

Statistical lattice design and topological selection of a 300–600 MeV scaling fixed-field alternating gradient proton driver at CSNS Phase-II upgrade

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A 300–600 MeV high-intensity scaling fixed-field alternating gradient (FFA) facility is proposed as a multipurpose proton driver for the China Spallation Neutron Source (CSNS) Phase-II upgrade. To identify a viable lattice configuration under strict engineering constraints, a statistical analysis methodology based on large-sample ray-tracing simulations comprising over 360,000 parameter configurations is applied. A two-stage scanning pipeline evaluates the topological limits of both FDF triplet and FD doublet structures under realistic multisource error budgets. Factoring in phase-space geometry, error tolerance, and facility integration, the findings indicate that a spiral FD doublet lattice with a super-periodicity of $N = 16$ is preferred as the structural baseline; it avoids parameter bottlenecks and yields a large, resonance-sparse stable region. To characterize the coupled high-dimensional dynamics quantitatively, a machine learning feature attribution framework is applied. The framework identifies the spiral edge angle and the full-ring vertical tune as the primary drivers of beam survivability, and maps the underlying resonance stopbands from tracking data alone. Based on these results, a baseline configuration is established with a wide dynamic tune allowance that maintains long-term beam survivability within the physical and engineering boundaries of the facility.

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I. INTRODUCTION

The planned Phase-II upgrade of the China Spallation Neutron Source (CSNS) will increase its linac energy to 300 MeV and repetition rate to 50 Hz [1,2]. This presents a unique opportunity: while the existing rapid cycling synchrotron (RCS) continues to operate with a 25 Hz beam, a second, high-power 25 Hz proton beam becomes available for new applications. To leverage this new capability, a 300–600 MeV high-intensity fixed-field alternating gradient (FFA) proof-of-principle facility is proposed. This multipurpose driver is designed to support diverse downstream applications with distinct beam demands: medical radioisotope production benefits from a high average current and flexible pulse structures, while a white neutron source requires fast extraction and precise

bunch compression [3,4]. As a multipurpose driver, an FFA offers distinct advantages over a conventional RCS by avoiding dynamic AC losses and providing enhanced flexibility in beam time structure and extraction timing. Furthermore, while nonscaling FFAs cross multiple resonance lines during acceleration [5], a scaling-FFA maintains constant transverse tunes across the full energy range via spatial field scaling [6], theoretically eliminating structural resonance crossing.

Historically, scaling-FFAs have been successfully developed and explored for various proton applications. A pioneering example is the 150 MeV radial-type scaling-FFA at Kyoto University (KURRI), which demonstrated high-repetition-rate proton acceleration and ADSR beam injection [7]. More recently, high-power scaling and non-scaling FFA designs have been extensively investigated for multi-GeV proton drivers (such as the ISIS upgrade studies [8]) and compact hadron therapy rings [9,10]. Compared with these machines, the CSNS Phase-II driver operates in a distinct parameter regime: unlike the KURRI ring with moderate beam energy or the ISIS-FFA with large ring circumferences, the CSNS-FFA accelerates intense protons across 300–600 MeV under stringent footprint constraints ($R_0 = 12$ m) and a highly restricted radial beam

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envelope ($\Delta r \leq 0.8$ m). These tight spatial and hardware boundaries turn the scaling-FFA lattice design into a highly constrained optimization problem. To maintain global stability and independent tune control within such a compact perimeter, each cell utilizes an alternating sequence of focusing (F) and defocusing (D) magnets. Additionally, to supply sufficient vertical focusing within this tightly constrained layout, a spiral edge angle is introduced as a critical degree of freedom. This additional edge focusing suppresses multipole field saturation at high intensity and keeps the vertical beam envelope small [11]. Under the mandatory 0.8 m radial beam envelope limit and the 1.66 T peak magnetic field ceiling, high-order non-linear fringe fields inevitably couple with multisource manufacturing and alignment errors. Consequently, isolated local optimization around nominal tunes is insufficient, and a quantitative statistical analysis across a large parameter space is required to establish robust topological boundaries and optimal focusing regions.

To find the optimal configuration under the CSNS constraints, this study replaces local optimization around isolated nominal tunes with a statistical methodology based on large-sample simulations. The compact focusing-defocusing (FD) doublet and the focusing-defocusing-focusing (FDF) triplet (which offers a longer straight section) serve as lattice probes. The scanning pipeline covers over 360,000 parameter configurations, and error budgets are included. In this study, full evaluation of 3D space-charge effects across a parameter space of this size is computationally prohibitive. Based on operational experience with the CSNS RCS, the maximum space-charge tune shift (ΔQ) for this proton driver is estimated to remain below 0.2 [12]. We therefore adopt a margin-based design strategy. The bare-lattice nonlinear dynamics are mapped, and configurations with large dynamic aperture (DA) stable islands are selected, providing a tune tolerance margin sufficient for the anticipated 0.2 tune depression. The statistical trends provide the quantitative justification and parameter guidance for the CSNS baseline lattice.

The remainder of this paper is organized as follows. Section II describes the analytical physics model, the CSNS engineering boundaries, and the two-stage statistical methodology. Section III evaluates the geometric and topological constraints of the FDF and FD structures to determine the baseline super-periodicities. Section IV investigates parameter subspaces to identify the nonlinear dynamic mechanisms that affect beam survival. Section V presents a topological comparison under common error constraints, leading to the baseline selection. Section VI uses machine learning feature attribution to analyze the high-dimensional dynamics. Section VII presents the baseline lattice configuration and optical verification. Section VIII discusses the methodological boundaries and outlines future hardware-oriented optimization. Concluding remarks follow in Sec. IX.

II. LATTICE MODEL, ENGINEERING CONSTRAINTS, AND METHODS

The macroscopic baseline parameters of the CSNS multipurpose scaling-FFA are summarized in Table I. The facility accelerates a proton beam of 2.08×10^{13} particles per pulse from 300 MeV to 600 MeV at a repetition rate of 25 Hz. The ring has a reference radius of $R_0 = 12$ m and must accommodate drift sections exceeding 1.8 m for injection and extraction hardware. These requirements impose tight geometric boundaries on the lattice topology. Given this spatial constraint, the core challenge is the single-cell magnetic design.

In the physical model of a scaling-FFA, maintaining a constant transverse tune relies on the mathematical description of the primary magnetic field and its fringe-field falloff [13]. We performed the ray-tracing simulations using the FixField code [14], which evaluates overlapping magnetic fields by spatial vector superposition of the analytical F and D components. This approach follows the same methodology used by Zgoubi and OPAL [15,16]. Within this framework, the midplane magnetic field distribution of an individual magnet follows the classical spatial scaling law:

$$B_z(r, \theta) = B_0 \left(\frac{r}{r_0} \right)^K F \left(\theta - \zeta \ln \frac{r}{r_0} \right), \quad (1)$$

where (r, θ) denotes the cylindrical coordinate system with its origin at the center of the ring. Here, $r_0 = 12$ m is defined as the reference radius, and $B_0 = 1.65$ T represents the nominal peak midplane magnetic field at this radius. The parameter K is the constant geometric field index providing horizontal linear focusing, and ζ is the spiral angle governing the edge focusing. The function F represents the azimuthal modulation of the magnetic field, which dictates the alternating-gradient structure and the fringe-field profile. A fundamental physical consequence of this scaling law is the geometric similarity of closed orbits; orbits at different momenta are simply scaled radially, which constrains the total radial beam envelope Δr within the vacuum chamber.

TABLE I. Design parameters for CSNS-FFA.

Parameters	Design value
Circumference (m)	75.40
Injection energy (MeV)	300
Extraction energy (MeV)	600
Injection beam power (kW)	25
Average beam current (μ A)	83.4
Particles per pulse	2.08×10^{13}
Repetition rate (Hz)	25

To analytically model the fringe-field falloff embedded within the azimuthal function F , the Enge function is employed:

$$g(s) = \left[1 + \exp \left(C_0 + \sum_{n=1}^5 C_n \left(\frac{s}{\lambda} \right)^n \right) \right]^{-1}, \quad (2)$$

where s denotes the perpendicular (normal) distance measured from the effective field boundary (EFB) of the magnet.

The fringe-field falloff in this study is based on a 3D magnetic simulation. Guided by previous prototype studies [17], a 3D F magnet model was simulated with the OPERA solver to resolve spatial leakage and local saturation effects at the acute pole tips before the large-scale parameter scan. The analytical Enge coefficients (C_0 through C_5) used in the FixField ray-tracing model were obtained by fitting the fringe-field profile extracted from this 3D OPERA model. Specifically, a multiparameter optimization algorithm was employed to systematically adjust the Enge parameters, minimizing the discrepancy between the analytical FixField model and the OPERA 3D field map. Figure 1 shows the resulting comparison between the fitted analytical model and the 3D OPERA simulation, evaluated along circular paths of constant radius ($r = 11.3$ m and $r = 11.8$ m). The analytical model successfully captures the peak field amplitudes and the primary gradients of the 3D simulation. Observable differences exist: at the 11.8 m radius, the OPERA profile is slightly broader near its peak, a characteristic of the onset of local magnetic saturation. This representation is considered a conservative foundation for the tracking studies. The fitted Enge model parameters used in the FixField code, which are identical

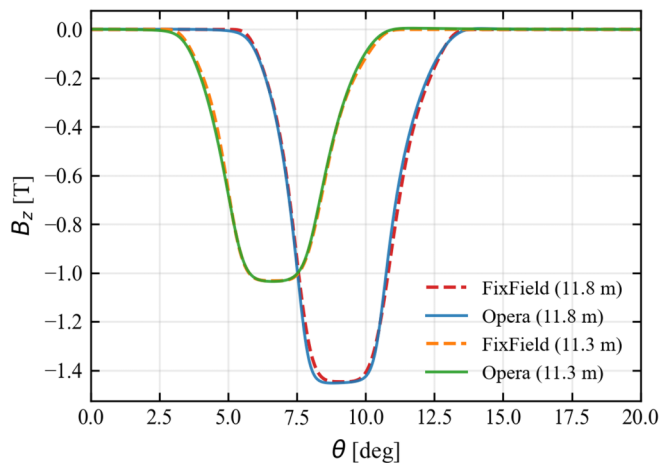


FIG. 1. Comparison of the vertical magnetic field (B_z) profiles along the azimuthal angle between the analytical Enge model (FixField) and the 3D magnetostatic calculation (OPERA). The dashed and solid lines correspond to the FixField analytical Enge model and the 3D OPERA model, respectively.

for both the magnet entrance and exit edges, are listed as follows: $\lambda = 0.3$ m, $C_0 = 0$, $C_1 = 2.8$, $C_2 = -1.8$, $C_3 = 1.5$, and $C_4 = C_5 = 0$.

Six geometric parameters define the single-cell analytical field. The field index K determines the radial gradient of the scaling field and provides horizontal linear focusing. The field ratio R_{DF} , defined as the absolute ratio of the peak magnetic fields of the D and F magnets ($R_{DF} = |B_D^0|/|B_F^0|$), sets the alternating-gradient balance and the vertical tune. The spiral angle ζ defines the geometric skew angle of the magnet pole faces, using fringe-field effects to provide vertical focusing [11].

Figure 2 shows the schematic layout of a single cell. To parameterize the azimuthal layout and quantify the degree of fringe-field superposition within a single cell, the structural geometry is defined by four quantities: the azimuthal widths of the focusing (θ_{M1}) and defocusing (θ_{M2}) magnets, the short drift spacing between them (θ_S), and the half-width of the long drift section (θ_L). From these spatial metrics, the overall azimuthal compactness of the lattice is described by the packing factor α , formulated as the ratio of the magnetic field region to the total single-cell extent:

$$\alpha = \frac{\theta_{M1} + \theta_S + \theta_{M2}}{\theta_{M1} + \theta_S + \theta_{M2} + 2\theta_L}. \quad (3)$$

Two reference spacing factors define the internal geometric balance. The first, $R_{sm1} = \theta_{M2}/\theta_{M1}$, specifies the relative azimuthal footprint of the D magnet against the F magnet. The second, $R_{sm2} = \theta_S/\theta_{M1}$, normalizes the intracell short drift. These geometric variables particularly R_{sm2} control the magnetic field overlap between adjacent F and D magnets, which influences the nonlinear optical behavior.

Configuration screening must respect the engineering framework of the CSNS multipurpose proton driver. First, the radial beam envelope must satisfy $\Delta r \leq 0.8$ m, which represents the absolute physical envelope necessary for manufacturing the ultrawide vacuum chamber. It is defined as $\Delta r = r_{\max}(E_{\text{ext}}) - r_{\min}(E_{\text{inj}})$, which includes the

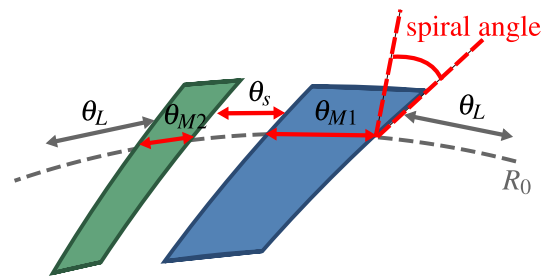


FIG. 2. Schematic layout of a single cell, illustrating the spiral magnet shape and the core azimuthal geometric parameters.

intracell orbit scallop (amplitude modulation) characteristic of alternating gradient focusing. Second, at the 600 MeV extraction energy, the maximum magnetic field on closed orbit is capped at $B_{\max} < 1.66$ T to avoid non-linear saturation of the silicon steel poles, which controls the overall construction cost. Third, the narrow vertical magnet gaps constrain the vertical beta function at the injection point to $\beta_z(s=0) < 8$ m to contain the vertical beam envelope. Fourth, the straight section angle within the analytical magnetic field boundaries exceeds 9° (approximately $\Delta s \geq 1.8$ m at the reference radius). This accommodates the injection stripping foils, radio-frequency cavities, and diagnostic equipment.

Direct evaluation of the DA across a continuous multi-dimensional parameter space is computationally expensive. A two-stage statistical scanning strategy is used. The first stage uses Latin-hypercube sampling over wide continuous bounds to identify the linear-optical feasible region that satisfies the vertical beam envelope, and straight-section requirements [18]. This filters out unviable configurations. Having defined the feasible domain, the second stage applies focused sampling within this region using 2D polygons or 3D convex hulls. During the multiparticle tracking evaluation in this stage, simultaneous perturbations are injected into each configuration: a 0.5% relative magnetic field random error applied to K , B_0 , and ζ , and a 0.5 mrad absolute alignment error encompassing spatial translation and three-dimensional rotation. To efficiently determine the stable phase-space boundary, a bisection search algorithm is employed to calculate the DA [19]. To consolidate the multidimensional phase-space boundary into a single comparative scalar metric for the large-scale statistical scan, the initial particle amplitudes are constrained such that the horizontal and vertical physical emittances are kept equal ($\epsilon_x = \epsilon_z = \epsilon$). The bisection algorithm iteratively searches along this diagonal path to find the maximum surviving emittance $\epsilon_{\max}$. The multiparticle tracking is evaluated at the 300 MeV injection energy over 1000 turns, a physical duration specifically corresponding to the charge-exchange beam injection phase of the CSNS facility. With these simultaneous perturbations applied, the resulting DA ($\epsilon_{\max}$) can effectively serve as a measure of the high-order dynamic robustness of each configuration against engineering symmetry breaking.

To compare topologies with different super-periodicities (N) on a common scale, the measurement criteria are modified. Because independent random errors are applied to each cell, the ideal super-periodicity of the lattice is broken [20]. This study uses the full-ring tune (Q_x, Q_z) rather than the single-cell phase advance as the primary metric for transverse dynamics. This approach describes the global resonance behavior over a complete revolution and enables cross-topology comparisons under symmetry-breaking conditions.

III. TOPOLOGICAL CONSTRAINTS ON SUPER-PERIODICITY AND BASELINE ESTABLISHMENT

The evaluation of topological boundaries begins with the geometric feasibility of the FDF triplet structure. While increasing the super-periodicity N generally reduces the betatron phase advance per cell and might appear dynamically favorable, the FDF topology is restricted by spatial and cost limits within the CSNS engineering framework. Because each FDF cell incorporates three distinct magnets, attempting to configure the lattice with $N > 12$ results in a total magnet count that violates the maximum ring circumference limit and the straight-section spatial budget required for injection and extraction hardware. Configurations with higher super-periodicities are thus filtered out by linear optical constraints. The baseline for the FDF structure is therefore set at $N = 12$.

Conversely, the structurally compact FD doublet requires only two magnets per cell, geometrically permitting the exploration of higher super-periodicities within the range of $N \in [12, 16]$. For each discrete super-periodicity, exactly 20,000 viable parameter configurations—having successfully passed the first-stage screening for linear optics and closed-orbit geometric limits—were subjected to the second-stage multiparticle tracking evaluated under the relative error budget. To systematically evaluate the intrinsic topological robustness of different super-periodicities, we introduce the “DA failure rate” as a primary comparative metric. This rate is defined as the percentage of simulated configurations within the 20,000-sample pool that completely fail to maintain a viable DA under the 0.5% relative error budget. By measuring this metric, we can directly quantify the probability of beam loss driven by realistic engineering symmetry breaking.

As illustrated in Fig. 3, the statistical results reveal a strong dependence of beam survivability on the super-periodicity N : configurations with $N \leq 14$ exhibit a DA failure rate that exceeds the 10% engineering threshold, whereas a systematic statistical enhancement in both survival probability and the median DA is observed as N increases toward 16.

The physical origin of this statistical divergence is examined in Fig. 4, which projects the evaluated DA distributions onto both the global tune space (Q_x, Q_z) [Figs. 4(a)–4(e)] and the control parameter subspace (K, R_{DF}) [Figs. 4(f)–4(j)]. The bottom row of Fig. 4 shows that the constraint governing beam survivability is a “parameter bottleneck” in the accessible field index K . This bottleneck originates from the interplay between the radial excursion limit and the spatial scaling law of the FFA. Within the CSNS engineering framework, this geometric definition constrains different topologies to different operational domains.

For super-periodicities like $N = 12$, the longer single-cell length requires a higher betatron phase advance to

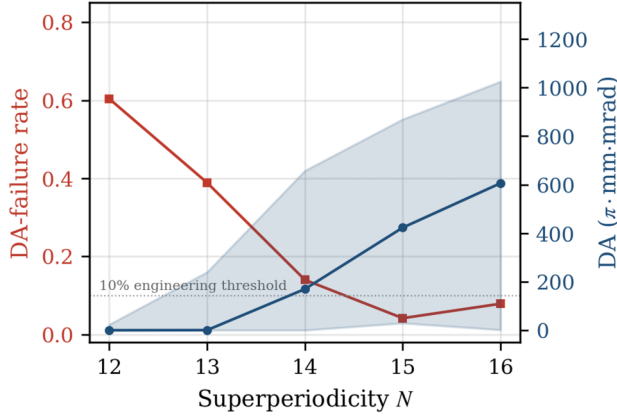


FIG. 3. Statistical dependence of beam survivability on the lattice super-periodicity (N). The red curve with square markers (left axis) illustrates the DA failure rate, with the horizontal dotted line marking the 10% strict engineering acceptance threshold. The solid blue curve and the corresponding shaded region (right axis) represent the median DA and its 10th–90th percentile distribution band, respectively.

maintain global stability, amplifying the intracell orbit scallop. To accommodate this amplified scallop without violating the 0.8 m vacuum envelope budget, the full radial extent of the closed orbits (Δr_{co}) must be restricted.

According to the spatial scaling law [21], Δr_{co} is determined by

$$\Delta r_{co} = r_1 \left[\left(\frac{B\rho_2}{B\rho_1} \right)^{\frac{1}{K+1}} - 1 \right], \quad (4)$$

where r_1 is the reference injection radius, and $B\rho_1$, $B\rho_2$ represent the magnetic rigidities at injection and extraction, respectively. Since the designed energy range is set, a sufficiently large field index K is mandatory to compress this radial extent, securing the necessary spatial margin to fit within the vacuum chamber. The viable operational window is constrained to a high-gradient regime, with $K \in [9.8, 12]$ [Fig. 4(f)]. In the $N = 16$ lattice, the shorter cells require less phase advance, suppressing the scallop amplitude. This geometric relaxation allows the viable field index to drop to a broader range of $K \in [6.8, 12]$ [Fig. 4(j)]. The escalation of K determines the global resonance behavior. According to the spatial scaling law $B \propto r^K$, an inflated field index amplifies high-order multipole strengths—specifically sextupole and octupole gradients. For the $N = 12$ lattice, these strong driving terms cause the resonance stopbands to broaden and overlap, covering the tune footprint [Fig. 4(a)]. This resonance overlap

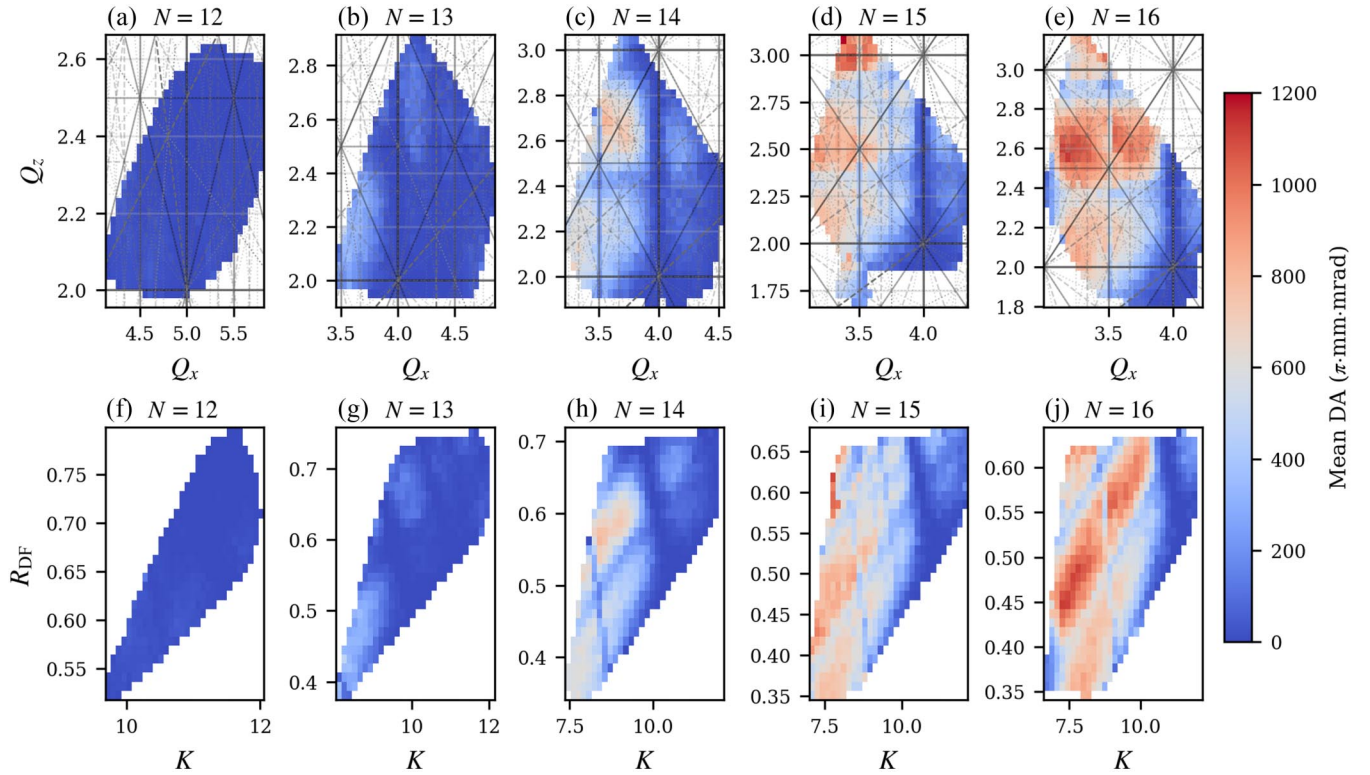


FIG. 4. DA distributions in the global tune space (Q_x , Q_z) [top row, (a)–(e)] and the parameter subspace (K , R_{DF}) [bottom row, (f)–(j)] for varying super-periodicities $N = 8, 10, 12, 14, 16$. The color gradient indicates the mean DA, demonstrating the progressive expansion of resonance-sparse stable regions as N increases.

leaves the accessible tune domain as a region of zero DA throughout. In the lower K regime of the $N = 16$ lattice, the reduced multipole strengths cause the stopbands to contract. Cross-referencing Figs. 4(j) and 4(e), these narrowed stopbands manifest as distinct blue diagonal resonance bands traversing the parameter space. As the stopbands shrink, resonance-sparse stable regions emerge, forming a high-DA parameter corridor that supports long-term dynamic robustness. These dynamic constraints indicate that the FD $N = 16$ lattice provides a more favorable structural baseline.

Furthermore, a closer inspection of the tune footprints in Figs. 4(d) and 4(e) reveals a highly nonlinear phenomenon regarding the vertical half-integer resonance ($Q_z = 2.5$). In these plots, the axes represent the linear (small-amplitude) tunes. However, in spiral scaling-FFAs, the vertical focusing is predominantly governed by the edge fringe fields, which intrinsically introduce strong high-order multipole components [22]. These edge nonlinearities drive a pronounced amplitude-dependent tune shift. As a result, configurations with a nominal linear tune of exactly $Q_z = 2.5$ experience a rapid tune shift away from the half-integer stopband as the particle amplitude increases, allowing them to survive and maintain a large DA. Conversely, configurations with linear tunes initially offset from the resonance are shifted into the stopband at large amplitudes and are lost. This dynamic mechanism manifests visually as a downward displacement (by approximately $\Delta Q_z \approx 0.1$) of the actual resonance depletion zone in the linear tune space.

To further secure this operational baseline, the subsequent analysis in Section IV focuses on evaluating the internal control parameters of the $N = 16$ lattice.

IV. NONLINEAR DYNAMICS DISCOVERY IN THE PARAMETER SUBSPACE

To evaluate the DA across the parameter subspace, the spiral angle ζ was scanned over $\zeta \in [38^\circ, 60^\circ]$ as the parameter governing vertical edge focusing. For each angle, the DA distribution across the configuration pool was evaluated via violin plots [Fig. 5(a)]. The corresponding full-ring transverse tunes (Q_x, Q_z) and their variation bands under engineering perturbations were extracted as a function of ζ [Fig. 5(b)].

The operational DA shows a nonmonotonic dependence on the spiral angle, peaking within $\zeta \in [40^\circ, 46^\circ]$. The DA variation with ζ is determined by the resonance structure in tune space. As ζ increases, the vertical edge focusing strengthens, and the full-ring vertical tune Q_z increases monotonically [Fig. 5(b)]. The DA decrease near $\zeta \approx 50^\circ$ coincides with Q_z crossing the $Q_z = 2.5$ half-integer resonance, and the sharp decrease at $\zeta \geq 56^\circ$ coincides with Q_z approaching the $Q_z = 3$ integer limit. The optimal spiral angle is thus bounded by the widest resonance-sparse tune window.

In addition to the spiral angle, the long-term stability of the $N = 16$ lattice depends on the packing factor α , the field index K , and the field ratio R_{DF} . A parameter scan was performed across packing factors $\alpha \in [0.46, 0.52]$ to map the stability boundaries. The DA distributions are projected onto the tune space (Q_x, Q_z) [Figs. 6(a)–6(d)] and the (K, R_{DF}) parameter subspace [Figs. 6(e)–6(h)].

The tune space projections show a consistent topological structure across different α values. Bounded by structural resonance lines, the viable operational space divides into two high-DA regions: a lower band at $Q_x \in [3.0, 3.5], Q_z \in [1.8, 2.3]$ and an upper band at

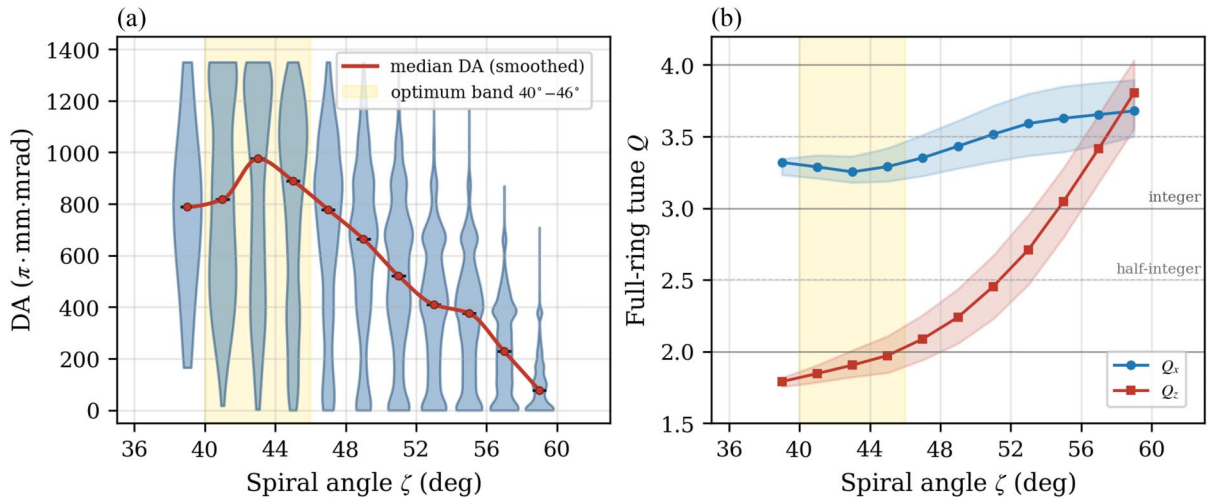


FIG. 5. Dependence of the DA and tunes on the spiral angle ζ . (a) Violin plots illustrating the DA distribution, with the red curve indicating the smoothed median values. The optimal operational band is highlighted in yellow. (b) The corresponding evolution of the full-ring transverse tunes (Q_x, Q_z) as a function of ζ . The shaded regions represent the tune footprint spread, explicitly demonstrating the severe DA degradation when Q_z intersects the 2.5 half-integer and 3.0 integer resonance lines.

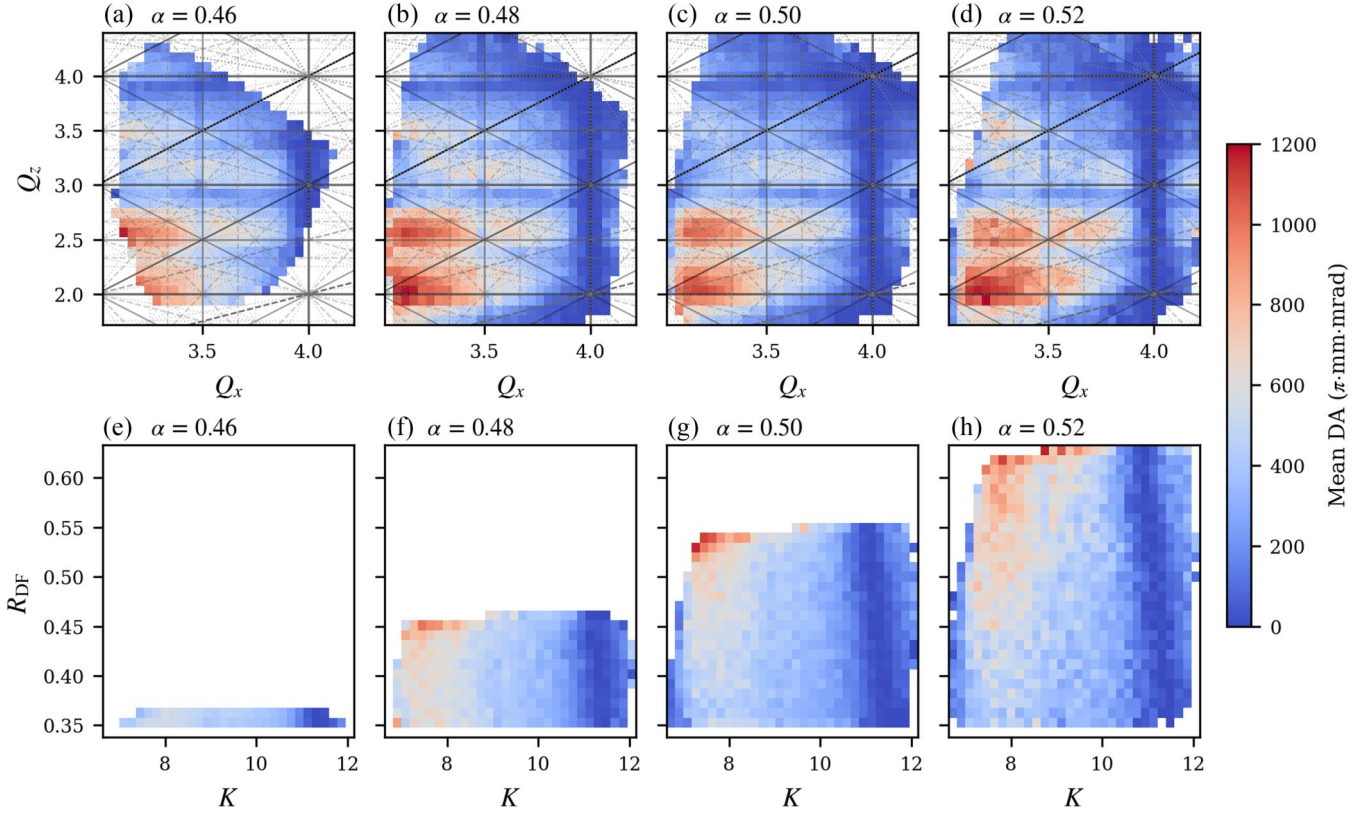


FIG. 6. Multidimensional mapping of the DA for the FD $N = 16$ lattice across different packing factors α . The top row (a)–(d) illustrates the robust convergence of high-DA regions within specific bands in the global tune space (Q_x, Q_z). The bottom row (e)–(h) demonstrates the progressive expansion of the viable control parameter subspace (K, R_{DF}) as α increases, reflecting the strict constraints of the physical engineering boundaries.

$Q_z \in [2.4, 2.8]$. Within these regions, the operational DA exceeds 1000π mm mrad under multisource engineering errors, providing selection margin for space-charge tune depression mitigation.

The bottom row of Fig. 6 shows the constraints imposed by the single-cell geometry. At lower packing factors (e.g., $\alpha = 0.46$), the shorter magnetic lengths cause the required gradients to exceed the 1.66 T peak field limit or the radial orbit excursion envelope, leaving almost no viable parameter subspace [Fig. 6(e)]. As α increases to 0.52, the accessible parameter range widens, defining the optimal gradient ranges for K and R_{DF} . Securing these high-DA tune regions requires a larger packing factor to provide the needed parameter range.

V. TOPOLOGICAL CHOICE: FD VS FDF UNDER UNIFIED CONSTRAINTS

The FDF triplet topology is also a viable lattice option. This section evaluates the parameter space of the FDF structure and compares its performance with the FD doublet. For a consistent comparison, the packing factor is set to $\alpha = 0.52$ for FDF topologies, at which the FD layout exhibits its optimal beam dynamics performance.

Like Sec. IV, a parameter scan mapping the FDF structure at $\alpha = 0.52$ is provided, allowing for a direct comparison with the FD doublet.

By comparing Fig. 7 with Fig. 6(d), it is evident that both topologies demonstrate robust nonlinear beam dynamics. They provide broad, similarly sized stable regions in the tune space where the DA exceeds 1000π mm mrad. These parameter islands are sufficiently wide to accommodate the expected space-charge tune spread without pushing the beam into major resonance stopbands. Since the phase-space capacities of the two designs are comparable, the dynamic tracking results alone do not dictate a clear preference. The topological selection is instead determined by physical hardware limitations.

The most immediate constraint of the FDF triplet involves the injection geometry. Because the current scheme utilizes the main FFA magnets for beam deflection rather than relying on a dedicated injection bump, the stripping foil must be carefully positioned within the lattice cell. As depicted in Fig. 8(a), the FDF layout requires installing the foil assembly in the narrow intracell gap between the F and D magnets. This local space is already densely occupied by the main excitation coils, leaving almost no clearance for additional hardware.

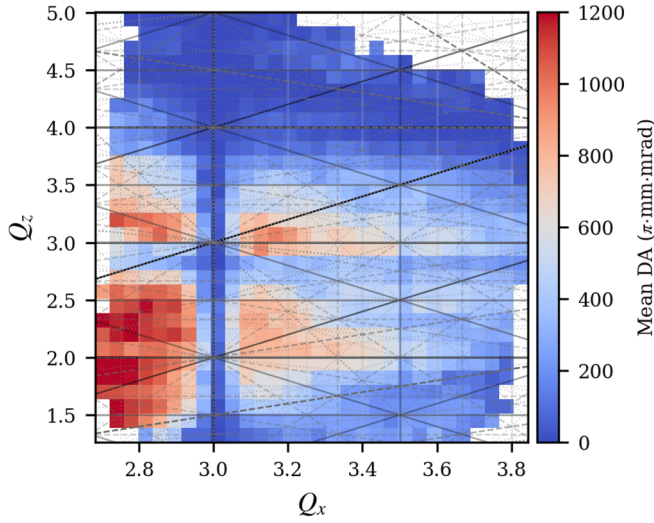


FIG. 7. DA distribution in the global tune space (Q_x, Q_z) of the FDF ($N = 12$) topology at $\alpha = 0.52$. A comparable area of high-DA regions is observed when compared with the FD layout under the same packing factor shown in Fig. 6(d).

Conversely, the FD doublet avoids this spatial conflict; as shown in Fig. 8(b), it allows the foil to be placed in the field-free straight section outside the magnet bodies.

Magnet fabrication presents another practical boundary. The required peak field of 1.64 T pushes the iron yokes close to magnetic saturation. Achieving the exact analytical fringe-field falloff requires mitigating the magnetic cross-talk between adjacent poles during 3D OPERA optimizations. Controlling this field leakage is demanding for an FD doublet, but it becomes significantly more difficult for an FDF triplet, where the mutual coupling among three saturated poles heavily distorts the field boundaries. When combined with the lower total magnet count—32 magnets for the FD ring versus 36 for the FDF ring—the FD doublet offers a more realistic path for engineering integration and cost control.

VI. HIGH-DIMENSIONAL FEATURE ATTRIBUTION VIA MACHINE LEARNING

In a scaling-FFA lattice under multisource perturbations, the coupling of geometric parameters (ζ, α, K, R_{DF}) and random symmetry-breaking errors creates a nonlinear system not easily addressed by traditional multivariable analytical derivation. Interpreting the high-dimensional tracking data is needed to extract sensitivity coefficients for practical engineering tolerance budgets. This section applies a machine learning-based feature attribution framework to identify individual parameter weights and the coupled nonlinear mechanisms that determine beam loss [23].

Specifically, to conduct this systematic evaluation, each lattice configuration is characterized using a mixed physics-informed feature basis. Rather than relying on the

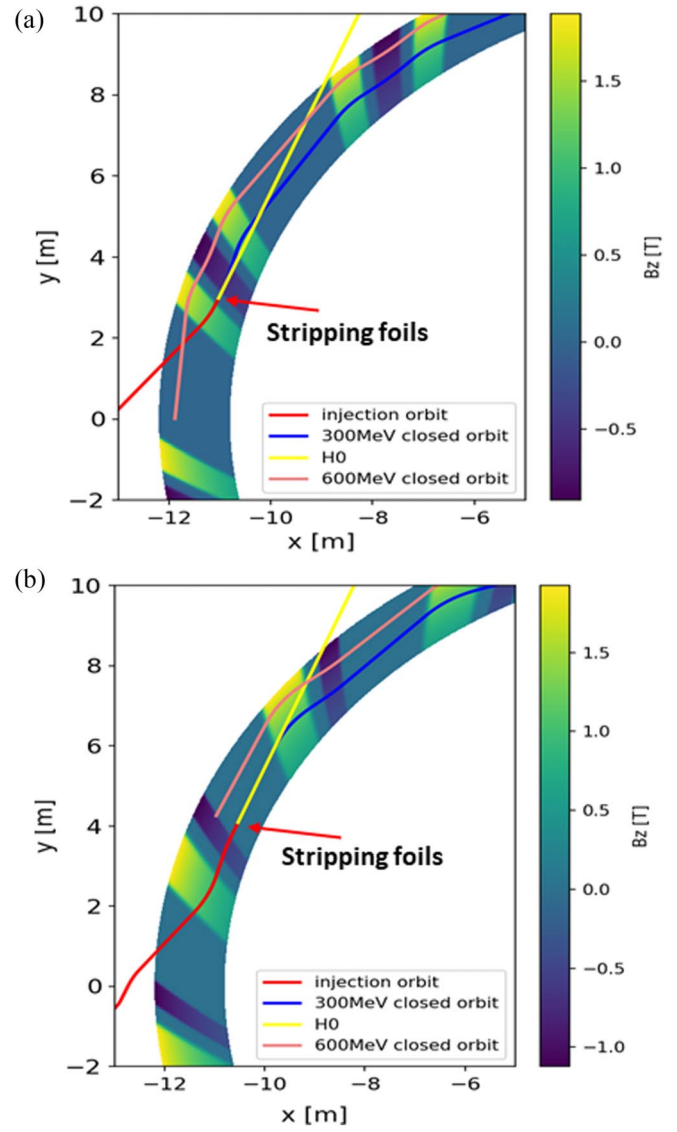


FIG. 8. Schematic of the injection orbits and stripping foil placements for the (a) FDF triplet and (b) FD doublet topologies. The background contour represents the midplane magnetic field B_z . The solid curves denote the injection orbit (red), the 300 MeV closed orbit (blue), the 600 MeV closed orbit (pink), and the unstripped H0 trajectory (yellow).

full set of raw geometric parameters—which are highly nonlinearly coupled—purely internal auxiliary spacing parameters (such as R_{sm1} and R_{sm2}) are replaced by the fundamental optical and dynamical observables they dictate. These include the full-ring tunes (Q_x, Q_z), the radial beam envelope (Δr), the horizontal tune diffusion rate $|D_x|$, which is extracted via frequency map analysis to proxy amplitude-dependent tune shifts [24,25], and the Euclidean distances to the nearest third-order and fourth-order resonance lines ($d_{res,3}$ and $d_{res,4}$). Meanwhile, primary macroscopic geometric drivers (such as the spiral angle ζ , field index K , field ratio R_{DF} , and packing factor α) are retained. By projecting the coupled internal

geometries onto these intermediate dynamical observables, the ML algorithm can explicitly untangle and interpret the actual physical mechanisms causing beam loss—such as structural resonance crossing or chaotic phase-space diffusion—rather than acting as a black-box regressor. To evaluate these feature contributions, this study employs an eXtreme Gradient Boosting (XGBoost) model coupled with Shapley Additive Explanations (SHAP) [26,27]. To ensure targeted feature attribution for our optimal baseline, the model was trained strictly on the FD $N = 16$ subset, utilizing 152,550 valid configurations ($DA > 0$).

The global feature importance ranking resolved by the mean absolute SHAP values is presented in Fig. 9(a). The attribution analysis demonstrates that the spiral angle ζ and the full-ring vertical tune Q_z are the two most dominant drivers governing the DA, followed by the horizontal tune diffusion rate $|D_x|$ and radial beam envelope Δr . This ranking indicates that vertical phase-space dynamics and edge-focusing parameters exert a higher relative impact on beam survivability than horizontal or purely geometric linear parameters within this lattice configuration space. Physically, this dominance highlights the intrinsic focusing asymmetry in compact scaling-FFAs: while horizontal focusing is broadly sustained by the main field body (K), vertical focusing relies predominantly on the fringe fields via the spiral angle (ζ). Because these edge regions inherently harbor the strongest nonlinear geometric multipoles, the vertical dynamic aperture naturally becomes the primary bottleneck restricting overall beam survival.

To resolve the exact physical dependence, the marginal effects of the top two features are mapped in Figs. 9(b) and 9(c). As shown in the ζ dependency plot [Fig. 9(b)], the SHAP value remains positive within the $\zeta \in [40^\circ, 48^\circ]$ domain, confirming a favorable contribution to the DA.

Beyond $\zeta \approx 50^\circ$, the SHAP value exhibits a sharp downward slope, turning deeply negative and indicating severe phase-space degradation. The color-coded field index K reveals that this detrimental trend is further exacerbated at higher gradient regimes ($K > 9$, shown in red), where the enhanced nonlinear multipoles couple with the steep edge angles.

This degradation mechanism is confirmed by the Q_z dependency plot [Fig. 9(c)]. The model captures the oscillatory structure of the resonance network from the tracking data. Distinct local minima (dips) in the SHAP value align with the major integer and half-integer structural resonance lines ($Q_z = 2.0, 2.5, 3.0$). The deep valley below $Q_z = 2.0$ represents the integer resonance crossing driven by random symmetry breaking. This correlation indicates the ML framework identifies the resonance stopbands as the physical constraint on long-term beam survival. The feature attribution confirms the role of vertical dynamic boundaries in compact scaling-FFA designs, consistent with the baseline parameter selection.

VII. BASELINE LATTICE CONFIGURATION AND OPTICAL VERIFICATION

Guided by the quantitative sensitivity rankings and resonance-sparse corridors established via the machine learning framework, a specific lattice configuration has been selected to serve as the design baseline. It must be emphasized that this baseline configuration represents a viable and robust operational point that satisfies all current structural and engineering constraints, rather than a mathematically absolute global optimum across the entire multidimensional parameter space. To further map the performance limits of this topology, a comprehensive multiobjective optimization utilizing evolutionary

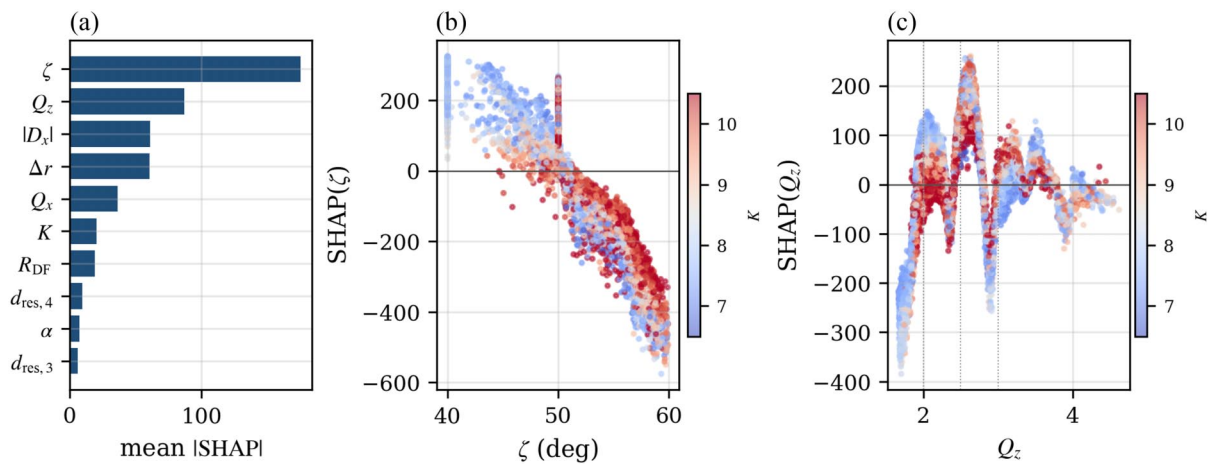


FIG. 9. Machine learning-based feature attribution using the XGBoost-SHAP framework for the FD $N = 16$ lattice ensemble. (a) Global feature importance ranking quantified by the mean absolute SHAP values ($|\text{SHAP}|$). SHAP dependency plots illustrate the isolated marginal effects on the DA for (b) the spiral angle ζ and (c) the full-ring vertical tune Q_z . The color gradient represents the localized variation of the field index K .

TABLE II. Lattice parameters for CSNS-FFA.

Parameters	Design value
Number of elements	16
Reference radius r_0 (m)	12
Reference magnetic field strength B_0 (T)	1.65
$B_{\max}$ of F magnet at 600 MeV (T)	1.64
$B_{\max}$ of D magnet at 600 MeV (T)	0.865
Spiral angle ($^\circ$)	45.227
Field index K	8.029
EFB width of F magnet ($^\circ$)	5.928
EFB width of D magnet ($^\circ$)	2.954
Angle between F and D magnet ($^\circ$)	6.818
Index of air gap variation gradient	8.029
Reference air gap height g_0 at r_0 (mm)	108

algorithms (e.g., NSGA-II) is currently underway and will be reported in a future study.

For the selected $N = 16$ FD doublet lattice, the core structural parameters are configured as: a spiral angle $\zeta = 45.22729^\circ$, a packing factor $\alpha = 0.5$, a scaling field index $K = 8.0292$, and a field ratio $R_{\text{DF}} = 0.528036$. The auxiliary geometric scaling factors are set to $R_{\text{sm1}} = 0.5$ and $R_{\text{sm2}} = 0.4041$. This specific configuration positions the full-ring operational tune at $(Q_x, Q_z) = (3.32, 2.21)$, placing it safely within the broad, high-DA stable island identified in Sec. IV. The detailed lattice parameters are listed in Table II.

The fundamental optical and magnetic profiles of the selected baseline lattice across the full acceleration cycle are evaluated and verified in Fig. 10. Figure 10(a) illustrates the closed-orbit radius r as a function of the azimuthal angle θ . The closed orbit evolves smoothly from approximately 11.3 m at injection energy to 12.0 m at

extraction energy. The resulting radial beam envelope Δr is 0.75 m, which satisfies the rigid 0.8 m engineering envelope budget required for the ultrawide vacuum chamber while naturally accommodating the alternating-gradient orbit scalloping.

The corresponding midplane magnetic field B_z profiles are plotted in Fig. 10(b). Driven by the spatial scaling law, the magnetic field waveforms scale self-similarly with increasing beam energy. The peak midplane field reaches a maximum value of 1.64 T at the 600 MeV extraction boundary. This peak value remains below the 1.66 T hardware limit, which avoids magnetic saturation, ensuring a safe operating margin for conventional magnet manufacturing and power supply stability.

Furthermore, Fig. 10(c) presents the horizontal and vertical beta functions (β_x, β_z) mapped against the path length s for the injection (300 MeV) and extraction (600 MeV) energies. The betatron functions exhibit stable and well-behaved periodic oscillations across the entire acceleration range. The maximum vertical beta function is bounded at $\beta_{z,\max} = 7.240$ m, effectively minimizing the transverse beam envelope and reducing the physical aperture clearance required for the magnets. Crucially, even when subjected to the multisource random engineering error budget, multiparticle tracking over 1000 turns confirms that the DA consistently remains above 1000π mm mrad. This tracking verification demonstrates that the chosen baseline configuration delivers exceptional high-order dynamical robustness under realistic engineering symmetry breaking.

Finally, it should be noted that in the current FixField ray-tracing model, the fringe field profiles are mathematically scaled with the reference radius, inherently yielding perfectly constant linear tunes across the 300–600 MeV

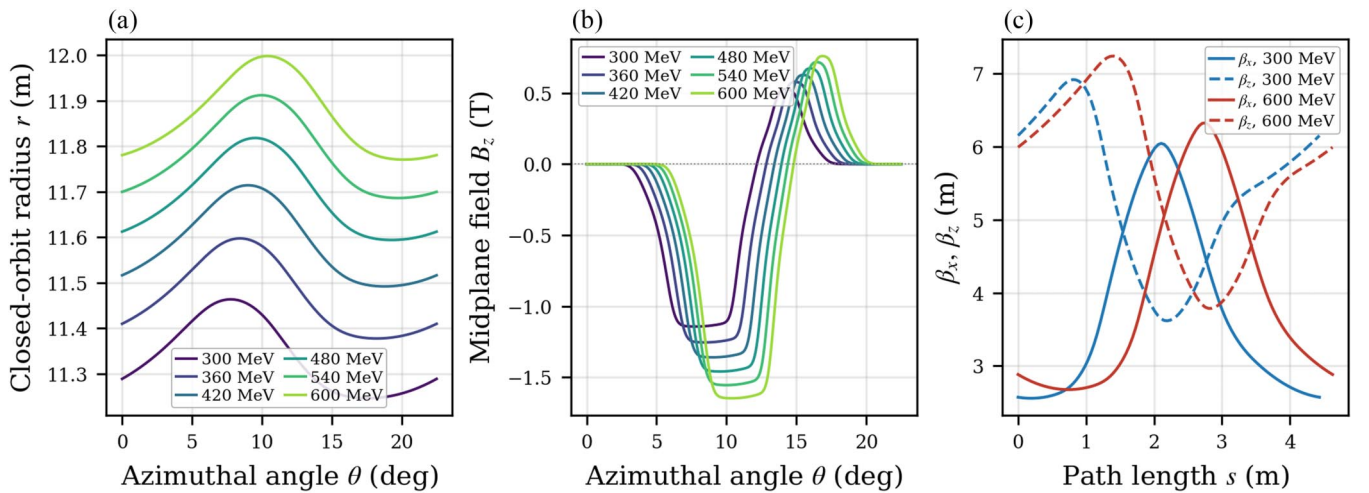


FIG. 10. Linear optics and magnetic field profiles of the selected baseline $N = 16$ FD scaling-FFA lattice during the acceleration cycle from 300 to 600 MeV. (a) Azimuthal modulation of the closed-orbit radius r , confirming a total radial beam envelope of $\Delta r = 0.75$ m. (b) Midplane magnetic field B_z distributions, showing self-similar scaling with a peak field of 1.64 T. (c) Horizontal (β_x) and vertical (β_z) beta functions evaluated at the injection and extraction energy boundaries.

energy range. In practical implementation, however, the proposed variable-gap design of the FD magnets will introduce localized scaling law violations in the 3D fringe fields. To strictly maintain the constant-tune property across the full acceleration cycle, precise unequal chamfering at the spiral pole edges is required. The detailed 3D engineering design of these magnets and the corresponding high-fidelity field tracking are currently under development and will be presented in a subsequent dedicated study.

VIII. METHODOLOGICAL BOUNDARIES AND FUTURE PATHWAY

While the multidimensional framework identifies a robust operational regime, translating this theoretical design into physical hardware requires acknowledging the limitations of our tracking model. Our simulations rely on an analytical Enge fringe-field model, which was benchmarked against a 3D OPERA prototype. However, as the spiral angle ζ and field index K vary across a wide parameter space, real-world local magnetic saturation and 3D flux leakage will inevitably cause the actual field distribution to deviate from this static analytical approximation.

This limitation has notable implications for the vertical phase-space dynamics. As quantified by the machine learning feature attribution in Sec. VI, the DA is strongly governed by the spiral angle ζ and its associated vertical edge focusing. Because the vertical tune Q_z is highly sensitive to the exact fringe-field gradient profile, physical deviations from the analytical model can induce tune shifts that potentially drive the operating point into the major resonance stopbands identified in Fig. 9(c).

Consequently, the baseline parameters presented in Sec. VII should be interpreted as dynamic target specifications rather than final manufacturing blueprints. The subsequent phase of this research necessitates a transition to hardware-in-the-loop engineering, combining passive 3D field shaping with active magnetic correction. Passively, iterative 3D field simulations, precise pole chamfering, and field clamps will be deployed to tailor the physical fringe-field falloff to replicate the validated analytical profiles. Actively, distributed pole-face trim coils will be integrated into the magnet design to enable *in situ* adjustment of the field index $K(r)$ and edge gradients after fabrication. This active tunability provides essential operational redundancy to compensate for unexpected machining errors and steer the tune away from destructive resonance stopbands during physical beam commissioning.

IX. CONCLUSION

This study evaluated the topological boundaries and nonlinear dynamic mechanisms of a 300–600 MeV scaling-FFA lattice designed for the CSNS Phase-II upgrade. By deploying a large-sample statistical scanning

framework encompassing over 360,000 parameter configurations under multisource engineering perturbations, the physical limitations of FDF triplet and FD doublet structures were quantitatively characterized.

The statistical results indicate that the 0.8 m radial excursion limit imposes a parameter bottleneck on the field index (K), which amplifies high-order multipole gradients in lower super-periodicity lattices due to exaggerated intracell orbit scalloping. Increasing the super-periodicity to $N = 16$ within an FD doublet topology relaxes this geometric constraint, contracting resonance stopbands and establishing a continuous, stable phase-space corridor. While the FDF $N = 12$ triplet demonstrates comparable peak DA metrics, its stable operating domain is confined to a narrow diagonal pocket vulnerable to space-charge tune depressions. Factoring in magnet economy and hardware integration with the existing CSNS infrastructure, the $N = 16$ FD doublet is established as the optimal baseline topology.

A machine learning feature attribution framework based on XGBoost and SHAP values identified the spiral angle (ζ) and the full-ring vertical tune (Q_z) as the dominant drivers governing beam survivability, accurately mapping the underlying resonance network. Guided by these sensitivity landscapes, a baseline configuration was selected. Optical and magnetic tracking verified that this baseline achieves a radial excursion of 0.75 m, a peak midplane field of 1.64 T, and a perturbed DA exceeding 1000π mm mrad, satisfying all engineering prerequisites.

While the analytical Enge model provides a conservative foundation for this scoping study, localized magnetic saturation and three-dimensional flux leakage introduce physical boundaries that deviate from static formulations. Future development will transition to hardware-in-the-loop engineering, focusing on iterative three-dimensional field mapping and physical magnet shaping—such as pole chamfering and field clamps—to replicate these validated analytical profiles and ensure resonance-sparse operation under high-intensity conditions.

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DATA AVAILABILITY

There are no publicly available research data or software supporting this manuscript. Requests for further information or data should be sent to the authors.

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