

Three-dimensional collimator design for suppressing transverse single-bunch instabilities in crab-waist colliders

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The collimator system is a significant source of impedance in ring colliders and can drive transverse mode coupling instabilities (TMCI) through vertical dipolar wakefields. In this study, we investigate the impact of collimator geometry on wake potentials using electromagnetic simulations based on simplified and realistic models. Various taper shapes, jaw widths, and structural modifications such as hollowed and asymmetric configurations are evaluated with a focus on reducing the vertical dipolar kick factor. Three mitigation strategies are proposed and benchmarked using the PyHEADTAIL tracking code, with the SuperKEKB Low Energy Ring used as a case study. Our results show that the implementation of hollowed exponential taper jaws, especially in one-sided layouts, leads to a substantial reduction in the total β_y -weighted dipolar kick factor and increases the TMCI threshold by up to approximately 90%.

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I. INTRODUCTION

In modern circular e^+e^- colliders, movable collimators are essential vacuum components strategically positioned to intercept particles in the non-Gaussian tails of the circulating beam. These halo particles, if left uncontrolled, can generate significant background in the interaction region and pose serious risks to sensitive accelerator components through localized beam losses. Collimators therefore serve a dual role: they act as protective devices that safeguard the machine against uncontrolled particle losses and as background suppression systems that preserve the experimental environment for detectors. At the same time, their integration into the accelerator lattice is far from trivial. Collimators must be designed with narrow gaps to achieve sufficient cleaning efficiency, yet this unavoidably increases their impact on the circulating beam through impedance effects. Striking an optimal balance between machine protection, background control, and beam stability is thus a central challenge in the design and operation of high-luminosity colliders.

A particularly important context for these considerations is the advent of the crab-waist (CW) collision scheme, first

proposed by Raimondi [1] and successfully demonstrated at DAΦNE [2]. This innovative scheme has reshaped the landscape of circular e^+e^- collider design, enabling significant luminosity enhancement without requiring prohibitively high beam currents. Since its implementation at SuperKEKB [3], the world's highest-luminosity collider, the CW scheme has become a defining feature of next-generation facilities such as FCC-ee [4], CEPC [5], SCTF [6], and STCF [7]. Compared with earlier colliders such as KEKB [8] and PEP-II [9], CW colliders operate with much smaller vertical emittance and tighter focusing at the interaction point, but these advances come at the cost of reduced dynamic aperture and increased nonlinearities in the interaction region. As a consequence, collimators in CW colliders must be positioned closer to the beam core, leading to exceptionally small jaw openings. While this geometry is necessary to intercept halo particles effectively, it also makes the collimators dominant contributors to the overall impedance budget of the ring [10,11]. Recent beam-beam simulations incorporating impedance effects have indicated that interactions between beam-beam forces and impedance can degrade the luminosity performance of CW colliders [12–14]. Therefore, suppressing collimator wakefields is a prerequisite for maintaining design luminosity in CW colliders.

The electromagnetic properties of collimators are strongly governed by their geometry. Most designs employ smooth tapers to transition the chamber cross section from the nominal aperture to the narrow jaw opening, thereby minimizing abrupt discontinuities. The impedance of such

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tapered structures has been extensively studied, both analytically for simplified geometries [15,16] and numerically for realistic three-dimensional designs [17–19]. Among the resulting beam dynamics effects, the transverse mode-coupling instability (TMCI) driven by the dipolar wakefield is the most critical for single-bunch stability. In the following, we focus on the simplest mode-coupling mechanism underlying TMCI with zero chromaticity. If the vertical betatron tune is denoted by ν_y , the coherent tune shift due to vertical impedance can be expressed as

$$\Delta\nu_y = -\frac{I_b T_0}{4\pi(E/e)} \sum_i \beta_{y,i} k_{y,i}(\sigma_z), \quad (1)$$

where I_b is the bunch current, E/e is the beam energy, and T_0 is the revolution period. The quantities $\beta_{y,i}$ and $k_{y,i}(\sigma_z)$ represent, respectively, the vertical beta function and the dipolar kick factor of the i th impedance source. The dipolar kick factor depends on the bunch length σ_z , and the summation extends over all vertical impedance sources in the ring.

As the bunch current increases such that $\Delta\nu_y$ approaches the synchrotron tune ν_s , coupling occurs between the 0 mode (ν_y) and the -1 mode ($\nu_y - \nu_s$), resulting in TMCI. The dipolar component of the wakefield is the primary driver of this instability, whereas the quadrupolar component mainly contributes to tune shift and can therefore influence beam stability (see, e.g., [20,21]). In the present study, the effect of the quadrupolar wakefield is assumed to be weak. Accordingly, strategies to mitigate TMCI focus primarily on reducing the dipolar wakefield of collimators, as indicated by Eq. (1). In addition, a nonzero chromaticity introduces a tune spread along the bunch and can therefore influence the onset of TMCI. While this effect is included in our simulations, it is not investigated in detail here.

In this study, we perform systematic electromagnetic simulations to examine the impact of collimator geometry on wake potentials, beginning with simplified models to isolate the role of individual geometric parameters. These simulations, carried out with GdfidL and ECHO3D [22,23], guide the development of design strategies that minimize the dipolar kick factor while preserving a narrow aperture for efficient halo interception. The effectiveness of these mitigation strategies is then evaluated in the context of single-bunch dynamics through PyHEADTAIL [24] simulations, using the low energy ring (LER) of SuperKEKB [25] as a representative case study. By combining detailed electromagnetic modeling with instability analysis, we aim to establish practical guidelines for collimator design in CW colliders.

The remainder of this paper is structured as follows. Section II introduces the simplified rectangular collimator geometry and explores how key design parameters affect wakefields. Section III applies the impedance reduction concepts to optimize the three-dimensional geometry of

SuperKEKB collimators. Sections IV and V present, respectively, impedance modeling results and single-bunch simulations that quantify the benefits of the proposed strategies in suppressing TMCI in the SuperKEKB LER. Finally, Sec. VI summarizes the findings and outlines directions for future work.

II. IMPACT OF COLLIMATOR GEOMETRY

To gain insight into how key design parameters influence the wakefield, we begin with a simplified geometry before extending the discussion to more realistic three-dimensional structures. This stepwise approach allows us to disentangle the contributions of taper profile, aperture size, and jaw shaping to the dipolar impedance, which in turn governs the TMCI threshold.

Figure 1 shows the simplified collimator model adopted as a starting point. The cross section is rectangular with a width of $2b = 90$ mm, and the jaws taper symmetrically in the vertical (y) direction with respect to the beam axis (z). Two taper profiles are considered: a linear taper and an exponential taper. The half-gap at the jaw tip is denoted by a , the taper length by l , and the half jaw width by w . In addition, the shaded region in Fig. 1 illustrates a recessed structure characterized by its depth d and an inward offset h . The detailed effects of this structure on the wakefields will be discussed later in Sec. II B. Unless otherwise specified, we use $l = 175$ mm, $a = 2$ mm, and a flat tip length $g = 10$ mm, values that are representative of the SuperKEKB collimator design [26].

The exponential taper follows the profile proposed by Yokoya [15] for minimizing the dipolar wakefield

$$A(z) = a \left(\frac{b}{a} \right)^{z/l}, \quad (2)$$

where $A(z)$ is the half-aperture as a function of longitudinal position z .

For a rectangular collimator, analytical expressions for the low-frequency impedance are available [16]. With $w = b$, the longitudinal monopolar impedance is

$$Z_{\parallel}^{\text{rec}}(k) = -\frac{7\zeta(3)}{2\pi^2} \frac{iZ_0 k}{4\pi} \int_{-\infty}^{\infty} dz A'(z)^2, \quad (3)$$

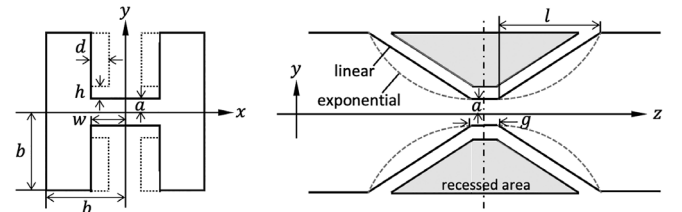


FIG. 1. Dimensions of the simplified symmetric collimator model.

where $\zeta(3)$ is the Riemann zeta function. The corresponding vertical dipolar impedance is

$$Z_{\text{yd}}^{\text{rec}}(k) = -\frac{iZ_0b}{2} \int_{-\infty}^{\infty} dz \frac{A'(z)^2}{A(z)^3}. \quad (4)$$

The above equation is valid in the regime $a \ll 2b \ll l$ and $4kb^2 \ll l$ [27]. The second regime imposes a tight condition on the applicability of Eq. (4): for a Gaussian bunch, the bunch length must satisfy $\sigma_z \gg 4b^2/l$ in order to ensure a valid evaluation of the dipolar wake.

For comparison, the impedances of round collimators take the form [16]:

$$Z_{\parallel}^{\text{md}}(k) = -\frac{iZ_0k}{4\pi} \int_{-\infty}^{\infty} dz A'(z)^2, \quad (5)$$

$$Z_{\text{xd}}^{\text{md}}(k) = Z_{\text{yd}}^{\text{md}}(k) \approx -\frac{iZ_0}{2\pi} \int_{-\infty}^{\infty} dz \frac{A'(z)^2}{A(z)^2}. \quad (6)$$

Equations (3)–(6) show that smoothly tapered collimators contribute mainly inductive impedance at low frequency. For the longitudinal impedance, a flat rectangular collimator has an inductance of about 43% of that of a round collimator with the same tapering function $A(z)$. The specific taper profile also plays a critical role. For instance, the ratio of the longitudinal impedance for an exponential taper relative to a linear taper is $\frac{1}{2} \ln R$, where $R = b/a$; this can be greater or less than unity depending on the taper ratio R .

For Gaussian bunches and linear tapering, the kick factors corresponding to Eqs. (4) and (6) are

$$k_{\text{yd,lin}}^{\text{rec}} = \frac{cZ_0(R^2 - 1)(R - 1)}{4\sqrt{\pi}Rl\sigma_z}, \quad (7)$$

$$k_{\text{yd,lin}}^{\text{md}} = \frac{cZ_0(R - 1)^2}{2\pi^{3/2}Rl\sigma_z}. \quad (8)$$

For exponential tapering, the kick factors corresponding to Eqs. (4) and (6) are

$$k_{\text{yd,exp}}^{\text{rec}} = \frac{cZ_0(R - 1)}{2\sqrt{\pi}l\sigma_z} \ln R, \quad (9)$$

$$k_{\text{yd,exp}}^{\text{md}} = \frac{cZ_0}{2\pi^{3/2}l\sigma_z} \ln^2 R. \quad (10)$$

Using $w = 45$ mm, $\sigma_z = 6$ mm, and the geometric parameters specified previously, Fig. 2 compares the normalized kick factors $k_{\text{yd}}l\sigma_z/(cZ_0)$ calculated from Eqs. (7)–(10) with numerical results obtained from ECHO2D [23]. Specifically, ECHO2D is designed to compute wakefields in structures with simple geometries, such as rotationally symmetric configurations or rectangular cross

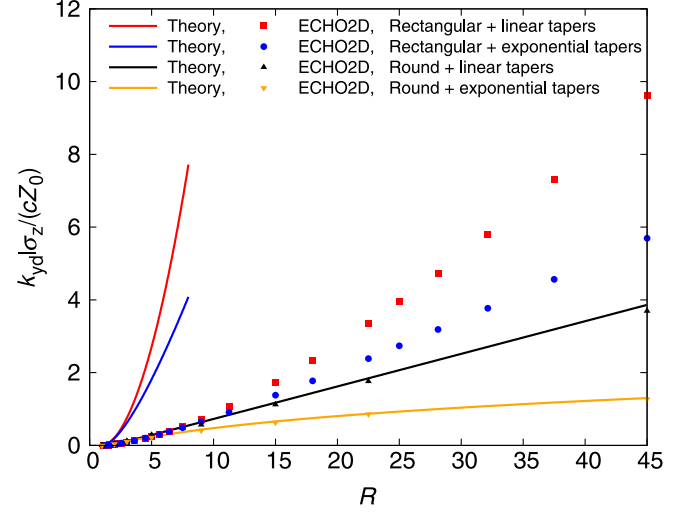


FIG. 2. Normalized dipolar kick factor as a function of the taper ratio $R = b/a$, comparing rectangular and round collimators with linear and exponential taper profiles. All results are shown for the case $w = b$.

sections. Good agreement between the analytical predictions and numerical simulations is observed for round collimators. In contrast, for rectangular collimators, the analytical model significantly overestimates the kick factors, indicating that the impedance models in Eqs. (4) and (6) are not applicable for the chosen collimator geometries with $R \gg 1$ and bunch length in this case. This limitation, discussed in Ref. [28] for a rectangular collimator, arises from the fact that Eqs. (4) and (6) are derived in the low-frequency limit; at higher frequencies, the transverse impedance decreases, leading to an overestimation when using the low-frequency approximation. Despite this limitation, the overall trends clearly indicate that exponential tapering is generally more effective than linear tapering in suppressing the dipolar kick factor, particularly in the regime of small collimator gaps, where $R \gg 1$.

While the simplified model provides valuable analytical scaling laws, practical collimators inevitably involve more complex three-dimensional features and cannot be classified as purely rectangular or round. In particular, the jaw width w and the presence of hollowed side structures (depth d) can substantially alter the wake potential. These effects are explored in the next subsections using numerical simulations, with the goal of identifying realistic design modifications that minimize vertical dipolar impedance without compromising collimation efficiency.

A. Jaw width

We first investigate how the jaw width influences the wake potential using a model without recessed structures ($d = 0$ mm). Figure 3 shows the dependence of the loss factor k_z , vertical dipolar kick factor k_{yd} , and vertical quadrupolar kick factor k_{yq} on the jaw half-width w from

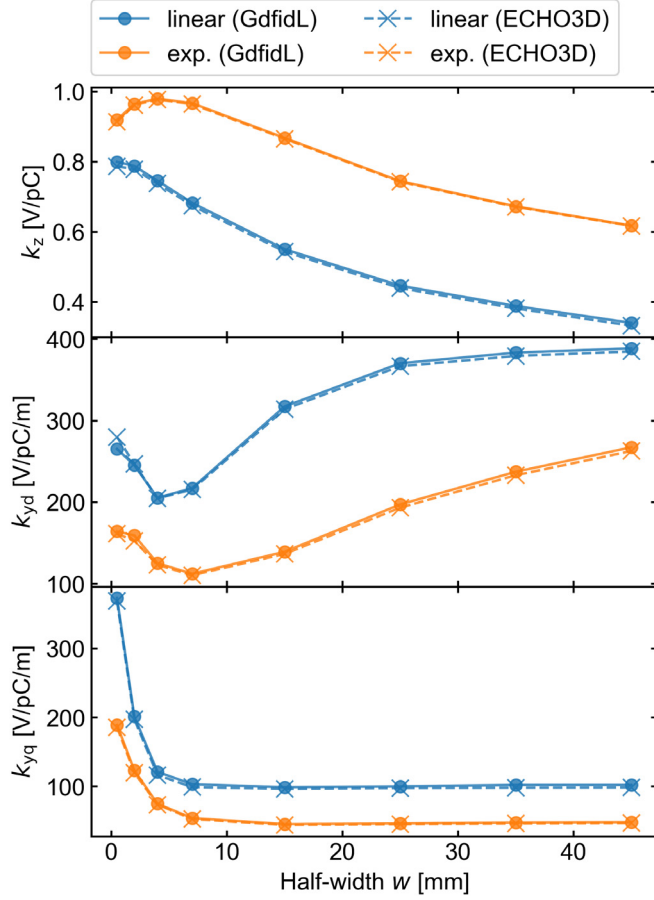


FIG. 3. Dependence of loss factor, vertical dipolar kick factor, and quadrupolar kick factor on the jaw half-width ($\sigma_z = 6.0$ mm).

0.5 to 45 mm obtained from GdfidL and ECHO3D. Since the chamber width is $b = 45$ mm, setting $w = 45$ mm eliminates the gap between the jaws and the chamber wall. The bunch length is fixed at $\sigma_z = 6.0$ mm, which corresponds to the nominal value in the SuperKEKB LER.

It is also observed that the results from the two codes are in good agreement, although GdfidL generally requires finer meshes than ECHO3D to achieve comparable accuracy [29].

The loss factor increases as the half-width decreases, reaching a maximum at a value of approximately 4 mm for the linear taper and 0.5 mm for the exponential taper within the range of this plot. The exponential taper yields consistently higher loss factors than the linear taper, indicating that the linear profile is more favorable for minimizing energy loss, consistent with the findings in [15].

For the dipolar kick factor, the minimum value occurs at a half-width of approximately 4 mm for the linear taper and around 7 mm for the exponential taper. As w increases beyond this minimum, the dipolar kick factor increases with jaw width, particularly for the linear taper. The minimum reduction factor $k_{yd,exp}/k_{yd,lin}$ is observed at $w = 15$ mm, where the ratio reaches 0.44. For $w = 7$ mm, the reduction factor is approximately 0.51.

Theoretical prediction [15] for round collimators gives:

$$\frac{k_{yd,exp}^{rd}}{k_{yd,lin}^{rd}} = \frac{R(\ln R)^2}{(R-1)^2}, \quad (11)$$

For this model, $R = 22.5$, yielding a theoretical reduction factor of 0.47, which is almost equal to the ratio obtained in the simulation with $w = 15$ mm.

B. Recessed and hollowed structure

To mitigate higher-order modes (HOMs), recessed regions have been proposed on the outer sides of the jaws, into which ferrite blocks can be brazed. Beyond their role in HOM damping, we find that these recesses also help reduce the dipolar kick factor.

Figure 4 illustrates the dependence of the loss, dipolar, and quadrupolar kick factors on the recess depth d for both linear and exponential tapers. In this study, the jaw half-width is fixed at $w = 7$ mm, and the recess follows the same taper profile as the outer jaw surface, offset inward by $h = 5$ mm. At $d = 7$ mm, the recess extends fully through the jaw, resulting in a completely hollowed structure.

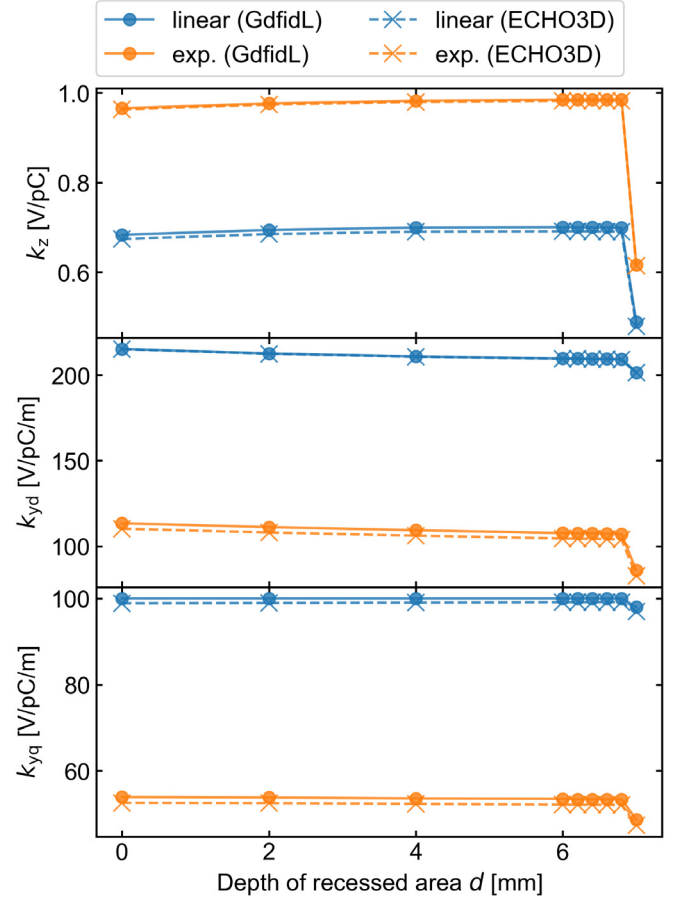


FIG. 4. Dependence of loss, vertical dipolar, and quadrupolar kick factors on recess depth ($\sigma_z = 6.0$ mm).

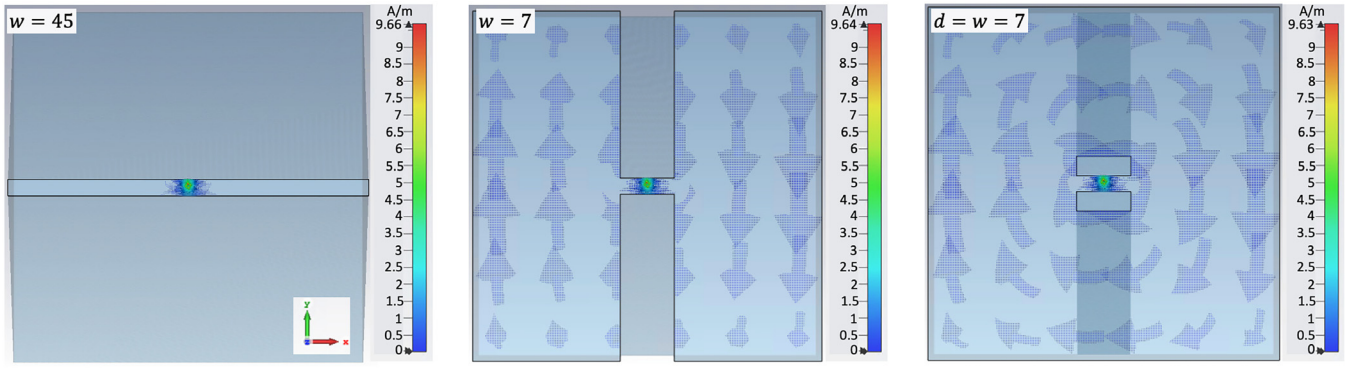


FIG. 5. Magnetic field at the center plane of the collimator with the linear taper for $w = 45, 7$ mm, and the hollowed structure ($d = w = 7$ mm) calculated using CST Wakefield Solver.

The loss factor increases gradually with recess depth, but drops sharply once the structure becomes fully hollowed at $d = 7$ mm. The dipolar kick factor decreases slightly but steadily as the recess deepens, with the strongest suppression observed in the hollowed configuration. By contrast, the quadrupolar kick factor remains largely insensitive to depth until the hollow is complete, at which point a noticeable reduction appears. Overall, the exponential taper benefits more from the hollowed design than the linear taper, as reflected consistently across all three factors.

For validation, Fig. 4 compares the GdfidL results with those obtained using ECHO3D. These results exhibit good agreement; the dependence of loss, dipolar, and quadrupolar factors on recess depth is consistent between the two codes, confirming the robustness of the observed trends.

Figure 5 shows the arrow plot of the magnetic field calculated using CST Wakefield Solver [30]. In the case of $w = 7$ mm, the magnetic field generated by the bunch circulates into the gap formed between the collimator protrusion and the chamber wall. As a result, the magnetic field in the vicinity of the bunch (and consequently the electric field) becomes weaker, leading to a reduction in the dipolar wakefield.

The quadrupolar kick factor increases as the jaw width decreases and shows a rapid rise, particularly when the half-width falls below approximately 7 mm, as shown in Fig. 3. This behavior can be understood from the field distribution near the aperture. When the jaw width is large, the vertical side walls of the jaws lie far from the beam, and the transverse field is nearly uniform, so the quadrupolar component remains weak. As the jaw width decreases, however, the proximity of the side walls enhances the transverse field distortion in the horizontal plane. This steepens the focusing/defocusing gradient near the aperture, leading to a pronounced increase in the quadrupolar impedance and, hence, the quadrupolar kick factor.

As shown in Fig. 5, the pronounced difference between recessed and fully hollowed cases can be understood in terms of electromagnetic boundary conditions. For recessed geometries, the jaw maintains a continuous conducting

boundary, so the fields are largely confined and terminate on the metallic surface. In contrast, when the structure is completely hollowed, the boundary condition changes fundamentally: instead of a solid conductor enclosing the aperture, the opening provides a channel through which magnetic flux can pass. This qualitative change in boundary conditions alters the field distribution. As a result, the hollowed structure behaves more like a distributed transition than a cavity, reducing the magnetic field contribution to the loss factor and dipolar kick factor.

C. One- and both-sided layouts

Movable collimators can be configured to approach the beam either from one side or symmetrically from both sides. One-sided configurations have been used at KEKB [31], while both-sided layouts are employed at PEP-II and SuperKEKB [26,32]. A both-sided design can intercept particles with transverse positions beyond approximately $\pm a$ using a single device, whereas the one-sided layout requires an additional collimator with a phase advance of $\Delta\phi \sim \pi$ (same orientation) or $\Delta\phi \sim 2\pi$ (opposite orientation) to achieve equivalent coverage. Hence, the both-sided layout is more efficient in terms of spatial constraints in the ring.

From the beam dynamics perspective, one-sided collimators introduce asymmetric wakefields, including a monopole component that generates a longitudinally dependent transverse kick, leading to beam tilt and potential emittance growth [33]. Such adverse effects can be mitigated by arranging one-sided collimators in properly phased pairs, again with $\Delta\phi \sim \pi$ or 2π , so that the wakefield-induced kicks cancel to first order, as discussed in [33]. This mitigation scheme was implemented in KEKB through careful optimization of the phase advances between the paired collimators.

Figure 6 illustrates the one-sided model with a linear taper. This model is based on the collimator geometry developed for KEKB [31], which maintains a constant cross-sectional shape (a square in this study) while being

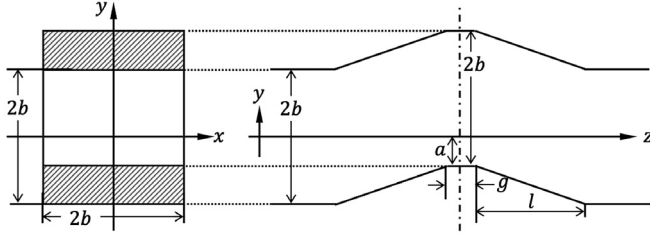


FIG. 6. Dimensions of the simple one-sided (asymmetric) collimator model with linear taper.

tapered along the beam axis, and the entire structure can be moved via bellows chambers installed on both sides. Consequently, parameters such as the jaw width w and the dimensions d and h of the recessed structure are not defined for this geometry. Using the same dimensional parameters as before ($b = 45$ mm, $l = 175$ mm, $a = 2$ mm, $g = 10$ mm), we performed calculations and compared the one- and both-sided layouts.

Figure 7 compares the longitudinal, vertical dipolar, and quadrupolar wake potentials for the one- and both-sided

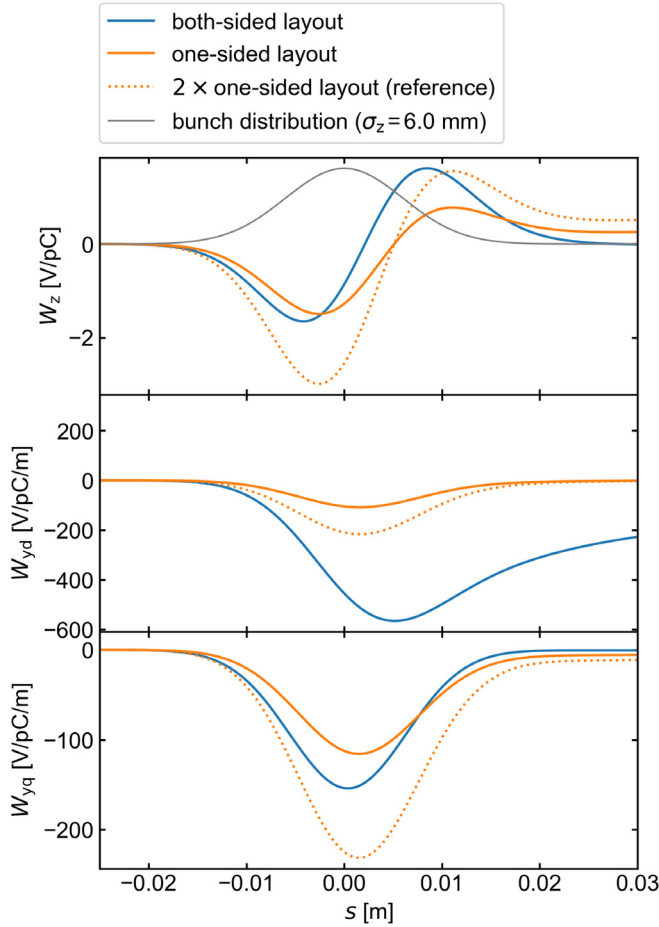


FIG. 7. Wake potentials (longitudinal, vertical dipolar, and quadrupolar) for both-sided and one-sided layouts ($\sigma_z = 6.0$ mm). The one-sided result is scaled for comparison.

models. For fair comparison, the wake potential of the one-sided model is scaled by a factor of 2, assuming a pair of such collimators installed $\Delta\phi \sim \pi$ apart.

The longitudinal wake W_z shows similar structures for both layouts, though the negative peak is slightly smaller in the one-sided case. The dipolar wake W_{yd} is significantly smaller (approximately 70%) in the one-sided layout. Even with two one-sided collimators, the total W_{yd} remains approximately 40% smaller than that of a single both-sided collimator. This indicates that, from the standpoint of dipolar wake potential and the associated kick factor, the one-sided layout is advantageous. However, as shown in Eq. (1), it is important to note that the coherent tune shift depends not only on the kick factor but also on the value of β_y at the collimator's location.

The quadrupolar wake W_{yq} is slightly smaller in the one-sided case, but once doubled, the both-sided configuration yields lower values.

III. 3D GEOMETRY OPTIMIZATIONS FOR SUPERKEKB COLLIMATORS

Based on the results and discussion in the previous sections, we propose several strategies to reduce the term $\sum_i \beta_{y,i} k_{yd,i}$ in the ring. We then quantitatively assess how these strategies affect the TMCI threshold through single-bunch instability simulations, taking the SuperKEKB LER as a case study.

Figure 8 shows the simulation model of the vertical collimator currently used in the SuperKEKB LER. In this model, movable jaws with a width of $w = 7$ mm are inserted into a beam pipe with a circular cross section of diameter 90 mm. The jaws approach the circulating beam symmetrically from above and below [26]. All currently

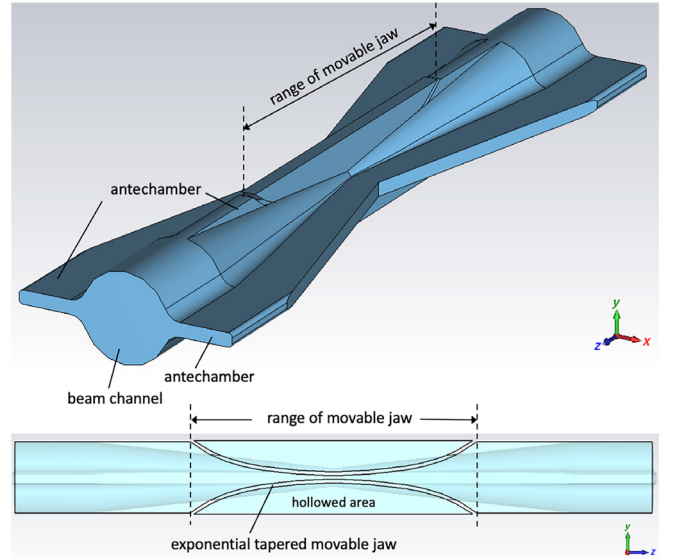


FIG. 8. Simulation model of the vertical collimator in the SuperKEKB LER with $a = 2$ mm.

installed movable jaws use linear tapers with dimensions $l = 175$ mm and $g = 10$ mm, consistent with the simple model discussed earlier.

Ideally, the exponential taper shape should vary depending on the collimator setting a , as described in Eq. (2). However, in practice, it is not feasible to change the jaw shape dynamically. Therefore, in this model, the jaw retains a fixed exponential profile defined for $a = 2$ mm, and the entire structure moves to adjust the aperture.

A major difference from the simple model is that the SuperKEKB LER collimator includes antechambers on both horizontal sides. Additionally, to suppress trapped modes, the inner diameter of both the beam pipe and the antechambers is tapered toward the center of the collimator. In the vertical direction, the movable jaws are also housed within an ante-chamber-like structure with a full width of 14 mm, preventing the beam from directly seeing the step between the jaw and chamber wall.

We consider the following three mitigation strategies: (i) Replacing the linear taper with an exponential taper on the jaws; (ii) applying a hollowed structure to the exponentially tapered jaws; (iii) doubling the number of

collimators and adopting an asymmetric aperture configuration.

Figure 9 compares the wake potentials for each geometry when these strategies are applied. All models assume $a = 2$ mm. In the asymmetric configuration, the lower jaw is used for collimation, while the upper jaw is opened to $a = 22$ mm. The longitudinal wake potentials show similar profiles, with the exponential taper yielding the highest peak, and the hollowed exponential taper showing the lowest.

The geometry has a pronounced impact on the dipolar wake potential. The hollowed structure leads to a significant reduction in amplitude, and the asymmetric layout with hollowed exponential taper achieves the lowest value. A similar trend is observed for the quadrupolar wake, where the lowest amplitudes are also found in the hollowed exponential taper models with symmetric layout. To cross-check these results, we also calculated the wake potentials using ECHO3D. The results are overlaid as black dashed lines in Fig. 9 and show overall good agreement with the GdfidL calculations.

IV. IMPEDANCE MODELING FOR SUPERKEKB LER

Impedance modeling of the main rings has been conducted in previous studies, and the resulting tune shifts showed good agreement with experimental measurements [10,34]. In this study, we follow a similar approach to model the impedance, focusing on the vertical collimators in the LER.

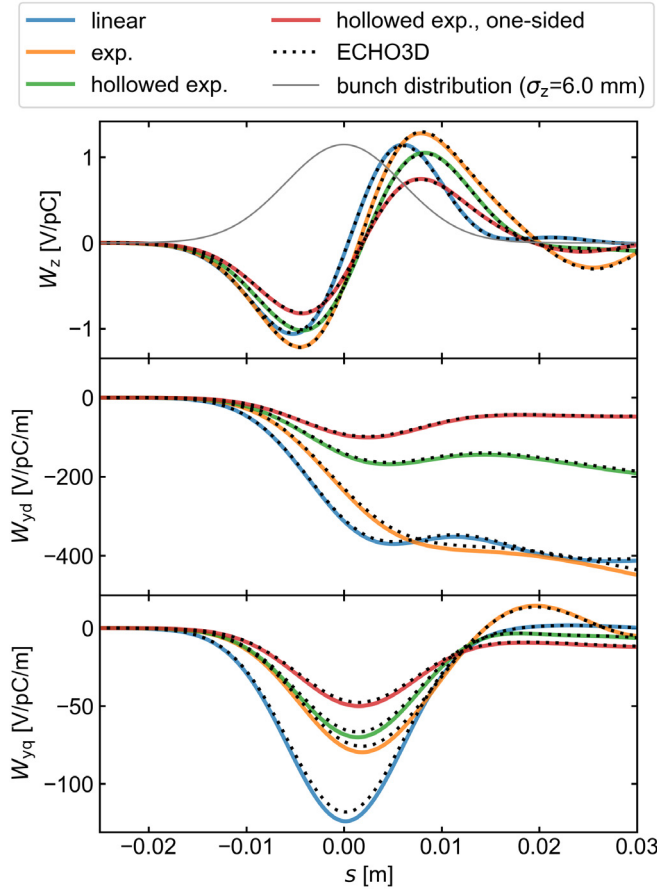


FIG. 9. Longitudinal, vertical dipolar, and quadrupolar wake potentials for each geometry in the SuperKEKB LER collimator ($\sigma_z = 6.0$ mm).

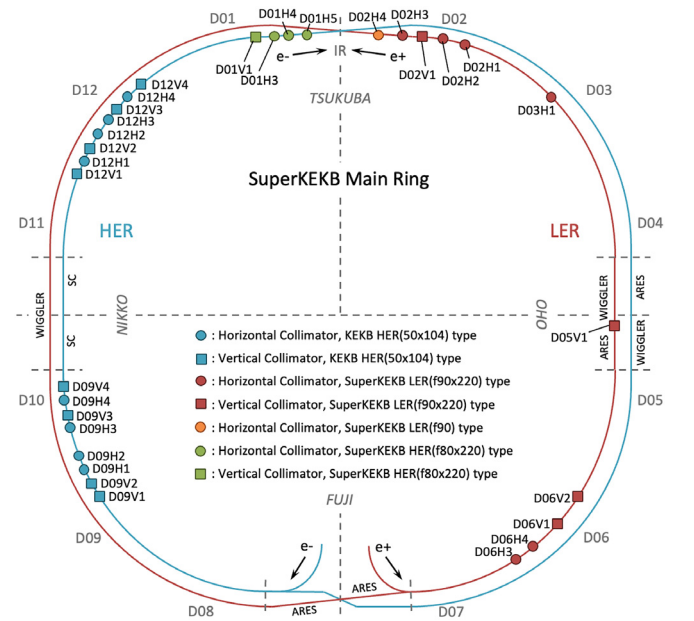


FIG. 10. Locations of collimators in the SuperKEKB main rings. The letters “H” and “V” denote horizontal and vertical collimators, respectively.

Figure 10 shows the locations of the collimators installed in the SuperKEKB main rings. The LER contains four vertical collimators: D02V1, D06V1, D06V2, and D05V1. The D02V1 collimator is located in the interaction region (IR), being the closest vertical collimator to the interaction point (IP), and is mainly used to suppress background arising from the stored beam. The D06V1 collimator, positioned just downstream of the injection point, targets background from injected beams. The D06V2 collimator provides additional suppression in support of D06V1.

The D05V1 collimator, situated in the Oho straight section, is part of the newly introduced nonlinear collimator (NLC) system [35,36]. This system utilizes vertical nonlinear kicks generated by a pair of skew sextupole magnets, allowing for a wide aperture and low β_y at D05V1. While the mechanical structure of this collimator is the same as others, it was installed during the long-term shutdown (LS1) specifically to reduce vertical impedance and has been in full operation since the 2024 winter run, mainly to suppress storage-beam-induced backgrounds [37]. In this study, all impedance reduction strategies are applied only to collimators other than D05V1.

Table I summarizes example aperture settings for the vertical collimators during the 2024 physics run, assuming an optics of $\beta_y^* = 1.0$ mm, which is a vertical beta function at the IP. The aperture values are given in millimeters and in units of σ_y , representing the beam size at each location. The final focusing magnet QC1RP, with an inner beam pipe diameter of only 27 mm, is located in a region of large β_y and becomes the most restrictive aperture in terms of σ_y except for the collimators [38]. As this region lies within the detector, it also constitutes a background hotspot. Therefore, collimator apertures are generally set to be smaller than the local σ_y at QC1RP.

The D06V1 collimator, due to its small aperture and high β_y , contributes the most to the vertical impedance. For D05V1, the effective aperture is calculated based on the strength of the skew sextupole magnets (SNAP) and the R_{34} element of the transfer matrix from the upstream SNAP to the D05V1 location [36]. These settings serve as the basis for impedance modeling in this study.

TABLE I. Aperture settings of vertical collimators and the QC1RP beam pipe during the 2024 physics run. Values are given in millimeters and as multiples of σ_y . Kick factors are evaluated for $\sigma_z = 6.0$ mm.

Name	β_y (m)	Aperture (mm)	σ_y	$\beta_y k_{yd}$ [10^{15} V/C]
QC1RP	764.0	± 13.5	69.1	4.66
D02V1	11.9	± 1.2	49.2	7.41
D06V1	67.4	± 2.2	37.9	16.17
D06V2	20.6	± 3.0	93.5	3.18
D05V1	4.05	-2.5	34.4	0.76

For the scenario involving one-sided hollowed exponential taper jaws, we assume that a second collimator of identical geometry is installed at a location $\Delta\phi \sim \pi$ from each original collimator (except D05V1). The aperture of the additional collimator is scaled according to the square root of the local β_y , and the vertical wake contribution is weighted using the corresponding β_y . Although this scenario is conceptual, actual implementation would be complicated by interference with magnets, diagnostics, and other components. Therefore, full adoption of this mitigation measure would likely require reoptimization of the lattice and surrounding hardware layout.

V. SINGLE-BUNCH TRACKING SIMULATIONS

Particle tracking simulations were carried out using PyHEADTAIL, incorporating the wake potentials evaluated for the collimators in the previous sections, together with contributions from other vacuum components as reported in [10,34]. The longitudinal impedance includes monopolar components, including coherent synchrotron radiation (CSR) and coherent wiggler radiation, while the transverse impedance includes both dipolar and quadrupolar terms. The input of the wake potentials for the PyHEADTAIL simulations was configured with $\sigma_z = 0.5$ mm.

Table II lists the main machine parameters used in the simulations for the $\beta_y^* = 1.0$ mm optics. The vertical chromaticity value was taken from measurements, while all other parameters were based on accelerator models. Each simulation was performed with 10^6 macroparticles tracked for 25,000 turns, and the longitudinal beam distribution was divided into 500 slices. A bunch-by-bunch transverse feedback system was included with a damping time of 100 turns, corresponding to 1 ms [39].

TABLE II. Main machine parameters used for PyHEADTAIL simulations with $\beta_y^* = 1.0$ mm optics. Damping times τ_x , τ_y , and τ_z correspond to the horizontal, vertical, and longitudinal directions, respectively. The average β_y excludes the high- β_y region near the IR.

Circumference (m)	3016.3
Beam energy (GeV)	4
rf voltage (MV)	9.12
Energy loss per turn (MV)	1.76
Harmonic number	5120
Bunch length at 0 mA (mm)	4.54
Synchrotron tune	0.0233
Momentum compaction factor	2.98×10^{-4}
Energy spread at 0 mA	7.52×10^{-4}
Vertical chromaticity	1.53
Average β_y (m)	19.1
Damping times $\tau_x/\tau_y/\tau_z$ (ms)	53.0/53.0/26.5

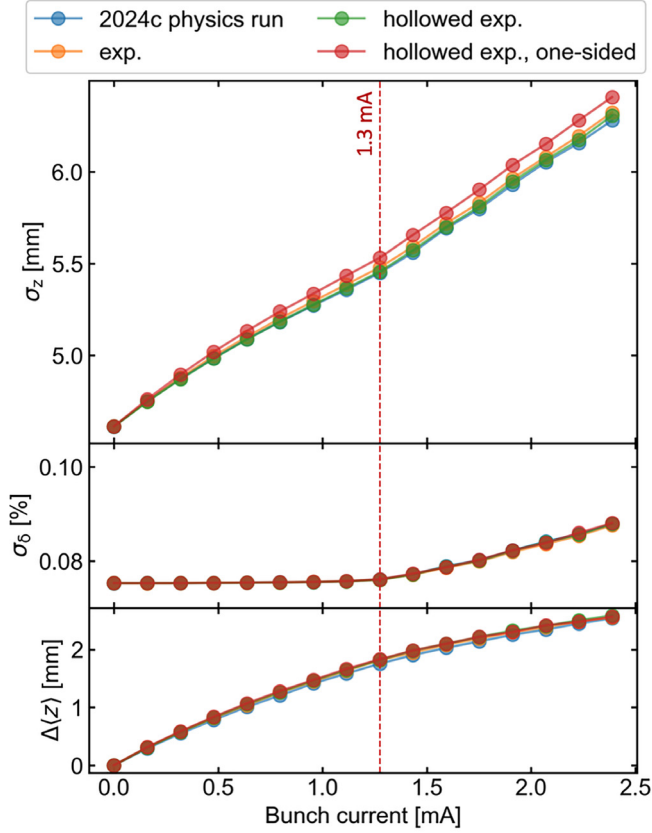


FIG. 11. Bunch length σ_z (top), energy spread σ_δ (middle), and longitudinal centroid shift $\Delta\langle z \rangle$ (bottom) as a function of bunch current for each mitigation scenario. The vertical dashed line indicates the MWI threshold.

A. Longitudinal direction

Figure 11 shows the evolution of bunch length (σ_z) and energy spread (σ_δ) as a function of increasing bunch current for each mitigation strategy. Compared to the linear taper case (labeled “2024c physics run”), the exponential taper tends to yield slightly longer bunch lengths, owing to its larger contribution to the longitudinal inductive impedance. In the case of the one-sided layout with a hollowed exponential taper, the number of installed collimators effectively doubles, further increasing the longitudinal inductive impedance and resulting in the most pronounced bunch lengthening.

In all cases, the energy spread increases sharply beyond approximately 1.3 mA, indicating that the threshold of the microwave instability (MWI) lies near this current, while the differences among the mitigation schemes are negligible. As discussed in Ref. [10], this is because the instability is predominantly driven by CSR.

It is observed that the behavior of the centroid shift $\Delta\langle z \rangle$ is nearly identical across all mitigation scenarios, with no significant differences among the various collimator configurations. This result is consistent with the fact that the proposed geometry optimizations primarily modify the

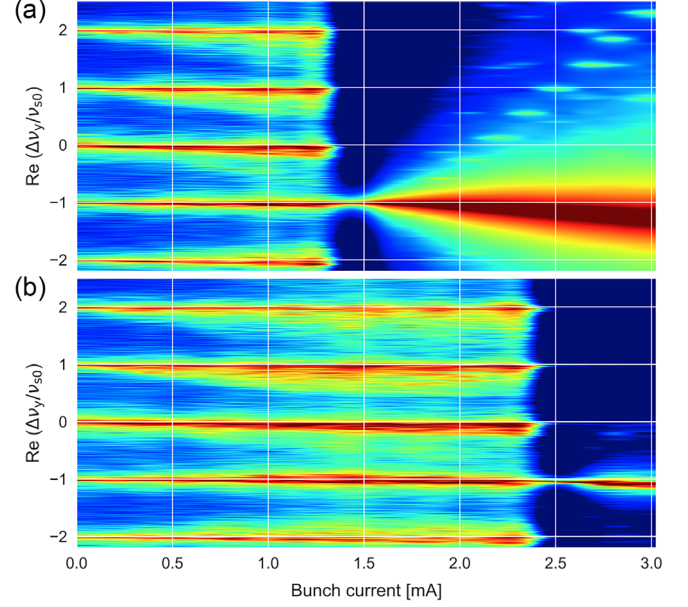


FIG. 12. Real part of vertical coherent mode frequency versus bunch current: (a) 2024 physics run configuration, (b) hollowed exponential taper with one-sided layout.

inductive component of the longitudinal impedance, which induces bunch lengthening. In contrast, the impact on the resistive component—the primary driver of the longitudinal phase shift—is negligible, as the resistive impedance is dominated by other vacuum components such as the rf cavities. Therefore, the proposed modifications effectively mitigate transverse instabilities without significantly affecting the longitudinal centroid dynamics.

B. Vertical direction

Figure 12 presents the real part of the vertical betatron tune for coherent transverse modes as a function of bunch current, as simulated using `PyHEADTAIL`. Two scenarios are shown here: (a) the current 2024 physics run configuration and (b) the hollowed exponential taper with a one-sided layout.

These results were obtained via Fourier analysis of the first ten moments of the particle distribution, following the method in [40]. In the 2024 configuration, the 0 mode shifts with increasing bunch current, although this trend is not very pronounced, likely due to the presence of nonzero vertical chromaticity, and eventually couples with the -1 mode at around 1.3 mA, marking the TMCI threshold. When the hollowed exponential taper with a one-sided layout is applied, the threshold increases to approximately 2.5 mA, demonstrating its strong mitigation effect.

Figure 13 shows the total β_y -weighted dipolar kick factor ($\sum_i \beta_{y,i} k_{y,d,i}$) at $\sigma_z = 6.0$ mm, alongside the corresponding TMCI threshold for each mitigation strategy. The stacked bars break down contributions into three categories: vertical collimators, IR, and other components. Since only the

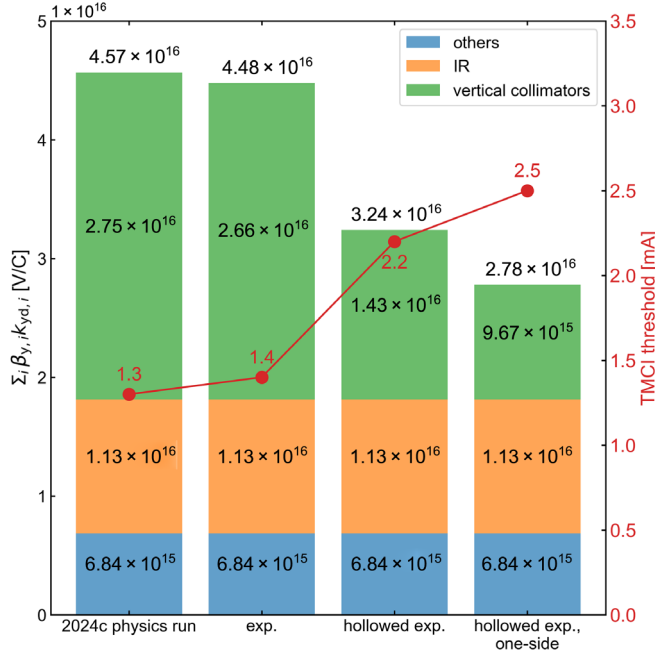


FIG. 13. Total β_y -weighted dipolar kick factor at $\sigma_z = 6.0$ mm and corresponding TMCI threshold for each mitigation strategy.

vertical collimators are modified in this study, the IR and other contributions remain constant.

In the 2024 configuration, the total vertical dipolar term is 4.57×10^{16} V/C. Switching to an exponential taper reduces this to 4.48×10^{16} V/C, and further adopting a hollowed taper decreases it to 3.24×10^{16} V/C. Previous work has shown that the NLC implementation can reduce this sum by up to approximately 1×10^{16} V/C [34], and the hollowed exponential structure achieves even greater reduction.

Applying a one-sided layout with the hollowed exponential taper reduces the total further to 2.78×10^{16} V/C, confirming the effectiveness of this approach in mitigating vertical wakefields and raising the TMCI threshold.

VI. SUMMARY AND CONCLUSION

The influence of collimator geometry on wakefields was investigated using simplified models. Based on these results, three mitigation strategies were proposed to reduce the vertical dipolar kick factor and raise the TMCI threshold. Their effectiveness was demonstrated through simulations using the SuperKEKB LER as a case study.

Among the tested configurations, the hollowed exponential taper structure, especially when combined with a one-sided layout, proved to be highly effective. This configuration achieved a greater reduction in $\sum_i \beta_{y,i} k_{y,d,i}$ than the NLC system, leading to an increase in the TMCI threshold of up to approximately 90%.

However, in order to implement these measures in practice, several engineering constraints must be addressed.

While a hollow structure can effectively reduce the dipolar kick factor, its mechanical strength is generally lower than that of conventional designs, and thermal displacements near the tip may become more pronounced. In addition, cooling structures must be incorporated to remove the thermal load. Although the influence of the hollow structure dimensions on the wakefields is not examined in detail here, in practice, the design freedom will be constrained by such mechanical limitations.

Moreover, introducing a hollow structure creates a gap between the jaw and the chamber, which may give rise to secondary effects such as electrical discharges and the excitation of trapped modes that could drive multibunch instabilities. Although the present study focuses on short-range wakefields and single-bunch dynamics, addressing these long-range effects is essential for practical implementation. As discussed in Sec. III, this recessed (hollowed) geometry, which also contributes to reducing the dipolar kick factor, was originally investigated for housing HOM absorbers. By integrating damping materials, such as ferrite blocks, into this structure, such trapped modes could potentially be suppressed. Consequently, future research will comprehensively evaluate these long-range wakefields alongside practical engineering challenges, including the optimization of absorber materials, robust bonding techniques, and detailed thermal and mechanical stress analyses.

The one-sided configuration also offers strong mitigation benefits, but it presents challenges related to installation space. Integrating such collimators into an existing ring would likely require reoptimization of the lattice and relocation of nearby components. Additionally, because particle shielding would be spatially distributed over a wider area in the ring, the potential impact on detector backgrounds should be carefully assessed.

It should be noted, however, that adopting a one-sided configuration offers an advantage in terms of redundancy, since the unused jaw can be reserved as a spare. In a two-sided configuration with a narrow set aperture of only a few millimeters, if the beam strikes one of the jaws due to beam losses, material from that jaw may scatter and adhere to the opposing jaw. In such cases, the background-suppression capability of both jaws can deteriorate [41]. By contrast, a one-sided configuration avoids this risk and can therefore provide a more robust collimation system, particularly in high-current storage ring operations at the ampere level.

At present, the verification is primarily based on simulations, and we plan to conduct complementary studies from the theoretical perspective in parallel. In addition, it should be noted that the asymmetric geometry of the one-sided collimators introduces a monopole transverse wakefield. This wakefield can impart a longitudinally dependent transverse kick to the on-axis beam, which may potentially lead to emittance growth. A detailed quantitative evaluation of this effect falls outside the scope of the present study and

will be a subject of future investigations, following the framework established in [33].

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DATA AVAILABILITY

The data that support the findings of this article are not publicly available. The data are available from the authors upon reasonable request.

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