

Design study of a high-brightness high-repetition rate thermionic injector for free-electron laser application. II. Gun technology

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The SPring-8 Angstrom Compact free-electron LAsER (SACLA) linear accelerator utilizes a dc gun with a thermionic cathode, known for delivering high beam quality, excellent operational stability, and low maintenance requirements. In the second part of our design study, we present a comprehensive beam dynamics analysis of the SACLA dc electron gun and outline a roadmap toward achieving submicrometer (0.1 μm) beam emittance. We provide both analytical and numerical estimates of four key sources contributing to emittance growth: (i) image = charge effects on the cathode, (ii) nonlinear space-charge forces arising from imperfect beam edge geometry, (iii) aberrations in the accelerating field, and (iv) solenoid field aberrations due to longitudinal–transverse momentum exchange. The analysis yields a closed-form expression for the superposition of *correlated* emittance growth mechanisms, highlighting an “interference” term that amplifies the net emittance when space-charge and solenoid effects coexist. To address the challenge of emittance growth, we propose a compact hybrid magnet that ensures strong longitudinal confinement of the magnetic field. This confinement allows placement of the hybrid magnet in close proximity to the gun exit, thus mitigating emittance growth effectively, while having a negligible magnetic field on the cathode. The hybrid magnet design is achieved by symmetrically positioning permanent ring magnets on either side of a solenoid, enabling fine control over the field distribution and “squeezing in” it longitudinally. Our findings offer new insights applicable to low-voltage continuous-wave very high-frequency guns and superconducting radio-frequency electron guns.

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I. INTRODUCTION

The electron gun is the key element of any injector, as it sets the initial beam brightness [1], which cannot be increased without particle loss or complex cooling schemes [2,3]. In this part of the design study we therefore examine, in detail, the beam dynamics in the SPring-8 Ångström Compact free-electron LAsER (SACLA) gun. With the notable exception of SACLA, operational x-ray free-electron lasers employ photocathode guns to generate high-brightness electron bunches. SACLA instead uses a pulsed dc thermionic gun that produces microsecond-long electron pulses with submicrometer normalized emittance [4–6]. A nanosecond bunch is extracted from the long pulse by

a high-voltage chopper [7]. Over two decades of gun routine operation have demonstrated high reliability and stability, straightforward maintenance, and excellent beam quality [8]. Building on this success, the new injector will employ an upgraded pulsed DC thermionic gun. One of the key objectives of the upgrade is to halve the rms normalized emittance, targeting 0.3 mm mrad for 1-A, 500-pC bunches.

Beam dynamics in low-voltage, low-gradient electron guns is reasonably well understood [9–12]. However, effects that were negligible in lower-brightness regimes now become critical as brightness increases. Beam dynamics simulations, coupled with multiobjective optimization, allow one to minimize emittance growth for a given layout, yet such simulations rarely reveal why the emittance grows, nor which specific layout changes or auxiliary components would deliver substantial improvements in brightness. To address this gap, we develop a detailed analytical framework of beam dynamics in a dc gun. The analysis complements existing numerical results in the literature and provides mechanism-level insight that informs design choices. The conclusions are also relevant to superconducting radio-frequency (rf) and normal-conducting, very-high-frequency (VHF) guns, which share similarly low

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accelerating gradients and beam energies and face limitations for emittance compensation [13–17].

We further show that a hybrid magnet—combining a solenoid with permanent magnet rings—results in smaller aberration and can preserve the initial beam brightness. In the current SACLA operation, a circular collimator downstream of the chopper removes the outer portion of the beam exhibiting strong nonlinear phase-space correlations. While this scraping reduces emittance, it also depresses the peak current to below 50% of the 1-A emission current. In contrast, our study demonstrates that careful optimization of the gun region and mitigation of aberrations can improve brightness without particle removal, thereby maintaining both emittance and peak current.

II. GUN TECHNOLOGY

We start by discussing the technical components of the gun, and in the next section, proceed to beam dynamics analysis.

A. Thermionic cathode

The SACLA gun utilizes a *single-crystal cerium hexaboride* (CeB_6) thermionic cathode for generating high-quality electron beams. This cathode technology ensures the production of stable, low-emittance electron beams necessary for FEL operations. CeB_6 offers several advantages over other thermionic emitters, including an extremely smooth surface, a lower work function, thermal stability, and high emission density, making it well suited for use in high-performance electron injectors [5]. (i) *Emission properties and work function.* The work function of CeB_6 is approximately 2.4 eV, which is lower than that of LaB_6 , enabling efficient operation at reduced temperatures of around 1400–1500 °C [18,19]. This lower operating temperature minimizes thermal stress on the crystal and extends the cathode’s operational lifespan. X-ray spectroscopy at the SPring-8 synchrotron radiation facility has been used to monitor the surface status of the CeB_6 crystal, detecting surface degradation such as carbon contamination, which could affect emission performance [20]. (ii) *Thermal stability and emission control.* In the nominal regime, the CeB_6 cathode generates a peak current of 1 A at 500 keV beam energy with a pulse width of 3 μs . The smooth surface of the single-crystal CeB_6 ensures uniform emission, maintaining the normalized thermal emittance as low as 0.4 μm [5]. The cathode assembly incorporates a *graphite heater*, which offers excellent thermal stability, consistent power delivery, and precise control of the operating temperature. This setup ensures long-term stability and reduces emission fluctuations [21]. (iii) *Minimizing space-charge effects.* The SACLA electron injector applies a 500 kV high-voltage pulse to minimize space-charge effects, which can lead to emittance growth and degrade beam quality. Additionally, advanced heater

designs at SACLA help reduce residual magnetic fields, thus preserving the stability of the electron beam throughout its trajectory [21]. (iv) *Operational challenges and lifetime management.* While the CeB_6 cathode offers excellent stability, operational challenges, such as *backward-accelerated electrons*, have been identified as potential causes for reduced lifetime. To address these challenges, a system was developed for *in-situ work function monitoring*, which enables real-time tracking of cathode degradation and optimizes replacement schedules [19,22].

B. High-voltage technology

The SACLA gun’s high-voltage system, modeled after klystron tanks, generates 500 kV pulses essential for beam acceleration. A C-band klystron modulator supplies a 24 kV pulse, which is stepped up to 500 kV using a 1:21 pulse transformer. A 1.9 k Ω dummy load is connected in parallel with the cathode to match impedance with the modulator’s pulse-forming network.

The cathode heater, requiring over 30 A, is powered via a high-voltage insulation transformer. A 7:1 step-down transformer minimizes losses along the transmission line. To prevent electrical discharge, key components such as the ceramic insulator, pulse transformer, and dummy load are immersed in insulating oil. The high-voltage tank is evacuated before operation to eliminate residual gases.

The applied 500 kV pulse produces a 500 keV electron beam with a peak current of 1 A. The beam current is monitored by a current transformer positioned after the gun. The pulse flattop lasts approximately 1 μs , ensuring the generation of nanosecond electron bunches. The compact high-voltage tank has been in regular operation since the 2000s.

C. Nanosecond pulser

The gun includes a beam chopper to extract 1-ns electron bunches from a longer, microsecond-scale electron pulse. The chopper system consists of two strip-line electrodes housed in a dedicated chopper chamber. The two strip-line electrodes, each 150 mm in length, are separated by a 20 mm gap and maintain a characteristic impedance of 50 Ω .

A dc magnetic field, with a field integral of 190 Gcm, is constantly applied for beam deflection. In addition, a fast-switching avalanche-type pulser generates a high-voltage rectangular pulse of 3-kV amplitude and 2-ns duration, which is applied to the upper electrode in the direction opposite to the beam propagation at the peak of the beam pulse. This configuration ensures that only the 1-ns segment of the long electron pulse, where the Lorentz forces from the static magnetic field and the force from the pulsed electric field are balanced, is allowed to propagate straight along the beam axis. The remaining portion of the beam is eliminated by a circular collimator with a hole diameter of 4 mm.

The performance of the chopper is monitored using a wall current monitor positioned 2 m downstream from the

gun exit [7]. The measured charge of the resulting electron bunch is 0.8 nC, with a full width at half maximum of 1.8 ns. Due to the finite rise time and filling time of the high-voltage pulse, and the transit time of the electrons through the chopper, the width of the electron bunch that passes through the collimator is approximately 1 ns shorter than the applied pulse duration. In the future, we plan to reduce the bunch duration to less than 0.5-ns flattop [23].

III. BEAM DYNAMICS IN THE SACLA GUN

The purpose of this section is to introduce an analytical model of beam dynamics in the SACLA gun, derive scaling laws for beam parameters, and elucidate the mechanisms of emittance growth. In the study, we always deal with the *rms normalized emittance*.

A. SACLA gun configuration

The problem under study is the beam dynamics in the electric field of a dc gun and the magnetic field of a solenoid, in the presence of space-charge forces. The on-axis field distributions and the evolution of the mean beam energy, the rms beam size, as well as the slice and total emittances, are presented in Fig. 1. The peak electric field at the gun cathode is 12.4 MV/m, while the solenoid field strength of 0.095 T is optimized to minimize emittance. The results have been obtained using the simulation code ASTRA [24]. As shown in the Appendix, the simulation model reproduces the experimental situation at the SACLA gun test stand very well, which gives us strong confidence that the predicted performance of the proposed upgraded gun can be achieved in practice.

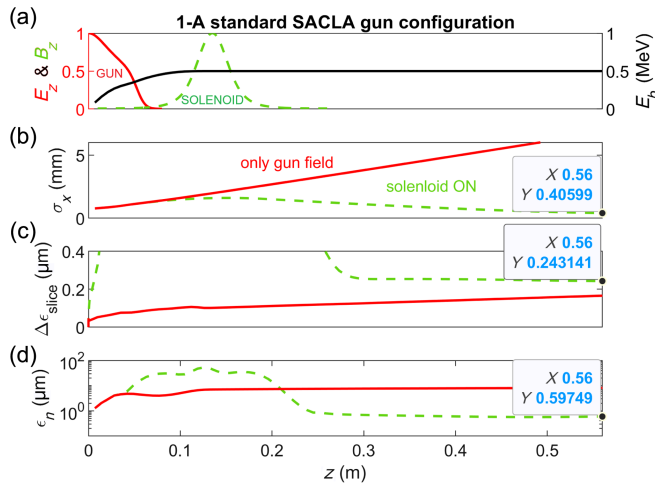


FIG. 1. Panel (a): on-axis normalized distributions of the gun electric (red) and solenoid magnetic (green dashed) fields; evolution of the mean beam energy (black). Panel (b): evolution of the rms beam size in the gun field alone (red) and in the additional presence of the solenoid field (green dashed). Panels (c) and (d): evolution of the *slice emittance growth* and total beam emittance.

In panels (c) and (d) of Fig. 1, one can observe a substantial growth of the slice emittance and a 50% increase in the total emittance from the thermal level, $\epsilon_{th} = 0.4 \mu\text{m}$. To calculate the slice emittance growth, in simulations, a 0.5-ns bunch distribution, representing the beam after the high-voltage chopper, is first binned into slices. Then, the mean slice emittance, $\bar{\epsilon}_n$, is computed as the average of the emittances for each slice. Finally, the slice emittance *growth* is estimated as $\Delta\epsilon_{\text{slice}} = \sqrt{\bar{\epsilon}_n^2 - \epsilon_{th}^2}$. The figure-of-merit $\Delta\epsilon_{\text{slice}}$ allows one to see more clearly the fine details of emittance evolution in simulations.

We now develop an analytical framework that elucidates the mechanisms of emittance growth, provides simple yet accurate quantitative estimates, and suggests a promising strategy for achieving lower emittances.

B. Gun geometry

Gun geometry has a big impact on beam dynamics, and we start our analysis with this aspect. It is common to deploy a focusing Wehnelt plate geometry, but a small transverse misalignment of the cathode with respect to the symmetry axis of the Wehnelt plate might lead to an emittance growth that is difficult to correct. Because of this potential misalignment-driven emittance growth, *the present SACLA gun uses a flat Wehnelt plate*, but the downside is an increased beam divergence, which leads to considerable emittance growth for high beam currents because of aberrations in a collimating solenoid [25]. Hence, we will analyze beam dynamics not only for the present SACLA gun geometry but also consider an option of the focusing Wehnelt plate geometry.

Within the analytical framework, calculations are done in the approximation of an infinitely long bunch so that the results for the emittance should be interpreted as an rms slice emittance.

C. Defocusing effect of the gun field

The impact of the dc gun geometry on the beam emittance was studied numerically by Bazarov and co-authors in [9]. Using the algorithm of genetic optimization, they investigated how the Pierce electrode inclination angle and the cathode-anode gap affect beam focusing and final emittance. It is pertinent to quote the findings of the study [9]: “An increased angle provides greater focusing, beneficial to counteract space charge, but also decreases the field at the photocathode surface. Decreasing the gap will strengthen the field at the photocathode surface, but will also increase the intrinsic effect of anode defocusing.”

In this subsection, we introduce a simple and instructive analytical model that describes the impact of the gun geometry on the beam divergence in the *limit of zero space-charge field*. The model allows one to quickly make a rough design of the gun geometry without

electromagnetic simulations. The equations of motion of an electron leaving the cathode at radial position r_0 and with zero initial slope $r'_0 = 0$ read:

$$\frac{dp_r}{dz} = \frac{eE_r}{c\beta_z}, \quad \frac{d\gamma}{dz} = \frac{eE_z}{mc^2}. \quad (1)$$

Here, β_z is the longitudinal velocity normalized to the speed of light c , γ is the beam energy in the units of the rest mass mc^2 . The longitudinal and radial electric fields E_z and E_r are connected to each other in the *linear* paraxial approximation as

$$E_r = -\frac{r}{2} \frac{\partial E_z}{\partial z}. \quad (2)$$

The radial kick $r' = p_r/p_z$ can be calculated as follows:

$$r' = -\frac{r}{2\gamma\beta_z} \int_0^z \frac{\gamma'}{\beta_z} dz'. \quad (3)$$

Let us integrate (3). To this end, we make an essential approximation and treat β_z in the integrand as a constant. The result of integration is surprisingly accurate, and the simplification can be justified using the following reasoning. On the metallic surface of the cathode, the radial field vanishes, $E_r|_{z=0} = 0$, and E_r must also be small in the vicinity of the cathode. Since the divergence of $\vec{E}$ must be zero ($\nabla \cdot \vec{E} = 0$), we can assume that $E_z \approx \text{const}$ in the vicinity of the cathode. Then, $\gamma' \approx 0$ near the cathode, and the exact form of β_z is not relevant since the integrand is zero anyway. On the other hand, away from the cathode, β_z first rapidly increases with the distance from the cathode and then saturates. Hence, the resulting evolution of the radial kick is

$$\frac{r'}{r} \approx -\frac{\gamma' - \gamma'_0}{2\gamma\beta_z^2}, \quad \gamma'_0 = \frac{eE_z|_{z=0}}{mc^2} = \frac{2eV/L}{mc^2}. \quad (4)$$

At the gun exit, the above expression simplifies to

$$\frac{r'}{r} \approx \frac{2eV/L}{mc^2} \frac{1}{2\gamma\beta_z^2}. \quad (5)$$

This is a very important result: *for a fixed voltage V , a too short accelerating distance L leads to an unacceptably large radial kick despite the increased field on the cathode!* Here, the effective gun length L includes both the accelerating gap region and the gun exit aperture region.

We want to highlight that the expression (4) gives at most 10%–20% discrepancy with the results of direct numerical simulations, and therefore it is useful for comparing different gun geometries. Consider the practical case of the SACLA gun. The longitudinal field distribution and energy gain along the gun are depicted in Fig. 2. A simple

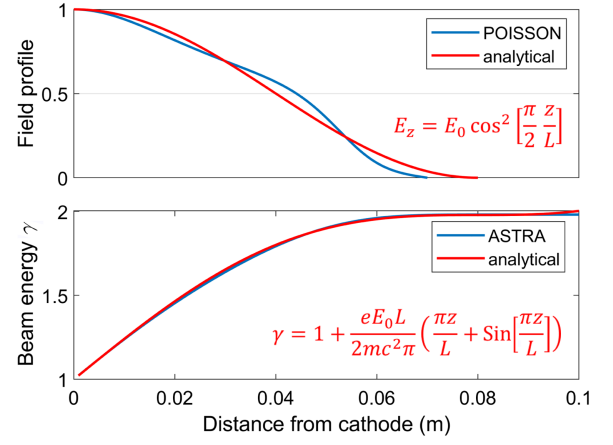


FIG. 2. Top plot: numerically computed (blue curve) distribution of the longitudinal electric field in the SACLA gun. An analytical model of the field distribution is shown in red. The model is motivated by the solution of the Laplace equation for the electric potential in a semiopen conducting cylinder. Bottom plot: numerically computed (blue) and analytically calculated (red) energy gain of the gun. The solution for γ is the direct result of the energy conservation.

analytical model of the field results in a nearly perfect prediction for the energy gain. Then, using Eq. (3), the evolution of the kick of an electron leaving the cathode at the position corresponding to the rms beam size is calculated and shown in Fig. 3. For comparison, the rms kick computed using the simulation code ASTRA [24] is also depicted. For this simple field distribution, the discrepancy between the numerical and analytical result is of the order of 1%.

Our next step is to establish a connection between a more complex gun geometry and the field distribution. We consider the study of Bazarov *et al.* [9] to be important for understanding how the gun geometry affects the field

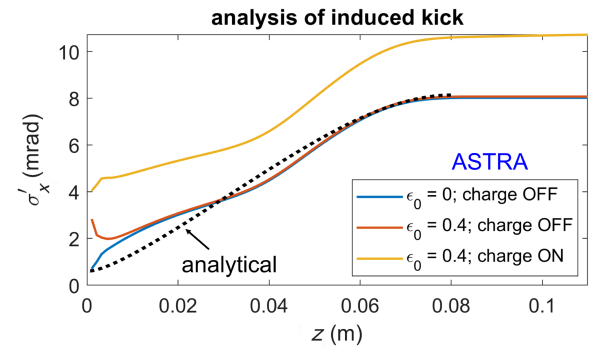


FIG. 3. Radial kick in the SACLA gun computed with the help of ASTRA and calculated using the analytical model (3). The simulation with ASTRA includes three cases: (i) zero emittance and switched off space-charge effect; (ii) initial emittance of 0.4 μm and switched off space-charge effect; and (iii) initial emittance of 0.4 μm and switched on space-charge effect. The initial emittance has a negligible impact on the kick.

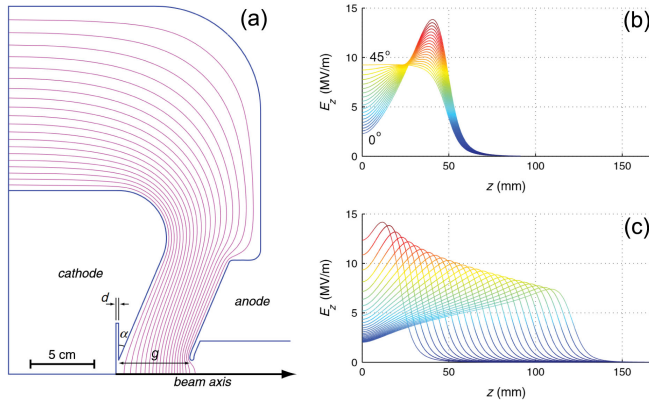


FIG. 4. Geometry of the dc gun (a) and the field distributions for inclination angles α from 0 to 45° (b) and different distances of the cathode-anode gap g (c). In images (b) and (c), color coding is used to better distinguish the curves. The larger the angle α , the lower the field on the cathode for a fixed gun voltage. Adopted from [9].

distribution and, therefore, with the permission of the publisher, we replicate Bazarov's Fig. 1 as Fig. 4 in the present paper. Making the Pierce electrode inclined dramatically changes the field distribution and, as we shall see, changes the evolution of the kick induced by the gun field. Let us develop an analytical model.

Consider the longitudinal field distribution in the form:

$$E_z/E_p = U_l \left(1 - \varepsilon + \varepsilon \sin^2 \left[\frac{\pi z}{2z_0} \right] \right) + U_r \cos^2 \left[\frac{\pi(z - z_0)}{2(L - z_0)} \right], \quad (6)$$

where $U_l = U(z_0 - z)$ and $U_r = U(z - z_0)$ are the compact notations for the unit step function $U(z)$. The indices l and r serve as a reminder that the unit step function is nonzero on the left and on the right of z_0 , respectively. This model can describe different field distributions analyzed numerically by Bazarov *et al.* [9]. Here L is the effective length of the gun, which depends on the radius of the exit aperture of the gun. The position z_0 gives the coordinate of the peak field E_p in a focusing gun geometry and strongly depends on the inclination angle of the focusing electrode. The parameter ε shows how much the longitudinal electric field sags near the cathode, and it also defines the transverse focusing strength of the radial field. For an order-of-magnitude estimate, we can assume $\varepsilon \sim \sin \alpha$ and $z_0 \sim L/2$.

From the condition that the gun voltage is equal to the integral of the field, $V = \int_0^L E_z dz$, we obtain

$$E_p = \frac{2V}{L - \varepsilon z_0 + z_0}. \quad (7)$$

For a *flat* cathode geometry $\varepsilon = 0$ and $z_0 = 0$ so that $E_0 = E_p = 2V/L$.

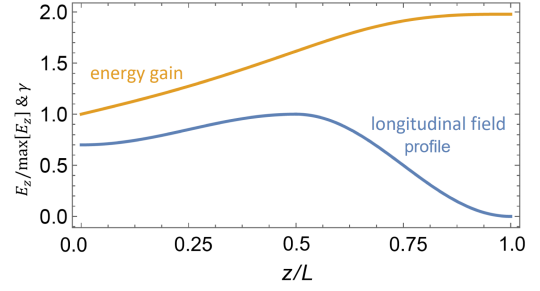


FIG. 5. Example of the longitudinal field distribution (blue) for a focusing gun geometry with $z_0 = L/2$ and $\varepsilon = 0.3$. The energy gain is shown in yellow. The effective length of the SACLAL gun, L , is 0.08 m.

The energy gain in the field (6) is given by

$$\gamma = 1 + U_l \frac{f_g(2 - \varepsilon)z_0}{2L} + U_l \frac{f_g z_0}{L} \left(\frac{z}{z_0} - \frac{\varepsilon z}{2z_0} - \frac{\varepsilon}{2\pi} \sin \left[\frac{\pi z}{z_0} \right] \right) + U_r \frac{f_g}{2\pi L} \left((L - z_0) \sin \left[\frac{\pi(L - z)}{L - z_0} \right] + \pi(z - z_0) \right). \quad (8)$$

Here $f_g = eE_p L / mc^2$ is the normalized field strength.

We can again use Eq. (3) to estimate the kick induced by the gun field, but *the field on the cathode takes on a more complex form*:

$$E_z|_{z=0} = \frac{(1 - \varepsilon)2V}{L + z_0(1 - \varepsilon)}. \quad (9)$$

The evolution of the kick along the gun in a focusing gun geometry, see Fig. 5, is shown in Fig. 6. The maximum focusing (maximum negative value of r') is achieved at the longitudinal position $z \approx z_0$. The maximum kick at this position can be evaluated as

$$\left. \frac{r'}{r} \right|_{z=z_0}^{\text{focus}} \approx \frac{2eV/L \varepsilon}{mc^2 \frac{3}{2}}. \quad (10)$$

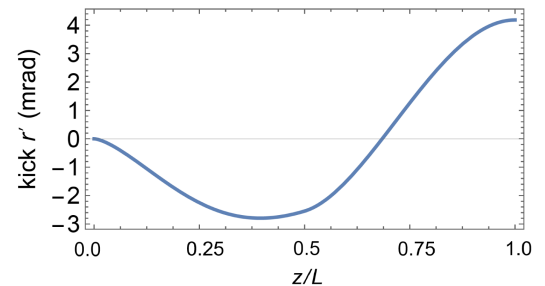


FIG. 6. Evolution of the radial kick in the focusing gun geometry. See Fig. 5 for the parameters.

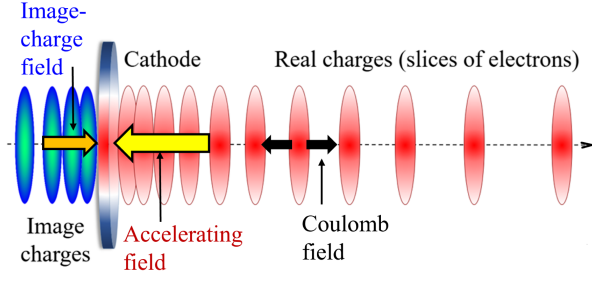


FIG. 7. Illustration of the electron emission and the fields acting on on-axis electrons.

The focusing geometry can be used to counteract the defocusing space-charge effect for high beam currents.

D. Maximum current density and image-charge effect

The particles leaving the cathode create a so-called space-charge field that consists of two parts: (i) the repulsive free-space Coulomb field and (ii) the image-charge field established by image charges induced on the metallic surface of the cathode, see Fig. 7. The space-charge field imposes a limitation on the maximum current $I_{\max}$ of particles that can leave the cathode. In the simplest approach of a uniform transverse density distribution, it reads [26,27]

$$I_{\max} \approx \frac{\pi \epsilon_0}{2} \left(\frac{2e}{m} \right)^{1/2} (E_0 R_c)^{3/2}, \quad (11)$$

where ϵ_0 is the vacuum permittivity, R_c is the cathode radius, $E_0 = 2V/L$ is the accelerating (longitudinal) field magnitude on the cathode. The numerical factor $1/2$ is different from the traditional factor of $4/9$ in the 1D Child-Langmuir law [28,29] because of the dimensionality effect, first analytically calculated in [30], and then refined in [27].

The maximum extractable current for a 1.5 mm cathode radius (SACLA case) versus the accelerating field on the cathode is shown in Fig. 8. The peak field in the SACLA gun is 12.6 MV/m, and the maximum extractable current is around 20 A. Normally, the gun is operated at 1 A in the regime of temperature-limited emission.

Let us investigate in detail the behavior of the beam in the cathode region. The top plot of Fig. 9 presents the

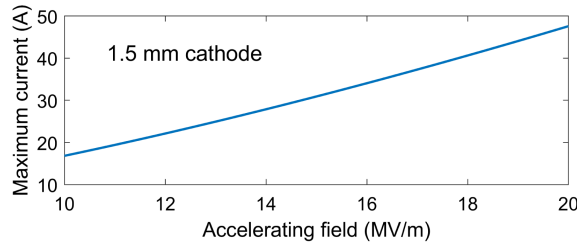


FIG. 8. Maximum extractable current vs accelerating field on the cathode.

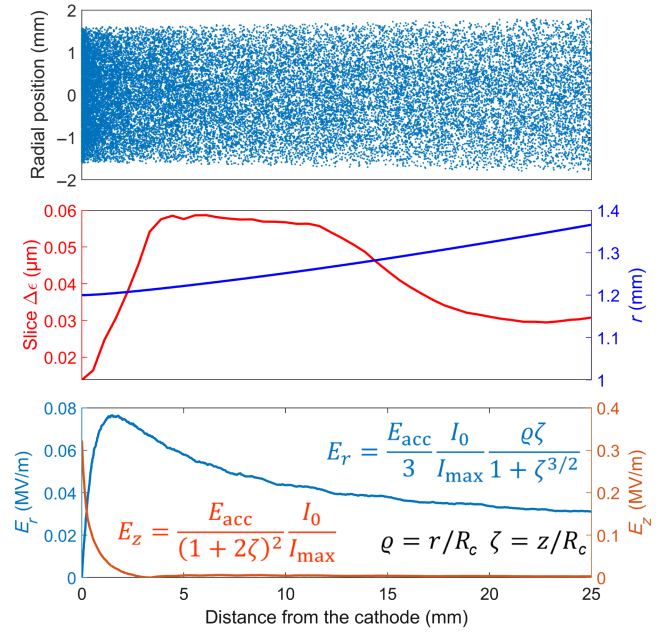


FIG. 9. Top plot: distribution of particles in space after leaving the cathode located at $z = 0$. Middle plot: evolution of the slice emittance (red) and rms beam size (blue) along the gun. The initial emittance is set to zero to reveal the emittance growth induced by nonlinear effects such as the image-charge field. Bottom plot: distributions of the radial (blue) and longitudinal (red) components of the space-charge electric field. The analytical formulas reproduce the field profiles with deviations less than 10%. The analytical model for the field is taken from [27].

simulated particle distribution after the cathode. The particle density quickly diminishes with the distance from the cathode since the current density is constant—due to the charge conservation—while the speed of particles increases. The middle plot of Fig. 9 shows the evolution of the macroscopic beam parameters, such as the slice emittance and beam size. The beam expands because of the defocusing effects of the gun field and the space-charge field. In simulations, the initial emittance is set to zero, and one can clearly see that there is a rapid induced emittance growth in the vicinity of the cathode. The bottom plot of Fig. 9 gives the distribution of the space-charge field along the gun. The radial component is zero on the cathode, but rapidly increases with the distance from the cathode and reaches the maximum at $z = R_c$. After that, it slowly diminishes. In contrast, the longitudinal component of the space-charge field attains its maximum value on the cathode and then rapidly diminishes. It is vanishingly small for $z > 2R_c$.

The emittance dynamics near the cathode, shown in Fig. 9, is rich in physics and interesting. Near the cathode, the accelerating field is very uniform, and all emittance growth is solely due to the nonuniformity of the space-charge field. Let us examine its radial component E_r^{charge} in detail. The profiles of E_r^{charge} for different longitudinal

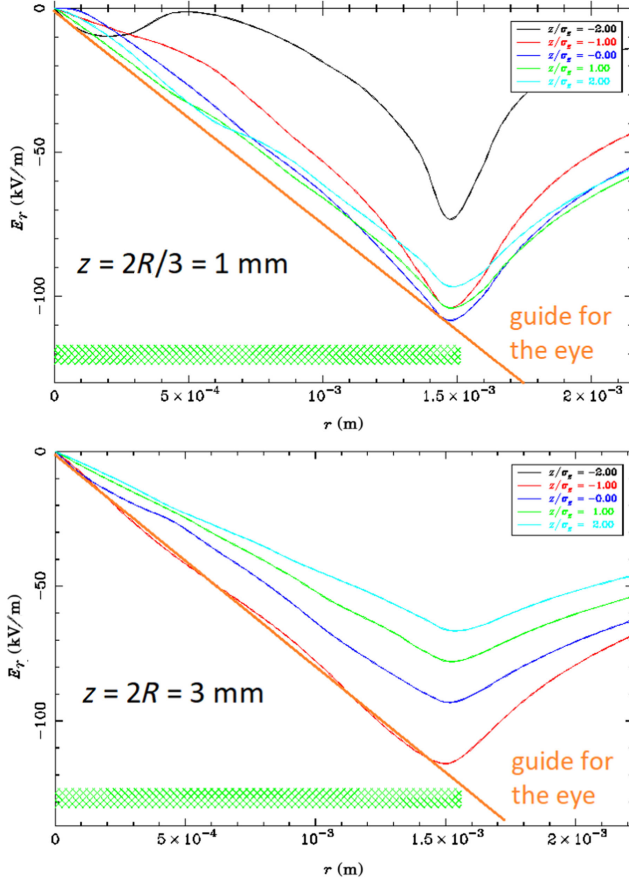


FIG. 10. Dependence of the radial component of the space-charge field on the radial coordinate for different longitudinal positions in the beam, and for two positions of the beam center: (i) 1 mm from the cathode (upper plot) and (ii) 3 mm from the cathode (bottom plot). The green bar at the bottom of the plot indicates the bunch radius. Here, σ_z is the bunch's rms length.

positions in the beam are depicted in Fig. 10. Right after the cathode, E_r^{charge} exhibits a Z-shaped dependence on r , see Figs. 4 and 6 in [31] and explanations therein. This nonlinearity in E_r^{charge} leads to Z-shaped correlation in the beam transverse phase space and a corresponding emittance growth. For $z \gg R_c$, the impact of the image charge becomes negligible, and the space-charge field profile is determined by the local charge density distribution. Because of the space-charge force and diffusion of the edge particles caused by thermal emittance, the initial sharp edge of the square-box transverse distribution of the beam density is smeared. The transverse density distribution becomes a bell-like one, and the space-charge field profile acquires an S-shaped distribution. This introduces an S-shaped correlation into the beam transverse phase space, which, to a large extent, compensates the Z-shaped one imprinted onto the transverse phase-space distribution of the beam near the cathode. As a result, the slice emittance attains a minimum value in the region around 22 mm from the cathode. After that, the slice emittance starts to grow

again, dominated by the S-shaped correlation of E_r^{charge} . For low currents, the emittance growth caused by the image space-charge field is negligible compared with the thermal emittance.

E. Linear space-charge effect in the gun

The space-charge-driven radial dynamics of the beam can approximately be described as

$$\frac{dp_r}{dz} \approx \frac{eE_r^{\text{charge}}}{c\gamma^2\beta_z}, \quad (12)$$

where γ^2 takes into account the reduction of the Lorentz force due to the relativistic effect, and E_r^{charge} is the radial component of the space-charge field. The latter, using the model developed in [27], can be written in the form

$$E_r^{\text{charge}} = \frac{E_0}{3} \frac{I_b}{I_{\text{max}}} \frac{\rho\zeta}{1 + \zeta^{3/2}}, \quad (13)$$

where $\rho = r/R_c$ and $\zeta = z/R_c$ with R_c being the cathode radius. Then, the expression for the slope of a test electron takes on the form:

$$\frac{r'}{r} = \frac{2eV}{3mc^2} \frac{1}{\gamma\beta} \frac{I_b}{I_{\text{max}}} \frac{J(\zeta)}{L}, \quad (14)$$

where

$$J(\zeta) = \int_0^{\zeta/R_c} \frac{f_r(\zeta)}{\gamma^2(\zeta)\beta(\zeta)} d\zeta, \quad f_r(\zeta) = \frac{\zeta}{1 + \zeta^{3/2}}. \quad (15)$$

The main contribution to the space-charge-driven radial kick occurs in the vicinity of the cathode because the longitudinal velocity is small and the electron spends a longer time in a stronger space-charge field. Therefore, we will limit our analysis to the most interesting region $z \leq L/4$. In this region, the energy gain is described by a very simple expression,

$$\gamma(\zeta) = 1 + G\zeta, \quad G = \frac{2eV R_c}{mc^2 L}, \quad (16)$$

and the integral in $J(\zeta)$ can be calculated analytically. The exact solution is very cumbersome; hence, an approximate solution is given

$$J(\zeta) \approx \frac{1}{3\sqrt{2G}} \left(\log \left[\frac{2\zeta^{3/2} + \zeta^3 + 1}{(\zeta G + 1)^3} \right] + \frac{3}{\zeta G + 1} - 3 \right). \quad (17)$$

Equations (14), (16), and (17) allow one to quickly estimate the space-charge induced kick. A graphical visualization of the evolution of the radial kick using an exact solution to

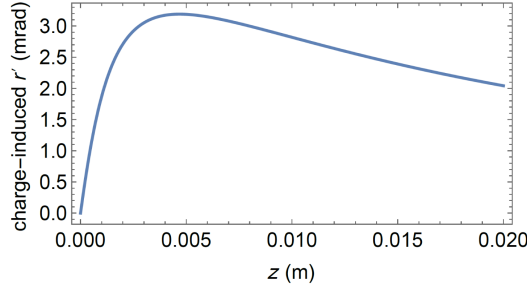


FIG. 11. Evolution of the radial kick induced by the linear space-charge force.

the integral $J(\zeta)$ is presented in Fig. 11. The kick rapidly grows in the vicinity of the cathode because of a large space-charge field, but then decreases because of the increased longitudinal velocity and shorter interaction time of the electron with the space-charge field.

F. Nonlinearity of the space-charge field

In this subsection, we analyze emittance growth arising from inhomogeneities in the electron-density distribution. Consider a beam moving in the presence of only linear forces. In the idealized limit of vanishing thermal emittance and negligible image-charge effects, the electrons' trajectories would be laminar and the density would remain uniform; the space-charge field would then be linear and emittance growth negligible. In practice, however, the edge of the beam becomes diffused because of the image charge and thermal emittance. Hence, the density becomes non-uniform as the beam propagates, giving rise to a nonlinear space-charge force. We begin by developing a *simple model for the evolution of the electron-density profile*.

Consider an axially symmetric beam having a uniform initial density distribution and a Gaussian spread of electrons' transverse momenta:

$$f_b(r_0, \theta_0) = \frac{U(R_{b0} - r_0)}{\pi R_b^2} \frac{1}{\sqrt{2\pi}\sigma_\theta} e^{-\frac{\theta_0^2}{2\sigma_\theta^2}}. \quad (18)$$

Here, $U(R_{b0} - r_0)$ is the unit step function, R_{b0} is the initial beam radius in the hard-edge approximation and σ_θ is the rms angular divergence of the electron beam. In the limit of zero beam divergence, $\sigma_\theta = 0$, the flow of electrons is laminar and the trajectories are described as $r = [1 + \alpha(z)]r_0$ and $\theta = \alpha'(z)r_0$, where $\theta = dr/dz$ is simply the slope of the trajectory. For nonzero σ_θ , the transformation of the electron's coordinates is as follows:

$$r = [1 + \alpha(z)]r_0 + \theta_0 z, \quad \theta = \alpha'(z)r_0 + \theta_0. \quad (19)$$

Consider the simplest case of a nearly collimated beam. Then, we impose the constraints $\alpha'(z)R_{b,0} \ll \sigma_\theta$ and $z \ll R_{b,0}/\sigma_\theta$. The numerical example will be given later.

The distribution function Eq. (18) is mapped onto a new position by replacing the variables according to $\theta_0 \approx \theta$, $r_0 = (r - \theta_0 z)/\alpha$ and multiplying it by the Jacobian of coordinate transformation. After integrating $f_b(r, \theta)$ over θ , it turns out that the radial density profile of the electron beam at an arbitrary position z is described by a Gaussian error function:

$$\rho(z)/\rho_0 = \text{erf} \left[\frac{1 - r/R_b(z)}{\sqrt{2}z\sigma_\theta/R_b(z)} \right], \quad (20)$$

where $R_b(z) = [1 + \alpha(z)]R_{b,0}$ is the beam radius at the position z .

Unfortunately, the error function is not convenient for analytical calculations because the integrals containing it are difficult to compute. Therefore, inspired by the model of a refractive index distribution in optical fibers, we replace the error function by a simpler function of the form

$$\rho(z)/\rho_0 \approx 1 - [r/R_b(z)]^p, \quad (21)$$

where p is the steepness parameter that determines how quickly the charge density diminishes at the edge; see the example of the $\rho(r)/\rho_0$ distribution in Fig. 12. The two distributions given by Eqs. (20) and (21) reproduce very well each other, if $p = R_b(z)/z\sigma_\theta$. The simple model, Eqs. (18)–(21), provides an educated guess about the value of the steepness parameter p . We found that the reasonable range for p after the gun is between 20 and 30.

Now, let us compute the *space-charge field created by the density profile (21)*. In the approximation of a long beam, the space-charge field can be found from Gauss's law as

$$E_r^{\text{charge}} = \frac{mc^2}{e} \frac{2I}{I_\alpha \beta_z} \begin{cases} \frac{r}{R_b} \left[1 - \frac{2}{p+2} \frac{r^p}{R_b^p} \right], & r \leq R_b, \\ \frac{1}{r} \left[1 - \frac{2}{p+2} \right], & r \geq R_b, \end{cases} \quad (22)$$

where $I \equiv \rho_0 c \beta_z \pi R_b^2$ is the beam current, $I_\alpha = 17$ kA is the Alfvén current.

In the region of the “soft” beam edge in Fig. 12, E_r^{charge} exhibits an S-shaped correlation, which leads to an

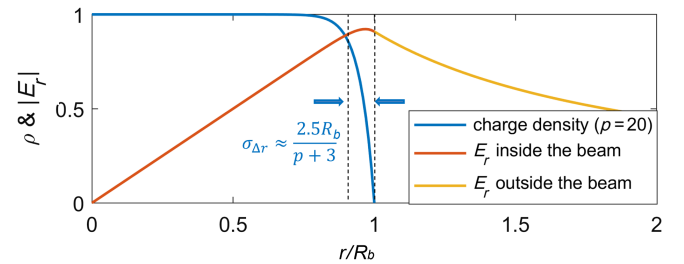


FIG. 12. Profiles of the charge distribution (blue) and the resulting space-charge field (yellow). The region of the “soft” edge of the beam is indicated by dashed lines.

S-shaped deformation of the transverse phase-space and associated emittance growth. As we shall see, this phase-space deformation can be enhanced by the aberration of a collimating solenoid at the gun exit that corrects the beam divergence.

Figure 13 demonstrates the distribution of the space-charge field at 0.14 and 0.7 m away from the cathode. The beam is mildly focused by the solenoid [see the rms beam size evolution in Fig. 1(b)]. While at the shorter distance, the beam density is still highly uniform and the space-charge field distribution is linear, at the longer distance, the space-charge field distribution exhibits a strong nonlinearity because of the inhomogeneous density distribution [Eqs. (20)–(22)].

Now, we will compute the emittance growth associated with the nonuniformity of the charge density. Using the model (22), the motion equation of the test electron takes on the form

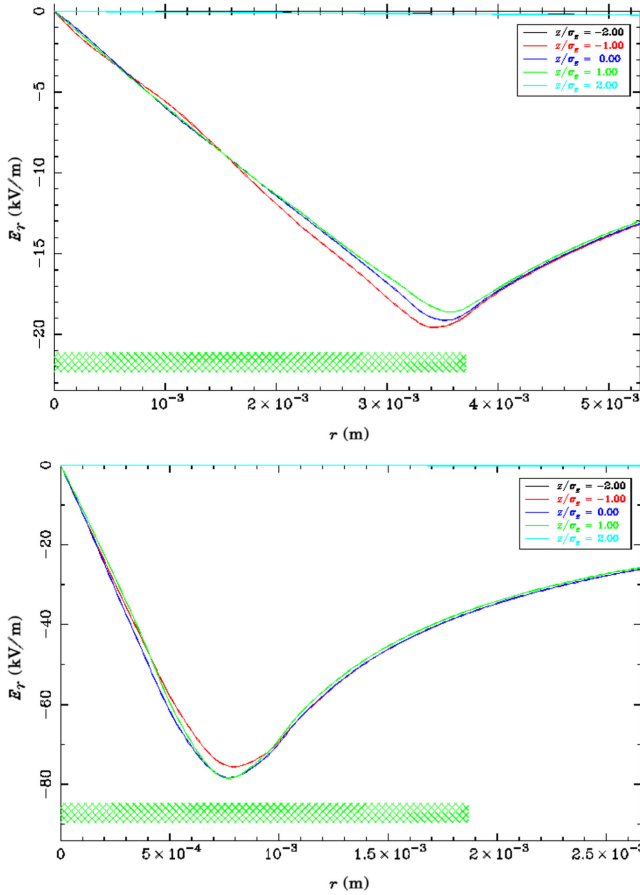


FIG. 13. Radial distributions of the space-charge field for several longitudinal slices: $z = 0$, $z = \pm\sigma_z$, and $z = \pm 2\sigma_z$, where σ_z is the rms bunch length. The results in upper plot are computed at the distance 0.14 m from the cathode, whereas for the bottom plot at 0.7 m, see Fig. 1(b) for the rms beam size profile. The corresponding steepness parameter in the analytical Eq. (21) is 30 and 4, respectively. The green box lying on the abscissa axis indicates the beam distribution.

$$\frac{dp_r}{dz} = mc \frac{2I}{I_\alpha (\gamma\beta_z)^2} \frac{r}{R_b^2} \left[1 - \frac{2}{p+2} \frac{r^p}{R_b^p} \right], \quad (23)$$

Since only the nonlinear part of the radial momentum contributes to emittance growth, let us separate the momentum into two parts—linear $p_r^{(\text{lin})} = a_1 r$ and nonlinear $p_r^{(\text{nl})} = a_{p+1} r^{p+1}$, and estimate the latter

$$p_r^{(\text{nl})} = -mc \frac{2I}{I_\alpha} \frac{2}{p+2} \int_{4R_{b0}}^z \left(\frac{r}{R_b} \right)^{p+1} \frac{dz'}{(\gamma\beta_z)^2 R_b}, \quad (24)$$

where z is the distance from the cathode. The case of emittance growth outside the gun is treated separately.

In the gun region $p \gg 1$, and under the assumption of a quasiuniform beam expansion ($r/R_b = \text{const}$), we obtain the following approximation of Eq. (24):

$$\begin{aligned} p_r^{(\text{nl})} &= a_{p+1} r^{p+1} \\ &\approx -\frac{2Imc}{I_\alpha \bar{\gamma}'} \left(\frac{1}{R_b} \right) \log \left[\frac{2}{\gamma+1} \sqrt{\frac{\gamma^2-1}{\gamma(4R_{b0})^2-1}} \right] \\ &\quad \frac{2}{p+2} \left(\frac{r}{R_b} \right)^{p+1}, \end{aligned} \quad (25)$$

where $\left(\frac{1}{R_b} \right) \approx \frac{R_b+R_{b0}}{2R_{b0}R_b}$ is the mean of $1/R_b$, and $\bar{\gamma}' \approx \frac{\gamma-1}{L}$ is mean acceleration rate over the gun gap L .

The contribution of the nonlinear momentum to the rms slice emittance growth reads:

$$\begin{aligned} \Delta\epsilon_{n,x} &= \frac{1}{2mc} \sqrt{\langle r^2 \rangle \langle p_r^2 \rangle - \langle r p_r \rangle^2} \\ &= \frac{|a_{p+1}| R_b^{p+2}}{2mc} \sqrt{\frac{1}{2(p+2)} - \frac{4}{(p+4)^2}}. \end{aligned} \quad (26)$$

While computing the expectation values $\langle r^m \rangle = 1/(\pi R_b^2) \int_0^{R_b} r^m r dr d\phi = 2R_b^m/(m+2)$, we treat the density distribution as a uniform one and ignore the edge diffusion, which leads to a negligible higher-order corrections. Inserting the coefficient a_{p+1} from Eq. (25) into Eq. (26) and estimating the dimensionless momentum as $\sqrt{\gamma(4R_{b0})^2-1} \approx 2\sqrt{2eE_0 R_c/mc^2}$ one obtains:

$$\begin{aligned} \Delta\epsilon_{n,x} &\approx \frac{2IR_b}{I_\alpha \bar{\gamma}'} \left(\frac{1}{R_b} \right) \log \left[\sqrt{\frac{\gamma-1}{\gamma+1}} \sqrt{\frac{mc^2}{2eE_0 R_c}} \right] \\ &\quad \frac{1}{p+2} \sqrt{\frac{1}{2(p+2)^3} - \frac{4}{(p+4)^4}}. \end{aligned} \quad (27)$$

In the limit $p \rightarrow \infty$, the emittance growth vanishes, $\Delta\epsilon_{n,x} \rightarrow 0$, which is expected since this scenario

corresponds to a perfect square-box density profile. We also note that the emittance growth is inversely proportional to the mean accelerating gradient $\bar{\gamma}$.

The steepness parameter p for a 1-A beam is estimated to be around 20 from the model, see Eqs. (20) and (21). This value of p is supported by the experimental data presented in [5]. The estimation for the SACLA gun parameters ($R_{b0} = R_c = 1.5$ mm, $R_b = 3.6$ mm, $\bar{\gamma}mc^2 = 10$ MeV/m, $E_0 = 12.67$ MV/m, $(\gamma - 1)mc^2 = 500$ keV) predicts the emittance growth to be around $\Delta\epsilon_{n,x} \approx 0.043$ μm . This value is close to the simulation result in Fig. 1.

To estimate *emittance growth in the space beyond the gun*, we assume a collimated beam and all parameters to be independent of z . Hence, the integration in Eq. (24) is trivial and yields:

$$p_r^{(\text{nl})} = -\frac{2I}{I_\alpha} \frac{mc}{(\gamma\beta)^2 \bar{R}_b} \left(\frac{r}{\bar{R}_b}\right)^{p+1} \frac{2(L-z)}{p+2}. \quad (28)$$

The emittance growth reads

$$\Delta\epsilon_{n,x} \approx \frac{I}{I_\alpha} \frac{z-L}{(\gamma\beta)^2} \frac{2R_b}{p+2} \sqrt{\frac{1}{2(p+2)^3} - \frac{4}{(p+4)^4}}, \quad (29)$$

and it vanishes for $\sigma_\theta = 0$ ($p \rightarrow \infty$).

G. Electrostatic aberration

At the exit aperture of the gun, the longitudinal accelerating field decreases in a nonlinear fashion with respect to z , which gives rise to a nonlinear component of the radial electric field

$$E_r(r, z) \approx -\frac{r}{2} \frac{dE_z(0, z)}{dz} + \frac{r^3}{16} \frac{d^3E_z(0, z)}{dz^3}. \quad (30)$$

This is the effect of electrostatic aberration. The radial motion equation of the test electron takes on the form

$$\frac{dp_r}{dz} = -\frac{er}{2v_z(z)} \frac{dE_z(0, z)}{dz} + \frac{er^3}{16v_z(z)} \frac{d^3E_z(0, z)}{dz^3}. \quad (31)$$

Since only the nonlinear part of the radial momentum contributes to emittance growth, as in the previous paragraph, we separate the momentum p_r into the linear $p_r^{(\text{lin})} = b_1 r$, and the nonlinear $p_r^{(\text{nl})} = b_3 r^3$ parts, and estimate the latter

$$p_r^{(\text{nl})} = b_3 r^3 = \frac{e}{16R_b^3} \int_0^{L+\Delta L} \frac{R_b(z)^3}{16v_z(z)} \frac{d^3E_z(0, z)}{dz^3} dz, \quad (32)$$

where L_g is the length of the accelerating gun gap, ΔL is the aperture length, $R_b(z)$ is the envelop beam radius. Here we assume that the transverse beam expansion is uniform so that r/R_b can be regarded as constant. Within this

model, the total accelerating region $L = L_g + \Delta L$ and for the standard SACLA gun configuration $L_g \approx 0.7L$.

Integrating the right-hand side of Eq. (32) by parts and assuming that on the cathode $E_z(0, 0) = E_0$, $\frac{d^2E_z(0, z)}{dz^2}|_{z=0} = 0$, and beyond the aperture $E_z(0, L_g + \Delta L) = 0$, $\frac{d^2E_z(0, z)}{dz^2}|_{z=L_g+\Delta L} = 0$, and taking into account the following $\frac{d^2}{dz^2} \left[\frac{R_b(z)^3}{v_z(z)} \right] |_{z=L_g+\Delta L} \approx 6 \frac{R_b(L_g)R'_b(L_g)}{v_z(L_g)}$, we can obtain the nonlinear momentum as

$$p_r^{(\text{nl})} = b_3 r^3 \approx -\frac{3}{8} \frac{eE_0}{v_z(L_g)} \left[\frac{R'_b(L_g)}{R_b(L_g)} \right]^2 r^3. \quad (33)$$

The contribution of the nonlinear momentum to the rms emittance growth reads:

$$\Delta\epsilon_{n,x} = \frac{1}{16\sqrt{2}} \frac{|b_3| R_b^4}{mc}. \quad (34)$$

Inserting the coefficient b_3 from Eq. (33) into Eq. (35), we, finally, obtain the slice rms emittance growth caused by the gun electrostatic field

$$\Delta\epsilon_{n,x} \approx \frac{1}{8\sqrt{2}} \frac{eV}{mc^2} \frac{[R'_b(L_g)R_b(L_g)]^2}{L\beta_z(L_g)}. \quad (35)$$

This emittance growth depends on the beam parameters at the gun aperture entrance. However, since we are interested in the scaling laws and a simple estimate, one can simplify the result Eq. (35) and replace L_g with L .

Note that the radial divergence of the beam, R'_b , and the corresponding envelope radius, R_b , can be estimated from the sum of the radial kicks induced by the gun field (5) and the linear space-charge force (14). The estimation for the SACLA gun parameters: $R_b(L)' = 22$ mrad, $R_b(L) = 3.6$ mm, $\beta(L) = 0.863$, $E_0 = 12.67$ MV/m, results in the emittance growth close to $\Delta\epsilon_{n,x} \approx 0.008$ μm .

The result in Eq. (35) predicts interesting and somewhat counterintuitive conclusions. Since R'_b is inversely proportional to the accelerating gap length L , reducing the gap while keeping the gun voltage constant leads to a strong emittance growth $\Delta\epsilon_{n,x} \sim 1/L^3$. This is because the accelerating field distribution is not flat in the accelerating gap, see Fig. 2, but instead is decreasing quite rapidly as a function of the distance from the cathode.

The contribution to the radial divergence from the space-charge force (14) is proportional to the beam current, $R'_b \sim I_b$, and large currents might substantially drive the emittance growth $\Delta\epsilon_{n,x} \sim I_b^2$.

In the limit of the energy gain much larger than the electron rest mass, $eV \gg mc^2$, the radial kicks induced by the gun field (5) and the linear space-charge force (14) become independent of the beam energy. However, the

emittance growth (34) is still dependent, in a linear fashion, on the energy gain in the gun.

Overall, the emittance growth due to the space-charge field and exit aperture aberration is not a problem in the standard SACLA gun operation, but the operation at higher beam currents might pose some challenges. The simple estimates (27) and (35) can also be applied for the design of dc, VHF rf, and SRF guns [15–17], which share with SACLA similar field profiles (QWR cavities) and low accelerating gradients.

H. Magnetostatic aberration

In the magnetic field of a solenoid, the beam quality might deteriorate due to a number of aberrations [2,32–34]. *Outside the gun region*, in a static longitudinal magnetic field, $B_z(z)$, the ray equation of the test electron can be written in the form [25]:

$$r' + \frac{1+r'^2}{1-k_c^2 r^2} \left\{ [k_c^2(z) - k_p^2(r)]r - k_c^2 \frac{k'_c}{k_c} r^2 r' \right\} = 0, \quad (36)$$

where $k_c = eB_z(z)/2p_0$ is the *cyclotron* wavenumber related to the magnetic field focusing strength; p_0 is the electron's momentum; and k_p is the *plasma* wavenumber related to the space-charge defocusing strength,

$$k_p^2 = \frac{K}{R_b^2} \left[1 - \frac{2}{p+2} \frac{r^p}{R_b^p} \right], \quad K = \frac{2I}{I_\alpha(p_0/mc)^3}. \quad (37)$$

Here K is the normalized beam perveance with $I_\alpha = 17$ kA being the Alfvén current. If the space-charge force is negligible, the dynamics of the radial momentum of the test electron simplifies to

$$\frac{dp_r}{dz} = -r \left[\frac{eB_z(z)}{2p_z(r)} \right]^2. \quad (38)$$

The momentum exchange between the transverse and longitudinal degrees of freedom of electrons in a magnetic field can result in significant beam emittance growth. The mechanism of it can be described in three key steps [25]: (1) the accumulation of a nonlinear $r - p_r$ correlation induced by the space-charge field and/or the gun field; (2) the development of an $r - p_z$ correlation within the solenoid magnetic field; and (3) nonlinear, chromatic r -dependent focusing by the magnetic field Eq. (38). Thus, the chromatic aberration in the solenoid field leads to an additional nonlinear deformation of the transverse phase space as the bunch is transported through the solenoid. The resulting bunch's distribution in the transverse phase space exhibits higher-order nonlinear correlations. The mechanism of emittance growth is visualized in Fig. 14.

In order to disentangle different mechanisms of emittance growth, for now we neglect the space-charge force in the lens and consider the combined effect in the next section.

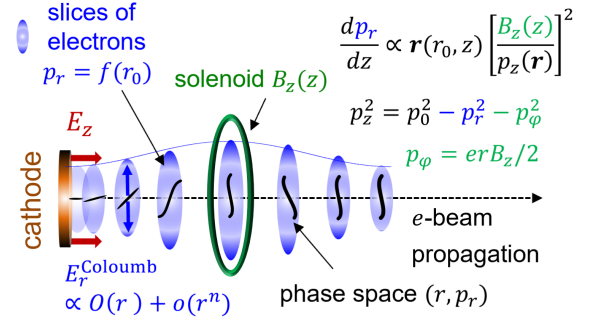


FIG. 14. Conceptual visualization of the chromatic magnetostatic aberration in a solenoid. The Coulomb force creates a nonlinear $r - p_r$ correlation in the bunch's distribution. The solenoid magnetic field transfers this nonlinear $r - p_r$ correlation into the $r - p_z$ plane, which leads to a chromatic aberration. After the lens, the nonlinear $r - p_r$ correlation appears to be enhanced.

The important parameter of the interaction of the test electron with the magnetic field of the lens, described by Eqs. (36) and (38) is a small parameter $\epsilon = R_{in}/\rho_b$, where R_{in} is the beam radius at the entrance to the solenoid, and $\rho_b = 2p_0/eB_{max}$ is the radius of the electron's orbit in the magnetic field. The typical value of ρ_b is 10 cm. This allows us to simplify Eq. (36) as

$$r' + \epsilon^2 \frac{f(z)^2}{R_{in}^2} r + \epsilon^2 \frac{f(z)^2}{R_{in}^2} r(r')^2 + \epsilon^2 \frac{f(z)}{R_{in}^2} f'(z) r' r^2 = 0, \quad (39)$$

where $f(z) = B(z)/B_{max}$ is the profile of the on-axis magnetic field. To estimate the emittance growth, the analytical solution of Eq. (39) can be expressed as a series expanded in powers of the small parameter ϵ . Using successive iterations, the solutions to Eq. (39) with the accuracy $O(\epsilon^4)$ are given as

$$r' \approx \left[\alpha - \frac{\epsilon^2}{R_{in}^2} F'(z, \alpha) \right] r_0 - 2\alpha^2 \frac{\epsilon^2}{R_{in}^2} F'(z, \alpha) r_0^3, \quad (40)$$

$$r \approx \left[1 + \alpha z - \frac{\epsilon^2}{R_{in}^2} F(z, \alpha) \right] r_0 - \alpha \frac{\epsilon^2}{R_{in}^2} \left[2\alpha F(z, \alpha) - \frac{1}{2} g(z, \alpha) \right] r_0^3, \quad (41)$$

where $\alpha = r'_0/r_0$ is the tangent of the slope of the initial correlated beam distribution in the (r'_0, r_0) phase-space, $F(z, \alpha)$ and $g(z, \alpha)$ are the following functions

$$F(z, \alpha) = \int_0^z ds \int_0^s ds' [f(s')]^2 (1 + \alpha s'), \quad (42)$$

$$g(z, \alpha) = \int_0^z ds [f(s)]^2 (1 + \alpha s)^2. \quad (43)$$

It can be shown that if the transverse phase space depends on the initial radial coordinates as: $r' = ar_0 + br_0^3$, $r = cr_0 + dr_0^3$, the emittance growth is obtained as

$$\Delta\epsilon_{n,x} = \frac{p_z}{2mc} \sqrt{\langle r^2 \rangle \langle r'^2 \rangle - \langle rr' \rangle^2} = \frac{p_0 R_{in}^4 |ad - bc|}{6\sqrt{2} 2mc}. \quad (44)$$

Here, we used the approximation of a uniformly charged beam $\langle r^m \rangle = 1/(\pi R_b^2) \int_0^{R_b} r^m r dr d\phi = 2R_b^m/(m+2)$. Inserting the coefficient a, b, c, d established from Eqs. (40) and (41) into Eq. (44), finally, we can obtain the slice rms emittance growth caused by magnetic-aberration effects

$$\Delta\epsilon_{n,x} = \frac{5}{24\sqrt{2}} \frac{p_0}{mc} \left[\frac{eB_{\max} R_{in}}{2p_0} \right]^2 R_{in}'^2 g(z, \alpha). \quad (45)$$

Here, R_{in}' is the rms beam divergence at the entrance to the solenoid lens.

For a solenoid much shorter than its focal length, the function $g(z, \alpha)$ simplifies to

$$g(z, \alpha) \approx \sqrt{\pi} \sigma_z (1 + \alpha z_s)^2, \quad (46)$$

where σ_z is the rms width of the on-axis magnetic field distribution, $f(z)$; z_s is the longitudinal coordinate at which the magnetic field attains its maximum value, $B(z_s) = B_{\max}$. The estimation for the SACLA gun parameters ($R_{in}' = 22$ mrad, $R_{in} = 3.6$ mm, $(\gamma_{in} - 1)mc^2 = 500$ keV, where $p_0 = mc\sqrt{\gamma_{in}^2 - 1}$) and the magnetic field profile shown in Fig. 15 (see the blue line, $B_{\max} = 0.1$ T, $\sigma_z = 0.0256$ m, $z_s = 0.125$ m) results in the emittance growth close to $\Delta\epsilon_{n,x} \approx 0.1$ μm .

Note that the emittance growth is sensitive to the initial beam divergence as $\Delta\epsilon_{n,x} \sim R_{in}'^2 [1 + (R_{in}'/R_{in})z_s]^2$, see Eq. (46). Therefore, to avoid emittance growth due to self-aberration effects in a magnetic lens, the initial beam divergence should be minimized.

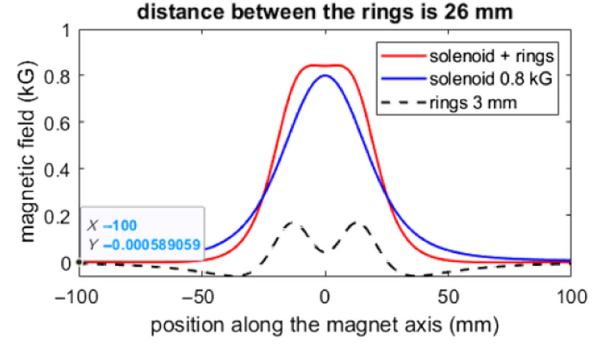


FIG. 15. Measured magnetic field distribution of the solenoid (blue) and of the rings (dashed black), and the computed sum of the two distributions (red).

To mitigate this emittance growth, a thin magnetic lens with the field concentrated inside the lens can be placed near the gun exit aperture. This configuration helps to suppress the radial beam divergence caused by the defocusing electric fields of the gun and of the space charge, see Eqs. (5) and (14).

I. Comparison of emittance growth mechanisms: Superposition of correlated emittance growth

So far, we have treated different mechanisms of emittance growth independently, which, strictly speaking, is not correct, as several nonlinear correlations add up in the beam transverse phase space. In general, the superposition of phase-space correlations leads to a much larger net emittance growth. One can draw an analogy with wave optics, where the superposition of waves might lead to strong variations in intensity via the interference effect. A similar effect takes place for the net correlated emittance growth.

The two dominant mechanisms of emittance growth are the magnetostatic aberration and the space-charge-driven aberration. It suffices to consider the superposition of the two only. Using the method of successive iterations, we obtain the total emittance growth in the form:

$$\Delta\epsilon_x^{(\text{tot})} = \sqrt{\left[1 + \frac{\sqrt{\pi} \sigma_s z_s}{\rho_b^2}\right]^2 (\Delta\epsilon_x^{(\text{charge})})^2 + \frac{8\sqrt{\bar{p}} + 2}{\bar{p} + 6} \left[1 + \frac{\sqrt{\pi} \sigma_s z_s}{\rho^2}\right] \Delta\epsilon_x^{(\text{charge})} \Delta\epsilon_x^{(\text{mag})} + (\Delta\epsilon_x^{(\text{mag})})^2}, \quad (47)$$

where $\Delta\epsilon_x^{(\text{charge})}$ is the emittance growth caused by the space-charge field alone, $\Delta\epsilon_x^{(\text{mag})}$ is the emittance growth due to the magnetostatic aberration alone. Here, we remind that $\rho_b = eB_{\max}/(2p_0)$ is the electron's orbit radius in the maximum of the solenoid magnetic field, $B_{\max}$; $\bar{p}$ is the average steepness parameter. There are two interesting observations from Eq. (47): (i) there is an “interference” term proportional to the product of the two emittances calculated separately, and (ii) the space-charge-driven

emittance growth is enhanced by the solenoid field (the factor in the square brackets).

The result Eq. (47) is well expected, but to our best knowledge, it is demonstrated for the first time in the literature in this simple and clear form. A quantitative comparison of different mechanisms of emittance growth is presented in Table I. Emittance growth is also computed numerically from ASTRA simulations whenever it is possible to single out a specific emittance growth mechanism.

TABLE I. Comparative summary of different emittance growth mechanisms. Content of the columns: (i) classification of the emittance growth mechanisms; (ii) the main origin of the emittance growth; (iii) references to key equations; (iv) results of analytical estimates for different emittance growth mechanisms; and (v) results of numerical simulations whenever the studied mechanism can be singled out.

Emittance growth mechanism	Physical origin	Key equations	Analytical estimate $\Delta\epsilon$ (nm)	Numerical result $\Delta\epsilon$ (nm)
Nonlinear space charge	Nonuniformity of the electron density at the beam edge	(27)	(28–43) ^a	≈ 40 (the two effects can not be disentangled in simulations)
Electrostatic aberration	Nonuniformity of the accelerating field	(35)	8	
Magnetostatic aberration	Nonuniformity of the magnetic field and coupling between the electron's transverse and longitudinal degrees of freedom	(45)	100	The effect can not be singled out
Cumulative correlated emittance growth	“Coherent” superposition of nonlinear correlations in the beam transverse phase space	(47)	(130–180) ^a	≈ 240

^aThe analytical estimate depends on the steepness parameter p , which is difficult to estimate accurately and we have taken it in the range from 10 to 20. Hence, the analytical result is also given in a range.

Overall, analytical estimates agree well with the simulated results. The main conclusion is that in order to suppress the net emittance growth, one needs to suppress the solenoid-driven enhancement of the space-charge emittance growth. This can be achieved by placing a thin solenoid lens right after the gun exit, as demonstrated in the next section. Care must be taken to ensure zero magnetic field on the cathode.

J. Hybrid magnetic lens

At present, the minimum distance between the first solenoid and the cathode is limited to 13 cm owing to the initial emittance growth caused by the Busch effect in a nonzero magnetic field. Over this distance, however, the beam divergence and size increase substantially, leading to pronounced aberrations in the solenoid and a considerable rise in slice emittance at high current density. To position a magnetic lens close to the gun exit without degrading the initial emittance, the solenoid field must be well confined longitudinally. In practice, a magnetic field distribution with a flattop and rapidly decaying edges can be realized through a combination of the solenoid and two permanent-magnet rings placed on either side. The short solenoid provides the main contribution to the field, characterized by a central peak with a moderate gradient, while the rings compensate for the edges of the distribution. It should be noted that the magnetic field of a standard solenoid tapers gradually at the ends and extends over a region several times the solenoid diameter.

Figure 15 shows the distributions of the longitudinal magnetic field of a short solenoid, of two ring magnets, and the numerically computed superposition of the two distributions. Note that the magnetic field distributions of the solenoid and the rings were actually measured. By placing two ring magnets symmetrically on either side of the

solenoid, one can adjust the magnetic field distribution and “squeeze in” the field longitudinally.

Figure 15 shows the combined magnetic field for only one configuration of the solenoid and the ring magnets. The distance from the center of the ring to the center of the solenoid is fixed at 26 mm. A more general result can be obtained by scanning the solenoid-to-ring distance. An example of such a scan is presented in the contour plot in Fig. 16. The value of the ordinate axis defines the solenoid-to-ring distance, whereas the resulting on-axis field distribution is plotted along the abscissa axis. The peak magnetic field of the ring corresponds to 22% of the solenoid peak field. For the ordinate value of 26 mm in the plot of Fig. 16, the field distribution along the abscissa axis corresponds to the red curve in Fig. 15. For reference, the magnetic field distribution with zero ring field is depicted in the bottom plot of Fig. 16 and simply corresponds to the blue curve in Fig. 15. By comparing the top and bottom plots in Fig. 16, one can conclude that the width of the combined solenoid and ring field is greatly reduced. In other words, the combined magnetic field is concentrated in the region of the solenoid.

IV. ROADMAP TO LOWER EMITTANCES

In order to reduce beam emittance, the combination of three strategies is needed: (i) decrease the cathode size in order to reduce thermal emittance; (ii) employ a higher accelerating gradient for counteracting the space-charge effect near the cathode; and (iii) place a hybrid magnetic lens very close to the cathode in order to cancel the radial kick at the gun exit, limit the growth of beam size and divergence, and mitigate aberration-driven emittance growth in the magnetic lens itself. An extensive campaign of numerical simulations with the code ASTRA has been conducted, and the summary is presented in Table II.

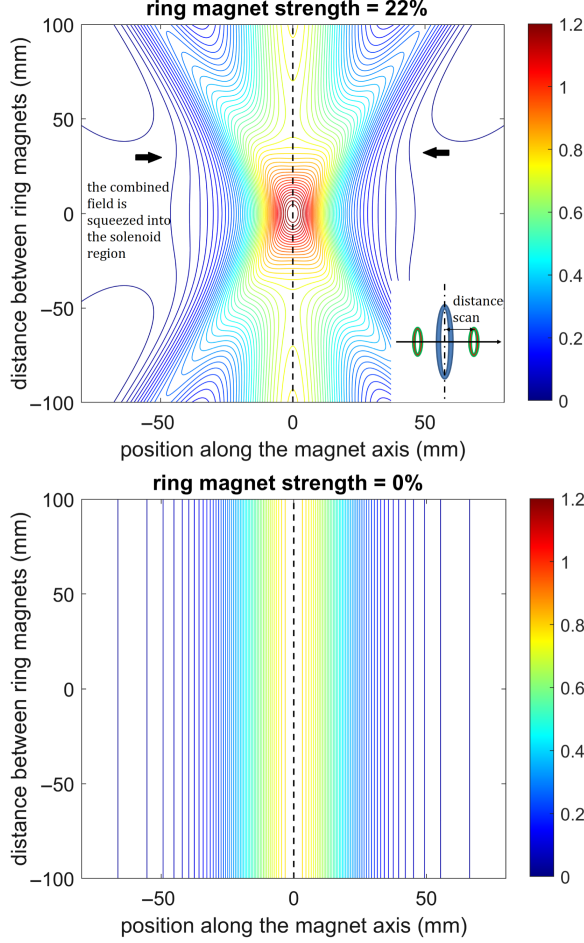


FIG. 16. Top plot: computed sum of the field distributions of the solenoid and two rings vs the distance between the rings, see the text for detailed explanations. Bottom plot: the field distribution of the solenoid for reference.

TABLE II. Table showing the projected rms beam emittance after the focusing solenoid for different initial cathode radii and two values of the accelerating gradient. The number in the curly brackets is the slice emittance. The bunch duration after the HV chopper is 500 ps.

Cathode radius (mm)	Thermal level ϵ_n (μm)	Accelerating gap 50 (mm) 12.6 MV/m	Accelerating gap 20 (mm) 28.5 MV/m
Standard solenoid			
1.5	0.4	0.6 (0.49 slice)	0.9 (0.8)
1.0	0.27	0.45 (0.36)	0.7 (0.6)
0.5	0.13	0.38 (0.33)	0.27 (0.22)
Hybrid magnetic lens			
1.5	0.4	0.52 (0.42)	0.62 (0.57)
1.0	0.27	0.355 (0.29)	0.37 (0.29)
0.5	0.13	0.3 (0.22)	0.24 (0.15)

In the nominal regime, the cathode radius is 1.5 mm, thermal emittance is 0.4 μm , and the accelerating field on the cathode is 12.6 MV/m. In the gun configuration with the standard solenoid, there is a noticeable growth of slice emittance to the level 0.49 μm . Assuming the emittance growth not to be correlated to the thermal emittance, the emittance growth alone can be estimated as $\sqrt{(0.49^2 - 0.4^2)} = 0.28 \mu\text{m}$. Employing a higher accelerating field at the expense of a shorter accelerating gap makes the situation even worse: the radial defocusing kick from the gun field (5) enhances aberrations in the focusing solenoid.

Reducing the cathode radius, while keeping the beam current constant at the level of 1 A, allows one to reduce the slice emittance thanks to a smaller thermal emittance. For the cathode radius 0.5 mm, the resulting slice emittance is dominated by the emittance growth in the solenoid. We want to stress that the spherical aberration of the solenoid magnetic field gives a small contribution to the emittance growth. The main contribution comes from a $r - p_r$ correlation in the beam prior to the lens, and the electrons' momentum exchange in the solenoid.

Using the hybrid magnetic lens described above allows suppressing emittance growth, and in combination with a smaller cathode and a higher accelerating field, much smaller slice emittance can be achieved. Experimental studies are underway and will be reported elsewhere. In fact, we measured the transverse phase-space distribution of electrons with the standard solenoid and calculated beam brightness as a fraction of particles following the procedure from [35]. Thus, confirming the high brightness of the beam core.

V. SUMMARY

This paper (part 2 of the design study) analyses the gun technology and beam dynamics of the SACLA pulsed dc thermionic injector with the aim of reaching submicrometer emittance at ampere-class peak current. We have developed an analytical framework and benchmarked it against ASTRA simulations. Four principal sources of emittance growth in the SACLA gun are identified: image-charge effect, non-linear space-charge forces arising from beam edge diffusion, electrostatic aberrations of the accelerating field, and magnetostatic aberrations in the solenoid due to chromatic focusing and longitudinal-transverse momentum exchange. The analysis yields compact scaling laws and a closed-form expression for the superposition of correlated growth mechanisms, highlighting an “interference” term that amplifies the net emittance when space-charge and solenoid effects coexist.

Guided by these insights, we propose a compact hybrid magnetic lens—two permanent-magnet rings symmetrically placed around a short solenoid—to confine the axial field longitudinally. This enables placement of strong focusing immediately after the gun exit while maintaining

negligible field on the cathode, thereby suppressing beam divergence growth and mitigating aberrations. Magnetic-field measurements and numerical synthesis of the measured field distributions confirm that the combined profile can be “squeezed” into the solenoid region for enhanced performance.

For representative SACLA parameters (500 kV, 1 A), the model explains the observed increase from the thermal emittance ($0.4 \mu\text{m}$). The optimization strategy, which makes use of a smaller cathode, a higher effective accelerating gradient near the cathode, and the hybrid lens, allowed us to halve the slice emittance while retaining peak current. The roadmap indicates the feasibility of $0.15 \mu\text{m}$ slice emittance for 1-A, 500-pC 0.5-ns bunches and provides informed guidance for the design of low-gradient cw VHF and SRF guns. Experimental studies are underway and will be published elsewhere.

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DATA AVAILABILITY

The data that support the findings of this article are openly available [36].

APPENDIX: COMPARISON OF ASTRA SIMULATION RESULTS WITH BEAM MEASUREMENT RESULTS AT THE SACLA GUN TEST STAND

ASTRA is a well-benchmarked simulation code for studying beam dynamics in rf photocathode guns and for particle tracking through accelerator components in the space-charge-dominated regime. The SACLA gun is, however, different as it uses a thermionic cathode-based pulsed dc gun and a HV pulser to extract a 1-ns bunch from a μs -long electron pulse produced by the gun. Therefore, in this section, we compare measured and computed beam parameters for several scenarios of SACLA gun operation. We shall see that the agreement is excellent.

First of all, we note that the ASTRA code does not include the option of thermionic emission, and therefore, we used the built-in generator of a photoemission-produced bunch distribution. The distribution obeys the Fermi-Dirac statistics and has the emittance equivalent to the case of thermionic emission. The real bunch from the SACLA gun is a few microseconds long, and a 1-ns bunch is extracted by means of an HV chopper. This has been modeled by producing a several ns-long bunch in ASTRA and then cutting out a 1-ns core in MATLAB. Another

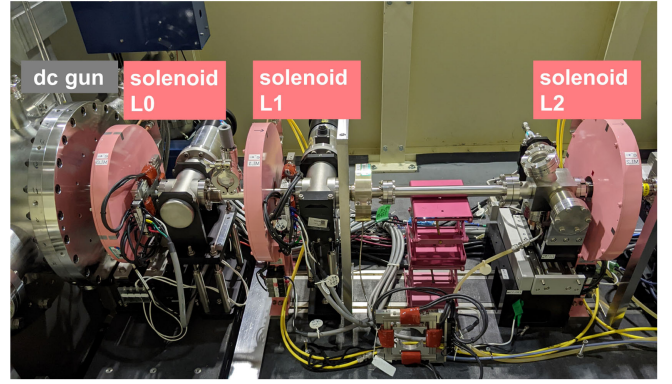


FIG. 17. Photo of the test stand for characterizing the beam produced by the SACLA gun. Only the accelerating region (large cylinder on the left) and the anode plate of the 500 kV gun tank are shown in the photo. The beam pipe and solenoids can clearly be seen.

approach is to directly specify a 1-ns bunch distribution in the ASTRA generator. No differences between the two approaches in terms of the beam parameters have been observed. Hence, we model the SACLA bunch as a photoemission ns distribution.

Figure 17 shows the photo of the experimental test stand at the gun test facility of SACLA. The test stand comprises a gun, three solenoids, and a slits-based emittance meter. The HV chopper located between the second solenoid and the third solenoid was removed in these experiments. The gun was operated with a current of 2.1 A, and the beam size was scanned using the second solenoid located at 0.435 m from the cathode. We show two cases: mild focusing and strong focusing.

The measurement and simulation results are presented in Fig. 18. The solenoid current was changed from 10 to 15 A. In the region of the beam waist, the agreement is excellent. Outside the beam waist, the measurement capability is

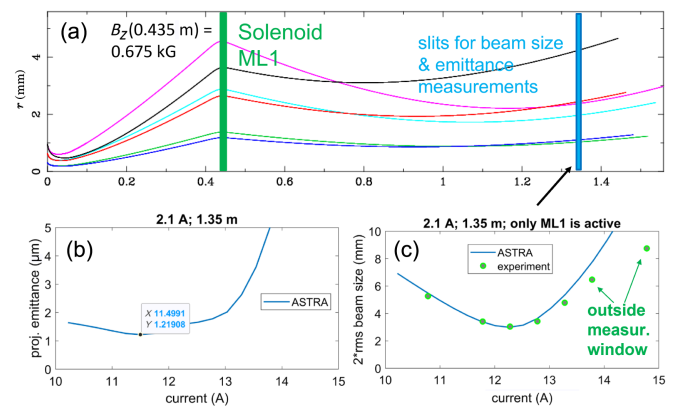


FIG. 18. Plot (a): electron trajectories simulated with ASTRA. Plot (b): computed projected emittance as a function of the solenoid current. Plot (c): computed (blue) and measured (green) beam size vs solenoid current.

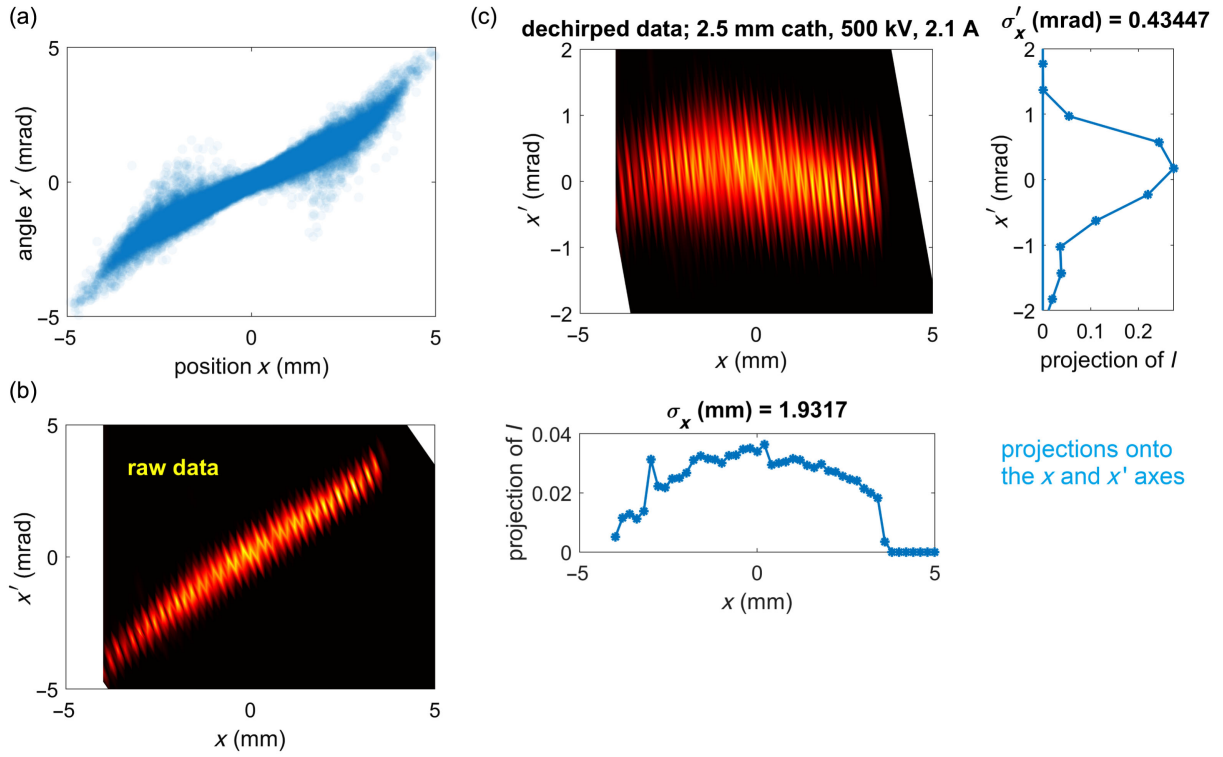


FIG. 19. Plot (a): transverse phase-space distribution computed with ASTRA. The simulation configuration corresponds to the scenario shown in Fig. 18(a). Plot (b): measured transverse phase space. The vertical strip-line structure is caused by the image processing using the limited data points. A rotation is applied to the raw data presented in plot (c). The projections of the “dechirped” measured distribution are also depicted to the right and below the plot (c).

limited by the maximum stroke of the slits; therefore, large beam sizes cannot be correctly measured.

Figure 19 presents the results of the measured and computed transverse phase-space distribution. In the measurement configuration, only the second solenoid is activated and the field strength is set to 0.675 kG.

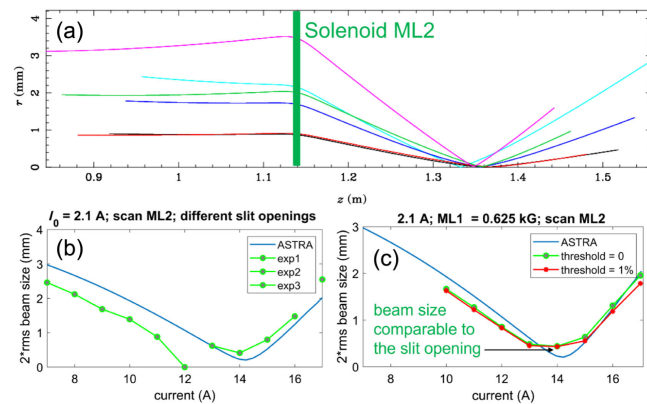


FIG. 20. Plot (a): electron trajectories simulated with ASTRA for a second measurement configuration. Plot (b): computed (blue) and measured (green) beam sizes vs solenoid current for three different widths of the measurement slits. Plot (c): computed and measured beam sizes vs solenoid current with and without 1% noise cut.

Plot (c) of Fig. 19 shows the raw measured data, which were processed to remove the linear correlation between x and x' . The vertical strip-line structure is caused by the image processing using the limited data points. The resulting distribution is depicted in plot (b) and is in a good agreement with the distribution computed with ASTRA. The latter was also modified to remove the linear correlation.

Figure 20 shows the results of a beam size scan for a second measurement configuration. The magnetic field of the second solenoid (located at 0.435 m) was set to 0.675 kG, and the beam size was scanned using the third solenoid located at 1.14 m from the cathode. The measurement and simulation results are presented in Fig. 20. The solenoid current was changed from 7 to 17 A. Measurements with three different opening width of the measurement slits were performed and the results are presented in plot (b). The standard measurement slit width of 50 μm yields results that are in a good agreement with those predicted by the simulations with ASTRA. The measurement was repeated again with the standard slit width to ensure the reproducibility, see plot (c). In post-processing of the measured data, a 1% noise threshold cut was applied to see whether the result is sensitive. A small visual difference is observed because the noise cut affects the beam halo.

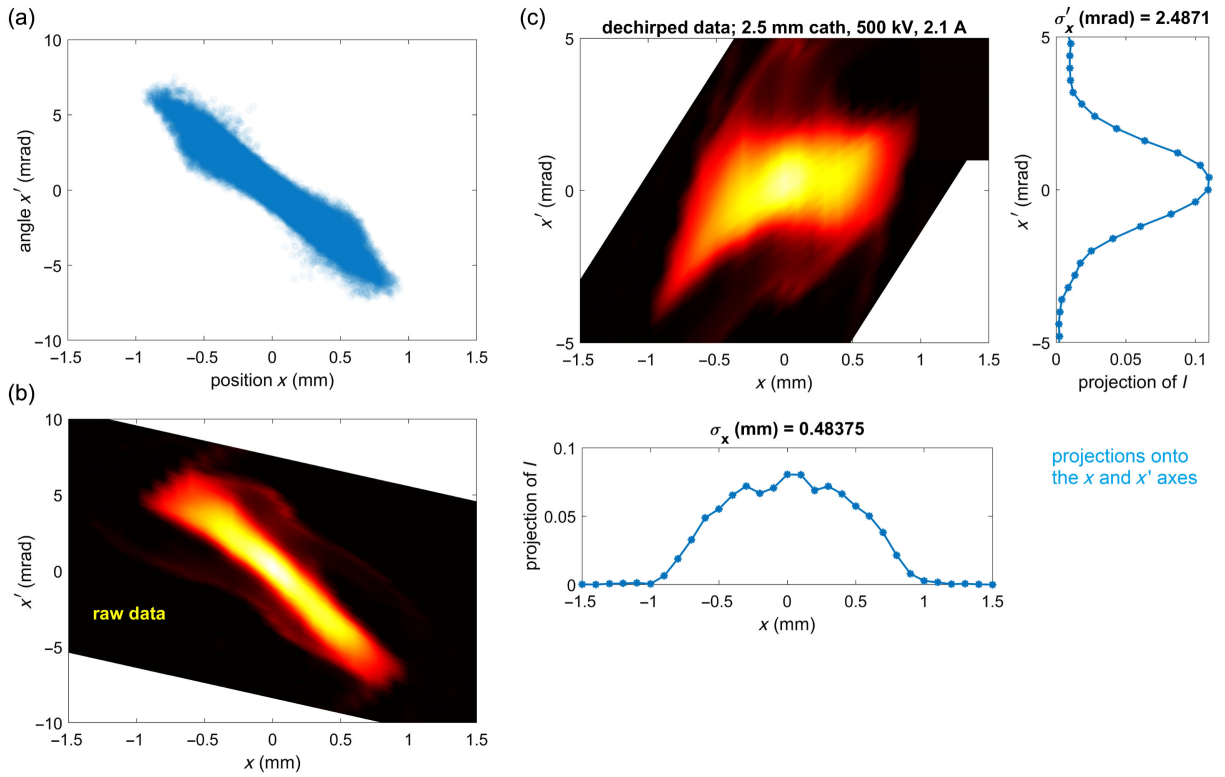


FIG. 21. Plot (a): transverse phase-space distribution computed with ASTRA. The simulation configuration corresponds to the scenario shown in Fig. 20(a). Plot (b): measured transverse phase space. A rotation is applied to the raw data presented in plot (c). The projections of the “dechirped” measured distribution are also depicted to the right and below the plot (c).

Figure 21 presents the results of the measured and computed transverse phase-space distribution for the second measurement configuration shown in plot (a) of Fig. 20. The raw and processed measured data, with the removed linear correlation between x and x' , are presented in plots (c) and (b), respectively. The distribution computed with ASTRA is depicted in plot (a) and is in a good agreement with the measured distribution. A large halo is observed, which may be the result of parasitic emission from the Wehnelt plate. A detailed discussion of the beam halo is outside of the scope of the present publication, and we just mention that the cathode was slightly back-shifted with respect to the Wehnelt plate and a ring of another material is used in between the cathode and cathode holder. This ring might also be the cause of the parasitic electron emission. In the standard configuration, the cathode surface is aligned with Wehnelt plate; therefore, further investigation of the beam halo is unnecessary.

- [1] J. S. Fraser, R. L. Sheffield, E. R. Gray, and G. W. Rodenz, High-brightness photoemitter injector for electron accelerators, *IEEE Trans. Nucl. Sci.* **32**, 1791 (1985).
- [2] M. Reiser, *Theory and Design of Charged Particle Beams* (Wiley–VCH Verlag, Darmstadt, 2008).

- [3] A. W. Chao, M. Tigner, H. Weise, and F. Zimmermann, *Handbook of Accelerator Physics and Engineering* (World Scientific, Singapore, 2023).
- [4] T. Shintake, H. Matsumoto, T. Ishikawa, and H. Kitamura, SPring-8 compact SASE source (SCSS), in *Optics for Fourth-Generation X-Ray Sources*, edited by R. O. Tatchyn, A. K. Freund, and T. Matsushita (SPIE, San Diego, CA, 2001), Vol. 4500, pp. 12–23.
- [5] K. Togawa, T. Shintake, T. Inagaki, K. Onoe, T. Tanaka, H. Baba, and H. Matsumoto, CeB₆ electron gun for low-emittance injector, *Phys. Rev. ST Accel. Beams* **10**, 020703 (2007).
- [6] T. Shintake *et al.*, A compact free-electron laser for generating coherent radiation in the extreme ultraviolet region, *Nat. Photonics* **2**, 555 (2008).
- [7] K. Togawa, Slice emittance measurements using a slit-grid system and a fast wall-current monitor, *Rev. Sci. Instrum.* **95**, 043304 (2024).
- [8] H. Tanaka *et al.*, Green upgrading of spring-8 to produce stable, ultrabright hard x-ray beams, *J. Synchrotron Radiat.* **31**, 1420 (2024).
- [9] I. V. Bazarov, A. Kim, M. N. Lakshmanan, and J. M. Maxson, Comparison of dc and superconducting rf photoemission guns for high brightness high average current beam production, *Phys. Rev. ST Accel. Beams* **14**, 072001 (2011).
- [10] C. Gulliford, A. Bartnik, I. Bazarov, L. Cultrera, J. Dobbins, B. Dunham, F. Gonzalez, S. Karkare, H. Lee, H. Li, Y. Li, X. Liu, J. Maxson, C. Nguyen, K. Smolenski,

- and Z. Zhao, Demonstration of low emittance in the cornell energy recovery linac injector prototype, *Phys. Rev. ST Accel. Beams* **16**, 073401 (2013).
- [11] J. Maxson, I. Bazarov, B. Dunham, J. Dobbins, X. Liu, and K. Smolenski, Design, conditioning, and performance of a high voltage, high brightness dc photoelectron gun with variable gap, *Rev. Sci. Instrum.* **85**, 093306 (2014).
- [12] H. Chen, L.-M. Zheng, B. Gao, Z.-Z. Li, Y.-C. Du, R.-K. Li, W.-H. Huang, C.-X. Tang, D. Gu, Z. Qi, M. Zhang, H.-X. Deng, Q. Gu, and D. Wang, Beam dynamics optimization of very-high-frequency gun photoinjector, *Nucl. Sci. Tech.* **33**, 116 (2022).
- [13] F. Sannibale, D. Filippetto, and C.F. Papadopoulos, Schemes and challenges for electron injectors operating in high repetition rate x-ray FELs, *J. Mod. Opt.* **58**, 1419 (2011).
- [14] R.P. Wells, W. Ghiorso, J. Staples, T.M. Huang, F. Sannibale, and T.D. Kramasz, Mechanical design and fabrication of the VHF-gun, the Berkeley normal-conducting continuous-wave high-brightness electron source, *Rev. Sci. Instrum.* **87**, 023302 (2016).
- [15] F. Zhou, C. Adolphsen, D. Dowell, and R. Xiang, Overview of cw electron guns and LCLC-II rf gun performance, *Front. Phys.* **11**, 1150809 (2023).
- [16] R. Xiang, A. Arnold, and J. W. Lewellen, Superconducting radio frequency photoinjectors for CW-XFEL, *Front. Phys.* **11**, 1166179 (2023).
- [17] F. Sannibale, High-brightness electron injectors for high-duty cycle x-ray free electron lasers, *Front. Phys.* **11**, 1187346 (2023).
- [18] M. Yoshitake, Measurement of work function, in *Work Function and Band Alignment of Electrode Materials* (Springer, Tokyo, Japan, 2020), pp. 71–95.
- [19] T. Magome, K. Togawa, and H. Tanaka, Precise measurement of the work function of a hot CeB₆ thermionic cathode through photoelectron yield spectroscopy using a tunable pulsed laser, *J. Appl. Phys.* **133**, 165107 (2023).
- [20] T. Ohkochi, T. Muro, E. Ikenaga, K. Togawa, A. Yasui, M. Kotsugi, M. Oura, and H. Tanaka, Multilateral surface analysis of the CeB₆ electron-gun cathode used at SACLA XFEL, *J. Synchrotron Radiat.* **28**, 1729 (2021).
- [21] K. Togawa, A. Higashiya, and T. Shintake, Graphite heater optimized for a low-emittance CeB₆ cathode, in *Proceedings of the 2007 IEEE Particle Accelerator Conference (PAC)* (IEEE, New York, 2007), Vol. 10, pp. 1013–1015.
- [22] K. Togawa, Lifetime improvement of the CeB₆ thermionic cathode at the x-ray free-electron laser facility SACLA by avoiding backward-accelerated electrons, in *Proceedings of the International Particle Accelerator Conference (IPAC'23)*, Venice, Italy (JACoW, Geneva, Switzerland, 2023), TUPA021, 10.18429/JACoW-IPAC2023.
- [23] K. Togawa, H. Maesaka, R. Kobana, and H. Tanaka, Frequency-segmented power amplification using multi-band radio frequency amplifiers to produce a high-voltage pulse, *Rev. Sci. Instrum.* **93**, 073304 (2022).
- [24] K. Floettmann, ASTRA—A Space Charge Tracking Algorithm (DESY, 2017), <http://www.desy.de/~mpyflo/>.
- [25] V. Goryashko, A. Opanasenko, and K. Togawa, Self-aberration in high-brightness uniformly charged particle beams, *Results Phys.* **68**, 108096 (2025).
- [26] D. Filippetto, P. Musumeci, M. Zolotarev, and G. Stupakov, Maximum current density and beam brightness achievable by laser-driven electron sources, *Phys. Rev. ST Accel. Beams* **17**, 024201 (2014).
- [27] G. Shamuilov, A. Mak, K. Pepitone, and V. Goryashko, Child-Langmuir law for photoinjectors, *Appl. Phys. Lett.* **113**, 204103 (2018).
- [28] C. D. Child, Discharge from hot CaO, *Phys. Rev.* **32**, 492 (1911).
- [29] I. Langmuir, The effect of space charge and initial velocities on the potential distribution and thermionic current between parallel plane electrodes, *Phys. Rev.* **21**, 419 (1923).
- [30] Y. Y. Lau, Simple theory for the two-dimensional child-Langmuir law, *Phys. Rev. Lett.* **87**, 278301 (2001).
- [31] G. Shamuilov, A. Opanasenko, K. Pepitone, Z. Tibai, and V. Goryashko, Emittance self-compensation in blow-out mode, *New J. Phys.* **24**, 123008 (2022).
- [32] J. D. Lawson, *The Physics of Charged-Particle Beams* (Oxford University Press, Oxford, 1988).
- [33] T. Gorlov, Paraxial optics of charged particles in solenoids, *Phys. Rev. Accel. Beams* **23**, 034001 (2020).
- [34] P. Denham and P. Musumeci, Space-charge aberrations in single-shot time-resolved transmission electron microscopy, *Phys. Rev. Appl.* **15**, 024050 (2021).
- [35] I. V. Bazarov, B. M. Dunham, and C. K. Sinclair, Maximum achievable beam brightness from photoinjectors, *Phys. Rev. Lett.* **102**, 104801 (2009).
- [36] V. Goryashko, Measurement results of the beam size scan and phase space at the SACLA gun test facility, Zenodo (2025), 10.5281/zenodo.17349925.