

Mean transverse energy of electrons generated from mixed thermionic, multiphoton, and field emissions

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The brightness of an electron emitter is fundamentally limited by the intrinsic emittance of the electrons born at a cathode. Different electron-emission mechanisms from a cathode give different values of mean transverse energy for calculating the intrinsic emittance of emitted electrons. This study reports a theory for calculating the mean transverse energy of electrons generated at a cathode via mixed thermionic, multiphoton, and field emissions. The theory successfully reveals the details of thermionically assisted multiphoton emission from a copper cathode excited by an intense infrared laser. An extension of the theory to calculate the intrinsic emittance and brightness of an electron emitter is also presented.

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I. INTRODUCTION

The mean transverse energy (MTE) is a fundamental parameter governing the intrinsic emittance and brightness of electron beams emitted from photocathodes. It quantifies the average kinetic energy associated with the transverse motion of photoemitted electrons and sets a lower bound on beam emittance, independent of external focusing optics. As such, MTE plays a critical role in determining the performance of high-brightness electron sources used in accelerator-based applications.

Recent advances in photocathode research have significantly enhanced our understanding of photoemission physics and its implications for beam quality. Kachwala *et al.* demonstrated that cesium antimonide (Cs₃Sb) photocathodes can approach the thermal limit of MTE under near-threshold excitation, establishing a benchmark for intrinsic emittance performance [1]. Investigations into ultrananocrystalline diamond (UNCD) photocathodes have revealed favorable quantum efficiency and low-MTE characteristics, underscoring the potential of engineered carbon-based surfaces for robust photoemission [2]. Parallel efforts in plasmonic enhancement have enabled nanoscale ultrafast electron sources, where interference-driven field localization facilitates subwavelength emission control and brightness scaling [3,4]. These developments highlight the growing importance of material-specific photoemission

studies in tailoring cathode properties for next-generation accelerator technologies. Earlier work on GaAs-based negative electron affinity (NEA) photocathodes also provides a valuable reference for benchmarking MTE and modeling emission across diverse material systems [5].

Cryogenic and near-threshold photoemission techniques have recently enabled the generation of ultracold electron beams with exceptionally low MTE. Cryocooled single-crystal Cu(100) photocathodes have achieved a record-low MTE of 5 meV, with an energy spread below 11.5 meV, representing the coldest electron source reported to date [6]. Multialkali photocathodes have demonstrated subthermal MTE values near 9 meV at room temperature, attributed to electron refraction effects at the surface [7]. Cs₃Sb photocathodes have also been shown to reach the thermal limit of MTE (~25 meV) near the photoemission threshold, challenging prior assumptions regarding surface roughness limitations [1]. These results underscore the potential of ultracold photoemission for ultrafast and high-resolution applications such as ultrafast electron diffraction (UED) [8].

Different electron emission mechanisms yield distinct MTE scaling behaviors. In thermionic emission, the MTE asymptotically approaches the classical thermal limit, $k_B T$, where k_B is the Boltzmann constant and T is the electron temperature [9–14]. In contrast, for photoemission above the work-function threshold, the MTE depends on the excess photon energy beyond the effective emission barrier Φ_{eff} [15–19] and is given by

$$\text{MTE} = \frac{h\nu - \Phi_{\text{eff}}}{3}, \quad (1)$$

where $h\nu$ is the incident photon energy and Φ_{eff} includes Schottky barrier reduction [12–14]. For finite-temperature

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emission from single-crystal metals [18] and disordered surfaces [19], assuming a constant density of states, the MTE can be expressed using polylogarithmic functions:

$$\text{MTE} = k_B T \frac{Li_3 \left[-\exp\left(\frac{h\nu - \Phi_{\text{eff}}}{k_B T}\right) \right]}{Li_2 \left[-\exp\left(\frac{h\nu - \Phi_{\text{eff}}}{k_B T}\right) \right]}, \quad (2)$$

where $Li_s(z) \equiv \sum_{k=1}^{\infty} \frac{z^k}{k^s}$ is the polylogarithm function. Equation (2) reduces to Eq. (1) as $h\nu > \Phi_{\text{eff}}$ under the zero-temperature approximation.

Recent studies have shown that under ultrashort infrared laser excitation, nonequilibrium carrier dynamics and multiphoton processes can lead to significant deviations from idealized MTE expressions [20–22]. These effects necessitate the inclusion of both thermal and nonthermal components to accurately model MTE in photocathodes. A hybrid framework that accounts for multiple emission mechanisms is particularly relevant for interpreting MTE in metallic [20] and semiconductor [23] photocathodes irradiated by intense infrared lasers.

In this study, we first adopt the theoretical models developed by Fowler, DuBridge [24–26], and Bechtel [27,28] to calculate the temperature-dependent photocurrent from photocathodes under single- and multiphoton excitation. We then develop a generalized theory to calculate the MTE of electrons emitted via thermionically assisted multiphoton processes, incorporating Schottky barrier reduction to account for field emission effects. Finally, we extend the MTE framework to evaluate intrinsic emittance and brightness for photocathodes involving multiple emission pathways.

II. PHOTOCURRENT

The transition of electrons from a bound state to a free state above the vacuum level is facilitated by three primary mechanisms: thermionic emission, photoemission, and field emission. The Fowler-DuBridge-Bechtel theory describes the total emission current of a photocathode as the summation of individual partial currents, J_n , where n denotes the number of photons involved in the emission process. This relationship is mathematically represented as

$$J = \sum_{n=0}^{\infty} J_n. \quad (3)$$

with

$$J_n = a_n A_0 \left(\frac{e}{h\nu} \right)^n (1 - R)^n I^n T^2 F[\mu], \quad (4)$$

where a_n is an empirical coefficient, R is the reflectance of the cathode surface, I is the incident laser intensity, T is the cathode-surface temperature, $A_0 = \frac{4\pi e m_e k_B}{h^3}$ is the

Richardson constant, m_e is the electron mass, e is the electron charge, h is the Planck constant, ν is the frequency of the incident laser, and $F[\mu]$ is the so-called Fowler function. Specifically,

$$F[\mu] = \begin{cases} \sum_{k=1}^{\infty} (-1)^{k-1} \frac{e^{k\mu}}{k^2}, & \mu \leq 0 \\ \frac{\pi^2}{6} + \frac{\mu^2}{2} - \sum_{k=1}^{\infty} (-1)^{k-1} \frac{e^{-k\mu}}{k^2}, & \mu \geq 0 \end{cases} \quad (5)$$

where $\mu = \frac{h\nu - \Phi_{\text{eff}}}{k_B T}$ with $\Phi_{\text{eff}} = \Phi - \Delta\Phi_{\text{Schottky}}$ being the effective work function, Φ is the work function of the cathode material, and $\Delta\Phi_{\text{Schottky}} = e\sqrt{\frac{eF_a}{4\pi\epsilon_0}}$ is the Schottky barrier reduction by an image-charge and applied field F_a , and ϵ_0 is the vacuum permittivity.

In the high temperature limit, where the thermal energy of an electron $\sim k_B T$ significantly exceeds the energy difference between the n -photon excitation and the effective work function or $nh\nu - \Phi_{\text{eff}}$, the Fowler function approaches the value $F[\mu] \rightarrow \frac{\pi^2}{12}$, allowing Eq. (4) to be approximated as

$$J_n \rightarrow a_n A_0 \left(\frac{e}{h\nu} \right)^n (1 - R)^n I^n T^2 \frac{\pi^2}{12}. \quad (6)$$

For the energy difference greater than thermal energy, we can categorize the electron-emission process into two cases depending on whether the photon energy can surpass the effective work function or not. The two cases give different signs for the value of μ .

If $\mu > 0$, the contribution from J_n can be approximated as

$$J_n \rightarrow a_n A_0 \left(\frac{e}{h\nu} \right)^n (1 - R)^n I^n \frac{1}{2} \left(\frac{nh\nu - \Phi_{\text{eff}}}{k_B T} \right)^2. \quad (7)$$

This indicates that the electron emission is independent of temperature but is more relevant to n -photon excitation I^n with the n -photon energy exceeding the effective work function $(nh\nu - \Phi_{\text{eff}})^2$.

If $\mu < 0$, the contribution from J_n can be approximated as

$$J_n \rightarrow a_n A_0 \left(\frac{e}{h\nu} \right)^n (1 - R)^n I^n T^2 \exp\left(\frac{nh\nu - \Phi_{\text{eff}}}{k_B T}\right). \quad (8)$$

In this case, both temperature and n -photon excitation I^n play important roles in the emission process through the term $T^2 \exp(\frac{nh\nu - \Phi_{\text{eff}}}{k_B T})$. The electron current exhibits exponential growth as the n -photon excitation energy approaches the effective work function. This case is in the regime of thermionically assisted photoemission. Furthermore, if we consider $n = 0$ (pure thermal emission), the formula reduces to the Richardson-Dushman equation [11–13].

III. INTRINSIC THERMAL EMITTANCE

To calculate the brightness of an electron emitter, it is necessary to calculate the intrinsic emittance of an emitted electron current. A high-quality beam is characterized by a low emittance value. The geometric emittance of an electron beam (ϵ_{rms}) is defined as $\epsilon_{\text{rms}} = \sqrt{\langle x^2 \rangle \langle x'^2 \rangle - \langle x \cdot x' \rangle^2}$, where x and x' are the transverse position and the emitting angle of an electron, respectively, and $\langle A \rangle$ denotes averaging for the quantity A over many electrons. In the linear-emission regime with negligible space-charge fields, the angle-position correlation $\langle x \cdot x' \rangle$ can be ignored for electrons emitting from a planar cathode. The expression of the geometric emittance reduces to $\epsilon_{\text{rms}} \sim \sigma_x \sigma_{x'}$ where σ_x and $\sigma_{x'}$ are the standard deviations of the electron's position and angle, respectively. In a high-energy accelerator, the normalized emittance is defined as $\epsilon_N = \beta_z \gamma \epsilon_{\text{rms}}$, where β_z is the electron's longitudinal speed normalized to the vacuum speed of light c and γ is the Lorentz factor. The normalized emittance is an invariant quantity under particle acceleration. At the cathode with $\gamma \sim 1$, the normalized emittance is given by

$$\epsilon_N \sim \sigma_x \frac{\sqrt{\langle v_x^2 \rangle}}{c} = \sigma_x \sqrt{\frac{\text{MTE}}{m_e c^2}}. \quad (9)$$

To calculate the emittance, it is necessary to first determine the MTE of the electrons.

As illustrated in Figs. 1(a) and 1(b), the total energy of an electron is decomposed into two orthogonal components: the longitudinal energy E_z , defined along the normal surface, and the transverse energy E_T , defined parallel to the emission surface. With the Fermi energy E_F as the reference, all electron states can be represented along the line $E = E_z + E_T$. The vacuum level E_V is shown as a horizontal line intersecting the E_z axis and is reduced to an effective vacuum level $E_{V,\text{eff}} = E_V - \Delta\Phi_{\text{Schottky}}$ due to Schottky barrier lowering. Electrons follow Fermi-Dirac statistics, and the energy spread around the Fermi level is on the order of $k_B T$. Upon absorption of n photons, the entire electron distribution is shifted upward by $nh\nu$. Electrons with total energy exceeding $E_{V,\text{eff}}$ contribute to photoemission. The mean transverse energy of these electrons will be derived in the following sections. Figure 1(a) depicts the energy diagram for the case of zero temperature and above-threshold excitation ($nh\nu > \Phi_{\text{eff}}$), while Fig. 1(b) illustrates the finite-temperature case with subthreshold excitation ($nh\nu < \Phi_{\text{eff}}$).

In the zero-temperature limit, the excited electron distribution $n_{\text{exc}}(E_z, E_T)$ can be defined for n -photon excitation as

$$n_{\text{exc}}(E_z, E_T) = \begin{cases} C_{\text{exc}}, & E_z + E_T \leq nh\nu \\ 0, & E_z + E_T > nh\nu \end{cases}, \quad (10)$$

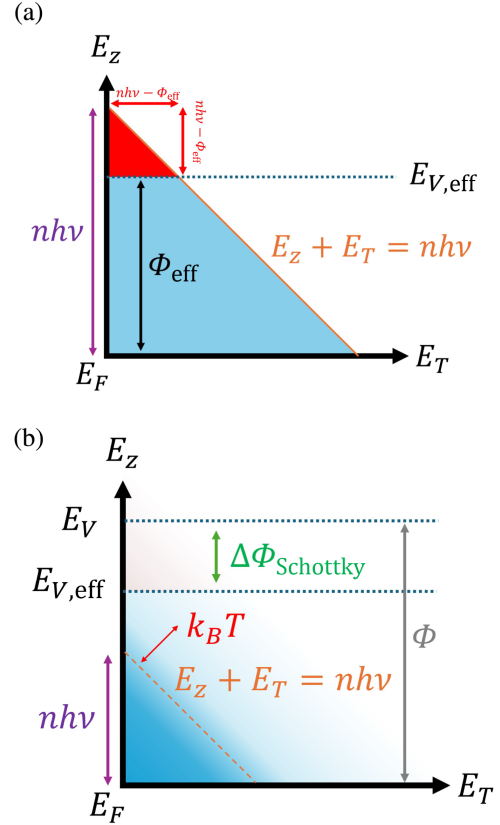


FIG. 1. The energy diagram to facilitate the derivation of our MTE theory including thermionic, multiphoton, and field emissions. E_F , Fermi energy; E_z , longitudinal energy normal to the emission surface; E_T , transverse energy parallel to the emission surface; E_V , vacuum level; $E_{V,\text{eff}}$, effective vacuum level; $nh\nu$, n -photon excitation; $\Delta\Phi_{\text{Schottky}}$, Schottky barrier reduction. (a) Above transition with zero temperature. (b) Below transition with finite temperature.

where C_{exc} is a constant. This distribution corresponds to the triangular region shown in Fig. 1(a), with the red portion indicating electrons that possess sufficient longitudinal energy to overcome the effective work function Φ_{eff} .

The number of emitted electrons is proportional to the area of this red triangle:

$$\int_{E_z > \Phi_{\text{eff}}} n_{\text{exc}}(E_z, E_T) dE_z dE_T = \frac{C_{\text{exc}}}{2} (nh\nu - \Phi_{\text{eff}})^2 \quad (11)$$

which is consistent with prior studies showing that the quantum efficiency and photocurrent scale as $(h\nu - \Phi_{\text{eff}})^2$ [15,16,24–28].

The mean transverse energy (MTE) is then calculated as the average transverse energy of the electrons within this emission region:

$$\text{MTE} = \frac{\int_{E_z > \Phi_{\text{eff}}} E_T n_{\text{exc}}(E_z, E_T) dE_z dE_T}{\int_{E_z > \Phi_{\text{eff}}} n_{\text{exc}}(E_z, E_T) dE_z dE_T} = \frac{nh\nu - \Phi_{\text{eff}}}{3}. \quad (12)$$

When the multiphoton energy is insufficient to surpass the effective work function ($n\hbar\nu < \Phi_{\text{eff}}$), the derivation is more elaborating. In a noninteracting electron gas adhering to the Fermi-Dirac statistics, the number of electrons with given velocity components and the density of states can be expressed as

$$n_e(v_\rho, \theta, v_z) dv_\rho d\theta dv_z = 2 \left(\frac{m_e}{h} \right)^3 \frac{v_\rho dv_\rho d\theta dv_z}{\exp\left(\frac{\frac{1}{2}m_e(v_\rho^2 + v_z^2) - E_F}{k_B T}\right) + 1}, \quad (13)$$

where v_ρ is the radial velocity of the electron. By integrating the transverse energy over the population of electrons with sufficient longitudinal energy to escape into vacuum, the MTE is given by

$$\text{MTE} = \frac{\int_{\frac{1}{2}m_e v_z^2 + n\hbar\nu = E_{V,\text{eff}}}^{\infty} \int_0^{\infty} \int_0^{2\pi} \frac{1}{2} m_e v_\rho^2 n_e(v_\rho, \theta, v_z) dv_\rho d\theta dv_z}{\int_{\frac{1}{2}m_e v_z^2 + n\hbar\nu = E_{V,\text{eff}}}^{\infty} \int_0^{\infty} \int_0^{2\pi} n_e(v_\rho, \theta, v_z) dv_\rho d\theta dv_z}. \quad (14)$$

In the near-threshold regime, where $E_z + n\hbar\nu \approx E_{V,\text{eff}}$, this expression can be approximated as

$$\text{MTE} \approx k_B T \frac{G[\mu]}{F[\mu]}, \quad (15)$$

with $\mu = \frac{n\hbar\nu - \Phi_{\text{eff}}}{k_B T}$. Here, $F[\mu]$ corresponds to the Fowler function defined in Eq. (5), and the numerator function $G[\mu]$ is given by

$$G[\mu] = \begin{cases} -\sum_{k=1}^{\infty} (-1)^{k-1} \frac{e^{k\mu}}{k^3}, & \mu \leq 0 \\ \frac{1}{6}\mu(\mu^2 + \pi^2) - \sum_{k=1}^{\infty} (-1)^{k-1} \frac{e^{-k\mu}}{k^3}, & \mu \geq 0 \end{cases}. \quad (16)$$

In Eq. (16), the expressions of $G[\mu]$ for the above and below transitions can be demonstrated by using the polylogarithmic identity in Eq. (17):

$$\begin{aligned} Li_m[-e^\mu] + (-1)^m Li_m[-e^{-\mu}] \\ = \sum_{k=0}^m \frac{\mu^k}{k!} (Li_{m-k}[-1] + (-1)^{m-k} Li_{m-k}[-1]) \\ \forall n \in \text{integers}. \end{aligned} \quad (17)$$

In the limit of single-photon excitation, where $n = 1$, Eq. (15) reduces to the form presented in Eq. (2), confirming consistency with the established scaling behavior of MTE near the photoemission threshold.

Compared to Eq. (2), our model incorporates multiphoton excitation under the same assumption adopted in Fowler's original work, namely that electron states are uniformly distributed in three-dimensional (3D) k space. This approach is consistent with a 3D Fermi gas, rather than

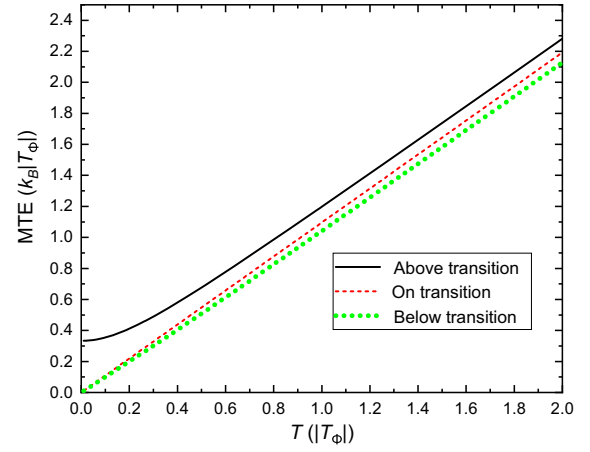


FIG. 2. MTE versus temperature T in units of T_ϕ defined in $n\hbar\nu - \Phi_{\text{eff}} = k_B T_\phi$ for the above-transition case $\mu > 0$, on-transition case $\mu = 0$, and below-transition case $\mu < 0$.

assuming a constant density of states as in a two-dimensional (2D) Fermi gas. Furthermore, the formulation in Eq. (15) offers a more tractable analytical form in the limit as $\mu \rightarrow \infty$.

We plot the MTE in Fig. 2 for three cases: the above-transition case ($\mu > 0$), the on-transition case ($\mu = 0$), and the below-transition case ($\mu < 0$). When plotting, we define the temperature T_ϕ in $n\hbar\nu - \Phi_{\text{eff}} = k_B T_\phi$ to normalize the scale of the horizontal axis. All three curves monotonically grow with the cathode-surface temperature.

In the low-temperature limit, the thermal energy $k_B T$ is significantly lower than the energy difference between n -photon excitation and the effective work function $n\hbar\nu - \Phi_{\text{eff}}$. For the above-transition case $\frac{n\hbar\nu - \Phi_{\text{eff}}}{k_B T} = \mu > 0$, as $T \rightarrow 0$, $\mu \rightarrow \infty$, one has

$$\text{MTE} \rightarrow \frac{n\hbar\nu - \Phi_{\text{eff}}}{3}. \quad (18)$$

This result matches Eq. (12). In the specific case of single-photon excitation ($n = 1$), this expression simplifies to

$$\text{MTE} = \frac{\hbar\nu - \Phi_{\text{eff}}}{3}, \quad (19)$$

which is identical to Eq. (1). This result is consistent with prior theoretical models and experimental observations, reinforcing the validity of the excess-energy scaling law for MTE under single-photon photoemission conditions.

For the below-transition case, $\frac{n\hbar\nu - \Phi_{\text{eff}}}{k_B T} = \mu < 0$, as $T \rightarrow 0$, $\mu \rightarrow -\infty$, one has

$$\text{MTE} \rightarrow k_B T, \quad (20)$$

which is expected for negligible photoemission from a cathode.

In the high-temperature limit, the electron thermal energy $k_B T$ exceeds the energy difference between n -photon excitation and the effective work function $nh\nu - \Phi_{\text{eff}}$, the MTE reduces to

$$\text{MTE} \rightarrow \frac{9\zeta(3)}{\pi^2} k_B T \approx 1.1 k_B T, \quad (21)$$

where $\zeta(s) = \sum_{k=1}^{\infty} k^{-s}$ is the Riemann zeta function. This result is expected for thermionically dominated electron emission.

IV. BRIGHTNESS

The brightness of an electron beam is an important parameter determining the usefulness of an electron source for various applications. It is expressed by the formula:

$$B = \frac{I}{\epsilon_x \epsilon_y}, \quad (22)$$

where I represents the beam current, and ϵ_x and ϵ_y are the transverse emittance in the x and y directions, respectively. Equation (22) can also be reformulated in terms of current density and MTE:

$$B = \frac{m_e c^2}{\pi} \frac{J}{\text{MTE}}. \quad (23)$$

By substituting the current density from Eq. (4) and MTE from Eq. (15) into Eq. (23), the brightness of a cathode can be expressed as

$$B = \frac{a_n m_e c^2 A_0 \left(\frac{e}{h\nu}\right)^n (1-R)^n I^n}{\pi k_B} \frac{TF[\mu]^2}{G[\mu]}. \quad (24)$$

Figure 3 illustrates the relationship between the cathode brightness and the incident n -photon energy relative to the work function. Under the low-temperature approximation, two scenarios can be considered based on whether the photoexcitation surpasses the effective work function or not. If the photoexcitation is below the transition threshold ($\mu < 0$), the contributions of $\frac{TF[\mu]^2}{G[\mu]}$ from Eq. (24) reduce to

$$\frac{TF[\mu]^2}{G[\mu]} = T e^{\mu}. \quad (25)$$

This case suggests an exponential growth for the brightness, as depicted in Fig. 3(a). It also indicates that the Schottky barrier reduction by an external field F_a results in a horizontal shift in the figure. The second scenario is associated with the above-transition photoexcitation ($\mu > 0$). In this case, the cathode brightness is plotted in Fig. 3(b) with a linear scale, as the photocurrent starts to grow quadratically.

Figure 4 plots the cathode brightness versus temperature below transition ($\mu < 0$, green dotted curve), on transition

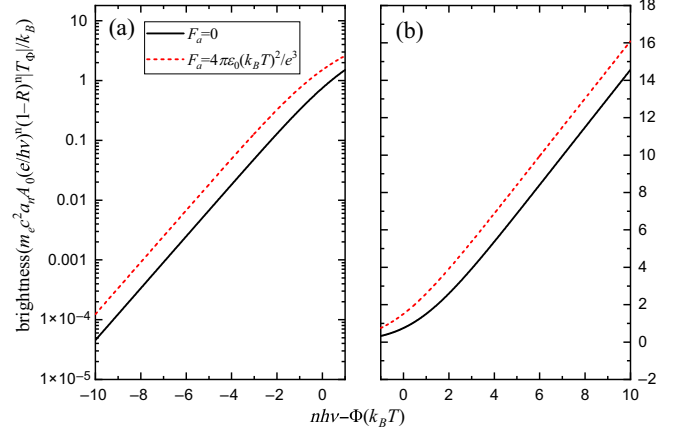


FIG. 3. Cathode brightness versus n -photon energy relative to the work function for external fields $F_a = 0$ (black solid curve) and $F_a = \frac{4\pi\epsilon_0(k_B T)^2}{e^3}$ (red dashed curve): (a) in logarithmic scale for $\mu < 0$, (b) in linear scale for $\mu > 0$. Below transition, the brightness increases exponentially, although the value is small.

($\mu = 0$, red dashed curve), and above transition ($\mu > 0$, black solid curve). Below and on transition ($\mu \leq 0$), the brightness increases monotonically with T , as the electron thermal energy $k_B T$ is comparable to or exceeds ($nh\nu - \Phi_{\text{eff}}$). When the temperature approaches zero, the cathode brightness approaches zero due to negligible thermionic emission.

Above transition $\mu > 0$, the contribution of $\frac{TF[\mu]^2}{G[\mu]}$ from Eq. (23) can be approximated as

$$\frac{TF[\mu]^2}{G[\mu]} = \frac{(nh\nu - \Phi_{\text{eff}})}{k_B} \frac{\left(\frac{\pi^2}{6} + \frac{\mu^2}{2}\right)^2}{\frac{1}{6}\mu^2(\mu^2 + \pi^2)}. \quad (26)$$

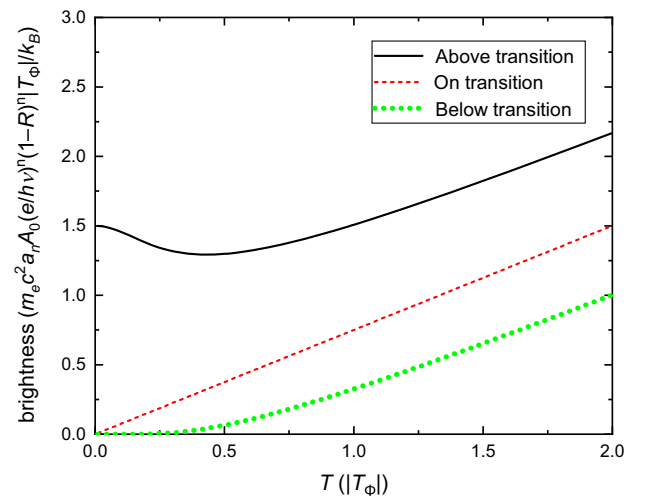


FIG. 4. Temperature-dependent brightness above transition ($\mu > 0$, black solid curve), on transition ($\mu = 0$, red dashed curve), and below transition ($\mu < 0$, green dotted curve).

This approximation indicates that when $(nh\nu - \Phi_{\text{eff}})$ is much larger than $k_B T$, the cathode brightness is proportional to $(nh\nu - \Phi_{\text{eff}})$, and the Schottky barrier reduction by an external field F_a results in a horizontal shift of the brightness curve in Fig. 3(b). As the temperature increases from zero, the cathode brightness starts from a nonzero value due to photoemission and settles to a local minimum at $\mu = \pi$, as predicted by Eq. (26). The condition $\mu = \pi$ marks a transition between an emittance-dominated regime and a current-dominated regime. As illustrated in Fig. 1(b), the thermal energy $k_B T$ broadens the energy distribution of electrons, creating a so-called “fuzzy zone” governed by the Fermi-Dirac distribution. When this fuzzy zone does not yet encompass the origin (i.e., the Fermi level E_F), the number of thermally excited electrons remains limited, resulting in only a modest increase in current. However, thermal broadening contributes significantly to the transverse energy spread, thereby increasing the emittance. Once the fuzzy zone extends below E_F , a larger population of electrons gains sufficient energy to surmount the vacuum level, leading to a more pronounced increase in emitted current and brightness.

In the high-temperature limit, $\mu \rightarrow 0$, the contribution of $\frac{TF[\mu]^2}{G[\mu]}$ from Eq. (24) is given by

$$\frac{TF[\mu]^2}{G[\mu]} = T \frac{\pi^4}{108\zeta(3)}. \quad (27)$$

This result implies that, no matter whether the photoexcitation overcomes the effective work function or not, the cathode brightness is linearly proportional to temperature, as expected for thermionic emission.

Our theoretical model reveals that near-threshold single-photon excitation generally yields lower mean transverse energy (MTE) due to the minimal excess energy imparted to photoemitted electrons. However, multiphoton excitation, despite producing higher electron temperatures, can offer distinct advantages. Specifically, elevated electron temperatures in this regime correlate with enhanced beam brightness, as predicted by our model. To counteract the associated increase in emittance, we propose the implementation of spatial apertures to selectively filter high-angle electrons. While this approach entails a trade-off in total charge, it enables the preservation of low emittance, thereby optimizing beam quality. Such a strategy is consistent with established practices in ultrafast electron diffraction and beamline design, where spatial filtering is routinely employed to balance brightness and coherence.

The MTE framework presented in this work is grounded in the semiclassical photoemission theory originally developed by Fowler, DuBridge [24–26] and later extended by Bechtel *et al.* [27,28]. This model assumes a noninteracting electron gas and does not account for quantum degeneracy pressure or collective effects among electrons [29]. Under high current density conditions,

space charge effects can significantly suppress Schottky barrier lowering, thereby increasing the effective work function and limiting achievable beam brightness prior to the onset of quantum degeneracy. This self-limiting behavior imposes additional constraints on the applicability of classical photoemission models.

V. INFRARED-LASER EXCITED GRAPHITE PHOTOCATHODE

To test our theory, we apply our model to understand the measured photoemission of a copper cathode excited by an infrared laser at 800 nm [30]. Although the paper concluded a three-photon emission process for the photocurrent, the measured emittance values were approximately twice as large as those predicted by the theoretical estimates based on energy considerations [30].

To interpret the photoemission data within our theoretical framework, we consider the ultrafast nature of the laser pulses used in the experiment [30] (80–400 fs), which are significantly shorter than the electron-phonon scattering timescale in metals. Under such conditions, the electronic subsystem rapidly thermalizes, and the lattice remains largely unaffected during the excitation. As a result, the electronic heat capacity dominates the thermal response of the system. For temperatures below the Fermi temperature ($T_F \approx 80\,000$ K for copper [11]), the electronic heat capacity can be approximated as $\gamma \frac{T}{T_F}$, where γ is a material-specific constant and T is the electron temperature.

In our data fitting procedure, the electron temperature is modeled using the relation $T^2 = \frac{2PE}{C_0} + T_0^2$, where PE is the incident pulse energy, C_0 is an effective parameter incorporating γ and the complex refractive index of copper, and T_0 is the initial temperature prior to excitation.

This temperature model allows us to establish a relationship between laser fluence and electron temperature, which we then used to calculate the mean transverse energy (MTE) and corresponding emittance for J_2 and J_3 . Since J_3 is largely insensitive to temperature and more influenced by excess energy, its emittance scales nearly linearly with laser spot size. In contrast, J_2 exhibits a more complex behavior: while its emittance decreases with smaller beam sizes, the MTE increases due to the rise in electron temperature. This interplay results in a sublinear trend in the J_2 curve, explaining why the unexpectedly large emittance value found in the claimed three-photon emission process in Ref. [30].

By using our theory, we found that both the partial currents J_2 and J_3 contribute to the emitted photocurrent. Figure 5 plots the emittance of the photocurrent versus rms laser spot size for the reported data (square dots fitted with a green line) and our theoretical predictions associated with J_2 (black curve) and J_3 .

As shown in the figure, the inclusion of the J_2 term offers improved alignment with the reported spot-size-dependent

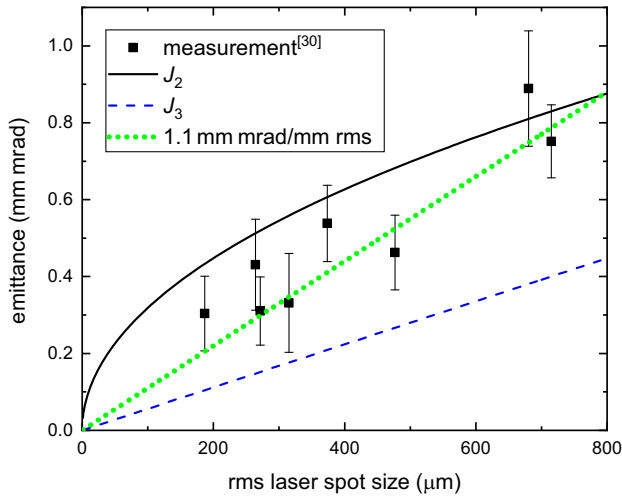


FIG. 5. The emittance of the photocurrent versus rms laser spot size for the reported data (square dots fitted with a green line) in Ref. [30] and our theoretical predictions associated with J_2 (black curve) and J_3 (blue dashed line). The measured data lie between the theoretical curves associated with J_2 and J_3 .

emittance—a feature not explicitly addressed in the original analysis by Ref. [30]. This improvement arises from the fact that J_2n incorporates thermal contributions to the MTE, which become relevant under a finite electron temperature. The two-photon energy corresponds to a near-UV excitation for the incident wavelength at 800 nm in Ref. [30]. This result echoes the thermionically assisted three-photon emission process measured from a copper photocathode excited by an infrared laser at 1064 nm [31].

VI. CONCLUSIONS

In summary, a theoretical framework is established for multiple electron-emission processes in a cathode, encompassing thermionic, multiphoton, and field emissions. The MTE of the emitted electrons exhibits significant variation depending on the interplay between incident photon energy, cathode work function, and cathode-surface temperature. When the incident -photon energy exceeds the effective work function (above transition), MTE is predominantly determined by the excess energy of the photons. Below the transition threshold, MTE is heavily influenced by the cathode-surface temperature, which is characterized as thermionically assisted photoemission. At an elevated cathode temperature, MTE becomes proportional to temperature, independent of photon energy.

In this work, we extend our MTE theory to the calculation of the brightness of a photoemitter. Below the transition, the brightness increases exponentially with rising photon energy and temperature. Above the transition, the brightness is proportional to the photon-energy excess and decreases to a local minimum when the photon-energy excess is comparable to thermal energy. In the high-temperature limit, the brightness is proportional to the

cathode temperature, irrespective of photon energy, because thermionic emission dominates the electron emission process.

Finally, we applied our theory to study a claimed three-photoemission process from the infrared-laser-excited copper photocathode reported in Ref. [30]. Our theory reveals that both the partial currents, J_2 and J_3 , contribute to the measured photocurrent. The inclusion of the thermionically assisted two-photon emission term, J_2 , and the corresponding temperature-dependent MTE of the electrons improves the agreement in the measured spot-size-dependent emittance. The theory derived in this work can be very useful in calculating the intrinsic emittance and brightness of a cathode operating in the mixed regime of thermionic, multiphoton, and field emission.

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DATA AVAILABILITY

The data that support the findings of this article are openly available [30].

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