

Engineering analysis of the medium energy beam transport of front-end test facility for an Accelerator Driven System

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Beam quality in accelerator transport lines can be significantly degraded by component imperfections such as alignment errors, magnetic field deviations, and rf phase fluctuations. In complex accelerator systems, accurately quantifying the influence of these errors and their interactions is essential for performance optimization. In this study, a variance-based global sensitivity analysis using the Sobol method is applied to the medium energy beam transport section of the CAFE facility to systematically evaluate the impact of multiple error sources on key beam quality metrics, including centroid position, envelope, Twiss parameters, and emittance. Two complementary strategies are employed: a component-level analysis that considers all error types of each element collectively, and an individual-error analysis that examines each source separately. The results show that centroid, envelope, and Twiss parameters are primarily influenced by independent error contributions, dominated by quadrupole translational and magnetic field strength errors, whereas emittance change arises from strong coupling among multiple sources. The model-independent nature of the Sobol method enables comprehensive exploration of parameter interactions, allowing clear identification of the most influential errors and providing guidance for tolerance definition, targeted correction, and system optimization. This work highlights the potential of global sensitivity analysis as a powerful framework for performance evaluation in accelerator design, commissioning, and operation.

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I. INTRODUCTION

In superconducting accelerators, the primary sources of beam loss include nonlinear space-charge effects, beam mismatch, and various imperfections in accelerator components [1–4]. After the design phase, inevitable errors such as alignment inaccuracies may arise during component manufacturing and installation. Additionally, during routine operation, environmental disturbances (e.g., helium pressure fluctuations and rf power source instabilities) can introduce random variations, leading to magnetic field errors.

These imperfections can result in beam centroid shifts, emittance growth, and even beam loss. Beam loss must be strictly controlled [5,6], and this requirement is particularly critical in superconducting accelerators. Therefore,

simulating the impact of various hardware errors and, based on beam quality metrics, defining hardware parameter tolerances together with appropriate control strategies are essential to ensuring the reliable and successful operation of the accelerator.

Currently, error analysis in accelerators is commonly carried out through extensive simulation studies to evaluate the impact of various types of errors on beam transport. A typical procedure involves analyzing individual error sources first, followed by their combined effects, and determining acceptable tolerance limits based on beam quality requirements [7–11].

This simulation-driven approach is straightforward and effective for assessing overall beam performance and identifying regions where beam degradation or beam loss occurs. However, direct inspection of simulation results often provides limited insight into the underlying physical mechanisms responsible for emittance growth or beam loss [12]. Consequently, more advanced data analysis techniques are required to systematically extract physical insight from large-scale simulation data and to support informed optimization of accelerator structures and parameters.

From a broader methodological perspective, error analysis and tolerance studies can be viewed as part of the general framework of uncertainty quantification (UQ),

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which aims to characterize how uncertainties in model inputs propagate to uncertainties in system outputs. Within this framework, sensitivity analysis plays a central role by quantifying the relative importance of input parameters and their interactions. Uncertainty quantification (UQ) is an effective approach for data analysis and has been widely applied in fields such as environmental science, healthcare, and architecture [13,14]. In recent years, UQ and global sensitivity analysis techniques have been increasingly explored in accelerator modeling, demonstrating their potential for systematic and model-independent assessment of beam dynamics uncertainties [14].

Sensitivity analysis methods can be broadly classified into local and global approaches. Local sensitivity analysis evaluates the response of model outputs to small perturbations around a nominal operating point and is appropriate for nearly linear systems. In contrast, global sensitivity analysis explores the full parameter space and is capable of capturing nonlinearities and interaction effects, making it particularly suitable for high-intensity linear accelerators with strong coupling among components [15,16].

Among global sensitivity analysis techniques, the Sobol method is a variance-based, model-independent approach that decomposes the output variance into contributions from individual parameters and their higher-order interactions [17]. This capability enables a quantitative assessment of both direct and coupled effects of hardware errors on beam quality, while minimizing subjective bias in the analysis process. For these reasons, the Sobol method has attracted increasing attention in UQ studies, although its application to accelerator systems remains relatively limited due to the large number of simulation samples required [18].

For linear accelerators, the high computational cost of high-fidelity beam dynamics simulations has historically been a major obstacle to applying global sensitivity analysis. However, recent developments in GPU-accelerated beam simulation tools have significantly improved computational efficiency, making large-scale sensitivity studies feasible [19]. In this work, we take advantage of the GPU-enabled version of the AVAS beam dynamics code developed at the Institute of Modern Physics to perform comprehensive Sobol sensitivity analysis for a realistic accelerator subsystem.

This study focuses on the medium energy beam transport (MEBT) section of the CAFE front-end test facility, a precursor to the China Initiative Accelerator Driven System (CIADS) [20]. The MEBT section consists of six quadrupoles and two bunchers [21,22].

By combining component-level and individual-error-level Sobol sensitivity analyses, this work aims to establish a direct and quantitative mapping between hardware error sources and beam quality degradation. The results provide insights into dominant error contributors and interaction effects, offering practical guidance for tolerance allocation and system optimization in superconducting linear accelerators.

The paper is organized as follows. In Sec. II, we present the principle of the Sobol sensitivity analysis approach as well as the modeling and analysis approach for the MEBT segment. The results and discussion are provided in Sec. III, where we analyze the impact of different accelerator components and errors on the quality of the output beam and discuss the corresponding physical principles. In Sec. IV, we conclude with a summary and outlook.

II. METHOD

A. Functional variance decomposition and Sobol indices

The Sobol method is a variance-based global sensitivity analysis technique that quantifies the contribution of each input parameter to the output variance, including both its independent effect and its interactions with other parameters. Here, $X = (X_1, X_2, \dots, X_d)$ denotes the vector of model input parameters, and $Y = f(X)$ represents the corresponding model output of interest. The Sobol method is based on the variance decomposition identity:

$$V(Y) = V[\mathbb{E}(Y|X_u)] + \mathbb{E}[V(Y|X_u)], \quad (1)$$

which separates the total output variance into the variance of the conditional expectation (explaining systematic variation due to X_u) and the expected conditional variance (representing residual uncertainty after fixing X_u).

Specifically, the model $f(\mathbf{x})$ can be expressed as

$$f(\mathbf{x}) = f_0 + f_u(x_u) + f_v(x_v) + f_{uv}(x_u, x_v), \quad (2)$$

where f_0 denotes the overall mean of the output, f_u and f_v represent the variations induced solely by the input subsets x_u and x_v , respectively, and f_{uv} captures their interaction effect. If this decomposition satisfies variance orthogonality—meaning that the components are uncorrelated [17,23] when computing the total variance—then the output variance can be uniquely decomposed as

$$\text{Var}(Y) = \text{Var}(f_u) + \text{Var}(f_v) + \text{Var}(f_{uv}), \quad (3)$$

indicating that the output uncertainty can be explicitly attributed to the independent contributions of each input and to their interaction effects.

From here, two sensitivity indices are defined: (i) Main-effect (first-order) index:

$$S_{X_u} = \frac{\text{Var}[\mathbb{E}(Y|X_u)]}{\text{Var}(Y)}, \quad (4)$$

measuring the independent contribution of X_u .

(ii) Total-effect index:

$$S_{X_u}^T = 1 - \frac{\text{Var}[\mathbb{E}(Y|X_v)]}{\text{Var}(Y)}, \quad (5)$$

measuring the total contribution of X_u , including all interaction effects.

The main-effect index S_{X_u} quantifies the contribution of the input variable X_u alone to the output variance. In contrast, the total-effect index $S_{X_u}^T$ accounts not only for the direct effect of X_u but also for its interactions with all other variables X_ν . This explains the difference between the two indices: if there is no coupling between variables (i.e., $\text{Var}[f(X_u, X_\nu)] = 0$), then $S_{X_u}^T = S_{X_u}$. If interactions exist, $S_{X_u}^T > S_{X_u}$, and the greater the interaction, the larger the difference between the two.

If the accelerator system is regarded as a mathematical function $f(\mathbf{X})$, where the input vector $\mathbf{X}$ represents a set of accelerator parameters (e.g., alignment errors, magnetic field deviations, rf phase errors), the influence of each parameter on the system output—such as beam centroid, envelope, or emittance—can be quantitatively evaluated. This influence includes both the independent contribution of individual parameters and the coupled effects arising from their interactions.

B. Numerical implementation

If the analytical expression of $f(x)$ is known, sensitivity indices can be directly calculated using analytical methods. However, in practice, accelerator models are highly complex and typically lack explicit mathematical expressions. In such cases, simulation software can be used to generate large quantities of sample data within a predefined parameter space. Sensitivity indices can then be estimated by computing the variances of the decomposed terms and the total variance based on statistical analysis of the simulation results.

For such model-independent systems, the Monte Carlo method [23] is commonly adopted to sample the parameter space, allowing a relatively small number of samples to represent the entire input domain. The detailed procedure is as follows:

Assume the model is represented as $y = f(x)$, where x is an n -dimensional input vector and y is a scalar output. Two independent $N \times n$ input matrices A and B are generated by random sampling from the input space. Each matrix contains N samples, and each sample has n input variables.

The matrix A is structured as follows (matrix B has a similar structure):

$$A = \begin{bmatrix} x_{11} & x_{12} & \cdots & x_{1n} \\ x_{21} & x_{22} & \cdots & x_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ x_{N1} & x_{N2} & \cdots & x_{Nn} \end{bmatrix}.$$

For a selected input variable u , with the remaining variables denoted as ν , the samples can be represented as

$$\begin{aligned} x_n^A &= (u_n^A, \nu_n^A), & y_n^A &= f(u_n^A, \nu_n^A), \\ x_n^B &= (u_n^B, \nu_n^B), & y_n^B &= f(u_n^B, \nu_n^B). \end{aligned}$$

By exchanging the u th column of matrices A and B , two new input matrices are constructed:

$$\begin{aligned} x_n^{(A_u)} &= (u_n^A, \nu_n^B), & y_n^{(A_u)} &= f(u_n^A, \nu_n^B), \\ x_n^{(B_u)} &= (u_n^B, \nu_n^A), & y_n^{(B_u)} &= f(u_n^B, \nu_n^A). \end{aligned}$$

Based on these definitions, the total variance and conditional variance can be estimated as [24]:

$$\text{Var}(y) \approx \frac{1}{2N} \sum_{n=1}^N \left[(y_n^A - y_n^B)^2 + (y_n^{(A_u)} - y_n^{(B_u)})^2 \right], \quad (6)$$

$$\mathbb{E}[\text{Var}(y|\nu)] \approx \frac{1}{2N} \sum_{n=1}^N \left[(y_n^A - y_n^{(B_u)})^2 + (y_n^{(A_u)} - y_n^B)^2 \right]. \quad (7)$$

Accordingly, the sensitivity indices can be computed as

$$S_u^T = \frac{\sum_{n=1}^N \left[(y_n^B - y_n^{(A_u)})^2 + (y_n^A - y_n^{(B_u)})^2 \right]}{\sum_{n=1}^N \left[(y_n^A - y_n^B)^2 + (y_n^{(A_u)} - y_n^{(B_u)})^2 \right]}, \quad (8)$$

$$S_u = 1 - S_\nu^T = \frac{2 \sum_{n=1}^N (y_n^{(A_u)} - y_n^B)(y_n^A - y_n^{(B_u)})}{\sum_{n=1}^N \left[(y_n^A - y_n^B)^2 + (y_n^{(A_u)} - y_n^{(B_u)})^2 \right]}. \quad (9)$$

This approach is particularly suitable for complex systems such as accelerators, where explicit models are unavailable but large volumes of simulation data can be generated for sensitivity analysis.

C. Convergence criterion

The accuracy of the Sobol index estimates depends on the sample size N , with larger values leading to improved statistical convergence. In the standard Sobol sampling scheme, the total number of model evaluations scales as $N(2d + 2)$, where d denotes the number of uncertain input parameters. Consequently, the computational cost increases with the dimensionality of the problem, a behavior that is not unique to the Sobol method but is also inherent to conventional Monte Carlo-based uncertainty analyses.

Based on convergence tests (see Fig. 1), a sample size of $N = 500$ was adopted in the present study, resulting in approximately 4.9×10^4 model evaluations for the full Sobol analysis. For comparison, traditional error analyses typically require a number of simulations that is roughly an order of magnitude smaller than that of the Sobol-based approach. Although the Sobol-based analysis entails a higher

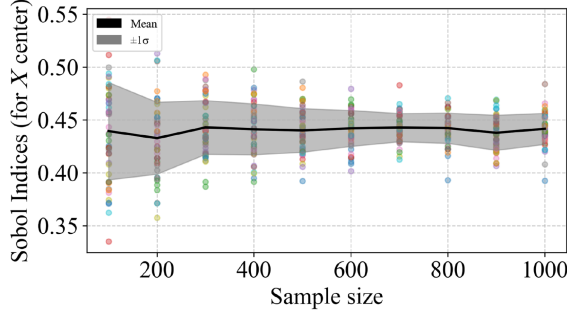


FIG. 1. Convergence behavior of the sensitivity factors for the beam centroid in the X direction.

simulation cost, the overall computational burden remains manageable owing to the substantial acceleration provided by the GPU-enabled version of AVAS. This improves simulation throughput and allows the Sobol analysis to scale to large uncertainty quantification studies.

D. MEBT section description

The simulation target in this study is the medium energy beam transport (MEBT) section of the CAFE facility [25], a front-end prototype for the China Initiative Accelerator Driven System (CIADS). The primary function of the MEBT is to match the beam output from the RFQ to the input requirements of the downstream superconducting linac. Additionally, it handles beam bunching, energy modulation, and partial beam collimation.

As shown in Fig. 2, the MEBT section consists of six quadrupole magnets and two bunchers, which serve the following purposes: (i) Quadrupoles: provide transverse focusing of the beam. Main sources of error include magnetic gradient deviations and mechanical misalignments. (ii) Bunchers: provide longitudinal bunching and energy adjustment. Typical errors include field amplitude errors, rf phase errors, and alignment offsets.

In this study, these component errors are parameterized and sampled to generate a wide range of error configurations. For each configuration, beam dynamics simulations are carried out to extract output metrics such as centroid

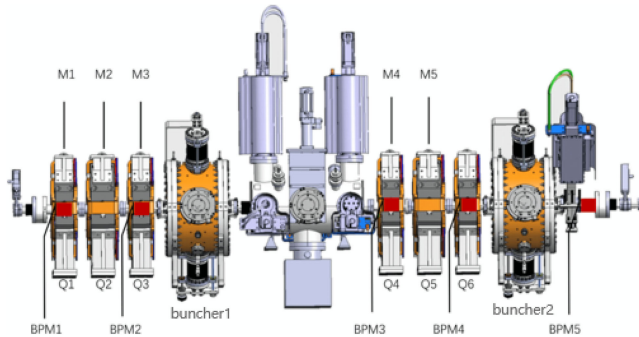


FIG. 2. Structure of the MEBT section of CAFE II. Q1–Q6 are quadrupole magnets.

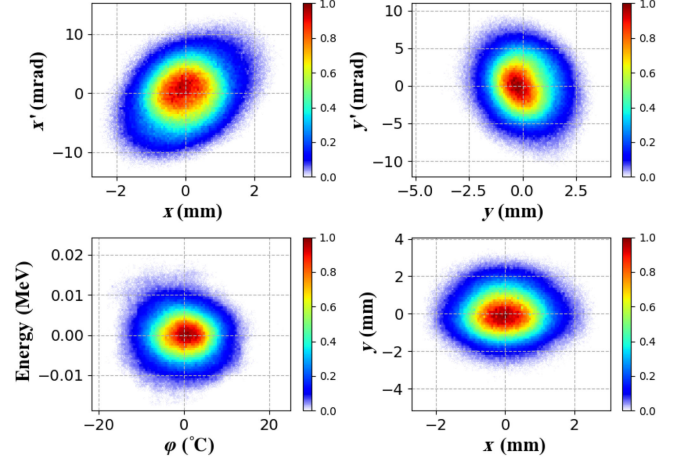


FIG. 3. Phase space at the entrance.

position, beam envelope, Twiss parameters, and emittance. These outputs are then used in sensitivity analysis to quantify the influence of each input error on beam quality.

We employed the multiparticle tracking method to simulate the transmission of a proton bunch through the MEBT section, with space-charge effects taken into account. Figure 3 shows the phase-space distributions at the entrance.

The simulation parameters are as follows: the particle type is proton, the beam current is 0.27 mA, the bunch repetition frequency is 162.5 MHz, and the beam energy is 1.3568 MeV. A total of 10^6 particles were tracked, with the initial particle distribution following a six-dimensional Gaussian distribution.

According to the operating conditions of the MEBT section, the error settings for each component are defined as follows: (1) Magnet elements: (i) Translation error (d_x, d_y): ± 0.5 mm; (ii) Rotation error (ϕ_x, ϕ_y, ϕ_z): ± 2 mrad; and (iii) Field amplitude error (ΔG): $\pm 0.50\%$. (2) Buncher elements: (i) Translation error (d_x, d_y): ± 0.5 mm; (ii) Rotation error (ϕ_x, ϕ_y): ± 2 mrad; (iii) rf phase error (ϕ): $\pm 0.1^\circ$; and (iv) Amplitude error (ΔE): $\pm 0.50\%$.

For magnet elements, six error types are included, namely two transverse translation errors, three rotation errors, and one field amplitude error, while rf phase errors are not applicable. For buncher elements, six error types are considered, including two transverse translation errors, two rotation errors, one rf phase error, and one field amplitude error, whereas rotation about the longitudinal (z) axis is not included.

TABLE I. Parameters of the input beam.

	ϵ_{rms} (π mm mrad)	β (mm/ π mrad)	α
$X - X'$	0.1532	0.1960	-0.3080
$Y - Y'$	0.1438	0.3280	0.1665
$Z - Z'$	0.1265	0.8216	-0.0721

The parameters of the input beam are summarized in Table I, while the corresponding phase-space distribution at the entrance is illustrated in Fig. 3.

III. RESULT AND DISCUSSION

In this study, the Sobol method is employed to perform a global sensitivity analysis on the MEBT section of the CAFe facility. Two complementary strategies are used to investigate and compare the influence of various errors on beam parameters: (i) Treating the six error types of each component as a whole, we analyze the relative sensitivity of different components to the beam parameters. This approach provides a rapid preliminary assessment of which components the beam parameters are most sensitive to. (ii) Examining each individual error type to determine its specific influence on beam dynamics. This more detailed analysis allows for the establishment of explicit mappings between specific error sources and corresponding beam parameters. In addition, a comparison is made with local sensitivity analysis: In this approach, only one error source is introduced at a time, while all other errors are set to zero. By statistically evaluating the average response of beam parameters within the variation range of each individual error, this method quantitatively assesses the specific influence of that error on the target beam parameter.

Figure 4 shows the statistical distributions of the resulting beam parameters at the MEBT exit, including the beam centroid positions, transverse beam envelopes, and Twiss parameters. In addition, the figure illustrates the emittance growth at the exit relative to the corresponding values at the entrance. It can be observed that under the influence of random errors in the MEBT section, the beam centroid,

envelope, and Twiss parameters tend to follow a Gaussian-like distribution, while the variations in beam emittance remain relatively small.

A. Component-level sensitivity analysis

The centroid position reflects the overall displacement of the beam. As shown in Fig. 5, for both the x and y directions, the main-effect sensitivity indices and total-effect sensitivity indices are nearly identical, indicating that under the current error settings, the influence of each component on the beam centroid is largely independent and noncoupled.

For the x -direction centroid, Q5's sensitivity index exceeds 0.4, indicating a substantial impact. For the y -direction centroid, Q2, Q4, and Q5 show high sensitivity indices, suggesting that these elements exert a strong influence on the beam centroid position. It is evident that the buncher errors have a negligible effect on the beam trajectory.

The beam envelope and the Twiss β parameter are closely related in physical meaning, both reflecting the divergence and focusing properties of the beam during transport. As shown in Fig. 6, their sensitivity indices across different components follow similar trends, confirming their intrinsic correlation.

Notably, the main-effect and total-effect sensitivity indices are almost equal, indicating that the effects of different components on the beam size are largely independent, with no significant nonlinear coupling observed.

Specifically, in the x direction, Q3 and Q5 show relatively high sensitivity indices for both the envelope and β_x . In the y direction, Q2 and Q4 dominate, suggesting that these components play a key role in modulating the transverse beam size.

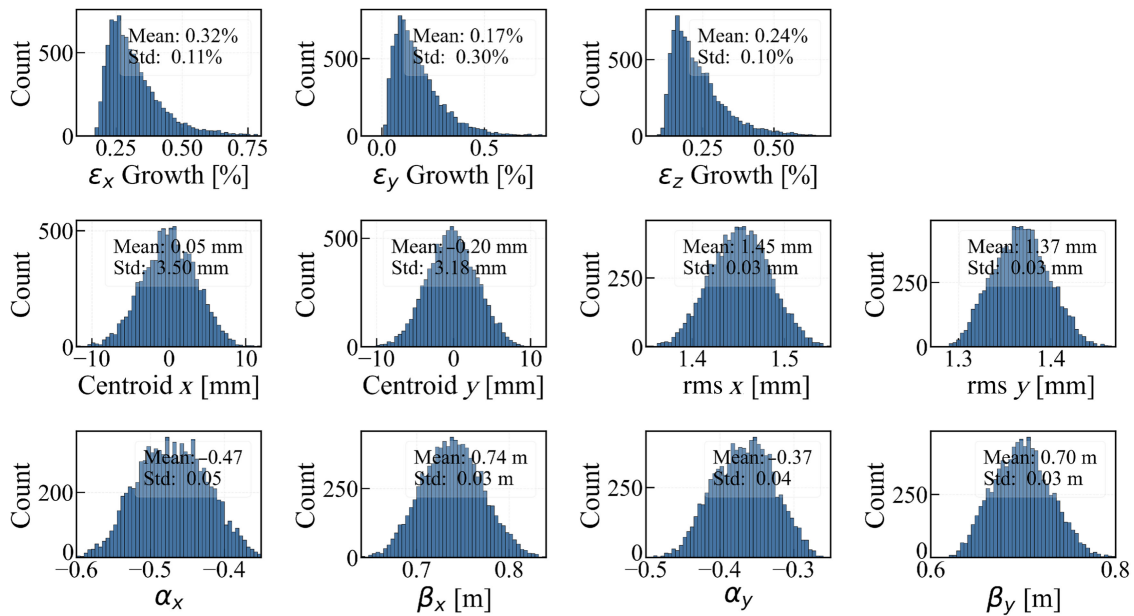


FIG. 4. Distribution of beam parameters in sensitivity analysis.

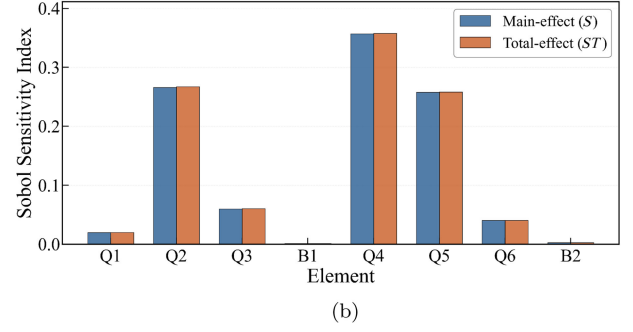
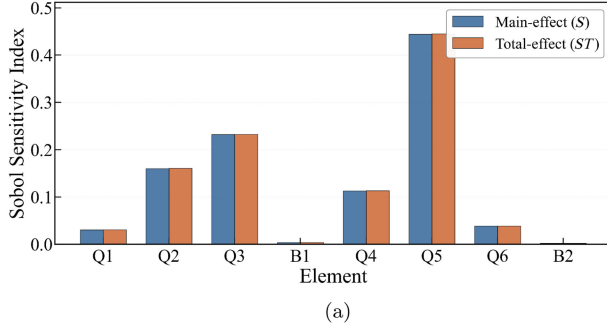


FIG. 5. Sensitivity indices for the beam centroid with respect to various accelerator components: (a) centroid in the X direction; (b) centroid in the Y direction.

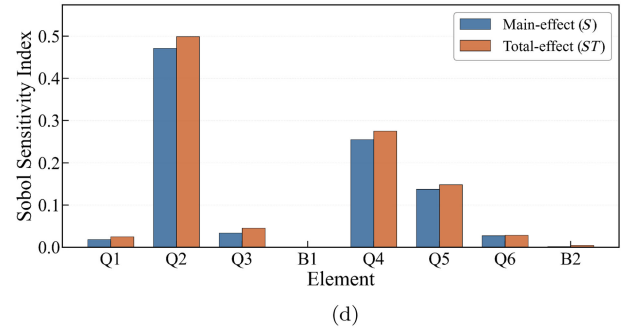
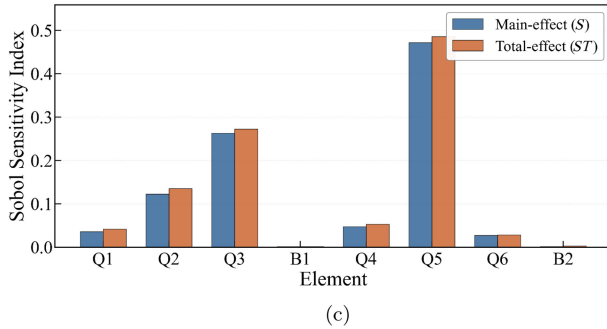
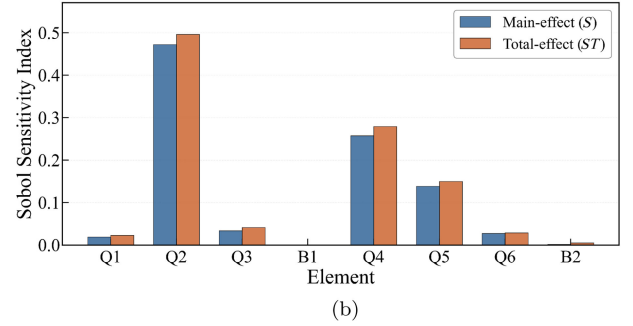
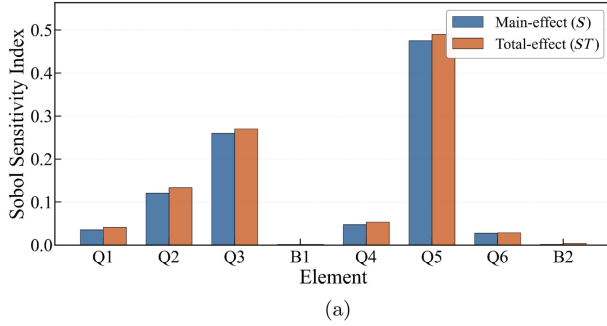


FIG. 6. Sensitivity indices for the beam envelope and Twiss β with respect to various accelerator components: (a) envelope in the X direction; (b) envelope in the Y direction; (c) Twiss β in the X direction; (d) Twiss β in the Y direction.

The Twiss α parameter reflects the slope of the beam envelope function and characterizes the beam's focusing or defocusing trend. Physically, $\alpha = 0$ indicates the beam is at a waist, $\alpha < 0$ implies the beam is diverging, and $\alpha > 0$ suggests the beam is converging.

As shown in Fig. 7, the sensitivity distribution of the Twiss α parameter is relatively more complex, and the influence of different components exhibits strong directional dependence. In the x direction, Q3 and Q5 have large main-effect sensitivity indices, indicating their dominant impact on α_x . In the y direction, Q2 and Q4 show higher sensitivity indices.

Moreover, the main-effect and total-effect indices remain closely aligned, suggesting that under the current error settings, the influence of each component on the Twiss α parameter is mainly independent, with no significant nonlinear coupling observed.

Emittance is a key parameter that characterizes the size and shape of the beam in phase space, reflecting the overall beam quality. Compared with the beam centroid and Twiss parameters, emittance more effectively captures the irreversible expansion caused by nonlinear errors and coupling effects in the system. In this work, the emittance is evaluated using the rms definition and is normalized by the relativistic factors $\beta\gamma$.

Ideally, emittance should remain conserved during transport. However, simulation results show that error perturbations can lead to emittance fluctuations within the MEBT section.

As shown in Fig. 8, the total-effect Sobol indices of all components are significantly larger than their corresponding main-effect indices, indicating the presence of strong nonlinear coupling effects in the system. Under error

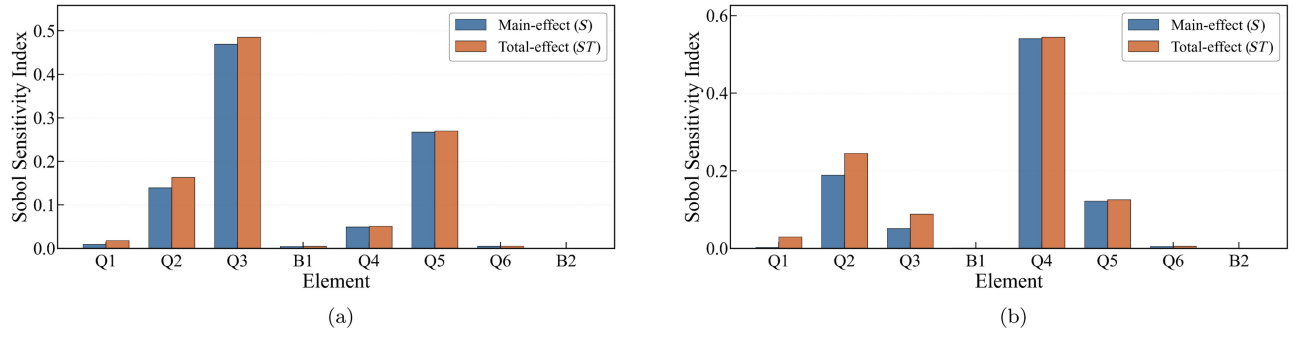


FIG. 7. Sensitivity indices for the Twiss parameter α with respect to various accelerator components: (a) Twiss α in the X direction; (b) Twiss α in the Y direction.

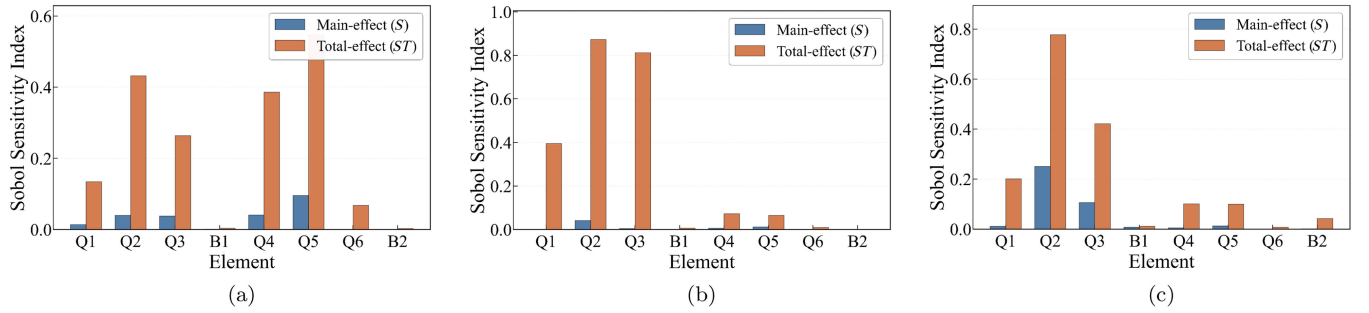


FIG. 8. Sensitivity indices for the beam emittance with respect to various accelerator components: (a) emittance in the phase space (X, X'); (b) emittance in the phase space (Y, Y'); (c) emittance in the phase space (Z, Z').

perturbations, the transverse emittance in the MEBT section exhibits distinct sensitivity characteristics. In the x direction, the emittance is mainly influenced by quadrupoles Q2, Q3, Q4, and Q5, while the contributions from Q1, Q6, and the bunchers are relatively weak. In contrast, both the y - and z -direction emittances show pronounced sensitivity to Q2 and Q3, suggesting that these upstream quadrupoles play a dominant role in driving coupled beam dynamics.

Further analysis of particle tracking data along the beamline reveals that in approximately 1% of the simulated cases, a negative growth of the y -direction emittance is observed, whereas such behavior does not occur in the x direction. A closer inspection indicates that, in these cases, a small fraction of beam loss (less than 1%) starts to appear at approximately 2 m downstream, near the entrance of Q4. This beam loss becomes much more severe at the second buncher, where 20%–30% of the particles are lost. Meanwhile, a substantial centroid offset is observed in the y direction, reaching values of up to 20 mm, while the centroid displacement in the x direction remains below 5 mm. These observations suggest that Q2 and Q3 primarily affect the emittance statistics through inducing large y -direction centroid deviations and subsequent beam loss, whereas Q4 and Q5 mainly influence the emittance in the x direction without causing significant particle loss.

It is worth noting that although the bunchers directly participate in z -direction phase modulation, their impact on the z -direction emittance remains relatively small. This implies that, under the current configuration, the evolution of the z -direction emittance is dominated by coupling effects originating from transverse (x - y) beam dynamics rather than by direct rf errors in the bunchers, thereby shaping the overall emittance evolution.

B. Individual-error-level sensitivity analysis

To further investigate the specific effects of various error sources on beam parameters, this study systematically investigates the effects of typical errors—including translational and rotational misalignments of the magnets, as well as field gradient errors—on the beam centroid, envelope, and emittance.

Figure 9 presents the distributions of the Sobol sensitivity indices for the beam centroid positions in the x and y directions. Several salient features can be identified from the results.

First, for both transverse planes, the main-effect sensitivity indices and the total-effect indices nearly overlap for all considered components and error sources. This behavior indicates that nonlinear interactions among different error sources are negligible and that the centroid motion exhibits

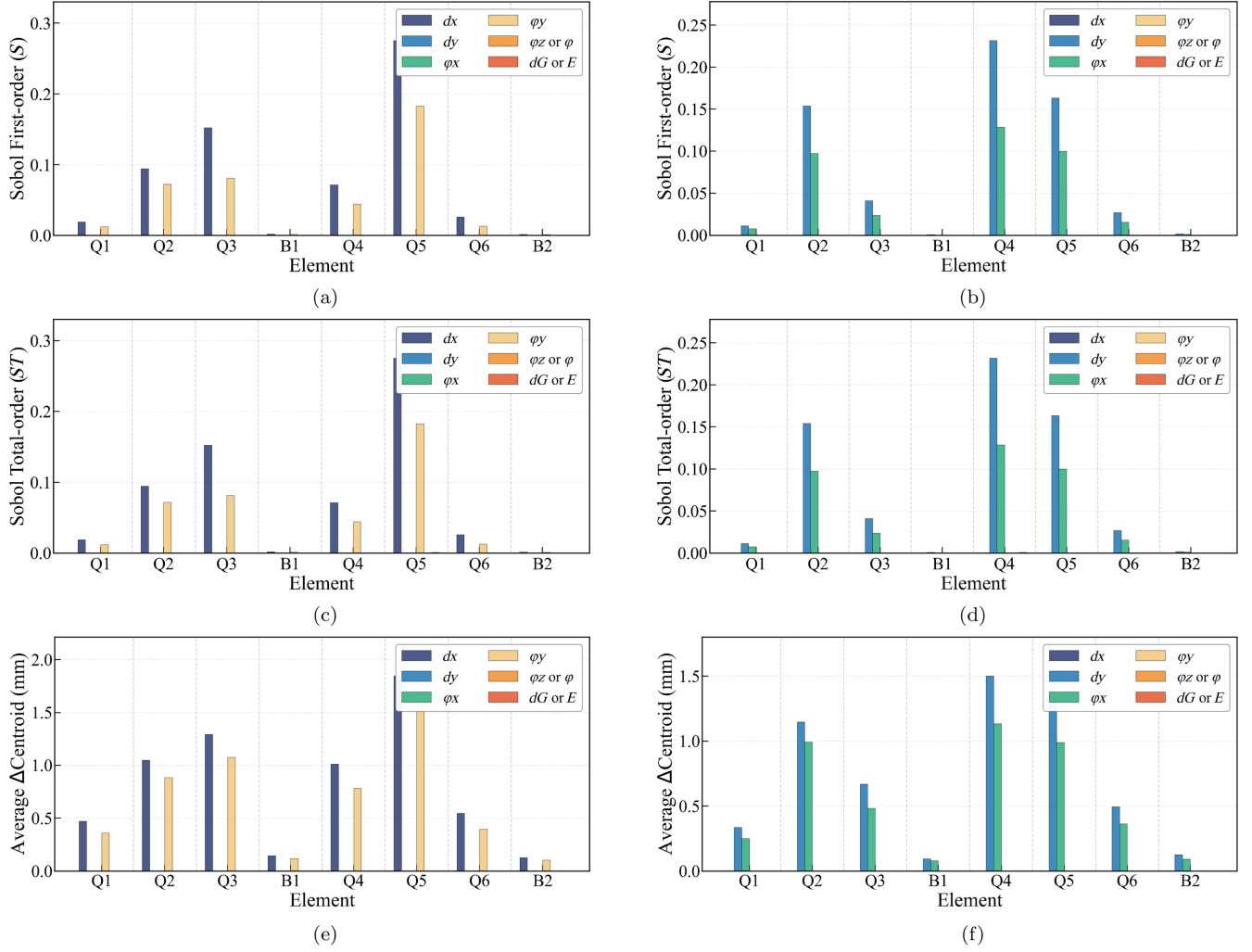


FIG. 9. Sensitivity indices for the beam centroid with respect to individual error types: (a),(b) main-effect indices S in the X and Y directions; (c),(d) total-effect indices ST in the X and Y directions; (e),(f) local sensitivity analysis showing the influence of each individual error on the centroid in the X and Y directions.

a predominantly linear response within the investigated error ranges.

Second, a clear directional dependence of the sensitivity is observed. The centroid position in the x direction is mainly influenced by misalignments along the x axis (dx) and rotations about the y axis (ϕ_y). Among the quadrupole magnets, the dx error of Q5 shows the largest contribution to the x -centroid displacement, followed by that of Q3. In contrast, centroid position in the y direction is primarily driven by misalignments along the y axis (dy) and rotations about the x axis (ϕ_x), with Q4 and Q5 being the most influential elements. In particular, the dy error of Q4 dominates the sensitivity of the y -centroid.

Finally, the overall sensitivity ranking demonstrates that centroid offsets are governed mainly by translational misalignments (dx , dy), while rotational errors play a secondary role. Other error sources, such as magnetic field

gradient errors (dG) and rf-related errors, have a negligible impact on the centroid positions. These findings are consistent with expectations from conventional local sensitivity analyses and provide a clear physical basis for alignment requirements: precise positioning of Q5 is critical for controlling centroid deviations in the x direction, whereas centroid stability in the x direction is most sensitive to the alignment accuracy of Q4.

Similarly, a sensitivity analysis was conducted for the beam envelope, as shown in Fig. 10. The results indicate that the envelope is also primarily affected by magnetic field strength errors, with minimal coupling observed between different error sources. This suggests that the system still exhibits predominantly linear response behavior. Notably, due to the alternating focusing structure in the x and y directions, a distinct cross-symmetry appears in the sensitivity factor distribution: the front half of the

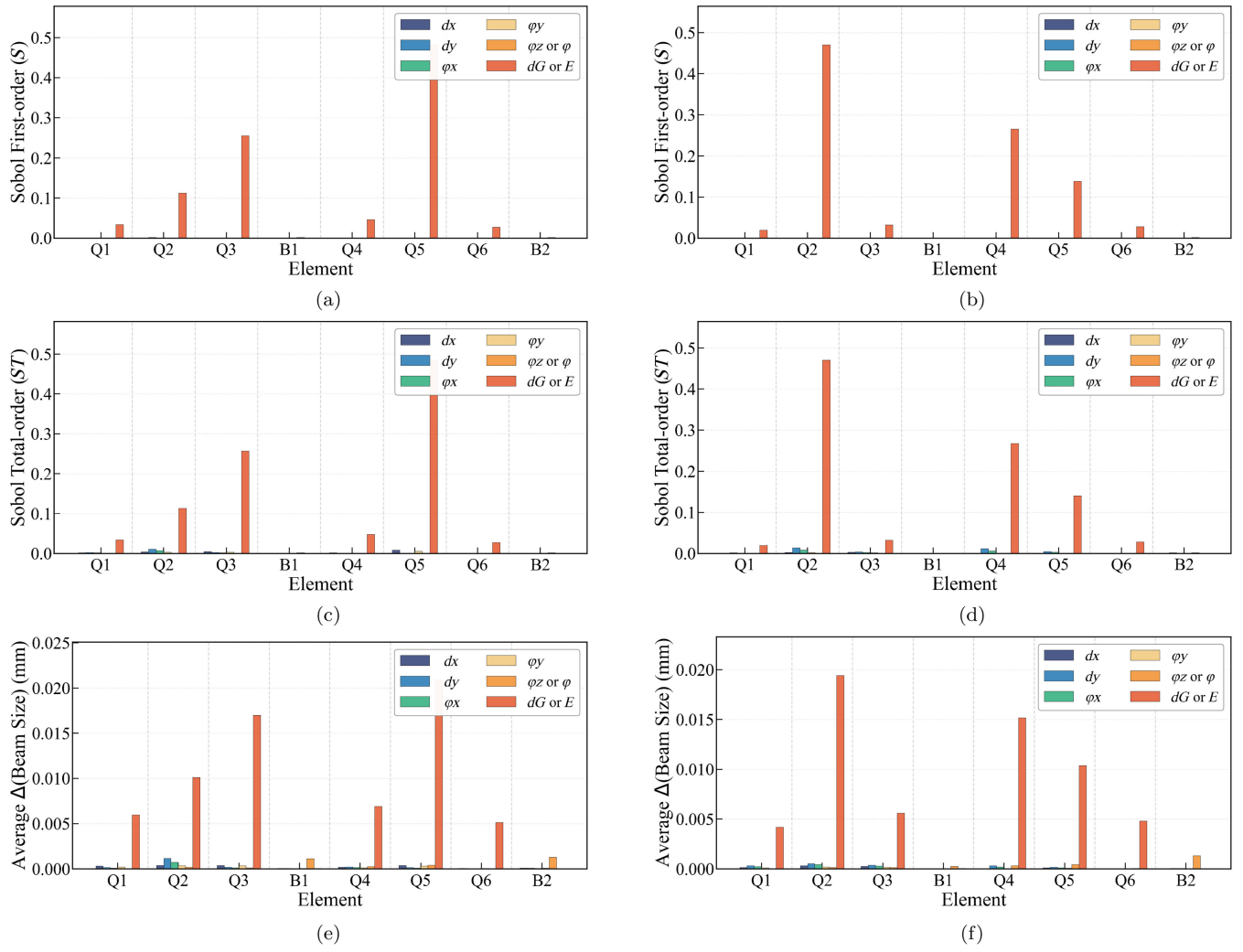


FIG. 10. Sensitivity indices for the beam envelope with respect to individual error types: (a),(b) main-effect indices S in the X and Y directions; (c),(d) total-effect indices ST in the X and Y directions; (e),(f) local sensitivity analysis showing the influence of each individual error on the envelope in the X and Y directions.

x -direction distribution resembles the rear half of the y -direction, while the rear half of the x -direction mirrors the front half of the y -direction.

When the beam emittance is taken as the output quantity of interest, the sensitivity analysis results at the individual error level are summarized in Fig. 11. A prominent feature is that, for all considered error sources, the total-effect Sobol indices are significantly larger than the corresponding main-effect indices. This indicates that emittance evolution in the MEBT section is dominated by strong nonlinear coupling among multiple error sources, rather than by the independent contribution of any single error.

For the z -direction emittance, the dominant contributions originate from translational misalignments of the upstream quadrupole magnets. In particular, the y -direction alignment error and the x -axis rotation error of Q2 show the highest sensitivity. More generally, the z -direction emittance is mainly influenced by the alignment and rotation errors of

Q1, Q2, and Q3. This behavior is consistent with the component-level analysis, which identified the upstream quadrupoles as the primary drivers of z -direction emittance variations through coupled x - y - z beam dynamics.

In contrast, the x -direction emittance shows a markedly different sensitivity pattern. It is highly sensitive to rotational errors of the downstream quadrupoles, with roll errors about the z axis (ϕ_z) of Q4 and Q5 being the dominant contributors. This indicates that these elements mainly modify the x -direction phase-space distribution without inducing significant particle loss, in agreement with the observations from particle tracking.

The y -direction emittance exhibits a more complex coupling behavior. In addition to the rotational errors of downstream quadrupoles, it is also sensitive to translation errors in the y direction (dy) and rotations about the x axis (ϕ_x) of the upstream quadrupoles, particularly Q2 and Q3. This combined sensitivity confirms that y -direction

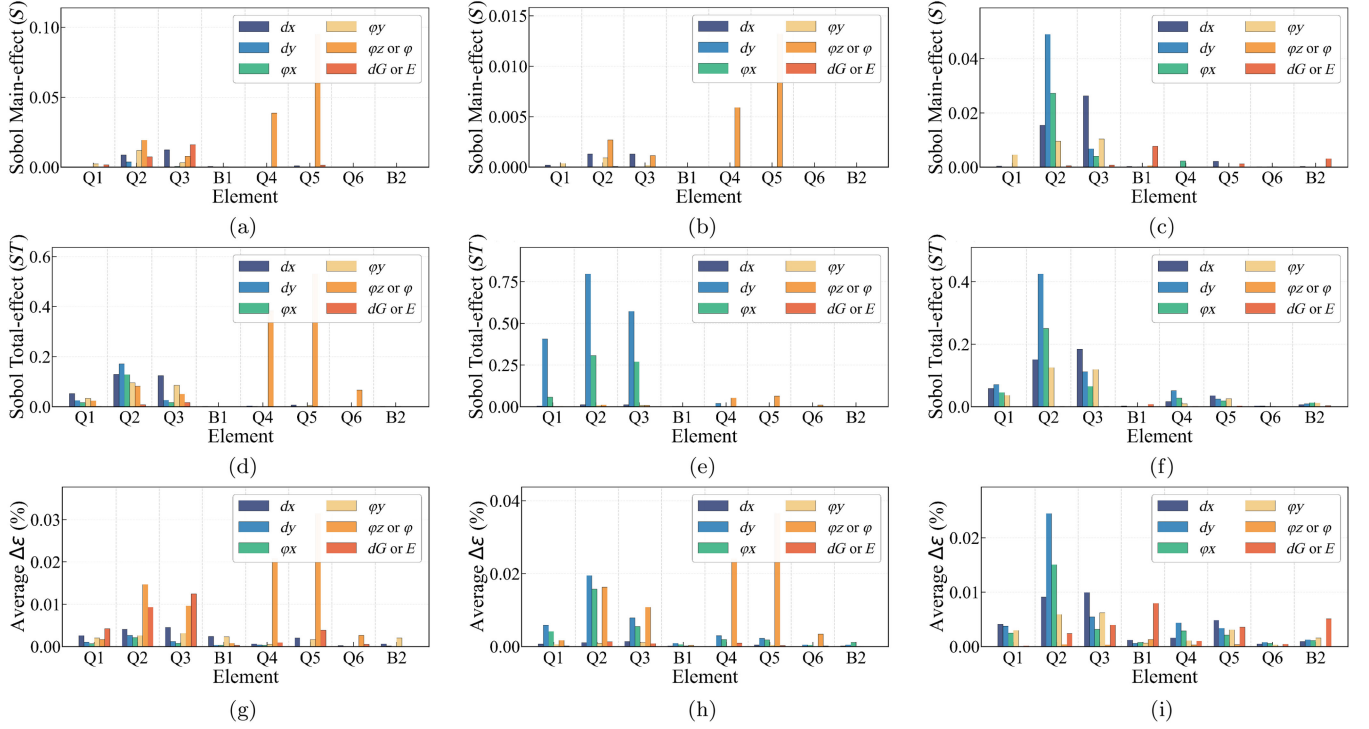


FIG. 11. Sensitivity indices for the beam emittance with respect to individual error types: (a)–(c) main-effect indices S for emittance in the phase spaces (X, X') , (Y, Y') , and (Z, Z') ; (d)–(f) total-effect indices ST for emittance in the phase spaces (X, X') , (Y, Y') , and (Z, Z') ; (g)–(i) local sensitivity analysis showing the influence of each individual error on the emittance in the phase spaces (X, X') , (Y, Y') , and (Z, Z') .

misalignments and rotations about the x axis in the upstream quadrupoles induce large centroid deviations in the y direction, which subsequently lead to beam loss and strongly affect the statistical evaluation of the y -direction emittance. This mechanism is fully consistent with the component-level analysis, in which Q2 and Q3 were identified as the dominant elements governing the y -direction emittance behavior.

IV. CONCLUSION

This work applies the model-independent Sobol sensitivity analysis to the CAFé MEBT to quantify the impact of alignment, magnetic field, and phase errors on beam quality. The results indicate that centroid, envelope, and Twiss parameters are dominated by independent error contributions, whereas both transverse and longitudinal emittances are strongly affected by error coupling, underscoring the importance of considering error interdependencies in accelerator design.

Sensitivity analysis plays a critical role in accelerator design and optimization. In practice, optimization objectives can be broadly categorized into two directions: improving overall system robustness and optimizing specific beam performance metrics.

From a robustness perspective, sensitivity information can be used to redistribute the system response such that

dominant sensitivities are concentrated on a limited number of well-controlled parameters. By appropriately tuning lattice settings or alignment strategies, the influence of less controllable error sources can be suppressed, thereby enhancing operational stability.

From a performance-oriented perspective, sensitivity analysis enables the identification of accelerator parameters that have the strongest impact on a given beam metric, such as emittance growth, centroid stability, or transmission. Targeted optimization of these sensitive parameters allows the dimensionality of the optimization problem to be effectively reduced, leading to faster convergence, lower computational cost, and improved optimization efficiency in high-dimensional accelerator systems.

Within this context, the proposed Sobol-based framework provides a systematic pathway for integrating sensitivity analysis into future accelerator design, commissioning, and control optimization workflows.

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DATA AVAILABILITY

The data that support the findings of this article are not publicly available. The data are available from the authors upon reasonable request.

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