

Design of a storage ring based on a fixed-field alternating-gradient configuration with an internal target for heavy-ion beams with stochastic charge state conversions

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In heavy-ion accelerators used for radioactive isotope production, the accelerated beam is typically directed onto a target and then discarded in a beam dump. To make more efficient use of the beam, recycling of the beam passed through the target is proposed in the framework of the so-called energy recovery internal target (ERIT). In the ERIT system, the target is irradiated by a circulating beam, while the energy lost in the target is recovered using rf cavities. So far, such a system has been realized only for proton beams. Here, an ERIT system for heavy-ion beams is demonstrated for the first time. A major challenge is the circulation of ions with multiple atomic charge states. After passing through the target, ions rapidly reach an equilibrated charge-state distribution, independent of the initial charge state. This stochastic charge-state conversion (SCSC) leads to rapid beam-emittance growth. To mitigate this effect, we develop a method to match the closed orbits and betatron functions of different charge states at the target location, based on a scaling fixed-field alternating-gradient (FFA) lattice. We present the design of such an FFA ring and show, through 6D beam-tracking simulations, that the transverse emittance growth induced by SCSC can be significantly reduced over a practical number of turns, whereas longitudinal emittance growth remains significant and is discussed separately.

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I. INTRODUCTION

Heavy-ion accelerators have been the subject of research and development for decades because of their versatile applications [1–10], ranging from fundamental physics research programs, such as superheavy element production, to medical isotope production [11,12]. The production rate of the desired nuclides/isotopes depends on the number of beam particles passing through the target per unit time interval. In the traditional method, accelerated heavy ions are spoiled into a beam dump after traversing the target. The only way to increase the reaction rate is to develop a high-intensity ion source and dedicated accelerator system.

When one attempts to create a new element with $Z = 120$, for example, in the reaction of a ^{50}Ti beam with a ^{249}Cf target, the predicted cross section varies by 2 orders

of magnitude, from a femtobarn (fb) to 100 fb depending on the theory [13]. The 1-fb experiment requires approximately 10 years using a 1 pμA beam and a 500 μg/cm² target, which are typical conditions [14–16] of the current setup for superheavy element research.

To significantly increase a reaction rate and reduce the beam time, a method to repeatedly use the beam which passed through the target can be considered. Such an idea is called the energy recovery internal target (ERIT) [17] system using a storage ring, in which the internal target is irradiated by a circulating beam. The energy lost in the target is recovered by the rf cavities. So far, this approach has been adopted only for proton beams. To adopt this approach for heavy-ion beams, for example, in the field of superheavy element research at a beam energy of approximately 5–6 MeV/u, all the beams with different atomic charge states must be circulated with a proper beam area/size at the target.

If the ion in the beam has a small nuclear charge Z and sufficient energy, the electrons of the atoms are fully stripped. However, if the beam has a large Z and a relatively low energy insufficient to fully strip the electrons, the atomic charge states q have an equilibrated distribution

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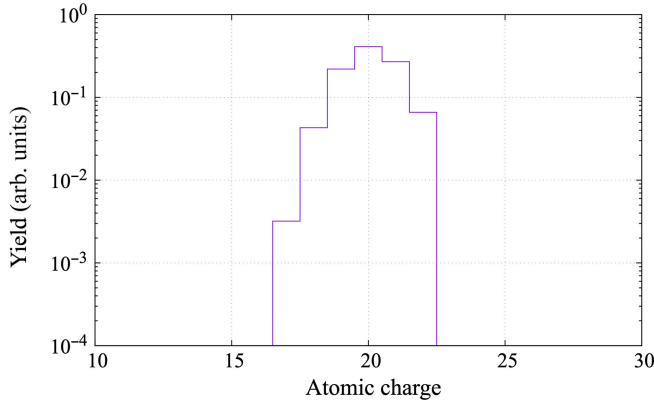


FIG. 1. Equilibrium atomic charge state distribution calculated using Shima's formula for the case of a ^{50}Ti beam of 5.5 MeV/u.

with a Gaussian-like shape described by Shima formula [18], irrespective of the initial charge state, after passing through the target. This stochastic charge state conversion (SCSC) at the target poses a challenge for heavy-ion ERIT systems. In this article, we show that the fixed-field alternating-gradient (FFA) configuration can be used to realize an ERIT system for heavy-ion beams, even in the presence of SCSC.

Figure 1 shows the atomic charge state distribution for a ^{50}Ti beam of 5.5 MeV/u, one of the candidate beams to create superheavy elements, calculated using the Shima formula. It has the maximum fraction at $q = 20^+$ and ranges from 17^+ to 22^+ . This corresponds to a momentum spread of $\Delta p/p$ from -15% to $+10\%$ around $q = 20^+$, which leads to a betatron tune (hereafter simply referred to as “tune”) spread due to chromaticity. In the conventional separated-function lattices, nonlinear elements such as sextupole magnets are used to correct chromaticity. However, it is difficult to compensate for a wide spread of rigidity exceeding $\pm 10\%$, as in the present case. Even when correction is effective, the dynamic aperture can be significantly reduced due to strong nonlinearities. In contrast, scaling FFA rings inherently maintain constant tunes regardless of beam rigidity, since the chromaticity is zero [19]. Another notable advantage of scaling FFA rings is their large dynamic aperture, which is preserved even under zero-chromaticity conditions.

In general, the magnetic field distribution in the median plane of a scaling FFA ring is [19]

$$B(r) \propto r^{k_0}, \quad (1)$$

where k_0 is a magnetic field index that is constant in the horizontal r and azimuthal θ directions. The definitions of the variables used in this study are summarized in Table I. As shown in Fig. 2, the closed orbit for charge state q is denoted by $\tilde{r}_q(\theta)$. The charge difference relative to q_0 is denoted by Δq , where q_0 is the nominal charge—i.e., the most probable charge (20^+ in the present case). The orbit

TABLE I. Definition of the variables.

	Radial coordinate in the global cylindrical coordinate system
r	Radial coordinate in the global cylindrical coordinate system
$\tilde{r}(\theta)$	Radius of the closed beam orbit as a function of θ
$\tilde{r}_q(\theta)$	Radius of the closed beam orbit in the charge state q
$\tilde{r}_{q_0}(\theta)$	Radius of the closed beam orbit in the nominal charge state
$\Delta\tilde{r}_q(\theta)$	Difference between $\tilde{r}_{q_0-\Delta q}(\theta)$ and $\tilde{r}_{q_0}(\theta)$
θ	Azimuthal coordinate in the global cylindrical coordinate system
θ_F	θ evaluated at the center of the F magnet
θ_D	θ evaluated at the center of the D magnet
θ_T	θ evaluated at the target position
x	Radial (or horizontal) coordinate in the local Frenet-Serret coordinate system on the closed orbit
y	Binormal (or vertical) coordinate in the local Frenet-Serret coordinate system on the closed orbit
s	Tangential (or along the beam axis) coordinate in the local Frenet-Serret coordinate system on the closed orbit
x_{cod}	Horizontal closed orbit distortion (closed orbit displacement in x)
$2J_x$	Horizontal betatron amplitude: $2J_x = \gamma_x x^2 + 2\alpha_x x x' + \beta_x x'^2$, $\alpha_x = -\frac{1}{2} \frac{d\beta_x}{ds}$, $\gamma_x = \frac{1+\alpha_x^2}{\beta_x}$
$2J_y$	Vertical betatron amplitude: $2J_y = \gamma_y y^2 + 2\alpha_y y y' + \beta_y y'^2$, $\alpha_y = -\frac{1}{2} \frac{d\beta_y}{ds}$, $\gamma_y = \frac{1+\alpha_y^2}{\beta_y}$

separation between q_0 and $q_0 \pm \Delta q$ is represented by $\Delta\tilde{r}_q(\theta)$.

From Eq. (1), the relationship between the charge difference and the closed orbit separation is given by (see Appendix A),

$$\Delta\tilde{r}_q(\theta) = -\frac{\Delta q}{q_0} \frac{\tilde{r}_{q_0}(\theta)}{k_0 + 1}, \quad (2)$$

where $\tilde{r}_{q_0}(\theta)/(k_0 + 1)$ corresponds to the dispersion function.

As shown in Fig. 3, the closed orbit point—i.e., the center of the Courant-Snyder ellipse—in transverse phase space at the target fluctuates when the SCSC occurs. As a result, the betatron amplitude changes accordingly.

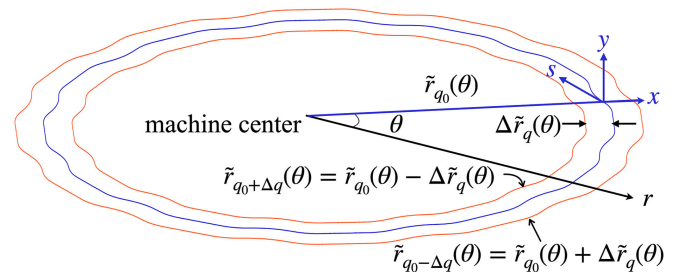


FIG. 2. Definition of the coordinate system and closed orbits. The orbit is drawn for the FFA having 12 FDF sectors.

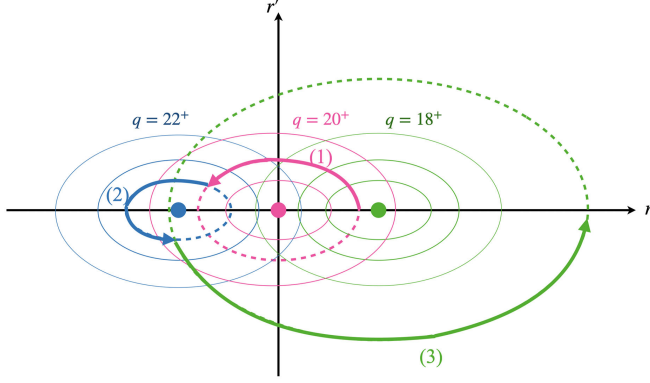


FIG. 3. Example of the change of betatron amplitude resulting from charge state changes: $q = 20^+ \rightarrow 22^+ \rightarrow 18^+$, corresponding to the amplitude shift (1) $\rightarrow$ (2) $\rightarrow$ (3).

Although the betatron amplitude increases or decreases depending on the betatron phase at the target, it tends to grow through long-term stochastic collision processes (see Fig. 6). To suppress this growth, the closed orbits for different charge states must satisfy the following achromatic conditions:

$$\tilde{r}_{q_0}(\theta_T) = \tilde{r}_{q_i}(\theta_T), \quad (3)$$

$$\tilde{r}'_{q_0}(\theta_T) = \tilde{r}'_{q_i}(\theta_T), \quad (4)$$

where θ_T is the azimuthal angle at the target location, q_i is any possible charge state, and $\tilde{r}'$ denotes the derivative of $\tilde{r}$ with respect to the longitudinal coordinate θ .

Achromatic lattices using FFA-type magnets have been studied, and several types of dispersion suppressors have been proposed [20–23]. The lattice introduced in Ref. [20] provides a dispersion suppressor in the straight beam transport line, which is not suitable for our purpose. According to studies in Ref. [21–23], the horizontal phase advance in the arc must be an integer multiple of 2π to suppress the dispersion to zero. This implies that a tune-adjusting straight section is required to avoid an integer horizontal tune. However, such straight sections, when designed to be dispersion-free, break the condition of zero chromaticity (an example of a zero-chromaticity racetrack FFA ring with dispersive straight sections has been demonstrated in the NORMA study [24]). Furthermore, this approach is difficult to apply to storage rings, which require both zero chromaticity and a horizontal tune sufficiently far from integer values. Therefore, we propose an alternative method to satisfy the conditions in Eqs. (3) and (4).

The remainder of this paper is organized as follows: Sec. II describes the lattice construction method; Sec. III presents the results of the beam simulations; Sec. IV discusses beam stability associated with the use of modulated k and the space-charge effects; and Sec. V provides the conclusion.

TABLE II. Basic parameters of the test ring.

Beam species	$^{50}\text{Ti}^{q+}$ ($q = 17 - 22$)	
Beam energy	5.5 MeV/u	
Lattice	Radial sector FDF triplet	
Symmetry	12	
Field index	k_0	2.16
Betatron tune	(ν_x, ν_y)	(1.88, 3.07)
$B_0(= B(r_0))$	F magnet	0.7235 T
	D magnet	−0.4698 T
Reference radius r_0	F magnet	5.96 m ($= \tilde{r}_{q_0}(\theta_F)$) ^a
	D magnet	5.82 m ($= \tilde{r}_{q_0}(\theta_D)$)
Opening angle	Long straight	10.5°
	Short straight	1.7°
	F magnet	5.12°
	D magnet	6.86°

^aRadius of the closed orbit at the center of F magnet defined as in Table I.

II. METHOD OF THE LATTICE CONSTRUCTION

In this section, we describe the design procedure of a heavy-ion storage ring intended for the production of superheavy elements. The proposed “test ring” is based on a 12-fold symmetric radial scaling FFA and comprises 12 FDF (focus-defocus-focus) triplets. This type of lattice has been previously adopted and experimentally tested in the ERIT system at the Institute for Integrated Radiation and Nuclear Science, Kyoto University (KURNS) [25,26]. The main design parameters of the test ring are summarized in Table II.

The magnetic field distribution in the test ring takes a more specific form of Eq. (1), given by

$$B(r) = B_0 \left(\frac{r}{r_0} \right)^{k_0}, \quad (5)$$

where B_0 and r_0 are the reference magnetic field and reference radius, respectively, specified for the F and D magnets as listed in Table II.

A. Matching of closed orbit and betatron functions

To fulfill the condition of Eq. (3), we intentionally use a closed orbit distortion (COD) $x_{\text{cod}}(\theta)$, as shown in Fig. 4, so that

$$\Delta \tilde{r}_q(\theta_T) + x_{\text{cod}}(\theta_T) = 0. \quad (6)$$

The condition of Eq. (4) is automatically satisfied if the target is located at the point where the mirror symmetry of the lattice is maintained.

Now, we consider the case in which the field index is not strictly constant with k_0 but modulated in θ direction as

$$k(\theta) = k_0 + \Delta k(\theta). \quad (7)$$

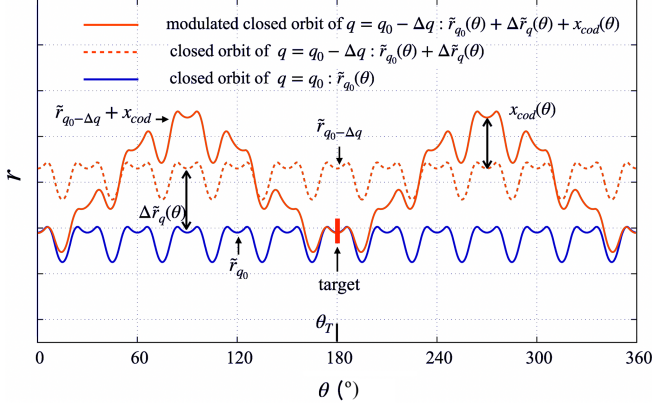


FIG. 4. Closed orbit matching. The blue line indicates the closed orbit of the beam in the nominal charge state q_0 . The dashed/solid red lines indicate the unmodulated/modulated closed orbits of the charge $q_0 - \Delta q$ beam.

Using this k modulation, $x_{\text{cod}}(\theta)$ is obtained as (see Appendix B)

$$x_{\text{cod}}(\theta) = -\frac{\Delta q}{q_0(k_0 + 1)} \frac{\sqrt{\beta_x(\theta)}}{2\pi B\rho} \sum_{n=-\infty}^{\infty} \frac{\nu_x^2 e^{in\phi_x(\theta)}}{\nu_x^2 - n^2} \Phi_n[\Delta k], \quad (8)$$

where we define

$$\Phi_n[\Delta k] := \oint B(\tilde{r}_{q_0}(\phi_x), \phi_x) \beta_x^{3/2}(\phi_x) \Delta k(\phi_x) e^{-in\phi_x} d\phi_x. \quad (9)$$

Substituting $x_{\text{cod}}(\theta_T)$ from Eq. (8) and $\Delta\tilde{r}_q(\theta_T)$ from Eq. (2) into the convergence condition given by Eq. (6), the following relation is obtained:

$$\frac{\Delta q}{q_0(k_0 + 1)} \left[\tilde{r}_{q_0}(\theta_T) + \frac{\sqrt{\beta(\theta_T)}}{2\pi B\rho} \sum_n \frac{e^{in\phi_x(\theta_T)} \Phi_n[\Delta k]}{\nu_x^2 - n^2} \right] = 0. \quad (10)$$

Because the condition for $\Phi_n[\Delta k(\theta)]$ in Eq. (10) is independent of Δq , the closed orbit in any charge state at the target location can converge to a point if we appropriately set the modulation $\Delta k(\theta)$.

To prevent SCSC-induced stochastic fluctuations of the Courant-Snyder ellipse and the resulting growth in betatron amplitude, it is essential to match not only the center of the ellipse (i.e., the closed orbit), but also its shape. This requires that the Courant-Snyder parameters β_x , β_y , α_x , and α_y be matched at the target location. In the test ring, which employs an FDF triplet lattice, the alpha functions vanish at the center of the long straight section, where the target is placed. Therefore, we only need to match β_x and β_y . Just as modulation of the dipole component leads to a COD,

modulation of the quadrupole component affects the betatron functions. The resulting modulation of β_x is given by [27]:

$$\frac{\Delta\beta_x}{\beta_x} = -\frac{\nu_x}{2} \sum_{p_x=-\infty}^{+\infty} \frac{\tilde{g}_{p_x} e^{ip_x\phi_x}}{\nu_x^2 - (\frac{p_x}{2})^2}, \quad (11)$$

where $\tilde{g}_{p_x}$ is the Fourier coefficient corresponding to harmonic number p_x , given by

$$\tilde{g}_{p_x} = \frac{\nu_x}{2\pi} \int_0^{2\pi} \Delta K_x(\phi_x) \beta_x^2(\phi_x) e^{-ip_x\phi_x} d\phi_x, \quad (12)$$

and ΔK_x denotes the modulation of the quadrupole component of the magnetic field. The same expression applies to the vertical beta function β_y by replacing the subscript x with y . Since modulation of the field index k also introduces quadrupole modulation, it can be utilized to control and match the betatron functions.

B. Optimization of the k modulation

Using the harmonic numbers n , p_x , and p_y , we apply the following modulation to $k(\theta)$:

$$k(\theta) = k_0 \{ 1 + \lambda_{\text{cod}} \cos[n\phi_x(\theta)] + \lambda_{\beta_x} \cos[p_x\phi_x(\theta)] + \lambda_{\beta_y} \cos[p_y\phi_y(\theta)] \}. \quad (13)$$

where λ_{cod} , λ_{β_x} , and λ_{β_y} are the perturbation amplitudes for each harmonic. According to Eqs. (10) and (11), n and $p_x/2$ should be close to ν_x to ensure efficient modulation with minimal perturbation. Similarly, $p_y/2$ should be close to ν_y for effective modulation of β_y .

The procedure for selecting these parameters is as follows. To achieve a wide dynamic aperture by minimizing nonlinear effects (see Sec. IV), we choose the value of k_0 to be as small as possible. Under this condition, requiring at least two achromatic points—one at the target and one at the rf cavity—leads to $n = 2$, which in turn results in $\nu_x \approx 2$.

To independently modulate β_x and β_y , we further require ν_y to differ from ν_x by at least an integer. Accordingly, the tunes are set to $\nu_x = 1.88$ and $\nu_y = 3.07$, as listed in Table II. Consequently, the harmonic numbers are determined to be $p_x = 4$ and $p_y = 6$.

Next, we determined λ_{cod} , λ_{β_x} , and λ_{β_y} . The parameters appearing in Eq. (13) are not mutually independent. The modulation introduced by the parameter n generates a closed-orbit distortion (COD), which is essential for orbit matching. However, in the presence of COD, the intrinsic sextupole components of the FFA magnetic field produce additional effective quadrupole fields through feed-down effects, resulting in a modulation of the beta functions,

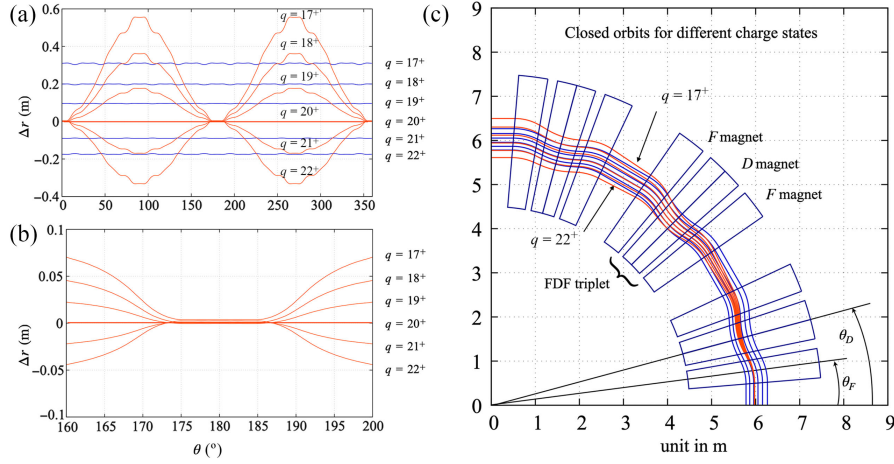


FIG. 5. Plots in (a) show the deviation of the closed orbits of various charge states from 17^+ to 22^+ concerning the closed orbits of the 20^+ charge state, with blue and red lines for the constant k and modulated k rings, respectively. Panel (b) is a close-up view of (a). In (c), the identical closed orbits in (a) are plotted as footprints.

as shown in Figs. 7(a) and 7(b). To compensate for this beta-function modulation, additional modulations controlled by p_x and p_y are applied to the focusing strength k . While these parameters are optimized to achieve the desired beta functions at the target location, they simultaneously modify the COD. Consequently, the parameters n , p_x , and p_y must be optimized in a coupled manner.

Based on this coupled dependence among the modulation parameters, the optimization was aimed at minimizing the growth of the horizontal and vertical root mean square (rms) emittances, ϵ_x and ϵ_y , over 1000 turns. The optimization was performed using the particle tracking code EARLIETIMES [28], operated in fourth-order Runge-Kutta mode under the hard-edge magnetic field approximation, where fringe-field effects were neglected. In this simulation, 1000 macroparticles were initialized with betatron amplitudes of $2J_x = 2J_y = 100$ mm mrad, uniformly distributed along the Courant-Snyder ellipse in phase space.

The corresponding rms emittance of this initial distribution was 50 mm mrad. Each particle was initially assigned a charge state of 20^+ , and tracking began from $\theta = 0$, which corresponds to 180° upstream from the target. At each target traversal, the particle's charge state was randomly reassigned according to the probability distribution shown in Fig. 1. Energy loss, multiple Coulomb scattering in the target, and synchrotron motion were all neglected in this optimization. Space-charge effects were also ignored, as the resulting tune shift is negligible under the current beam conditions (see Sec. IV). The optimized perturbation amplitudes were found to be: $\lambda_{\text{cod}} = -0.236$, $\lambda_{\beta_x} = -0.065$, and $\lambda_{\beta_y} = 0.044$.

Although the modulation of $k(\theta)$ should ideally be continuous, implementing such a modulation in magnet hardware is difficult. Therefore, we adopt an approach in which k is assumed to take a constant value within each magnet sector, defined by its value at the sector center.

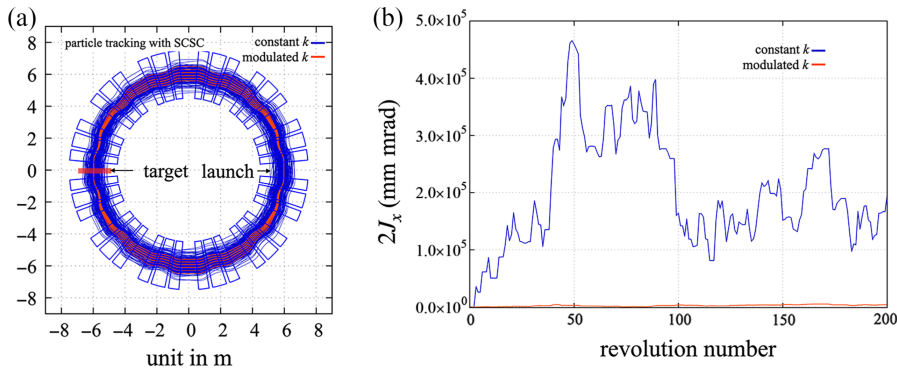


FIG. 6. Plots in (a) show the footprints of single particle tracking during 200 turns, considering the SCSC with blue and red lines for the constant k and modulated k rings, respectively. The plots in (b) show the horizontal betatron oscillation amplitude $2J_x$ as a function of the revolution number.

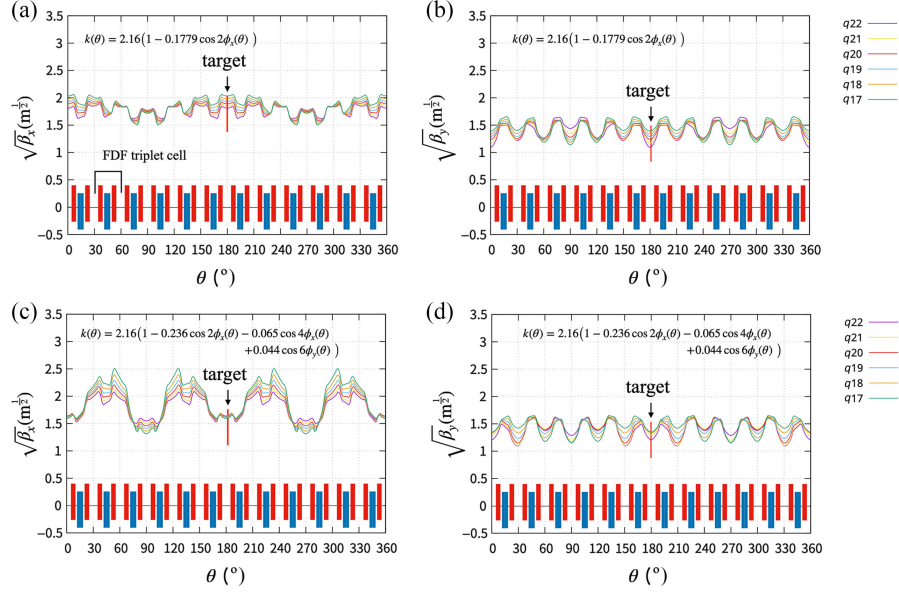


FIG. 7. Betatron functions for charge states ranging from $q = 17^+$ to 22^+ . Panels (a) and (b) show β_x and β_y , respectively, for the case in which only a single harmonic in k is included. Panels (c) and (d), on the other hand, show β_x and β_y when three harmonics in k are included.

C. Behavior of closed orbits, betatron functions, and emittance growths

Figure 5 shows the closed orbits for charge states ranging from $q = 17^+$ to 22^+ under constant k (blue) and modulated k (red) configurations. Due to the use of harmonic number $n = 2$ for COD excitation, the modulated k ring generates two convergence/focus points around the ring, effectively guiding particles toward stable orbits. (Using a higher harmonic number (e.g., $n = 3$) would generate three convergence points, which could be advantageously used to place rf cavities and would also reduce orbit excursions for different charge states. However, achieving this requires the horizontal tune ν_x to be close to 3, which in turn demands a higher k_0 , introducing strong nonlinearities into the system.)

To evaluate the orbital stability, we examined the motion of a test particle initialized with zero betatron amplitude (i.e., $2J_x = 0$). Figure 6(a) shows the footprints of the test particle over 200 turns: the blue line represents the case with constant k , while the red line corresponds to the modulated k configuration. Figure 6(b) shows the evolution of the betatron amplitude $2J_x$ for both cases. In the constant k ring, $2J_x$ grows rapidly, indicating strong instability. In contrast, the modulated k ring effectively suppresses the amplitude growth, and the particle remains oscillating in the vicinity of the closed orbit, demonstrating significant improvement in orbit confinement.

Figure 7 presents the betatron functions β_x and β_y for charge states ranging from $q = 17^+$ to 22^+ . Panels (a) and (b) show β_x and β_y , respectively, for the case in which only a single harmonic in k , namely the closed-orbit

modulation harmonic n responsible for orbit matching, is included, with

$$k(\theta) = 2.16\{1 - 0.1779 \cos[2\phi_x(\theta)]\}. \quad (14)$$

In this case, a large spread in the betatron functions among different charge states is observed at the target location in both the horizontal and vertical directions. Panels (c) and (d), on the other hand, show β_x and β_y when three harmonics in k , i.e., n , p_x , and p_y , are included. The corresponding modulated field index $k(\theta)$ is shown in Fig. 8 and is given by

$$k(\theta) = 2.16\{1 - 0.236 \cos[2\phi_x(\theta)] - 0.065 \cos[4\phi_x(\theta)] + 0.044 \cos[6\phi_y(\theta)]\}. \quad (15)$$

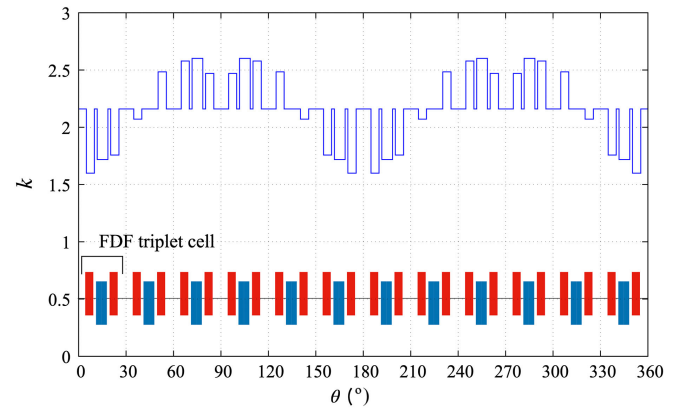


FIG. 8. Modulated k represented as a step-wise function of the azimuthal angle. The value of k is assumed to be constant within each magnet sector and is defined by its value at the sector center.

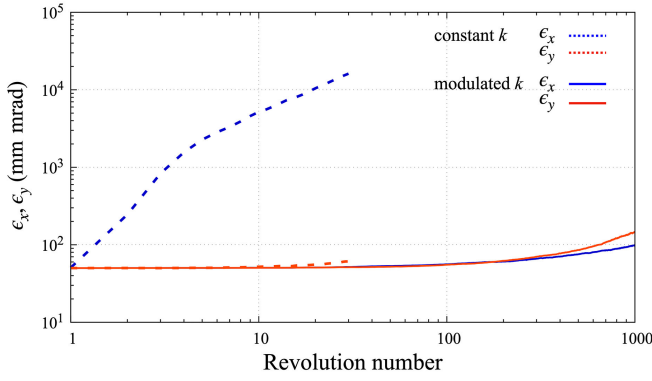


FIG. 9. Emittance growth in the x and y planes, with (solid) and without (dashed) k modulation.

Compared with the single-harmonic case, the betatron functions at the target location are well converged across the different charge states. This result confirms that the modulated k configuration ensures robust focusing and charge-state-independent transport at the target.

The effect of the modulation on beam stability is further illustrated in Fig. 9, which shows the evolution of ϵ_x (red) and ϵ_y (blue). Solid lines indicate the case with k modulation, while dashed lines represent the unmodulated case. Without k modulation, ϵ_x increases rapidly, reaching approximately 20,000 mm mrad within 50 turns. In contrast, with the optimized k modulation, the emittance growth in both planes is significantly mitigated. After 1000 turns, both ϵ_x and ϵ_y remain near 100 mm mrad—about only twice the initial value—clearly demonstrating the effectiveness of the modulation in preserving beam quality.

To examine the underlying dynamics in more detail, Fig. 10 illustrates the diffusion process of particles in phase space for the same simulations. Panels (a)–(d) show the horizontal phase space for the unmodulated k case, where particles are initially distributed along the edge of the Courant-Snyder ellipse corresponding to $2J_x = 100$ mm mrad. The distributions are shown after 1, 2, 5, and 10 turns. Owing to the stochastic charge-state conversion (SCSC), the phase-space ellipses rapidly diffuse. Panels (e)–(h) present the corresponding distributions in the vertical plane. Since the vertical motion is not directly affected by SCSC, the diffusion in this plane remains minimal. Panels (i)–(l) show the horizontal phase-space distributions for the optimized k -modulated case after 1, 100, 500, and 1000 turns. As the effect of SCSC is significantly mitigated, only a limited diffusion of the phase-space ellipse is observed, even after long-term tracking. Panels (m)–(p) present the corresponding results in the vertical plane, where similarly small diffusion effects are observed, consistent with the horizontal case.

III. BEAM SIMULATION

Based on the lattice described above, a 6D beam-tracking simulation was performed. The application under consideration is a storage ring for the production of superheavy elements. We assumed that a $^{50}\text{Ti}^{20+}$ beam was incident on a ^{249}Bk target to produce element 119. Here, the thickness of the target is chosen to be $100 \mu\text{g}/\text{cm}^2$. The main beam, lattice, and target parameters used in the simulations are summarized in Table III. We note that this material was used to produce superheavy element 117 (Tennessine) using a ^{48}Ca beam [29]. To account for beam-target interactions, the following effects were incorporated into the simulation: (1) energy loss and straggling of the beam, and (2) energy recovery by an rf cavity. The SCSC process was switched on and off to evaluate its impact. For energy loss and straggling, the Monte Carlo code TRIM [30] was used. Figure 11(a) shows the energy loss distribution of the ^{50}Ti beam (5.5 MeV/u) after passing through the target at the incident beam energy of 275 MeV, while Fig. 11(b) shows the corresponding scattering angle distribution. The rf cavity was located at $\theta = 0^\circ$.

The simulation results are presented in Figs. 12 and 13. Figure 12 shows the time evolution of the longitudinal phase space ($\Delta E - \phi_{\text{rf}}$). The upper and lower rows correspond to the cases without and with SCSC effects, respectively. Due to energy loss at the target, the synchronous phase was set to 8.7° , as listed in Table III, and an acceleration bucket was formed accordingly. The separatrices shown correspond to the charge states ranged from 17^+ to 22^+ . As shown in panels (a) and (d), the beam was injected with a 20% duty factor. A comparison between (b) and (e) clearly shows that SCSC acts as a diffusive process in longitudinal phase space, driving particles beyond the separatrix. Once particles leave the stable region, the ERIT mechanism can no longer compensate for their energy loss at the target. Consequently, these particles continue to lose energy and their orbits shift inward. Eventually, they reach the radial aperture limit, as indicated by the red dots in panel (f).

Figure 13 shows the evolution of the rms emittance in the transverse (ϵ_x, ϵ_y), and longitudinal (ϵ_s) directions, as well as the survival rate ξ_i at each turn i . Solid and dashed lines represent cases with and without SCSC, respectively.

In the absence of SCSC, ϵ_s increased during the first ~ 200 turns due to filamentation in phase space, and then saturated below 1 eVs. Due to the scattering effects, ϵ_x and ϵ_y also increased up to several hundred mm mrad and then saturated. Their growth was mitigated by ionization cooling [25], resulting in no beam loss.

When SCSC was included, ϵ_s increased rapidly due to diffusion in the longitudinal phase space (as shown in Fig. 12). The survival rate ξ_i began to decrease after

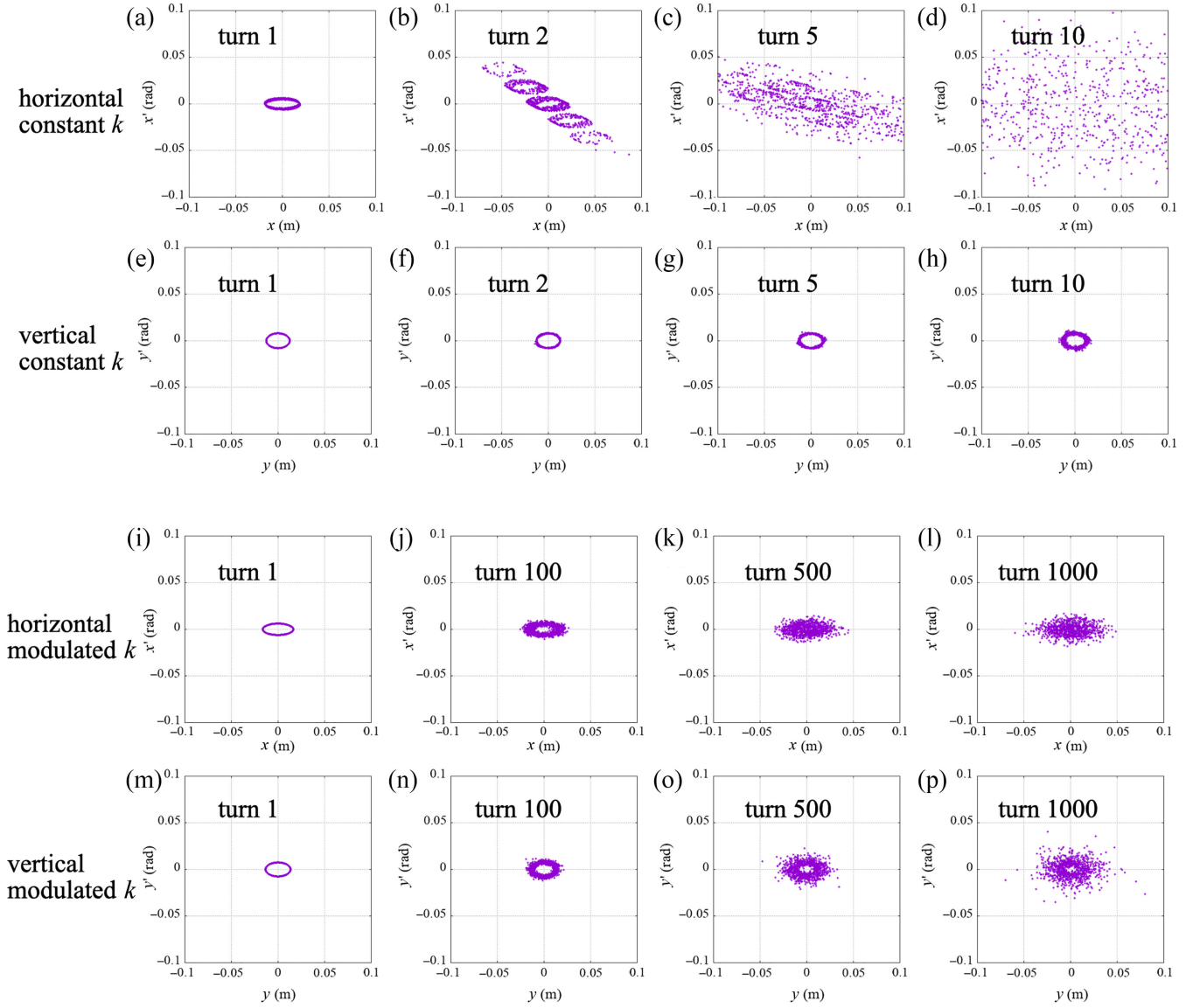


FIG. 10. Evolution of phase-space ellipses. Panels (a)–(d) show the horizontal phase space for the unmodulated k case, where particles are initially distributed along the edge of the Courant-Snyder ellipse corresponding to $2J_x = 100$ mm mrad. The distributions are shown after 1, 2, 5, and 10 turns. Panels (e)–(h) present the corresponding distributions in the vertical plane. Panels (i)–(l) show the horizontal phase-space distributions for the optimized k -modulated case after 1, 100, 500, and 1000 turns. Panels (m)–(p) present the corresponding results in the vertical plane.

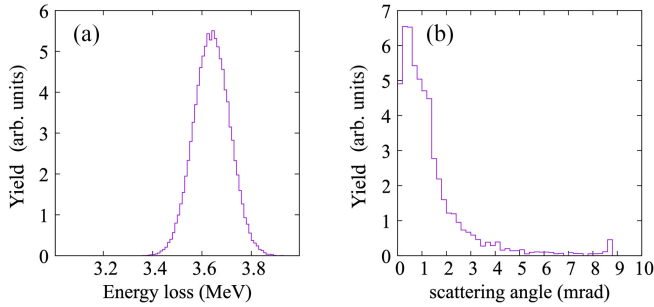


FIG. 11. Energy loss and scattering angle distributions used for the beam simulation, obtained by TRIM.

approximately 100 turns, indicating beam loss caused by this diffusion process. Nevertheless, it is noteworthy that about 15% of the particles still survived after 1000 turns.

In typical ERIT operation, a continuous beam is injected from an upstream accelerator into the storage ring to irradiate the internal target. Accordingly, the effective number of turns N_i is defined as the weighted sum of the surviving beam at each turn:

$$N_i = \sum_j^i j \xi_j. \quad (16)$$

TABLE III. Parameters used in the beam simulation.

Beam species	$^{50}\text{Ti}^{q+} (q = 17 - 22)$
Beam energy	275 MeV
rf cavity voltage	1.2 MV
rf cavity frequency	0.861 MHz
Harmonic number	1
Synchronous phase	8.7°
Slippage factor	-0.676
Target material	^{249}Bk
Target thickness	$100 \mu\text{g}/\text{cm}^2$
Injection beam duty factor	20%
Number of macroparticles	1000
Physical aperture	$4.5 \text{ m} < r < 7.5 \text{ m}$ $-0.15 \text{ m} < y < 0.15 \text{ m}$

Figure 14 shows ξ_i (red dashed) and the accumulated effective turns N_i (blue solid). The results show that N_i saturates at approximately 600 around the 1500th turn.

IV. DISCUSSION

In this section, we discuss the chromaticity and transverse beam stability associated with the use of modulated k . We also address the space-charge effects, which are unavoidable in low-energy storage rings.

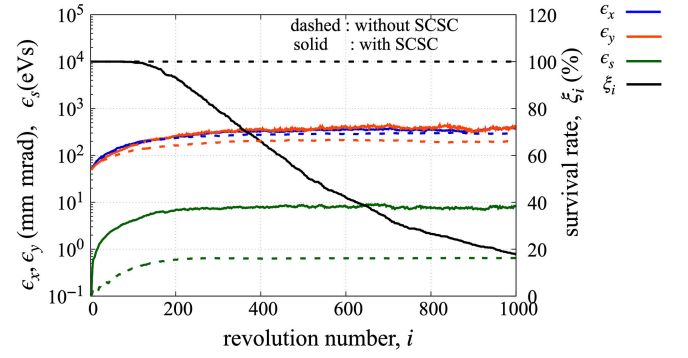


FIG. 13. Emittance growth and survival rate. Blue, red, and green lines represent ϵ_x , ϵ_y , and ϵ_s , respectively. Black lines represent the survival rate ξ_i . Dashed and solid lines correspond to cases without and with SCSC effects, respectively.

A. Chromaticity and transverse beam stability

An ideal scaling FFA ring exhibits zero chromaticity, ensuring identical tunes for beams with different charge states. However, when the scaling condition is violated—such as through the modulation of k —this chromaticity is no longer guaranteed. It is therefore essential to evaluate how the tune varies with charge state under such conditions.

Figure 15 shows the horizontal and vertical tunes (ν_x and ν_y) for six charge states ranging from 17^+ to 22^+ (see also

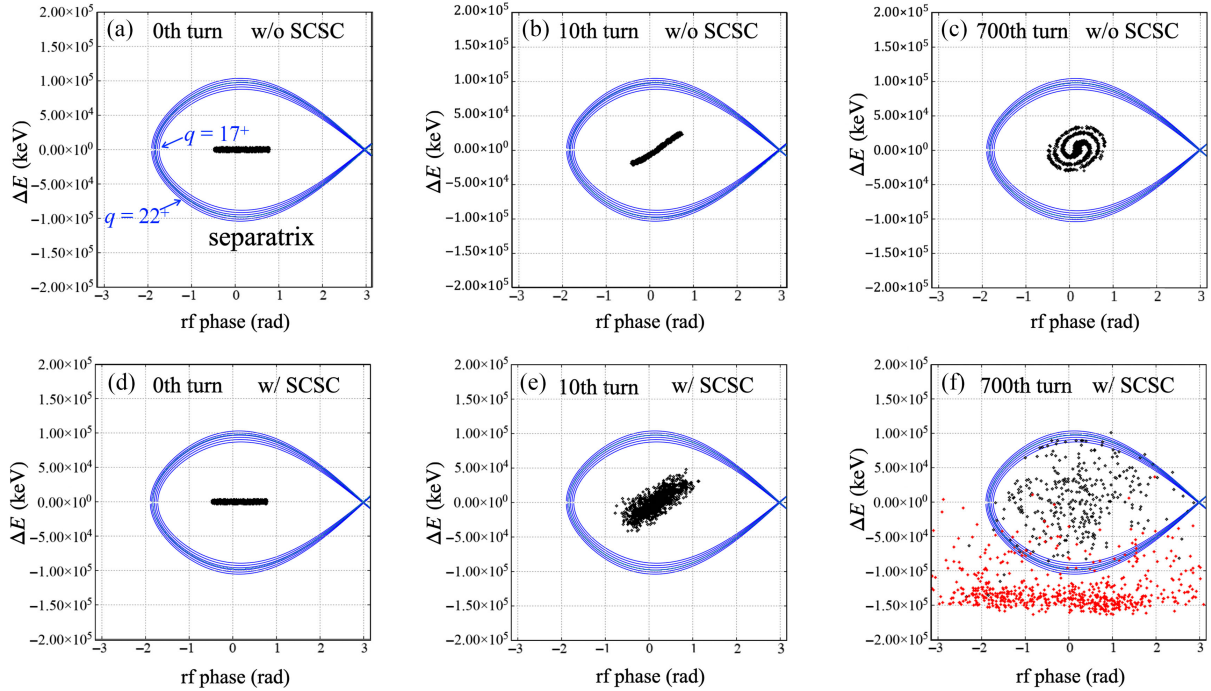


FIG. 12. Time evolution of the longitudinal phase space ($\Delta E - \phi_{\text{rf}}$). Panels (a)–(c) show snapshots at the 0th, 10th, and 700th turns without SCSC effects. Panels (d)–(f) show corresponding snapshots with SCSC effects included. Red dots indicate particles that exceeded the radial aperture limit. Blue lines denote the separatrices for the charge states ranged from 17^+ to 22^+ . Raw tracking data are available at Ref. [31].

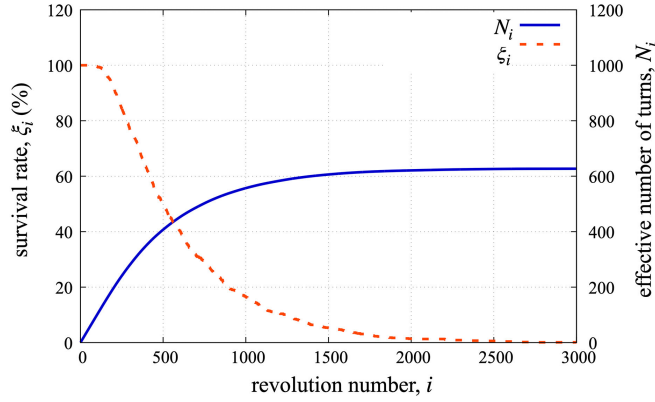
FIG. 14. Survival rate ξ_i and effective number of turns N_i .

Table II for the reference charge state q_0). Blue dots indicate tunes calculated at small amplitude (initial displacements are 0.1 mm in both x and y), while red crosses correspond to a large betatron amplitude of $2J_{x,y} = 4000$ mm mrad. The resulting tune variation among different charge states is approximately 0.02, which is sufficiently small to conclude that the deviation from ideal zero chromaticity poses no significant issue. Furthermore, the tunes at large betatron amplitudes are

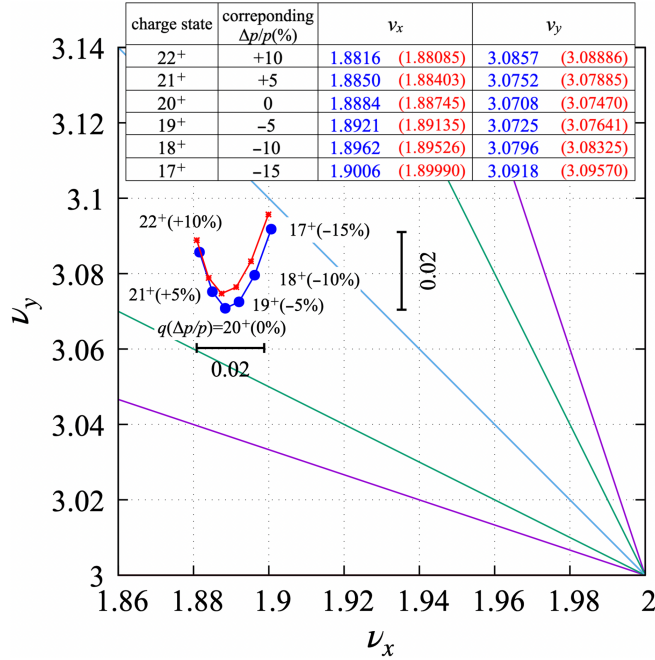


FIG. 15. Tune variation for beams with different charge states plotted on the ring's tune diagram. The diagram includes resonance lines up to fourth order for a ring with superperiodicity 1. Blue dots indicate tunes calculated at small amplitudes (initial displacements of 0.1 mm in both x and y), while red crosses correspond to a large betatron amplitude of $2J_{x,y} = 4000$ mm mrad. In the table, blue numbers denote tunes at small amplitude, while red numbers indicate those at large amplitude.

shifted away from the integer resonances, indicating that amplitude-dependent tune shifts in the nonlinear field do not drive particles toward unstable integer working points under the present operating conditions.

Integer resonances are employed for closed-orbit matching, while half-integer resonances are utilized for beta function matching. The tunes are chosen close to these resonance harmonics so that modulations of the closed orbit and beta functions are enhanced with minimal perturbation. Under this condition, the magnetic field distribution effectively excites the relevant resonances. Nonetheless, as demonstrated in Figs. 4 and 7, these excitations remain sufficiently controlled: there is no instability in the closed orbit due to integer resonance, nor divergence in amplitude from half-integer resonance excitation.

The introduction of k modulation breaks the ring symmetry, reducing it from 12-fold to 2-fold symmetry in the test ring. This prompts an important question: to what extent does the reduced symmetry and associated resonance excitation affect transverse phase-space stability?

Figure 16 compares the phase-space structures before and after the k modulation. The plots show 1000-turn Poincaré maps at the target location, obtained by tracking a single particle with a charge state of $q = 20^+$. The red ellipses indicate the apertures $x = \pm 50$ mm, $y = \pm 50$ mm. Despite the lowered symmetry, a sufficiently large stable region is maintained in both horizontal and vertical directions, indicating that transverse stability is preserved under modulation.

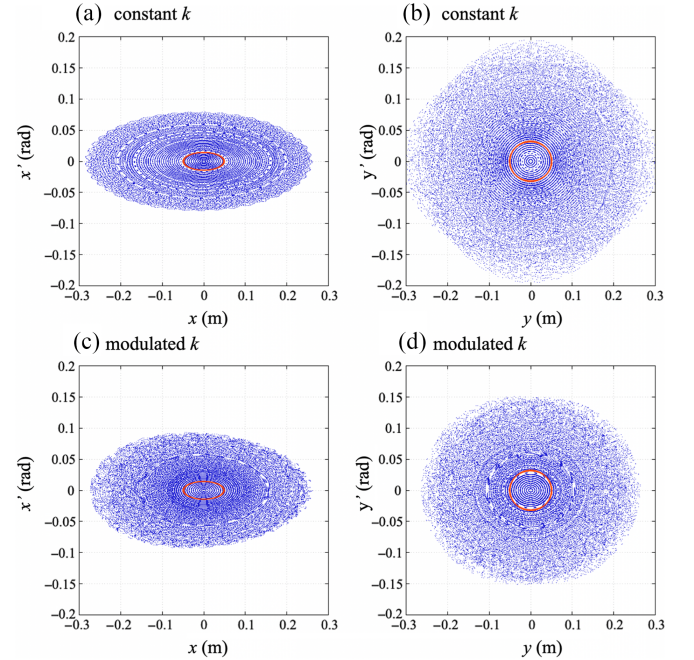


FIG. 16. Transverse phase-space structure at the target location without SCSC effects. Upper: constant k . Lower: modulated k . The green ellipses correspond to the apertures, $x = \pm 50$ mm, $y = \pm 50$ mm.

B. Space charge effects

The influence of space-charge on the tune is estimated using the following expressions:

$$\begin{aligned}\Delta\nu_x &= -\frac{r_l n R}{\nu_x \pi \beta^2 \gamma^3} \frac{1}{\sigma_x (\sigma_x + \sigma_y) B_f}, \\ \Delta\nu_y &= -\frac{r_l n R}{\nu_y \pi \beta^2 \gamma^3} \frac{1}{\sigma_y (\sigma_x + \sigma_y) B_f}, \\ \sigma_x &= \sqrt{\tilde{\beta}_x \epsilon_x + [D_x (\Delta p / p_0)]^2}, \\ \sigma_y &= \sqrt{\tilde{\beta}_y \epsilon_y},\end{aligned}\quad (17)$$

where r_l is the classical radius of the ion, n is the total number of stored particles, R is the average radius of the ring, and B_f is the bunching factor. The transverse beam sizes σ_x and σ_y include contributions from both emittance and dispersion of rigidity (both momentum and charge). A $^{50}\text{Ti}^{20+}$ beam with an energy of 275 MeV and a current of 1 μA was assumed in the evaluation. The average beta functions were set to $\tilde{\beta}_x = 3.2$ m and $\tilde{\beta}_y = 1.9$ m, with transverse emittances of 400 mm mrad in both planes. The bunching factor was assumed to be $B_f = 0.2$. Allowing for a maximum space-charge tune shift of 0.1, the number of stored particles could reach up to 1.5×10^{10} . However, in the test ring, beam losses limited the effective number of turns to about 600, well below the space-charge limit. Thus, beam accumulation was constrained more by particle loss than by space-charge effects.

It should be noted that the parameters used in the space-charge evaluation serve a different purpose from those employed in the modulation optimization studies discussed earlier. The tune shift of approximately 0.02 shown in Fig. 14 originates from the breaking of the zero-chromaticity condition due to the adoption of the modulated k scheme, and does not include space-charge effects. This level of tune shift is sufficiently small and acceptable from the viewpoint of beam stability. In contrast, the maximum allowed tune shift of 0.1 adopted in the present space-charge evaluation is used as a conservative upper bound for estimating the space-charge limit. This value is intentionally set somewhat lower than the commonly used empirical limit of approximately 0.25 and therefore provides a reasonable safety margin. Similarly, the transverse emittance of ~ 100 mm mrad discussed in the modulation optimization corresponds to the emittance growth observed in 1000-turn tracking simulations including SCSC, starting from an initial ring-shaped distribution with $2J_x = 2J_y = 100$ mm mrad (rms emittance of 50 mm mrad). On the other hand, the larger emittance value of 400 mm mrad used in the present space-charge calculation represents an expected upper bound obtained from simulations that include additional effects such as energy loss, scattering, synchrotron motion, and physical aperture limitations.

V. CONCLUSION

The ERIT-type heavy-ion storage ring is a promising device for efficient production of secondary particles in rare nuclear reactions. One of the major challenges for stable ring operation is the SCSC that occurs when heavy-ion beams interact with the internal target. This process induces random changes in charge states, which in turn cause significant emittance growth and beam loss.

To address this issue, we developed a method for closed-orbit matching at the target location that accommodates multiple charge states by utilizing coherent CODs. This approach is implemented within an adapted-scaling FFA optics framework.

Although the field index k is globally constant in a scaling FFA, introducing a modulation of k enables orbit convergence at the target. A similar modulation strategy was applied to match the beta function. We confirmed that beta-function matching is feasible by taking advantage of coherent beta-beat resonances induced by the modulated k . In simulations using a validation lattice, emittance growth due to SCSC was effectively suppressed: the initial transverse emittance of $\epsilon_{x/y} = 50$ mm mrad increased only to ~ 100 mm mrad after 1000 turns. In these simulations, only the SCSC effect was considered, while energy loss, scattering in the target, and synchrotron motion were neglected.

Based on this lattice, we conducted full 6D beam tracking simulations. The results demonstrated that the system can accumulate beam equivalent to over 600 effective turns.

The chromaticity in this system remains extremely small due to the scaling nature of the FFA optics. Even for a broad charge state distribution ranging from $q = 17^+$ to 22^+ —corresponding to a $\pm 12.8\%$ rigidity deviation—the tune variation was limited to only 0.02. In addition, we confirmed that the wide dynamic aperture, a key advantage of scaling FFAs, was preserved even with the modulated k .

When applied to superheavy element production (e.g., elements 119 or 120), the space-charge tune shift remains below 0.1, allowing for up to 2000 turns of accumulation. Since the effective number of turns achieved in this study was 600, beam loss—not space charge—was the dominant limiting factor.

In contrast to the transverse direction, no comprehensive solution has yet been found for mitigating longitudinal beam diffusion due to SCSC. Future work will explore potential remedies such as second-harmonic rf systems, barrier buckets, and negative- k lattices to suppress longitudinal diffusion.

In summary, our results demonstrate that adapted-scaling FFA rings with modulated k offer a viable and robust approach for heavy-ion storage rings where SCSC is unavoidable.

DATA AVAILABILITY

The data that support the findings of this article are openly available [31].

APPENDIX A: ORBIT COMPACTION

Because of the photographic similarity of the closed orbits in a scaling FFA ring, the radius of curvature in the magnet, ρ , is proportional to the radius of the closed orbit, r (i.e., $\rho \propto r$). Using the relationship among the momentum p , the particle charge eq , and the magnetic rigidity $B\rho$, Eq. (1) can be rewritten as

$$B\rho = \frac{p}{eq} = Ar^{k_0+1}, \quad (\text{A1})$$

$$\frac{p}{e(q_0 + \Delta q)} = A(r_0 + \Delta r)^{k_0+1}, \quad (\text{A2})$$

where A is a proportionality constant expressed in terms of the reference orbit r_0 and the nominal charge state q_0 as

$$A = \frac{p}{eq_0} \left(\frac{1}{r_0} \right)^{k_0+1}. \quad (\text{A3})$$

Then Eq. (A2) becomes

$$\frac{r_0 + \Delta r}{r_0} = \left(\frac{q_0 + \Delta q}{q_0} \right)^{-\alpha_p}, \quad (\text{A4})$$

where $\alpha_p = 1/(k_0 + 1)$. Expanding $\Delta r/r_0$ as a power series in $\Delta q/q_0$, we obtain

$$\frac{\Delta r}{r_0} = -\alpha_p \frac{\Delta q}{q_0} + \frac{1}{2} \alpha_p (\alpha_p + 1) \left(\frac{\Delta q}{q_0} \right)^2 + \dots \quad (\text{A5})$$

Substituting the numerical values for the test ring, $k_0 = 2.16$, $\Delta q = 3$, and $q_0 = 20$, the first- and second-order terms on the right-hand side of Eq. (A5) are -4.75×10^{-2} and 4.69×10^{-3} , respectively. The second-order term is approximately one order of magnitude smaller than the first-order term, indicating that the first-order approximation in $\Delta q/q_0$ is sufficiently accurate for the present analysis. By rewriting $\Delta r \rightarrow \Delta \tilde{r}_q$ and $r_0 \rightarrow \tilde{r}_{q_0}$, we obtain Eq. (2).

APPENDIX B: COD WITH MODULATED k

The magnetic field with modulated k defined in Eq. (7) is expressed as

$$B_m(r, \theta) = B(\tilde{r}_{q_0}(\theta), \theta) \left(\frac{r}{\tilde{r}_{q_0}(\theta)} \right)^{k(\theta)}, \quad (\text{B1})$$

where $\tilde{r}_{q_0}$ is the radius of the closed orbit of the beam with the charge q_0 . Let us evaluate the difference in the magnetic

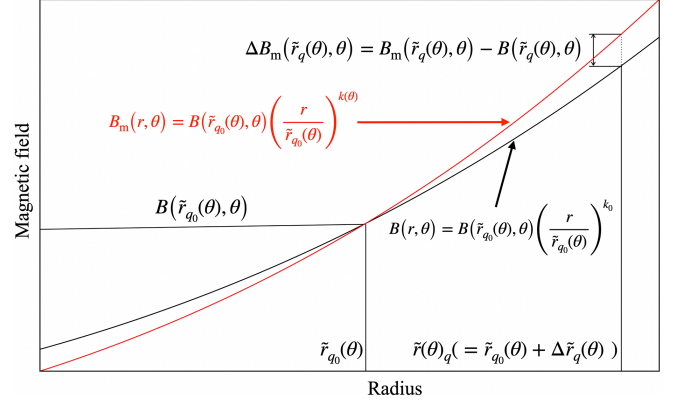


FIG. 17. The r -dependence of the magnetic field at the center of the F magnet as an example. The black and red lines describe the magnetic field as functions of r , obtained using constant k and modulated k , respectively.

field caused by the modulated k . Figure 17 shows the r -dependence of the magnetic field at selected azimuthal angles θ within the magnet, which may be either an F -magnet or a D -magnet. The black and red lines describe $B(r)$ with a constant $k(=k_0)$ and $B_m(r, \theta)$ with a modulated $k(=k(\theta))$, respectively. Along the closed orbit of the beam with charge $q(=q_0 - \Delta q)$ of which radius is $\tilde{r}_q(\theta) = \tilde{r}_{q_0}(\theta) + \Delta \tilde{r}_q(\theta)$ the magnetic field modulation which should be introduced is given by

$$\begin{aligned} \Delta B_m(\tilde{r}_q(\theta), \theta) &= B(\tilde{r}_{q_0}(\theta), \theta) \left[\left(1 + \frac{\Delta \tilde{r}_q(\theta)}{\tilde{r}_{q_0}(\theta)} \right)^{k(\theta)} - \left(1 + \frac{\Delta \tilde{r}_q(\theta)}{\tilde{r}_{q_0}(\theta)} \right)^{k_0} \right] \\ &= B(\tilde{r}_{q_0}(\theta), \theta) \left(\frac{\Delta \tilde{r}_q(\theta)}{\tilde{r}_{q_0}(\theta)} \right) \Delta k(\theta) + \mathcal{O} \left[\left(\frac{\Delta \tilde{r}_q(\theta)}{\tilde{r}_{q_0}(\theta)} \right)^2 \right] \\ &\approx -B(\tilde{r}_{q_0}(\theta), \theta) \frac{\Delta k(\theta) \Delta q}{k_0 + 1 q_0}, \end{aligned} \quad (\text{B2})$$

where we used the general characteristics of the scaling FFA described in Eq. (2).

In general, if there is an extra dipole field distributed as $\mathcal{E}(\theta)(= \Delta B(\theta)/B\rho)$ along the closed orbit, a COD will emerge, and the displacement from the original closed orbit $x_{\text{cod}}(\theta)$ is represented by the Fourier series as [27,32]

$$x_{\text{cod}}(\theta) = \sqrt{\beta_x(\theta)} \sum_{n=-\infty}^{\infty} \frac{\nu_x^2}{\nu_x^2 - n^2} \tilde{f}_n e^{in\phi_x(\theta)}, \quad (\text{B3})$$

where ν_x is the horizontal tune, β_x is the horizontal betatron function, and ϕ_x is the normalized phase advance

$$\phi_x(s) = \frac{1}{\nu_x} \int_0^s \frac{ds'}{\beta_x(s')}, \quad (\text{B4})$$

which advances 2π per turn in the ring.

In Eq. (B3), $\tilde{f}_n$ is the n th Fourier component, defined as

$$\tilde{f}_n = \frac{1}{2\pi} \oint \beta^{3/2}(\phi_x) \mathcal{E}(\phi_x) e^{-in\phi_x} d\phi_x, \quad (\text{B5})$$

where $B\rho$ is the magnetic rigidity of the beam with charge state $q_0 - \Delta q$. By substituting

$$\frac{\Delta B_m(\tilde{r}_{q_0}(\phi_x) + \Delta \tilde{r}_q(\phi_x), \phi_x)}{B\rho} \rightarrow \mathcal{E}(\phi_x), \quad (\text{B6})$$

Equation (B5) is rewritten as

$$\tilde{f}_n = -\frac{1}{2\pi B\rho} \frac{\Delta q}{q_0} \frac{1}{k_0 + 1} \oint B(\tilde{r}_{q_0}(\phi_x), \phi_x) \beta^{3/2}(\phi_x) \Delta k(\phi_x) e^{-in\phi_x} d\phi_x. \quad (\text{B7})$$

APPENDIX C: SIMULATION CONDITIONS

A summary of the calculation conditions in simulation studies is given in Table IV.

TABLE IV. Simulation conditions.

Figure	Panel	Description	Legend	Used harmonics for k modulation	SCSC	rf, E_{loss} scattering
5	(a)–(c)	Closed orbits	Blue	None	No	No
			Red	n, p_x, p_y	No	No
6	(a)	Footprints of single particle tracking	Blue	None	Yes	No
	(b)	Amplitude growth in single particle tracking	Red	n, p_x, p_y	Yes	No
			Blue	None	Yes	No
			Red	n, p_x, p_y	Yes	No
7	(a)	β_x		n	No	No
	(b)	β_y		n	No	No
	(c)	β_x		n, p_x, p_y	No	No
	(d)	β_y		n, p_x, p_y	No	No
8		$k(\theta)$	Blue	n, p_x, p_y	No	No
9		ϵ_x (blue), ϵ_y (red)	Dashed	None	Yes	No
		ϵ_x (blue), ϵ_y (red)	Solid	n, p_x, p_y	Yes	No
10	(a)–(d)	Edge $2J_x = 100$ mm mrad	Purple	None	Yes	No
	(e)–(h)	Edge $2J_y = 100$ mm mrad	Purple	None	Yes	No
	(i)–(l)	Edge $2J_x = 100$ mm mrad	Purple	n, p_x, p_y	Yes	No
	(m)–(p)	Edge $2J_y = 100$ mm mrad	Purple	n, p_x, p_y	Yes	No
12	(a)	$\phi - \Delta E$ at turn 0	Black	n, p_x, p_y	No	Yes
	(b)	$\phi - \Delta E$ at turn 10	Black	n, p_x, p_y	No	Yes
	(c)	$\phi - \Delta E$ at turn 700	Black	n, p_x, p_y	No	Yes
	(d)	$\phi - \Delta E$ at turn 0	Black	n, p_x, p_y	Yes	Yes
	(e)	$\phi - \Delta E$ at turn 10	Black	n, p_x, p_y	Yes	Yes
	(f)	$\phi - \Delta E$ at turn 700 (particle lost)	Black (red)	n, p_x, p_y	Yes	Yes
13		$\epsilon_x, \epsilon_y, \epsilon_s, \xi_s$	Dashed	n, p_x, p_y	No	Yes
		$\epsilon_x, \epsilon_y, \epsilon_s, \xi_s$	Solid	n, p_x, p_y	Yes	Yes
14		Survival rate	Red dashed	n, p_x, p_y	Yes	Yes
		Effective revolution	Blue solid	n, p_x, p_y	Yes	Yes
15		Tune with small amplitude	Blue	n, p_x, p_y	No	No
		Tune with large amplitude	Red	n, p_x, p_y	No	No
16	(a)	Poincaré maps of $x - x'$	Blue	None	No	No
	(b)	Poincaré maps of $y - y'$	Blue	None	No	No
	(c)	Poincaré maps of $x - x'$	Blue	n, p_x, p_y	No	No
	(d)	Poincaré maps of $y - y'$	Blue	n, p_x, p_y	No	No

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