

Modern signal processing techniques for efficient transverse feedback systems operation in hadron accelerators

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Transverse feedback systems are essential for stable operation of all hadron accelerators with high intensity beams. An important component of such a system is a powerful digital signal processing unit. Current digital technology enables the implementation of advanced algorithms to process signals and analyze key parameters of both the accelerator and its feedback systems. In the paper, we present a new digital filter, which is suitable for the construction of a single pickup feedback with a larger tune acceptance and higher gain than can be achieved by the traditionally used Hilbert phase-shifter technique. We propose a method to extract key parameters required for optimal feedback operation, such as the closed-loop phase and delay, as well as accelerator parameters like bunch-by-bunch damping time and machine tune. This is achieved solely through active manipulation of the beam by the transverse feedback system and its analysis of acquired data. Accurate knowledge of these parameters is essential for ensuring beam stability and the reliable operation of the accelerator. Easy access to them is important for automated TFB setting up, performance monitoring, or efficient troubleshooting.

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I. INTRODUCTION

Transverse feedback systems (TFBs) are an essential part of each high energy circular particle accelerator, regardless if proton [1–3], heavy ion [4,5], electron-positron storage ring [6,7], or a synchrotron light source [8,9] to name only few.

Their task is to stabilize high intensity beams by suppressing any transverse instabilities driven by machine impedance [10,11], reduce transverse motion induced by external perturbations [12,13], or preserve emittance of the injected beam by rapid damping of injection oscillation transients [14,15].

With the arrival of very powerful programmable digital electronics more than two decades ago, the data and signal

processing hardware is no longer the limiting factor to the complexity of implemented algorithms and digital signal treatment. This opened completely new possibilities for transverse feedback systems in accelerators. The damping or stabilizing function of the transverse feedback is no longer the only service these systems are expected to provide. The TFBs are used now, for example, for beam cleaning [16–18], real time transverse activity monitoring, and instability detection [19] as well as beam transfer function measurement [20]. Transverse feedback can be a source of controlled machine impedance, allowing for special measurements such as direct Landau damping strength measurement [21], impedance measurement of individual accelerator components (e.g., low-impedance collimators) by means of high precision tune shift detection techniques [22], or impedance measurements of insertion devices in light sources by the grow-damp method [23]. With full-resolution beam position data now available in digital form [24], the focus has shifted beyond the core feedback functionality (which remains necessary) toward value-added services aimed at enhancing accelerator performance and operation.

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II. TFB PRINCIPLE OF OPERATION

Key components of a transverse feedback system are outlined in Fig. 1. The beam position is measured every turn by one or more position monitors composed of a coupling device to the beam (pickup) and signal conditioning electronics [25,26]. This combination is often referred to as a beam position monitor or a beam position measurement module (BPM).

A correction signal is then calculated based on the position data and transformed to the point in the accelerator where the transverse kicker is located such that the overall phase in the feedback loop is negative and the loop stabilizes the beam. In the early days, the calculation of feedback signals was fully analog, using filters and delay lines, often hundreds of meters [1,27], or even a few kilometers long [28]. The time alignment of the processing chain delay T_{signal} to the beam time-of-flight T_{beam} between the position monitors and the transverse kicker is a crucial factor for the transverse feedback to work correctly. Fixed cable delays are not compatible with accelerating beams, so complicated tricks needed to be implemented to cope with a decreasing time-of-flight delay T_{beam} during the energy ramp [29].

If multiple beam position monitors can be installed in the machine with an ideal 90° betatron phase advance between them, the required feedback phase can be obtained by (analog) vector combination of the two (or more) position signals creating a “virtual pickup” [30].

The correction signals are finally amplified and fed to kickers, which deflect the beam and close the feedback. When a lower deflection voltage and high bandwidth is needed, typically terminated stripline kickers and solid-

state amplifiers are used [31]. If a high deflection is needed, e.g., for highly relativistic beams, high power tetrode-based amplifiers feeding unterminated E-field kickers are often used [32].

The availability of the first fast analog to digital converters and digital memories immediately allowed for the replacement of the bulky fixed cable delay lines and introduced the possibility for the TFB to follow the accelerating cycle. For example, at CERN’s Super Proton Synchrotron, these efforts date back to the beginning of the 1980s [33–35]. With advancements in digital technology in the 1990s, the next natural step was to introduce full digital signal processing with digital filtering [36].

In hadron machines, bunch length is comparable with the length of the rf bucket. In smaller synchrotrons, this is typically hundreds of nanoseconds. Digital low-level rf systems therefore can sample the passing bunch at many points and can easily detect an intrabunch motion. If such a TFB is equipped with pickups and deflectors with sufficient bandwidth, an intrabunch feedback can be realized to stabilize the head-tail motion. A great example of upgrade of a regular “mode-0 only” transverse feedback to a 64-slice intrabunch feedback is the J-PARC’s Main Ring TFB project [37]. Bunch length in the largest synchrotrons is on the order of nanoseconds, but modern multi-gigasample-per-second digital systems and GHz- bandwidth amplifiers and kickers allow us to also realize intrabunch feedbacks capable of stabilizing head-tail motion of such short bunches [38].

Today’s digital technology offers virtually unlimited resources for advanced signal processing and data storage.

III. DIGITAL SIGNAL PROCESSING FOR TRANSVERSE FEEDBACKS

A. Vector-sum feedback

The simplest transverse feedback can be realized using a single pickup and a single deflector/kicker. The correct phase shift required between the position measurement and application of the correction kick can be obtained by integrating the system into properly selected positions in the accelerator lattice. However, such a scheme makes the optics design, TFB, and accelerator operation complicated, as there is no flexibility in any of the parameters.

The problem can be overcome by using two beam pickups, which are conveniently placed in the lattice to have an ideal $\pi/2$ betatron phase advance between them. The two orthogonal components can be used to rotate the measured position vector by any phase by simple multiplication as illustrated in Fig. 2, often referenced to as *pickup vector sum*:

$$y_k[n] = p_1[n]b_1 + p_2[n]b_2 = p_1[n]\cos\varphi + p_2[n]\sin\varphi, \quad (1)$$

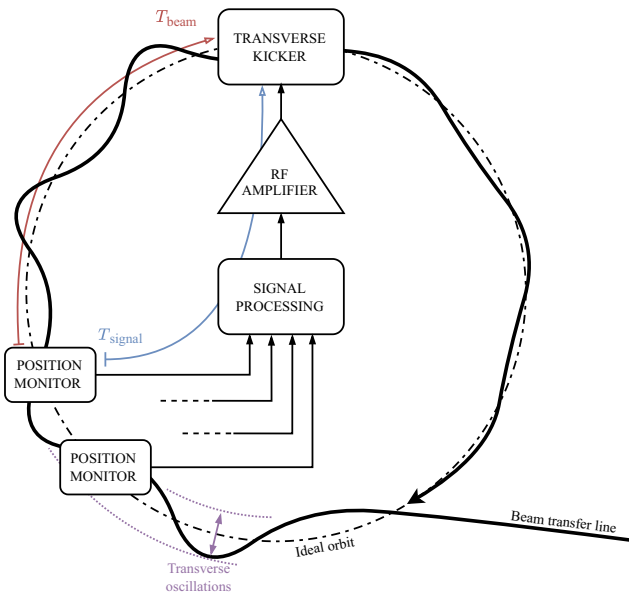


FIG. 1. Principle blocks of a transverse feedback system in a circular particle accelerator.

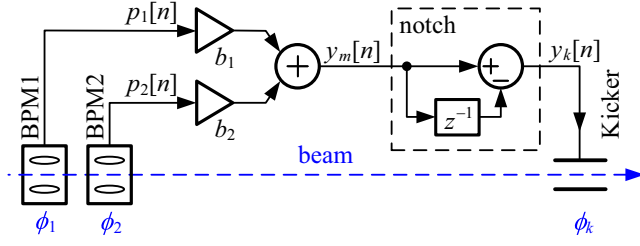


FIG. 2. TFB signal processing using two pickups spaced exactly by $\pi/2$ (or close to) betatron phase advance. Sampling is once per turn per bunch passage, and the structure is implemented for each individual bunch.

where φ is the required phase rotation angle. For the combination of two pickup signals that have a phase advance different from $\pi/2$, a more general solution has been formulated in [39]. The two pickup mixing coefficients follow the form:

$$b_{1,2} = -\frac{1}{2} \left(\frac{\cos(\Delta\phi_{Qkm})}{\cos(\Delta\phi/2)} \mp \frac{\sin(\Delta\phi_{Qkm})}{\sin(\Delta\phi/2)} \right), \quad (2)$$

where $\Delta\phi = \phi_2 - \phi_1$ describes the phase advance between the two pickups, and

$$\Delta\phi_{Qkm} = -3\pi Q_f + \phi_k - \frac{\phi_2 + \phi_1}{2}. \quad (3)$$

The fractional tune is denoted as Q_f , and ϕ_1 , ϕ_2 , and ϕ_k are the betatron phase advances at pickup and kicker locations with respect to some fixed reference. Equation (3) is readily taking into account one-turn delay and the phase shift of the notch filter.

The pickup vector-sum method provides the shortest processing delay. The correction kick is applied within the same turn or the next turn, depending if the pickup is upstream or downstream of the deflector. A very short delay also allows for a larger TFB tune spread acceptance which is important for some applications. For correct operation, there must be a minimum of two pickups and the pickup signals $p_1[n]$, $p_2[n]$ must have the same amplitude (i.e., locations with the same β values, and attenuation of different length rf transmission lines compensated). Using information from two pickups allows us to slightly reduce the pickup measurement noise. The reduction factor is lower than $\sqrt{2}$ as the two individual orthogonal signals do not contribute by the same amplitude.

B. Single pickup feedback with digital phase shifter

Using more complex digital filters, a TFB can be realized with a single pickup placed at any location in the lattice. The correction kick is calculated from data acquired from one location, but over multiple turns. The Hilbert transform, which provides two orthogonal vector components—commonly referred to in signal processing as the in-phase and quadrature Q components—is typically used, followed by a phase rotation. The scheme is often referred to as a

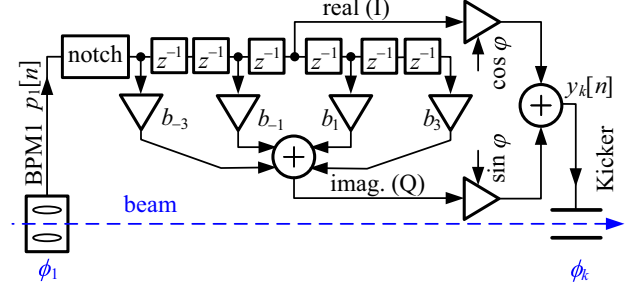


FIG. 3. Typical signal processing for single pickup feedback using Hilbert phase shifter (7 taps). Sampling is once per turn per bunch passage, and the structure is implemented for each individual bunch.

Hilbert phase shifter. A typical TFB architecture (for one pickup) using this principle with the Hilbert transform realized by a finite impulse response (FIR) filter is depicted in Fig. 3. The method is not limited to one pickup only. Any number of pickups can be processed in parallel and the resulting correction kick is obtained as a sum of individual pickup contributions. The transverse feedback system in the Large Hadron Collider is typically operated in this mode using four pickups per beam per plane.

Using the Hilbert transformer coefficients [40] and phase rotation terms, the FIR filter coefficients will be

$$\begin{aligned} b_n &= \frac{2}{\pi n} \sin \varphi \quad \text{for } n \text{ odd} \\ &= 0 \quad \text{for } n \text{ even} \\ &= \cos \varphi \quad \text{for } n = 0, \end{aligned} \quad (4)$$

where φ is the required phase rotation angle between pickup and kicker:

$$\varphi = \phi_k - \phi_1 + Q(\tau_{\text{Hilbert}} + \tau_{\text{Notch}} + \tau_{\text{Other}}). \quad (5)$$

Q is tune, ϕ_1 and ϕ_k are the betatron phase advances at pickup and kicker locations with respect to some fixed reference, τ_{Hilbert} is group delay (in turns) of the used Hilbert filter (in the example of Fig. 3, $\tau_{\text{Hilbert}} = 3$), τ_{Notch} is group delay (in turns) of the used notch filter (in this example, $\tau_{\text{Notch}} = 0.5$) and τ_{Other} is other hardware or algorithm specific processing delay (in turns).

The frequency response of the Hilbert phase shifter must be taken into account when using this method. Short filters exhibit a non-negligible ripple in the response that can be detrimental for certain values of fractional tune. Longer filters have a flatter response but introduce more delay into the feedback loop. Operating close to integer or half-integer tunes is practically impossible. As multiple samples from a number of consecutive turns are used for every correction kick calculation, pickup measurement noise is partly reduced.

The longer processing delay of the Hilbert phase-shifter method compared to the vector-sum method limits the

maximum feedback loop gain; the tune acceptance range is also narrower. Nevertheless, the Hilbert phase-shifter method is very popular in modern digital TFBs, as it provides flexibility, it is easy to follow machine parameter evolution during the accelerating cycle (e.g., tune or optics change), and if multiple pickups are used, it also provides an operational redundancy, since such a TFB can run with a single pickup.

C. New filter for single pickup signal processing with large tune acceptance range and high gain

The performance of TFB digital filters affects tune acceptance and gain margin of the feedback system. Larger tune acceptance provides operational flexibility and potentially improves beam stability due to better damping of bunch trains with large tune spreads (i.e., due to electron cloud or resistive wall effects). Better gain margin allows shorter damping times for injection oscillations and coupled bunch instabilities. To overcome the problem of processing delay, a new type of digital filter was developed for CERN SPS and LHC transverse feedbacks. The shortest possible filter providing the closed orbit suppression (notch filter) and arbitrary phase shift functionality for a single pickup and single kicker TFB system is a 3-tap long FIR (general structure depicted in Fig. 4).

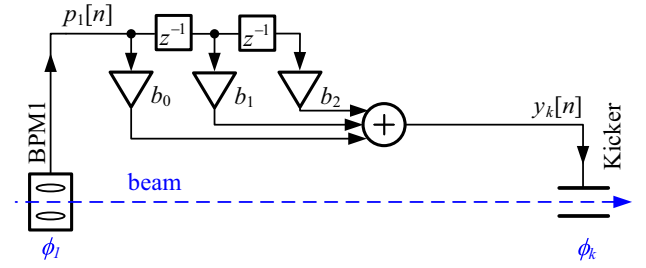


FIG. 4. Shortest possible digital filter for a single pickup feedback with full phase rotation flexibility using the new type 3-tap FIR filter. Sampling is once per turn per bunch passage, and the structure is implemented for each individual bunch.

A generic construction method for 3-tap filters based on a purely analytical solution from beam rotation matrices is presented in the Appendix. The method calculates an exact correction kick for any phase advance from three position measurements at different phase advances. The method allows straightforward construction of FIR filters of arbitrary length as a linear combination of multiple 3-tap filters. The time domain solution also generates FIR filters for above half-integer tunes, which are sometimes more challenging to construct in the frequency domain due to the Nyquist sampling limit.¹

Values of the filter coefficients are as follows:

$$\begin{aligned} b_0 &= \sqrt{\beta_{ki}\beta_{pu}} \frac{\cos(\theta_{kick})[\cos(\theta_Q) - 1] + \sin(\theta_{kick})\sin(\theta_Q)}{2\sin\theta_Q(\cos\theta_Q - 1)} \\ b_1 &= \sqrt{\beta_{ki}\beta_{pu}} \frac{\cos(\theta_{kick})[1 - \cos(2\theta_Q)] - \sin(\theta_{kick})\sin(2\theta_Q)}{2\sin\theta_Q(\cos\theta_Q - 1)} \\ b_2 &= \sqrt{\beta_{ki}\beta_{pu}} \frac{\cos(\theta_{kick})[\cos(2\theta_Q) - \cos(\theta_Q)] + \sin(\theta_{kick})[\sin(2\theta_Q) - \sin(\theta_Q)]}{2\sin\theta_Q(\cos\theta_Q - 1)}, \end{aligned} \quad (6)$$

where β_{ki} is the beta function value at the kicker and β_{pu} at the pickup locations. Angles θ_Q and θ_{kick} are as follows:

$$\begin{aligned} \theta_Q &= 2\pi Q \\ \theta_{kick} &= \theta_Q(d + 2) + \Delta\theta, \end{aligned} \quad (7)$$

where Q is the transverse tune, d is the signal processing delay in turns (typically $d = 1$), and the $\Delta\theta$ is the phase advance between the pickup and the kicker in radians.

The coefficients in Eq. (6) are finite, when the divisor $2\sin\theta_Q(\cos\theta_Q - 1)$ is nonzero. This means that values at the integer and half-integer tunes are forbidden as with any FIR phase shifter utilizing measurements from a single pickup. It is worth noting that the useful range can be extended closer to these forbidden tune values by using measurements from every second or third turn, which, however, increases the group delay and decreases the tune

acceptance. This can be anticipated from Eq. (7) due to faster tune roll off, when θ_Q is $4\pi Q$ or $6\pi Q$ with the measurements from every second or third turn, respectively.

A performance comparison of the traditional notch + Hilbert filter and the new 3-tap filter by means of stability maps is shown in Fig. 5. The contours have been calculated using single bunch PyHEADTAIL [41] simulation. The new filter was also tested with beam in the LHC TFB. The performance was evaluated by measuring the tune shift induced by the TFB and the damping time when varying the machine tune. Results are shown in Fig. 6.

D. TFB signal processing in the Large Hadron Collider

A simplified block diagram of the digital signal processing of the LHC transverse feedback is shown in Fig. 7. The

¹After its inventor, this type of filter is referenced to as the Kompula filter in CERN TFBs.

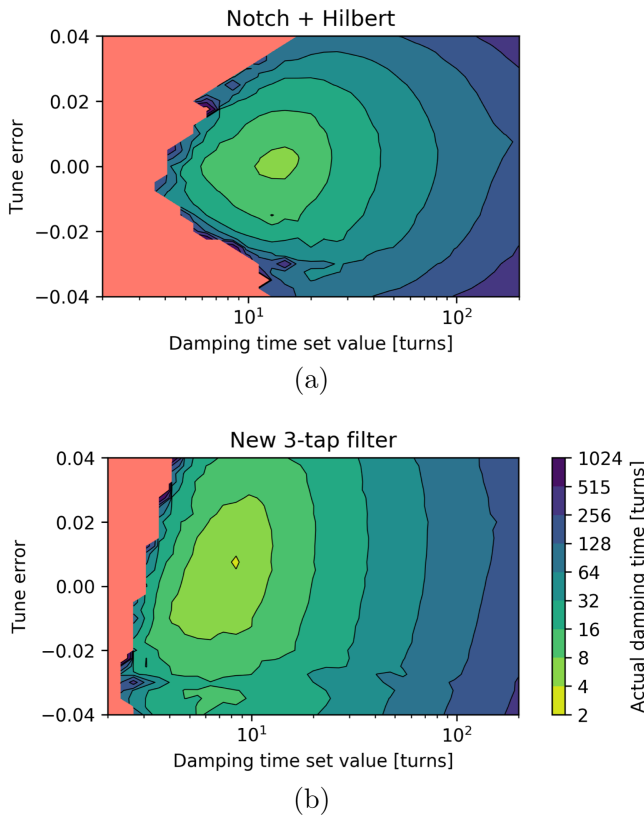


FIG. 5. Turn-by-turn filter stability maps for the LHC TFB with the notch + Hilbert scheme (a), and the newly developed 3-tap filter (b). The maps are based on single bunch PyHEADTAIL simulations.

TFBs for each beam (1/2) and each plane (H/V) use four pickups located in the arcs around the former interaction point 4 (IP4), and their tetrode power amplifiers and deflectors are located in the LHC radiofrequency zone at IP4. Due to the TFB complexity, technical functionality is split into two modules. Beam position measurement is realized by a dedicated, very low noise beam position measurement module (BPM) providing a 1 Gbps data-stream of one position sample per bunch per pickup passage (one data point every 25 ns) [42]. The BPM module interfaces to the rf signals from the pickups, adjusts the input gain to optimally measure bunches of all intensities used in LHC (from 1×10^9 to 4×10^{11} charges per bunch), and performs position normalization with respect to bunch intensity.

Four digital streams of normalized beam positions are sent over fiber optic links to the digital signal processing module. In this module, the streams from all pickups are synchronized, and a notch filter is applied to suppress the closed orbit information. With a beam synchronous sampling clock, the notch filter also suppresses all other static signals, errors, and imperfections seen by the beam position module and extracts only the oscillatory component relevant for TFB operation. Therefore, beam synchronous

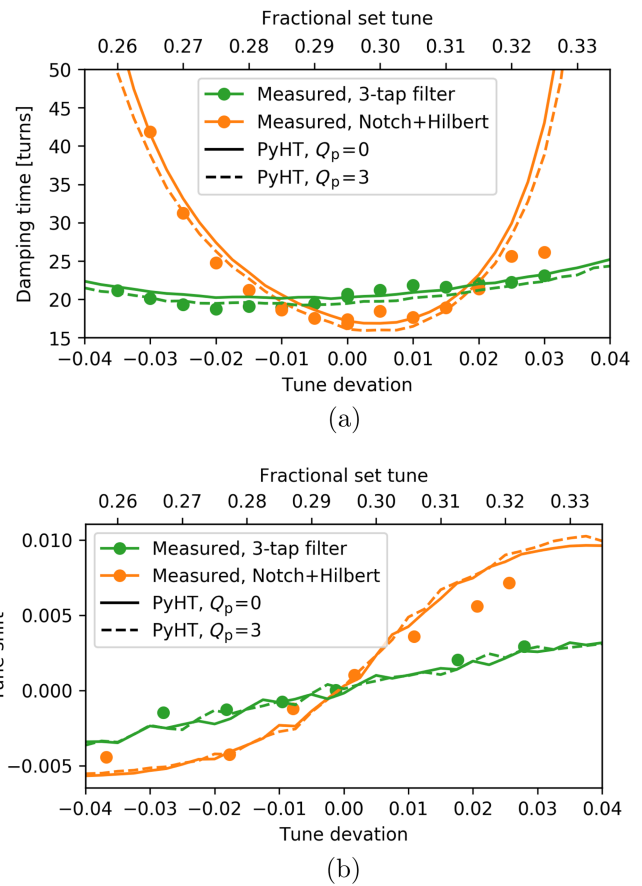


FIG. 6. Measured damping times (a) and TFB induced tune shifts (b), with the currently used filter scheme (notch + Hilbert) and the new 3-tap filter.

sampling and the notch filter relaxes already very demanding requirements for BPM electronics.

The LHC TFB has all three previously mentioned signal processing schemes implemented: one Hilbert phase shifter for each pickup controlled by a real-time function, vector-sum mode for pairs of pickups controlled by a real-time function, and an FIR filter with programmable coefficients for any other operational mode (e.g., the Kompola filters). The standard mode of LHC TFB operation is phase-shifter mode for each pickup, as it is compatible with fast damping times (10–15 turns), and it can follow the dynamic LHC optics in real time (filter coefficients are recalculated and updated approximately every 12 turns). Four pickups per beam per plane provide quadruple redundancy.

The LHC TFB signal processing supports four independent feedback loops (“activity masks” in Fig. 7), where groups of bunches (within one turn) can be in real time assigned to loops with different dynamics or other feedback features. Separate loops are used, for example, to treat the main LHC physics beam differently to the group of noncolliding, so-called witness bunches which are needed for accelerator operation. Bunches can have strong

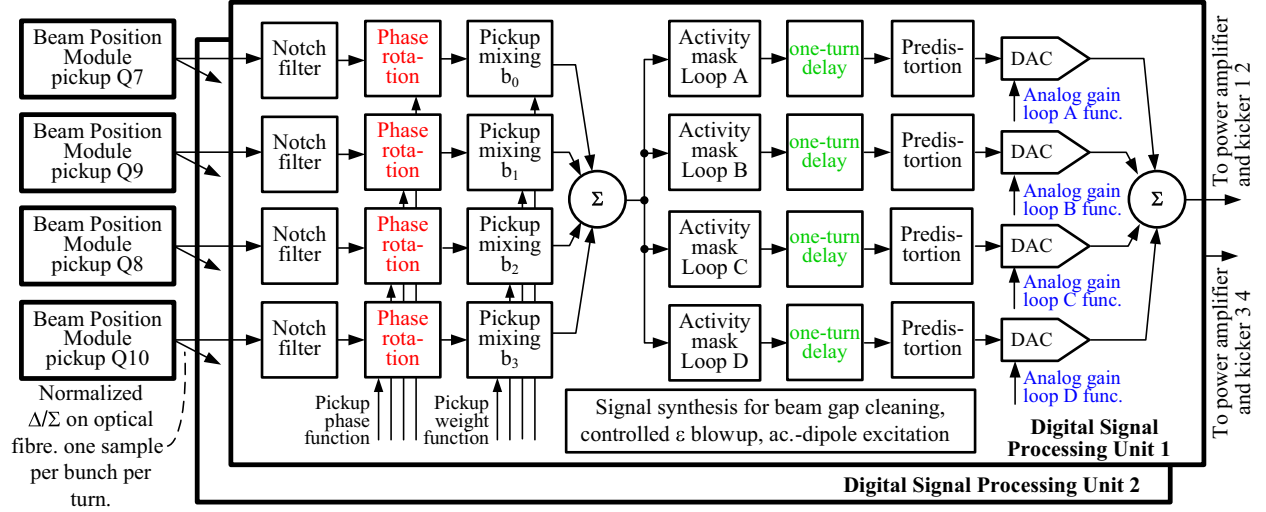


FIG. 7. Simplified block diagram of the LHC transverse feedback signal processing. Two redundant signal processing units drive four sets of power amplifiers and kickers. Adjustment of parameters shown in color is critical for TFB performance.

damping of injection oscillations and then be handed over to a regular loop with lower gain.

The signal synthesis blocks generate signals for beam cleaning, white, or colored noise for controlled emittance blowup, or ac-dipole like excitation for optics measurements. Signals can be injected into any of the four feedback channels. Each of the four outputs has individual predistortion filters to shape the signals and compensate for the power system transfer function. Signals can be generated with a large bandwidth to, for example, manipulate single bunches within a 25 ns spaced bunch train or provide an “ideal” bunch-by-bunch damper. Reduced bandwidth provides high strength kicks, e.g., to use the TFB as ac-dipole excitation to probe dynamic aperture, or to generate sustained 1 MW beam losses on primary collimators for the purpose of a magnet quench test.

Possibilities are virtually endless.

IV. TFB PARAMETERS

In the past, TFB setting up was performed mostly empirically using precalculated parameters and trial and error (loop stable/unstable). Later, external instruments such as vector network analyzers were employed to extract parameters from repeated measurements. The standard method for setting up TFBs relies on repeated transverse excitations of the beam at betatron resonance frequencies which is very laborious and time consuming, often leading to emittance blowup and beam losses due to repeated measurements. Beam losses in hadron accelerators are detrimental, as they lead to contamination by radiation.

Following data extraction poses additional challenges, as dedicated cycles with sufficiently long flat plateaus are necessary, as demonstrated, for example, in the CERN PSB [43]. The limited quality and representation of the data require expertise to accurately interpret the measurements,

and results can be subjective. Furthermore, this method offers only a limited resolution at high frequencies due to bandwidth limitations and cutoff frequencies inherent to the TFB power systems. This is particularly problematic when adjusting delay settings, which are critical for achieving optimal TFB performance in counteracting coupled bunch oscillations.

Our goal was to research methods to enable the TFB system to autonomously identify and adjust its parameters or verify its performance using only the beam manipulations or data it generates itself. The techniques involve exciting the beam using the TFB to induce controlled oscillations that facilitate subsequent analysis, but without generating noticeable emittance blowup, or generating losses. We aim to reconstruct the transverse phase space of the beam by analyzing its response, providing valuable insights into its dynamics.

Key transverse damper parameters that we will focus on include the recommended feedback phase, the measurement and setup of loop delay, and the feedback gain or damping time. To achieve this, we will utilize the newly developed ultralow noise beam position monitors (BPMs), which are capable of extracting data even from low-amplitude signals [42]. The data collection and processing will be conducted using the so-called ObsBox system [24], which integrates full-rate, real-time information from all BPMs to facilitate a comprehensive analysis.

In a feedback system, the relationship between the closed-loop transfer function and the open-loop (OL) transfer function can be expressed as a function of frequency ω :

$$CL(\omega) = \frac{OL(\omega)}{1 + OL(\omega)}. \quad (8)$$

In the context of a transverse feedback system, the open-loop transfer function $OL(\omega)$ is defined as the combined transfer function of the beam, represented as G , along with the sensor, controller, and actuator [44,45]. In these studies, we assume that both the sensor and the actuator can treat bunches independently (“ideal damper”), meaning they do not contribute to the overall system transfer function:

$$OL(\omega) = G \times K e^{j(\omega T + \phi_{PK})}. \quad (9)$$

To ensure closed-loop stability in the system, it is crucial that the denominator of Eq. (8) does not equal zero, therefore, the following stability criteria must be satisfied:

$$|G| \times K < 1 \quad (10)$$

$$\arg\{G\} + \omega T + \phi_{PK} = -\pi \pm \text{tol}. \quad (11)$$

Transverse feedback can be understood as a complex impedance acting on the beam. When purely resistive, it stabilizes the beam. However, when the impedance generated by the TFB is not resistive anymore, it can shift the tune [46] or even drive the beam unstable. Modern TFBs can be used as a source of controlled machine impedance to perform specialized measurements [21] or shift the tune of individual bunches within a train in a controlled way to generate a tune spread.

The key parameters that significantly influence the gain and phase margins, and consequently the overall stability of the transverse feedback loop, include the loop gain K , the feedback phase ϕ_{PK} , and the total loop delay ωT . Knowing the actual values of these parameters throughout the machine cycle and making precise adjustments are essential for optimizing the performance of the TFB.

V. PARAMETER EXTRACTION METHODS

A. Phase space reconstruction

For the analysis, experimental data were collected in April 2024 from the TFB in CERN’s Large Hadron Collider (LHC), referenced to as ADT. The LHC TFB uses four beam position modules for each beam and plane (low-noise pickups, referenced as Q7–Q10) together with two redundant digital signal processing units (module 1 and module 2). Resulting TFB correction signals are sampled at $f_{\text{Sample}} = 3/10 f_{\text{rf}} = 120.24$ MHz, converted into the analog domain, amplified by tetrode amplifiers, and applied to the beam by means of E-field kickers. Figure 8 shows the raw data acquired from the LHC TFB, each being the normalized transverse position in the particular pickup. At a sampling rate of 40 Msps (or 25 ns per sample, 1 sample per bunch per turn), the x axis record length is approximately 147 turns or 13.1 ms. The y axis is represented in 16-bit signed binary codes, ranging from $[-32,768, 32,767]$, corresponding to a transverse beam movement

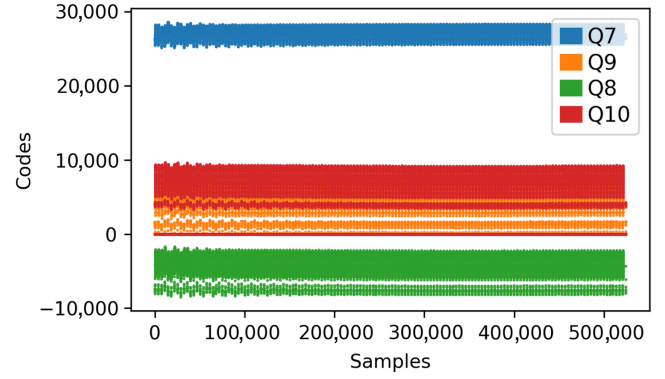


FIG. 8. Example of raw data of 156 bunches observed for 146 turns, from four pickups (Q7–Q10) as captured by the LHC TFB observation system. Such data are used for the analyses.

in the TFB pickup of ± 1.5 mm ($\pm 1.7\sigma$ at injection energy and $\pm 6.8\sigma$ at flattop).

A low-amplitude oscillation, present during the first 60 turns is caused by a short, controlled excitation at the beginning of the record, generated by the TFB controller and applied to the beam using the TFB kicker. Figure 9 provides a detailed snapshot of the excitation. The green trace shows the transverse positions recorded during the first three turns. At turn 2, the TFB triggers a precisely timed single excitation (orange trace) lasting one turn, or 3564 samples, with the amplitude dropping to zero before recording the position data for turn 3. Periodic waveforms, such as the cosine excitation pattern with $N = 1$ (one oscillation period per turn), are intentionally selected to ensure a mean excitation voltage of zero due to ac coupling of the TFB power system.

Continuous readings from Fig. 8 are reorganized to form a two-dimensional matrix: turns $\times$ bunches (146×156 for this demonstration). This allows us to visualize the time evolution of each bunch on a turn-by-turn basis, as shown in Fig. 10. The applied one-turn excitation causes all bunches to oscillate coherently with amplitude lower than

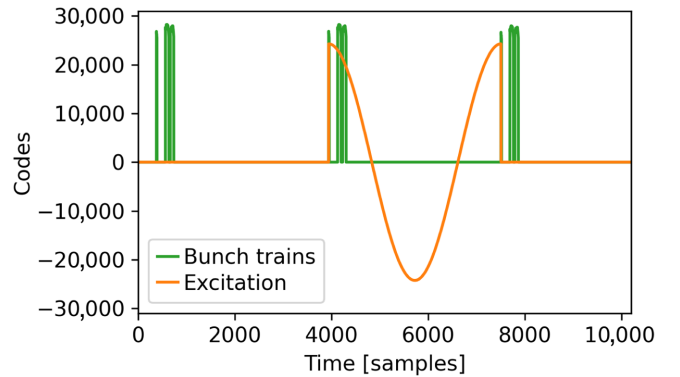


FIG. 9. First 3 turns of pickup data with cosine excitation pattern ($N = 1$) applied during the second turn. 1 turn equals 3564 samples.

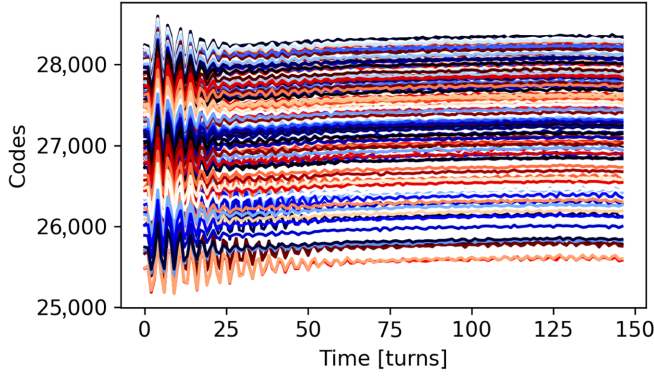


FIG. 10. Data from a single pickup. One curve is shown for each of the 156 bunches and over 146 turns. Baseline offset is due to the beam being not centered in this pickup and slightly drifting. The excitation amplitude is very low; 1000 codes on vertical axis corresponds to 0.05σ movement.

0.05σ , while the TFB action dampens this oscillation over approximately 20 to 60 turns.

Each of these bunch-by-bunch traces is utilized to reconstruct the transverse phase space by means of two individual digital FIR filters. One is typically a Hilbert transformer, and the second is a simple all-pass to compensate for the group delay of the Hilbert transformer, as described in Sec. III B. The two branches with filter kernels— $h_I[n]$ for the in-phase component and $h_Q[n]$ for the quadrature component—are combined to a final analytic signal $c[n]$, representing a reconstruction of the transverse phase space:

$$c[n] = y_I[n] + jy_Q[n] = y[n] * (h_I + jh_Q). \quad (12)$$

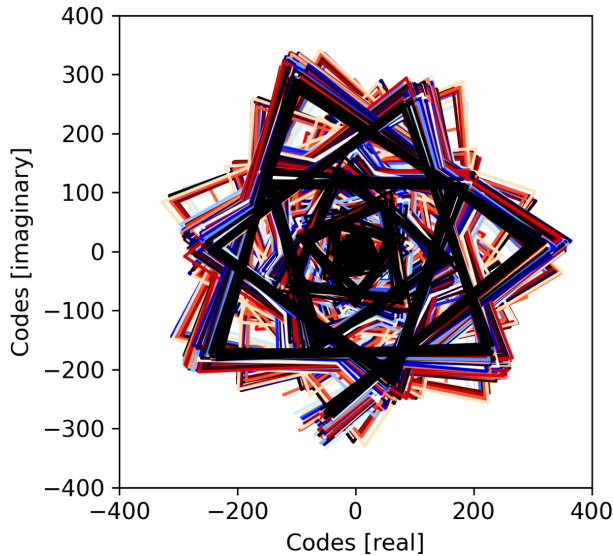
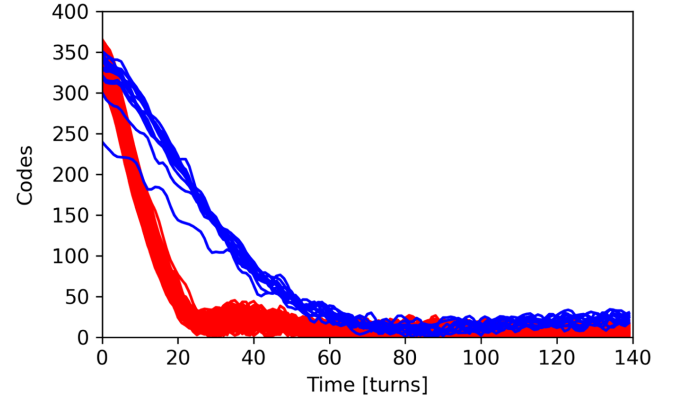
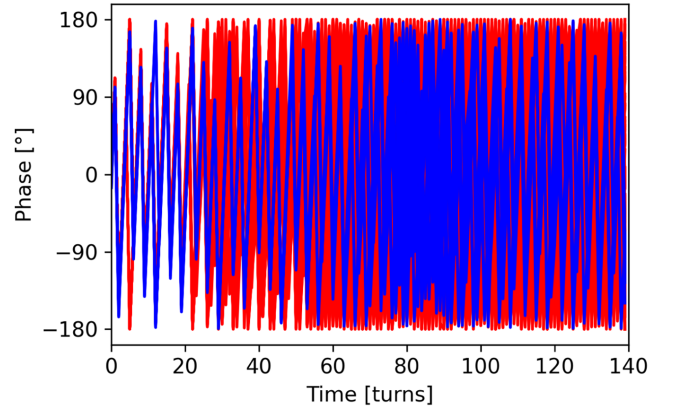


FIG. 11. Reconstructed transverse phase space plot for all bunches in real-imaginary format.



(a)



(b)

FIG. 12. Transverse phase space analysis: (a) amplitude and (b) phase. Movement of 400 codes corresponds to 0.02σ .

The reconstructed transverse phase space plot is illustrated in Fig. 11. The spiraling behavior toward the center reflects the TFB's stabilization process (damping). We are particularly interested in analyzing these traces, as they provide valuable insights into the feedback system, its performance, and specific parameters affecting accelerator operation. The same data are plotted in Fig. 12 in amplitude-phase format.

B. Transverse activity, damping time, and feedback gain

Using the reconstructed phase space as input, we define the transverse activity as the magnitude of the vector $c[n]$,

$$A[n] = \text{abs}\{c[n]\}, \quad (13)$$

which is a measure of the instantaneous oscillation amplitude in the normalized transverse phase space. An optimally performing TFB system should quickly dampen oscillations, resulting in a rapid (exponential) decay of the amplitude. Conversely, prolonged oscillations or slow decay may suggest that the TFB system is not functioning as intended.

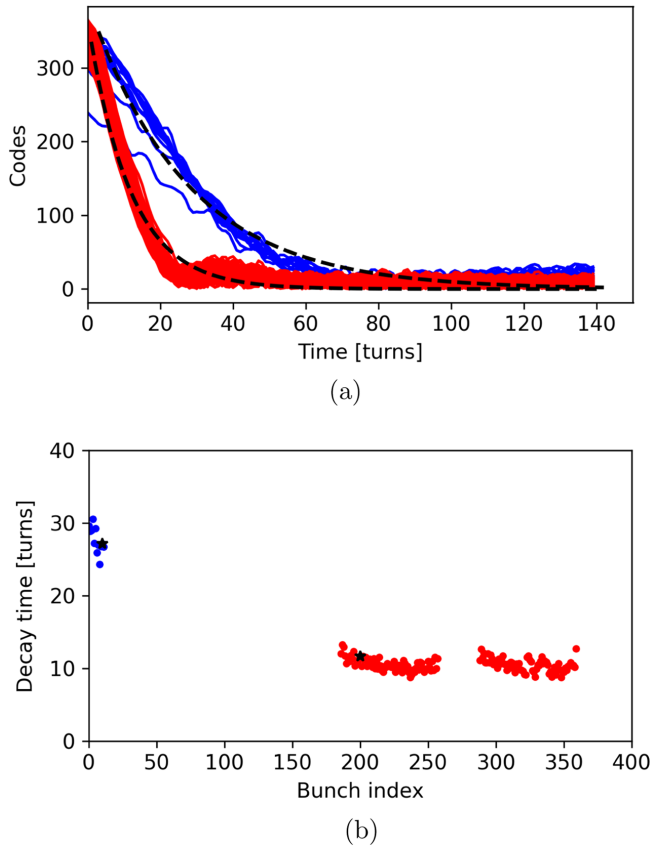


FIG. 13. Transverse activity: (a) Oscillation amplitude decay over time. The black dashed lines represent exponential decay curves for bunches 10 and 200. (b) Exponential decay time estimation per bunch.

The plot of Fig. 13(a) evaluates transverse activity for all 156 bunches. Two distinct clusters indicate different damping regimes. In the LHC, witness bunches (blue traces) at the beginning of the turn deliberately experience lower TFB gain compared to the main bunches (red traces) to provide sufficient signal levels for various beam instrumentation systems, which would otherwise not be available due to strong TFB.

Using the derived transverse activity per bunch, we can analyze the behavior of a well-adjusted TFB. A pure P-controller applies corrective actions that are proportional to the error input. As a result, the change in amplitude follows an exponential decay, represented as e^{-1/τ_d} . To derive the decay time τ_d from the transverse activity, we compare two time instances, n_1 and n_2 , using the following equation:

$$\tau_d = (n_2 - n_1) \left(\log \frac{A[n_1]}{A[n_2]} \right)^{-1}. \quad (14)$$

The damping time τ_d is related to the damping ratio and the natural frequency. This relationship is fundamental for understanding how feedback influences system dynamics,

particularly regarding response speed and stability [47]. For weak internal damping, this relationship can be expressed as

$$K = \frac{2}{\tau_d}. \quad (15)$$

Two black dashed curves overlaid on Fig. 13(a) represent the estimated damping times based on an exponential decay model for bunches 10 (in the witness region) and 200 (in the main region). The decay time constant for all bunches can be obtained from Fig. 13. The decay model clearly identifies two damping regimes. Regular monitoring of system performance is essential for proactively identifying and addressing potential issues. Thanks to the signal processing method presented in this study, automatic analysis of damping time for every injection into LHC was already implemented.

It is important to note that proper selection of samples from the excitation transient is critical to obtain correct results. Digital filters used to reconstruct the phase space need a certain number of samples to fill, until the result is valid and these need to be discarded. Equally at the decay side, only samples with sufficient signal-to-noise ratio should be used for reconstruction. Typically, two time constants worth of good data are necessary to measure the damping time with useful accuracy. If the damping time is very short (for example, 10 turns like in LHC) we need either an ultralow noise BPM or increased excitation amplitude. Alternatively, where available, multiple pickups at $\pi/2$ phase advance can be used to reconstruct the phase space (Sec. III A).

C. Recommended feedback phase

In this section, we shift our focus to evaluating the argument of the reconstructed phase space. Figure 12(a) depicts the transverse activity, while Fig. 12(b) the corresponding phase.

The comparison of the two plots reveals that, when noise is present, the quality of the extracted phase data deteriorates as the initial oscillation amplitude approaches the measurement noise floor of the beam position module. For main bunches (red traces), an oscillation amplitude of 100 codes or higher (out of a full scale $\pm 32,768$ codes, i.e., 0.005σ) yields a satisfactory phase reading just before turn 15. Witness bunches (blue traces) offer a good signal-to-noise ratio until approximately turn 40.

In the very same turn when the TFB excitation kick has been applied, i.e., $n = 0$, the betatron phase advance between kicker and pickup effectively transforms the transverse deflection in normalized coordinates, thus leading to

$$\phi_0 = \frac{\pi}{2} - \arg c[0]. \quad (16)$$

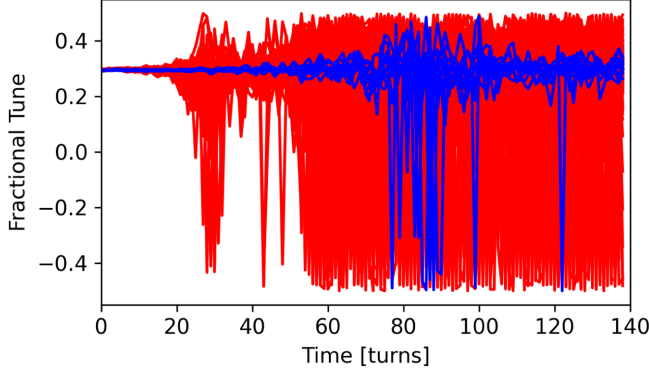


FIG. 14. Bunch-by-bunch tune estimation from reconstructed transverse phase space. In presence of measurement noise and 0.02σ excitation, 15 and 40 turns of usable data are available for main (red) and witness (blue) bunches.

Subsequent beam oscillations recorded by the pickup will advance by the value of the fractional tune. After a turn, the phase space has advanced in phase by $\Delta\varphi = \omega_0 \equiv 2\pi Q$. Therefore, by taking the ratio over two consecutive turns, we can express the per-turn or instantaneous fractional tune, $Q[n]$, as

$$Q[n] = \frac{1}{2\pi} \arg \frac{c[n-1]}{c[n]}. \quad (17)$$

Figure 14 illustrates the evolution of the calculated fractional tune for each individual bunch. Calculating the difference between two phase readings can amplify the effects of measurement noise. Averaging over multiple consecutive turns can reduce the impact of random noise.

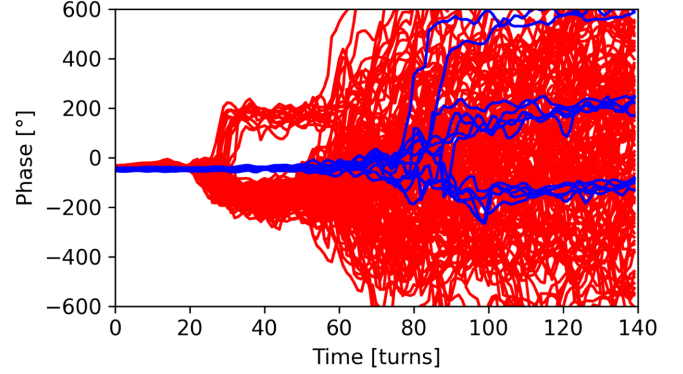
In Fig. 12, we examined the phase space, where the phase is wrapped within the range $[-180^\circ, 180^\circ]$. The beam oscillations recorded by the pickup advance the phase by the fractional tune with each turn, as shown in Fig. 14. Using Eq. (17), we can now also determine the initial phase for later turns, $\psi[n]$, from the argument of the analytic signal by including a linear phase term,

$$\psi[n] \doteq \frac{\pi}{2} - \arg c[n] \times e^{j2\pi Q \times n}. \quad (18)$$

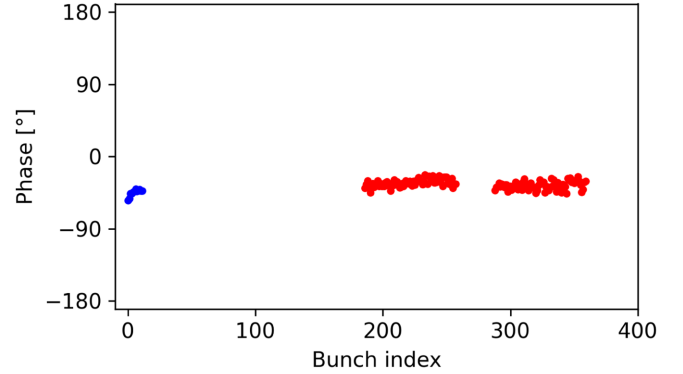
Taking this effect into account, we apply a compensation, the results of which are illustrated in Fig. 15(a). From this, we instantly obtain the phase advance between the kicker and the pickup by averaging over M consecutive turns,

$$\zeta = \frac{1}{M} \sum_{k=0}^{M-1} \psi[k]. \quad (19)$$

By averaging multiple readings over consecutive turns, we enhance the precision of our measurements. The phase advance between the kicker and pickup, measured for each



(a)



(b)

FIG. 15. Unfolded phase from reconstructed transverse phase space (a), with phase advance between kicker and pickup using averaged turn-by-turn data over 10 turns (b).

bunch, is shown in Fig. 15(b). These data were averaged using $M = 10$ turns in Eq. (19).

With the phase advance between the kicker and pickup extracted from our measurements, we can now determine the recommended TFB phase setting for the pickup that recorded this measurement. The feedback phase is, however, influenced by both processing delay and group delay, each expressed in terms of the number of turns. Additionally, we must consider the phase shift introduced by the notch filter, the phase rotation necessary to convert position data to slope, and a 180° phase shift to generate a damping correction signal.

By accounting for all these factors in the context of the LHC TFB, we derive the recommended transverse feedback phase, which is the optimal setting for using this pickup, as illustrated in Fig. 16. In this graph, the black horizontal dashed line represents the current phase setting from the control system, which aligns closely with the recommended phase for this system.

D. Loop delay measurement method

For proper operation, transverse feedback systems require precise temporal alignment of the kick signal with

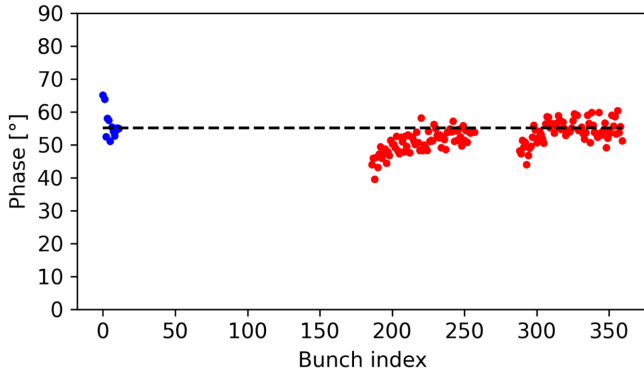


FIG. 16. Recommended transverse feedback phase using single-turn excitation and beam position data from one pickup.

the arrival times of the bunches. If we consider a kick signal modulated in amplitude over one turn, then a sinusoidal kick waveform with M periods per machine turn (revolution frequency) is sampled by a bunch with index k depending on the time of arrival at the location of the kicker. The resulting bunch oscillation magnitude, described as,

$$A_k = A_0 \times \cos\left(\frac{2\pi M}{h} \times k + 2\pi M \times \eta\right), \quad (20)$$

is then recorded as betatron oscillation decay at a downstream pickup. Here, the harmonic number h represents the maximum number of bunch slots per turn, and a delay offset factor, $\eta = \Delta T / T_{\text{Rev}}$, defined as the ratio between the kicker delay offset, ΔT , and the revolution period, T_{Rev} .

By applying quadrature excitation signals, denoted as A_I and A_Q , we can reconstruct an IQ footprint of the traversing bunches at the kicker as

$$\chi[k, n] = A_I[k, n] + jA_Q[k, n]. \quad (21)$$

In Fig. 17, each dot represents the result for one bunch based on two orthogonal excitation measurements. Dots of the same color indicate data obtained from the same turn.

Bunches are equally distributed around a circle with constant radius. Therefore, unwinding the phase response of Eq. (21) by taking into account a linear position-dependent phase term,

$$\rho[n] = \arg \chi[k, n] \times e^{-2\pi M k / h}, \quad (22)$$

this alignment produces a straight line. The parameter of interest, the delay offset ΔT , is defined by the orientation, or phase θ , of this line, applying averaging over populated bunches and N turns results:

$$\theta = \frac{1}{N} \sum_{m=0}^{N-1} \rho[m] \doteq 2\pi M \times \eta. \quad (23)$$

In Fig. 18, we average over populated bunches. The dots represent the mean values of bunch position data at turn n ,

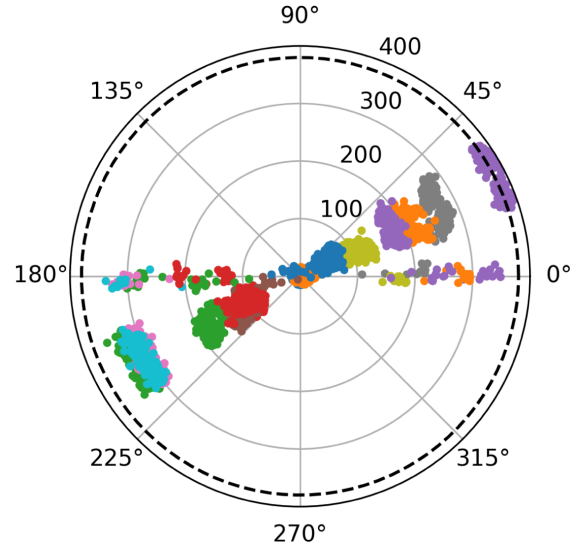


FIG. 17. Polar grid projection of bunch position signals.

with colors indicating different pickups. To estimate the delay offsets, we fit straight lines through these data points. Additionally, the black dashed rays illustrate the delay error associated with the TFB coarse delay, expressed in terms of the number of clock ticks.

Since $\eta = \Delta T / T_{\text{Rev}}$, it follows for the delay offset:

$$\Delta T = \frac{\theta}{2\pi M} \times T_{\text{Rev}}. \quad (24)$$

To further enhance measurement resolution, the excitation parameter M is increased, enhancing sensitivity to delay offsets down to a fraction of a nanosecond. Selection of $M = 400$ results in an excitation frequency of 5 MHz, increasing the measurement sensitivity by a factor of 400, as illustrated in Fig. 19.

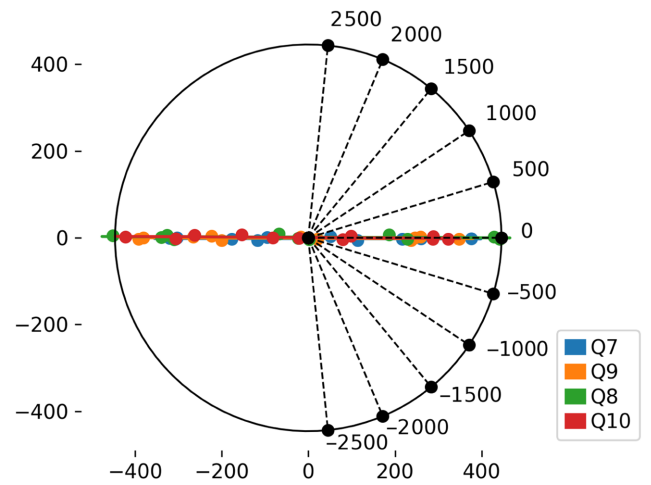


FIG. 18. Delay data analysis. Black dashed rays represent the delay error in terms of TFB system's digital signal processing clock ticks.

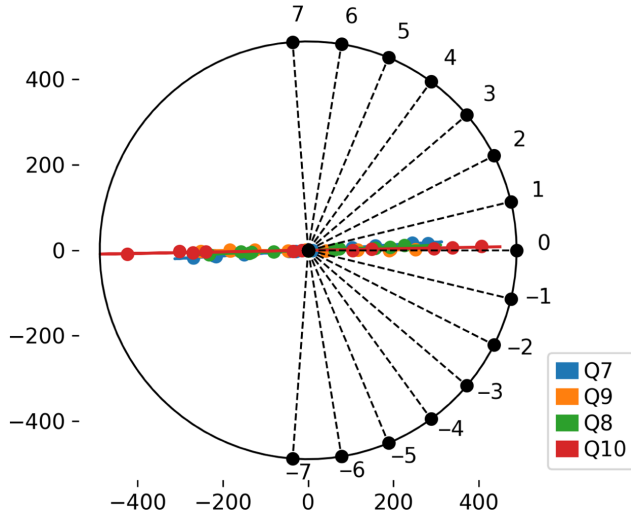


FIG. 19. Delay analysis for $N = 400$, with rays indicating the delay error in the TFB coarse delay. Grid spacing is now 1 clocktick, i.e., 8.3 ns.

VI. TFB PARAMETER MEASUREMENT RESULTS AND ITS SIGNIFICANCE FOR ACCELERATOR OPERATION

This section applies the methods introduced for extracting essential TFB parameters to evaluate performance of the TFB system and identify potential problems or areas requiring optimization. Data used in this section are real data from the transverse feedback system in the Large Hadron Collider, obtained in the TFB recommissioning session after the 2023–2024 winter stop. All data were generated and acquired exclusively by the TFB system itself, without the use of any external instruments. The acquired data were processed by Python scripts, and the results applied back to the TFB system.

In the past, commissioning a TFB after a technical stop or a major intervention had taken multiple shifts. Measurements were processed manually and parameters adjusted in converging incremental steps. The data presented here were collected in less than a minute, and once the analysis scripts are implemented into the control system, the results and proposed corrections will be available in seconds. The presented data demonstrate how powerful and useful the proposed methods are and their significance for the accelerator operation.

We will specifically analyze the closed-loop gain, feedback phase, and loop delay across the two distinct TFB gain regions, referred to as main gain and witness gain, as previously described. Each plot presents a comparative analysis of the results obtained for each pickup (16x), each signal processing module (8x), both planes (H/V), and both beams (1/2). Data from each pickup are distinctly color-coded for clarity. In every graph, we present a mean value derived from a series of measurements, accompanied with the corresponding root mean square (rms) values, which are

also color-coded to align with the respective pickups. The black lines indicate the minimum and maximum values of the data samples, providing insights into the measurement spread and highlighting any outliers. Through this analysis, we will discuss the performance of each digital signal processing system (DSPU), explore potential causes for deviations from nominal behavior, and identify opportunities for optimization. In all graphs, the first module of beam 2, vertical plane was off-line and could not be used for data evaluation.

A. Feedback gain

Comparison of the closed-loop feedback gain measured for main bunches with excitation set to $N = 1$ is presented in Fig. 20. The plots indicate that all modules for beam 1 (Hor B1 Mod.1, Hor B1 Mod.2, Ver B1 Mod.1, Ver B1 Mod.2), and one module of beam 2 (Ver B2 Mod.2) are operating within a gain range of 0.1 to 0.15. Notably, the majority of the pickups exhibit relatively small scatter in the rms values and peak excursions, suggesting that the obtained results are close to the true values. However, an outlier is observed in the horizontal pickup Q8 for beam 1, which displays a significantly larger rms scatter and substantial peak excursions in the measurements. Given that the same position data from this pickup are shared between the two concerned DSPUs, the similar results observed in these measurements imply that the issue is related to the beam position module of this pickup or its settings.

Furthermore, a clear deviation from the nominal gain parameter (typical value 0.1) is evident for both horizontal modules of beam 2 (mean values 0.3–0.4), accompanied by a considerably larger scatter as indicated by the rms values. Both modules exhibit similar measurement data for the same pickups. These data are obtained through independent measurements conducted by each signal processing unit individually, suggesting that the observed phenomena are related to a common factor affecting both modules and require expert attention.

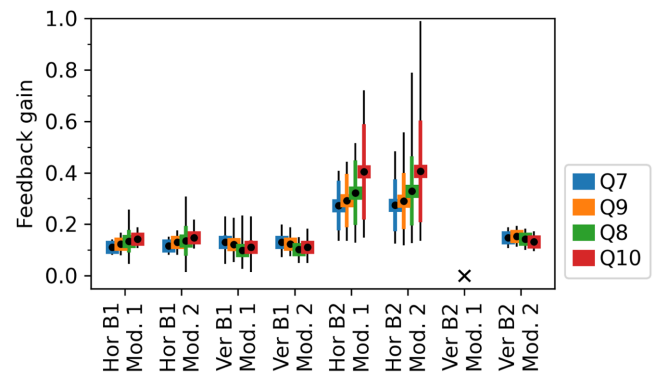


FIG. 20. Comparison of LHC TFB closed-loop feedback gain, measured for main bunches and excitation $N = 1$.

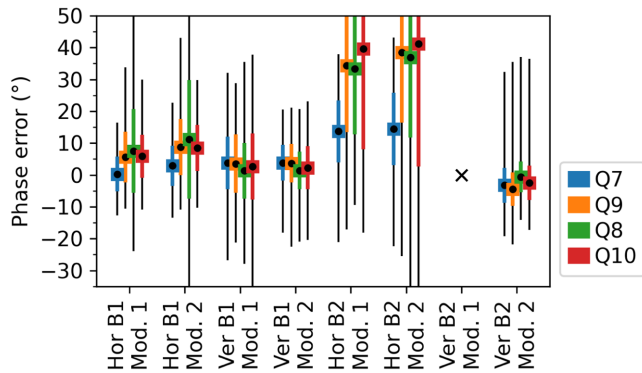


FIG. 21. ADT feedback phase error analysis, measured for excitation $N = 1$.

B. Feedback phase

Figure 21 presents the analysis of the TFB feedback phase error measured for excitation set to $N = 1$ (lowest frequency). The graphs illustrate the phase error for each pickup, i.e., comparing the current pickup set phase to the recommended phase setting determined by the signal processing algorithm. For the TFB systems operating beam 1, there is overall very good agreement, with phase errors not exceeding 10° and an rms spread of only 5° . However, an exception is noted for the horizontal pickup Q8 of beam 1, which exhibits a significant rms spread of nearly 20° and a phase error exceeding 10° for module 2. Suboptimal settings of the pickup Q8 Hor B1 were identified as the root cause.

In the case of beam 2, the candidates for concern are both horizontal modules. The data for pickup Q7 indicate a phase error of 15° , while the other three pickups show phase errors closer to 40° . The latter measurements display an rms spread of more than 20° , suggesting a general issue with horizontal, beam 2 TFB system. The observation is compatible with the earlier noticed lower TFB gain for this unit. It turned out, the accelerator was “unhappy” with HorB2 TFB, so the operations group preemptively lowered the gain of the unit and the situation became quieter. Nevertheless, the root cause of the problem has still not been identified.

C. Feedback loop delay

A delay error in the TFB loop leads to a phase error. For the example of the highest coupled bunch mode of 20 MHz in the LHC, this corresponds to $360^\circ \times 20 \text{ MHz} \times 1 \text{ ns}$, or 7.2° per nanosecond.

Figure 22 illustrates the loop delay error measurement (in nanoseconds) for each TFB system, with the excitation pattern set to $N = 400$. It is important to note that the delay error is related to the kicker timing, so individual measurements for each pickup are combined to form the overall result.

For beam 1, the delay errors are within acceptable limits, not exceeding 2 ns (14°). Three out of four pickups show

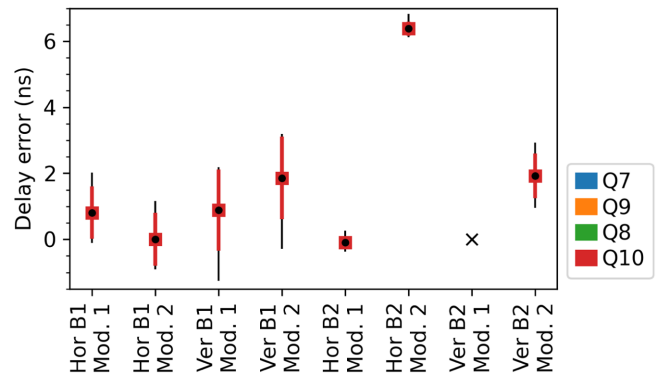


FIG. 22. ADT fine delay error. Excitation pattern $N = 400$.

errors better than 1 ns (7°), and one pickup has nearly zero delay error.

Interestingly, both modules in beam 2 horizontal show remarkably high data quality, with rms values well below 0.25 ns. While the delay setting for signal processing unit Hor B2 Mod.1 is nearly optimal, the second redundant unit Hor B2 Mod.2 has a delay error of 6.3 ns, which is significantly off target. The delay error for Hor B2 Mod.2 results in a phase error of 45° at 20 MHz. Such a significant phase error could lead to challenges or even drive the beam unstable. If one module is not properly adjusted, it will attempt to dampen these oscillations but may divide its action into two components: resistive and reactive. The reactive component could inadvertently shift the fractional tune of the already oscillating bunches.

In LHC, there are two redundant TFB modules operating on the same beam, both aiming to stabilize it (as illustrated in Fig. 7). However, with differing settings, these modules may compete with one another. A well-adjusted module could help compensate for some imperfections in the other module. While the combined function of both modules may still provide net damping, it is less effective than a system configured with optimal settings. Finally, thanks to the presented methods, the root cause of the observed difficulties with the horizontal system of beam 2 was identified.

VII. SUMMARY AND CONCLUSIONS

Digital signal processing for transverse feedback systems with typical configurations in terms of number of pickups, kickers, tune or machine optics was presented. We provide equations to calculate pickup mixing coefficients in vector-sum mode for optics, where betatron phase advance between pickups is different from the ideal $\pi/2$ radians.

A new, shortest possible digital filter for a single pickup TFB providing the full closed orbit suppression and arbitrary phase shift was developed at CERN. This filter provides a much larger tune acceptance and higher feedback gain with respect to the traditional Hilbert phase-shifter scheme. A full derivation of filter coefficients from beam rotation matrices was presented to allow construction

of either shortest possible filters or more sophisticated stacked versions for advanced functionalities.

Based on formalisms of signals and systems theory, we present methods to reconstruct the transverse phase space using only the data already available in the transverse feedback system. The techniques allow us to measure and extract a number of vital TFB parameters and system performance such as damping time, feedback gain, real-time transverse activity, recommended feedback phase or loop delay.

Thanks to a new generation of very low noise beam position measurement modules developed for the High Luminosity LHC, we are able to extract the TFB parameters using almost negligible beam oscillation amplitude. Typically a transverse excitation in the order of 0.02σ already provides good quality data for parameter extraction, TFB verification, or setting up. The techniques presented in this work can be used virtually transparently to the beam, opening possibilities for operational performance checks even with physics beams circulating in the machine. Enabled by readily available, very powerful digital technology, all this is now possible using only the TFB system itself, without the involvement of any external instruments and laborious and very time-consuming analysis by experts.

A full set of measurements to characterize a TFB system in CERN's LHC, comprising of 16 pickups and 8 signal processing units, takes less than a minute. The proposed techniques allow us not only to shorten the TFB commissioning time from several shifts to less than a minute but also regular, fully automated checks of TFB system can now be performed in case of suspected suboptimal performance, saving significant amounts of precious machine time.

ACKNOWLEDGMENTS

The authors would like to thank Robert Borner and Sergey Antipov. Without their support, this paper would not have been possible. We would also like to thank the Referee A, who inspired us to introduce a more sophisticated algorithm to select the input data for parameter extraction which provides more accurate results when different groups of bunches in the machine experience very different TFB dynamics.

DATA AVAILABILITY

The data that support the findings of this article are not publicly available upon publication because it is not technically feasible and/or the cost of preparing, depositing, and hosting the data would be prohibitive within the terms of this research project. The data are available from the authors upon reasonable request.

APPENDIX: GENERIC SOLUTION FOR THE 3-TAP FILTER

The algorithm uses three measurements (x_1 , x_2 , and x_3) from arbitrary phase advances θ_1 , θ_2 , and θ_3 . The phase advance difference between the measurements x_1 and x_2 is $\theta_{1,2}$ and between the measurements x_1 and x_3 is $\theta_{1,3}$. Furthermore, it is assumed that the beta function values β_1 , β_2 , and β_3 are known on each measurement point.

In the case of linear optics and perfect measurements, the beam oscillations can be rotated between measurements with rotation matrices,

$$\begin{vmatrix} x_2 \\ xp_2 \end{vmatrix} = \begin{vmatrix} \sqrt{\frac{\beta_2}{\beta_1}} \cos \theta_{12} & \sqrt{\beta_1 \beta_2} \sin \theta_{12} \\ -\sqrt{\frac{1}{\beta_1 \beta_2}} \sin \theta_{12} & \sqrt{\frac{\beta_1}{\beta_2}} \cos \theta_{12} \end{vmatrix} \begin{vmatrix} x_1 \\ xp_1 \end{vmatrix} \quad (\text{A1})$$

$$\begin{vmatrix} x_3 \\ xp_3 \end{vmatrix} = \begin{vmatrix} \sqrt{\frac{\beta_3}{\beta_1}} \cos \theta_{13} & \sqrt{\beta_1 \beta_3} \sin \theta_{13} \\ -\sqrt{\frac{1}{\beta_1 \beta_3}} \sin \theta_{13} & \sqrt{\frac{\beta_1}{\beta_3}} \cos \theta_{13} \end{vmatrix} \begin{vmatrix} x_1 \\ xp_1 \end{vmatrix}. \quad (\text{A2})$$

When all the measurements include the same, constant, offset error Δx , the same rotation matrices can be used after this unknown offset is subtracted from the measurements, i.e.,

$$\begin{vmatrix} x_2 - \Delta x \\ xp_2 \end{vmatrix} = \begin{vmatrix} \sqrt{\frac{\beta_2}{\beta_1}} \cos \theta_{12} & \sqrt{\beta_1 \beta_2} \sin \theta_{12} \\ -\sqrt{\frac{1}{\beta_1 \beta_2}} \sin \theta_{12} & \sqrt{\frac{\beta_1}{\beta_2}} \cos \theta_{12} \end{vmatrix} \begin{vmatrix} x_1 - \Delta x \\ xp_1 \end{vmatrix} \quad (\text{A3})$$

$$\begin{vmatrix} x_3 - \Delta x \\ xp_3 \end{vmatrix} = \begin{vmatrix} \sqrt{\frac{\beta_3}{\beta_1}} \cos \theta_{13} & \sqrt{\beta_1 \beta_3} \sin \theta_{13} \\ -\sqrt{\frac{1}{\beta_1 \beta_3}} \sin \theta_{13} & \sqrt{\frac{\beta_1}{\beta_3}} \cos \theta_{13} \end{vmatrix} \begin{vmatrix} x_1 - \Delta x \\ xp_1 \end{vmatrix}. \quad (\text{A4})$$

These matrices can be rewritten as four independent equations. Two of these equations are particularly useful because they include only two unknown variables, xp_1 and Δx :

$$x_2 - \Delta x = \sqrt{\frac{\beta_2}{\beta_1}} \cos \theta_{12} (x_1 - \Delta x) + \sqrt{\beta_1 \beta_2} \sin \theta_{12} xp_1 \quad (\text{A5})$$

$$x_3 - \Delta x = \sqrt{\frac{\beta_3}{\beta_1}} \cos \theta_{13} (x_1 - \Delta x) + \sqrt{\beta_1 \beta_3} \sin \theta_{13} xp_1. \quad (\text{A6})$$

By rewriting known constant values as

$$C_{12} = \sqrt{\frac{\beta_2}{\beta_1}} \cos \theta_{12} \quad (\text{A7})$$

$$S_{12} = \sqrt{\beta_1 \beta_2} \sin \theta_{12} \quad (\text{A8})$$

$$C_{13} = \sqrt{\frac{\beta_3}{\beta_1}} \cos \theta_{13} \quad (\text{A9})$$

$$S_{13} = \sqrt{\beta_1 \beta_3} \sin \theta_{13}, \quad (\text{A10})$$

the Eqs. (A5) and (A6) can be presented in simpler forms

$$x_2 - \Delta x = C_{12}(x_1 - \Delta x) + S_{12}xp_1 \quad (\text{A11})$$

$$x_3 - \Delta x = C_{13}(x_1 - \Delta x) + S_{13}xp_1. \quad (\text{A12})$$

The unknown variables Δx and xp_1 can be solved from these equations, which gives

$$\begin{aligned} \Delta x &= \frac{(C_{12}S_{13} - C_{13}S_{12})x_1 + S_{12}x_3 - S_{13}x_2}{C_{12}S_{13} - C_{13}S_{12} + S_{12} - S_{13}} \\ &\equiv \frac{(C_{12}S_{13} - C_{13}S_{12})x_1 + S_{12}x_3 - S_{13}x_2}{D} \end{aligned} \quad (\text{A13})$$

$$\begin{aligned} xp_1 &= \frac{C_{12}(x_3 - x_1) + C_{13}(x_1 - x_2) - x_3 + x_2}{C_{12}S_{13} - C_{13}S_{12} + S_{12} - S_{13}} \\ &\equiv \frac{(C_{13} - C_{12})x_1 + (1 - C_{13})x_2 + (C_{12} - 1)x_3}{D}. \end{aligned} \quad (\text{A14})$$

Now all the beam coordinate components (x_1 , xp_1 , and Δx) are known for measurement 1. This allows beam coordinate prediction for any phase advance by using the rotation matrix again.

Beam coordinates can be predicted for an arbitrary location 4, where the beta function value is β_4 and the phase advance from the measurement 1 is θ_{14} . By marking constant values as

$$C_{14} = \sqrt{\frac{\beta_4}{\beta_1}} \cos \theta_{14} \quad (\text{A15})$$

$$S_{14} = \sqrt{\beta_1 \beta_4} \sin \theta_{14}, \quad (\text{A16})$$

the rotation matrix for the beam can be written as

$$\begin{vmatrix} x_4 - \Delta x \\ xp_4 \end{vmatrix} = \begin{vmatrix} C_{14} & S_{14} \\ -S_{14} & C_{14} \end{vmatrix} \begin{vmatrix} x_1 - \Delta x \\ xp_1 \end{vmatrix} \quad (\text{A17})$$

By substituting Eqs. (A13) and (A14), this matrix product gives the beam coordinates

$$\begin{aligned} x_4 &= \frac{1}{D} [(-C_{12}C_{14}S_{13} - C_{12}S_{14} + C_{13}C_{14}S_{12} + C_{13}S_{14} + C_{14}D)x_1 \\ &\quad + (-C_{13}S_{14} + C_{14}S_{13} + S_{14})x_2 + (C_{12}S_{14} - C_{14}S_{12} - S_{14})x_3] \end{aligned} \quad (\text{A18})$$

$$\begin{aligned} xp_4 &= \frac{1}{D} [(-C_{12}C_{14} + C_{12}S_{13}S_{14} + C_{13}C_{14} - C_{13}S_{12}S_{14} - S_{14}D)x_1 \\ &\quad + (-C_{13}C_{14} + S_{14}S_{13} + C_{14})x_2 + (C_{12}C_{14} + S_{14}S_{12} - C_{14})x_3]. \end{aligned} \quad (\text{A19})$$

The Eqs. (A18) and (A19) are generic solutions, which allow an analytical beam coordinate prediction for any phase advance when measurements from three different turns or locations are available.

1. FIR filter

For the CERN SPS and LHC transverse feedback systems, the total correction kick is a linear combination of signals from different pickups. This means that each pickup-kicker pair is processed independently utilizing measurements from the previous turns of individual pickups.

The generic solution for the 3-tap filter presented in Eq. (A19) can be simplified when values from three subsequent turns from a single pickup are used.

The advance differences between the measurements are

$$\theta_{13} = 2\theta_{12} = 2\theta_Q = 2Q_{\text{frac}}2\pi, \quad (\text{A20})$$

where Q_{frac} is the fractional tune and θ_Q is the betatron phase change per turn. The phase advance between the first measurement and the kick can be marked as

$$\theta_{\text{kick}} = \theta_4 = \theta_{ki} - \theta_{pu} + (d + 2)\theta_Q \quad (\text{A21})$$

where θ_{ki} and θ_{pu} are phase advances of the pickup and kicker locations and d is the additional signal processing delay in turns. The beta function value is the same for all the measurements, i.e.,

$$\beta_{pu} = \beta_1 = \beta_2 = \beta_3 \quad (\text{A22})$$

and for the kicker

$$\beta_{ki} = \beta_4. \quad (\text{A23})$$

This simplifies beta function dependencies so that the Eq. (A18) is only proportional $\sqrt{\frac{\beta_{ki}}{\beta_{pu}}}$ and Eq. (A19) is only proportional to $\sqrt{\beta_{ki}\beta_{pu}}$. By using all these markings, the correction kick of Eq. (A19) can be presented as

$$\begin{aligned} b_0 &= \sqrt{\beta_{ki}\beta_{pu}} \frac{\cos(\theta_{\text{kick}})[\cos(\theta_Q) - 1] + \sin(\theta_{\text{kick}}) \sin(\theta_Q)}{2 \sin \theta_Q (\cos \theta_Q - 1)} \\ b_1 &= \sqrt{\beta_{ki}\beta_{pu}} \frac{\cos(\theta_{\text{kick}})[1 - \cos(2\theta_Q)] - \sin(\theta_{\text{kick}}) \sin(2\theta_Q)}{2 \sin \theta_Q (\cos \theta_Q - 1)} \\ b_2 &= \sqrt{\beta_{ki}\beta_{pu}} \frac{\cos(\theta_{\text{kick}})[\cos(2\theta_Q) - \cos(\theta_Q)] + \sin(\theta_{\text{kick}})[\sin(2\theta_Q) - \sin(\theta_Q)]}{2 \sin \theta_Q (\cos \theta_Q - 1)}, \end{aligned} \quad (\text{A24})$$

The coefficients in Eq. (A24) are finite, when the divisor $2 \sin \theta_Q (\cos \theta_Q - 1)$ is nonzero, i.e., values at the integer and half-integer tunes are forbidden.

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