

Plasma dynamics in transient electrostatic shocks driven by relativistic few-cycle laser pulses

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The rapid advancement of laser compression techniques has brought increasing attention to few-cycle laser-plasma interactions. Through comprehensive particle-in-cell simulations, we demonstrate that compressing laser pulses to sub-10-femtosecond durations significantly enhances transient electrostatic shocks. Crucially, the plasma dynamics are strongly modulated by the laser carrier-envelope phase (Φ_{CEP}). Our results reveal a pronounced cosinelike dependence of the proton cutoff energy on Φ_{CEP} , with certain values of Φ_{CEP} yielding superior performance due to enhanced electron displacement and stronger charge-separation fields. Phase-space analysis uncovers expanding “source”-like electron orbits, whose gyration radii correlate with Φ_{CEP} -dependent energy transfer. We further develop an analytical model connecting the electrostatic shock strength, inferred from the proton energy, to the laser-plasma interaction parameters. This model is validated through extensive parameter scans over target density, thickness, laser intensity, and Φ_{CEP} . Our findings establish a predictive framework for Φ_{CEP} -controlled plasma dynamics in short-pulse laser interactions, providing insight into mechanisms relevant to laboratory high-energy proton sources and origin of energetic cosmic rays in astrophysical environments.

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Over the past few decades, substantial progress has been made in reducing laser pulse durations to enhance peak intensity [1,2], leading to booming explorations of laser-matter interactions [3–7]. Through external compression techniques such as self-phase modulation, optical pulses can now be shortened to just a few cycles [8–12]. Further compression has been achieved by tailoring the oscillator characteristics [13] and synchronizing independently mode-locked lasers based on a balanced optical cross correlator [14]. In particular, the thin-film compressor technique proposed by Mourou *et al.* [15,16] enables the compression of high-energy, tens-of-femtosecond laser pulses down to the few-femtosecond regime. Moreover, few-cycle lasers can also be generated through laser-plasma interactions [17–23]. As a result, few-cycle lasers with extremely high peak power have become feasible. Given a fixed laser energy, pulse compression increases both the power and intensity, which facilitates ultrashort, ultraintense interactions [24–27].

These advances would potentially drive significant developments in ultrafast science, including nonlinear atomic and molecular dynamics [28,29], high-order harmonic generation [28,30,31], terahertz emission [32–34], and attosecond radiation [35–37]. More recently, the advent of few-cycle lasers has also enabled laboratory-scale investigations of acceleration in electrostatic shocks [38,39], which are intimately connected to the astrophysical generation of high-energy cosmic rays [40,41]. A thorough understanding of laser-driven ion acceleration is crucial for applications in inertial fusion energy and high-energy-density physics [42,43], helping in gaining profound insights into structural information of nuclear density and wave functions in nuclear physics research [44,45].

Few-cycle laser pulses have been widely employed to drive electrostatic shocks in gaseous targets [46–53]. Theoretical methods have been proposed to generate attosecond electron bunches [54], and experiments have been carried out to generate high-quality electron beams [47,55,56]. Furthermore, electron emission can be excited by few-cycle lasers in solid targets [57–59], where the laser carrier-envelope phase is a crucial factor. For more interactions of few-cycle lasers with solid targets [60,61], notably, Zhou *et al.* showed that single-cycle, ultrahigh-intensity lasers can suppress transverse instabilities and produce higher proton cutoff energies than longer-duration

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pulses [62]. Wu *et al.* further reported that few-cycle pulses achieve superior laser-to-ion energy conversion compared to multicycle regimes [63]. Despite these promising results, the influence of Φ_{CEP} on the plasma dynamics induced by ultrashort laser pulses in solid-density targets remains largely unexplored.

In this work, we investigate how Φ_{CEP} governs the plasma dynamics in transient electrostatic shocks driven by relativistic few-cycle laser pulses. We find that the areal density of the target determines which laser electric field peak first interacts effectively with the plasma electrons. When a stronger peak is selected, electrons are displaced farther, gaining more energy, and subsequently oscillate under the restoring field from the ion layer. These oscillations exhibit larger gyration radii in phase space, leading to enhanced electrostatic shock formation and more efficient proton acceleration. As a result, different target thicknesses can lead to distinct peaks in proton energy due to the selection of different laser field maxima. This study offers a theoretical foundation for understanding and controlling few-cycle laser-plasma interactions, with implications for laboratory astrophysics and high-energy proton sources.

To investigate the effects of Φ_{CEP} on electrostatic shock formation and proton acceleration, we performed a series of two-dimensional (2D) and one-dimensional (1D) particle-in-cell simulations using the EPOCH code [64]. In the 2D setup, a linearly polarized Gaussian laser pulse with a wavelength of $\lambda = 800$ nm and a focal spot radius of $3 \mu\text{m}$ is normally incident on a fully ionized solid-density target composed of carbon and proton species in equal proportions. The electron density is fixed at $200n_c$, where n_c is the critical density, and the target thickness L is varied from 30 to 80 nm, located $5 \mu\text{m}$ from the simulation boundary. As $t = 0$ corresponds to the simulation start time, the laser hits the target at $t = 16.7$ fs. The laser pulse duration full width at half maximum (FWHM) is varied among 3, 10, and 30 fs, while keeping the total laser energy constant. Correspondingly, the normalized laser amplitude is $a_0 = 30$ for 30 fs, $a_0 = 52$ for 10 fs, and $a_0 = 95$ for 3 fs, where $a_0 = eE_0/m\omega c$ is the normalized laser amplitude. The simulation domain spans $25 \times 16 \mu\text{m}$, resolved by 7500×1600 cells. We scan the carrier-envelope phase Φ_{CEP} from $-\pi/2$ to $\pi/2$ in steps of $\pi/10$, with an additional point at $\Phi_{\text{CEP}} = \pi$. For detailed analysis, 1D simulations are also performed under the same parameters as the 2D case, using 3 fs pulses with $\Phi_{\text{CEP}} = 0$ and π . The 1D simulation domain is $25 \mu\text{m}$, discretized into 10,000 cells with 100 macroparticles per species per cell. Extensive parameter scans are carried out, covering laser amplitudes a_0 from 10 to 1000, target densities n_e from $10n_c$ to $1000n_c$, target thicknesses from 10 to 1000 nm, and Φ_{CEP} from 0 to 2π , with 50 data points per parameter axis. To check if lasers with the same amplitude and different Φ_{CEP} will affect our results, we conduct additional simulations and the results are shown in Appendix A. We also conduct simulations with radiation reaction and find an insignificant difference, as described in Appendix B.

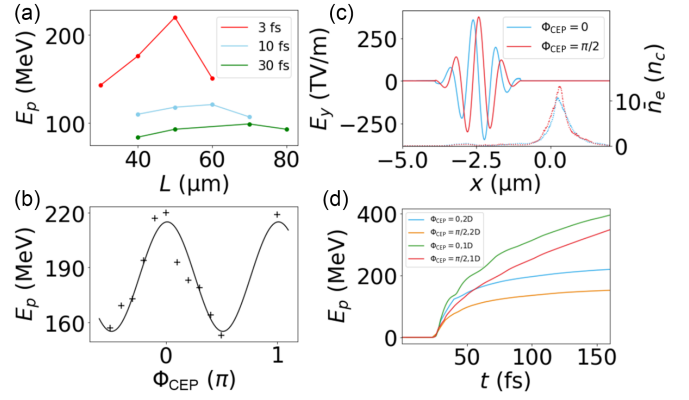


FIG. 1. (a) Variation of proton cutoff energy versus target thickness for laser pulse durations (FWHM) of 3, 10, and 30 fs in 2D simulations, demonstrating significant electrostatic shock enhancement through pulse compression. (b) Cosinelike dependence of proton cutoff energy on Φ_{CEP} from 2D simulations for the case with laser pulse duration of 3 fs. The 3 fs pulses are also employed in the following analysis. (c) Laser electric field distributions and target axial electron density distributions. Solid curves: laser pulse profiles at $\Phi_{\text{CEP}} = 0$ and $\Phi_{\text{CEP}} = \pi/2$; dashed curves: target expansion at 50 fs in 2D simulations. (d) Temporal evolution of maximum proton energy from both 1D and 2D simulations.

For a given laser energy, pulse compression enhances the laser intensity, thereby increasing the electrostatic shock and proton energy. Figure 1(a) shows the proton cutoff energy as a function of target thickness for different laser pulse durations. The proton cutoff energy E_p is derived from the maximum energy of protons at $t = 130$ fs. Shorter pulses yield significantly higher proton energies, with the 3 fs pulse enhancing the cutoff energy by 222% compared to the 30 fs case. Moreover, as the pulse duration decreases, the optimal target thickness for shock formation also becomes thinner, in agreement with previous studies [62,65]. When the pulse duration is reduced to a few cycles, Φ_{CEP} begins to significantly influence proton acceleration. Figure 1(b) reveals a clear cosinelike dependence of the proton cutoff energy on Φ_{CEP} for a 3 fs pulse at $a_0 = 95$ and a target thickness of 50 nm. The energy peaks when $\Phi_{\text{CEP}} = 0$, showing a periodicity of π .

To reveal the physical process, pulses with $\Phi_{\text{CEP}} = 0$ and $\Phi_{\text{CEP}} = \pi/2$ are analyzed, which are shown as solid lines in Fig. 1(c). The growth rates of proton energies from 20 to 40 fs are significantly higher for $\Phi_{\text{CEP}} = 0$, as demonstrated in Fig. 1(d), resulting in higher cutoff energies. This comes from a stronger electron expulsion at $\Phi_{\text{CEP}} = 0$, leading to an enhanced target expansion. This is confirmed by the dashed lines in Fig. 1(c), showing the electron density profiles at 50 fs. For clarity, 1D simulations with parameters identical to 2D simulations are chosen to analyze the proton energy evolution, as shown in Fig. 1(d).

To further elucidate the underlying physical mechanisms, we present the laser electric field distributions

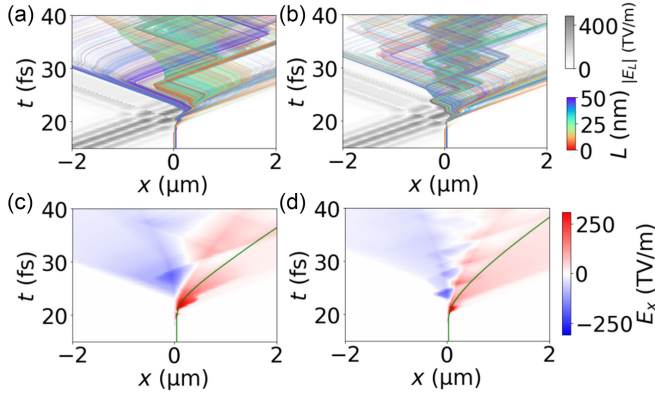


FIG. 2. Plot of electron location and time for (a) $\Phi_{\text{CEP}} = 0$ and (b) $\Phi_{\text{CEP}} = \pi/2$, with colors indicating their initial positions in the target. The oscillation amplitudes are larger at $\Phi_{\text{CEP}} = 0$. Gray scale represents laser electric field magnitude. The time evolution of electrostatic shock for (c) $\Phi_{\text{CEP}} = 0$ and (d) $\Phi_{\text{CEP}} = \pi/2$, showing stronger electrostatic shock at $\Phi_{\text{CEP}} = 0$ from $t = 20$ to $t = 40$ fs. Green curves show the traces of the highest-energy protons.

and target electron trajectories at different times for $\Phi_{\text{CEP}} = 0$ and $\Phi_{\text{CEP}} = \pi/2$ in Figs. 2(a) and 2(b), respectively. The gray shading represents the magnitude of the laser electric field, with the reflected wave exhibiting spectral broadening and a redshift. The electron trajectories, which are color-identified by their initial positions, reveal that most electrons oscillate near the target. This is because the laser electric field expels electrons forward, while the heavier ions remain quasistationary, establishing a charge-separation field. This field exerts a restoring force on the displaced electrons. When a laser electric field peak ceases to drive them effectively after passing them, the restoring force tends to pull the electrons back toward the target. Due to their residual momentum, the electrons pass the target backward before they are pulled forward again, resulting in sustained oscillations. As electrons gain energy from the laser field, the oscillation amplitude increases over time.

In particular, the electron oscillations for $\Phi_{\text{CEP}} = 0$ exhibit larger amplitudes than those for $\Phi_{\text{CEP}} = \pi/2$, confirmed by the stronger redshift of the reflected wave in Fig. 2(a). This difference arises because the first selected laser field peak determined by the target parameters to effectively push electrons at $\Phi_{\text{CEP}} = 0$ is stronger [Fig. 2(a)] and displaces electrons farther, generating a larger charge-separation field, as depicted in Figs. 2(a) and 2(c). In contrast, a weaker selected peak at $\Phi_{\text{CEP}} = \pi/2$ limits the electron displacement and subsequent acceleration, yielding smaller oscillations and a weaker charge-separation field in Fig. 2(d). From $t = 20$ to $t = 40$ fs, the electrostatic shock for $\Phi_{\text{CEP}} = 0$ significantly exceeds that of $\Phi_{\text{CEP}} = \pi/2$, which is described in Figs. 2(c) and 2(d), directly explaining the differences in the proton energy observed in Fig. 1(d).

Figures 3(a) and 3(b) show the phase-space trajectories of some electrons, with colors matching Figs. 2(a) and 2(b),

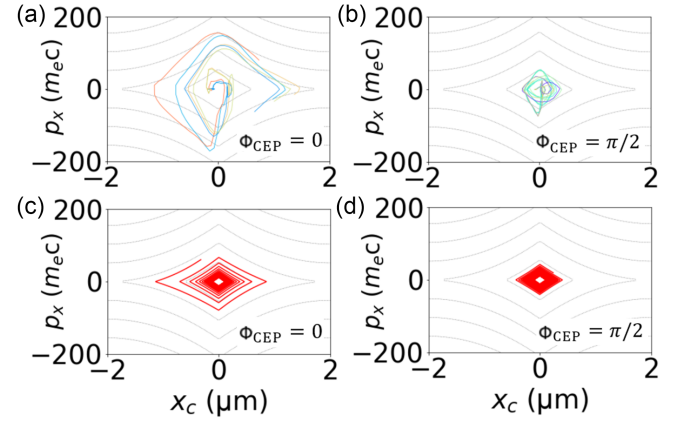


FIG. 3. Electron phase-space trajectories for (a) $\Phi_{\text{CEP}} = 0$ and (b) $\Phi_{\text{CEP}} = \pi/2$, where $x_c = 0$ is the gyration center, exhibiting expanding diamond-shaped orbits with larger radii at $\Phi_{\text{CEP}} = 0$. The colors of lines in (a) and (b) are the same as those in Figs. 2(a) and 2(b), respectively, representing the initial positions of particles in the target. Background dashed curves represent source-free Hamiltonian contours. Model-predicted electron trajectories in phase space for (c) $\Phi_{\text{CEP}} = 0$ and (d) $\Phi_{\text{CEP}} = \pi/2$.

respectively. The pushing forces of the laser electric field and the restoring forces induced by the charge-separation field result in spiral trajectories of electrons in phase space. While electrons would ideally trace contours of constant Hamiltonian (dashed lines), continuous energy gain from the laser field enlarges their gyration radii, forming a source pattern. For $\Phi_{\text{CEP}} = 0$, the stronger selected field peak transfers more energy to electrons, resulting in a larger gyration radius consistent with Fig. 2(a) and thus a stronger electrostatic shock for protons.

To describe the motion of the source in contrast to the attractor [66], we propose a theoretical model. Assuming the longitudinal electrostatic shock and potential satisfy:

$$E_x = \begin{cases} -\frac{E_0}{x_0}x - E_0, & -x_0 < x \leq 0 \\ -\frac{E_0}{x_0}x + E_0, & 0 < x \leq x_0 \\ 0, & \text{else} \end{cases} \quad \phi = \begin{cases} \frac{E_0}{2x_0}(x^2 - x_0^2) + E_0(x + x_0), & -x_0 < x \leq 0 \\ \frac{E_0}{2x_0}(x^2 - x_0^2) - E_0(x - x_0), & 0 < x \leq x_0 \\ 0, & \text{else} \end{cases} \quad (1)$$

where $E_0 = \eta_0 \frac{n_e e L}{2\epsilon_0}$ and $\eta_0 = 50\%$. In our model, $2x_0 = 4 \mu\text{m}$ is used to describe the electric field distribution. From the canonical equations $\dot{p}_x = -\partial H/\partial x = -eE_x$ and $\dot{x} = \partial H/\partial p_x = p_x c / \sqrt{p_x^2 + p_y^2 c^2 + m_e^2 c^2}$ [67,68], we can obtain the conserved Hamiltonian as

$$H = \sqrt{p_x^2 c^2 + p_y^2 c^2} + m_e^2 c^4 - e\phi. \quad (2)$$

Here, $e = 1.6 \times 10^{-19}$ C is the electron charge, $c = 3 \times 10^8$ m/s is the speed of light, and $m_e = 9.11 \times 10^{-31}$ kg is the electron mass. The Hamiltonian phase-space contours (dashed lines in Fig. 3) are centered at the equilibrium point $(x_c, 0)$. The longitudinal force is the Lorentz force experienced by an electron in the laser field: $F = k_0 e v_y B_z \approx k_0 e c B_z / \sqrt{2} \approx k_0 e E_y / \sqrt{2} = k_0 m_e \omega c a(x) / \sqrt{2} = k_0 m_e \omega c a_0 \exp(-4 \ln 2 x^2 / L_{\text{FWHM}}^2) / \sqrt{2}$, where k_0 is an effective coefficient. For an intense laser $a \gg 1$, the Lorentz factor is $\gamma = \sqrt{1 + a^2} \approx a$. Then we can obtain $F \approx k_0 m_e \omega c \gamma / \sqrt{2} \approx k_0 \omega \sqrt{m_e^2 c^2 + p_x^2 + p_y^2} / \sqrt{2} \approx k_0 \omega p_x$. Here, B_z is the laser magnetic field, E_y is the laser electric field with a maximum of $E_{y0} = 3.8 \times 10^{14}$ V/m, ω is the laser frequency, and L_{FWHM} is the full width at half maximum of the laser pulse. In our model, $v_y B_z \approx c B_z \approx E_y$, so $|p_y| = |p_x|$ in the laser field is used in this equation and further analysis. For an electron co-moving with the laser pulse at half maximum, $\partial F / \partial x = -\sqrt{2} \ln 2 k_0 e E_y / L_{\text{FWHM}}$. Taking into account the longitudinal force $F = k_0 \omega p_x$, the equations of motion for a nonconservative system become

$$\begin{aligned} \dot{p}_x &= -\frac{\partial H}{\partial x} + F = -eE_x + F = f(x, p_x), \\ \dot{x} &= \frac{\partial H}{\partial p_x} = \frac{p_x c}{\sqrt{p_x^2 + p_y^2 + m_e^2 c^2}} = g(x, p_x). \end{aligned} \quad (3)$$

The Jacobian matrix near the equilibrium point is

$$\begin{aligned} \mathbf{J}_a &= \begin{pmatrix} \frac{\partial g}{\partial x} & \frac{\partial g}{\partial p_x} \\ \frac{\partial f}{\partial x} & \frac{\partial f}{\partial p_x} \end{pmatrix} \\ &= \begin{pmatrix} 0 & \frac{p_y^2 c + m_e^2 c^3}{(p_x^2 + p_y^2 + m_e^2 c^2)^{3/2}} \\ \frac{eE_0}{x_0} - \frac{\sqrt{2} \ln 2 k_0 e E_{y0}}{L_{\text{FWHM}}} & k_0 \omega \end{pmatrix}, \end{aligned} \quad (4)$$

where $T = 2.67$ fs is the laser duration, and k_0 is a coefficient determined by the valid energy of the first effective field peak [colored area in Fig. 4(d)]. For $\Phi_{\text{CEP}} = 0$, $k_0 = 0.22$ is used to obtain Fig. 3(c); for $\Phi_{\text{CEP}} = \pi/2$, $k_0 = 0.14$ is utilized to acquire Fig. 3(d). Equation (3) can be written as $\mathbf{J}_a \mathbf{X} = \lambda \mathbf{X}$, whose eigenvalues are given by $\lambda_{\pm} = \frac{1}{2} (\text{Tr}(\mathbf{J}_a) \pm \sqrt{(\text{Tr}(\mathbf{J}_a))^2 - 4 \text{Det}(\mathbf{J}_a)})$. The general solution is $\mathbf{X} = (x, p_x)^T = c_1 e^{\lambda_+ t} \mathbf{X}_1 + c_2 e^{\lambda_- t} \mathbf{X}_2$, where c_1 and c_2 are the undefined constants and $\mathbf{X}_1$ and $\mathbf{X}_2$ are

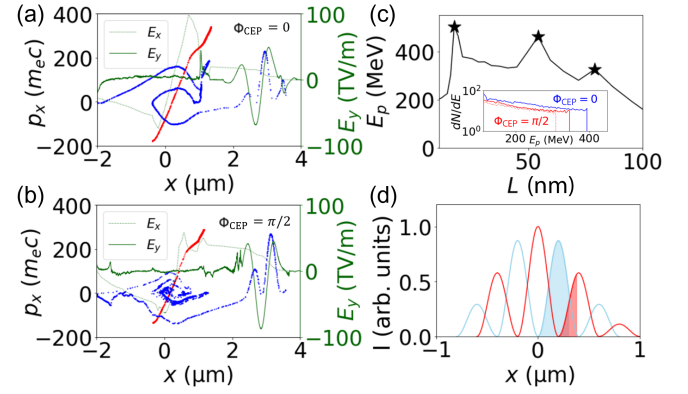


FIG. 4. Spatial distributions of laser electric fields (solid green line) and electrostatic shocks (dotted green line), along with electron (blue dots) and proton (red dots) phase-space distributions depicting acceleration dynamics for (a) $\Phi_{\text{CEP}} = 0$ and (b) $\Phi_{\text{CEP}} = \pi/2$. The longitudinal momentum of protons is modified by 1/3 and the laser electric field is modified by 1/4. (c) The variation of proton energy versus target thickness at $a_0 = 95$ and $n_e = 200 n_c$ for $\Phi_{\text{CEP}} = 0$, with the stars marking energy peaks. The inset figure shows the proton energy spectrum at 130 fs for $\Phi_{\text{CEP}} = 0$ and $\Phi_{\text{CEP}} = \pi/2$. The dotted line shows the energy spectrum for $\Phi_{\text{CEP}} = \pi/2$ with the same amplitude as $\Phi_{\text{CEP}} = 0$. (d) Schematic of theoretical models: blue curve shows the four-peak model for areal density effects when $\Phi_{\text{CEP}} = 0$, red curve shows the model for Φ_{CEP} effects when $\Phi_{\text{CEP}} = \pi/2$. Colored areas represent valid laser energy for electron acceleration.

the eigenvectors. Since $[\text{Tr}(J_a)]^2 - 4 \text{Det}(J_a) < 0$ at initial time, $\lambda_{\pm}$ can be rewritten as $\lambda_{\pm} = \text{Tr}(J_a)t/2 + i\Delta t/2$, where $\Delta = \sqrt{4 \text{Det}(J_a) - [\text{Tr}(J_a)]^2}$. For a positive eigenvalue, the linear independent solution is $e^{\lambda_+ t} \mathbf{X}_1 = e^{\text{Tr}(J_a)t/2 + i\Delta t/2} (x_1, p_{x1})^T = e^{\text{Tr}(J_a)t/2} (x_1, p_{x1})^T \cos(\frac{\Delta t}{2}) + i e^{\text{Tr}(J_a)t/2} (x_1, p_{x1})^T \sin(\frac{\Delta t}{2})$. The gyration radius varies with a factor $e^{\text{Tr}(J_a)t/2}$ and the direction is determined by $\Delta t/2$. So $\text{Tr}(J_a)$ determines whether it is a source or a sink and Δ or $\text{Det}(J_a)$ determines the gyration orbit. If $\text{Tr}(J_a)$ is positive, then we have a spiral source at the origin, otherwise we have a spiral sink at the origin. Moreover, if $\text{Det}(J_a)$ is negative, which means $(\text{Tr}(J_a))^2 - 4 \text{Det}(J_a) > 0$, there is a saddle at the origin. In our results, the conditions of $\text{Det}(\mathbf{J}_a) > 0$ and $\text{Tr}(\mathbf{J}_a) > 0$ imply a source at the origin [69]. The predicted electron trajectories, shown as diamond-shaped expanding traces in Figs. 3(c) and 3(d), agree with the simulation results, showing faster expansion in the gyration radius for $\Phi_{\text{CEP}} = 0$.

Figures 4(a) and 4(b) present the electron (blue dots) gyration and proton (red dots) acceleration processes in transient electrostatic shocks for $\Phi_{\text{CEP}} = 0$ and $\Phi_{\text{CEP}} = \pi/2$, respectively, along with the laser electric field (solid green line) and the electrostatic shock (dotted green line) distributions. For $\Phi_{\text{CEP}} = 0$, most electrons are accelerated forward to higher energies by the laser, resulting in larger gyration radii and consequently a stronger electrostatic shock, which enhances proton acceleration, as depicted by the proton energy spectrum in the inset figure of Fig. 4(c).

The Φ_{CEP} influences the electrostatic shock by determining the amplitude of the first effective electric field peak that drives the electrons, thereby affecting their gyration radii and electrostatic shocks. This theory suggests that the target density and thickness also play a critical role in selecting the effective driving field peak. Figure 4(c) shows the proton energy as a function of target thickness from 1D simulations at $a_0 = 95$, revealing three distinct peaks that correspond to the last three electric field peaks of the laser. The accelerated proton energy spectrum for $\Phi_{\text{CEP}} = 0$ and $\Phi_{\text{CEP}} = \pi/2$ is shown in the inset figure. The dotted line shows the energy spectrum for $\Phi_{\text{CEP}} = \pi/2$ with the same amplitude as $\Phi_{\text{CEP}} = 0$, verifying that the proton energy is affected by Φ_{CEP} rather than amplitude. To further elucidate the role of field peaks in the generation of electrostatic shocks indicated by the proton acceleration, we propose a theoretical model, as illustrated in Fig. 4(d).

The model employs a four-peak structure, shown as the blue curve in Fig. 4(d), representing the laser electric field distribution for $\Phi_{\text{CEP}} = 0$ (red curve for $\Phi_{\text{CEP}} = \pi/2$). Since target electron heating consumes part of the laser energy, which is depicted as the part before the blue area in Fig. 4(d), the remaining energy in the first effective field peak (blue area) drives electron motion. A larger blue area means a greater electron energy gain. Once this peak passes, the laser field reverses, terminating further electron acceleration and initiating gyration motion. The laser intensity in our model is given by

$$I(x) = C(\Phi_{\text{CEP}})E_p^2 \sin^2(2.5\pi x), \quad (5)$$

where $C(\Phi_{\text{CEP}}) = e^{-4x_{\text{peak}}^2 \ln 2/L_{\text{FWHM}}^2}$ is the envelope coefficient determined by ϕ_{cep} , and $x_{\text{peak}} = (\Phi_{\text{CEP}} + \pi/2 + k\pi)\lambda/2\pi$ is the position of the field peak, where $\Phi_{\text{CEP}} + \pi/2 + k\pi \in [-\pi/2, \pi/2]$. The proton energy depends on the residual energy of the first effective field peak [blue and red areas in Fig. 4(d)]:

$$E_p = C(\Phi_{\text{CEP}})k_1 a_0^2 \left(\int_{x_{I=0} < x_t(n_e, L)} \sin^2(2.5\pi x) dx + E_{p0} \right), \quad (6)$$

where $k_1 = 0.1$ is the acceleration coefficient and $E_{p0} = 0.05$ MeV is the energy constant. Here, $x_{I=0}$ is the first zero point of the light intensity I that satisfies $x_{I=0} < x_t$. For example, when $\Phi_{\text{CEP}} = 0$, $x_{I=0} = 0$. The position x_t is determined by the condition $k_2 \int_{x_t(n_e, L)}^\infty I dx = (\eta n_e L)^2$, where $\eta = 0.01$ quantifies the areal density efficiency for laser energy depletion and $x_0 = 0.2 \mu\text{m}$ represents the transitional length from heating to expelling. The heating efficiency factors k_2 take successive values of 1, 1, 10, and 100 for four consecutive field peaks.

To validate the effectiveness of our model, we performed extensive scans of the proton cutoff energy

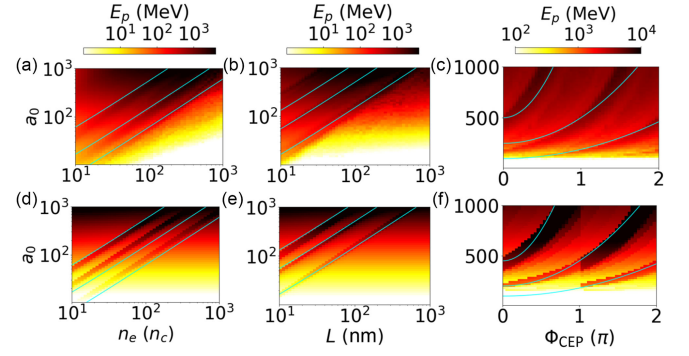


FIG. 5. Proton energy dependence on (a) target electron density, (b) target thickness, and (c) laser Φ_{CEP} at various laser intensities. The scans of the target electron density and the target thickness are conducted under $\Phi_{\text{CEP}} = 0$ and the scans of Φ_{CEP} are performed under $n_e = 200n_c$ and $L = 100$ nm. Corresponding model predictions are shown in (d)–(f), exhibiting good agreement with simulations. The cyan curves, which are the same in both simulations and theoretical predictions, indicate the reference positions of proton energy peaks.

under varying laser intensities (a_0), target densities (n_e), and thicknesses (L) for $\Phi_{\text{CEP}} = 0$, as shown in Figs. 5(a) and 5(b). An additional Φ_{CEP} scan (from 0 to 2π) is presented in Fig. 5(c). These figures reveal three distinct energy bands that correspond to the last three dominant field peaks effectively driving electrons. From Eq. (6), we can obtain the maximum proton energy $E_p = k_1 a_0^2 e^{-4((\Phi_{\text{CEP}} + \pi/2 + k\pi)_{[-\pi/2, \pi/2]} \lambda/2\pi)^2 \ln 2/L_{\text{FWHM}}^2} \times (\int_{x_{I=0} < x_t(n_e, L)} \sin^2(2.5\pi x) dx + E_{p0})$, where $x_t(n_e, L)$ can be derived from $k_2 \int_{x_t(n_e, L)}^\infty I dx = (\eta n_e L)^2$. Theoretical predictions of proton energies as a function of laser normalized amplitude a_0 , plasma density n_e , target thickness L , and Φ_{CEP} are plotted in Figs. 5(d)–5(f). The agreement between simulations and theory confirms the robustness of our model.

In summary, we have investigated the plasma dynamics underlying transient electrostatic shocks driven by relativistic few-cycle laser pulses, with a particular focus on the role of the laser carrier-envelope phase (Φ_{CEP}). Through extensive PIC simulations and analytical modeling, we demonstrate that Φ_{CEP} critically determines which peak of the laser electric field initiates electron displacement. This selection, in turn, governs the strength of the electrostatic shock and the resulting proton energy. Our analysis reveals that the effective energy available for electron acceleration is set by the combination of laser intensity (a_0), target areal density ($n_e L$), and Φ_{CEP} . Electrons driven by a stronger selected field peak exhibit larger gyration amplitudes in phase space, characterized as expanding source-like orbits, which enhance the charge-separation field and shock strength. We developed a theoretical model that captures this mechanism and accurately reproduces the trends observed in large-scale parameter scans. The model offers

a predictive framework for understanding and controlling shock-driven proton acceleration in solid-density plasmas under few-cycle laser irradiation. It also provides a general strategy for optimizing energy transfer by tailoring target thickness, density, and Φ_{CEP} to match specific electric field structures. These results not only support the advancement of compact high-energy ion sources but also shed light on fundamental plasma dynamics relevant to collisionless shocks and high-energy particle generation in astrophysical environments. As few-cycle laser technologies continue to mature, our study provides a theoretical foundation for probing and engineering laser-plasma interactions at ultra-short timescales and extreme field strengths.

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DATA AVAILABILITY

The data that support the findings of this article are not publicly available upon publication because it is not technically feasible and/or the cost of preparing, depositing, and hosting the data would be prohibitive within the terms of this research project. The data are available from the authors upon reasonable request.

APPENDIX A

We conduct additional simulations to check if lasers with the same amplitude and different Φ_{CEP} will affect our results. The control group with $\Phi_{\text{CEP}} = 0$ is the previous result. But the amplitude in the group with $\Phi_{\text{CEP}} = \pi/2$ multiplies a coefficient of 0.93 to achieve the same amplitude as the control group, as shown in Fig. 6(a). In Fig. 6(b), the proton cutoff energy is still lower in the case of $\Phi_{\text{CEP}} = \pi/2$ after multiplying 0.93 than that in $\Phi_{\text{CEP}} = 0$, similar to our original results. Furthermore, we conduct a series of 1D simulations for lasers with the same amplitude and different Φ_{CEP} , verifying that the proton energy is affected by Φ_{CEP} rather than amplitude. Due to a short laser duration, the minimum amplitude is only 0.93 times the maximum amplitude. As a result, the difference made by amplitude is insignificant.

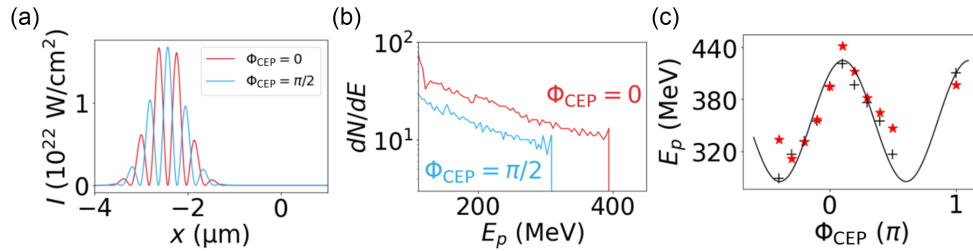


FIG. 6. (a) Laser electric field distribution with the same amplitude for $\Phi_{\text{CEP}} = 0$ and $\Phi_{\text{CEP}} = \pi/2$. (b) Proton energy spectrum for cases in (a), verifying that the proton energy is affected by Φ_{CEP} rather than amplitude. (c) Cosinlike dependence of proton cutoff energy on Φ_{CEP} from 1D simulations for the case with laser pulse duration of 3 fs with the same amplitude. The red stars represent cases with different amplitudes and Φ_{CEP} , verifying that the proton energy is affected by Φ_{CEP} rather than amplitude.

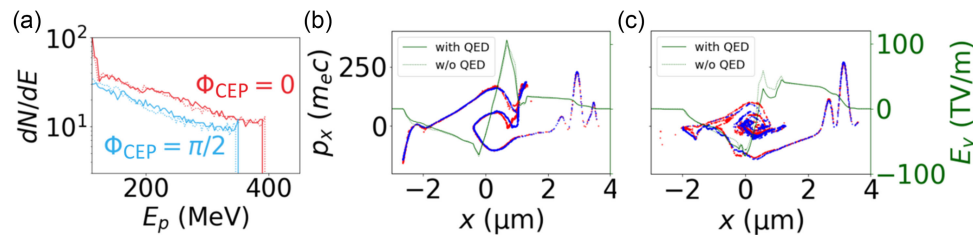


FIG. 7. (a) Proton energy spectrum for $\Phi_{\text{CEP}} = 0$ and $\Phi_{\text{CEP}} = \pi/2$. The solid lines show the results with radiation reaction, while the dotted lines show the results without radiation reaction. (b) Spatial distributions of electron phase-space distributions for $\Phi_{\text{CEP}} = 0$ with QED radiation reaction (red dots) and without QED radiation reaction (blue dots), along with longitudinal electric field distributions (green lines). (c) Spatial distributions of electron phase-space distributions for $\Phi_{\text{CEP}} = \pi/2$ with QED radiation reaction (red dots) and without QED radiation reaction (blue dots), along with longitudinal electric field distributions (green lines). The radiation reaction makes an insignificant difference.

APPENDIX B

To check whether the radiation reaction will make a difference, we conducted additional simulations with the radiation reaction of the quantum electrodynamics (QED) block of EPOCH code. The results are shown in Fig. 7. It can be found in Fig. 7(a) that there is an insignificant difference in the proton energy spectrum. The difference made by different Φ_{CEP} is still obvious. Moreover, the electron oscillations in Figs. 7(b) and 7(c) are similar to those in Fig. 4, showing an insignificant difference in the gyration radius. The acceleration electric fields with radiation reaction also resemble those without radiation reaction. Considering the quantum invariant parameter $\chi_e = 2\gamma E_L/E_{\text{sch}} \approx 2a_0^2/3.25 \times 10^5 \approx 0.0555 \ll 1$, the radiation reaction makes an insignificant difference to our results.

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