

Imaging calculation of coherent transition radiation Michelson interferometry for adaptive bunch profile reconstruction

K. Huang^{1,2,*} G. Andonian¹ A. Fukasawa¹ O. Williams¹
P. Manwani¹ and J. Rosenzweig¹

¹*Department of Physics and Astronomy, University of California, Los Angeles, California 90095, USA*

²*Kansai Institute for Photon Science (KPSI), National Institutes for Quantum Science and Technology (QST), 8-1-7 Umemidai, Kizugawa-city, Kyoto, 619-0215, Japan*



(Received 30 January 2026; accepted 9 July 2026; published 29 July 2026)

We present a methodological study for measuring electron-bunch temporal profiles using coherent transition radiation Michelson interferometry. A numerical frequency-filter model is established through imaging calculations of the experimental setup, accounting for the electron-bunch transverse profile, finite angular acceptance, and detector size. An adaptive genetic-algorithm-based forward-fitting approach is implemented to retrieve arbitrary bunch temporal profiles, demonstrating advantages over the classical Kramers-Kronig minimum-phase method. The frequency-filter model, together with the adaptive fitting framework, provides a robust tool for electron-bunch temporal profile characterization in beam physics studies.

DOI: [10.1103/v5xw-2nz6](https://doi.org/10.1103/v5xw-2nz6)

I. INTRODUCTION

The generation of high-peak-current electron bunches is a central topic in accelerator research, motivated by the development of plasma wakefield accelerators [1–5], x-ray free-electron lasers (XFELs) [6–9], investigations of strong-field quantum electrodynamics (QED) [10,11], and studies of particle physics with high-luminosity colliders [12]. To increase the peak current $I \propto Q/\sigma_t$, the optimization of the bunch duration σ_t would be essential, making the temporal profile monitor an indispensable tool for beam physics studies.

The Coulomb field of an electron bunch can be expressed as the convolution of its spatiotemporal charge distribution with the Coulomb field of a single electron. Consequently, measuring the spatial and temporal distributions of the electrons can be translated into detecting the bunch's Coulomb field in either the time or frequency domain. When a charged particle traverses the boundary between two media with different dielectric properties, transition radiation (TR) is emitted [13,14], as a result of the transient current induced by the Coulomb field at the interface. TR inherently possesses a broad spectrum.

The coherent component of TR is known as coherent transition radiation (CTR) [15]. The TR signal carries the spatiotemporal information of the electron-bunch distribution into the far field. In the case of modern picosecond electron beams, a substantial fraction of the emitted power is found in the terahertz (THz) range [16] due to these coherence effects. These terahertz signals enable various optical diagnostic techniques, including: (a) CTR spectral intensity measurement using a spectrometer [17–19], (b) CTR temporal field measurement via electro-optic (EO) sampling [20], and (c) CTR Fourier spectroscopy or Michelson interferometry [21–25].

Method (a) enables single-shot measurements over a wide range of bunch durations but requires multiple splitting of the TR pulse and measurement of its spectral intensity across a broad wavelength range. Method (b) provides single-shot detection of the TR electric field but relies on a synchronized femtosecond probe laser. Such a laser is readily available in photoinjector-based accelerators [26–28] and laser wakefield acceleration experiments [29–35] but is generally unavailable at conventional accelerator facilities. Method (c) requires scanning the optical path length of one arm of a Michelson interferometer over many shots and is therefore limited to relatively stable accelerator systems operating at sufficiently high repetition rates. Nevertheless, it offers several advantages, including straightforward experimental setup and direct measurement in the time domain, without the need for dispersive optical components such as gratings or an additional synchronized laser pulse. Furthermore, the

*Contact author: huang.kai@qst.go.jp

Published by the American Physical Society under the terms of the [Creative Commons Attribution 4.0 International license](https://creativecommons.org/licenses/by/4.0/). Further distribution of this work must maintain attribution to the author(s) and the published article's title, journal citation, and DOI.

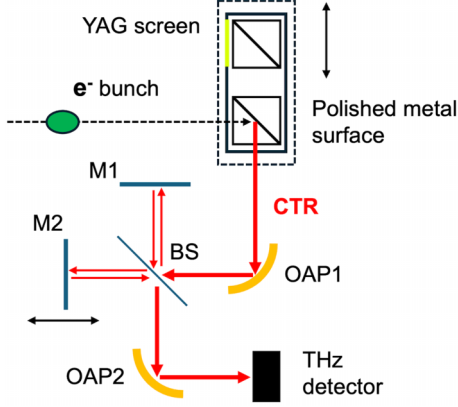


FIG. 1. Experimental concept of the CTRI. OAP: Off axis parabolic gold mirror; BS: Beam splitter; M1: a fixed gold mirror; M2: A movable mirror. The THz detector can be a pyroelectric sensor, a Golay cell, or a cryogenic bolometer.

temporal resolution can be improved simply by reducing the scanning step size.

In this article, we address two key issues in the application of coherent transition radiation Michelson interferometry (hereafter referred to as CTRI): (1) Imaging calculation of the frequency filter in CTRI measurements: the low-frequency losses in CTRI measurements were reported in Ref. [23], where a simple Gaussian (G) model was used to estimate the bunch duration on the picosecond scale. Here, following established approaches to TR transportation [36–40], we perform imaging-based calculations of the experimental setup to obtain CTRI signals from arbitrary electron-bunch temporal profiles. By accounting for the optical acceptance, detector size, and beam transverse profile, we develop a frequency-filter model that describes the frequency-dependent losses of the system. (2) Adaptive fitting with a genetic algorithm (GA) [41] for bunch shape reconstruction: the Kramers-Kronig (KK) method has been widely used to retrieve the minimum phase $\Phi_{KK}(\omega)$ of the complex longitudinal form factor amplitude [19,42]. However, the validity of the minimum

phase faces challenges when applied to data confined within a limited frequency range and when the electrons possess complicated temporal structures [17,43]. As an alternative, we employ a GA-based forward-fitting approach to reconstruct arbitrary bunch temporal profiles directly from CTRI signals in the time domain.

This paper is organized as follows: Section II introduces the concepts of CTRI and its theoretical framework. Section III presents the derivation of the frequency filter based on imaging calculations of the experimental setup under the Fraunhofer approximation. Section IV demonstrates the reconstruction of electron-bunch temporal profiles using the frequency-filter model, employing both the KK minimum-phase approach and the adaptive GA fitting method. The robustness of the GA fitting approach is further evaluated using an artificial CTRI signal. Finally, Sec. V presents the discussions and conclusions of this study.

II. THE CTR MICHELSON INTERFEROMETRY

A typical experimental setup of the CTR Michelson interferometer measurement system can be found in Fig. 1. A diagnostic ladder containing a yttrium–aluminum–garnet (YAG) screen and a polished metal surface is movable to measure the transverse electron-bunch size and generate TR at the same location. The importance of the transverse size in determining the longitudinal profile of short bunches will be elaborated upon in later sections. The TR pulse is collected by the first off axis parabolic gold mirror (OAP1) into the Michelson interferometer. The beam splitter separates the TR pulse into two pulses. The two pulses are then combined and focused by the OAP2 into the detector. The second mirror M2 is mounted on a linear motorized stage for the adjustment of the time delay between the two TR pulses.

Figure 2 illustrates the theoretical concept of the CTRI. The TR source is located at the focal point of OAP1. A Michelson interferometer, comprising two optical arms, is positioned between OAP1 and OAP2. As demonstrated in

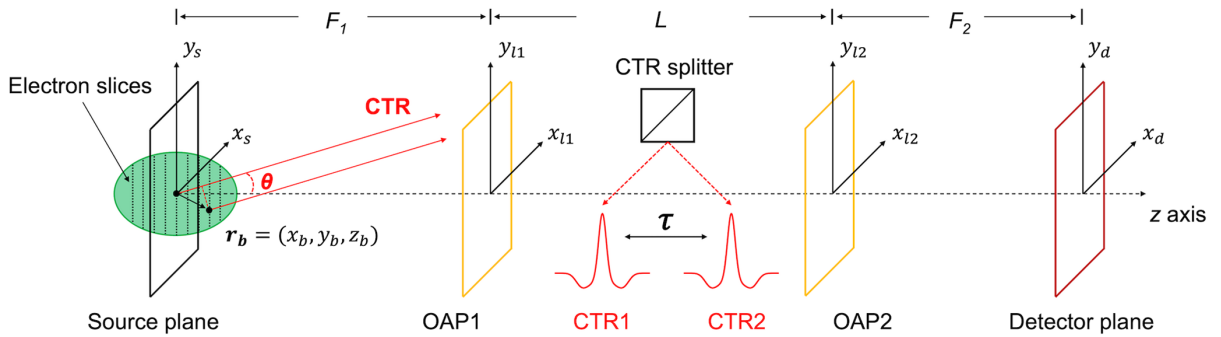


FIG. 2. The concept of the imaging calculation of TR of an electron bunch. OAP1 and OAP2 have effective focal lengths of F_1 and F_2 , respectively. The coordinates of the source plane, lens plane, and detector plane are denoted by (x_s, y_s) , (x_l, y_l) , and (x_d, y_d) , respectively. The coordinates of individual electrons inside a bunch are described by (x_b, y_b, z_b) . This figure is adapted from Ref. [44] with modifications.

previous numerical studies [44], the system can be treated as a thin-lens imaging configuration with high accuracy, satisfying $1/f = 1/F_1 + 1/F_2$, where f denotes the effective focal length of the overall optical system. The corresponding magnification of this setup is $M = F_2/F_1$.

The total CTR field on the detector plane is the sum of the two separated complex fields $E_{\text{TR}}(x_d, y_d)$ with a time gap of τ defined by the relative delay between the two arms of the Michelson Interferometer. The CTRI signal intensity at a certain point in the detector plane is calculated as:

$$\begin{aligned} S(\tau, x_d, y_d) &\propto \int |E_{\text{TR}}(\omega, x_d, y_d) \\ &\quad + E_{\text{TR}}(\omega, x_d, y_d)e^{i\omega\tau}|^2 d\omega \\ &= 2 \int [\cos(\omega\tau) + 1] |E_{\text{TR}}(\omega, x_d, y_d)|^2 d\omega. \end{aligned} \quad (1)$$

The signal exhibits an oscillatory behavior [23,24]. The background level of the interferogram is defined in the limit of a sufficiently large temporal separation between the two TR pulses, i.e., $\tau \rightarrow \infty$. In this limit, the CTRI intensity reduces to the sum of the intensities of the two TR pulses: $S_\infty = 2 \int |E_{\text{TR}}(\omega, x_d, y_d)|^2 d\omega$. It is readily seen that the signal peak intensity of the CTR interferometry at $\tau = 0$ is 2 times the background level, in an ideal measurement.

The CTR Michelson Interferometry signal is obtained by integrating all photons incident on the detector's acceptance area:

$$S(\tau) = \int S(\tau, x_d, y_d) dx_d dy_d. \quad (2)$$

In this methodological study, the frequency dependence of the beam splitter reflection and transmission, as well as the response of the detector and other optical components, is assumed to be weak over the frequency range of interest. For frequencies below a few terahertz (corresponding to bunch durations above several hundred femtoseconds), this approximation is often adequate when using, for example, a semitransparent metal-coated polymer film [45] or a high-resistivity float-zone silicon (HRFZ-Si) plate [46] as the beam splitter, together with pyroelectric detectors [47,48]. For ultrashort electron bunches with durations of a few to tens of femtoseconds, the spectral response of the entire optical system may require experimental characterization, for example, using blackbody radiation [24,25]. In this work, we focus on the methodological aspects of CTRI, particularly the frequency-dependent losses arising from diffractive propagation through finite-aperture optics and from the finite detector size.

III. IMAGING CALCULATION APPROACH FOR CTRI

A. Imaging of the TR field from an electron bunch

By considering the metal surface as an ideal conductor, the TR is mathematically treated as the total reflection of the Coulomb field. The Coulomb field of a single electron [13,14] incident onto the metal surface is calculated as Eq. (3) [36,37,43,44].

$$E_{x,y}^s(x, y, \omega) = -\frac{e\alpha}{2\pi\epsilon_0 v} \frac{(x, y)}{\sqrt{x^2 + y^2}} K_1\left(\alpha\sqrt{x^2 + y^2}\right), \quad (3)$$

where $\alpha = \omega/(\gamma v)$, ω is the angular frequency, γ is the Lorentz factor, and v is the electron velocity. The first-order modified Bessel function of the second kind is denoted by K_1 . The vacuum permittivity is denoted by ϵ_0 . The superscript “S” denotes quantities evaluated at the source plane. The derivation of an analytical formula in the frequency domain for TR imaging has been reported in previous work [38,39,44]. The overall process can be calculated using the Fraunhofer diffraction. With normalized transverse distance in the image plane $\zeta = k\rho/M$ ($\rho = \sqrt{x_d^2 + y_d^2}$, $k = 2\pi/\lambda$), the TR image field at the detector is derived as Eq. (4).

$$E_0^d(\rho, \phi_d) = -\frac{e}{\epsilon_0} \frac{(\cos \phi_d, \sin \phi_d)}{\lambda M v} f(\theta_m, \gamma, \zeta), \quad (4)$$

where $(\cos \phi_d, \sin \phi_d) = (x_d, y_d)/\rho$. The last term in Eq. (4) describes the diffraction caused by the limited aperture of the first imaging optics [38,44,49]:

$$f(\theta_m, \gamma, \zeta) \approx \zeta^{-1} [\gamma^{-1} \zeta K_1(\gamma^{-1} \zeta) - J_0(\zeta \theta_m)], \quad (5)$$

with condition $1/\gamma \ll \theta_m$. Here, the maximum acceptance angle limited by the OAP1 is $\theta_m = R_m/F_1$, with R_m the radius of OAP1. The Bessel function of the first kind of order zero is denoted by J_0 .

The TR field at a point (x_d, y_d) in the detector plane is the sum of the field from all electrons in the bunch,

$$\begin{aligned} E_{\text{TR}}(\omega, x_d, y_d) &\propto \Sigma \exp[i(k_x x_b + k_y y_b + k_z z_b)] \\ &\quad \times E_0^d(\omega, x_d - x_b, y_d - y_b), \end{aligned} \quad (6)$$

with individual electron locations (x_b, y_b, z_b) . The spatial-frequency distribution of the TR electric field at the detector plane is calculated as:

$$E_{\text{TR}}^{x,y}(\omega, x_d, y_d) = N \int dz_b g_{\parallel} \exp(i\omega z_b/v) (g_{\perp} * E_0^{x,y}). \quad (7)$$

The upper indices $\{x, y\}$ denote the transverse components of the field in orthogonal directions. The electron-bunch distribution function is: $g = g_{\perp}(x_b, y_b) \times g_{\parallel}(z_b)$. The symbol “ $\ast$ ” denotes convolution in Cartesian coordinates. The electron number is denoted by N . The normalized transverse and longitudinal distributions are described by $g_{\perp}(x_b, y_b)$ and $g_{\parallel}(z_b)$, respectively. This convolution approach has also been employed in previous studies [49,40]. A detailed description is provided in Appendix A.

The complex longitudinal form factor amplitude [42] is the Fourier transform of the bunch longitudinal profile:

$$A_z = \int dz_b g_{\parallel}(z_b) \exp(i\omega z_b/v). \quad (8)$$

The longitudinal form factor is calculated as $F_z = |A_z|^2$. When the electron bunch has a high charge, the TR intensity is dominated by the coherent part in the infrared to THz region. The CTR spatial-frequency distribution is thus calculated as:

$$I_{\text{CTR}}(\omega, x_d, y_d) = N(N-1)F_z \times (|g_{\perp} \ast E_{0,x}^d|^2 + |g_{\perp} \ast E_{0,y}^d|^2). \quad (9)$$

The intensity distribution $I_{\text{CTR}}(x_d, y_d)$ with a finite detector size of 5 mm was calculated by summing over all frequency components, as shown in Fig. 3. A comparison is made for electron-bunch temporal durations of 100 and 800 fs. As illustrated in Fig. 3(a), the I_{CTR} from the shorter electron bunch is confined to a smaller spatial

region. In contrast, for a longer bunch duration, the transition radiation contains a larger fraction of long-wavelength components, resulting in a larger I_{CTR} distribution in space, part of which is truncated by the finite detector size. The underlying physics is straightforward: for a given wavelength λ , the effective electron radius scales as $r_{\text{eff}} \sim \lambda\gamma$ [43]. Thus, longer wavelengths correspond to a more extended spatial span of the Coulomb field. Since the two-OAP configuration functions as an imaging system, the transition radiation image on the detector consequently expands over a larger area. Such spatial-frequency effects in CTR imaging measurements have also been noted in a numerical study [44].

B. The frequency filter

A numerical model of the frequency filter is developed for a given optical system. Using Eqs. (2) and (9), the CTRI signal can be expressed as

$$S(\tau) \propto \int [\cos(\omega\tau) + 1] F_z(\omega) G(\omega) d\omega. \quad (10)$$

In this expression, the frequency filter $G(\omega)$ is defined as

$$G(\omega) = \int_{-x_d^m/2}^{x_d^m/2} \int_{-y_d^m/2}^{y_d^m/2} \times (|g_{\perp} \ast E_{0,x}^d|^2 + |g_{\perp} \ast E_{0,y}^d|^2) dx_d dy_d, \quad (11)$$

where x_d^m and y_d^m denote the horizontal and vertical dimensions of the detector, respectively. Equation (11) can be computed numerically for arbitrary optical configurations and electron transverse profiles. In practical CTRI measurements, the frequency filter $G(\omega)$ can be precomputed as a constant array for a given setup, enabling rapid processing of CTRI data.

Based on the theoretical framework described above, the CTRI signal shape can be calculated. The results for various temporal electron-bunch profiles are presented in Fig. 4. CTRI signals were computed for both single-Gaussian [Figs. 4(a) and 4(b)] and multi-Gaussian [Figs. 4(c) and 4(d)] temporal bunch distributions. Individual Gaussian bunch durations of 200 and 300 fs were adopted. In the multibunch cases, the temporal separation between adjacent bunches was set to 1 ps. As shown in Figs. 4(a) and 4(b), variations in the CTRI signal shape can be clearly resolved at the hundred-femtosecond scale.

1. Simplified frequency filter with very small transverse beam size

Equation (11) requires two-dimensional convolutions between large matrices to achieve the desired accuracy. When the electron beam is focused to a transverse size of only a few tens of micrometers, the mesh grid must be

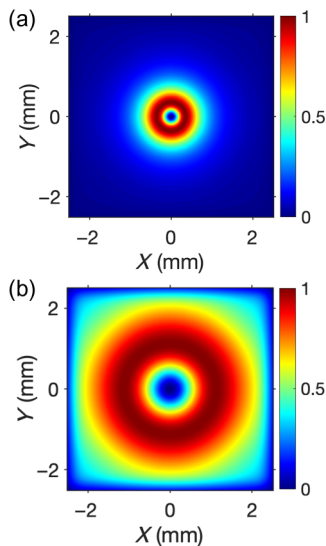


FIG. 3. Calculation results of normalized CTR intensity at the imaging plane. All the frequency components have been included. The transverse size of the electron bunch is 200 μm . (a) and (b) correspond to temporal durations of 100 and 800 fs (rms), respectively. The electron energy is 30 MeV.

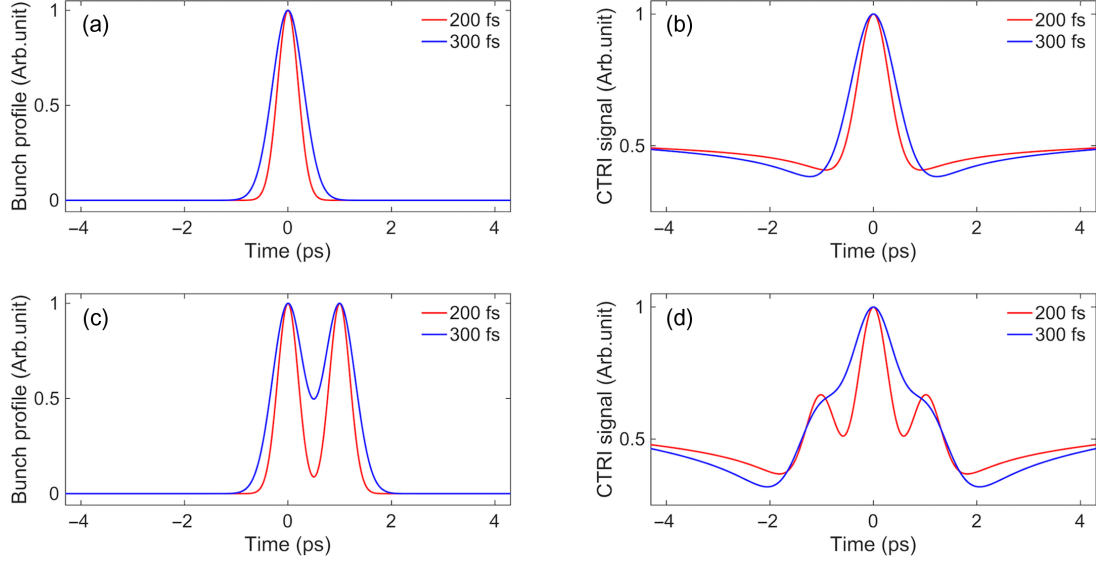


FIG. 4. CTRI signal profile calculation results. The red and blue curves represent the calculation results for individual Gaussian bunch durations of 200 and 300 fs (rms), respectively. (a) and (b) are the normalized bunch temporal profile and CTRI signal profile of single-Gaussian bunches, respectively. (c) and (d) are the results modeled by two Gaussian bunches. The time gap between two separate Gaussian bunches is set to be 1 ps. The electron beam has a transverse size of 200 μm (rms) and an energy of 30 MeV. The detector size is 5 mm.

refined to the micrometer scale, resulting in extremely large computational matrices. Nevertheless, a smaller transverse beam size leads to enhanced transverse coherence. In such cases, an infinitesimal transverse size can be assumed, allowing the frequency filter to be further simplified.

For an electron-bunch distribution with considerable cylindrical symmetry, Eq. (11) can be simplified in polar coordinates. Inserting Eq. (4) into Eq. (11), with $\zeta = k\rho/M = \frac{2\pi\rho}{\lambda M} = \frac{\omega\rho}{cM}$, we have:

$$G^\rho(\omega) \propto \omega^2 \int_0^{\rho_m} \rho d\rho \times |(g_\perp * f)(\rho)|^2, \quad (12)$$

where “ f ” is the diffraction term described by Eq. (5) and “ ρ_m ” is the maximum radius defined by the size of the detector. Here, the $*$ denotes a convolution in the polar coordinates. Further, with an infinitesimal transverse size: $g_\perp(\rho) = \delta^{(2)}(\rho)$ or $\sigma_\perp = 0$, the frequency filter is then calculated as:

$$G_0(\omega) = \int_0^{\rho_m} d\rho [\gamma^{-1}\zeta K_1(\gamma^{-1}\zeta) - J_0(\zeta\theta_m)]^2 / \rho. \quad (13)$$

C. The impacts of the beam transverse profile and the detector size

The frequency filters for different beam transverse sizes are compared in Fig. 5(a), where the colored curves represent the $G(\omega)$ of various $\sigma_\perp$ values. The curve labeled “ $\sigma_\perp = 0$ ” is calculated using the simplified filter described

by Eq. (13), while the others are obtained from Eq. (11). For reference, the longitudinal form factors corresponding to temporal durations of 200 and 800 fs are also plotted as black curves in Fig. 5(a). For long electron bunches, F_z is confined to the low-frequency region, and the influence of the transverse size is negligible. In contrast, for short electron bunches, F_z extends into the high-frequency range, and the transverse size effect needs to be considered to ensure accurate reconstruction.

Previous TR studies have developed models that primarily consider the emission process [36,43]. However, those models assume an infinitely large detector that collects all radiation within the acceptance of the first collecting lens. For the unambiguous evaluation, we calculated the CTR spectral intensity distribution for different detector sizes. The spectral intensity $I(\omega)$ is obtained by integrating $I_{\text{CTR}}(\omega, x_d, y_d)$ over the spatial area defined by the detector. As shown in Fig. 5(b), the normalized spectral intensity varies with different detector sizes. The long-wavelength loss is pronounced for small detector sizes. When the detector size becomes extremely large, however, the $I(\omega)$ distribution changes only slightly, since the majority of the frequency components have been included.

Although the exact analytical form of Eq. (11) or (12) is not obtained, a lookup table can be constructed to estimate the bunch duration from the width of the CTRI main peak. Figure 5(c) shows the fitted main-peak width (σ^{CTRI}) for different electron-bunch durations and detector sizes. For large detector sizes, σ^{CTRI} scales approximately linearly with the bunch duration, due to reduced loss of low-frequency components. The CTRI system’s linear response

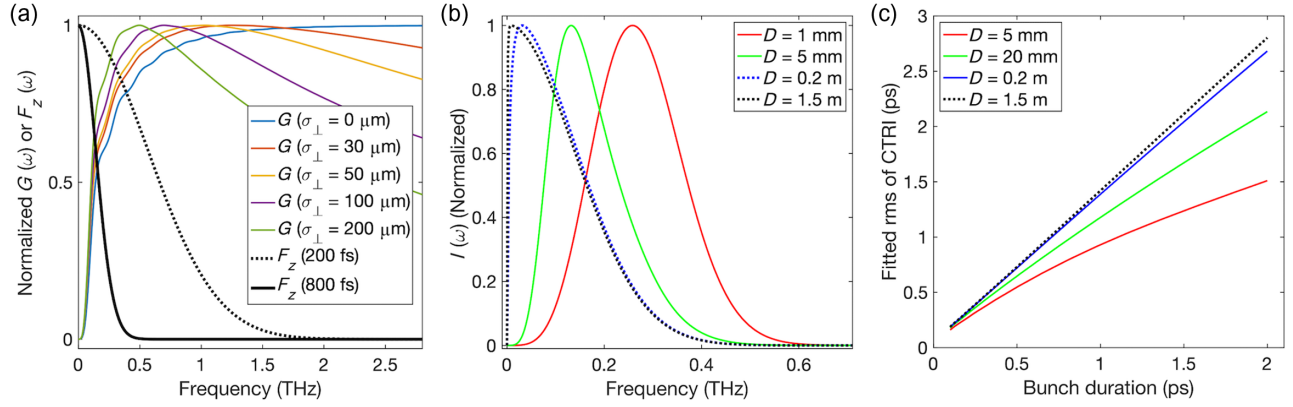


FIG. 5. Effects of beam transverse size and detector size. (a) Frequency filters and longitudinal form factors. Colored curves show $G(\omega)$ for different transverse beam sizes with a fixed detector size of 5 mm, while black curves show the longitudinal form factor $F_z(\omega)$. (b) Integrated CTR spectra with detector sizes of $D = \{1 \text{ mm}, 5 \text{ mm}, 0.2 \text{ m}, 1.5 \text{ m}\}$. The electron bunch has a temporal duration of 800 fs and a transverse size of 200 μm . (c) Gaussian-fitted rms widths σ of the main peaks in CTRI signals, as a function of the original bunch duration. The results from different detector sizes $D = \{5 \text{ mm}, 20 \text{ mm}, 0.2 \text{ m}, 1.5 \text{ m}\}$ have been calculated. The electron energy is 30 MeV. The maximum acceptance angle of OAP1 is $\theta_m = 0.5$.

is demonstrated by the monotonic increase of all the curves in Fig. 5(c).

IV. ANALYSIS OF CTRI DATA USING THE FREQUENCY FILTER MODEL

A demonstration of data analysis using the established frequency-filtering model was performed on CTRI data obtained at the Megavolt InTense High-gradient Research Accelerator (MITHRA) [27], an educational and research facility of the Particle Beam Physics Laboratory (PBPL) at UCLA. The CTRI data were achieved with the Bunch Length Interferometer System (BLIS, RadiaBeam Technologies) [50]. During the measurement, the electron bunches had energies of 30 MeV, charges of about 120 pC, and a transverse size of 200 μm at the source plane. A pyroelectric device with an acceptance of 5 mm was used as the detector.

In the experiment, OAP1 and OAP2 had diameters/effective focal lengths of 4/4 inches and 3/3 inches, respectively, forming an imaging system with a magnification of 3/4. The CTR imaging and frequency-filter calculations employed a two-dimensional mesh with a spatial grid size of 20 μm . The calculated frequency range was 0–42 THz with an interval of 0.082 THz, corresponding to a temporal resolution of 12 fs, which is sufficiently shorter than the bunch durations considered in this work.

A. Analysis of CTRI data using the KK relation

First, the analysis of CTRI data has been carried out using the Kramers-Kronig relation [43,51,52] to obtain the minimum phase, ϕ_{KK} . This approach has been widely adopted as an inverse method for reconstructing the longitudinal bunch profile, with a workflow illustrated in Fig. 6(a). To achieve the temporal profile of the electron

bunch, the complex form factor amplitude $A_z(\omega)$ is necessary, as indicated by Eq. (8). The Fourier transformation of the measured CTRI signal is: $S(\omega) \propto |A_z(\omega)|^2 G(\omega) + C$. The $|A_z(\omega)|$ is achieved with a pre-calculated $G(\omega)$ from Eq. (11), according to the detection setup in the experiment. The natural logarithm of $A_z(\omega)$ is: $\ln A_z(\omega) = \ln |A_z(\omega)| + i\phi(\omega)$. This is a form of a complex number, and the imaginary part is calculated with the real part via:

$$\phi_{KK}(\omega) = -\frac{2\omega}{\pi} \text{P.V.} \int_0^\infty \frac{\ln |A_z(\omega')| / |A_z(\omega)|}{\omega'^2 - \omega^2} d\omega'. \quad (14)$$

The notation P.V. means the principal value. In the data analysis, Eq. (14) was calculated as a discrete summation over the sampled spectral range. With the ϕ_{KK} , the complex spectral distribution is achieved. The electron-bunch longitudinal shape is:

$$g_{KK}(t) \propto \sum \cos[\omega_i t + \phi_{KK}(\omega_i)] |A_z(\omega_i)| \Delta\omega, \quad (15)$$

where ω_i denotes the discrete frequency components sampled during the data analysis. The experimental CTRI data are shown as blue asterisks in Fig. 7(a). The temporal profile reconstructed using the KK method is shown as black circles in Fig. 7(b). The CTRI signal calculated from this reconstructed profile is plotted as black circles in Fig. 7(a). The normalized longitudinal form factor amplitudes, $|A_z|$, are shown in Fig. 7(c).

The classical KK method illustrated in Fig. 6(a) assumes that the response function is analytic in the upper half of the complex frequency plane and satisfies the minimum-phase condition [13,43]. In practice, the measured spectrum is limited by the finite bandwidth of the detector system, requiring fitting and extrapolation of the form factor. Moreover, electron bunches with complicated

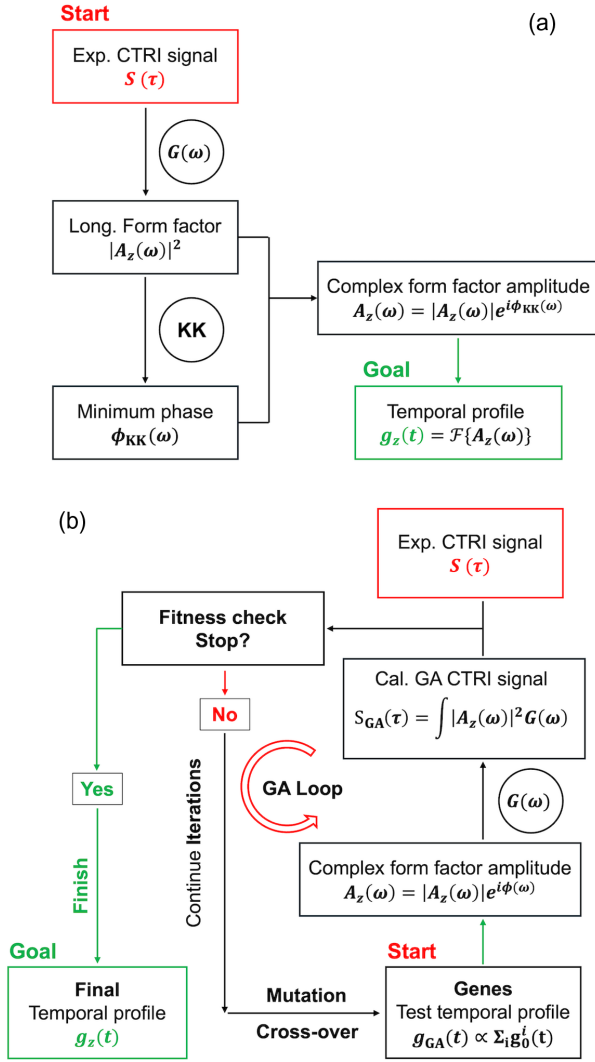


FIG. 6. The workflows of the reconstructions of the electron-bunch temporal profiles from the experimental CTRI data. (a) Kramers-Kronig phase reconstruction. (b) Adaptive fitting using genetic algorithm.

temporal structures may produce form factors with zeros in the upper half of the complex frequency plane, violating the minimum-phase condition. Consequently, the standard KK relation may yield unphysical negative values and residual oscillations in the reconstructed temporal profile, as shown in Fig. 7(b) and reported in previous studies [17,19,24,43].

Additional phase contributions, known as the Blaschke phase [43,53], may then be required. However, determination of the Blaschke phase generally relies on prior assumptions about the bunch structure, introducing additional uncertainty into the reconstruction.

B. Adaptive forward fitting using GA

While the KK relation is commonly employed in CTR spectral measurements—where the experimental signal is

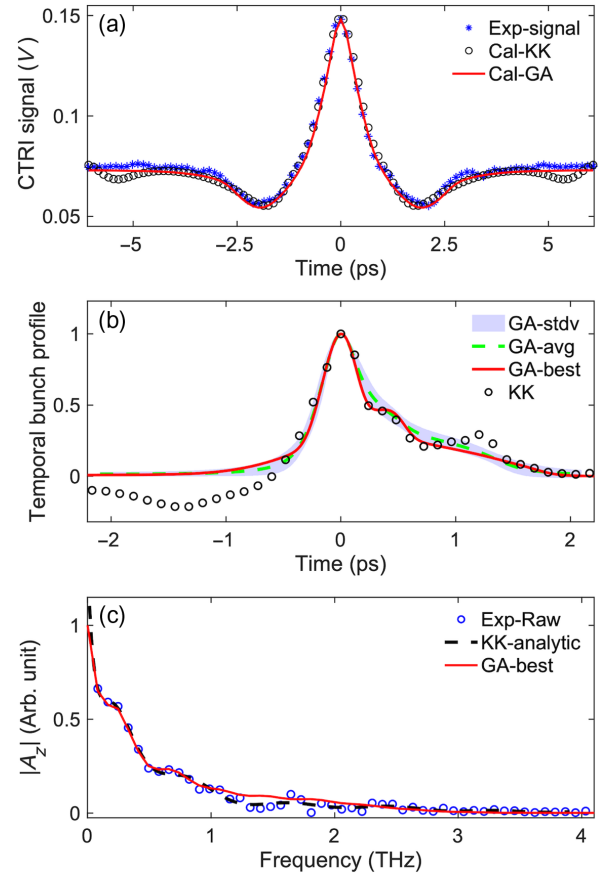


FIG. 7. CTRI experimental data analysis. (a) Measured CTRI signal (blue asterisks), together with the reconstructed signals obtained from the KK method (black circles) and GA fitting (red curve). (b) Comparison of the normalized bunch temporal profiles. The black circles represent the KK reconstruction, while the red curve shows the GA solution with the best fitness. The green dashed curve denotes the average profile from 10 independent GA runs, and the shaded region indicates the corresponding standard deviation. (c) Longitudinal form factor amplitude, $|A_z|$. The blue circles represent the experimental data, the black dashed curve shows the analytic extrapolation used for the KK reconstruction, and the red curve corresponds to the GA-reconstructed bunch temporal profile shown in (b).

obtained in the frequency or wavelength domain, the CTRI signal is measured directly in the temporal domain. To avoid excessive extrapolation and restrictive shape assumptions on the form factor in the frequency domain, we propose a forward-calculation approach that directly fits the experimental CTRI data rather than its Fourier-transformed counterpart, using a GA. The workflow of the GA fitting process is illustrated in Fig. 6(b). In this workflow, the procedure begins with a temporal bunch profile of an arbitrary but reasonable shape, represented by a chromosome composed of genes. A forward calculation is then performed to obtain the corresponding CTRI signal based on the numerical model described in the previous sections. The fitness value is calculated as

fitness = $\sqrt{\sum_i [I^{\text{exp}}(i) - I^{\text{GA}}(i)]^2 / N_{\text{sample}}}$, where i and N_{sample} denote the indices of and the total element number of the CTRI data array. This value represents the root-mean-square error (RMSE) between the GA calculation and the experimental CTRI data. The iteration is terminated when the difference in fitness values between the best solutions of two consecutive generations falls below 10^{-6} , corresponding to a relative variation of less than one over a million.

The genes are defined as the variables characterizing a skewed super-Gaussian (SG) profile, as described in Eq. (16):

$$g^{\text{SkewSG}}(t) = A e^{-\left[\frac{(t-t_0)^2}{2w^2}\right]^P} \times \{1 + \text{erf}[\eta(t-t_0)]\}, \quad (16)$$

where “ w ” and “ t_0 ” denote the temporal duration and central timing of the Gaussian. “ P ” is the order of the SG. The “erf” denotes the error function and “ η ” represents the skewness. The “ A ” denotes the weight of a gene. This gene structure is capable of representing asymmetric, flat-top, and near-Gaussian bunch distributions within a unified functional form. When $\eta = 0$, the profile reduces to a symmetric SG.

A single gene used in the GA calculation is therefore defined as $\text{gene} = \{A, t_0, w, P, \eta\}$. The variation ranges of each element of the gene are $[(0, 1), (-1.5, 1.5) \text{ ps}, (0.1, 3) \text{ ps}, (0.5, 2), (-0.2/\delta t, 0.2/\delta t)]$, with the time interval in the calculation denoted by “ δt ”. The variation range of the bunch duration is selected based on simulation results for the MITHRA accelerator system reported in Ref. [27]. The variation range of the bunch center timing is then determined according to the maximum bunch duration considered. Examples of individual skewed SG genes are shown in Fig. 8. A chromosome is constructed by the superposition of six genes, each with randomly assigned amplitude, center position, duration, skewness, and other parameters. A generation is defined as a group of 100 chromosomes.

The GA-fitted temporal profile corresponding to the best fitness is shown by the red curve in Fig. 7(b), with the corresponding calculated CTRI signal plotted in Fig. 7(a).

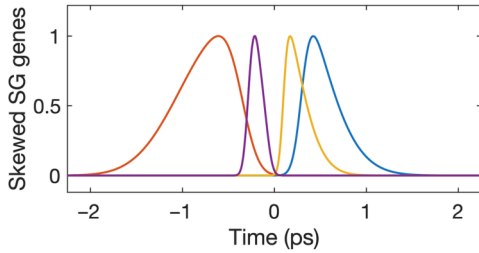


FIG. 8. Examples of individual temporal profiles described by genes, corresponding to various skewnesses and durations.

The “GA-best” profile reproduces the overall shape of the “KK minimum-phase” result while eliminating the unphysical negative values and oscillations. To assess the robustness of the fitting, 10 independent GA runs were performed. The average reconstructed profile is shown by the green dashed curve in Fig. 7(b), with the shaded region indicating the standard deviation. The small variation among the reconstructed profiles demonstrates good convergence and stability of the GA fitting. The reconstructed profile consists of a main peak accompanied by a pedestal. The main peak has a full width at half maximum (FWHM) of approximately 490 fs, corresponding to an rms duration of $\sigma_1 \sim 200 \text{ fs}$ (60 μm), while the rms duration of the entire profile, including the pedestal, is $\sigma_2 \sim 600 \text{ fs}$ (180 μm). These results indicate that, with appropriate gene structures and termination criteria, GA fitting provides a robust and physically consistent approach for reconstructing electron-bunch temporal profiles.

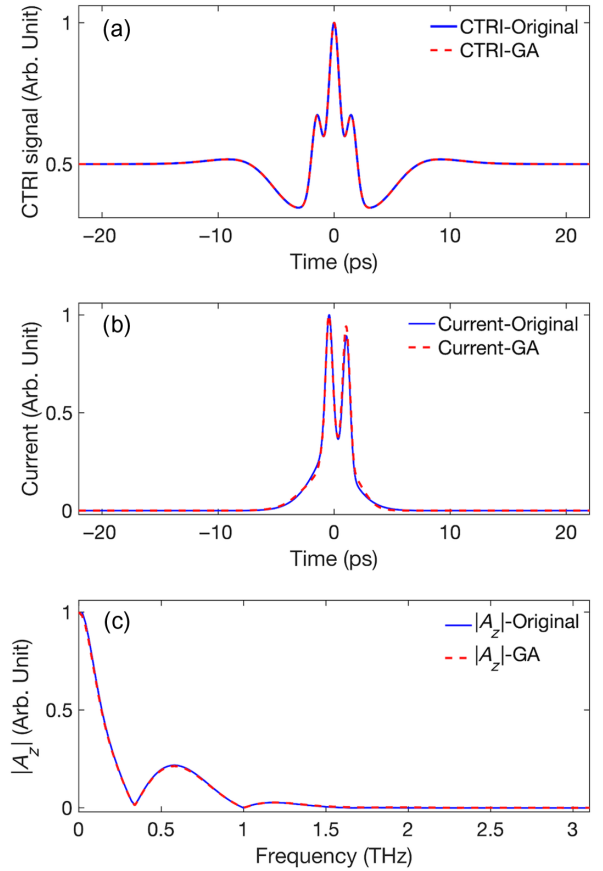


FIG. 9. A demonstration of GA forward fitting with artificially created CTRI signal from an electron bunch with multibunch structure. The blue curves and the red dashed curves represent the original setting and the results from GA fitting, respectively. (a) The CTRI signal shapes. (b) The electron-bunch temporal profiles. (c) The absolute values of longitudinal form factor amplitudes.

C. Numerical validation with an artificial electron-bunch profile

To further demonstrate the robustness of the proposed adaptive forward-fitting method, a GA fitting test was performed using artificially generated data, as shown in Fig. 9. The original temporal profile, shown by the blue curve in Fig. 9(b), is a multibunch distribution formed by the superposition of five Gaussian profiles. The corresponding artificial CTRI signal is plotted in Fig. 9(a). The normalized current profile reconstructed by the GA fitting (“Current GA”) and its corresponding CTRI signal are shown by the red dashed curves in Figs. 9(b) and 9(a), respectively. Figure 9(c) compares the longitudinal form factor amplitude, $|A_z|$, calculated from the original profile (blue) and reconstructed profile (red dashed). The excellent agreement among the current profiles, CTRI signals, and form factors demonstrates the capability of the proposed GA-based forward-fitting approach to recover complex electron-bunch temporal structures.

V. DISCUSSIONS AND CONCLUSION

The CTRI signal is symmetric because of the cosine function term in Eq. (1). Consequently, the temporal direction cannot be uniquely determined from the measured interferogram. Nevertheless, owing to the linearity of the forward calculation, each temporal bunch profile produces a unique CTRI signal, as demonstrated in Figs. 4, 5(c), 7, and 9. Therefore, the actual electron-bunch temporal profile is either the reconstructed profile or its time-reversed counterpart.

Regarding the comparison between the GA forward-fitting approach and the KK minimum-phase method, iterative phase-retrieval algorithms are beyond the scope of the present work. When the minimum-phase condition is violated, iterative methods such as the Gerchberg-Saxton (GS) algorithm [54] can be employed to recover a physically reasonable phase and have been applied to CTR spectral measurements [43,55]. However, the primary focus of this work is the analysis of CTRI signals measured directly in the time domain, rather than the reconstruction of bunch profiles from measured CTR spectra. From this perspective, if an iterative reconstruction strategy is to be adopted, fitting to the measured CTRI signal is a more natural and straightforward approach, as it converges directly to the experimental data.

In conclusion, for the measurement of electron temporal profiles using the coherent transition radiation Michelson interferometry, we have developed a frequency filter model based on imaging calculations that account for the electron-bunch transverse profile, the finite angular acceptance of the collection optics, and the limited detector size. The model has been applied to CTRI data analysis using both the KK minimum-phase and GA adaptive fitting approaches. The results demonstrate that

the frequency-filter model together with adaptive fitting provides a robust alternative for reconstructing the temporal profile. This work offers useful methodological tools for beam diagnostics in accelerator studies.

ACKNOWLEDGMENTS

This research was funded by U.S. DOE: DE-SC0009914. K. Huang acknowledges support by the JSPS KAKENHI Grants No. JP26K03219, No. JP23K17152, and the QST President’s Strategic Grant (Exploratory Research), Japan.

DATA AVAILABILITY

There are no publicly available research data or software supporting this manuscript. Requests for further information or data should be sent to the authors.

APPENDIX A: THE TRANSVERSE FORM FACTOR IN THE SCENARIO OF IMAGING CALCULATION

Consider an electron with transverse position (x_b, y_b) in the bunch, as illustrated in Fig. 2, the TR field emitted by this electron, from the source plane (x_s, y_s) to the lens plane (x_l, y_l) , is calculated by Fraunhofer diffraction as:

$$E_{\text{TR}}^l(x_l, y_l) = \frac{\exp(ikF_1)}{i\lambda F_1} \exp\left[\frac{ik}{2F_1}(x_l^2 + y_l^2)\right] \times \int_{-\infty}^{+\infty} dx_s \int_{-\infty}^{+\infty} dy_s E^s(x_s - x_b, y_s - y_b) \times \exp\left[-\frac{ik}{F_1}(x_s x_l + y_s y_l)\right] \times \exp\left[\frac{ik}{F_1}(x_b x_l + y_b y_l)\right]. \quad (\text{A1})$$

The last term in Eq. (A1) is then rewritten as:

$$\exp\left[\frac{ik}{F_1}(x_b x_l + y_b y_l)\right] = \exp\left(ik \frac{x_l}{F_1} x_b + ik \frac{y_l}{F_1} y_b\right) = \exp(ik_x x_b + ik_y y_b). \quad (\text{A2})$$

The overall CTR field of the bunch imaged to the detector is thus:

$$\Sigma \exp(ik_x x_b + ik_y y_b) \int \cdots dx_l dy_l dx_s dy_s = \Sigma \exp[i(k_x x_b + k_y y_b)] E_0^d(\omega, x_d - x_b, y_d - y_b). \quad (\text{A3})$$

The transverse form factor is included in the Fraunhofer diffraction calculations above, which demonstrates the

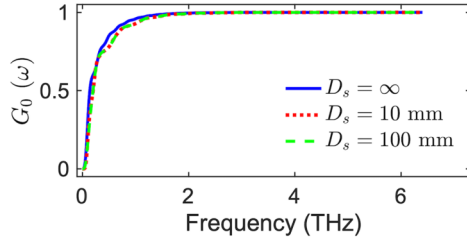


FIG. 10. The frequency filters $G_0(\omega)$ with different TR radiator sizes. The result with infinite radiator size is plotted as the blue curve. The results with radiator sizes of $D_s = 10$ and 100 mm are illustrated in red dots and green dashed curves, respectively. The detector has a size of 5 mm. The electron energy is 30 MeV.

validity of the “convolution approach” in imaging calculations of the CTR intensity.

APPENDIX B: ON THE FINITE SIZE OF A TR RADIATOR

The derivation of the frequency filter in this study neglects the frequency loss caused by the finite size of the TR radiator. This issue has been examined numerically in detail in a previous study [44], which has demonstrated that the analytical model described by Eq. (4) introduced errors only at the percentage level or smaller. Here, calculations of the frequency filters have been made for different TR radiator sizes, as shown in Fig. 10. The calculation was performed by integrating all field components within a finite radiator area characterized by a size parameter D_s . The differences of $G_0(\omega)$ between radiator diameters of $D_s = \{10, 100, \infty\}$ mm are negligible. Since the CTR field observed at the detector corresponds to the image of the source distribution, frequency-dependent losses are primarily determined by the detector size when the radiator is larger than the detector, a condition commonly satisfied in experiments [18,37].

APPENDIX C: THE IMPACT OF GENE DESIGN IN THE GA FITTING

We have conducted calculations with different gene structures: Skew-SG, SG, and G. The Skew-SG is described by Eq. (16). The SG is described by

$$g^{\text{SG}}(t) = Ae^{-\left[\frac{(t-t_0)^2}{2w^2}\right]^p}, \quad (\text{C1})$$

while the Gaussian profile is

$$g^{\text{G}}(t) = Ae^{-\left[\frac{(t-t_0)^2}{2w^2}\right]}. \quad (\text{C2})$$

For each gene type, two independent runs were performed, as shown in Fig. 11. The Skew-SG case exhibits a faster decrease in fitness and reaches the lowest final fitness value, while the results from the simple Gaussian

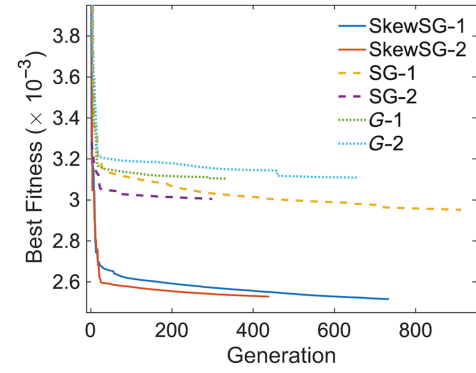


FIG. 11. GA fitting comparison for different gene structures. The solid, dashed, and dotted curves denote the trend of the best fitness values in each generation for the cases of the Skewed-Super-Gaussian, Super-Gaussian, and Gaussian, respectively. The results were obtained by fitting the experimentally measured CTRI signal using the GA algorithm.

case are less satisfactory. These results demonstrate that increasing the flexibility of the gene structure improves both convergence and fitting performance.

-
- [1] P. Chen, J. M. Dawson, R. W. Huff, and T. Katsouleas, *Phys. Rev. Lett.* **54**, 693 (1985).
 - [2] J. B. Rosenzweig, *Phys. Rev. Lett.* **58**, 555 (1987).
 - [3] J. B. Rosenzweig, D. B. Cline, B. Cole, H. Figueroa, W. Gai, R. Konecny, J. Norem, P. Schoessow, and J. Simpson, *Phys. Rev. Lett.* **61**, 98 (1988).
 - [4] I. Blumenfeld, C. E. Clayton, F.-J. Decker, M. J. Hogan, C. Huang, R. Ischebeck, R. Iverson, C. Joshi, T. Katsouleas, N. Kirby *et al.*, *Nature (London)* **445**, 741 (2007).
 - [5] E. Adli, A. Ahuja, O. Apsimon, R. Apsimon, A.-M. Bachmann, D. Barrientos, F. Batsch, J. Bauche, V. Berglyd Olsen, M. Bernardini *et al.*, *Nature (London)* **561**, 363 (2018).
 - [6] P. Emma, R. Akre, J. Arthur, R. Bionta, C. Bostedt, J. Bozek, A. Brachmann, P. Bucksbaum, R. Coffee, F.-J. Decker *et al.*, *Nat. Photonics* **4**, 641 (2010).
 - [7] T. Ishikawa, H. Aoyagi, T. Asaka, Y. Asano, N. Azumi, T. Bizen, H. Ego, K. Fukami, T. Fukui, Y. Furukawa *et al.*, *Nat. Photonics* **6**, 540 (2012).
 - [8] K. Yun, S. Kim, D. Kim, M. Chung, W. Jo, H. Hwang, D. Nam, S. Kim, J. Kim, S.-Y. Park *et al.*, *Sci. Rep.* **9**, 3300 (2019).
 - [9] W. Decking, S. Abeghyan, P. Abramian, A. Abramsky, A. Aguirre, C. Albrecht, P. Alou, M. Altarelli, P. Altmann, K. Amyan *et al.*, *Nat. Photonics* **14**, 391 (2020).
 - [10] V. Yakimenko, S. Meuren, F. Del Gaudio, C. Baumann, A. Fedotov, F. Fiuza, T. Grismayer, M. J. Hogan, A. Pukhov, L. O. Silva, and G. White, *Phys. Rev. Lett.* **122**, 190404 (2019).
 - [11] W. Zhang, T. Grismayer, and L. O. Silva, *Phys. Rev. Accel. Beams* **28**, 091001 (2025).
 - [12] K. Akai, K. Furukawa, H. Koiso, and S. accelerator team, *Nucl. Instrum. Methods Phys. Res., Sect. A* **907**, 188 (2018).

- [13] J. D. Jackson, *Classical Electrodynamics*, 3rd ed. (Wiley, New York, 1999).
- [14] M. L. Ter-Mikaelian, *High-Energy Electromagnetic Processes in Condensed Media* (John Wiley & Sons, New York, 1972), Vol. 29.
- [15] A. H. Lumpkin, B. X. Yang, W. J. Berg, Y. C. Chae, N. S. Sereno, R. J. Dejus, C. Benson, and E. Moog, *Nucl. Instrum. Methods Phys. Res., Sect. A* **475**, 462 (2001).
- [16] E. Chiadroni, A. Cianchi, M. Ferrario, A. Mostacci, R. Pompili, and V. Shpakov, *Condens. Matter* **5**, 40 (2020).
- [17] B. Schmidt, N. M. Lockmann, P. Schmüser, and S. Wesch, *Phys. Rev. Accel. Beams* **23**, 062801 (2020).
- [18] O. Zarini, J. C. Cabadağ, Y.-Y. Chang, A. Köhler, T. Kurz, S. Schöbel, W. Seidel, M. Bussmann, U. Schramm, A. Irman, and A. Debus, *Phys. Rev. Accel. Beams* **25**, 012801 (2022).
- [19] R. Lai, U. Happek, and A. J. Sievers, *Phys. Rev. E* **50**, R4294 (1994).
- [20] Q. Tian, Y. Du, H. Xu, Y. Liang, Y. Wang, Y. Tan, L. Yan, R. Li, C. Cheng, W. Huang, and C. Tang, *Phys. Rev. Accel. Beams* **23**, 102802 (2020).
- [21] W. Barry, in *Proceedings of the workshop on advanced beam instrumentation, KEK-PROC-91-2 (v. 1)* (High Energy Accelerator Research Organization (KEK), Tsukuba, Japan, 1991), pp. 224–235.
- [22] H.-C. Lihn, P. Kung, C. Settakorn, H. Wiedemann, and D. Bocek, *Phys. Rev. E* **53**, 6413 (1996).
- [23] A. Murokh, J. B. Rosenzweig, M. Hogan, H. Suk, G. Travish, and U. Happek, *Nucl. Instrum. Methods Phys. Res., Sect. A* **410**, 452 (1998).
- [24] T. Watanabe, M. Uesaka, J. Sugahara, T. Ueda, K. Yoshii, Y. Shibata, F. Sakai, S. Kondo, M. Kando, H. Kotaki, and K. Nakajima, *Nucl. Instrum. Methods Phys. Res., Sect. A* **437**, 1 (1999).
- [25] I. Nozawa, K. Kan, J. Yang, A. Ogata, T. Kondoh, M. Gohdo, K. Norizawa, H. Kobayashi, H. Shibata, S. Gonda, and Y. Yoshida, *Phys. Rev. ST Accel. Beams* **17**, 072803 (2014).
- [26] X. J. Wang, X. Qiu, and I. Ben-Zvi, *Phys. Rev. E* **54**, R3121 (1996).
- [27] O. Williams, A. Fukasawa, Y. Sakai, G. Andonian, F. Bosco, M. Carillo, P. Manwani, S. O'Tool, J. Pan, M. Yadav *et al.*, *Instruments* **7**, 54 (2023).
- [28] C. M. Scoby, P. Musumeci, J. T. Moody, and M. S. Gutierrez, *Phys. Rev. ST Accel. Beams* **13**, 022801 (2010).
- [29] T. Tajima and J. M. Dawson, *Phys. Rev. Lett.* **43**, 267 (1979).
- [30] J. Faure, Y. Glinec, A. Pukhov, S. Kiselev, S. Gordienko, E. Lefebvre, J.-P. Rousseau, F. Burgy, and V. Malka, *Nature (London)* **431**, 541 (2004).
- [31] C. G. R. Geddes, C. Toth, J. van Tilborg, E. Esarey, C. B. Schroeder, D. Bruhwiler, C. Nieter, J. Cary, and W. P. Leemans, *Nature (London)* **431**, 538 (2004).
- [32] S. P. Mangles, C. Murphy, Z. Najmudin, A. G. R. Thomas, J. Collier, A. E. Dangor, E. Divall, P. Foster, J. Gallacher, C. Hooker *et al.*, *Nature (London)* **431**, 535 (2004).
- [33] A. D. Debus *et al.*, *Phys. Rev. Lett.* **104**, 084802 (2010).
- [34] J. van Tilborg, C. B. Schroeder, C. V. Filip, C. Tóth, C. G. R. Geddes, G. Fubiani, R. Huber, R. A. Kaindl, E. Esarey, and W. P. Leemans, *Phys. Rev. Lett.* **96**, 014801 (2006).
- [35] K. Huang, Z. Jin, N. Nakanii, T. Hosokai, and M. Kando, *Light Sci. Appl.* **13**, 84 (2024).
- [36] S. Casalbuoni, B. Schmidt, and P. Schmusuer, TESLA FEL Reports, 2005-15, 2005.
- [37] S. Casalbuoni, B. Schmidt, P. Schmüser, and B. Steffen, TESLA FEL Reports, 2006-4, 2006.
- [38] D. Xiang and W.-H. Huang, *Nucl. Instrum. Methods Phys. Res., Sect. A* **570**, 357 (2007).
- [39] M. Castellano and V. A. Verzilov, *Phys. Rev. ST Accel. Beams* **1**, 062801 (1998).
- [40] A. Potylitsyn, L. Sukhikh, T. Gusvitskii, G. Kube, and A. Novokshonov, *Phys. Rev. Accel. Beams* **23**, 042804 (2020).
- [41] M. Mitchell, *An Introduction to Genetic Algorithms* (MIT Press, Cambridge, MA, 1998).
- [42] R. Lai and A. J. Sievers, *Phys. Rev. E* **50**, R3342 (1994).
- [43] B. Schmidt, S. Wesch, C. Behrens, T. Koevener, E. Hass, S. Casalbuoni, and P. Schmüser, Deutsches Elektronen-Synchrotron (DESY), Technical Report, 2018.
- [44] K. Huang, Z. Jin, N. Nakanii, T. Hosokai, and M. Kando, *Phys. Rev. Accel. Beams* **26**, 112801 (2023).
- [45] B. S.-Y. Ung, C. Fumeaux, H. Lin, B. M. Fischer, B. W.-H. Ng, and D. Abbott, *Opt. Express* **20**, 4968 (2012).
- [46] V. E. Rogalin, I. A. Kaplunov, and G. I. Kropotov, *Opt. Spectrosc.* **125**, 1053 (2018).
- [47] A. Fisher, Y. Park, M. Lenz, A. Ody, R. Agustsson, T. Hodgetts, A. Murokh, and P. Musumeci, *Nat. Photonics* **16**, 441 (2022).
- [48] G. Andonian, M. Dunning, E. Hemsing, J. Rosenzweig, A. Cook, A. Murokh, S. Reiche, M. Babzien, I. Ben-Zvi, K. Kutsche *et al.*, *Int. J. Mod. Phys. A* **22**, 4101 (2007).
- [49] H. Loos, R. Akre, A. Brachmann, F. Decker, Y. Ding, D. Dowell, P. Emma, J. Frisch, S. Gilevich, G. Hays *et al.*, in *Proceedings of the 2008 Free-Electron Laser Conference, Gyeongju, Korea* (Joint Accelerator Conferences Website (JACoW), Geneva, Switzerland, 2008).
- [50] RadiaBeam Technologies, <https://radiabeam.com/>.
- [51] R. Lai and A. J. Sievers, *Nucl. Instrum. Methods Phys. Res., Sect. A* **397**, 221 (1997).
- [52] J. S. Toll, *Phys. Rev.* **104**, 1760 (1956).
- [53] R. Lai and A. J. Sievers, *Phys. Rev. E* **52**, 4576 (1995).
- [54] R. W. Gerchberg, *Optik (Stuttgart)* **35**, 237 (1972).
- [55] S. I. Bajlekov, M. Heigoldt, A. Popp, J. Wenz, K. Khrennikov, S. Karsch, and S. M. Hooker, *Phys. Rev. ST Accel. Beams* **16**, 040701 (2013).