

Study of systematic effects in the proton EDM experiment with a symmetric-hybrid ring design

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The physics and ring design of the storage ring proton electric dipole moment (sr-pEDM) experiment, including the symmetries used to mitigate first-order and second-order systematic effects, are described in [Z. Omarov *et al.* Comprehensive symmetric-hybrid ring design for a proton edm experiment at below 10^{-29} e · cm, *Phys. Rev. D* **105**, 032001 (2022)]. The sr-pEDM experiment leverages a novel symmetric-hybrid, frozen-spin storage ring design using radial electric bending and alternate magnetic focusing to minimize systematic errors, enabling precise measurement of the vertical rotation of the polarization of polarized stored proton beams. In this work, we present a more systematic and detailed analysis of second-order effects, focusing in particular on the interplay between stray radial magnetic fields and electric quadrupole and sextupole fields. We demonstrate how the symmetries of the symmetric-hybrid ring design, including beam reversal and magnetic quadrupole polarity switching, can be used to suppress these effects to below the targeted sensitivity of the sr-pEDM experiment.

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I. INTRODUCTION

Storage ring experiments have emerged as promising approaches to measure hadronic electric dipole moments (EDMs) with unprecedented sensitivity [1–7]. A nonzero EDM of the proton would signal new sources of charge parity violation beyond the standard model and could provide critical insight into the origin of the baryon asymmetry of the universe. A comprehensive overview of storage ring EDM concepts, including alternative design strategies and experimental approaches, is given in the CERN feasibility study [8], which provides important context for the symmetric-hybrid approach considered here. Briefly, the story begins with measurements of the anomalous magnetic moment of the muon in the 1960–1970s at CERN. The CERN I [9] experiment was limited by statistics. The next breakthrough was to go to a magnetic storage ring. The CERN II result was then limited by the systematics of knowing the magnetic field

seen by the muons in the storage ring using the magnetic focusing.

The spin precession rate of the longitudinal component of the spin $\vec{S}$ of particles, i.e., the helicity, due to the magnetic dipole moment (MDM) $qg\vec{S}/2m$ in a storage ring with electric $\vec{E}$ and magnetic fields $\vec{B}$ is [10]:

$$\frac{d(\hat{\beta} \cdot \vec{S})}{dt} = -\frac{q}{m} \vec{S}_{\perp} \cdot \left[\left(\frac{g-2}{2} \right) \hat{\beta} \times \vec{B} + \left(\frac{g\beta}{2} - \frac{1}{\beta} \right) \frac{\vec{E}}{c} \right], \quad (1)$$

where the g -factor of the particle is denoted by g , its charge by q , its mass by m , and its velocity by $\vec{v} = \beta\vec{c}$; the speed of light is denoted by c ; and $\vec{S}_{\perp}$ is the component of $\vec{S}$ perpendicular to the velocity.

The CERN III [9,11] experiment used a bending magnetic field with electric quadrupoles for focusing at the “magic” momentum, given by $\beta^2 = 2/g$ —as shown in the electric field term in Eq. (1). The CERN III experiment and Brookhaven National Laboratory (BNL) E821 [12,13] were limited by statistics, not systematics. The Fermi National Accelerator Laboratory (FNAL) E989 result, $da/a = 127$ parts per billion, is still limited by statistics [14,15]. The storage ring EDM (sr-EDM) method has its origins in the measurements of the anomalous magnetic

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moment of the muon, where an electric field at the so-called magic momentum does not influence the particle $g-2$ precession. Rather, the electric field precesses the momentum and the spin at the same rate, so the difference is zero. The storage ring/magic momentum breakthrough gained a factor of 10^4 in systematic error. The BNL E821 set a “parasitic” limit on the EDM of the muon: $d_\mu < 1.9 \times 10^{-19} e \cdot \text{cm}$ [13]. The FNAL E989 is expected to improve this result.

The sr-EDM method is a direct conceptual extension of the muon $g-2$ experiments, which established precision control of spin motion in electromagnetic storage rings. In the muon $g-2$ case, operation at the “magic momentum” suppresses the electric-field contribution to spin precession relative to momentum, enabling a clean measurement of the anomalous magnetic moment. In the EDM case, the complementary frozen-spin condition suppresses the magnetic-dipole precession relative to momentum, so that any vertical spin precession can be attributed to an EDM signal. In this sense, the EDM experiment represents the next step: transforming a precision magnetic moment measurement into a test for charge parity violation using the same underlying methodology.

The storage ring proton electric dipole moment (sr-pEDM) experiment utilizes a symmetric-hybrid storage ring operating at the proton’s magic momentum ($\sim 0.7 \text{ GeV}/c$), employing electric bending and alternating magnetic focusing to achieve the frozen-spin condition, aiming to achieve a sensitivity of $10^{-29} e \cdot \text{cm}$ [16]. This design effectively shields the beam from external radial magnetic fields and minimizes systematic errors through simultaneous clockwise (CW) and counter-clockwise (CCW) beam storage. In this work, we present a symmetry-driven analysis of second-order systematic effects, emphasizing the role of the 24-fold lattice symmetry, CW and CCW beams comparison, and magnetic quadrupole polarity reversal as a unified framework for suppressing false EDM signals.

In the sr-pEDM concepts, first-order systematic effects arising from vertical electric fields (E_y) [3] were addressed in Ref. [16] through the symmetric-hybrid storage ring design. This work extends that analysis by providing an analytical explanation of how the symmetric storage of CW and CCW beams suppresses second-order systematics, particularly those involving electric quadrupole ($E_x = -k_E x$ and $E_y = k_E y$) and sextupole [$E_x = k^e(x^2 - y^2)$ and $E_y = -2k^e xy$] fields in the presence of stray radial magnetic fields (B_x).

Previous studies, by Omarov *et al.*, have analyzed these field configurations. The current treatment clarifies the underlying mechanism of false EDM signals and emphasizes the power of simultaneous CW and CCW storage in canceling them.

Specifically, when the closed orbit is distorted—either by magnetic or electric-field imperfections—a vertical

force can act on the stored protons. In the presence of a nonzero radial magnetic field, the resulting Lorentz force includes a component that causes vertical spin precession, mimicking the signature of a true EDM. This mechanism, not just the closed-orbit distortion (COD) itself, is the key source of systematic error. If uncorrected, such effects could exceed the target EDM sensitivity of 1 nrad/s , corresponding to $10^{-29} e \cdot \text{cm}$.

In the ideal case where no significant radial magnetic field exists (e.g., no offset magnetic quadrupoles), these false effects cancel when averaging CW and CCW spin precession. However, if a residual radial magnetic field is present, additional alignment is required. In such cases, spin-based beam alignment techniques will be applied to map and compensate for the stray B_x , as discussed in Ref. [16].

An instructive example is the vertical force due to gravity. Gravity pulls the protons downward, requiring a small radial magnetic field to maintain the beam in a stable orbit. This compensating field induces vertical spin precession, indistinguishable from an EDM-like signal. The effect from gravity alone can be over an order of magnitude larger than the signal from an EDM of $10^{-29} e \cdot \text{cm}$. Since gravity is an effectively uniform field, its induced precession cancels between CW and CCW operation [17].

The paper is organized as follows: Sec. II introduces the physics and ring design of the sr-pEDM experiment. Section III discusses the electric quadrupole and sextupole fields’ systematic effect. Section IV explores how beam-spin dynamics symmetries in the symmetric-hybrid storage ring design mitigate these systematic effects. Sections V and VI conclude with relevant discussions.

II. OVERVIEW OF THE STORAGE RING PROTON-EDM EXPERIMENT

A. Storage ring spin dynamics equations

The spin $\vec{S}$ precession rate for a proton at rest in the presence of magnetic field $\vec{B}$ and the electric field $\vec{E}$ is given by:

$$\frac{d\vec{S}}{dt} = \vec{\mu} \times \vec{B} + \vec{d} \times \vec{E}, \quad (2)$$

with the MDM of the proton as $\vec{\mu} = (gq/2m)\vec{S}$ and the electric dipole moment of the proton as $\vec{d} = (\eta q/2mc)\vec{S}$. The η , as a dimensionless factor, plays the same role in the EDM term as the g -factor g does in the MDM term.

Spin motion relative to the momentum for a proton with $\vec{\beta} = \vec{v}/c$ in a cylindrical coordinate system (standard Frenet–Serret (x, y, s) coordinates, with “ x ” as the horizontal coordinate, “ y ” as the vertical coordinate, and “ s ” as the longitudinal coordinate) is given as [18–20]:

$$\frac{d\vec{S}}{dt} = (\vec{\omega}_a + \vec{\omega}_\eta) \times \vec{S}, \quad (3)$$

with

$$\vec{\omega}_a = -\frac{q}{m} \left\{ G\vec{B} - \frac{G\gamma}{\gamma+1} \vec{\beta}(\vec{\beta} \cdot \vec{B}) - \left(G - \frac{1}{\gamma^2 - 1} \right) \frac{\vec{\beta} \times \vec{E}}{c} + \frac{1}{\gamma} \left[\vec{B}_{\parallel} - \frac{1}{c\beta^2} (\vec{\beta} \times \vec{E})_{\parallel} \right] \right\}, \quad (4a)$$

$$\vec{\omega}_\eta = -\frac{\eta q}{2m} \left(\frac{\vec{E}}{c} - \frac{\gamma}{\gamma+1} \frac{\vec{\beta}}{c} (\vec{\beta} \cdot \vec{E}) + \vec{\beta} \times \vec{B} \right), \quad (4b)$$

where $\vec{\omega}_a$ and $\vec{\omega}_\eta$ stand for precession due to magnetic and electric dipole moments, respectively, the quantity $G = (g-2)/2$ represents the magnetic anomaly for hadrons (in this case, the proton). For leptons, the magnetic anomaly is denoted by a . γ is the Lorentz factor; $\parallel$ indicates horizontal (in-plane) projection of a vector.

By setting $\vec{B} = 0$ and selecting the proton's momentum as the magic momentum, defined by $G = 1/(\gamma^2 - 1)$, with a purely horizontal electric field $\vec{E}$ and purely horizontal $\vec{\beta}$, all terms in Eq. (4a) vanish, resulting in:

$$\vec{\omega}_a = 0. \quad (5)$$

This condition is known as the frozen-spin condition. In this setup, the spin precesses into the vertical direction only due to the EDM contribution.

$$\frac{dS_x}{dt} = 0, \quad \frac{dS_s}{dt} \approx 0, \quad (6a)$$

$$\frac{dS_y}{dt} = \frac{\eta q}{2mc} E_x S_s, \quad (6b)$$

where S_s is the projection of the unit spin vector along the longitudinal direction.

Measurement of the out-of-plane spin precession rate (dS_y/dt) probes the intrinsic EDM of the particles. The target sensitivity of $d_p = 10^{-29} e \cdot \text{cm}$ corresponds to a vertical spin precession rate of $dS_y/dt = 1 \text{ nrad/s}$. Any non-EDM-originating vertical spin precession rate larger than 1 nrad/s is considered a potential systematic-error source.

Operationally, the frozen-spin condition corresponds to nulling the spin tune relative to the beam orbit. Residual departures from this condition will be tracked through continuous monitoring of the $g-2$ precession and corrected using feedback to maintain the required spin freeze [8].

In practice, deviations from the frozen-spin condition arise from momentum spread, residual transverse and longitudinal fields, and lattice imperfections. Maintaining $\omega_a \approx 0$ at the level required for EDM searches places stringent constraints on these effects. Experimental studies by the JEDI Collaboration [21] have demonstrated

high-precision control of the spin tune in a storage ring, corresponding to a relative tune stability at the 10^{-10} level.

A particularly important limitation arises from the beam momentum spread, which leads to dephasing of the spin ensemble and limits the spin coherence time (SCT). In the absence of longitudinal bunching, the SCT is typically on the order of a few milliseconds. The use of rf fields to maintain longitudinal bunching can extend the SCT to the 1–10 s range. Achieving an SCT of $\gtrsim 10^3 \text{ s}$, as required for target sensitivity in EDM searches, necessitates dedicated lattice optimization, including the deployment of sextupole fields at strategic locations to compensate for higher-order spin tune spread, as demonstrated in Refs. [16,22].

B. Symmetric-hybrid ring design

The current ring design of the sr-pEDM experiment incorporates the following features: (i) Simultaneous beams stored in both CW and CCW directions. (ii) Polarity reversal of quadrupole magnet currents for each fill. (iii) A 24-fold symmetric lattice structure.

In an ideal storage ring, the true EDM can be determined from the difference in precession rates between counter-rotating (CR) beams, expressed as:

$$\left(\frac{dS_y}{dt} \right)_{\text{EDM}} = \frac{1}{2} \left(\frac{dS_y}{dt} \right)_{\text{CW}} - \frac{1}{2} \left(\frac{dS_y}{dt} \right)_{\text{CCW}}. \quad (7)$$

When the ring design includes switching the polarity of the quadrupole magnets, the EDM is given by:

$$\left(\frac{dS_y}{dt} \right)_{\text{EDM}} = \frac{1}{2} \left[\frac{1}{2} \left(\frac{dS_y}{dt} \right)_{\text{CW}} - \frac{1}{2} \left(\frac{dS_y}{dt} \right)_{\text{CCW}} \right]_+ + \frac{1}{2} \left[\frac{1}{2} \left(\frac{dS_y}{dt} \right)_{\text{CW}} - \frac{1}{2} \left(\frac{dS_y}{dt} \right)_{\text{CCW}} \right]_-. \quad (8)$$

Here, the subscript “+” denotes the positive polarity of the current in the magnetic quadrupoles that produce magnetic focusing in the CW beam, while the subscript “−” denotes the negative polarity of the current in the magnetic quadrupoles that produce magnetic defocusing in the CW beam.

III. SYSTEMATIC EFFECTS RELATED TO THE PRESENCE OF ELECTRIC QUADRUPOLE FIELDS AND THE MISALIGNMENT OF SEXTUPOLE FIELDS

A. Systematic effects related to the presence of electric quadrupole fields

In an all-magnetic storage ring, and in the absence of nonmagnetic forces acting on the particles, the total vertical spin precession rate averages to zero. This condition implicitly requires precise radial alignment of the bending elements. Stray magnetic fields, such as Earth's field, can in principle induce transverse magnetic forces;

however, these are counteracted by magnetic quadrupole fields, ensuring an average horizontal field of $\langle B_{x,\text{co}} \rangle = 0$ along the closed orbit [23]. In contrast, when local electric forces are present, this cancellation is no longer guaranteed, and in addition, closed-orbit distortions may arise even in an otherwise ideal lattice. The general equation for the vertical closed-orbit distortion (y_{co}) around the ring is given by:

$$y_{\text{co}} = \eta_y(\phi_y) \sqrt{\beta_y}, \quad (9)$$

where the definitions of ϕ_y and $\eta_y(\phi_y)$ are given in Appendix A 1, and β_y denotes the vertical beta function in the symmetric-hybrid ring design of the sr-pEDM experiment. It can be approximated as:

$$\beta_y = \beta_0(1 \pm 0.5 \cos(24\phi_y)). \quad (10)$$

As an example, this approximation explicitly reflects the 24-fold symmetry of the sr-pEDM ring. The beam stored in the CW direction is denoted by the positive ϕ_y direction ($\phi_y \in [0, 2\pi]$), and the beam stored in the CCW direction is denoted by the negative ϕ_y direction ($\phi_y \in [0, -2\pi]$). The representative parameters used in the analysis can be found in Table I.

The unwanted radial magnetic field perturbation, denoted $F(\phi_y)$, can be expanded in a Fourier series around the ring:

$$F(\phi_y) = \sum_{N=0}^{\infty} [F_c \cos(N\phi_y) + F_s \sin(N\phi_y)], \quad (11)$$

where F_c and F_s are the cosine and sine coefficients of $F(\phi_y)$, respectively.

The corresponding closed-orbit solution, as derived from Eqs. (A7) and (9), can then be expressed for the cosine and sine components as:

$$y_{\text{co},N}(\phi_y) = a_{\text{co},c,N} \cos(N\phi_y) + b_{\text{co},c,N} \cos(L\phi_y) + c_{\text{co},c,N} \cos(M\phi_y) + \dots + \sin \text{ terms}, \quad (12)$$

TABLE I. Representative parameters used in the analysis.

Parameter	Value
Bending radius R_0	95.49 m
Number of lattice periods	24
Vertical tune ν_y	2.245
Horizontal tune ν_x	2.699
Beta-function model	$\beta_y(\phi_y) = \beta_0(1 \pm 0.5 \cos(24\phi_y))$
Quadrupole gradient k_E	$\sim \pm 1.5 \times 10^{-4} \text{ V/m}^2$
Quadrupole length l_E	0.4 m
Sextupole strength k^e	$\sim \pm 2 \times 10^2 \text{ V/m}^3$
Sextupole length l^e	0.4 m

where $a_{\text{co},c,N}$, $b_{\text{co},c,N}$, $c_{\text{co},c,N}$, etc., are the cosine coefficients of the closed-orbit solution and N denotes the primary driving harmonic index. For each N , D represents the dominant closed-orbit distortion harmonic index and is provided in Table III of the Appendix A 1. The quantities $L = 24 - D$ and $M = 24 + D$ correspond to the first lower and upper sideband alias harmonics, respectively, arising from the 24-fold symmetry of the lattice.

The maximum amplitude of the closed-orbit distortion upon $B_x = 1 \text{ nT}$ field N harmonic around the ring azimuth is shown in Fig. 1. The perturbation $F(\phi_y)$ does not have L and M Fourier harmonics, but the closed-orbit distortion does—as shown in Table III. This comes from Eq. (9) via the trigonometric identity: $2 \cos(A) \cos(B) = \cos(A+B) + \cos(A-B)$, etc.

In this study, a stray radial magnetic field magnitude of $B_x = 1 \text{ nT}$ was adopted. This radial magnetic field separates the CR beams by a relative distance of $\sim 14 \mu\text{m}$, as illustrated in Figs. 1 and 5. This beam splitting will be measured using electric beam position monitors (BPM) systems [24] and magnetic BPM systems with a target resolution of $10 \mu\text{m}$ [25].

For an ideal ring, the net electromagnetic force in the vertical direction, F_y , is expected to vanish. This condition is expressed as:

$$F_y = q(E_y + c\beta_s B_x) = 0. \quad (13)$$

The presence of a small undesired electric quadrupole field $E_y = k_E y$ gives a nonzero stray radial magnetic field on the closed orbit around the ring, and thus it gives unwanted vertical spin precession [26]:

$$\frac{dS_y}{dt} = \frac{q}{m} G \langle B_x \rangle S_s = -\frac{qG}{m} \frac{\langle k_E y_{\text{co}} \rangle}{v} S_s. \quad (14)$$

The unwanted electric quadrupole gradient $k_E(\phi_y)$ can be expanded in a Fourier series around the ring azimuth:

$$k_E(\phi_y) = \sum_{N=0}^{\infty} [k_{E,c} \cos(N\phi_y) + k_{E,s} \sin(N\phi_y)], \quad (15)$$

where $k_{E,c}$ and $k_{E,s}$ are the cosine and sine coefficients of $k_E(\phi_y)$, respectively.

The EDM-like background arises from the product of the electric quadrupole multipole and the closed-orbit distortion. For details on the Fourier selection rules and aliasing in a 24-fold lattice, as well as the expression of the EDM-like background in terms of Fourier harmonics, see Appendixes A 2 and A 3.

B. Systematic effects related to the misalignment of sextupole fields

The sr-pEDM experiment features both magnetic and electric sextupoles. By optimally incorporating these

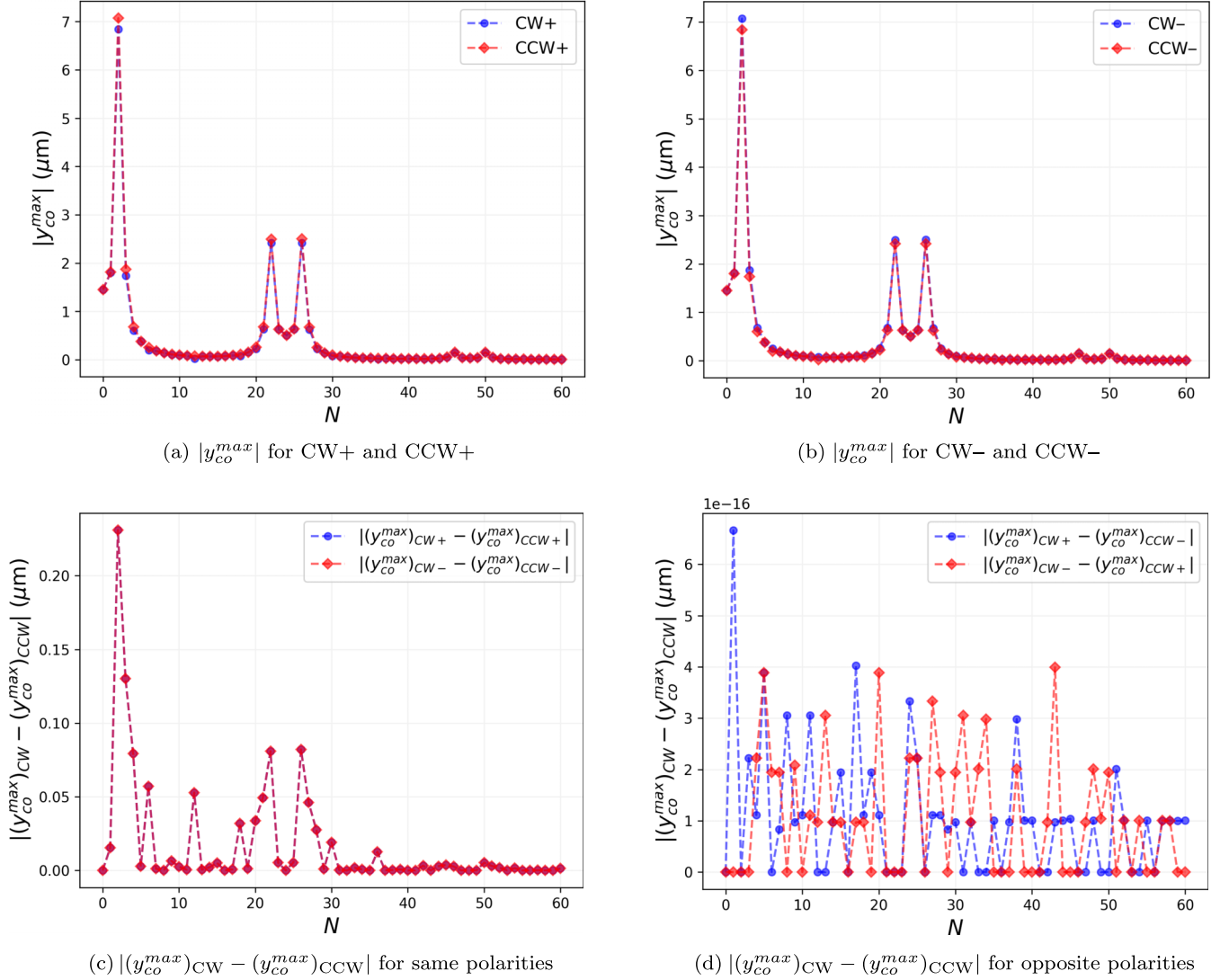


FIG. 1. The maximum amplitude of the closed-orbit distortion upon $B_x = 1$ nT field N harmonic around the ring azimuth. The betatron tune ν_y is 2.245. The three largest harmonics of the closed-orbit distortion occur at $N = 2$ (primary peak) and at $N = 24 \pm 2$ (aliasing sideband peaks) for both CW and CCW: (a) shows the maximum amplitude of the closed-orbit distortion upon $B_x = 1$ nT field N harmonic around the ring azimuth for both CW and CCW beams with positive polarity of the currents in the magnetic quadrupoles; (b) shows the same quantity for both beams with negative polarity; (c) presents the CW-CCW difference in maximum closed-orbit distortion for beams of the same polarity, i.e., (CW+)-(CCW+) and (CW-)-(CCW-); (d) shows the CW-CCW difference for beams of opposite polarity. With both CW-CCW and beams of opposite-polarity symmetries, the CW-CCW difference is reduced by a factor of $\sim 10^{16}$ in the opposite-polarity case.

sextupoles, the SCT could be improved for both CR beams [16].

For an electric sextupole, the transverse electric-field components are given by:

$$E_x = k^e(x^2 - y^2), \quad (16a)$$

$$E_y = -2k^e xy, \quad (16b)$$

where k^e is the electric sextupole strength factor, defined as $k^e = \partial^2 E_x / \partial y^2$, with units of V/m³.

The contribution of electric sextupole fields to the vertical spin precession arises in the presence of horizontal misalignment Δx , leading to an average vertical electric field:

$$\langle E_y \rangle = \langle -2k^e \Delta x y_{co} \rangle, \quad (17)$$

which induces a vertical spin precession rate:

$$\frac{dS_y}{dt} = \frac{q}{m} G \langle B_x \rangle S_s = \frac{2qG}{m} \frac{\langle k^e \Delta x y_{co} \rangle}{v} S_s. \quad (18)$$

Representative values for the sextupole strength are $k^e \sim 2 \times 10^2 \text{ V/m}^3$, as used in the present study. To suppress the resulting false EDM contribution below the target sensitivity of $\sim 1 \text{ nrad/s}$, the effective horizontal misalignment must be controlled at the level of

$$\Delta x \lesssim 0.1 \text{ mm}, \quad (19)$$

averaged over all sextupole elements.

The control of this contribution relies on beam-based alignment. The closed-orbit distortion $y_{\text{co}}(\phi_y)$ is measured using BPMs, and its harmonic components are extracted and correlated with known lattice elements. By varying sextupole settings and applying controlled perturbations, the coupling term $\langle k^e \Delta x y_{\text{co}} \rangle$ can be isolated and minimized iteratively.

The required BPM resolution is set by the need to resolve orbit distortions at the $\sim 10 \text{ } \mu\text{m}$ level, which is within the capability of modern BPM systems. Long-term stability is ensured through continuous monitoring and periodic reoptimization using beam-based and spin-based alignment procedures, making the suppression of sextupole-induced systematic effects robust against slow drifts.

IV. BEAM-SPIN DYNAMICS SYMMETRIES

The beam-spin dynamics symmetries associated with reversing the quadrupole magnet currents for each fill are examined. The normal electric quadrupole gradient is set to $k_E \approx 1.477 \times 10^{-4} \text{ V/m}^2$, corresponding to the upper limit of the sr-pEDM experiment designated measurable true EDM, $|dS_y/dt| = 1.0 \times 10^{-9} \text{ rad/s}$, as indicated by the solid red lines in Fig. 2. The Fourier terms for CW and CCW beams with flipped quadrupole magnet currents exhibit opposite symmetry compared to the EDM, as shown in Figs. 2(a) and 2(d) for $N = 2$ harmonic and Figs. 2(e) and 2(h) for $N = 26$ harmonic. Similarly, the CCW and CW beams with flipped quadrupole magnet currents display this opposite symmetry, as illustrated in Figs. 2(b) and 2(c) for $N = 2$ harmonic and Figs. 2(f) and 2(g) for $N = 26$ harmonic.

The EDM backgrounds discussed above are characterized by the following relations:

$$\left(\frac{dS_y}{dt}\right)_{\text{EDMB1}} = \frac{1}{2} \left(\frac{dS_y}{dt}\right)_{\text{CW}+} - \frac{1}{2} \left(\frac{dS_y}{dt}\right)_{\text{CCW}-} = 0, \quad (20a)$$

$$\left(\frac{dS_y}{dt}\right)_{\text{EDMB2}} = \frac{1}{2} \left(\frac{dS_y}{dt}\right)_{\text{CCW}+} - \frac{1}{2} \left(\frac{dS_y}{dt}\right)_{\text{CW}-} = 0. \quad (20b)$$

These two separate equations, Eqs. (20a) and (20b), give an additional symmetry besides Eq. (8). This is our most important result in this paper.

V. DISCUSSION

The electric field E_y systematic effect has been characterized and discussed in Ref. [16]. The Ref. [16] spin-based alignment procedure for the second-order systematic effect is as follows: The presence of E^{quad} can be monitored by controlling B_x . The combination of E^{quad} and B_x produces nonzero vertical spin precession rate dS_y/dt . B_x that could be made large on purpose. Being able to freely control B_x and all of its harmonics allows us to selectively (for each N harmonic) amplify and then reduce the effect of initially unknown E^{quad} .

We illustrate with an example how beam-spin dynamics symmetries and spin-based alignment analysis can mitigate the electric sextupole field arising from a misaligned electric sextupole. As shown in Figs. 2 and 3, the vertical spin precession effect arising from electric sextupole fields in the presence of horizontal misalignment—which produces a nonzero average vertical electric field—is identical to that of the electric quadrupole case.

The beam-spin dynamics symmetries associated with flipping the quadrupole magnet currents at every fill mitigate the electric sextupole field arising from misaligned electric sextupoles. As illustrated in Fig. 3, the electric sextupole strength factor is set to $k^e \approx 2 \times 10^2 \text{ V/m}^3$ [16], with the horizontal alignment precision for the electric sextupole configured at $\Delta x = 100 \text{ } \mu\text{m}$. The Fourier terms for CW and CCW beam configurations with reversed quadrupole magnet currents exhibit opposite symmetry compared to the EDM, as depicted in Figs. 3(a) and 3(d) for $N = 2$ harmonic and Figs. 3(e) and 3(h) for $N = 26$ harmonic. Similarly, the CCW-CW configurations with reversed quadrupole magnet currents show corresponding symmetries, as shown in Figs. 3(b) and 3(c) for $N = 2$ harmonic and Figs. 3(f) and 3(g) for $N = 26$ harmonic. For EDM backgrounds induced by misaligned electric sextupoles, the beam-spin dynamics symmetries and their mitigating effects are analytically described in Eqs. (20a) and (20b).

To mitigate this systematic effect, spin-based alignment analysis can be employed. Misaligned electric sextupoles produce fields given by $E_x = k^e(x^2 - y^2)$ and $E_y = -2k^e xy$. The spin-based alignment procedure [16] is as follows: The presence of E_y^{sext} can be monitored by controlling B_x . The combination of E_y^{sext} and B_x produces nonzero vertical spin precession rate dS_y/dt . By adjusting B_x , this precession rate can be driven to zero, which directly yields the value of E_y^{sext} and, in turn, the misalignment Δx . The ability to freely control B_x and all of its harmonics further allows us to selectively (for each harmonic order N) amplify and then suppress the effect of the initially unknown E_y^{sext} .

During spin-based alignment analysis, spin polarimetry measures the average vertical spin precession rate, induced by electric quadrupole and misaligned electric sextupole

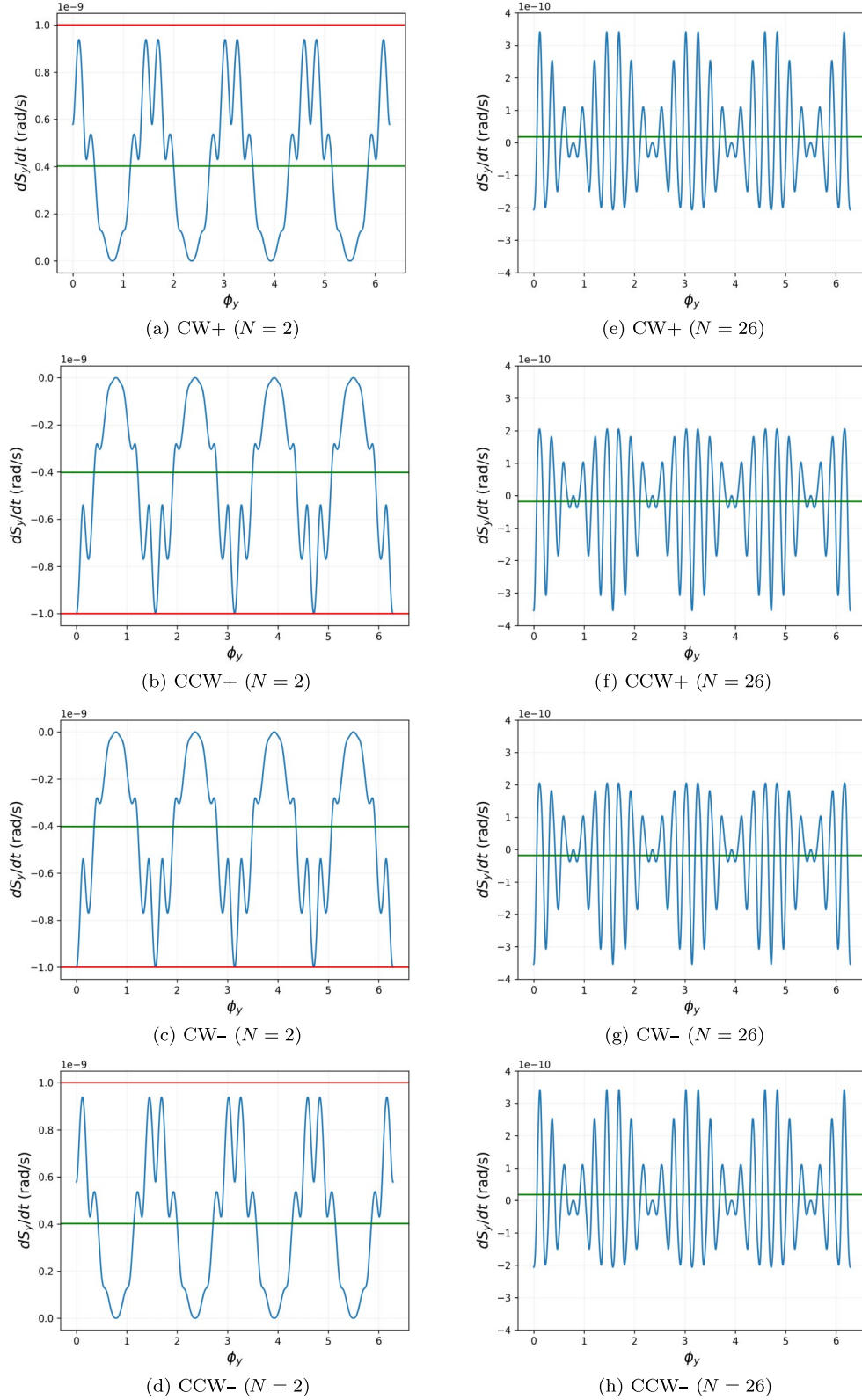


FIG. 2. Longitudinal polarization case. Vertical spin precession rate upon $B_x = 1$ nT field with the presented E_y -electric quadrupole field N harmonic around the ring azimuth ϕ_y for (a)–(d) $N = 2$ and (e)–(h) $N = 26$; the green solid line represents the averaged vertical spin precession rate $\langle dS_y/dt \rangle$ around the ring azimuth ϕ_y . Fourier terms of the CW and CCW beams with flipped quadrupole magnet currents show symmetry opposite to the EDM. This behavior appears for the $N = 2$ harmonic in (a) and (d) and for the $N = 26$ harmonic in (e) and (h). The same opposite symmetry is observed between the CCW and CW beams with flipped currents, as shown in (b) and (c) for $N = 2$ and (f) and (g) for $N = 26$.

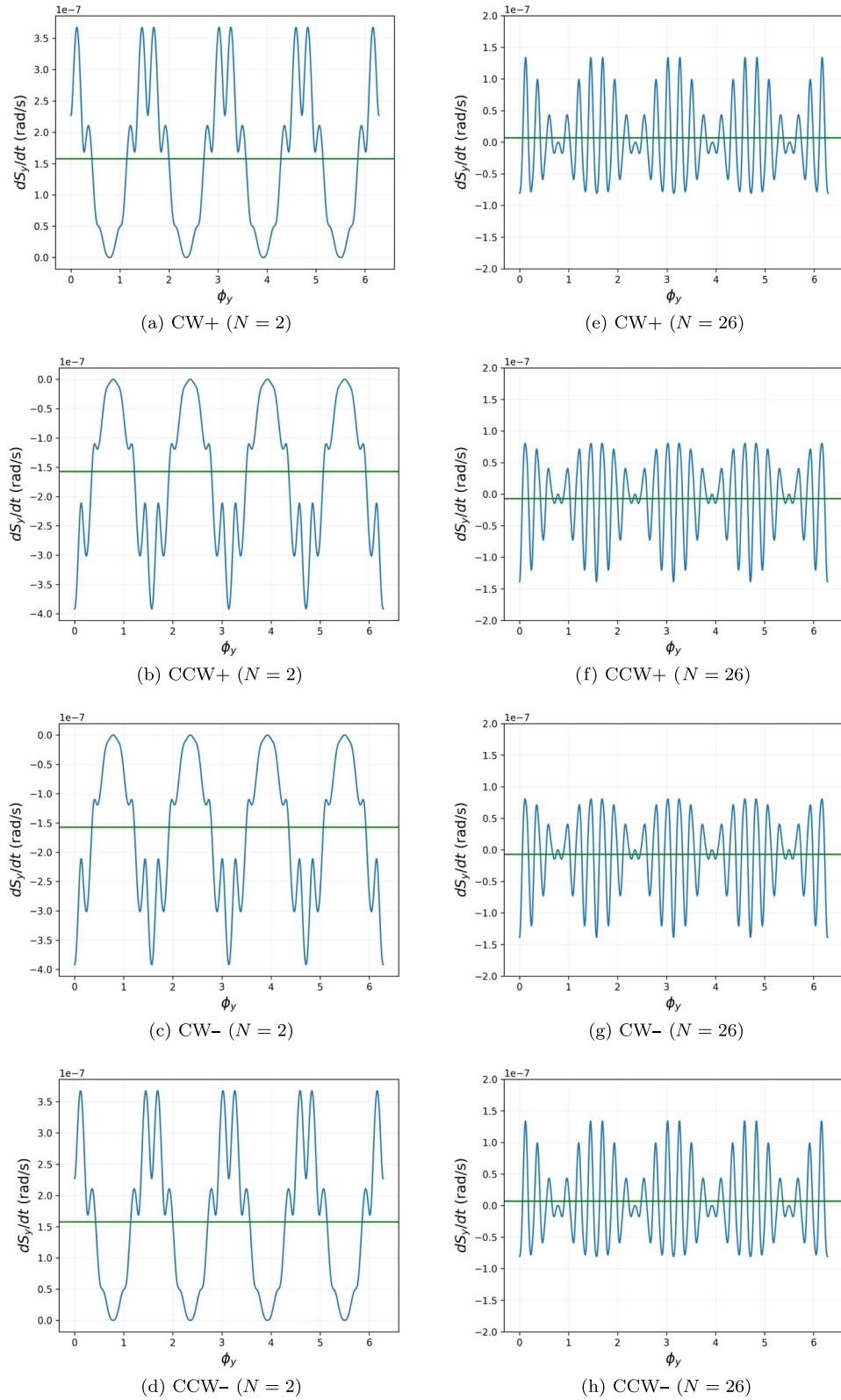


FIG. 3. Longitudinal polarization case. Vertical spin precession rate upon $B_x = 1$ nT field with the presented E_y -misaligned electric sextupole field N harmonic around the ring azimuth ϕ_y for (a)–(d) $N = 2$ and (e)–(h) $N = 26$. Fourier terms of the CW and CCW beams with flipped quadrupole magnet currents show symmetry opposite to the EDM. This behavior appears for the $N = 2$ harmonic in (a) and (d) and for the $N = 26$ harmonic in (e) and (h). The same opposite symmetry is observed between the CCW and CW beams with flipped currents, as shown in (b) and (c) for $N = 2$ and (f) and (g) for $N = 26$.

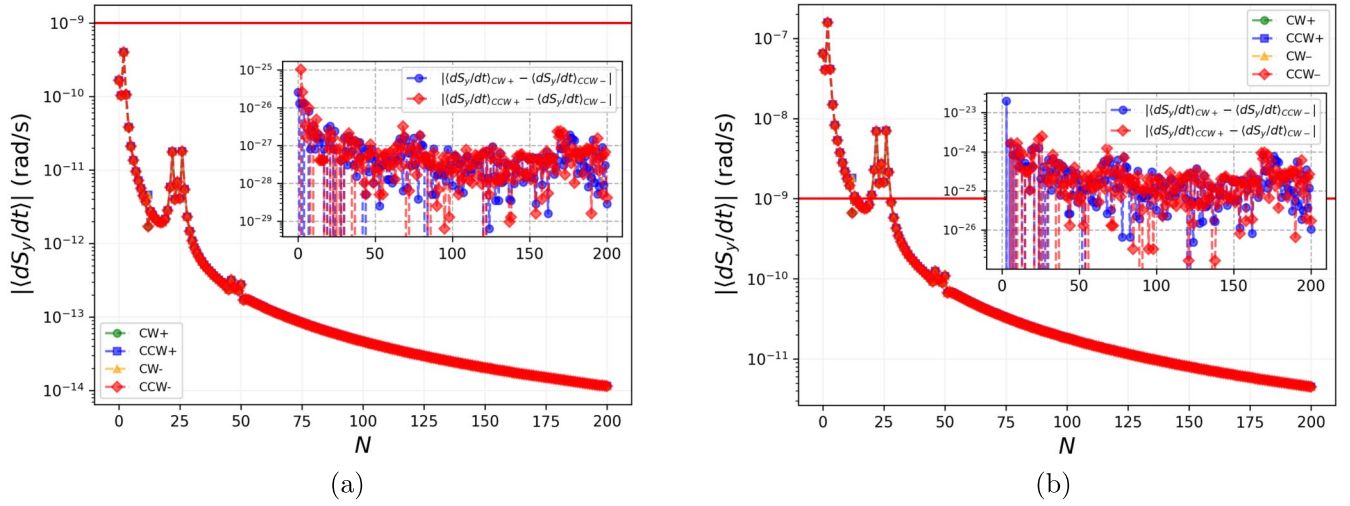


FIG. 4. Longitudinal polarization case. (a) Averaged vertical spin precession rate upon $B_x = 1$ nT field with the presented E_y -electric quadrupole field N harmonic around the ring azimuth for CW+, CCW+, CW- and CCW- beams. (b) Averaged vertical spin precession rate upon $B_x = 1$ nT field with E_y -misaligned electric sextupole field N harmonic around the ring azimuth for CW+, CCW+, CW- and CCW- beams. The electric sextupole strength factor is set to $k^e \approx 2 \times 10^2$ V/m³ [16], with the horizontal alignment precision for the electric sextupole configured at $\Delta x = 100$ μ m. Fourier terms of the CW and CCW beams with flipped quadrupole magnet currents show symmetry opposite to the EDM; the same opposite symmetry is observed between the CCW and CW beams with flipped currents for all harmonic number N . By applying both CW-CCW symmetry and opposite-polarity beam configurations, the averaged vertical spin precession rate is reduced by a factor of $\sim 10^{15}$ for both cases. The inset figures show the CW-CCW differences of the averaged vertical spin precession rate, $|\langle dS_y/dt \rangle_{CW+} - \langle dS_y/dt \rangle_{CCW-}|$ and $|\langle dS_y/dt \rangle_{CCW+} - \langle dS_y/dt \rangle_{CW-}|$, confirming the effectiveness of these key cancellations for both cases. High-field harmonics become negligible rapidly as the harmonic number N increases for both cases.

field harmonics around the ring azimuth ϕ_y [27]. Highfield harmonics become negligible rapidly as the harmonic number N increases, as illustrated in Fig. 4.

Normal electric quadrupole fields arise when electrode plates deviate from perfect parallelism and are rotated relative to the longitudinal axis. Electric sextupole fields emerge due to the misaligned electric sextupole. Higher-order electric multipole fields (e.g., octupole, decapole,...) are not considered in this study. A summary of the dominant second-order systematic effects considered in

this work, their representative scales, and the corresponding suppression mechanisms is given in Table II.

VI. CONCLUSION

In this work, we presented a detailed analysis of second-order systematic effects in the symmetric-hybrid storage ring proton EDM experiment, with particular emphasis on the interplay between stray radial magnetic fields and electric quadrupole and sextupole fields. The study

TABLE II. Representative second-order systematic-error budget.

Source	Representative scale/tolerance	Suppression/control	Residual bound on dS_y/dt
Stray B_x	1 nT benchmark	CW/CCW + alignment	<1 nrad/s
Electric quadrupole k_E	index $\lesssim 10^{-5}$	Symmetry + polarity flip	<1 nrad/s
Sextupole k^e , offset Δx	$k^e \sim 2 \times 10^2$ V/m ³ , $\Delta x \lesssim 0.1$ mm	Alignment + symmetry	<1 nrad/s
Quadrupole misalignment	1–10 μ m rms	Beam-based alignment	<1 nrad/s
Gravity	Uniform field	CW/CCW cancellation	$\ll 1$ nrad/s
Ring tilt/floor motion	~ 0.1 mm	HLS + orbit control	Subdominant
Beam separation (CW/CCW)	$\lesssim 10$ μ m	BPM monitoring	Subdominant
HV drift/sag	~ 100 ppm	symmetric design	Subdominant
Flip mismatch Δ_k	$\lesssim 10^{-5}$	Air-core + averaging	Subdominant
BPM offsets	Calibrated systems	Cross-checks	Subdominant
Higher multipoles	Beyond sextupole	Design + CDR validation	Neglected

demonstrates that the combination of simultaneous clockwise/counterclockwise (CW/CCW) beam storage, magnetic quadrupole polarity reversal, and the intrinsic 24-fold lattice symmetry provides a powerful framework for suppressing false EDM signals.

We showed analytically and numerically that the dominant EDM-like backgrounds generated by electric quadrupole fields and horizontally misaligned electric sextupoles exhibit symmetry properties opposite to those of a true EDM signal. As a consequence, these effects cancel to very high precision when the appropriate CW-CCW and opposite-polarity combinations are formed. The cancellation mechanism is robust across the relevant harmonic spectrum and remains effective in the presence of realistic lattice imperfections and closed-orbit distortions.

The analysis further demonstrates that the remaining residual effects can be controlled through spin-based alignment techniques together with beam-based orbit diagnostics. In particular, controlled manipulation of the radial magnetic field harmonics allows systematic amplification and subsequent suppression of unwanted electric-field-induced spin precession effects. The required beam position and alignment tolerances are within the capabilities of modern BPM and alignment systems.

An important outcome of this work is the identification of additional beam-spin dynamical symmetries, expressed in Eqs. (20a) and (20b), which provide an independent and highly constraining validation channel for distinguishing true EDM signals from systematic backgrounds. These symmetries substantially strengthen the overall robustness of the sr-pEDM experimental concept.

The results presented here support the conclusion that the symmetric-hybrid frozen-spin storage ring approach can suppress the dominant second-order systematic effects to levels compatible with the targeted sensitivity of $10^{-29} e \cdot \text{cm}$. More broadly, the methods developed in this work establish a general framework for precision spin-dynamics control in storage rings and may prove valuable for future EDM experiments involving protons, deuterons, and ^3He , as well as other high-precision spin-based searches for physics beyond the standard model.

ACKNOWLEDGMENTS

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DATA AVAILABILITY

The data that support the findings of this article are openly available [28].

APPENDIX A: MATHEMATICAL FOUNDATIONS

1. Storage ring beam dynamics equations

A storage ring employs dipole fields to bend charged particle beams and quadrupole magnetic fields for focusing. The transverse motion in the vertical direction is governed by Hill's equation:

$$\frac{d^2y}{ds^2} + k(s)y = 0, \quad (\text{A1})$$

where $k(s)$ gives the focusing strength around the ring, and it satisfies

$$k(s + C) = k(s),$$

where s is the longitudinal coordinate in the Frenet–Serret coordinate system and C is the ring circumference.

The solution is the ideal orbit ($y = 0$), with “betatron oscillations” about the closed orbit:

$$y(s) = y_{\text{co}} + \sqrt{\varepsilon_y \beta_y(s)} \sin(\psi_y(s) + \alpha_y), \quad (\text{A2a})$$

$$\beta_y(s + C) = \beta_y(s), \quad (\text{A2b})$$

$$\psi_y(s_1 \rightarrow s_2) = \int_{s_1}^{s_2} \frac{ds}{\beta_y(s)}, \quad (\text{A2c})$$

where the vertical beam emittance, ε_y , is a constant independent of the longitudinal coordinate s and the β_y is the vertical beta function, $\alpha_y = -\beta'_y/2$ and $\gamma_y = (1 + \alpha_y^2)/\beta_y$. $\{\alpha, \beta, \gamma\}$ are commonly called the Twiss parameters in the accelerator physics context; these parameters characterize the beam's optical properties. The function $\psi_y(s_1 \rightarrow s_2)$ is the phase advance from s_1 to s_2 .

The vertical betatron tune is given by:

$$\nu_y = \frac{1}{2\pi} \oint \frac{ds}{\beta_y(s)}. \quad (\text{A3})$$

Using the Courant-Snyder transformations:

$$\eta_y = \frac{y}{\sqrt{\beta_y}}, \quad (\text{A4a})$$

$$\phi_y = \int \frac{ds}{\nu_y \beta_y(s)}. \quad (\text{A4b})$$

The Hill's equation transforms into:

$$\frac{d^2\eta_y}{d\phi_y^2} + \nu_y^2\eta_y = 0. \quad (\text{A5})$$

Stray radial magnetic fields $\Delta B_x(s)$ introduce a field error:

$$F(s) = -\frac{\Delta B_x(s)}{B\rho}, \quad (\text{A6})$$

where $B\rho$ is the magnetic rigidity.

This field error perturbs the closed orbit; hence the Hill's equation for this closed-orbit distortion due to the field error is

$$\frac{d^2\eta_y}{d\phi_y^2} + \nu_y^2\eta_y = \nu_y^2\beta_y^{3/2}F(\phi_y), \quad (\text{A7})$$

where $F(s)$ is transformed to $F(\phi_y)$ via the Courant–Snyder transformations in Eq. (A4b).

The solution for the perturbed closed orbit is

$$\begin{aligned} \eta_y(\phi_y) = & \frac{\nu_y}{2\sin(\pi\nu_y)} \int_{\phi_y}^{\phi_y+2\pi} \beta_y^{3/2} F(\phi'_y) \\ & \times \cos[\nu_y(\pi + \phi_y - \phi'_y)] d\phi'_y. \end{aligned} \quad (\text{A8})$$

2. Fourier selection rules and aliasing in a 24-fold lattice

The 24-fold symmetry of the storage ring imposes characteristic selection rules on the coupling between field harmonics and the closed-orbit response. In particular, the modulation of the beta function with periodicity 24 leads to mixing between a primary field harmonic N and sideband harmonics at $24 \pm N$, $48 \pm N$, etc. As a result, even when the applied perturbation contains only a single Fourier component, the closed-orbit distortion and associated spin response exhibit additional “alias” harmonics.

This effect is evident in Eq. (12) and Table III, where a field harmonic at $N = 2$ generates dominant closed-orbit components at $N = 2$ together with sidebands at $N = 24 \pm 2$ and higher-order aliases. These sidebands arise from the product of the lattice periodicity and the field perturbation, and their amplitudes are governed by the beta-function modulation and tune-dependent response:

$$\begin{aligned} \cos(N\phi_y) \cos(24\phi_y) = & \frac{1}{2} [\cos((24 + N)\phi_y) \\ & + \cos((24 - N)\phi_y)]. \end{aligned} \quad (\text{A9})$$

TABLE III. The three largest COD harmonics (N , L , and M) as a function of the primary driving harmonic index N . Here, D denotes the dominant COD harmonic index, while $L = 24 - D$ and $M = 24 + D$ represent the first lower and upper sideband alias harmonics, respectively, arising from the 24-fold symmetry of the lattice. $y_{\text{co}}^{\text{max}}$ is the maximum amplitude of the closed-orbit distortion for each primary driving harmonic index N .

N	D	L	M	$(y_{\text{co}}^{\text{max}})_{\text{CW}} (\mu\text{m})$	$(y_{\text{co}}^{\text{max}})_{\text{CCW}} (\mu\text{m})$
0	0	24	0	1.45	1.45
1	1	23	25	1.80	1.81
2	2	22	26	6.84	7.07
3	3	21	27	1.74	1.87
4	4	20	28	0.60	0.68
5	5	19	29	0.38	0.38
6	6	18	30	0.19	0.25
7	7	17	31	0.18	0.18
8	8	16	32	0.14	0.14
9	9	15	33	0.10	0.11
10	10	14	34	0.09	0.09
11	11	13	35	0.08	0.08
12	12	12	36	0.02	0.07
13	13	11	37	0.07	0.07
14	14	10	38	0.07	0.07
15	9	15	33	0.06	0.07
16	8	16	32	0.04	0.08
17	7	17	31	0.09	0.09
18	6	18	30	0.08	0.11
19	5	19	29	0.15	0.15
20	4	20	28	0.23	0.26
21	3	21	27	0.63	0.68
22	2	22	26	2.42	2.50
23	1	23	25	0.63	0.63
24	0	24	24	0.50	0.50
25	1	23	25	0.63	0.63
26	2	22	26	2.42	2.50
27	3	21	27	0.63	0.67
28	4	20	28	0.22	0.25
29	5	19	29	0.14	0.14
30	6	18	30	0.08	0.09

Compared to lower-fold symmetric configurations (as discussed in Ref. [16]), the higher periodicity of the present lattice leads to a clearer separation between primary and alias harmonics. This separation is advantageous for systematic-error analysis, as it allows different contributions to be identified and disentangled using harmonic decomposition of the beam orbit and spin response.

Small deviations from exact 24-fold symmetry introduce additional weak harmonic mixing but do not qualitatively alter the selection rules. In practice, such effects appear as small corrections to the dominant N and $24 \pm N$ components and can be monitored and corrected using beam-based alignment. No near-degenerate resonances are expected for the working range of the vertical tune ν_y , and the alias structure remains well resolved.

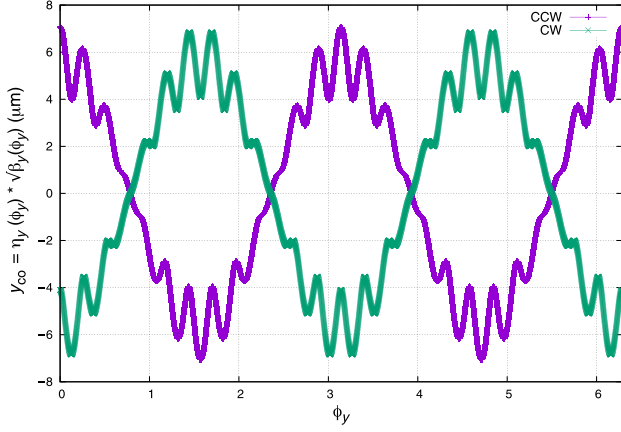
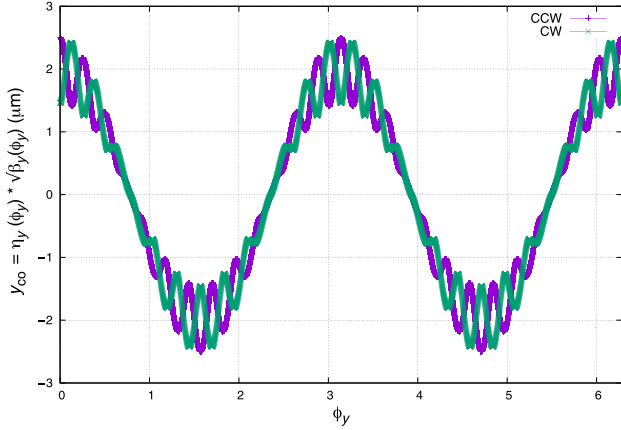
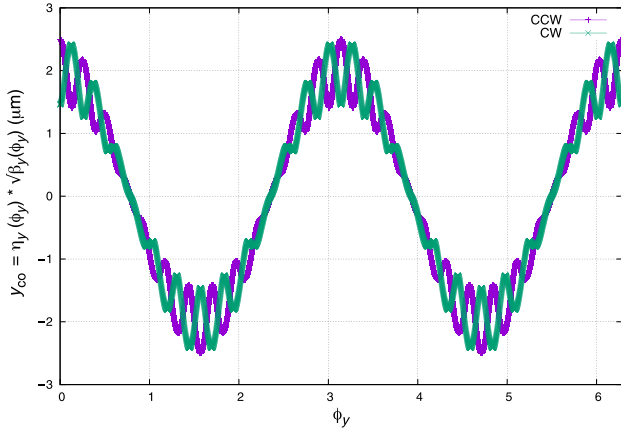
(a) Closed-orbit distortion for $N = 2$, $L = 22$, $M = 26$ (b) Closed-orbit distortion for $N = 26$, $L = 22$, $M = 26$ (c) Closed-orbit distortion for $N = 22$, $L = 22$, $M = 26$

FIG. 5. Vertical beam separation of CW-CCW beams around the ring azimuth ϕ_y upon $B_x = 1$ nT radial magnetic field. Both the $N = 26$ and $N = 22$ harmonics are clearly seen in the four crossings with negative interference at $\phi_y = \pi/4, 3\pi/4, 5\pi/4, 7\pi/4$. The CCW-oriented beam is mapped to the positive ϕ_y direction ($\phi_y \in [0, 2\pi]$), such that the ϕ_y coordinate corresponds to the same physical locations for both the CW and CCW beams. (a) The closed-orbit distortion with a $\cos(2\phi_y)$ term has opposite sign for the CW/CCW beams—an EDM background; (b) The $\cos(26\phi_y)$ and (c) the $\cos(22\phi_y)$ terms have opposite sign for the CW/CCW beams—an EDM background.

3. EDM-like background expressed via fourier harmonics

For instance, the $k_{E,c} \cos(2\phi_y)$ term couples with the closed-orbit terms corresponding to Fourier harmonics $N = 2$, $L = 22 = 24 - 2$, and $M = 26 = 24 + 2$, as shown below:

$$\begin{aligned}
 (k_E)_{N=2} \cdot y_{\text{co},N=2}(\phi_y) &= k_{E,c} \cos(2\phi_y) [a_{\text{co},c,2} \cos(2\phi_y) + b_{\text{co},c,2} \cos(22\phi_y) \\
 &\quad + c_{\text{co},c,2} \cos(26\phi_y) + \dots] + \sin \text{ terms} \\
 &= k_{E,c} \cos^2(2\phi_y) [a_{\text{co},c,2} + \cos(24\phi_y)(b_{\text{co},c,2} + c_{\text{co},c,2}) \\
 &\quad + \dots] + \sin \text{ terms}, \tag{A10}
 \end{aligned}$$

where the bracketed terms are depicted in Fig. 5, while the small terms $[(b_{\text{co},c,2} - c_{\text{co},c,2})/2] \cos(22\phi_y)$, $[(b_{\text{co},c,2} - c_{\text{co},c,2})/2] \cos(26\phi_y)$, and other higher-order terms are neglected. The EDM-like background arises from the product of the electric quadrupole multipole and the closed-orbit distortion.

The closed-orbit distortion in the top panel [Fig. 5(a)] exhibits a $\cos(2\phi_y)$ term with opposite signs for the CW and CCW beams, indicating an EDM background. Similarly, the $\cos(22\phi_y)$ and $\cos(26\phi_y)$ terms in the middle [Fig. 5(b)] and bottom panels [Fig. 5(c)] show opposite signs for the CW and CCW beams, also signifying an EDM background. However, the Fourier harmonics with $L = 22$ and $M = 26$ in the top panel, $L = 22$ and $M = 26$ in the middle panel, and $L = 22$ and $M = 26$ in the bottom panel do not contribute to the EDM background. By employing electric BPM systems [24] alongside magnetic BPM systems [25], one can differentiate the closed-orbit distortion effects arising from primary field harmonics ($N = 2$) and higher field harmonics ($N = 22$ and $N = 26$). These higher harmonics produce aliasinglike effects in combination with the 24 FODO sections in the storage ring, as illustrated in Fig. 5.

APPENDIX B: NUMERICAL ANALYSIS METHOD

Figure 6(a) presents the maximum amplitude of the closed-orbit distortion upon $B_x = 1$ nT field N harmonic around the ring azimuth, evaluated for different integration step sizes. Notably, for higher N , the numerical integration converges when the integration step size is ≤ 1 mrad, where the step size corresponds to $d\phi_y$ in Eq. (A8). For this work, an integration step size of 0.1 mrad was selected. The numerical convergence of the closed-orbit distortion calculation has been verified systematically by computing the relative convergence error,

$$\epsilon = \frac{|y_{\text{co}}^{\text{max}}(d\phi_y) - y_{\text{co}}^{\text{max}}(d\phi'_y)|}{|y_{\text{co}}^{\text{max}}(d\phi_y)|}$$

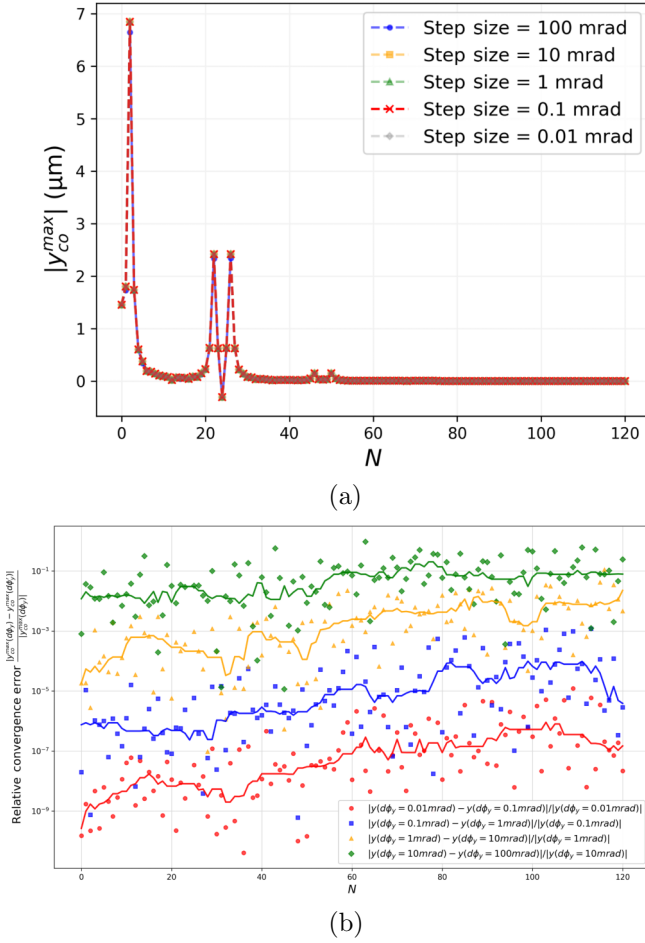


FIG. 6. (a) The maximum amplitude of the close-orbit distortion upon $B_x = 1$ nT field N harmonic around the ring azimuth for various integration step size. The CW+ rotating beam is shown as an example. Notably, for higher N , the numerical integration converges when the integration step size is ≤ 1 mrad. (b) The numerical convergence of the closed-orbit distortion calculation has been verified systematically by computing the relative convergence error for successive step size refinements. As shown, the adopted settings of $d\phi_y = 0.1$ mrad and QUADPACK tolerances of 10^{-14} yield a maximum relative convergence error below 10^{-5} (10 ppm), which is well within the required accuracy. The key results presented in the paper are stable under variation of these settings, as demonstrated by the convergence plot.

for successive step size refinements, where N is the harmonic index, $d\phi_y$ is the azimuthal step size and $d\phi'_y$ is the successive azimuthal step size. The results are shown in Fig. 6(b). The calculation was performed with step sizes ranging from $d\phi_y = 100$ mrad down to $d\phi_y = 0.01$ mrad, retaining harmonics up to $N = 120$. The integration at each evaluation point was carried out using the QUADPACK adaptive quadrature routine [29], with absolute and relative tolerances of 10^{-14} . As shown in Fig. 6(b), upon refinement from $d\phi_y = 0.1$ mrad to $d\phi_y = 0.01$ mrad (red circles), the relative convergence

error remains below 10^{-5} across all harmonic orders $N = 0$ –120, confirming that the result is well converged at $d\phi_y = 0.1$ mrad. The successive refinements from coarser step sizes (green diamonds, orange triangles, and blue squares) show systematic reduction of the relative convergence error by approximately two orders of magnitude per decade of step size reduction, consistent with the expected convergence behavior of the adaptive quadrature scheme. To address the concern regarding aliasing and dispersion, we note that the evaluation points ϕ_y serve only as sampling locations at which the integral [Eq. (A8)] is evaluated independently via adaptive quadrature. Since each evaluation is an independent numerical integration rather than a discrete Fourier sum, the result is not subject to aliasing in the classical sense. The smoothness and stability of the envelope solid lines in Fig. 6(b) across all N confirms the absence of dispersion artifacts. With respect to the ~ 1 mrad/s numerical goal, the adopted settings of $d\phi_y = 0.1$ mrad and QUADPACK tolerances of 10^{-14} yield a maximum relative convergence error below 10^{-5} (10 ppm), which is well within the required accuracy. The key results presented in the paper are stable under variation of these settings, as demonstrated by the convergence plot.

APPENDIX C: FIELD MAPPING, ALIGNMENT, AND B_x BENCHMARK

The value $B_x = 1$ nT used throughout this work serves as a representative benchmark for evaluating systematic effects, rather than a strict requirement for experimental operation. The primary requirement is instead the control of the relative orbit of the counter-rotating (CW-CCW) beams, which must be maintained at the level of $\mathcal{O}(10 \mu\text{m})$ around the ring.

As demonstrated in Ref. [16] (see Fig. 9), this level of control corresponds to quadrupole alignment tolerances of order 1–10 μm (rms). In particular, the vertical spin precession induced by lattice imperfections is strongly suppressed after CW-CCW cancellation when such alignment conditions are achieved, while the corresponding beam separation provides a direct diagnostic of residual systematic effects.

The effective radial magnetic field $B_x(\phi_y)$ can be expressed as a harmonic expansion around the ring azimuth, and its relevant components are those that couple to the dominant closed-orbit distortion modes. Experimentally, these harmonics will be inferred from the measured CW-CCW beam separation using beam position monitors, enabling beam-based alignment through harmonic decomposition.

In addition, spin-based alignment techniques will be employed, in which controlled external fields are applied to deliberately enhance the vertical spin precession signal. By varying the applied field harmonics, the underlying

$B_x(\phi_y)$ components and associated lattice imperfections can be mapped and corrected iteratively.

The scale of the B_x benchmark can be understood from typical lattice parameters. For quadrupole gradients of ~ 0.2 T/m and element lengths of ~ 0.4 m, a transverse misalignment at the level of $1\ \mu\text{m}$ corresponds to an effective field of order 10^{-7} T locally, which translates to an azimuthally averaged field at the nT level. This motivates the use of $B_x \sim 1$ nT as a representative scale for systematic studies.

Operationally, the experiment will proceed from an initial magnetic environment at the ~ 100 nT level, achieved using conventional magnetic shielding and monitoring, followed by beam-based and spin-based alignment procedures to reduce the effective field harmonics below the ~ 10 nT level and ultimately toward the nT regime after about 4000 separate injections, each one of them with reversed (air-core) quadrupole currents.

Long-term stability of the lattice elements is a critical requirement for achieving these conditions. In particular, quadrupole alignment at the micron level will be maintained using precision survey and stabilization systems such as hydrostatic leveling systems, as implemented in modern light sources [30].

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