

Embedded structure design of nanocrystalline soft magnetic alloy cores for high-power synchrotron rf cavities: Power density redistribution and thermal optimization

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Nanocrystalline soft magnetic alloy (MA) cores are essential magnetic loading components in proton/heavy-ion synchrotron rf cavities. While conventional MA cores fabricated from 16 μm ribbons offer reliable performance, further enhancing their shunt impedance and thermal stability to meet the demands of next-generation high-power synchrotrons presents significant challenges. Although the use of thinner ribbons (e.g., 13 μm) can improve high-frequency magnetic properties, their widespread application is hindered by manufacturing difficulties and reduced packing factor consistency. To address this, we propose an embedded MA core design that strategically integrates 13 μm ribbons in specific radial regions alongside normal 16 μm ribbons. This approach not only increases shunt impedance but also optimizes the electromagnetic field distribution, thereby reducing peak surface temperature and improving thermal stability. This paper first elucidates the physical mechanism through which the embedded structure improves temperature uniformity. Additionally, a two-dimensional heat transfer numerical model is developed and experimentally validated to assess the thermal performance of the embedded MA core under actual operating conditions. Experiments are conducted and the results demonstrate the embedded core's peak surface temperature is reduced by 2.5 $^{\circ}\text{C}$ compared to a normal MA core. Finally, parameter optimization of the embedded structure is conducted, demonstrating that optimizing the radial position, volume ratio, and thickness of thinner ribbons can achieve a temperature reduction of approximately 6 $^{\circ}\text{C}$ and a 12% impedance increase. The findings of this study provide a theoretical foundation for the performance optimization of MA cores in high-power synchrotrons.

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I. INTRODUCTION

Nanocrystalline soft magnetic alloys (MA)—with high saturation magnetic flux density [1,2], permeability, and broadband response—enable high acceleration gradients and stable wide-frequency operation without tuning. These properties establish them as the preferred magnetic loading material for radio frequency (rf) cavities in high-power proton/heavy-ion synchrotrons [3–8]. Table I summarizes the performance parameters of MA cores in typical synchrotron facilities.

As evidenced by Table I, some of proton/ion synchrotron facilities predominantly utilize large-diameter MA cores

(outer diameter: 800–900 mm) operating across a broad frequency range of 0.3–5 MHz to accommodate frequency-sweeping requirements during beam energy variation. Under such geometric scale and high-power-density operation, thermal management of MA cores has thus emerged as the primary technical bottleneck constraining further performance enhancement of rf systems.

Current MA-loaded cavities primarily employ large-scale MA cores fabricated from 16- μm -thick (average thickness, with fluctuations of approximately $\pm 1 \mu\text{m}$) nanocrystalline ribbons. The manufacturing protocol comprises three key steps: (1) applying 2- μm -thick insulating surface coatings to prevent interlayer breakdown and reduce eddy current losses, (2) transverse magnetic annealing under high-temperature conditions to enhance magnetic properties, and (3) precision finishing processes including curing and encapsulation [9]. Nevertheless, MA cores based on normal 16- μm -thick ribbons struggle to satisfy the escalating demands of next-generation high-power synchrotrons, which include simultaneous improvements in acceleration gradients and duty cycles, resulting in

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TABLE I. Technical parameters comparison of MA cores in typical synchrotron facilities [5–8].

Facility	Frequency (MHz)	Core size	Cooling method
J-PARC RCS (Japan)	0.94–1.67 (fundamental), 1.88–3.34 (second)	Ø854 × Ø371 × 25 mm	Water
J-PARC MR (Japan)	1.67–1.72 (fundamental), 3.34–3.44 (second)	Ø800 × Ø245 × 25 mm	Water
CSNS RCS (China)	1.02–2.44 (fundamental), 2.04–4.88 (second)	Ø850 × Ø316 × 25 mm	Water
CERN LEIR (Switzerland)	0.7–3.5	Ø670 mm	Water
HIAF BRing (China)	0.32–2.10	Ø660 × Ø290 × 25 mm	Oil
GSI SIS12/18 (Germany)	0.43–2.80	Ø660 × Ø295 × 25 mm	Oil

higher power density in the core. These requirements necessitate substantial advancements in the thermal stability of MA cores.

A critical performance parameter that requires significant enhancement is the shunt impedance of MA cores. Higher shunt impedance directly contributes to reduced power dissipation, which in turn leads to better temperature control and improved operational reliability under high-power conditions. This improvement is particularly crucial for next-generation synchrotrons where thermal management becomes increasingly challenging.

Another critical issue that needs to be addressed is the localized overheating in MA cores fabricated from uniform-thickness ribbons. Based on coaxial transmission line theory, the magnetic flux density distribution in such cores follows an inverse proportional relationship $B(r) \propto 1/r$, where r is the radial coordinate. This nonuniform distribution results in a power loss density $P(r) \propto B(r)^2 f$ (with f as frequency) predominantly concentrated in the inner diameter region. Consequently, this concentration generates substantial temperature gradients and localized thermal stress concentrations [10]. Such thermal inhomogeneity restricts the core's performance under high-power conditions.

A theoretically promising solution to these challenges involves utilizing thinner nanocrystalline ribbons for MA core fabrication. Theoretical analysis indicates that employing thinner ribbons (e.g., 13 μm in thickness) is an effective approach to improving shunt impedance, as reducing the ribbon thickness enhances material permeability and mitigates eddy current losses [11]. However, this approach is constrained by three critical limitations in practical fabrication processes. (1) Stringent dimensional requirements: thinner ribbons demand stringent quality control, with flatness, warpage, and thickness uniformity must be maintained within a tight tolerance of $\pm 1 \mu\text{m}$. Exceeding this tolerance can result in geometric deformation during large-core fabrication (outer diameter $> 500 \text{ mm}$) and even damage the integrity of interlayer insulation, thereby reducing product reliability. (2) Compromised packing efficiency: the stacking factor, simply defined as the ratio of ribbon thickness to the sum of ribbon thickness and coating thickness (2 μm), decreases from 0.88 to 0.86. This reduction in packing

density results in a 2.3% decrease in the effective volume fraction of magnetic material within the core, thereby partially negating the theoretical performance advantages associated with thinner ribbons. (3) Stress-induced performance degradation: experimental results indicate that the actual performance improvement of large-sized MA cores using 13 μm ribbons falls below theoretical expectations. This discrepancy primarily arises from the high mechanical stress sensitivity of 3 μm ribbon [12]. Subsequent processing steps, such as insulation coating and waterproof encapsulation, exacerbate performance degradation due to process-induced stress, ultimately limiting the full realization of 13 μm ribbons' potential advantages.

These challenges have motivated our research to develop a new practical solution. We propose a radial-embedded core structure that strategically integrates 13- μm -thick ribbons in specific radial regions while utilizing normal 16- μm -thick ribbons in other areas.

The embedded structure offers two significant advantages. First, the localized integration of thinner ribbons enhances the overall impedance of MA cores through improved permeability and reduced eddy current losses. Second, both theoretical analysis and numerical simulations demonstrate that the strategic placement of thinner ribbons modifies the conventional magnetic flux density distribution. This modification effectively mitigates thermal power concentration in the inner diameter region, resulting in a substantial reduction in peak surface temperature and improved thermal uniformity across the core.

In this study, we conduct a comprehensive investigation of multidimensional performance optimization for embedded MA core structure through theoretical analysis, numerical simulations, and experimental validation. The paper is organized as follows: Sec. II establishes a theoretical framework to analyze the mechanisms by which thinner ribbons enhance shunt impedance and redistribute power density in MA cores. Section III introduces high-power performance testing protocols for MA cores, with experimental results validating both the simulation accuracy and theoretical model effectiveness. Section IV conducts systematic parameter scanning and optimization analysis for multiple embedded core configurations based on the established theoretical model, providing practical design guidelines for engineering applications. Finally, the

conclusion summarizes key findings, discusses potential applications, and outlines future research directions.

II. THEORETICAL FOUNDATION

A. Relationship between magnetic permeability of MA materials and ribbon thickness

The magnetic properties of nanocrystalline soft MA under rf conditions are predominantly characterized by the complex permeability, expressed as $\mu_r = \mu'_s - j\mu''_s$, where the subscript s denotes the equivalent series circuit model. The real component, μ'_s , signifies the relative permeability, which quantifies the material's magnetization capability, whereas the imaginary component, μ''_s , represents the magnetic loss properties of the material.

In rf applications, the magnetic performance of MA cores is substantially influenced by eddy current effects within the ribbons. These effects are intricately linked to both the thickness of the ribbons and the operational frequency. Based on the theory of eddy current effects, Grandi *et al.* proposed a theoretical expression for the complex permeability of nanocrystalline ribbons without specific magnetic field annealing [13]:

$$\begin{aligned}\mu'_s &= \mu_i \frac{\delta \sinh(d/\delta) + \sin(d/\delta)}{d \cosh(d/\delta) + \cos(d/\delta)}, \\ \mu''_s &= \mu_i \frac{\delta \sinh(d/\delta) - \sin(d/\delta)}{d \cosh(d/\delta) + \cos(d/\delta)},\end{aligned}\quad (1)$$

where d represents the ribbon thickness, and μ_i denotes the dc permeability. The parameter δ corresponds to the skin depth, mathematically expressed as

$$\delta = \sqrt{\frac{2\rho}{\mu_0\mu_i\omega}}. \quad (2)$$

Here, ρ denotes the electrical resistivity of the ribbon, and ω represents the angular frequency. This theoretical model demonstrates good consistency with experimental measurements obtained from unannealed magnetic cores, thereby establishing a significant theoretical foundation for analyzing the impact of eddy current effects on the magnetic properties of nanocrystalline materials.

However, the pursuit of higher acceleration gradients and improved power efficiency of rf system necessitates MA cores with superior permeability and quality factors. To meet these stringent requirements, transverse magnetic annealing has emerged as a widely adopted technique. This specialized heat treatment process involves the application of a static magnetic field along the ribbon's transverse (width) direction. The fundamental mechanism of this process is to optimize the magnetic domain structure by inducing magnetic anisotropy. It can effectively suppress irregular motion of magnetic domains and reduce the

resistance to domain wall movement, consequently leading to a significant reduction in high-frequency losses and a substantial enhancement of overall magnetic performance [14]. Experimental investigations have demonstrated that transverse magnetic annealing is an effective process for increasing the permeability of nanocrystalline ribbons [15].

Although transverse magnetic annealing has achieved remarkable performance enhancement, the establishment of a comprehensive theoretical model accurately characterizing the relationship between permeability and ribbon thickness remains elusive. This challenge primarily stems from the intricate interplay of multiple factors, including the magnetic anisotropy field, magnetic domain structure modifications, and stress distribution.

From a theoretical perspective, the transverse magnetic treatment primarily modifies the material's magnetic domain configuration and magnetic anisotropy parameters, while the basic physical principles of eddy current effects as electromagnetic induction phenomena remain fundamentally unaltered by these structural changes. Consequently, we propose that the thickness dependence of permeability in transversely annealed ribbons should still fundamentally adhere to the $1/d$ scaling law associated with eddy current effects.

To validate this hypothesis, we conducted a systematic statistical analysis of thickness distribution characteristics and corresponding magnetic performance parameters for 60 large-sized transversely annealed MA cores (outer diameter: 850 mm, inner diameter: 316 mm, thickness: 25 mm). The thickness distribution characteristics of these ribbons mainly stem from the dispersion in the manufacturing process.

The average ribbon thickness of each core was determined through precise measurements of radial resistance and mass, with the detailed calculation method described in [11]. Figure 1 presents the statistical distribution of ribbon

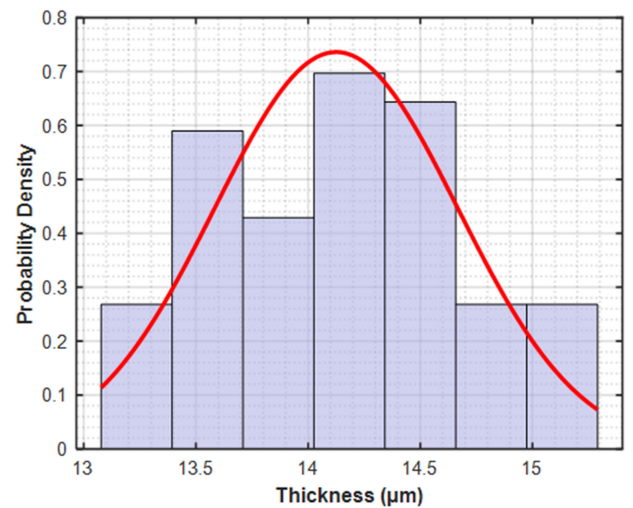


FIG. 1. Statistical characteristics of nanocrystalline ribbon thickness distribution.

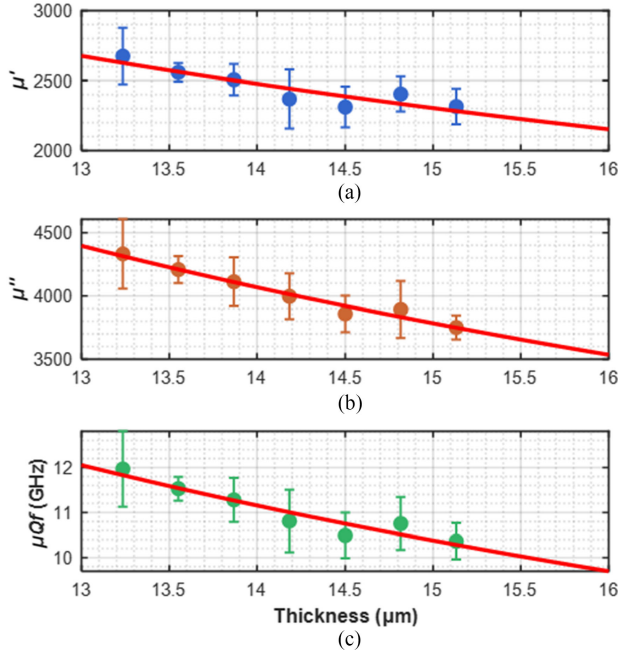


FIG. 2. Correlation analysis between magnetic parameters [(a) μ'_s , (b) μ''_s , and (c) μQf] and ribbon thickness. Measurements are represented by discrete circles, while the solid lines correspond to the theoretical $1/d$ relationship fitting.

thickness, exhibiting a characteristic Gaussian distribution with a peak thickness of 14.2 μm .

To rigorously characterize the thickness distribution, we conducted a Kolmogorov-Smirnov normality test, which yielded a statistically significant p value exceeding 0.05 ($p > 0.05$). This result demonstrates that the thickness distribution follows a Gaussian pattern, indicating that manufacturing variations primarily stem from random process fluctuations rather than systematic errors.

Based on the thickness statistical results, the cores were grouped at 0.5- μm thickness intervals, with the average magnetic performance parameters of each group measured at 2 MHz. This specific frequency was selected as it represents a critical operational point within the typical operating range of proton synchrotrons, as shown in Table I. The measurement results are presented in Fig. 2. In addition to analyzing the real component μ'_s and imaginary component μ''_s of permeability, particular attention was given to the μQf parameter, as it directly determines the core's shunt impedance characteristics [15].

We employed a nonlinear fitting approach using the function $y = a/x + b$ to analyze the measurement data, with detailed fitting results presented in Table II. The fitting coefficients for all three magnetic performance parameters exceed 0.8, which demonstrates that both μ'_s and μ''_s , as well as μQf parameter, exhibit a clear inverse correlation with ribbon thickness in transversely annealed nanocrystalline ribbons. These findings provide strong experimental validation for our theoretical framework.

TABLE II. Nonlinear fitting coefficients for magnetic parameters versus nanocrystalline ribbon thickness.

Parameters	a	b	R^2
μ'_s	36298.985	-115.793	0.823
μ''_s	59735.860	-198.713	0.969
μQf	163.334	-0.511	0.907

Utilizing this fitting function, we can obtain the magnetic characteristic parameters for the required ribbon thickness for embedded core analysis. For example, at an operating frequency of 2 MHz, the real and imaginary components of permeability for ribbon thicknesses of 13 and 16 μm are calculated as {2677.223, 4396.353} and {2152.890, 3534.778}, respectively. Correspondingly, the μQf values are determined to be 12.103 and 9.697 GHz, respectively.

B. Equivalent circuit model of embedded core

In rf systems, the power source delivers rf current into the acceleration gap, which subsequently propagates through the MA cores via the vacuum beam pipe, thereby generating electromagnetic fields within the core. Based on the principles of coaxial transmission line theory, the MA core can be modeled as a continuously distributed transmission line structure with radial symmetry. According to Ampère's circuital law, the magnetic field intensity at a radial position r within the toroidal core geometry is expressed as

$$H(r) = \frac{I}{2\pi r}. \quad (3)$$

The corresponding magnetic flux density is

$$B(r) = \mu_0 \mu_r(r) H(r) = \frac{\mu_0 \mu_r(r) I}{2\pi r}. \quad (4)$$

The total magnetic flux is obtained by radial integration:

$$\Phi = \int_{r_{\text{in}}}^{r_{\text{out}}} B(r) h \, dr = \frac{\mu_0 I h}{2\pi} \int_{r_{\text{in}}}^{r_{\text{out}}} \frac{\mu_r(r)}{r} \, dr. \quad (5)$$

For an embedded core comprising n regions with distinct permeability parameters along the radius, the integration can be decomposed into the sum of contributions from each region:

$$\Phi = \frac{\mu_0 I h}{2\pi} \sum_{i=1}^n \mu_{ri} \ln\left(\frac{r_{i+1}}{r_i}\right) = I \sum_{i=1}^n L_i, \quad (6)$$

where the inductance of each region is expressed as

$$L_i = \mu_{ri} L_{0,i}, \quad L_{0,i} = \frac{\mu_0 h}{2\pi} \ln\left(\frac{r_{i+1}}{r_i}\right), \quad (7)$$

The total inductance is the summation of the inductances of each region. For materials with complex permeability $\mu_r = \mu'_r - j\mu''_r$, the complex impedance of each region is given by

$$Z_i = j\omega L_i^* = \omega\mu''_{ri}L_{0,i} + j\omega\mu'_{ri}L_{0,i} = R_i + j\omega L_i. \quad (8)$$

Consequently, the total impedance corresponds to the series combination of the impedances of each region:

$$Z_{\text{total}} = \sum_{i=1}^n Z_i. \quad (9)$$

Based on the above series equivalent circuit model, under the excitation of a given total voltage V_{total} , the current flowing through the entire core is

$$I = \frac{V_{\text{total}}}{|Z_{\text{total}}|}. \quad (10)$$

According to the voltage division principle of series circuits, the voltage distribution in each region is given by

$$V_i = I \cdot |Z_i| = V_{\text{total}} \cdot \frac{|Z_i|}{|Z_{\text{total}}|}. \quad (11)$$

When a specific region incorporates thinner ribbons, both the real (μ'_{ri}) and imaginary (μ''_{ri}) components of permeability exhibit significant enhancement, leading to a corresponding increase in regional impedance $|Z_i|$. This results in a higher voltage share in the series voltage division.

C. Mechanism of magnetic flux density redistribution

The introduction of thinner ribbon regions within the embedded core induces a redistribution of the magnetic flux density throughout the core, which constitutes the fundamental physical mechanism for optimizing the core's power distribution. According to Faraday's law of electromagnetic induction, the induced voltage in each region is directly proportional to its magnetic flux:

$$V_i = j\omega\Phi_i. \quad (12)$$

Combining this relationship with the previously established magnetic flux expression $\Phi_i = \frac{\mu_0 h}{2\pi} \mu_{ri} I \ln\left(\frac{r_{i+1}}{r_i}\right)$, the magnetic field strength distribution in each region can be derived as

$$H_i = \frac{I}{2\pi r}, \quad B_i(r) = \mu_0 \mu_{ri} H_i = \frac{\mu_0 \mu_{ri} I}{2\pi r}. \quad (13)$$

The synergistic combination of two key factors—enhanced voltage allocation to the 13- μm ribbon region in the series circuit configuration and the higher complex permeability characteristics of 13- μm ribbons compared to

normal 16- μm ribbons—induces a significant magnetic flux density enhancement in the thinner ribbon region. This phenomenon manifests as a distinct magnetic flux density discontinuity, characterized by a sharp transition at the interface between different ribbon thickness regions.

As a result, the integration of thinner ribbons modifies the characteristic magnetic flux density distribution in MA cores fabricated from uniform-thickness ribbons, where power losses are predominantly concentrated in the inner diameter region. This innovative approach provides a promising technical solution for enhancing temperature uniformity across MA cores.

D. Theory of power density redistribution

To establish a theoretical framework for converting flux density distribution to power density distribution, it is necessary to analyze the power loss mechanism through the principles of electromagnetic field theory. In magnetic materials, the volumetric power loss density $P_{\text{vol}}(r)$ is predominantly determined by magnetic losses, which can be expressed as

$$P_{\text{vol}}(r) = \frac{1}{2} \omega \mu_0 \mu''_r(r) H^2(r). \quad (14)$$

Substituting the magnetic field strength expression $H(r) = \frac{I}{2\pi r}$ in Eq. (3) into the above equation yields:

$$P_{\text{vol}}(r) = \frac{1}{2} \omega \mu_0 \mu''_r(r) \left(\frac{I}{2\pi r} \right)^2 = \frac{\omega \mu_0 \mu''_r(r) I^2}{8\pi^2 r^2}. \quad (15)$$

Considering the spatial variation of permeability parameters in the embedded core, the volumetric power density for each distinct region is given by

$$P_{\text{vol},i}(r) = \frac{\omega \mu_0 \mu''_{ri} I^2}{8\pi^2 r^2} = \frac{C_i}{r^2}, \quad (16)$$

where the power density coefficient C_i is defined as

$$C_i = \frac{\omega \mu_0 \mu''_{ri} I^2}{8\pi^2}. \quad (17)$$

It can be seen from Eq. (17) that the power density coefficient C_i includes two critical parameters: the material's imaginary permeability component μ''_{ri} and the excitation current I . In the embedded core configuration, the thinner ribbon region has two distinguishing characteristics: (1) enhanced imaginary permeability and (2) potentially stronger localized excitation current, according to the voltage distribution law in Eq. (11). The superposition of these two effects significantly amplifies the power density coefficient C_i in this region, resulting in a steplike phenomenon in power density at the interface between the thinner ribbon and the normal ribbon. The shunt power

of each region can be obtained through radial integration of the power density:

$$P_{\text{shunt},i} = \int_{r_i}^{r_{i+1}} P_{\text{vol},i}(r) \cdot 2\pi r h \, dr = 2\pi h C_i \ln\left(\frac{r_{i+1}}{r_i}\right). \quad (18)$$

The core's overall power represents the summation of individual regional contributions:

$$P_{\text{total}} = \sum_{i=1}^n P_{\text{shunt},i} = \sum_{i=1}^n \int_{r_i}^{r_{i+1}} P_{\text{density},i}(r) \cdot 2\pi r h \, dr. \quad (19)$$

When the power density coefficient C_i in the thinner ribbon region increases, the power of other regions must be adjusted accordingly to maintain the total power P_{total} constant. This redistribution mechanism effectively transfers power from both the inner and outer regions toward the central part of the core, thereby mitigating thermal concentration and enhancing temperature uniformity across the core structure.

E. Theoretical model verification and analysis

To validate the effectiveness of the embedded core in optimizing power density distribution, we conducted a comprehensive analysis of an embedded configuration comprising two distinct nanocrystalline ribbon thicknesses: 13 and 16 μm . The selected core geometric parameters are: 850 mm for outer diameter, 316 mm for inner diameter, and 25 mm for thickness, which is a classic size core product applied in current high-power accelerators.

In the embedded core design, the utilization of different ribbon thicknesses partitions the core into $n = 3$ distinct radial regions: the inner radius region R_0-R_1 (158–188 mm) employs 16- μm -thick ribbons, the intermediate region R_1-R_2 (188–250 mm) incorporates 13- μm ribbons, and the outer diameter region $R_2 \sim R_3$ (250–425 mm) maintains the use of 16- μm -thick ribbons. This configuration strategy aims to leverage the high permeability of 13- μm ribbons while simultaneously managing manufacturing costs and process complexity.

To conduct a systematic performance evaluation of the embedded core, we first perform a comparative analysis of the power density distributions between the embedded core and the normal core (fabricated from 16 μm ribbons) under identical voltage conditions. By incorporating the complex permeability parameters into Eqs. (8)–(15), we calculate the power density distribution characteristics for both configurations, as illustrated in Fig. 3. In the figure, the horizontal axis represents the core radius position, while the vertical axis denotes the power density.

The blue solid line in Fig. 3 depicts the power density distribution of the normal core under a voltage of 1735 V,

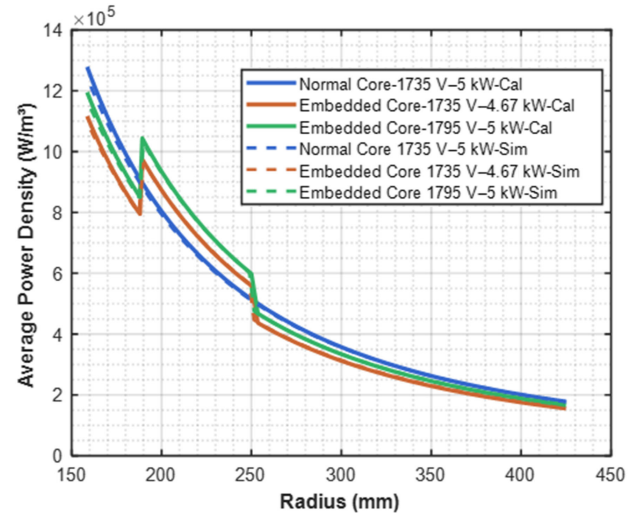


FIG. 3. Comparison and validation of power density distribution between theoretical model (solid lines) and electromagnetic simulation (dashed lines).

with a total power of approximately 5 kW. The result indicates that its power density strictly follows a $1/r^2$ distribution, resulting in an extremely high peak power density in the inner diameter region. In contrast, the embedded core (red solid line) exhibits a distinct power density jump phenomenon within the 13- μm ribbon region (R_1-R_2), while the peak power density in the inner diameter region is reduced by approximately 15%–20%. These findings align with the theoretical analysis presented in Sec. IID, confirming that the embedded structure effectively mitigates the power load concentration in the inner diameter region, thereby enhancing the radial temperature distribution uniformity of the core.

In addition, we increased the embedded core's excitation voltage to achieve a total power dissipation equivalent to that of the normal core (approximately 5 kW). The resulting power density distribution of the embedded core under this condition is shown by the green solid line in Fig. 3. The result reveals that the power density at the inner diameter ($r = 158$ mm) remains significantly lower than that of the normal core, with a reduction of approximately 7%–10%. It demonstrates that even under identical power loading conditions, the embedded structure effectively alleviates power concentration in the inner diameter region, a characteristic of substantial importance for enhancing the thermal performance and operational reliability of the core.

To further validate our theoretical model, we conducted a comparative analysis between the HFSS [16] electromagnetic simulation results (represented by dashed lines in Fig. 3) and the theoretical model calculations. The results demonstrate good agreement between the two approaches, thereby providing strong evidence for the validity and effectiveness of the proposed theoretical model.

From an application perspective, the power density distribution of the embedded core offers three key

advantages. First, the partial redistribution of power from the high-temperature inner diameter region to the middle diameter region contributes to reduced temperature gradients and improved thermal stress distribution. Second, the middle diameter region benefits from a larger heat dissipation surface area and more efficient thermal pathways, making the power transfer advantageous for overall thermal management. Finally, compared to cores utilizing exclusively 13- μm ribbons, the embedded core achieves comparable performance enhancements while significantly reducing the quantity of 13- μm ribbons required, thereby greatly alleviating the manufacturing difficulties.

While the optimization of power density distribution establishes a theoretical foundation for enhancing the thermal performance of the core, a comprehensive evaluation of the embedded core's practical application efficacy requires inputting the redistributed power density function as a heat source for temperature field analysis. Considering the complex anisotropic thermal conductivity characteristics of the core and the influence of various heat dissipation boundary conditions, it is necessary to develop a precise two-dimensional heat transfer numerical model to analyze the temperature response characteristics of the embedded core under different thinner ribbon configurations, thereby providing valuable guidance for optimized design.

III. TEMPERATURE DISTRIBUTION OF EMBEDDED MA CORES

To accurately assess the thermal performance of the embedded core under actual operating conditions, it is necessary to transform the power density distribution theory established in Sec. II D into specific temperature predictions. The temperature distribution within the core is intrinsically linked to its thermal stability and operational reliability during high-power operation, serving as a crucial metric for evaluating the optimization performance of the embedded structure. This section will develop a comprehensive heat transfer numerical model that accounts for the material's anisotropic characteristics, validate the model's accuracy through experimental verification, and conduct systematic optimization analysis of the embedded structure based on the validated model.

A. Anisotropic heat transfer numerical model

The manufacturing process of MA cores fundamentally determines their distinctive thermal conduction characteristics. During the ribbon winding process, the application of a 2- μm -thick SiO_2 insulating coating on the surface creates a significant thermal resistance layer, resulting in pronounced anisotropic thermal conductivity within the core. Based on experimental measurements, the thermal conductivity distribution of the core in the cylindrical coordinate system exhibits the following characteristics:

the thermal conductivity in the axial direction (z) and circumferential direction (θ) is 7.1 W/(m K), while that in the radial direction (r) is significantly reduced to 0.19 W/(m K) [17]. This substantial reduction in radial thermal conductivity, approximately 1/37 of the axial value, is primarily attributed to the thermal resistance effect generated by the interlayer insulating coating.

Given the rotational symmetry of the core structure, the temperature field is independent of the circumferential angle θ , permitting the use of a two-dimensional axisymmetric model for analysis. This simplification is not only physically reasonable but also significantly improves calculation efficiency, thereby facilitating subsequent parameter scanning and optimization analysis. The two-dimensional steady-state heat transfer control equation is expressed as

$$\frac{1}{r} \frac{\partial}{\partial r} \left(k_r r \frac{\partial T}{\partial r} \right) + \frac{\partial}{\partial z} \left(k_z \frac{\partial T}{\partial z} \right) + q(r, z) = 0, \quad (20)$$

where the heat source term $q(r, z)$ is determined according to the power density distribution theory established in Sec. II D. For the three-region embedded structure, the heat source distribution is given by

$$q(r, z) = \begin{cases} C_1/r^2, & r_{\text{in}} \leq r \leq r_1 \\ C_2/r^2, & r_1 < r \leq r_2 \\ C_3/r^2, & r_2 < r \leq r_{\text{out}} \end{cases} \quad (21)$$

Here, C_1 , C_2 , and C_3 represent the power density coefficients of each region, calculated by Eq. (17). The boundary conditions are specified as convective heat transfer on all external surfaces of the core:

$$-k_n \frac{\partial T}{\partial n} \Big|_{\text{surface}} = h(T_{\text{surface}} - T_{\text{ambient}}), \quad (22)$$

where h denotes the convective heat transfer coefficient, with typical values of approximately 5 W/(m² K) under natural convection conditions and approximately 500 W/(m² K) in a direct water-cooling environment [17].

B. Development of specialized numerical solution program

Given the requirement for systematic parameter scanning analysis of various embedded structure configurations in this study, which involves temperature field calculations for numerous r_1 and r_2 combinations (each configuration corresponds to a different multiregion power density distribution function), traditional commercial software exhibits significant limitations in handling such batch computations. The utilization of the commercial software necessitates repetitive modeling, mesh generation, and solution configuration, resulting in an inefficient workflow and inadequate flexibility to accommodate frequent

modifications to heat source boundary conditions and data visualization requirements.

To address these practical requirements, this study developed a specialized numerical solution program in MATLAB, specifically for the anisotropic heat transfer characteristics of cores [18]. The program employs the finite difference method for discretizing and solving the heat transfer equation, incorporating the following key technical features: (1) automatic reading of power density coefficients for each region with rapid heat source distribution reconstruction; (2) implementation of adaptive mesh optimization based on core geometric characteristics; and (3) support for batch parameter scanning calculations to enhance computational efficiency.

Considering the extremely high power density and temperature gradients in the inner radius region of the core, the mesh division strategy plays a crucial role in ensuring calculation accuracy. The developed program implements an adaptive nonuniform mesh division approach: the computational domain is partitioned into a high-gradient inner region ($r_{in} \sim 250$ mm) and a low-gradient outer region (250 mm $\sim r_{out}$) along the radial direction, with 75% of the mesh nodes allocated to the inner region to guarantee sufficient computational precision.

In the high-gradient inner region, a nonuniform mesh is generated using a geometric series distribution, with the adjacent mesh size ratio set to 1.015, thereby significantly increasing the mesh density near the inner radius. The size of the first control volume in the inner region is determined by the geometric series summation formula presented in Eq. (23), while the outer region employs a uniform mesh distribution.

$$\Delta r_1 = \frac{r_{critical} - r_{in}}{\sum_{i=1}^{n_{inner}} \text{ratio}^{i-1}} = \frac{r_{critical} - r_{in}}{(1 - \text{ratio}^{n_{inner}})/(1 - \text{ratio})}. \quad (23)$$

Here, Δr_1 represents the radial size of the first mesh, $r_{critical}$ denotes the boundary radius between the high-gradient region and the low-gradient region, ratio is the geometric series ratio (1.015), and n_{inner} indicates the number of meshes in the high-gradient region.

For the discretized equations, a five-point difference scheme is employed to handle internal nodes, while boundary nodes are specially treated according to the convective heat transfer conditions. The resulting linear system of equations $Ax = b$ is solved by a sparse matrix solver, where A represents the coefficient matrix, x denotes the temperature vector, and b is the heat source and boundary condition vector. The construction of the coefficient matrix comprehensively accounts for the influence of anisotropic thermal conductivity on heat flow in different directions, thereby ensuring the accuracy of the physical model.

C. Grid independence verification

The reliability of numerical calculation results is significantly influenced by the appropriateness of mesh division, particularly for complex heat transfer problems characterized by extremely high power density gradients in the inner radius region of the core. To ensure computational accuracy, this study performed a systematic grid independence verification on the heat transfer model, utilizing the core peak surface temperature as the primary evaluation metric.

The verification scheme adopted a strategic approach by maintaining a fixed number of 90 axial mesh lines while systematically varying the number of radial mesh lines from 50 to 500, resulting in a total node count ranging from 4500 to 45,000. Throughout the testing process, physical parameters, boundary conditions, and solution algorithms were rigorously maintained at consistent settings. An embedded core exhibiting complex multiregion power density distribution was selected as the test subject to ensure the representativeness and reliability of the verification results.

Figure 4 presents the detailed results of the grid independence test. Analysis of Fig. 4 reveals that when the total number of nodes is less than 1×10^4 , the peak surface temperature rapidly increases from 52.79 °C to 53.04 °C. However, as the node count exceeds 2×10^4 , the peak surface temperature tends to stabilize, ultimately converging to 53.15 °C. Further analysis of the relative temperature change rate indicates that as the grid density increases from 2.25×10^4 to 2.7×10^4 nodes, the temperature variation range decreases from 0.05% to below 0.02%.

Based on the above analysis, this study establishes a convergence criterion of relative temperature change below 0.02% and finally selects a grid configuration of 300 radial $\times$ 90 axial (total nodes 27,000). Under this

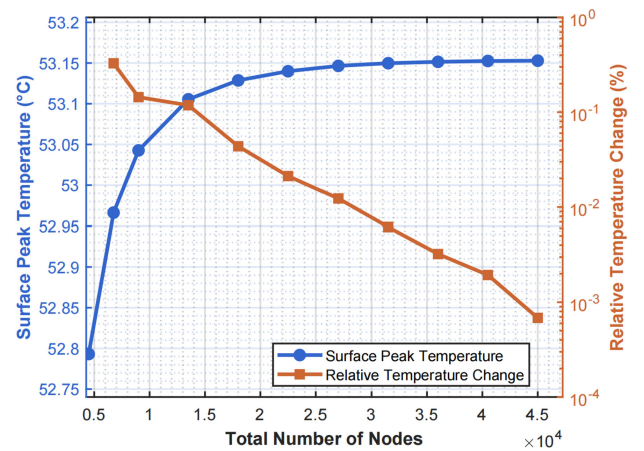


FIG. 4. Peak surface temperature convergence analysis: the blue solid line shows peak surface temperature variation with total node number, while the orange square line represents relative temperature change rate.

configuration, the peak surface temperature is 53.149°C , which differs from the calculation result of the densest grid (45,000 nodes) by only 0.001°C , with a relative error of less than 0.002% . Moreover, the calculation time is significantly reduced to one-third of that of the densest grid. Consequently, this approach achieves an effective balance between computational accuracy and efficiency.

D. Cross verification of simulation programs

To validate the computational accuracy of the self-developed heat transfer model, this study employed Ansys CFX commercial CFD software for cross verification [19]. The CFX simulation maintained identical geometric model, material parameters, and boundary condition settings as those used in the program.

In the CFX simulation, a three-dimensional embedded core geometric model was adopted, with the mesh division adhering to the grid independence verification results. The computational domain was discretized into 300 radial and 90 axial control volumes, yielding a total mesh count of approximately 270,000 elements. The heat source term was configured based on the measured power density distribution, and the heat transfer boundary condition incorporated the actual convective heat transfer coefficient $h = 500 \text{ W}/(\text{m}^2 \text{ K})$ of the water-cooled radiator. The material thermal property parameters were set as: radial thermal conductivity $k_r = 0.19 \text{ W}/(\text{m K})$, axial and circumferential thermal conductivity $k_z = k_\theta = 7.1 \text{ W}/(\text{m K})$. A steady-state heat transfer solver was employed with a convergence criterion of residual less than 1×10^{-6} .

Figure 5 compares the temperature distribution calculations on the core's bottom surface between the self-developed computational program and commercial

simulation software. The results demonstrate that the finite volume method program developed in this study is highly consistent with the CFX simulation results throughout the radial range, with a maximum deviation of 0.008°C and an average deviation of 0.006°C , corresponding to a relative error of less than 0.015% . Furthermore, the coefficient of determination R^2 reaches 0.9999. These findings indicate that the self-developed program demonstrates a good calculation accuracy in handling complex heat transfer problems associated with embedded cores.

E. Experimental validation and model calibration

Through grid independence verification and CFX cross validation, the convergence and consistency of the self-developed program in numerical calculations have been confirmed. However, both the finite volume method program developed in this study and the CFX simulation are fundamentally based on mathematical models for numerical computation. Although the two numerical methods demonstrate consistent calculation results, this only proves the reliability of the numerical algorithms, not the conformity of the simulation results with actual physical phenomena. Therefore, it is necessary to compare the simulations with experimentally measured temperature distributions to validate the effectiveness of the heat transfer model.

In high-power synchrotron rf systems, the direct water-cooling method is typically employed for MA cores. However, temperature measurement within the actual water-cooled cavity presents numerous technical challenges: the flow channel gap of only 3 mm prevents the installation of temperature sensors; the high-gradient electromagnetic fields inside the cavity may cause arcing and electromagnetic interference in sensor leads; furthermore, the direct water cooling environment makes it difficult to obtain detailed temperature distribution information on the core surface, thus unable to provide sufficient validation data for theoretical models. To address these issues, this study has designed a specialized experimental validation scheme where a direct power feed experiment was conducted under atmospheric conditions.

In this setup, the core relies on air convection and thermal radiation for heat dissipation. Although the power level is limited by the encapsulation layer temperature and cannot reach actual operating values, the core still operates under rf current excitation in an electromagnetic field environment, maintaining magnetic flux distribution patterns and relative power density distribution characteristics consistent with actual operating modes. Although the experimental conditions are simplified compared to actual water-cooling conditions, the key physical mechanisms remain consistent: (1) the electromagnetic field excitation mode is identical, ensuring the relative regularity of power density distribution; (2) the magnetic flux density redistribution mechanism of the embedded core is unaffected by heat dissipation boundary conditions; (3) the relative

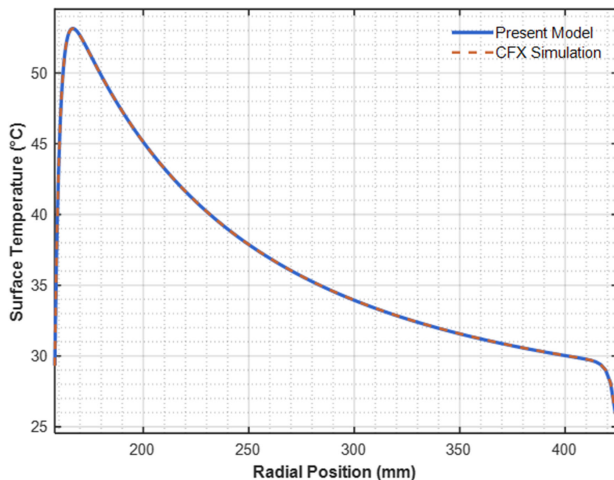


FIG. 5. Comparison of the temperature distribution calculations by the self-developed program and commercial software. The blue solid line represents finite volume method results of this study, while the orange dashed line shows Ansys CFX simulation results.

improvement trend of temperature distribution is universal. The advantage of this experimental design lies in the ability to directly measure the temperature distribution across the entire core surface using a high-precision thermal imager (FLUKE-TI45), providing detailed and reliable experimental data for theoretical model validation.

The primary objective of the experimental validation is to assess the validity of key assumptions in the numerical model, including the assigned values of anisotropic thermal conductivity, the computational accuracy of power density distribution, and the methodology for boundary condition treatment. Given that the convective heat transfer coefficient directly influences the prediction of temperature field distribution, the use of fixed empirical values may lead to significant discrepancies between simulation and experimental results, thereby compromising the evaluation of numerical algorithm accuracy. Consequently, this study adopts an experimental data-based calibration method for the convective heat transfer coefficient to eliminate uncertainties in boundary conditions that could interfere with algorithm validation.

The experiment uses a FLUKE-TI45 thermal imager for noncontact temperature measurement of the MA core surface. The core is suspended using a specialized fixed insulating bracket, with a 10 cm clearance from the platform at the rear to ensure adequate heat dissipation, while other surfaces are exposed to the ambient environment for natural convective heat transfer. Power is directly coupled to the core through magnetic induction using a solid-state power source connected to a single-turn copper strip wound around the core.

Thermal images were captured at 20-min intervals after power application to the MA core until thermal equilibrium was achieved, as shown in Fig. 6. A fixed temperature sampling path was established along the radial direction of the core (indicated by the white dashed line in the thermal image) to extract temperature time series data at various radial positions. Based on the exponential decay model derived from Newton's law of cooling [17], the temperature rise process at any point on the core surface can be expressed as

$$T(t) = T_{\infty} + (T_0 - T_{\infty}) \exp\left(-\frac{t}{\tau}\right), \quad (24)$$

where T_{∞} denotes the steady-state temperature, T_0 represents the initial temperature, and τ is the time constant. The relationship between the convective heat transfer coefficient and the time constant is given by

$$h = \frac{\rho c V_{\text{eff}}}{\tau A_{\text{eff}}}, \quad (25)$$

where ρ and c are the material density and specific heat capacity, respectively, while V_{eff} and A_{eff} represent the equivalent volume and heat transfer area.

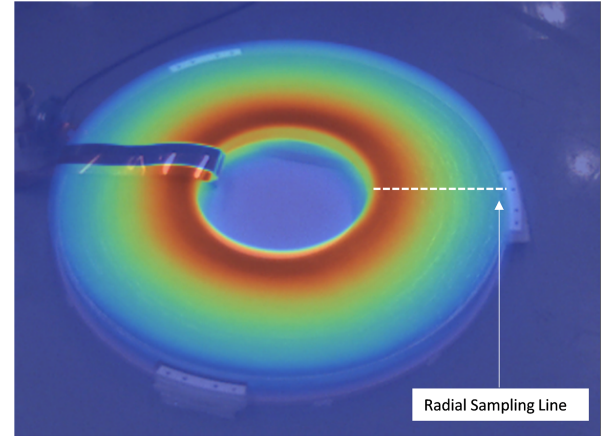


FIG. 6. Thermal imaging of the embedded MA core surface with radial sampling line indication. The thermal image illustrates the surface temperature distribution under 2 MHz, 300 V excitation conditions, where warm regions (red/orange) correspond to high temperature zones and cool regions (blue/green) indicate lower temperature areas. The white dashed line indicates the radial sampling line used for extracting temperature data along radial direction for quantitative analysis.

Figure 7 presents a typical temperature rise curve at the radial position $R = 250$ mm and its corresponding exponential fitting results. The experimental data exhibit strong agreement with the exponential decay model, with the determination coefficient R^2 exceeding 0.99, and the 95% confidence interval indicating that the fitting results have robust statistical reliability. Through systematic fitting analysis at various radial positions on the core surface, the radial distribution of the convective heat transfer coefficient was obtained, as illustrated in Fig. 8.

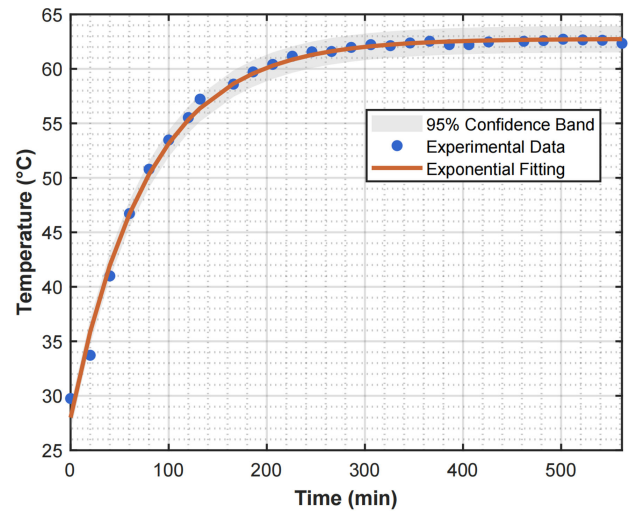


FIG. 7. Temperature rise curve and exponential fitting results at the radial position $R = 250$ mm on the MA core surface. The blue circles represent experimental measurement data, while the red solid line illustrates the exponential decay model fitting curve. The light gray shaded area denotes the 95% confidence interval.

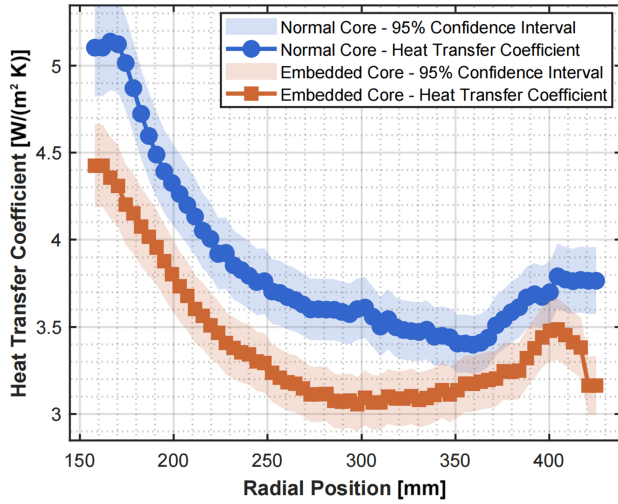


FIG. 8. Radial distribution of convective heat transfer coefficients obtained through the fitting method (results are averaged over two cores for each type). The blue line with its associated 95% confidence band represents the normal MA core, while the orange line with its corresponding 95% confidence band depicts the embedded MA core.

The results reveal that the distribution of the convective heat transfer coefficient closely follows the power density distribution pattern. The normal core displays significantly higher overall convective heat transfer coefficients compared to the embedded core, reaching a maximum value of approximately $5.2 \text{ W}/(\text{m}^2 \text{ K})$ in the inner diameter region, followed by a gradual radial decrease that stabilizes around $3.5 \text{ W}/(\text{m}^2 \text{ K})$. Conversely, the embedded structure MA core exhibits relatively lower convective heat transfer coefficients in the inner diameter region [approximately $4 \text{ W}/(\text{m}^2 \text{ K})$], while maintaining a similar decreasing trend along the radius and stabilizing in the outer diameter region at approximately $3.1 \text{ W}/(\text{m}^2 \text{ K})$.

This distribution difference aligns with the physical mechanisms of natural convection heat transfer and the theoretical analysis presented earlier: (1) According to the power density distribution theory in Sec. II D, the normal core strictly adheres to a $1/r^2$ power distribution, resulting in high power density in the inner diameter region and consequently significantly elevated surface temperatures; (2) The embedded structure redistributes power distribution through transferring partial power from the high-temperature inner diameter region to the middle diameter region, effectively reducing surface temperatures in the inner diameter region; and (3) The natural convection heat transfer coefficient is directly correlated with surface temperature, where higher surface temperatures induce stronger buoyancy-driven convection heat transfer and correspondingly larger heat transfer coefficients.

In addition to the convective heat transfer coefficient, accurate determination of the surface temperature distribution of the MA cores necessitates the acquisition of their

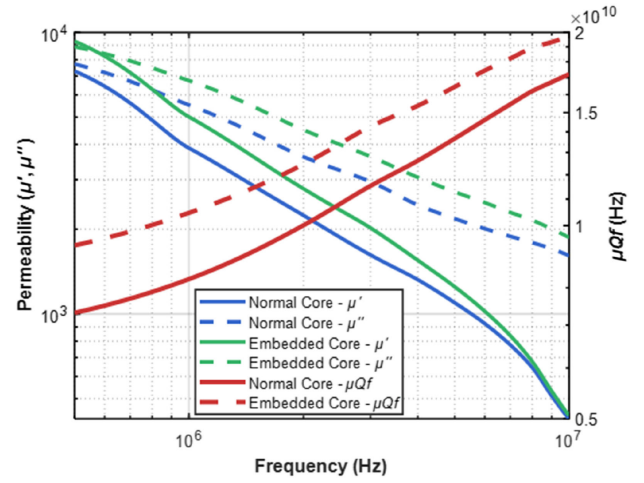


FIG. 9. Variation of the average permeability (real and imaginary parts) and the μQf value parameters with frequency for two types of MA cores (results are averaged over two cores of each type).

magnetic parameters. We measured the magnetic parameters of two normal cores and two embedded cores (with the thinner ribbon region positioned at 188–250 mm) using a network analyzer [11]. The study employed manufacturer-supplied ribbons with thickness specifications: 14–16 μm for normal ribbons and 12–14 μm for thinner ribbons. All MA cores share identical geometric dimensions and are coated with an approximately 300 μm -thick epoxy resin cured encapsulation layer.

Figure 9 presents the frequency-dependent characteristics of the average permeability real and imaginary parts, along with the μQf values, for both types of MA cores. The results indicate that the normal core exhibits real and imaginary permeabilities of 2437 and 3822 at 2 MHz, respectively, with a μQf value of approximately 10.7 GHz. In contrast, the overall equivalent real and imaginary permeabilities of the embedded core are approximately 2559 and 4089, respectively, and μQf is approximately 11.4 GHz, representing improvements of 5%, 5%, and 7% compared to the normal core. Due to the physical integration of thinner ribbons and normal ribbons in the embedded structure, it is not feasible to experimentally measure the individual magnetic properties of each region separately. To obtain reasonable material parameters for subsequent simulation analysis, this study employs the fitting equations (Table II) and the linear superposition theory introduced in Sec. II B to inversely calculate the equivalent permeability of thinner ribbons in the embedded core. Given the known parameters of the entire core and the thickness of normal ribbons, the magnetic properties of normal ribbons can be directly derived from Table II. Subsequently, the magnetic parameters of thinner ribbons are deduced based on the linear superposition theory.

Based on the geometric configuration of the embedded core (where normal ribbons are employed in the inner

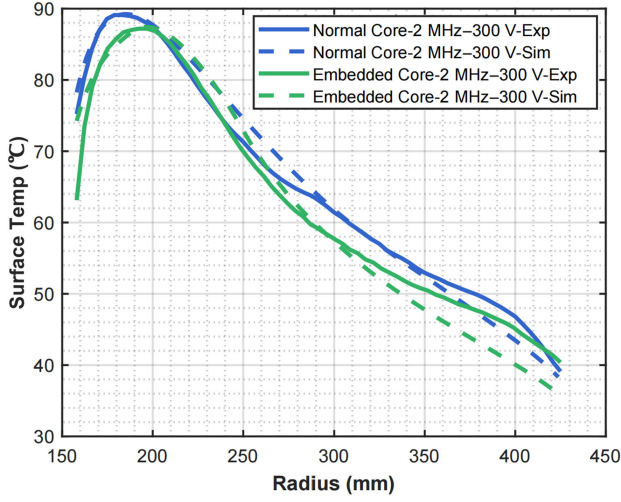


FIG. 10. Surface temperature distribution comparison between normal and embedded MA cores along the radial direction under 2 MHz, 300 V excitation conditions. The solid lines depict experimental measurement data (averaged over two cores of each type), while the dashed lines represent simulation results. The blue lines correspond to normal MA cores, and the green lines indicate embedded MA cores.

diameter region of 158–188 mm and the outer diameter region of 250–425 mm, while 13- μm ribbons are utilized in the intermediate region of 188–250 mm), and utilizing the linear superposition relationship discussed in Sec. II, the equivalent permeability parameters of the 13- μm ribbon region are inversely calculated as $\mu'_s \approx 2861$ and $\mu''_s \approx 4749$, corresponding to an equivalent thickness of approximately 12.1 μm and a μQf value of about 12.9 GHz.

Utilizing the fitted convective heat transfer coefficients and measured magnetic parameters, we conducted temperature distribution verification experiments on the embedded core. The experiments were performed at frequencies of 2 and 3 MHz, with an excitation voltage set at 300 V. Each MA core was tested three times, and the temperature data along the white sampling line in Fig. 6 at steady state were averaged. Figure 10 presents a comparative analysis of the surface temperature distribution along the radius between the normal core and the embedded core under 2 MHz, 300 V excitation conditions.

The experimental results reveal that the normal MA core exhibits its temperature peak near the inner diameter ($r \approx 160$ mm), while the embedded core demonstrates a temperature peak shift toward the middle radial region ($r \approx 200$ mm), which aligns with the power density redistribution mechanism analyzed in Sec. II D. Under the same excitation conditions, the embedded core exhibits a peak surface temperature reduction of approximately 3–4 $^{\circ}\text{C}$ compared to the normal core. The simulation results, incorporating the fitted convective heat transfer coefficients, demonstrate good agreement with experimental measurements in terms of temperature distribution trends.

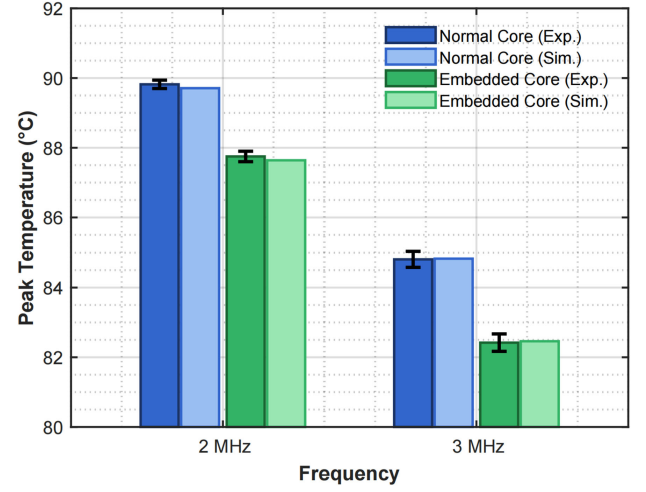


FIG. 11. Comparison of peak surface temperature between normal and embedded MA cores. Experimental data represent mean values from two cores of each type. The dark and light blue bars correspond to experimental data and simulation results for the normal core, respectively. The dark and light green bars denote experimental data and simulation results for the embedded core, respectively.

However, certain discrepancies between experimental and simulation results persist, primarily attributable to the following factors: (1) uncertainties in the convective heat transfer coefficient due to microscopic flow field fluctuations and surface conditions in practical environments; (2) influence of the core support structure on local temperature fields in experiments; and (3) propagation of measurement errors in material thermal property parameters to temperature prediction results.

Figure 11 further illustrates the comparison between experimental measurements and simulation results of peak surface temperatures for both normal and embedded cores at 2 and 3 MHz. The results indicate that the embedded structure demonstrates substantial temperature reduction effects at both frequencies, with peak temperature decreases about 2.5 $^{\circ}\text{C}$. The deviations between simulation predictions and experimental measurements are maintained within acceptable limits, thereby validating the reliability of the numerical model.

To thoroughly evaluate the performance advantages of the embedded structure under actual operating conditions, this study conducted comparative verification experiments in a high-power MA-loaded cavity under direct water-cooling conditions. The experiments simulated the real working environment of the MA cores, employing a closed-loop cooling system with a constant inlet water temperature of 25 $^{\circ}\text{C}$ and a cooling water flow rate of 18 L/min, corresponding to laminar flow conditions within the cavity. The four MA cores discussed above were used in the experiments, with each core installed in a standard coaxial cavity structure. Continuous excitation signals were provided through a rf power amplifier. During the experiments, the rf voltage was

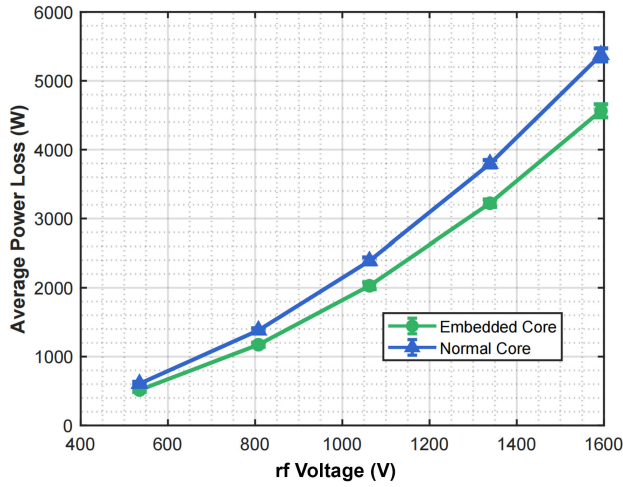


FIG. 12. Comparison of power losses between normal and embedded MA cores in directly water-cooled cavities (results are averaged over two cores for each type).

gradually increased, and precise thermocouples (accuracy $\pm 0.1^\circ\text{C}$) were utilized to continuously monitor the outlet water temperature until thermal equilibrium was reached (temperature change rate $< 0.05^\circ\text{C}/10\text{ min}$). The average power loss of the MA cores was calculated using flow rate and temperature rise formulas [6].

The experimental results, as illustrated in Fig. 12, compare the power losses of both types of MA cores under identical excitation voltages at 2 MHz. The results demonstrate that the normal MA core exhibits significantly higher power losses compared to the embedded core, with this discrepancy becoming more pronounced as the voltage increases. For instance, at a voltage of 1.6 kV, the power loss of the normal core reaches 5.37 kW, exceeding that of the embedded core by 15%. If the voltage is further increased, the surface temperature of the normal core may prematurely reach the softening temperature of the encapsulation layer, thereby increasing the risk of core damage.

From a holistic thermal equilibrium perspective, these results confirm the effectiveness of the embedded structure. Although direct measurement of the detailed temperature distribution on the core surface within the water-cooled cavity is not feasible, the reduction in average power loss indirectly reflects the mitigation of thermal load within the core. Combined with the aforementioned theoretical analysis and numerical simulation results, it can be concluded that the embedded structure effectively enhances thermal performance of MA cores under actual high-power operating conditions.

To further validate the performance of the embedded core under long-term high-power operation, it was installed in an rf cavity at the CSNS RCS. Under real operating conditions, the cavity runs at an accelerating gradient of 40 kV/m, with each core subjected to a peak voltage of 4 kV and operating at an average power density of $0.3\text{ W}/\text{cm}^3$.

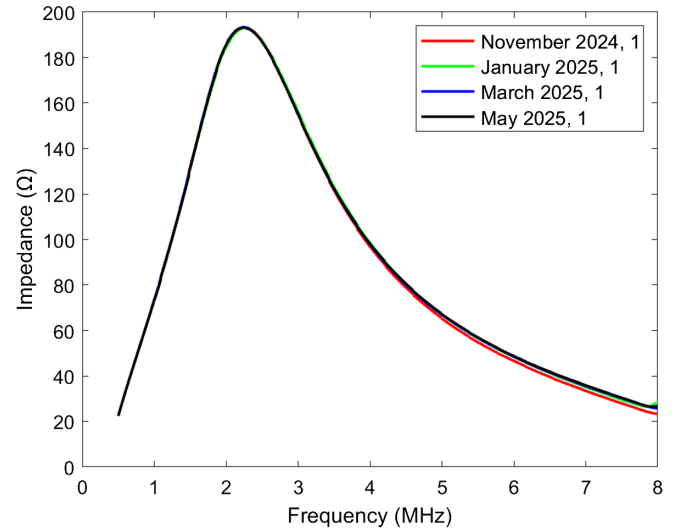


FIG. 13. Long-term stability of the cavity impedance measured over six months of operation. The consistent overlap of bi-monthly measurements demonstrates the stable performance of the embedded MA core under actual operating conditions.

To monitor the core's performance and stability, the cavity impedance was measured every two months. As shown in Fig. 13, all impedance curves obtained during this period remain consistent and stable, providing direct evidence of the embedded core's reliability and long-term stability under high electric field gradients and substantial power density. It should be noted that the impedance shown in Fig. 13 represents the whole cavity system, including busbar coupling and distributed parameters, measured *in situ*. The consistent overlap confirms the macroscopic stability of the rf system.

IV. PARAMETER OPTIMIZATION ANALYSIS OF EMBEDDED MA CORES

Building upon the validated numerical heat transfer model established in Sec. III, this section systematically analyzes the influence of different embedded configuration parameters on the temperature distribution of MA cores, providing design guidance for practical applications. We mainly focus on investigating the impact of the radial position, volume proportion, and thickness combinations of thinner ribbons on the thermal performance of embedded cores.

A. Parametric modeling and baseline performance analysis of embedded structure

To investigate the influence of thinner ribbon position in the embedded structure on the temperature distribution of MA cores, a geometric parameterization model based on constraint conditions was established. Given the weight W of the $13\text{-}\mu\text{m}$ ribbons, when the inner diameter boundary position r_1 is determined, the outer diameter boundary

TABLE III. Comparison of thermal performance of MA cores under different embedded structure configurations.

Type	rf voltage (V)	r_1/r_2 (mm)	Average power (kW)	Peak surface temperature (°C)
Normal core	1735.8	...	5	53.15
Embedded core 1	1735.8	188/210	4.862	51.67
Embedded core 2	1735.8	188/250	4.672	49.58
Embedded core 2	1795.9	188/250	5	51.31

position r_2 can be calculated through the material volume conservation relationship:

$$r_2 = \sqrt{\frac{W}{\rho \cdot \pi \cdot t \cdot P_f}} + r_1^2, \quad (26)$$

where W represents the weight of the thinner ribbons (kg), ρ denotes the ribbon density (7100 kg/m³), t is the axial thickness of the core (0.025 m), P_f signifies the packing factor (approximately 0.8), and r_1 and r_2 are the inner and outer boundary positions (m) of the thinner ribbon region, respectively. This parameterization method ensures consistency in the amount of thinner ribbons used across different configurations, providing a foundation for fair comparison of the performance of various embedding schemes.

Considering the practical application environment of MA cores in high-power accelerators, direct water-cooling boundary conditions were utilized in the calculations. The convective heat transfer coefficients for the core surface, inner diameter, and outer diameter regions were set to 500 W/(m² K), and the cooling water inlet temperature was 25°C. These settings are consistent with the cooling schemes employed in J-PARC and CSNS.

First, the thermal performance of the embedded core under fixed geometric configurations was analyzed. A starting position of $r_1 = 188$ mm was selected, and two thinner ribbon volume configurations of $r_2 = 210$ and 250 mm were considered for comparative analysis with the normal MA core. All MA cores share identical geometric dimensions: an outer diameter of 850 mm, an inner diameter of 316 mm, and a thickness of 25 mm. The parameter in the simulation that represents the rf voltage was adjusted to achieve a total power of 5 kW for the normal core, and the temperature distribution of the embedded core was then calculated under the same excitation voltage. Table III summarizes the key performance indicators for different configurations.

The results demonstrate that the embedded architecture effectively reduces the MA core's peak surface temperature under identical excitation voltages, with the temperature reduction effect becoming more significant as the thinner ribbon volume proportion increases. Further analysis reveals that even when the excitation voltage of embedded core 2 is increased to raise its power loss to the same level as the normal core (5 kW), it still maintains a peak surface

temperature nearly 2°C lower than its conventional counterpart. This indicates that the embedded structure effectively improves the temperature distribution uniformity of the MA core.

Figure 14 illustrates the simulated temperature distribution characteristics of the normal MA core and different embedded structures under an excitation voltage of 1735.8 V. The cross-sectional temperature distribution of the MA cores reveals a pronounced temperature concentration phenomenon in the inner diameter region of the normal core, which is consistent with the power density distribution pattern analyzed in Sec. II. The introduction of the embedded structure alters this temperature distribution pattern, with the power density jump in the thinner ribbon region causing the temperature peak position to shift toward the radial center, thereby effectively alleviating the thermal load in the inner diameter region. The results in Fig. 14 demonstrate that embedded core 2 achieves a peak temperature reduction of approximately 5°C in the inner diameter region compared to the normal structure, while simultaneously improving the overall uniformity of the temperature distribution.

B. Optimal position study of thinner ribbons

To determine the optimal radial position of the thinner ribbons, a systematic parameter scanning study was designed. Three weight levels of thinner ribbons were considered: 10, 20, and 30 kg, corresponding to different volume proportions and cost investments. For each weight configuration, the starting position r_1 of the thinner ribbon was scanned from the inner radius $r_0 = 158$ to 400 mm with a step of 2 mm, while simultaneously recording the variation in the peak surface temperature of the MA core for each configuration. Throughout the scanning process, the rf voltage was maintained at 1735.8 V to enable direct comparison with the performance of the normal MA core.

Figure 15(a) illustrates the variation of the peak surface temperature of the MA core with respect to the r_1 position for different thinner ribbon weights. The results reveal several important design principles. First, a temperature minimum occurs at $r_1 \approx 184$ mm for all weight configurations. This position is independent of the thinner ribbon weight, indicating that the optimal position is primarily determined by the geometric parameters of the MA core and the electromagnetic field distribution characteristics. Second, for smaller thinner ribbon weights (10 kg), when

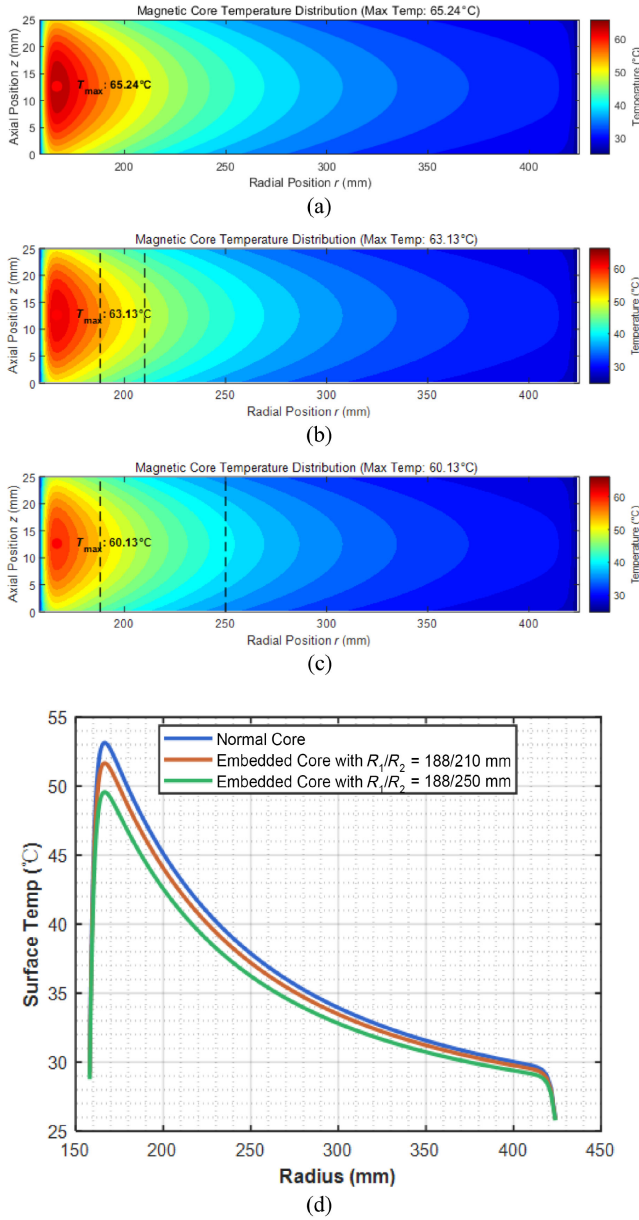


FIG. 14. Comparison of temperature field distribution of normal and embedded MA cores. (a) Cross-sectional temperature distribution of normal core: peak temperature of 53.34°C located at the inner radius region; (b) cross-sectional temperature distribution of embedded core 1: $r_1/r_2 = 188/210$ mm, peak temperature of 51.15°C; (c) cross-sectional temperature distribution of embedded core 2: $r_1/r_2 = 188/250$ mm, peak temperature of 49.13°C; (d) radial surface temperature distribution comparison: blue line represents normal core, red line indicates embedded core 1, and green line denotes embedded core 2.

$r_1 < 184$ mm, despite the better magnetic performance of the embedded core compared to the normal core, its peak temperature is instead higher. This phenomenon suggests that the use of excessively small amounts of thinner ribbons should be avoided in practical applications to reduce dependence on positional accuracy. Third, the temperature

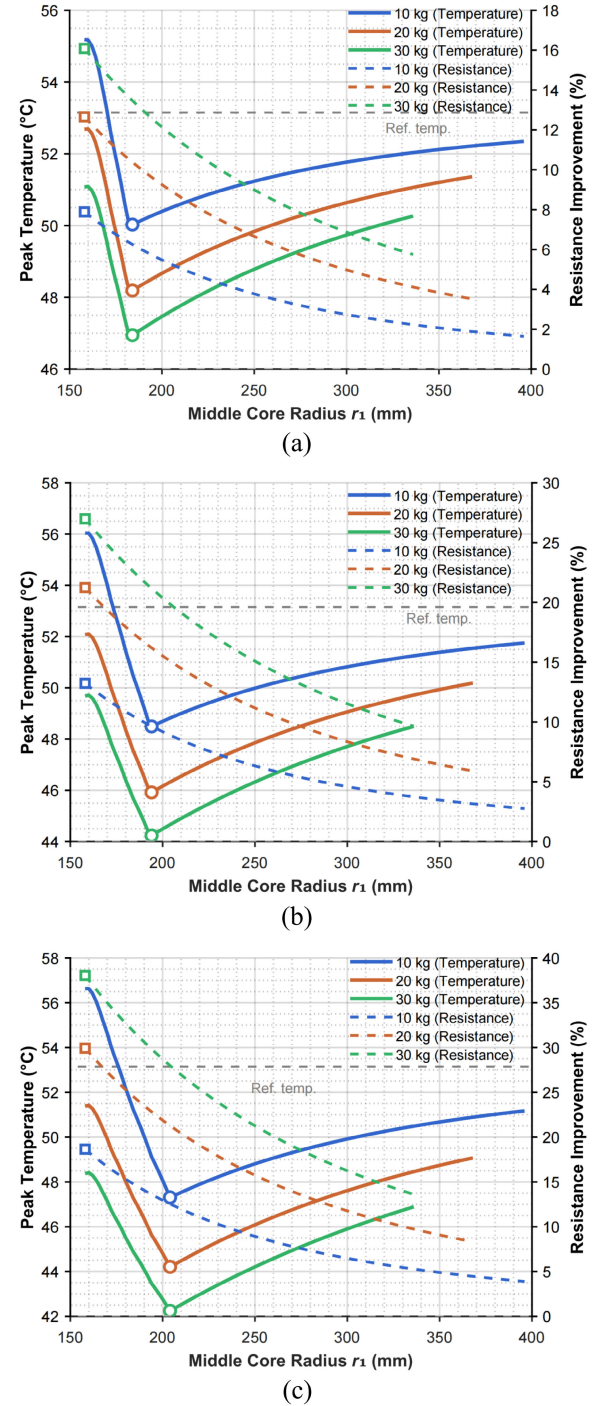


FIG. 15. Optimization parameter analysis of embedded MA cores with different material combinations. (a) 13 + 16 μ m combination: the blue solid line illustrates the variation in peak surface temperature, while the dashed line indicates the percentage of impedance enhancement, with the optimal position located at $r_1 \approx 184$ mm; (b) 13 + 18 μ m combination: the optimal position shifts rightward to $r_1 \approx 190$ mm with enhanced temperature improvement; (c) 13 + 20 μ m combination: the optimal position is at $r_1 \approx 206$ mm, where the increased material property difference significantly enhances the power redistribution effect. Different colored lines correspond to various thinner ribbon weights (10, 20, 30 kg).

response exhibits significant asymmetry on either side of the optimal position. On the right side of the optimal position, the temperature curve has a smaller slope, indicating lower sensitivity of the peak temperature to positional deviations, which provides tolerance for positional errors in the actual manufacturing process.

Based on the above analysis and considering the practical constraints of manufacturing tolerances and positional control accuracy, it is recommended to set the starting position of the thinner ribbon near $r_1 = 190$ mm, on the right side of the optimal point. This position can ensure good thermal performance while providing sufficient process tolerance, enabling stable temperature improvement effects across different thinner ribbon weights.

C. Optimization analysis of material combination schemes

To evaluate the impact of different material combinations on the performance of the embedded core, this study analyzed three thickness combination schemes ($13 + 16\ \mu\text{m}$, $13 + 18\ \mu\text{m}$, and $13 + 20\ \mu\text{m}$) based on the permeability-thickness relationship established in Sec. II. Figures 15(b) and 15(c) present the analysis results for the $13 + 18\ \mu\text{m}$ and $13 + 20\ \mu\text{m}$ combinations, respectively.

The analysis reveals that the temperature improvement effect becomes increasingly pronounced with greater disparities in material properties. For instance, considering the 10 kg thinner ribbon configuration, at the optimal position, the $13 + 16\ \mu\text{m}$ combination yields a temperature reduction of $3.1\ ^\circ\text{C}$, the $13 + 18\ \mu\text{m}$ combination results in a $4.74\ ^\circ\text{C}$ reduction, and the $13 + 20\ \mu\text{m}$ combination achieves a significant $5.86\ ^\circ\text{C}$ reduction. Correspondingly, the impedance performance improvement follows a similar trend, with enhancements of 6.25%, 9.6%, and 12.3%, respectively. A corresponding trend is observed in impedance performance enhancement, with improvements of 6.25%, 9.6%, and 12.3%, respectively. Crucially, as the disparity in material properties increases, the temperature gradient in the vicinity of the optimal position diminishes substantially, thereby relaxing manufacturing precision requirements. Furthermore, the optimal position is observed to shift toward a larger radius ($r_1 \approx 206$ mm), while maintaining independence from the thinner ribbon weight.

These results substantiate the theoretical mechanism proposed in Sec. II: the permeability differential between the two ribbon types directly determines the voltage distribution ratio, consequently influencing the magnitude of the magnetic flux density jump. A greater disparity in magnetic properties between the two materials not only yields a more pronounced power redistribution effect but also provides enhanced process tolerance.

D. Embedded core design guidance and discussion

Taking into account the fluctuations in material properties in actual production and the response characteristics of

different thickness combinations, it is recommended to adopt $r_1 = 200$ mm as a compromise solution. This position offers several advantages: for large property difference combinations (e.g., $13 + 20\ \mu\text{m}$), this position is close to the optimal point, enabling significant temperature improvement; for small property difference combinations (e.g., $13 + 16\ \mu\text{m}$), the temperature performance at this position is near optimal and exhibits low sensitivity to positional accuracy; furthermore, this position provides a unified design reference for different material combinations, simplifying parameter selection in engineering applications.

However, it should be noted that the current analysis is based on idealized material parameters, and batch-to-batch variations in material properties and environmental dependencies during actual manufacturing may affect the accuracy of the optimization results. The uniform convective heat transfer boundary condition employed represents a simplification of the actual complex cooling environment, and variations in local heat transfer coefficients in practical applications may influence the temperature distribution. Furthermore, while the current study focuses on steady-state thermal performance, the thermal fatigue characteristics and performance degradation patterns of the embedded structure during long-term operation require further investigation. Despite these limitations, the analytical methods established and the design principles derived in this study provide a crucial theoretical foundation and practical guidance for the application of embedded cores.

V. CONCLUSION

The proposed new embedded structure enhances the magnetic performance of MA cores while preserving structural feasibility through the strategic radial zoning of thinner ribbons. This performance improvement is explained by a power density redistribution mechanism, which utilizes the voltage distribution differences among materials with varying permeabilities in the series equivalent circuit to reconfigure the core's internal power flow and thereby improve temperature distribution.

Numerical and experimental results confirm that the optimal embedded structure reduces the peak temperature by $3\text{--}6\ ^\circ\text{C}$ and increases the core impedance by 6%–12%. Most importantly, it shifts the temperature peak from the high-gradient inner diameter region to the intermediate diameter region with better heat dissipation, mitigating the inherent “hot inside, cold outside” gradient. A half-year *in situ* test during regular CSNS operation demonstrated excellent long-term stability, confirming the design's practical reliability. Based on this success, plans are underway to implement these embedded cores in CSNS cavities to enhance performance. Furthermore, development of large-scale cores using entirely $13\ \mu\text{m}$ ribbons is actively pursued.

This study also reveals that the optimal radial position for thinner ribbons is governed by electromagnetic field

distribution rather than material usage, providing a foundation for standardized design. While increasing the magnetic property disparity enhances the redistribution effect, the current steady-state thermal analysis indicates the need for further investigation into thermal cycle effects, dynamic response, and the influence of complex boundary conditions in real operation.

Future work will also explore extending this embedded design concept to MA cores of various geometric dimensions for a wider range of accelerator applications.

ACKNOWLEDGMENTS

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DATA AVAILABILITY

The data that support the findings of this article are not publicly available. The data are available from the authors upon reasonable request.

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