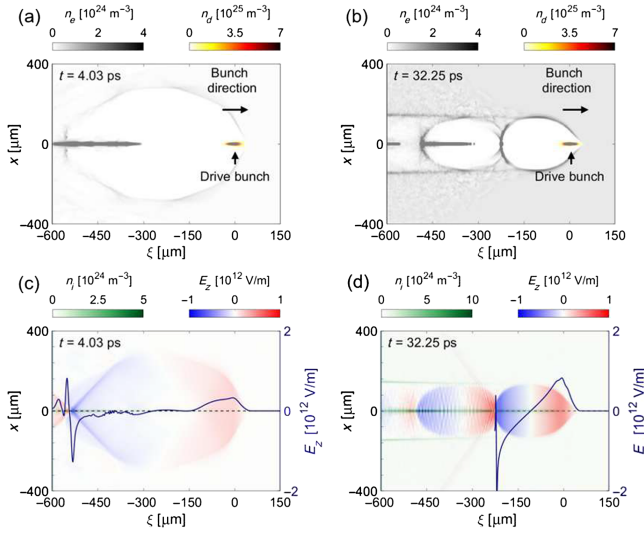


FIG. 2. 3D-PIC simulations of an electron beam driving a nonlinear wakefield in a plasma. (a) and (b) show the density distributions of the driver and plasma electrons during the injection at $t = 4.03$ ps and $t = 32.25$ ps, respectively. Panel (a), the driver is in the low plasma density region. Panel (b), it is in the high plasma density region. (c) and (d) show the longitudinal electric field distribution in the x - ξ plane and on the axis (dark blue curve), as well as the plasma ion density distribution at $t = 4.03$ ps and $t = 32.25$ ps, respectively.

strongly nonlinear conditions, a large amount of plasma electrons, on the order of tens of nC, can be injected into the bubble [47,48]. This induces a substantial beam-loading effect [49], which strongly modifies the original wakefield and adversely impacts muon acceleration. The ultrahigh-density driver attracts background ions in the plasma toward the axis [50–52], leading to a departure from the traditional electron bubble structure, as shown in Figs. 2(c) and 2(d). This mechanism provides a focusing force for muons throughout the bubble, enabling them to be safely decelerated first and then rapidly accelerated into the acceleration phase without being expelled.

To further analyze the dynamic process of muon longitudinal injection, we conducted single-particle simulations, as shown in Fig. 3(a). The driver and plasma density are configured as described previously, with the initial position of the muon set at $x = 17$ μm , $y = -127$ μm , and $z = 98$ μm . Before a propagation distance of 1.6 mm, the muon is unaffected by the nonlinear wakefield. From 1.6 to 2.3 mm, it experiences a decelerating field in the low-density plasma, reducing its energy from 300 to 117 MeV. As the longitudinal force reverses direction and grows in magnitude, the muon subsequently gains energy, being rapidly accelerated to 1.7 GeV over the next 1.6 mm, after which stable acceleration is sustained. A wakefield amplitude of $E_z > 1$ TV/m permits phase matching within the dephasing length and initiates sustained acceleration of muons at the bubble's accelerating edge with phase stability. Without the plasma density up-ramp, the muon

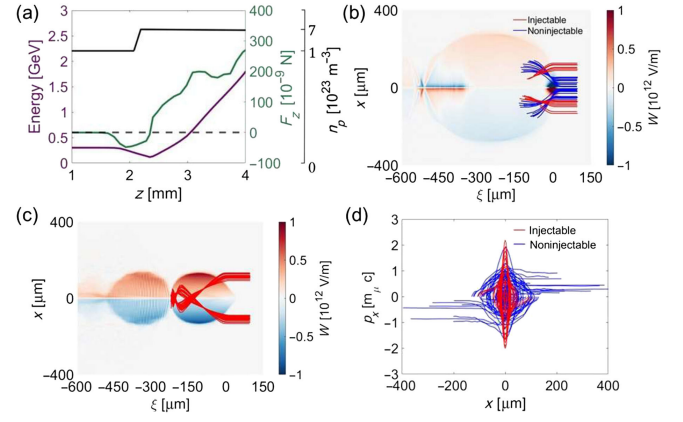


FIG. 3. (a) The black line depicts the distribution of plasma density as a function of direction of driver propagation distance. The green and purple curves show the longitudinal electric field force experienced by the muon and its resulting energy evolution. (b) Transverse wakefield distribution and trajectories of injectable (red) and noninjectable (blue) muons. (c) Transverse wakefield distribution and trajectories of injectable (red) muons. (d) Phase space trajectories of injectable (red) and noninjectable (blue) muons.

would be decelerated directly to 25.7 MeV. Due to the excessively low energy after deceleration, injection cannot be achieved. For a monoenergetic muon beam, based on the initial phase and relative velocity of the muon, the most optimal central position z_0 of the plasma density up-ramp can be calculated to ensure that the plasma density rises to a high value exactly when the muon enters the acceleration region.

Figures 3(b) and 3(c) visualize x - ξ plane spatial trajectories of the initially nonrelativistic muons. As illustrated in Fig. 3(b), the near-axis muons undergo defocusing in the transverse wakefield during the propagation in the low plasma density, leading to radial expulsion beyond the radius of the high-density bubble and injection loss. In contrast, the off-axis muons experience focusing fields, achieving radial convergence. After the transition to high plasma density, the transverse focusing strength increased, enabling muons to focus on the axis. Figure 3(d) shows the trajectory of the muons in the transverse phase space. Injectable muons form a continuously rotating ellipse in the transverse phase space. Muons that cannot rotate in phase space would directly exit the plasma channel, failing to be injected.

IV. INFLUENCE ANALYSIS

We conducted quasi-three-dimensional PIC simulations of the factors influencing injection. In the plasma wakefield acceleration theory, the driver charge Q_d is a key parameter governing the wakefield strength. Based on the nonlinear wakefield theoretical model, the maximum wake amplitude $E_{z\text{max}}$ can be further quantified as [53]

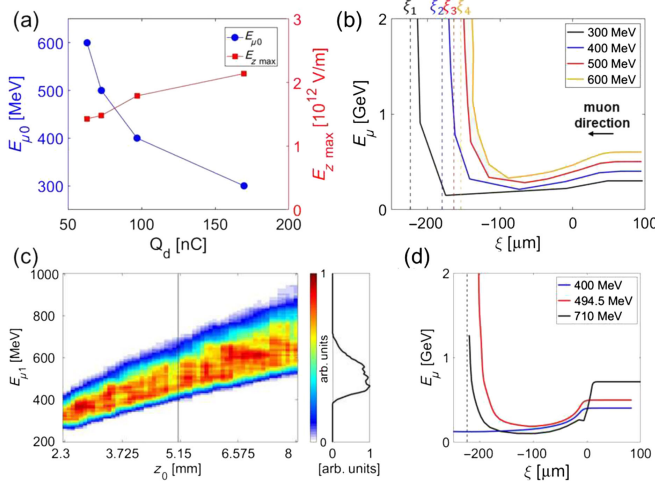


FIG. 4. (a) Scaling of $E_{\mu 0}$ (blue curve), $E_{z \max}$ (red curve) with driver charge Q_d obtained from simulations. (b) The energy evolution during injection for muon beams with initial energies of 300 MeV (black curve), 400 MeV (blue curve), 500 MeV (red curve), and 600 MeV (yellow curve), each at its optimal z_0 and Q_d . Dashed lines of different colors indicate the boundary of the accelerating regions for different Q_d . (c) A two-dimensional spectrum formed by the collection of $E_{\mu 1}$ corresponding to different values of z_0 . The black curve in the right frame shows the $E_{\mu 1}$ at $z_0 = 5.10$ mm (black dotted line). (d) Energy evolution of a single muon energy below (blue curve), within (red curve) and above (black curve) the injectable energy range at $z_0 = 5.10$ mm. The black dashed line is the boundary of the accelerating region.

$$\frac{eE_{z \max}}{mc\omega_p} \approx 1.3 \frac{q n_b}{e n_p} k_p^2 \sigma_r^2 \ln \left(\frac{1}{\sqrt{\Lambda}/10} \right), \quad (5)$$

where n_p is the plasma density, n_b is the electron-driven beam density, and $\Lambda \equiv \left(\frac{n_b}{n_p}\right) k_p^2 \sigma_r^2$ is the normalized charge per unit length of the beam. Simulations show that at a plasma density of $n_p = 7 \times 10^{23} \text{ m}^{-3}$, the maximum accelerating field for negatively charged particles $E_{z \max}$ is proportional to the driver charge Q_d , obtained by varying n_b as indicated by the red curve in Fig. 4(a), which is consistent with theory.

Under the condition of fixed plasma densities both before and after the density up-ramp region, the minimum injectable muon energy $E_{\mu 0}$ is determined by Q_d . The results show that as Q_d increases from 62 to 170 nC while $E_{\mu 0}$ decreases from 600 to 300 MeV, as clearly illustrated by the blue curve in Fig. 4(a). Corresponding to the four cases marked in Figs. 4(a) and 4(b) demonstrates the evolution of muons' energy through the injection processes. The dashed lines of different colors correspond to the acceleration boundaries ξ_k of the nonlinear wakefield. A large velocity mismatch in low-energy muons consequently demands a steeper longitudinal electric field and longer dephasing length for injection.

It is necessary to pinpoint the range of injectable energies at a fixed z_0 considering the injection of muons with

an initial energy spread. Figure 4(c) presents a one-dimensional spectrum formed by the collection of injectable energy ranges of muons $E_{\mu 1}$ corresponding to different values of z_0 . These ranges are obtained from an ensemble of simulations scanning z_0 at fixed intervals. As an example, the black curve in the right frame in Fig. 4(c) shows the injectable energy spectrum of $z_0 = 5.10$ mm that spans approximately 400–700 MeV. To further analyze the energy range of injectable muons, three single particles under identical conditions with energies below, within, and above this injectable energy range were simulated, as shown in Fig. 4(d). Muons below the injectable energy range reach the boundary prematurely and escape capture. Those within the injectable energy range synchronize perfectly with the wakefield, gaining 1.5 GeV and meeting the trapping condition as the plasma density ramps up. In contrast, muons above the injectable energy range experience prolonged deceleration due to a smaller relative speed difference. When the density changes at z_0 , they lose too much energy to be injected. For the injection of broad-spectrum muons, the injection efficiency is highest when the muon energy range matches z_0 . Simultaneously, the injection efficiency of broad-spectrum muons is also influenced by their initial density distribution.

We investigated the injection of muons with initial emittance through three-dimensional PIC simulations. Their initial phase-space distribution was defined as uniform over a transverse position range between -500 and $500 \mu\text{m}$, and a divergence angle range between -15° and 15° , considering an emittance similar to that of realistic muon sources [13]. Simulation results show that the transverse position distribution of injectable muons spans the full initial range, while their angular distribution is confined between -10° and 10° . This injectable phase-space distribution is predominantly concentrated and exhibits a left-handed ellipse, as shown in Fig. 5(a). Crucially, particles with large divergence angles undergo transverse oscillations with amplitudes exceeding the plasma bubble radius, which could not be effectively focused by the transverse field and were consequently lost. The evolution of the normalized root-mean-square (rms) transverse emittance ϵ_x and ϵ_y of the injectable muons during

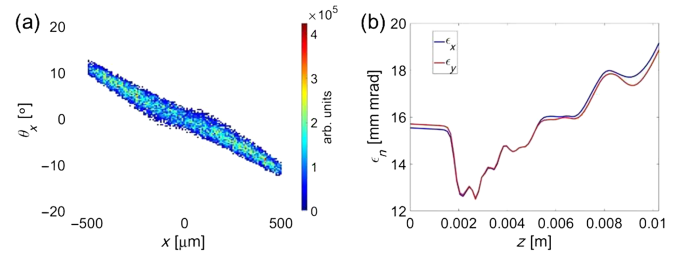


FIG. 5. (a) Initial divergence angle distribution of injectable muons. (b) The rms transverse emittance of the injectable muons during the injection process.

the injection process is shown in Fig. 5(b), where ϵ_x is defined as [54] $\epsilon_x = \frac{1}{m_{e,x}} \sqrt{\langle x^2 \rangle \langle p_x^2 \rangle - \langle x p_x \rangle^2}$, with x and p_x being the transverse position and momentum of a witness particle, respectively. The emittance first decreases as the muons' initially large transverse position is reduced upon injection, and then increases during the acceleration process. The strong focusing fields intrinsic to the plasma wakefield allowed muon injection within a limited transverse emittance acceptance, which relaxes the transverse cooling constraints required for the accelerated muon beam [3,4,55].

V. ACCELERATION RESULT

The initial energy spread and emittance affect only the injection efficiency but not the subsequent acceleration after injection. Therefore, we employed muons with zero initial energy spread and emittance as the injection source to calculate the ensuing acceleration results. Under the condition of a 23 GeV ultraintense electron beam driver, the three-dimensional PIC simulations show that the average muon energy increases from 300 MeV to 40 GeV, with an 18.85% energy spread and an 8.86 mm mrad normalized emittance, as presented in Figs. 6(a) and 6(b). The energy spectrum broadens due to muons' phase slip in the relatively small acceleration region, as shown in Fig. 6(c). The maximum energy of muons is determined by their injection and acceleration phases, which depend on the duration of deceleration and acceleration that the muons experience during injection. Direct factors influencing this process include z_0 and the initial energy of the muon beam. Due to the limitations of low-charge muons, the muon acceleration efficiency is 0.02%. In order to achieve a higher single-stage energy gain for a compact muon accelerator, a driver with higher energy can be employed. Our simulations

establish the relationship between the final muon energy and the initial driver energy. Specifically, when the driver energy is increased to 63 GeV, the final energy of the muon beam reaches 146 GeV, as shown in Fig. 6(d).

VI. CONCLUSION

In conclusion, we address the critical challenge of longitudinal injection of low-energy muons into the accelerating phase in PWFA by proposing a novel approach that utilizes an ultraintense electron driver and a plasma density up-ramp. This tailored profile spatially separates the deceleration and acceleration regions of the injected muons. This measure effectively reduces the energy loss experienced by muons in the decelerating region, enabling both successful injection and sustained acceleration of muons within a centimeter-scale plasma. The required charge of the driver drops to as low as 62 nC when the initial muon energy is increased to 600 MeV. Manipulation of z_0 provides active control for injecting muon beams with specific initial energies. Simulations confirm the successful injection and acceleration of muons with finite transverse emittance. When the driver energy is 23 GeV, a 300 MeV muon beam can be accelerated to 40 GeV. With a driver energy of 63 GeV, the muons can be accelerated to 146 GeV. The pathway to high-flux, high-energy muon beams involves continued progress in successful implementation of staging techniques.

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DATA AVAILABILITY

The data that support the findings of this article are not publicly available upon publication because it is not technically feasible and/or the cost of preparing, depositing, and hosting the data would be prohibitive within the terms of this research project. The data are available from the authors upon reasonable request.

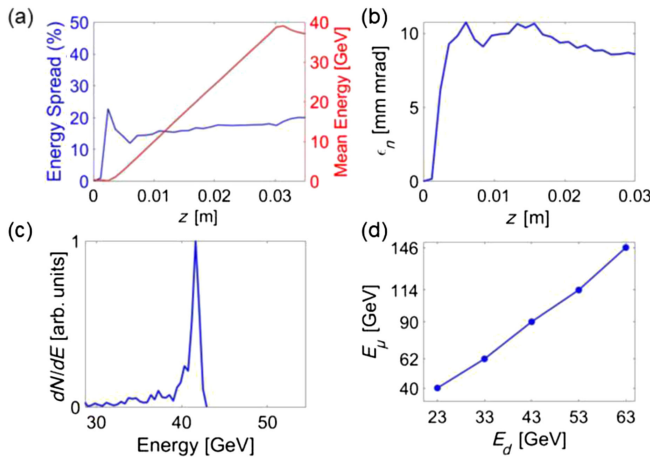


FIG. 6. At a driver energy of 23 GeV, (a) evolution of muons' energy and energy spread during injection and acceleration, (b) muons' emittance variation, (c) final muons' energy spectrum, and (d) scaling of final acceleration energy of muons with the energy of the driver.

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